Nonlinear localized modes in flatband lattices

Magnus Johansson

Department of Physics, Chemistry and Biology, Linköping University, Sweden

Daejeon, August 30, 2017

Purpose of the talk:

To review various dynamical effects resulting from interplay between nonlinearity and flat linear dispersion bands.

Specific model examples:

1D: Sawtooth, kagome chains and ladders (dimerized).

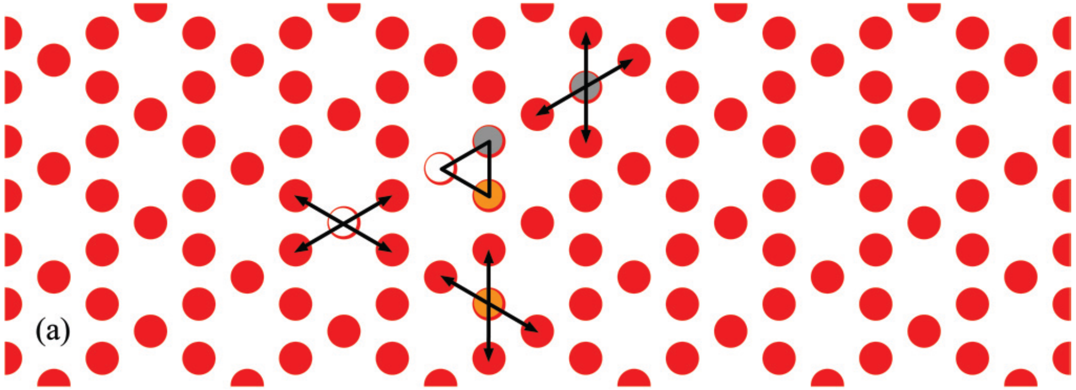
2D: Kagome, dimerized Lieb.

Some observed effects:

- Threshold-less bifurcation of nonlinear localized modes from linear band also in 2D.
- Symmetry-broken ground states.
- Mobility of strongly localized modes.
- Compactification tuning by nonlinearity.

R.A. Vicencio, MJ, PRA **87**, 061803(R) (2013)); MJ, U.Naether and R.A. Vicencio, PRE **92**, 032912 (2015); P.P. Beličev, G. Gligorić, A. Radosavljević, A. Maluckov, M. Stepić, R.A. Vicencio, and MJ, PRE **92**, 052916 (2015); P.P. Beličev et al, to be submitted (2017).

The Kagome lattice



Discrete Nonlinear Schrödinger (DNLS) model:

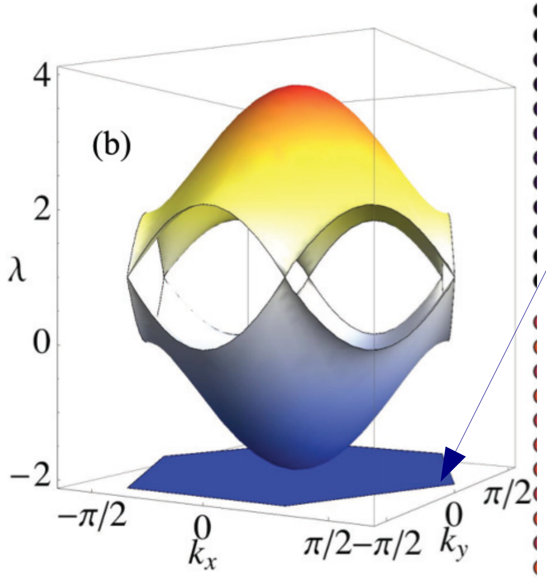
R.A. Vicencio, MJ, PRA **87**, 061803(R) (2013))

$$i\frac{\partial u_{\vec{n}}}{\partial z} + \sum_{\vec{m}} V_{\vec{n},\vec{m}} u_{\vec{m}} + \gamma |u_{\vec{n}}|^2 u_{\vec{n}} = 0,$$

Cubic on-site nonlinearity + linear nearest-neighbour interactions

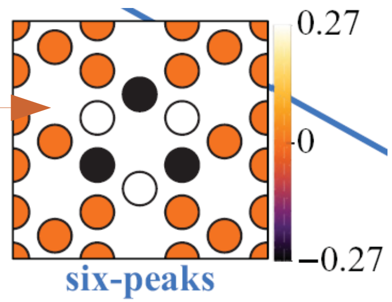
Defocusing nonlinearity: $\gamma = -V_{n.m} = -1$

Linear spectrum ($\gamma = 0$): (e.g., Bergman et al, PRB **78**, 125104 (2008))



Lower band exactly flat, built up from 6-site ring modes:

Equal amplitude, opposite phases, strictly zero background!



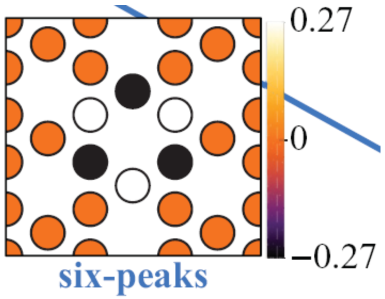
Nonlinear solutions bifurcate from flat band without excitation threshold!

DNLS conserved quantities: $P = \sum_{\vec{n}} |u_{\vec{n}}|^2$,

$$H = -\sum_{\vec{n}}\{\sum_{\vec{m}} V_{\vec{n},\vec{m}}(u_{\vec{m}}u_{\vec{n}}^* + u_{\vec{m}}^* u_{\vec{n}}) + (\gamma/2)|u_{\vec{n}}|^4\}$$

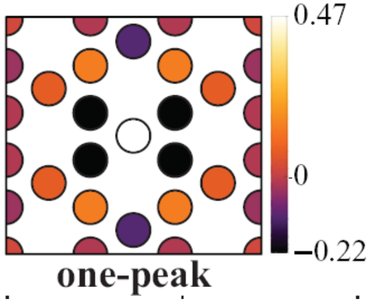
Stationary solutions: $u_{\vec{n}}(z) = u_{\vec{n}} \exp(i\lambda z)$.

Two families of fundamental nonlinear modes for $\gamma < 0$:



single 6-peak ring mode: same as linear case!

