Geometric Phase & Nonlinear Photonic Metasurface

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Outline

Introduction to Metasurface

I. Geometric Berry Phase and Metasurface

Spin and Orbital Angular Momentum

Metasurface Holography

II. Nonlinear Photonic Metasurfaces

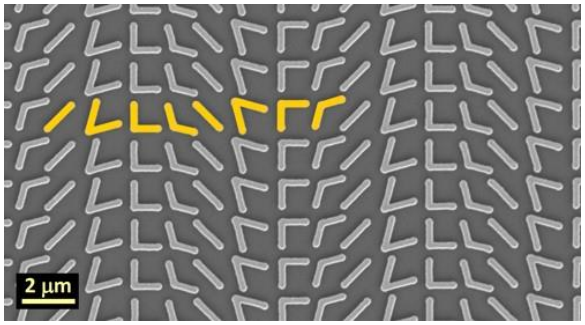
Nonlinear Geometric Berry Phase

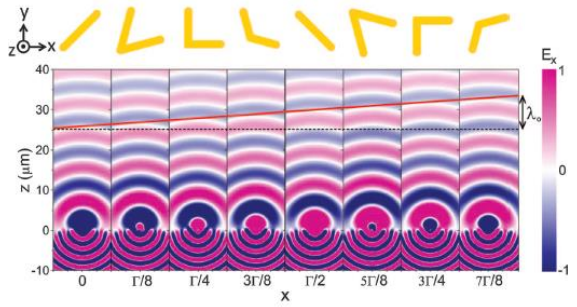
Nonlinear Spin-Orbit Interaction

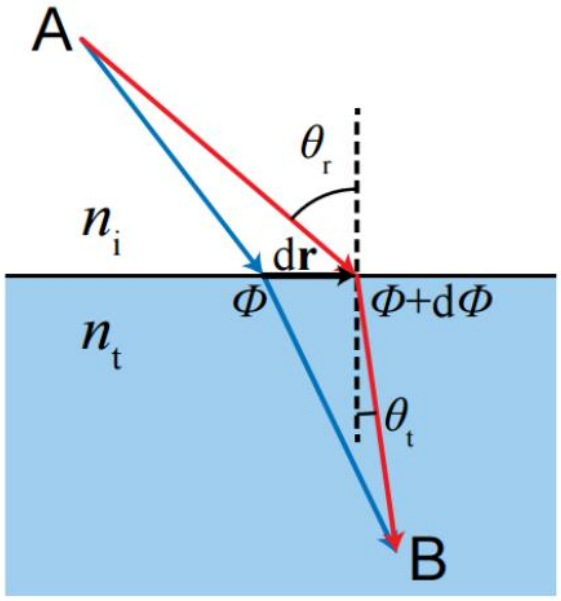
III. Nonlinear Rotational Doppler Effect

1. Flat Optics with Metasurface

Momentum is Conserved!





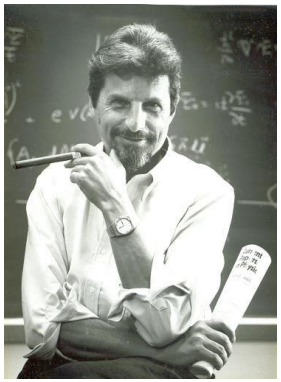


Generalized Snell's Law: $n_t \sin\theta_t - n_i \sin\theta_i = \frac{\lambda_0}{2\pi}\frac{d\Phi}{dx}$

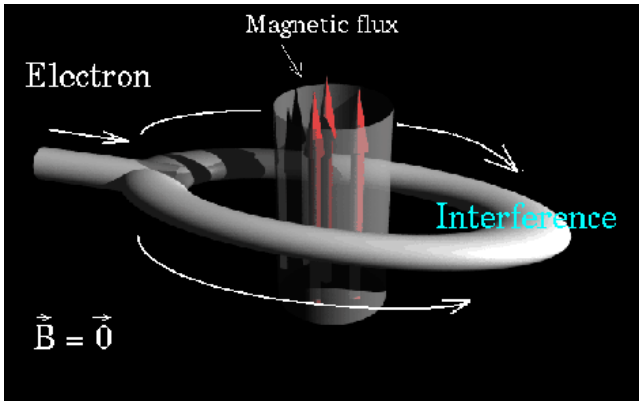
N. Yu, P. Genevet, M. A. Kats, F. Aieta, J. P. Tetienne, F. Capasso, and Z. Gaburro, **Science 334**, 333-337 (2011).

Aharonov-Bohm Effect and Berry Phase

In 1959, Aharonov and David Bohm proposed the thought experiment.



Yakir Aharonov



$$\Delta\phi = \frac{q}{\hbar}\oint Bds = \frac{q}{\hbar}\oint Adl$$

Y. Aharonov and D. Bohm, Significance of Electromagnetic Potentials in the Quantum Theory. **Phys. Rev.** 115, 485 (1959).

A exists even B is zero.

Sir Michael Berry

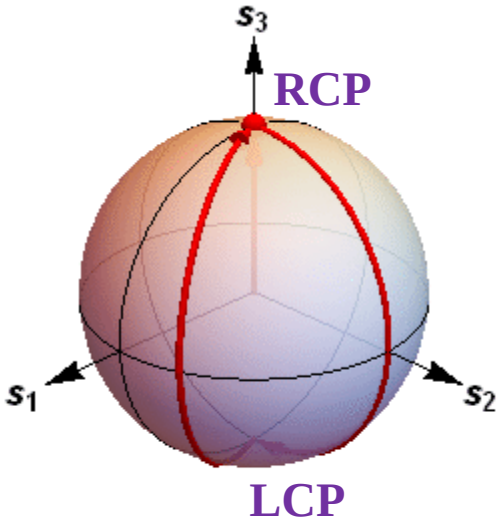
$$\Delta\phi = \oint_s \Omega(R)ds$$

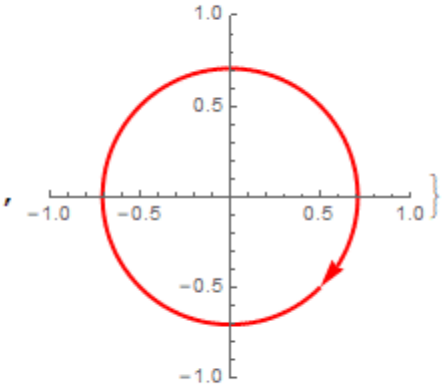
$\Omega(R)$ is Berry Curvature and R is any parameter in a Hilbert space.

M.V. Berry, Quantal phase factors accompanying adiabatic changes. **Proc. R. Soc. Lond. A** 392,45-57(1984).

Geometric P-B Phase and Poincare Sphere

Henri Poincare





'A-B' Effect on Pincare Sphere

Berry connection $\mathbf{A}(\theta,\varphi) = -\dfrac{(\ell+\sigma)\cos\theta}{2r\sin\theta}\hat{\mathbf{e}}_{\varphi}$

Berry curvature $\mathbf{V}(\mathbf{R}) = \nabla_{\mathbf{R}} \times \mathbf{A}(\mathbf{R})$

Pancharatnam-Berry Phase

$$\phi_{Berry} = \frac{\Omega}{2}$$

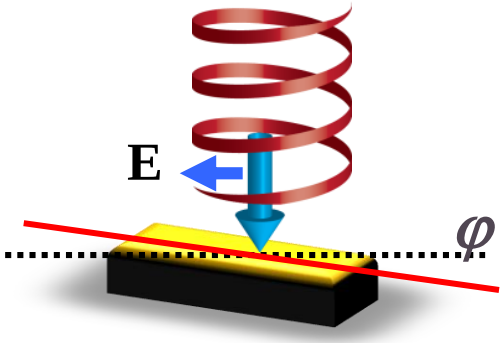
Acknowledgment

I thank Professor S. Ramaseshan and Professor R. Nityananda for telling me about Pancharatnam's work.

S. Pancharatnam, **Proc. Ind. Acad. Sci.** 44, 247 (1956). M.V. Berry, **J. Mod. Opt.** 34,1401-1407 (1987).

Geometric Phase and Metasurfaces

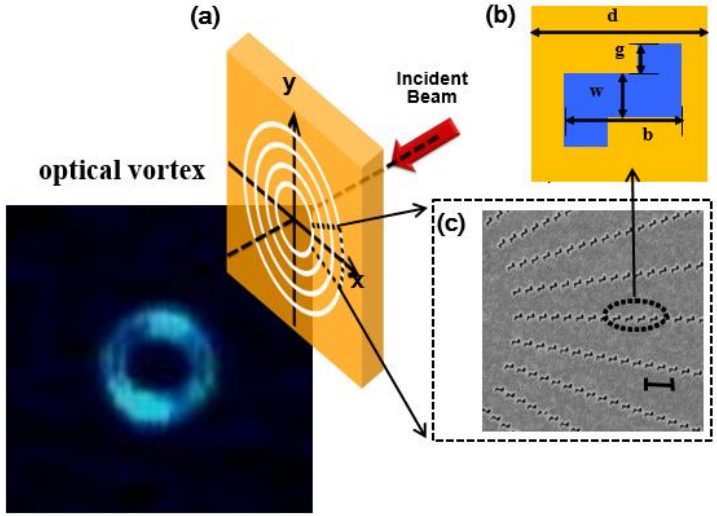
Meta-atom with Geometric Berry Phase



Input	Output
RCP	$LCP\cdot e^{i\cdot 2\varphi}$
LCP	$RCP\cdot e^{-i\cdot 2\varphi}$

$$\phi_{Berry} = \frac{\Omega}{2} = \pm 2\varphi$$

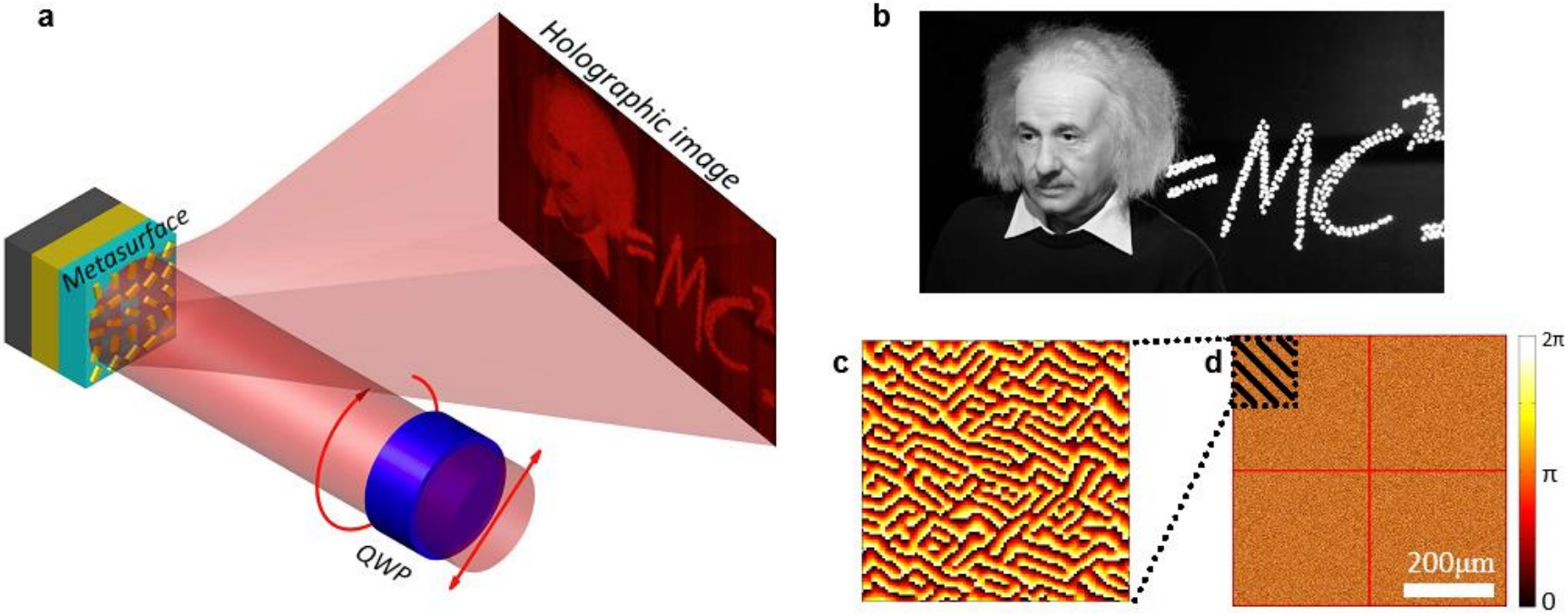
X. Chen, et. al. Dual polarity plasmonic lens. **Nature Communications** 3:1198 (2012).



G. X. Li, et al. Spin-enabled plasmonic metasurfaces for manipulating orbital angular momentum of light. **Nano Lett.** 13, 4148-4151 (2013).

J. H. Poynting, **Proc. R. Soc. Lond. A** 82, 560-567 (1909). L. Allen et al., **Phys. Rev. A** 45,8185-8189 (1992).

High Efficiency Metasurface Holography

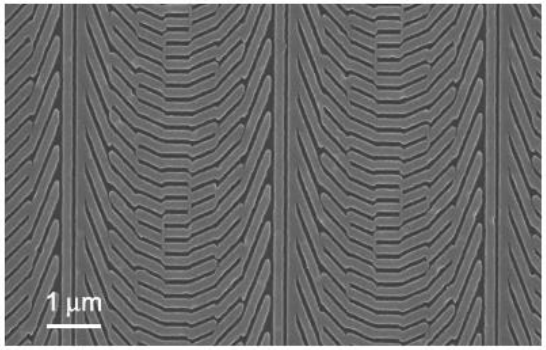


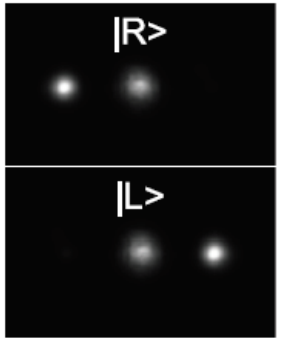
Working principle and phase distribution of the metasurface hologram.

G. Zheng, H. Mühlenbernd, M. Kenney, G. Li, T. Zentgraf, and S. Zhang, "Metasurface holograms reaching 80% efficiency", **Nature Nanotech.** 10, 308-312 (2015).

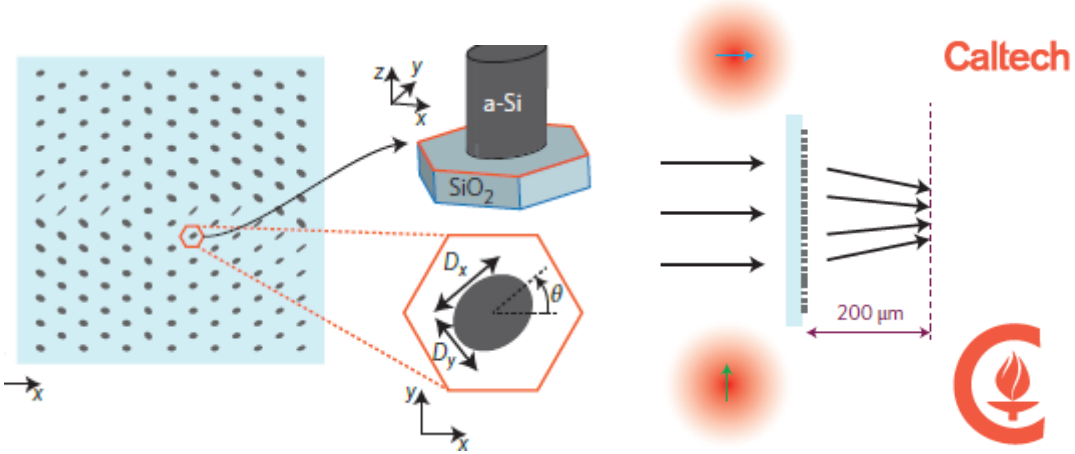
Dielectric Metasurface Holography

Materials：silicon; efficiency：up to ~ 90%； Wavelength：0.95 and 1.6μm

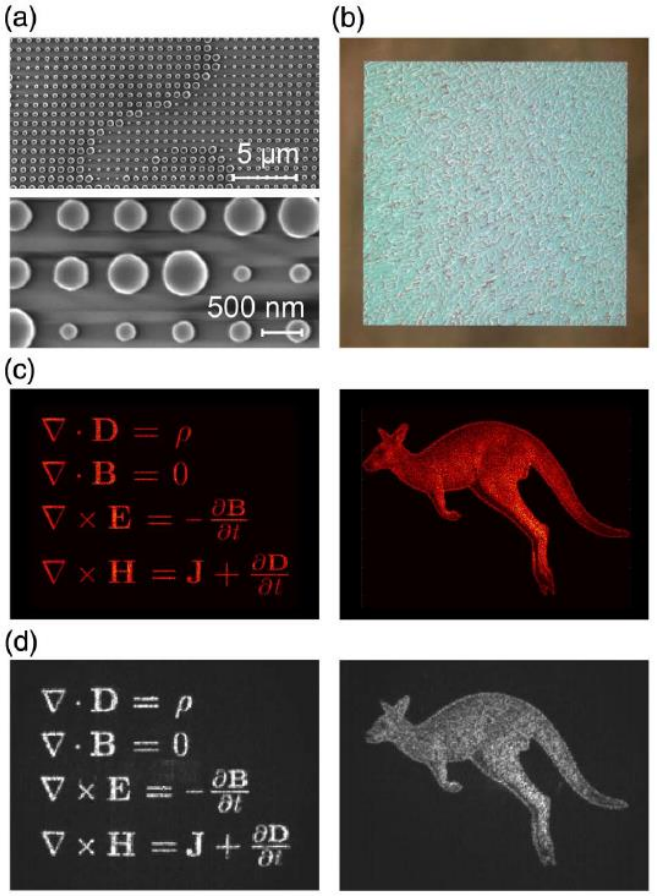




Dianmin Lin et al. **Science** 345, 298 (2014).