Exact discrete compacton! $P = 6(\lambda + 2)/\gamma$



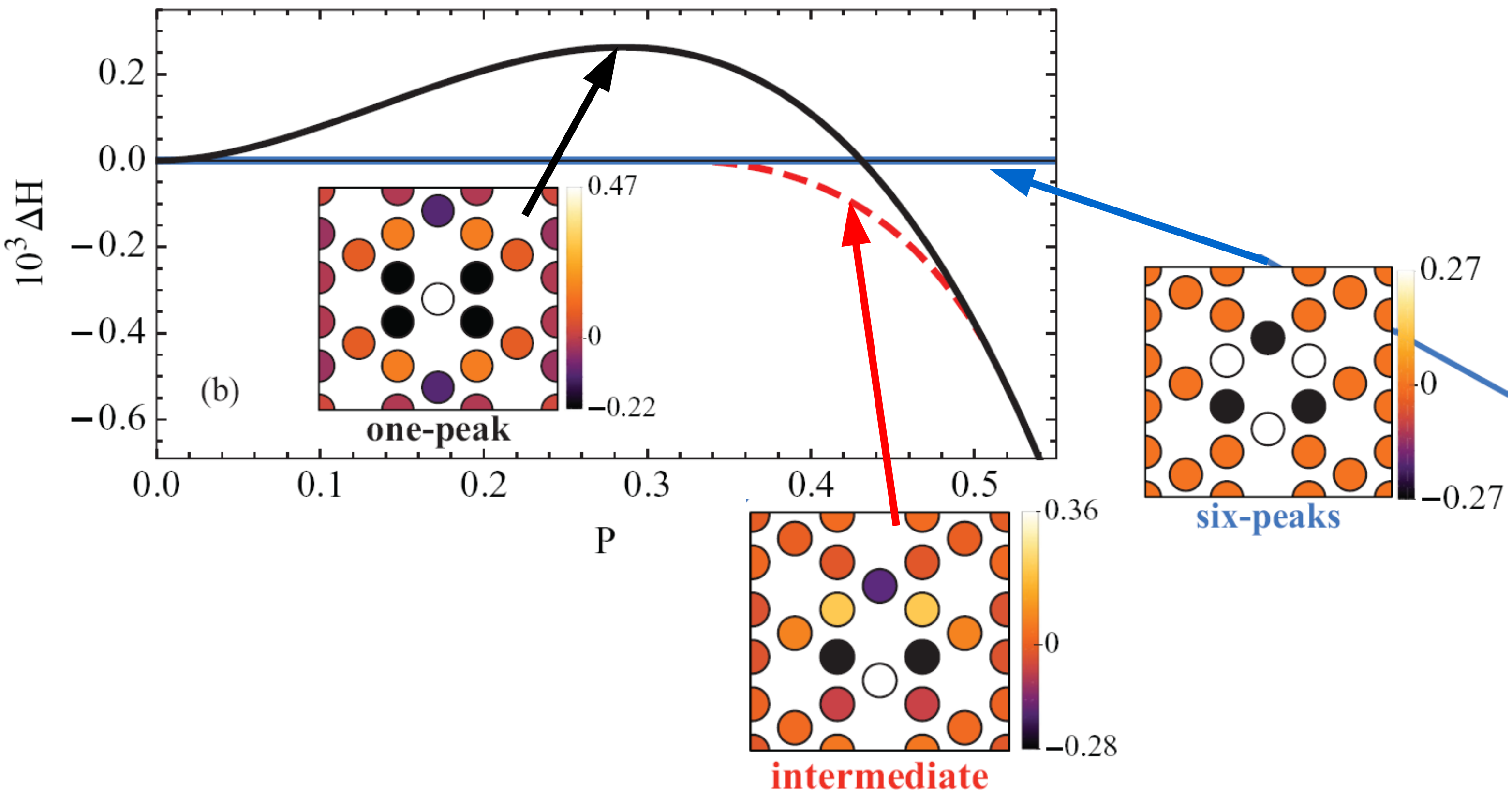
single-peak mode:

linear limit: two neighboring rings with one common site

“anticontinuous” limit ($|\gamma|P \to \infty$): single-site excitation

in-between: “ordinary” discrete soliton with exponential tails

Stability exchange between fundamental modes

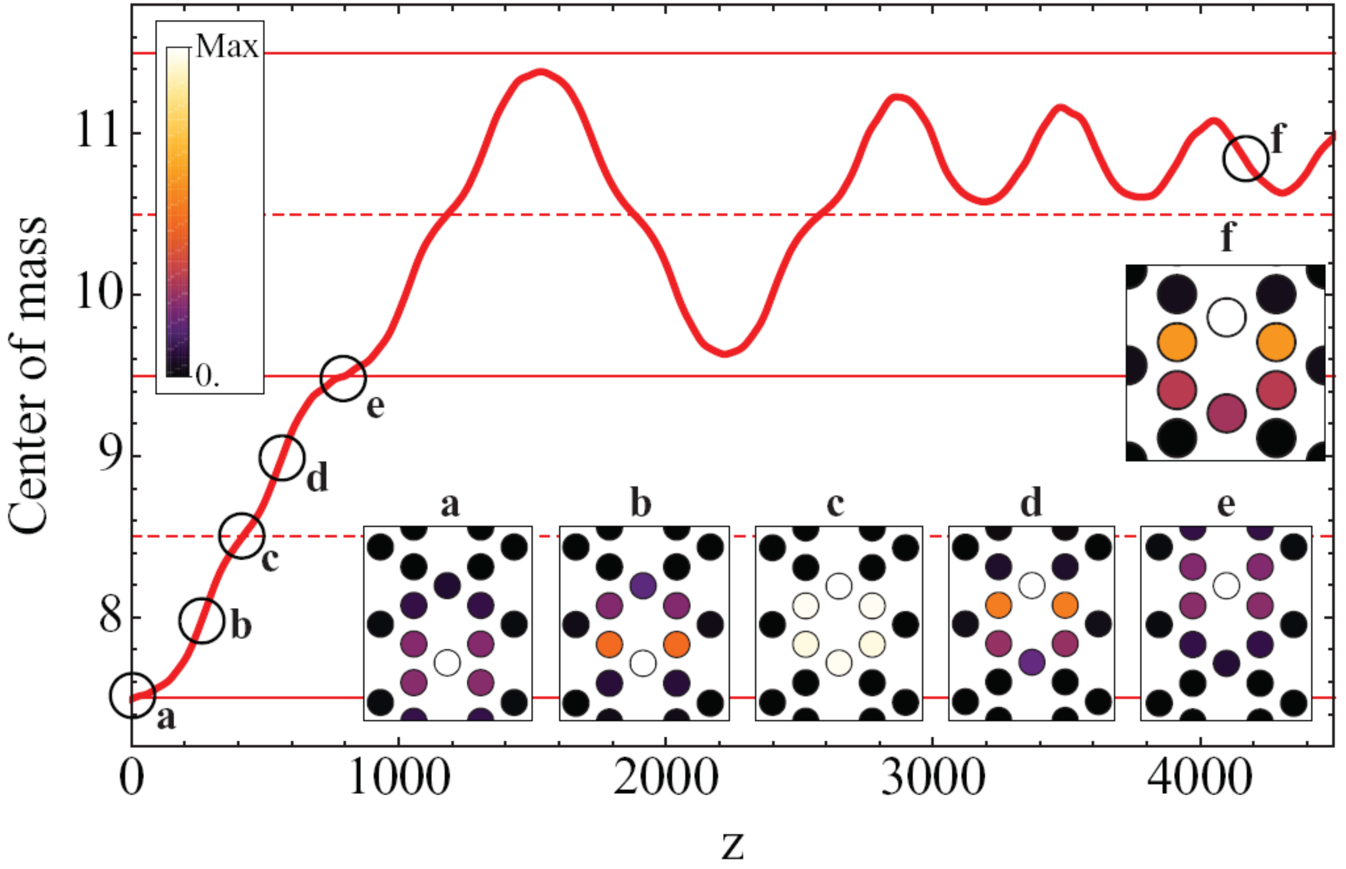


6-peak ring mode ground state for weak nonlinearity

1-peak mode ground state for strong nonlinearity

Symmetry-broken mode ground state in intermediate regime!

Moving fundamental solutions in stability-exchange regime



Small vertical kick (phase-gradient) on unstable 1-peak mode

Finally trapped around symmetry-broken ground state!

Stronger kicks give longer propagation distances.

The sawtooth lattice

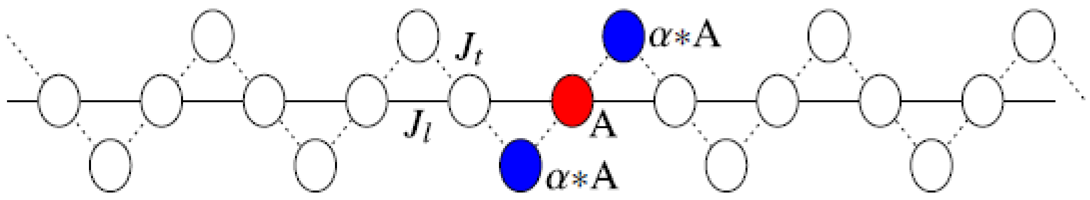


FIG. 1. (Color online) Geometry of the sawtooth lattice with its compact mode (white circles imply zero amplitudes).

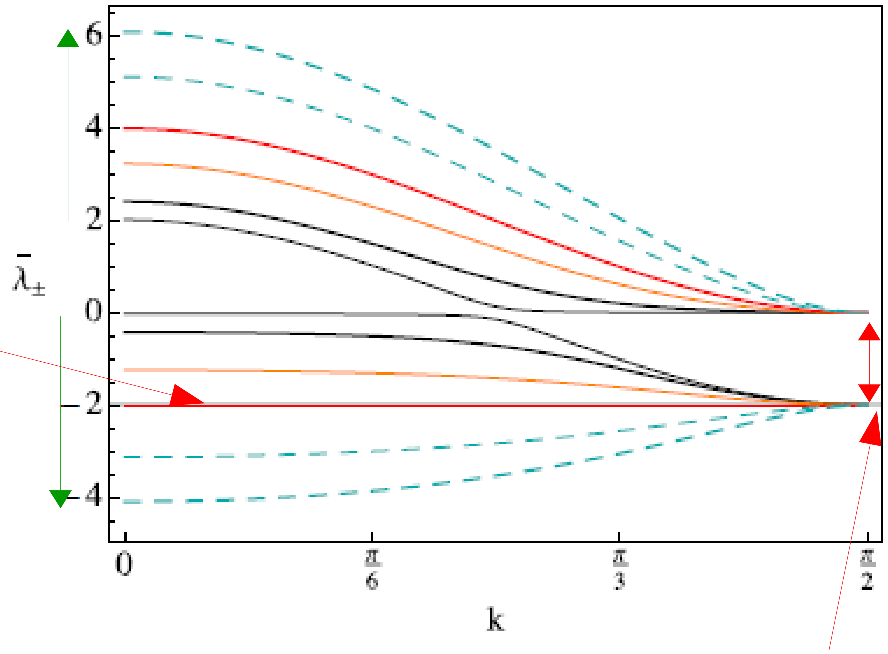
Linear dispersion relation:

(e.g., Derzhko et al, PRB **81**, 014421 (2010))

$$\bar{\lambda}_{\pm}(k;J) \equiv \lambda_{\pm}/J_l = \cos(2k) \pm \sqrt{\cos^2(2k) + 2J^2[1+\cos(2k)]}.$$

Band structure for increasing coupling ratio $J \equiv J_t / J_l$:

Lower band becomes flat for specific ratio $J = \sqrt{2}$



Note: flat band gapped from dispersive band.

3-site compact modes with amplitude ratio α = -1/J ($J = \sqrt{2}$)

Nonlinearity shifts the coupling ratio where compactons appear!