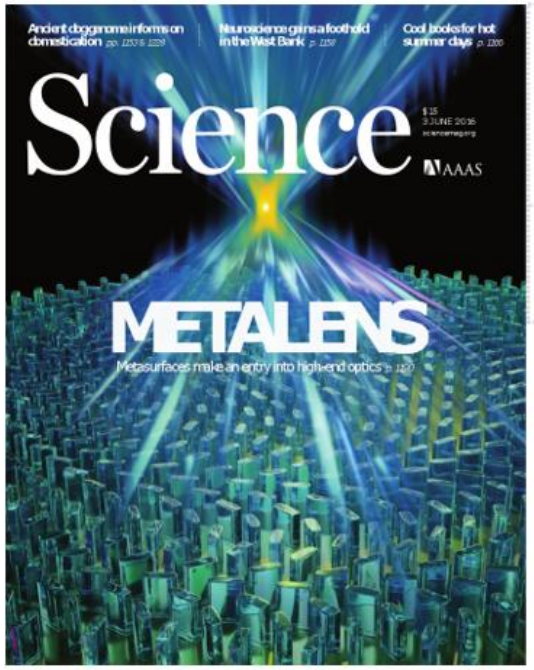


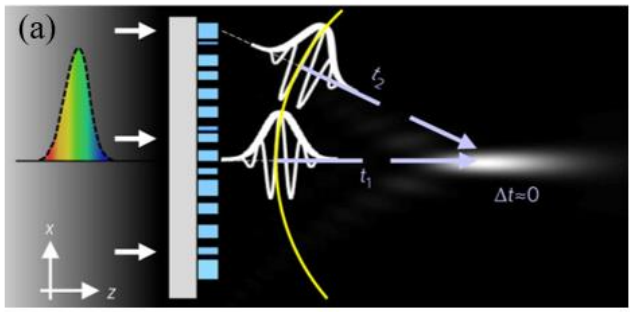
A. Arbabi, et al., **Nature Nanotech**. 10, 937 (2015).

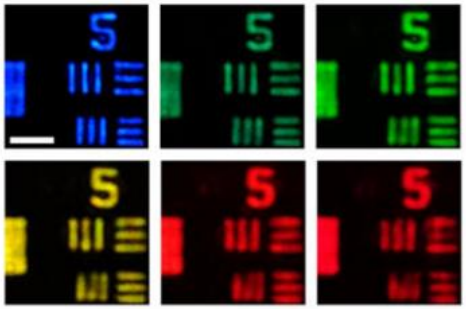


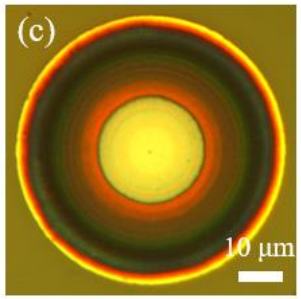
L. Wang et al. **Optica** 3:1504 (2016)

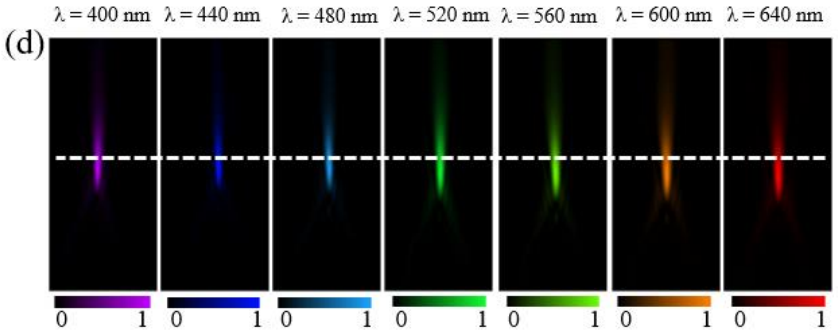
Meta-Lens











Geometric P-B Phase + Resonant Phase

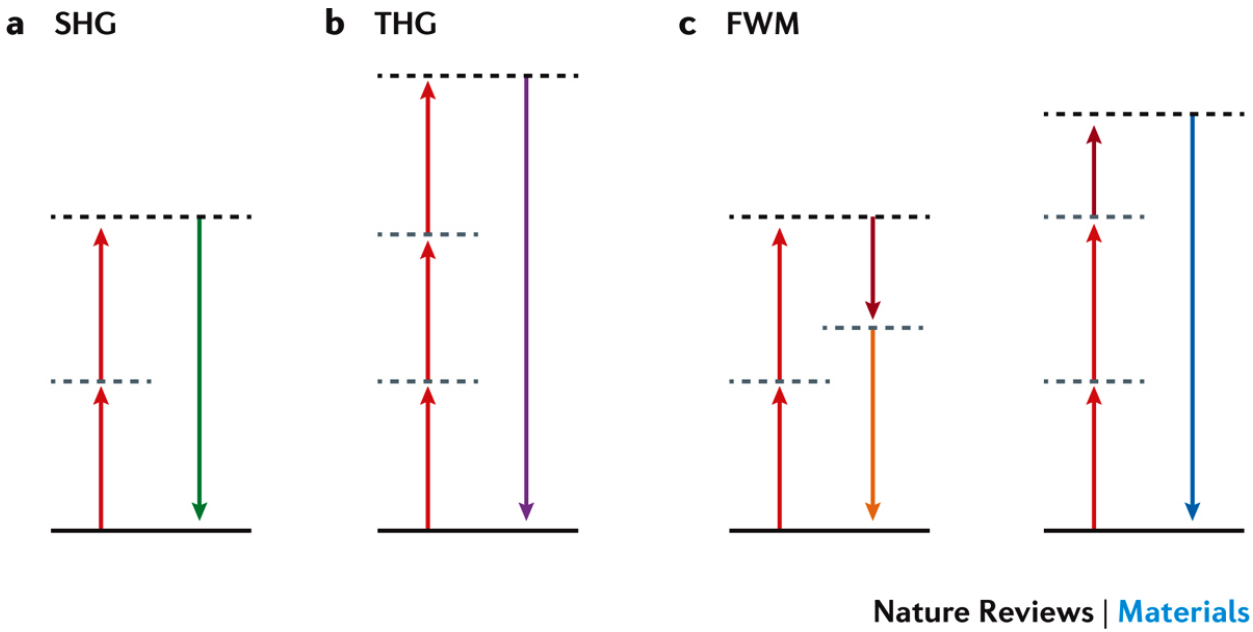
M. Khorasaninejad et. al. **Science** 352, 1190-1194 (2016).

S. M. Wang, et al., **Nature Communications**. 8: 187 (2017).

W. T. Chen, et al. **Nature Nanotechnology** doi:10.1038/s41565-017-0034-6 (2018).

S. M. Wang, et al. **Nature Nanotechnology** doi:10.1038/s41565-017-0052-4 (2018).

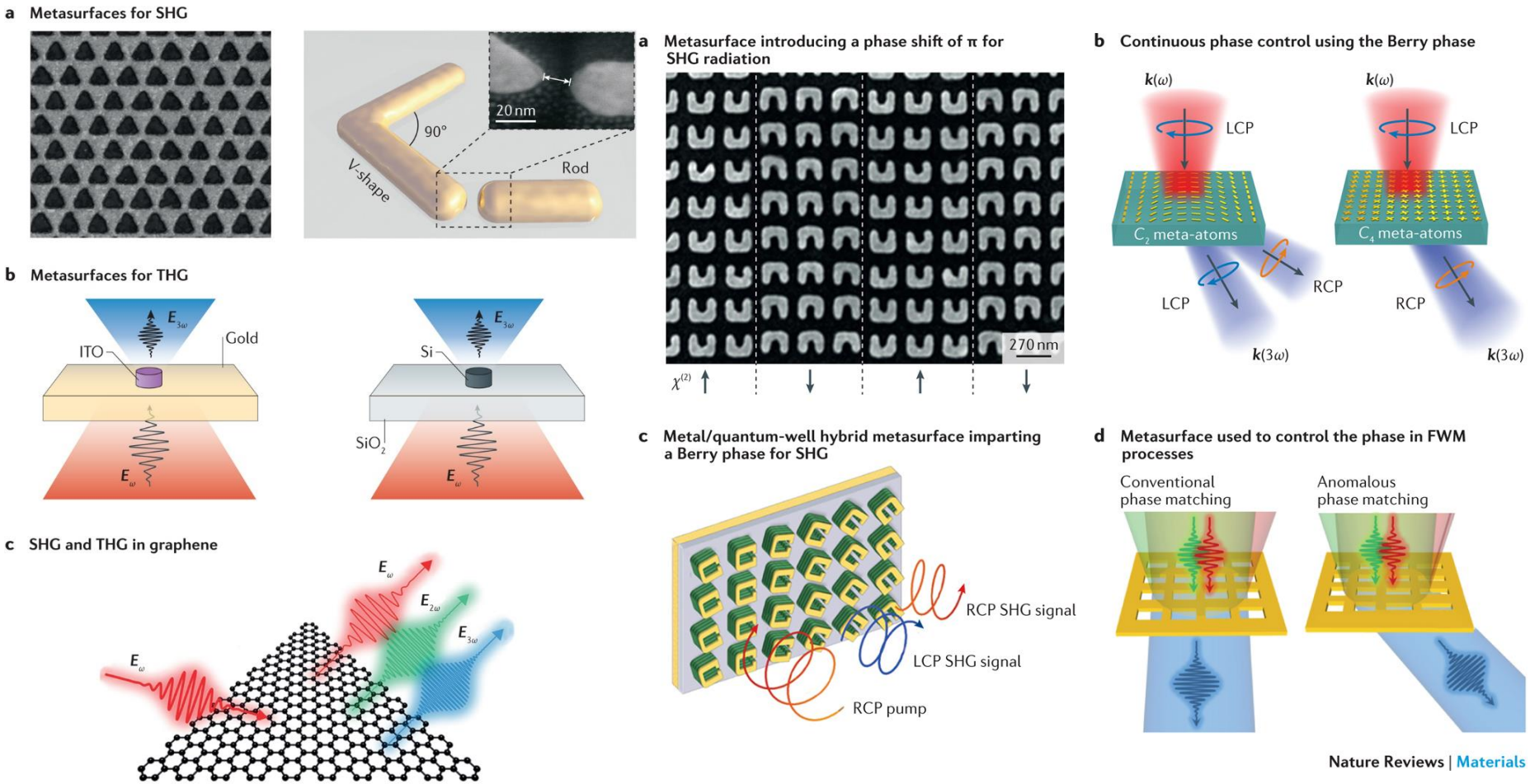
2. Nonlinear Photonic Metasurface



Schematic diagrams of nonlinear optical processes. The solid and dashed lines represent the electronic and virtual levels, respectively. (a) SHG; (b) THG; (c) and (d) nondegenerate FWM.

- Kauranen, M. & Zayats, A. V. Nonlinear plasmonics. ***Nat. Photonics*** **6**, 737–748 (2012).
- Lapine, M., Shadrivov, I. V. & Kivshar, Y. S. Colloquium: nonlinear metamaterials. ***Rev. Mod. Phys***. **86**, 1093–1123 (2014).
- Li, G., Zhang, S. & Zentgraf, T. Nonlinear photonic metasurfaces. ***Nat. Rev. Mater***. **2**, 17010 (2017).

Nonlinear Photonic Metasurfaces



Li, G. *et al.* (2017) Nonlinear photonic metasurfaces. ***Nat. Rev. Mater.*** doi:10.1038/natrevmats.2017.10

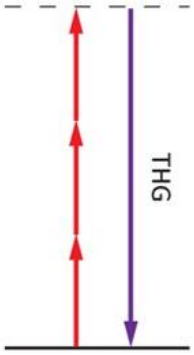
2.1 Symmetry & Harmonic Generations

$$n = ql \pm 1$$

'+' and '-' correspond to the harmonic generations with ***same*** *or* ***opposite*** *handiness*

q(q>2) is the symmetry number, l is an integer *N. Bloembergen et.al.,* ***Phys. Rev****. B,* ***4****, 3437(1971).*

THG n=3



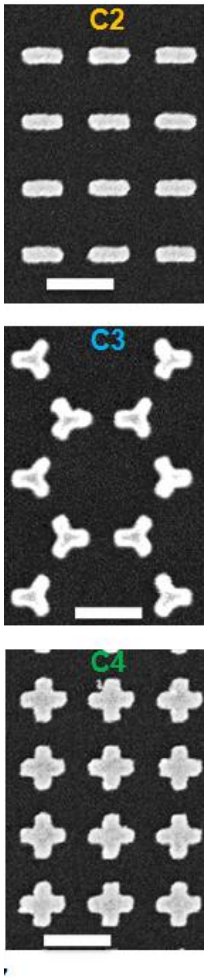
Symmetry Class	Isotropic	C2	C3	C4
q	1	2	3	4
RCP excitation	x	LCP/RCP	x	LCP

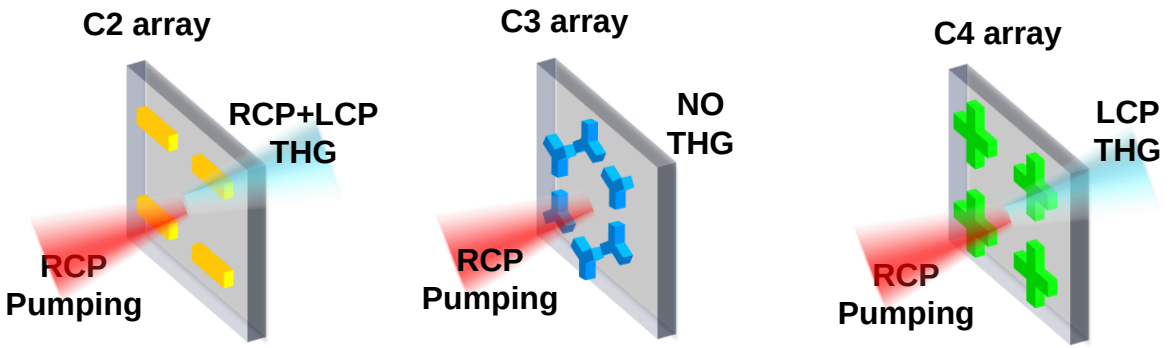
Nonlinear Optical Crystal (Atom Scale) → ***Selection rules*** → **Metacrystal (Sub-wavelength Scale)**

Design of Meta-Crystal

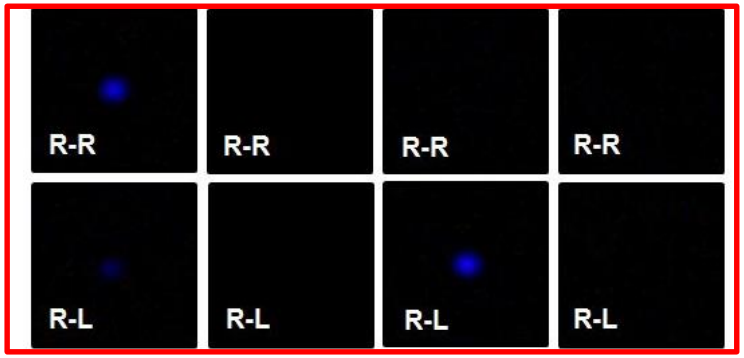
Symmetry	C2	C3	C4
Crystal Class	Orthorhombic	Trigonal	Tetragonal
Local Symmetry	L1 = 210nm w = 60nm	L2 = 110nm w	L3 = 230nm w
Global Symmetry	Period = 400nm	Period = 400nm	Period = 400nm

Spin Dependent THG





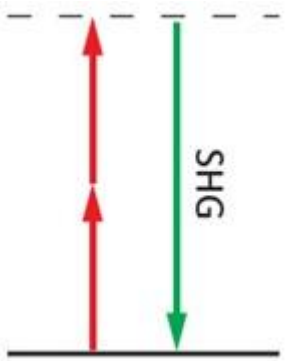
Circularly polarized pumping laser

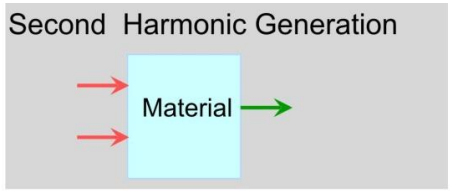


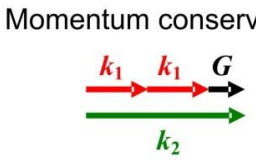
Fundamental Wavelength: 1250 nm

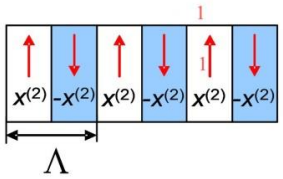
S. M. Chen, G. X. Li, F. Zeuner, W. H. Wong, E. Y. B. Pun, T. Zentgraf, K. W. Cheah, and S. Zhang, "Symmetry selective third harmonic generation from plasmonic metacrystals", ***Phys. Rev. Lett.*** 113, 033901 (2014).