General DNLS: $i\dot{u}_n + \sum_{m\neq n} J_{n,m}u_m + \gamma f(|u_n|^2)u_n = 0,$

General existence condition for stationary compactons: $\frac{\gamma}{J_l}[f(|A|^2) - f(|A/J|^2)] = 2 - J^2$

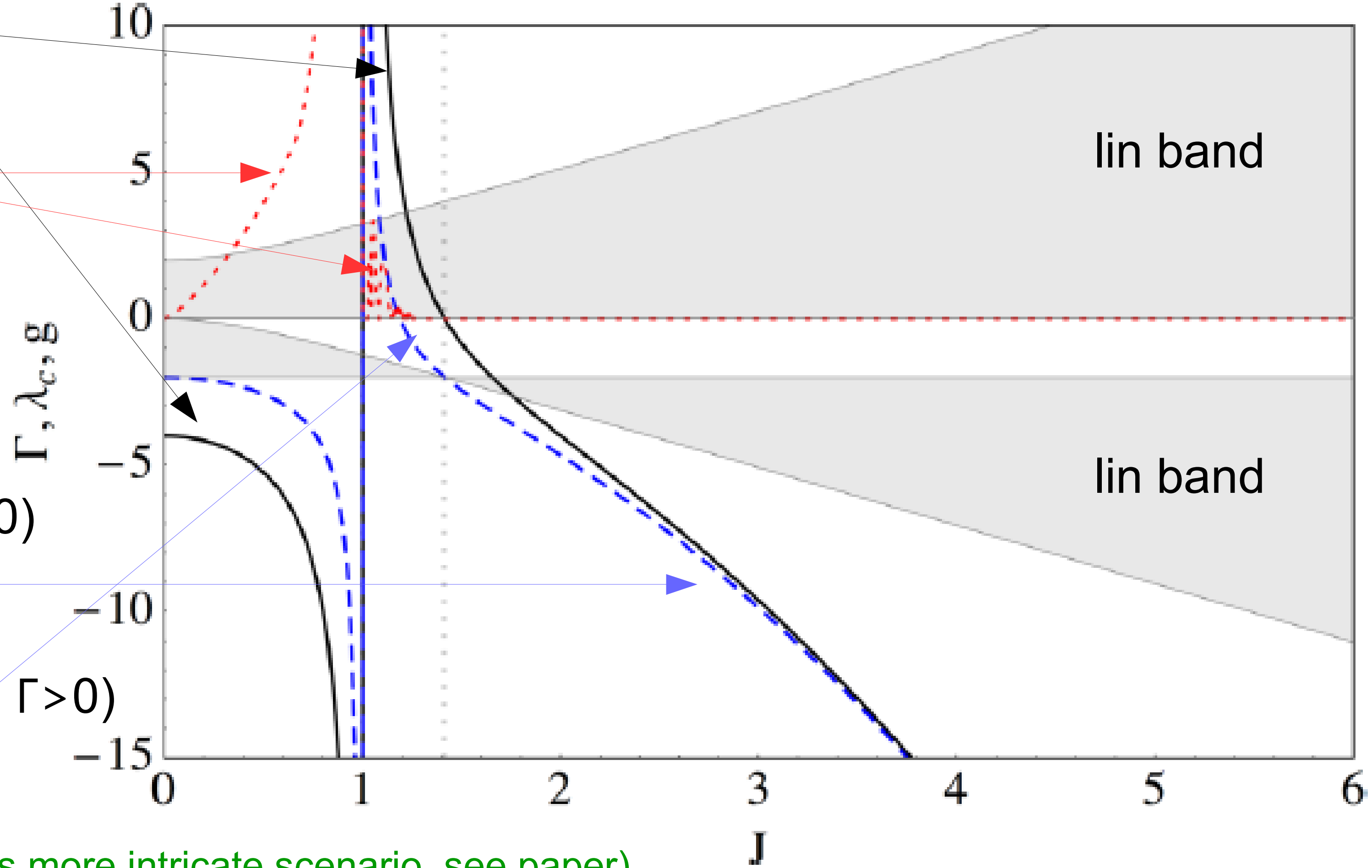
Simplifies for cubic nonlinearity:

MJ, U.Naether and R.A. Vicencio, PRE **92**, 032912 (2015)

$$\Gamma \equiv \frac{P\gamma}{J_l} = \frac{4-J^4}{J^2-1}, \quad P \equiv \sum_n |u_n|^2$$

Stability parameter:

(lin. stable when g = 0)

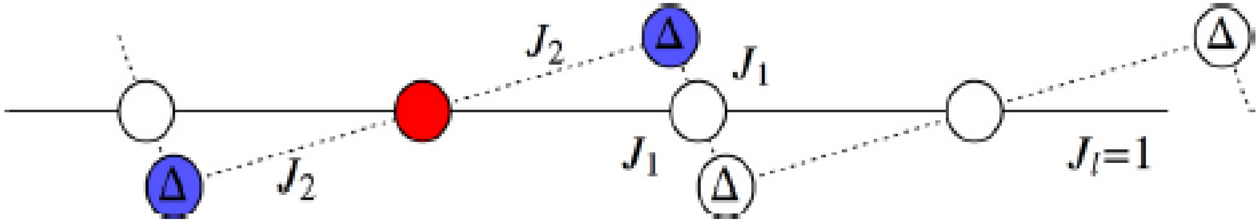


Stable compactons for:

- $J > \sqrt{2}$ (defocusing, $\Gamma<0$) frequency below band
- $1.27.. < J < \sqrt{2}$ (focusing, $\Gamma>0$) frequency in gap

(saturable nonlinearity gives more intricate scenario, see paper)

Stable nonlinear compactons exist also in more general sawtooth-like chains:



$$i\dot{u}_n + \Delta_n u_n + \sum_m J_{n,m} u_m + \gamma f(|u_n|^2) u_n = 0.$$

FIG. 9. (Color online) Sketch of the anisotropic geometry with pairwise alternating couplings and alternating on-site energies.