The Need of Nonlinear Phase

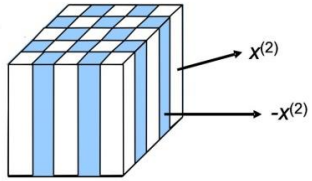






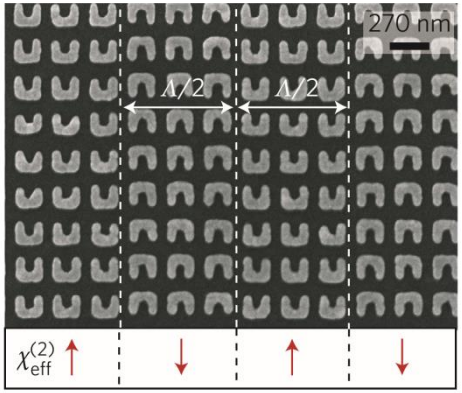


$G=2\pi/\Lambda$



2D Nonlinear Photonic Crystal

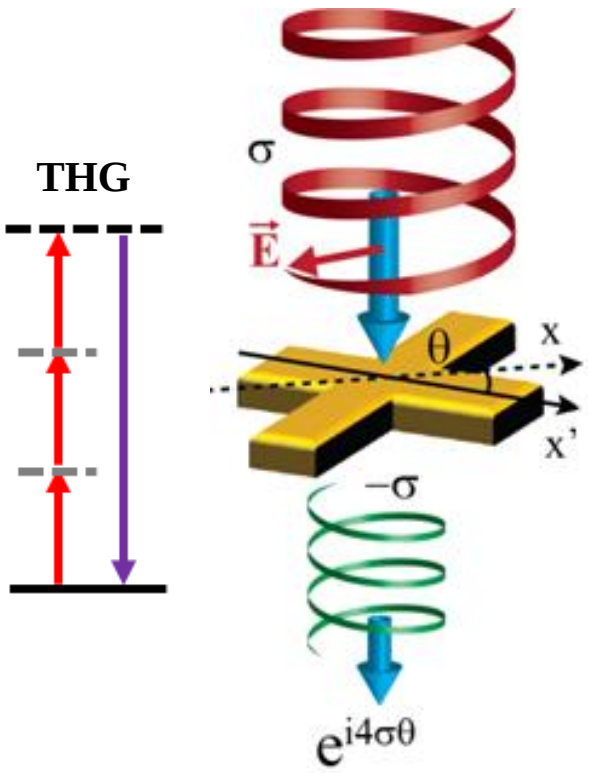
J. A. Armstrong, N. Bloembergen, J. Ducuing & P. S. Pershan, Interactions between light waves in a nonlinear dielectric. ***Phys. Rev.*** 127, 1918-1939 (1962). Xu, P. & Zhu, S. N. ***AIP Advances***, 2, 041401(2012). Zhu, S. N., Zhu, Y. Y. Qin, Y. Q., Wang, H. F., Ge, C. Z. & Ming, N. B. ***Phys. Rev. Lett***. 78, 2752-2755 (1997).



0 and π

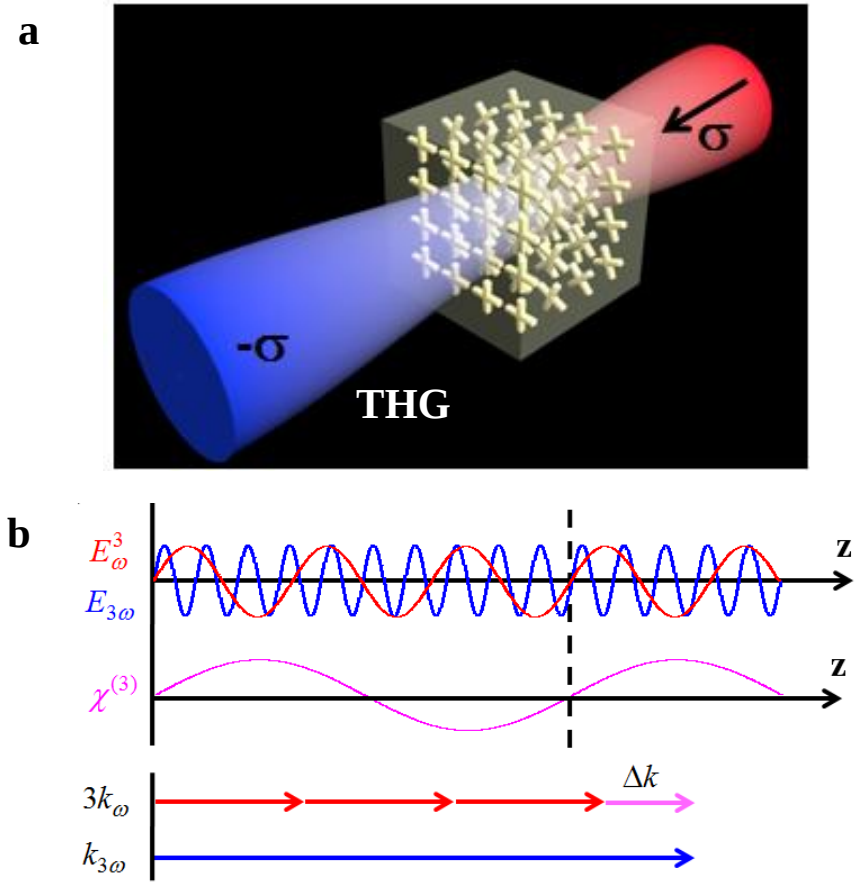
N. Segal, S. Keren-Zur, N. Hendler, T. Ellenbogen, Controlling light with metamaterial-based nonlinear photonic crystals, **Nature Photon.** 9, 180-184 (2015).

2.2 Nonlinear Geometric Berry Phase



THG has a phase of $4\sigma\theta$.

Continuous-Phase Matching

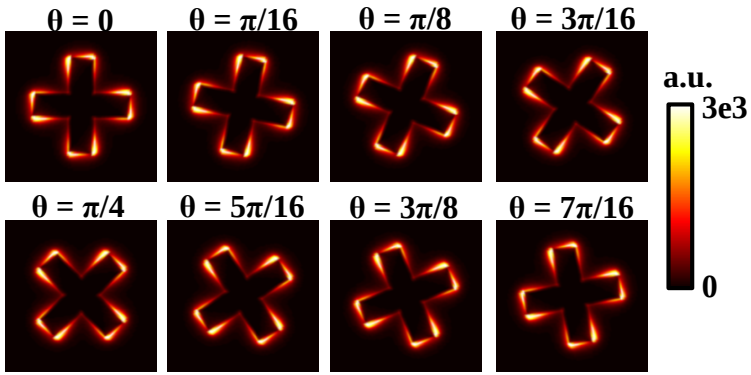


G. X. Li, et al. Continuous control of nonlinearity phase for harmonic generations. **Nature Materials** 14, 607-612 (2015).

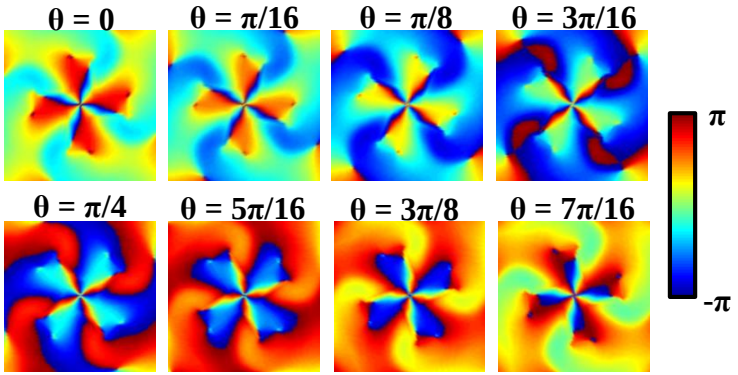
Confirm the Nonlinear Berry Phase!

Input/Output polarization: RCP-LCP

(a) Amplitude Distribution



(b) Phase Distribution

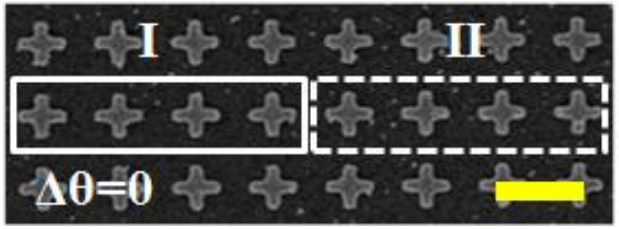


Local contribution of nonlinear polarizations for LCP THG. a, The amplitude distribution; **b**, the phase distribution.

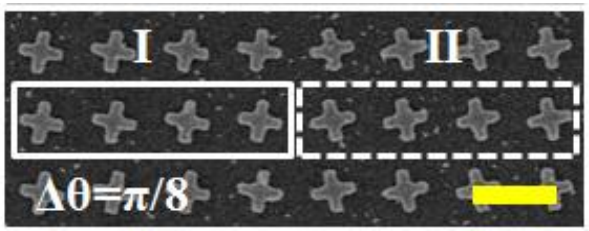
Nonlinear phase has a factor of 4θ!

Experimental Verification

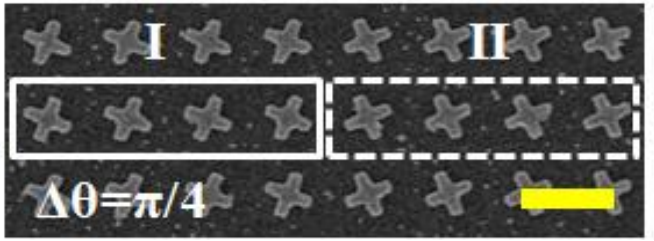
Sample A

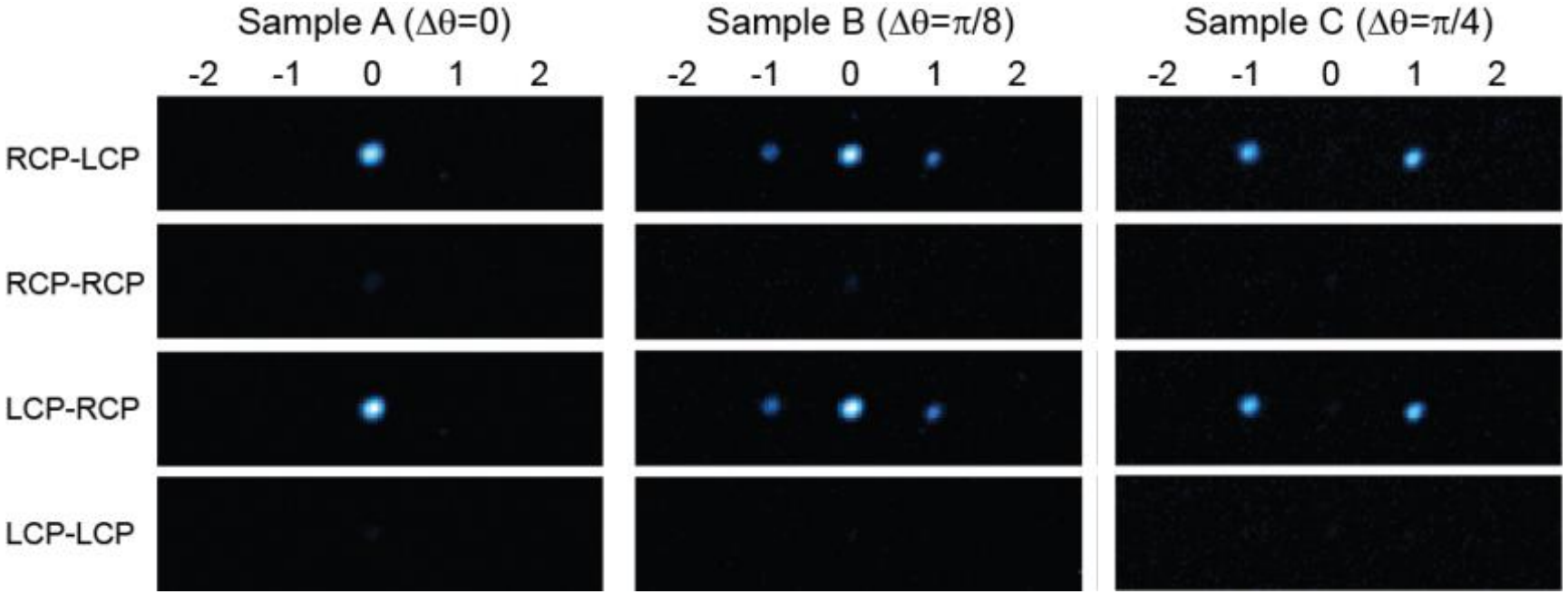


Sample B



Sample C

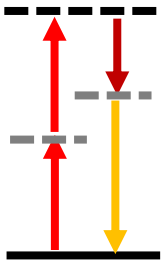




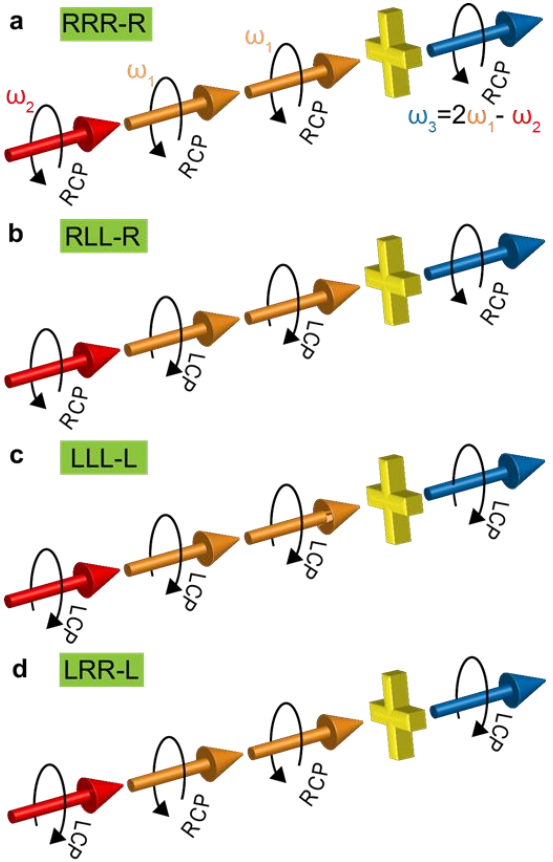
Sample B: $I_0^{3\omega} : I_{\pm 1}^{3\omega} = 2.8$ (*theory* ~ 2.4)

2.3 Spin Controlled Four Wave Mixing

$$\vec{P}^{FWM}(2\omega_1 - \omega_2) = \epsilon_0 \chi^{(3)} \vec{E}_{\omega_1} \vec{E}_{\omega_1} \vec{E}^*_{\omega_2}$$



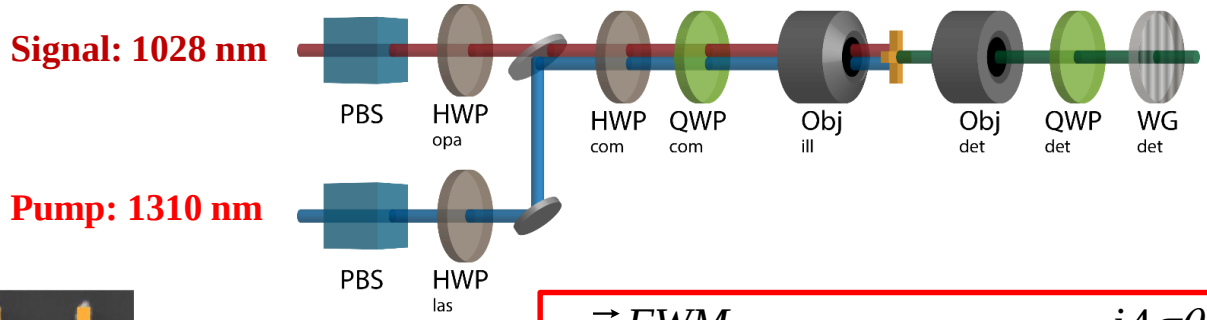
FWM

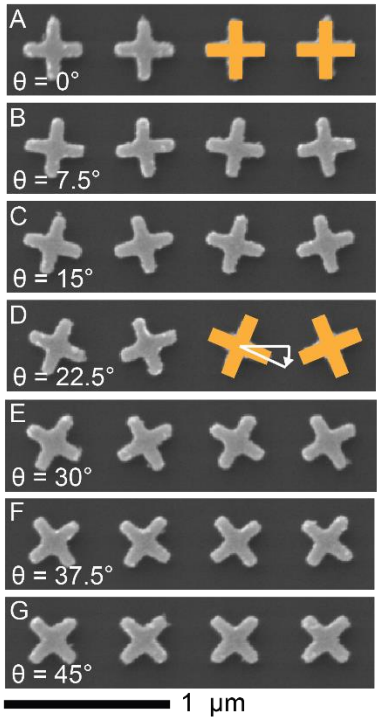


FWM selection rules from plasmonic meta-atoms with C4 symmetry.

G. X. Li et al. Spin and peometric phase control Four-Wave Mixing from metasurfaces. **Laser Photonics Rev.** DOI: 10.1002/lpor.201800034 (2018).

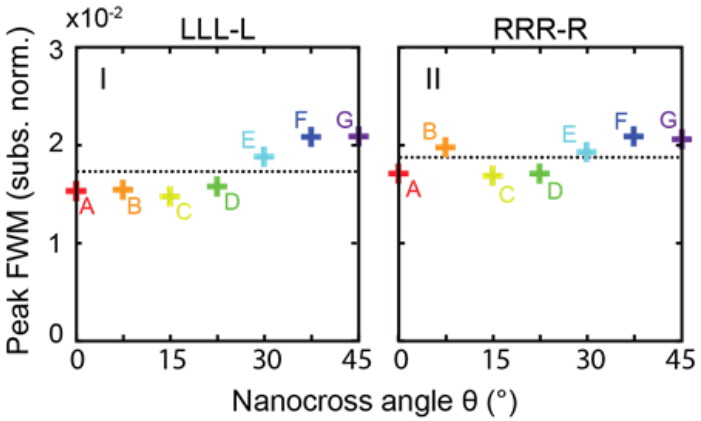
FWM & Geometric Berry Phase

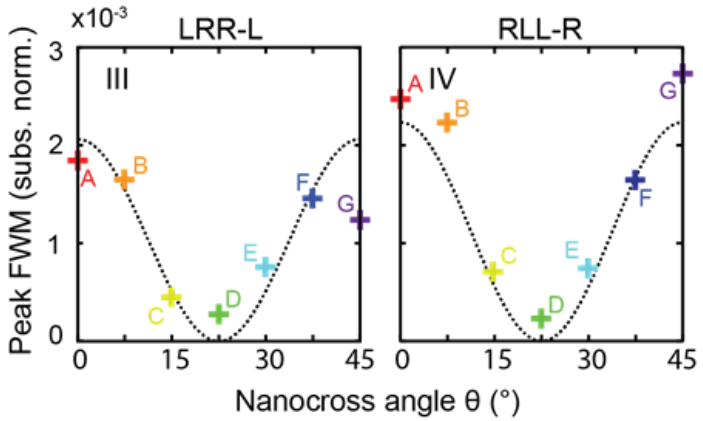




$$\vec{P}_{-\sigma}^{FWM}(2\omega_1-\omega_2)\propto e^{i4\sigma\theta}$$

$$I(2\omega_1-\omega_2)\propto\left|e^{i4\sigma\theta}+e^{-i4\sigma\theta}\right|^2$$