Exists for cubic nonlinearity when

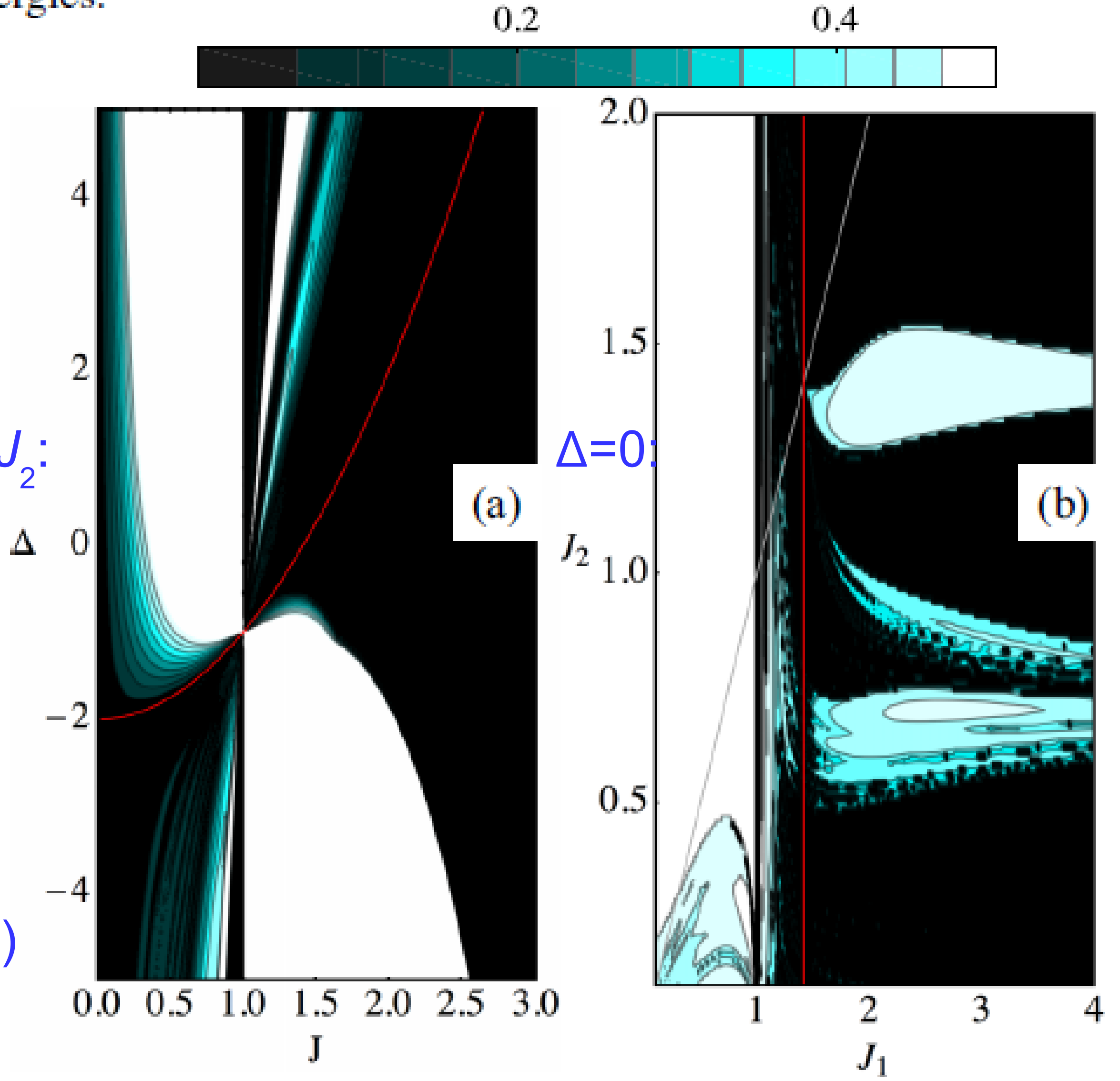
$$\Gamma_\Delta(J_1, J_2) = \frac{J_2(J_1^4 - 4) - J_1 \Delta (J_1^2 + 2)}{J_1 - J_1^3}$$

Examples of stability diagrams:

(Stable compactons in black areas)

$J_1=J_2$:

$\Delta=0$:



Particularly: For Δ < - 2, a focusing nonlinearity may stabilize compactons also with $J < 1$, with main localization on *tip* sites (since $\alpha = -1/J$).

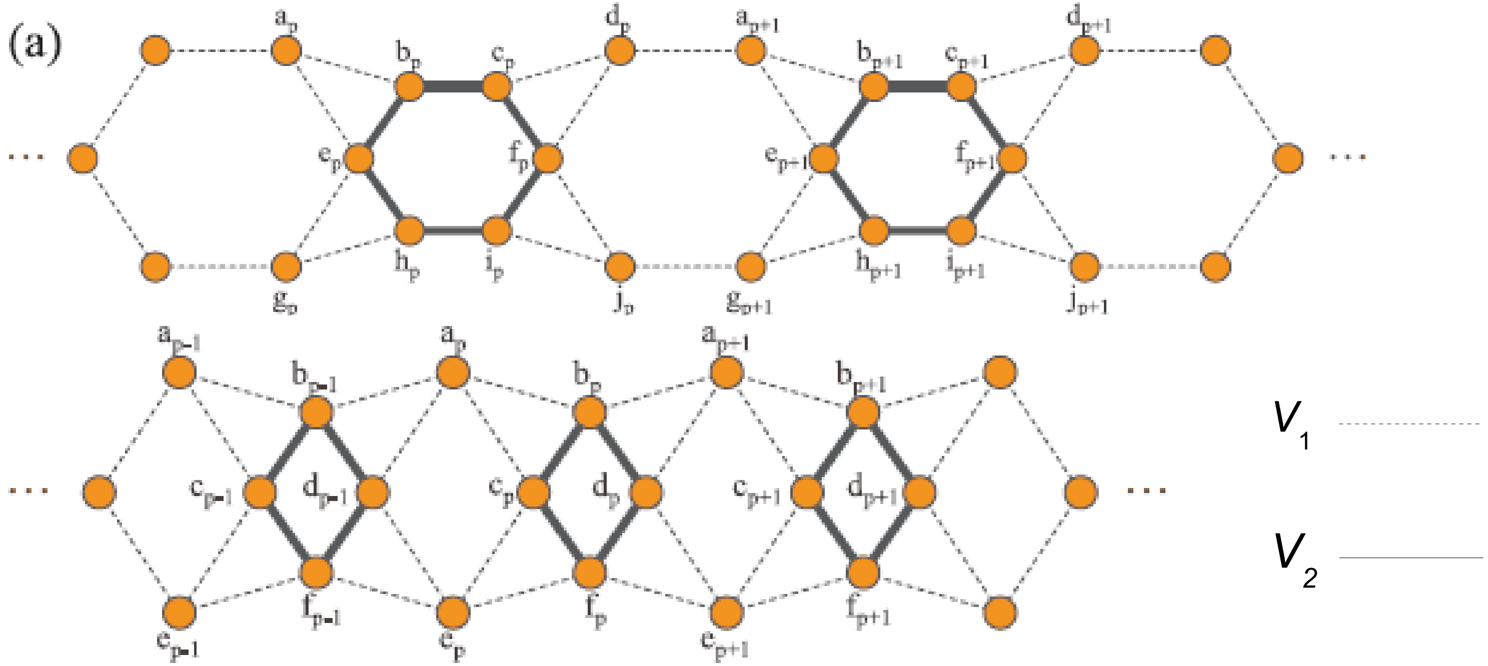
(Linear compacton exists only for Δ ≥ - 2.)