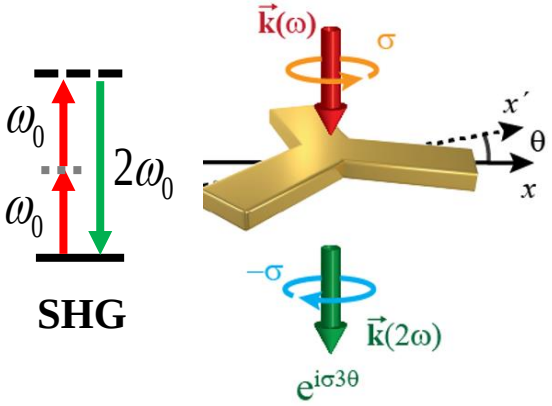




G. X. Li et al. Spin and geometric phase control Four-Wave Mixing from metasurfaces. **Laser Photonics Rev.** DOI: 10.1002/lpor.201800034 (2018).

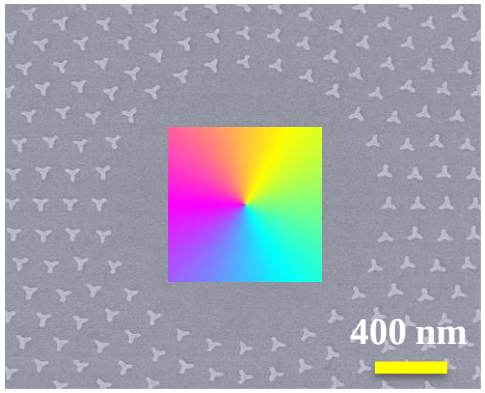
2.4 Nonlinear Spin-Orbit Interaction

Nonlinear Phase Element



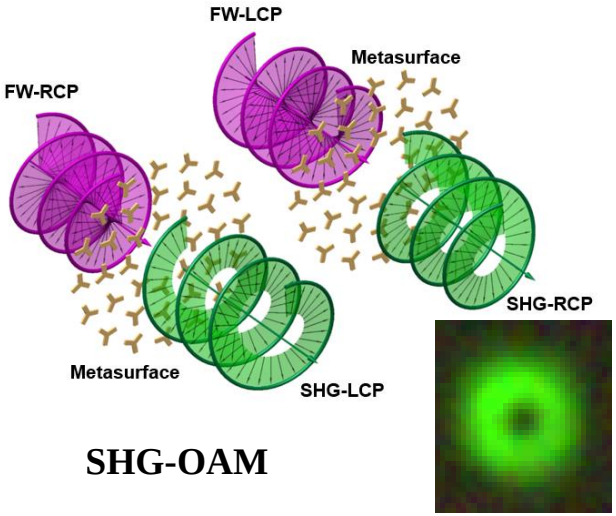
SHG Phase: $3\sigma\theta$

Rotational Metasurface



Introduce Phase Singularity! *$2\pi*q$*

Nonlinear Optics



SHG-OAM

Pumping Light	
SAM	$2\sigma\hbar$
OAM	$2l_{Pump}\hbar$

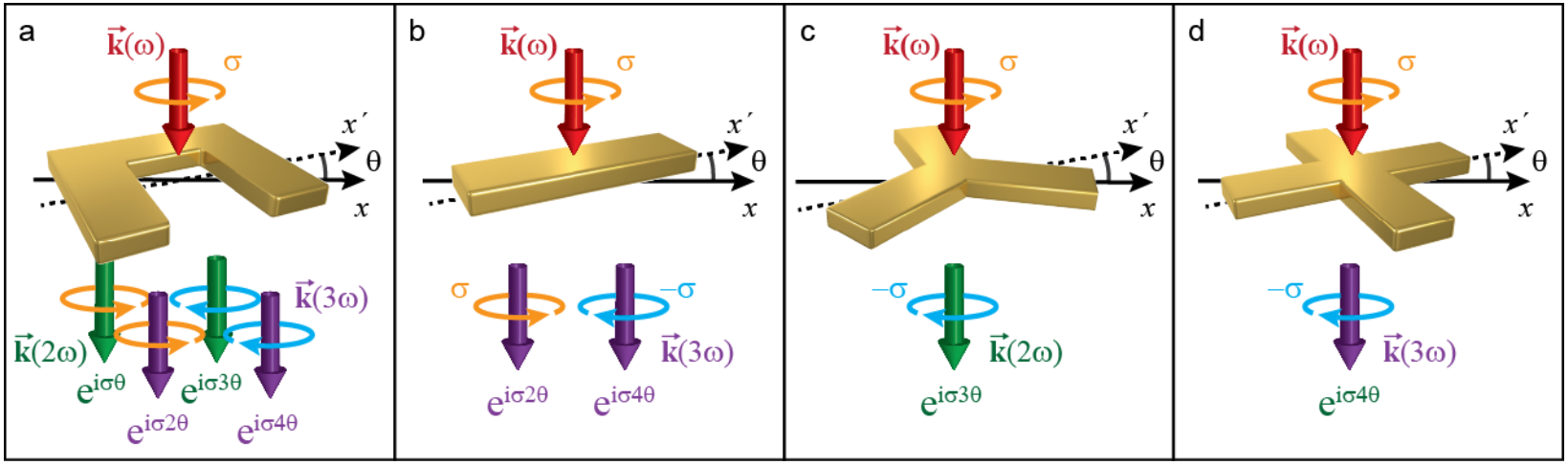
SHG	
SAM	$-\sigma\hbar$
OAM	$l_{SHG}\hbar$?

$$l_{SHG} = 3\sigma q$$

$$2\sigma\hbar + 2l_{Pump}\hbar = -\sigma\hbar + l_{SHG}\hbar ?$$

Li, G. X., et al. Nonlinear metasurface for simultaneous control of spin and orbital angular momentum in second harmonic generation. **Nano Letters**, 17, 7974-7979 (2017).