Binary (dimerized) kagome chains/ladders

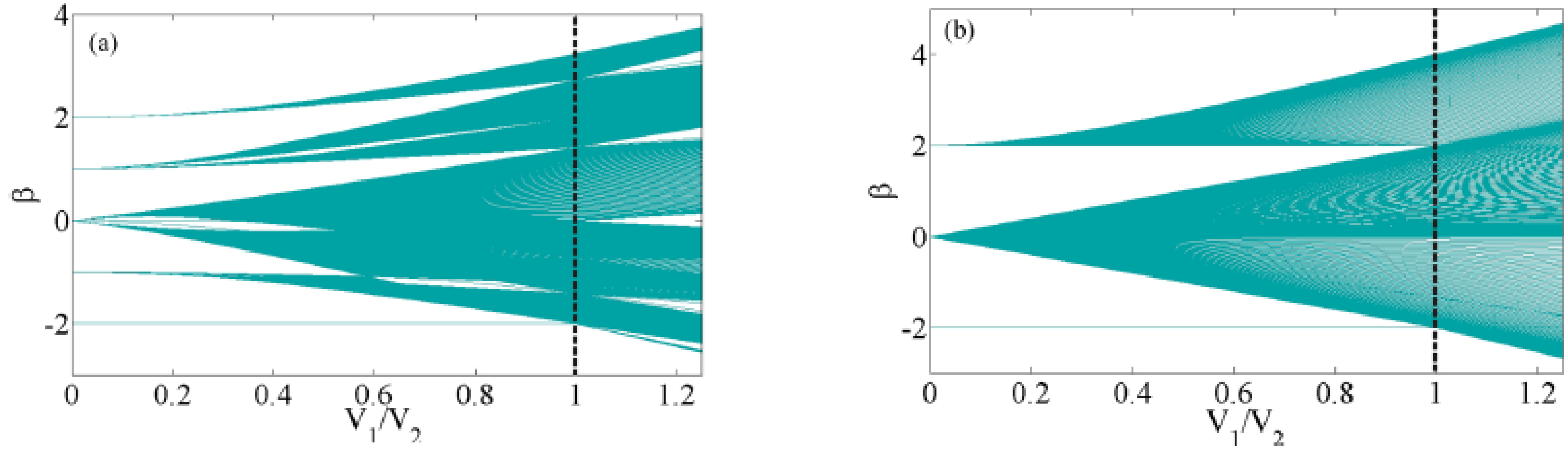
Two types of 1D chains with flat band for arbitrary coupling strengths: P.P. Beličev et al, PRE **92**, 052916 (2015)

(a) “Binary kagome strip”:

(b) “Binary kagome ladder”:



Linear dispersion relations: (uniform chains: Derzhko et al, PRB **81**, 014421 (2010))



Note: flat band gapped from dispersive bands when $V_2 > V_1$

Compact staggered ring-modes on rings coupled by V_2 only.

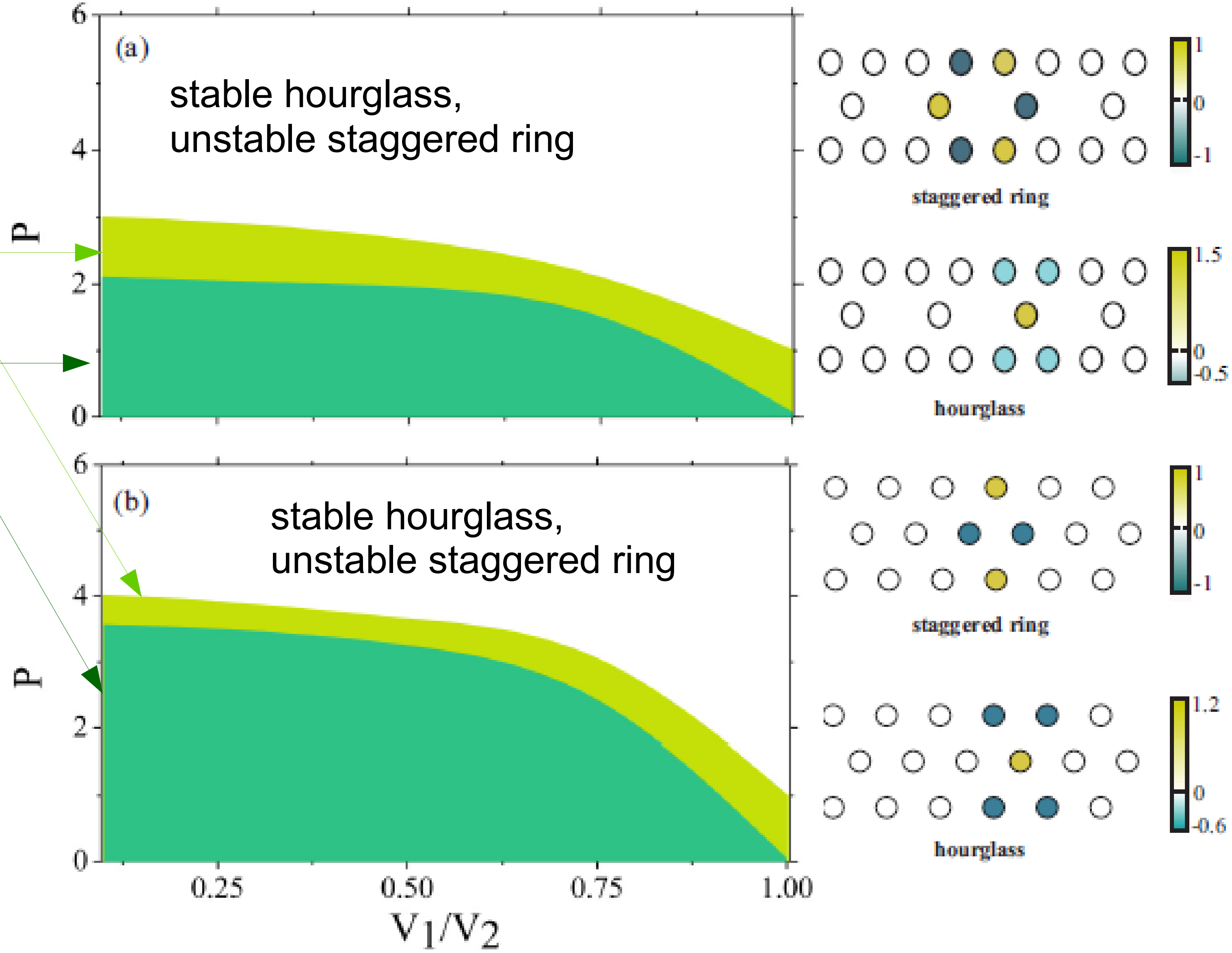
Stable compactons exist for focusing and defocusing nonlinearities when $V_2 > V_1$

Stability diagrams for

defocusing cases:

stable staggered ring,
unstable hourglass

stable staggered ring,
no hourglass



Staggered rings:

compact, thresholdless

Hourglass:

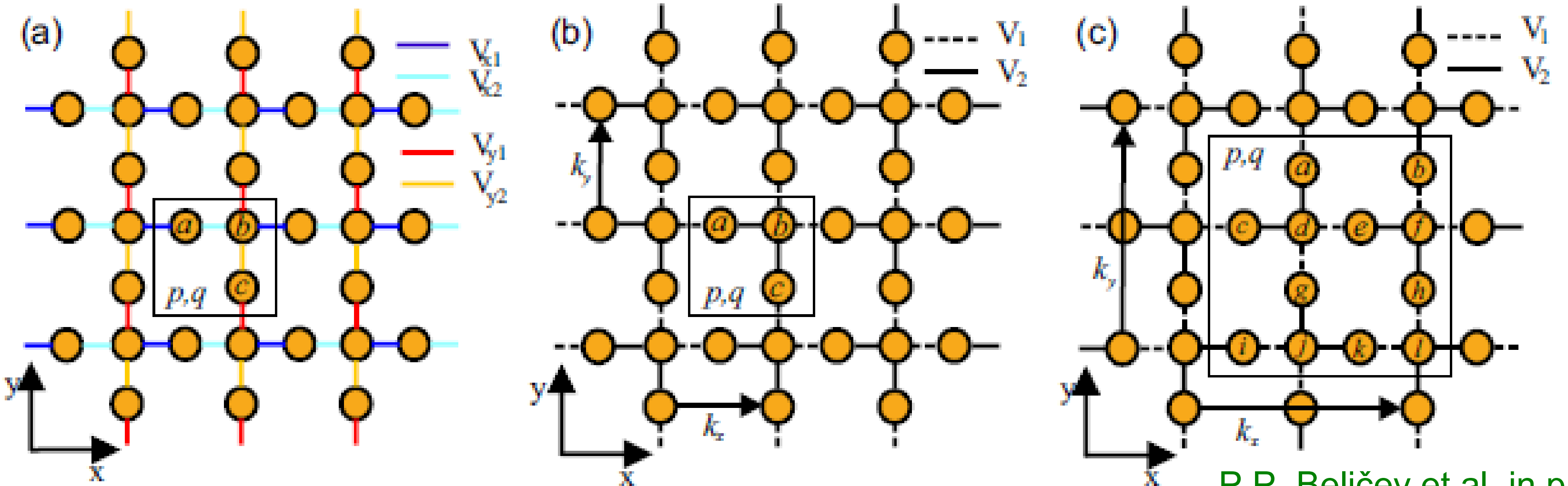
exponential tails,

threshold when $V_2 > V_1$

Focusing cases: Compact staggered rings stable for most frequencies β inside the first gap

(a) $P = 6(\beta + 2)/\gamma$ (b) $P = 4(\beta + 2)/\gamma$

Briefly about Lieb lattice dimerizations



P.P. Beličev et al, in preparation (2017)

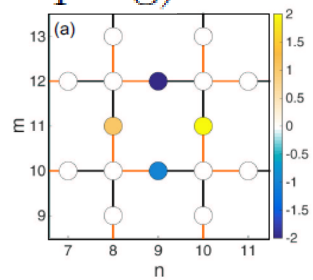
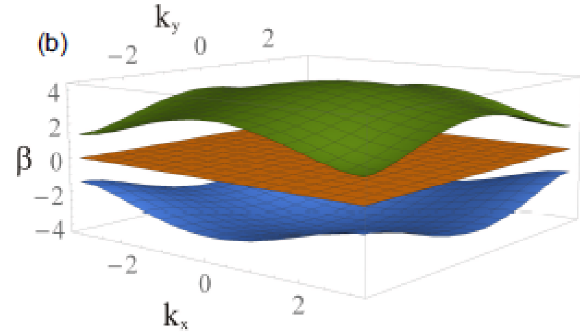
(a) 3-site unit cell and general coupling coefficients, dispersion relation:

$$\beta_0 = 0, \quad \beta_{1,2} = \pm\sqrt{V_{x1}^2 + V_{x2}^2 + V_{y1}^2 + V_{y2}^2 + 2V_{x1}V_{x2}\cos k_x + 2V_{y1}V_{y2}\cos k_y}$$

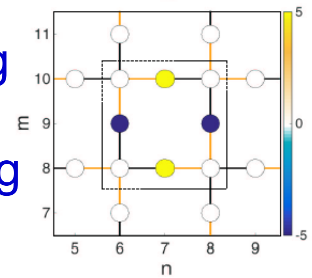
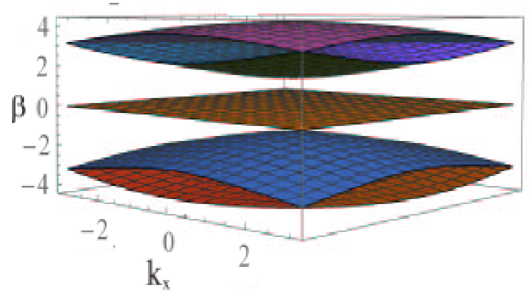
Compact linear 4-peak ring modes: $a_{p,q} = -\frac{V_{y2}}{V_{x2}}c_{p,q}, \quad a_{p,q-1} = -\frac{V_{y1}}{V_{x2}}c_{p,q}, \quad c_{p-1,q} = \frac{V_{x1}}{V_{x2}}c_{p,q}$

(b) "Type I": Special case of (a): $V_{x1} = V_{y1} = V_1$ (inter-cell coupling)
$V_{x2} = V_{y2} = V_2$ (intra-

e.g. Julku et al., PRL **117**, 045303 (2016)
Compact modes antisymmetric w.r.t diagonal but asymmetric w.r.t. antidiagonal

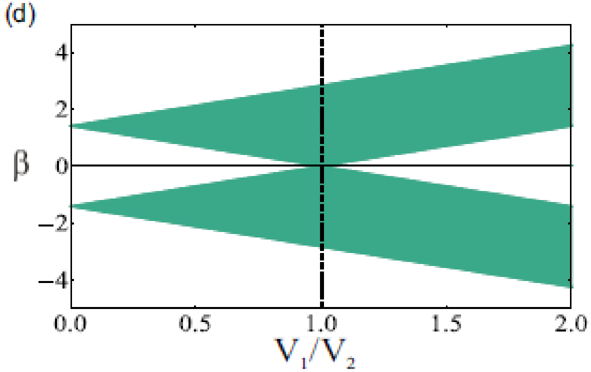
(c) "Type II": 12-site unit cell; alternation of vertical coupling also in horizontal direction
Fully (anti-)symmetric compact modes on every 2^{nd} ring

Nonlinear localized gap modes for both signs of nonlinearity

P.P. Beličev et al, in preparation (2017)

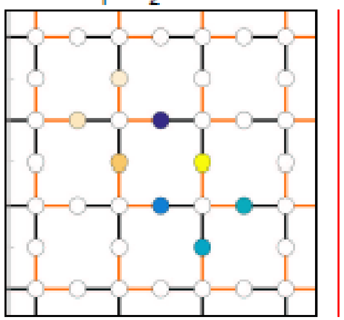
Gap width for both types of dimerization: $\Delta = \sqrt{2}|V_2 - V_1|$

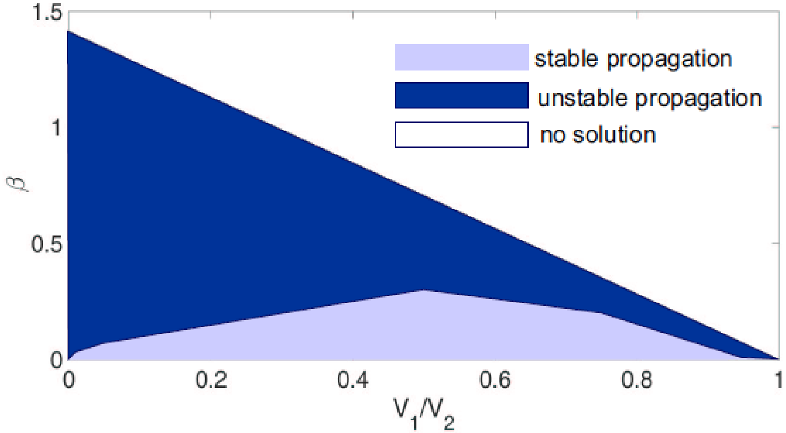


Type I: No compact stationary modes; site-dependent nonlinear frequency shift

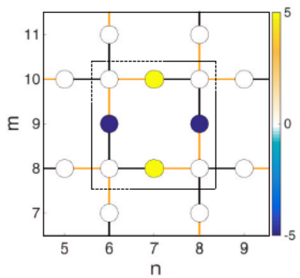
Exponentially decaying gap modes; stability diagram:

V_1/V_2=0.9



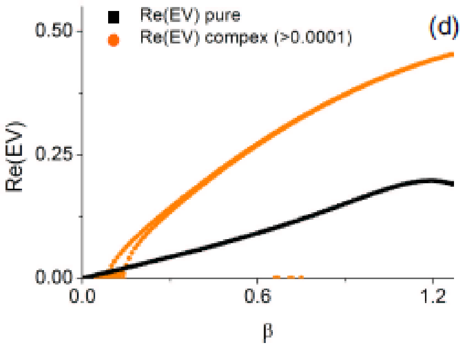


Type II: Nonlinear compact 4-site ring modes exist, with $P = 4\beta/\gamma$.

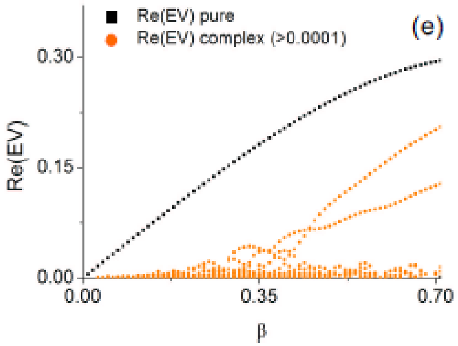


Generally unstable in gap due to various resonances (details in progress)!

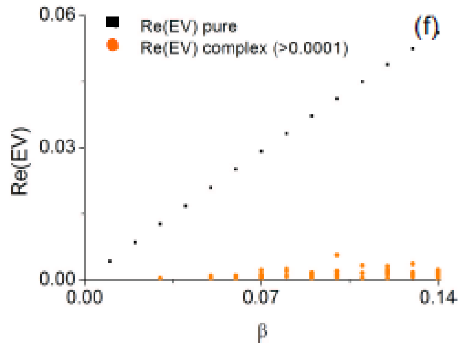
$V_1/V_2 = 0.1$



$V_1/V_2 = 0.5$



$V_1/V_2 = 0.9$



Conclusions:

- The Kagome lattice gives a 2D example where nonlinear localized modes bifurcate from a flat linear band **without excitation threshold** for “standard” cubic nonlinearities.

- Flat-band discrete solitons exchange stability, resulting in a regime with mobility and symmetry-broken ground state.

- Exchange regime appears for weak Kerr nonlinearity and involves strongly localized states $\Rightarrow$ potentially interesting for optics applications.

- In sawtooth lattice, nonlinearity can be used for tuning compactness for a broad range of coupling coefficients.

- Dimerizations of Kagome chains and Lieb lattices gap out the flat band, and allow for stable compact and/or strongly localized nonlinear modes also for focusing nonlinearities.

- Are experiments ready to reach into these nonlinear regimes yet...?

Thanks to Swedish Research Council for funding through the Swedish Research Links Programme!