Nonlinear Berry Phase Elements

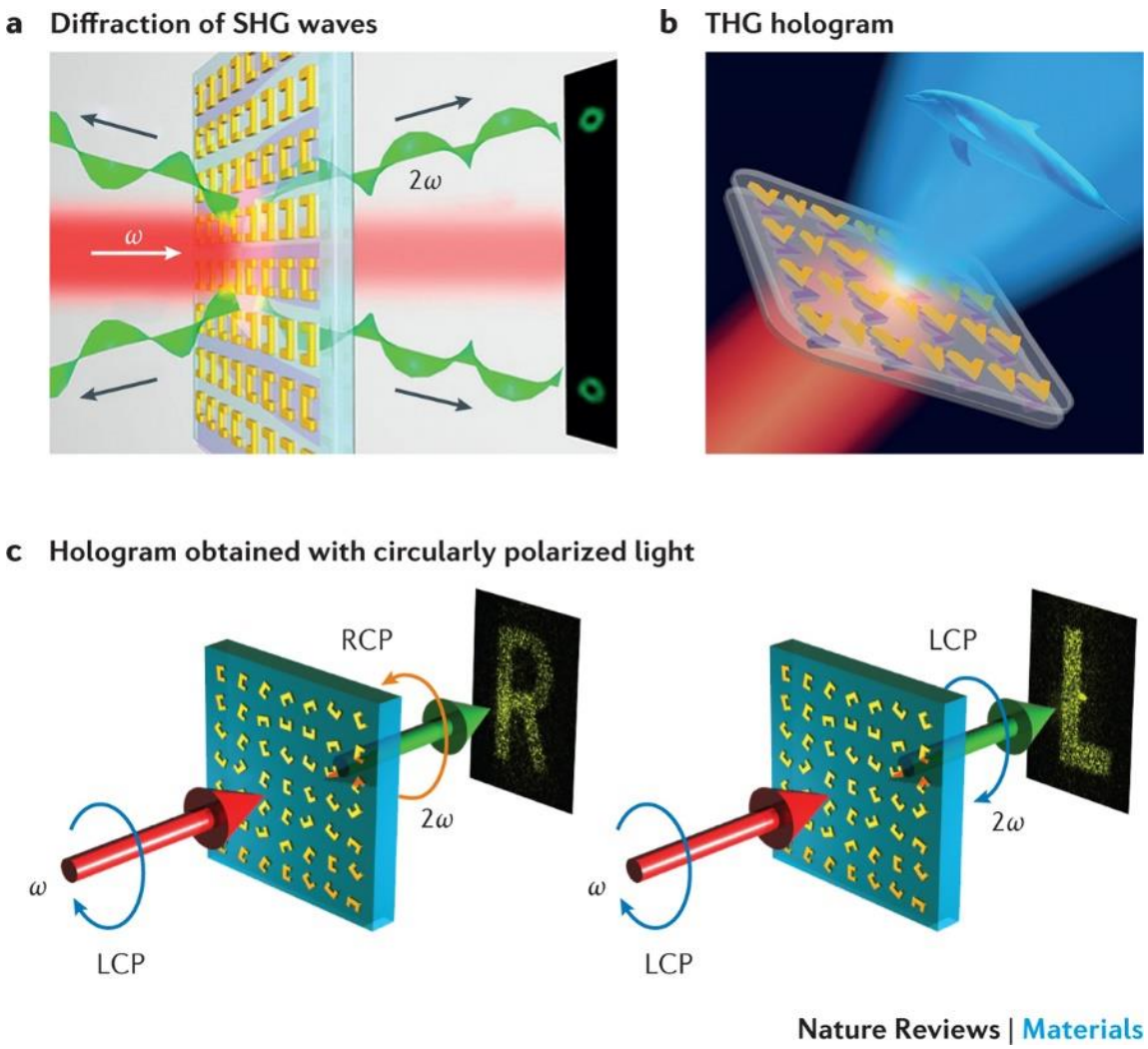


Nonlinear Meta-atoms for SHG and THG

(n+1) σθ or (n-1) σθ, n is the order of harmonic generation!

Li, G., Zhang, S. & Zentgraf, T. Nonlinear photonic metasurfaces. ***Nat. Rev. Mater.*** **2**, 17010 (2017).

Nonlinear Beam Shaping and Holography



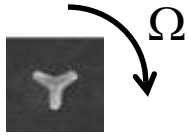
Li, G., Zhang, S. & Zentgraf, T. Nonlinear photonic metasurfaces. ***Nat. Rev. Mater***. **2**, 17010 (2017).

3. Rotational Doppler Effect for SHG

Gradient of orientation angle in space --- Momentum deflection

$$k_x = \pm 3\frac{d\theta}{dx}$$

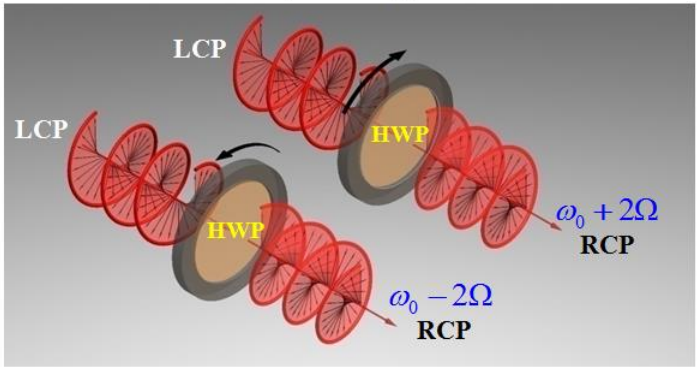
Gradient of orientation angle in time --- Energy (frequency) shift



$$\Delta\omega = 3\frac{d\theta}{dt} = \pm 3\Omega$$

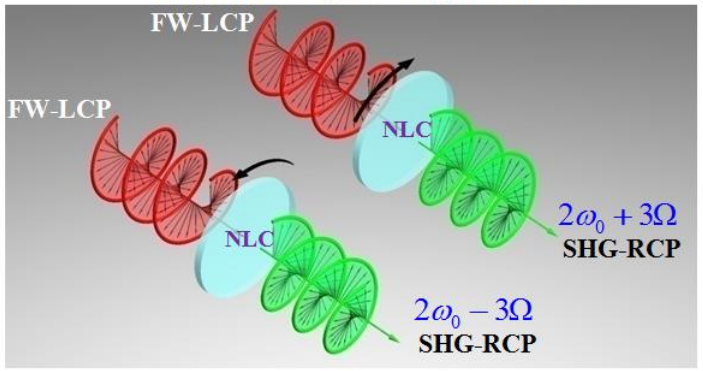
Nonlinear Rotational Doppler Shift

a



Garetz, B. A. Angular Doppler effect. *J. Opt. Soc. Am.* **71**, 609–611 (1980).

b



G. X. Li, T. Zentgraf, and S. Zhang, Rotational Doppler Effect in Nonlinear Optics. **Nat. Phys.** 12, 736-740 (2016).

News and Views:
Etienne Brasselet, "Optical physics: Harmonic angular Doppler effect", **Nat. Photon.** 10, 362-364 (2016).

RDE for Harmonic Generations

I: Momentum inertia of BBO crystal;
s: Rotational direction of BBO crystal;
Ω: Rotational frequency of BBO crystal
σ: Spin direction of light
L: Angular Momentum
E: Kinetic energy
$\hbar\omega_0$ *: Energy of pumping light*
$\hbar\omega_n$ *: Energy of harmonic generation*

1. Conservation Law of Angular Momentum

$$L = sI\Omega \qquad n\sigma\hbar + sI\Omega_0 = \pm\sigma\hbar + sI\Omega_n$$

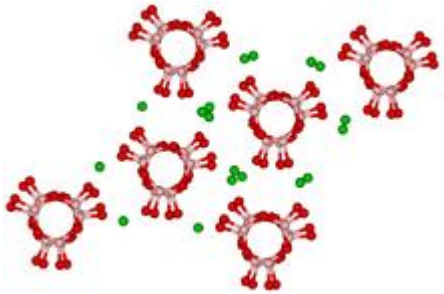
2. Conservation Law of Energy

$$E = I\Omega^2/2 \qquad n\hbar\omega_0 + I\Omega_0^2/2 = \hbar\omega_n + I\Omega_n^2/2$$

3. Rotational Doppler Shift $\Delta\omega_n = \omega_n - n\omega_0 = -(n \mp 1)s\sigma\Omega_0$

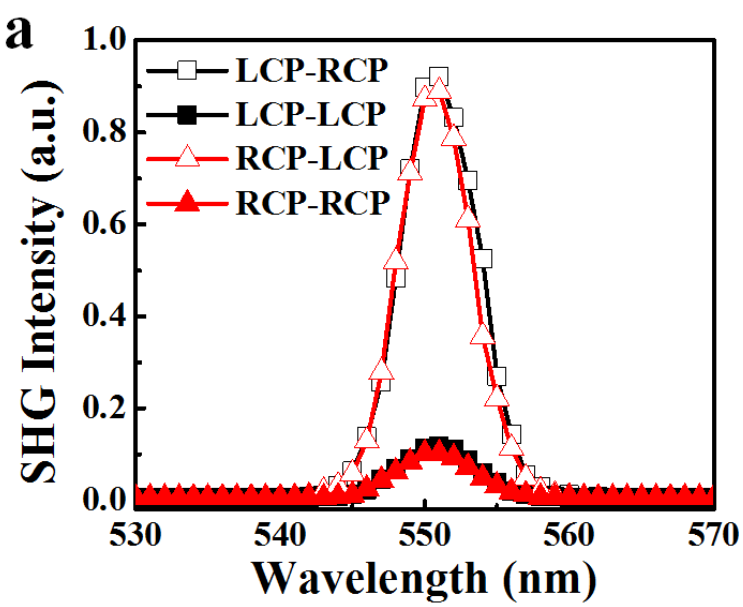
Second Harmonic Generation from β-BBO

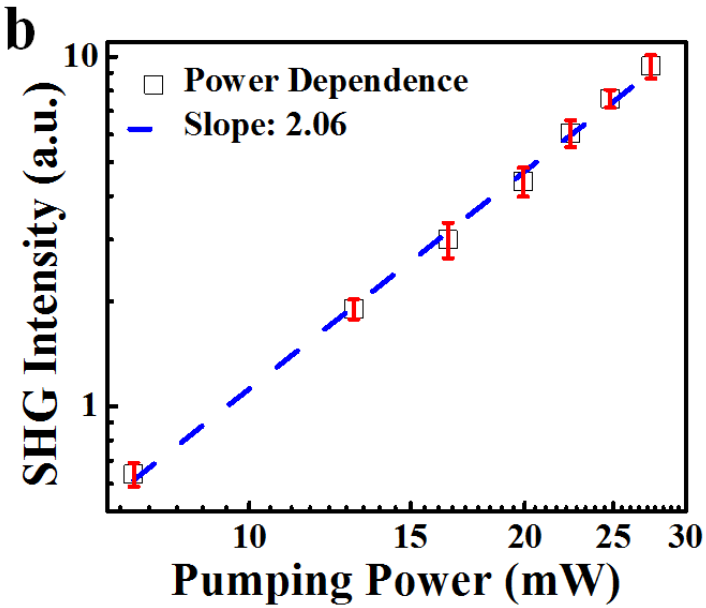
Gold nanorod



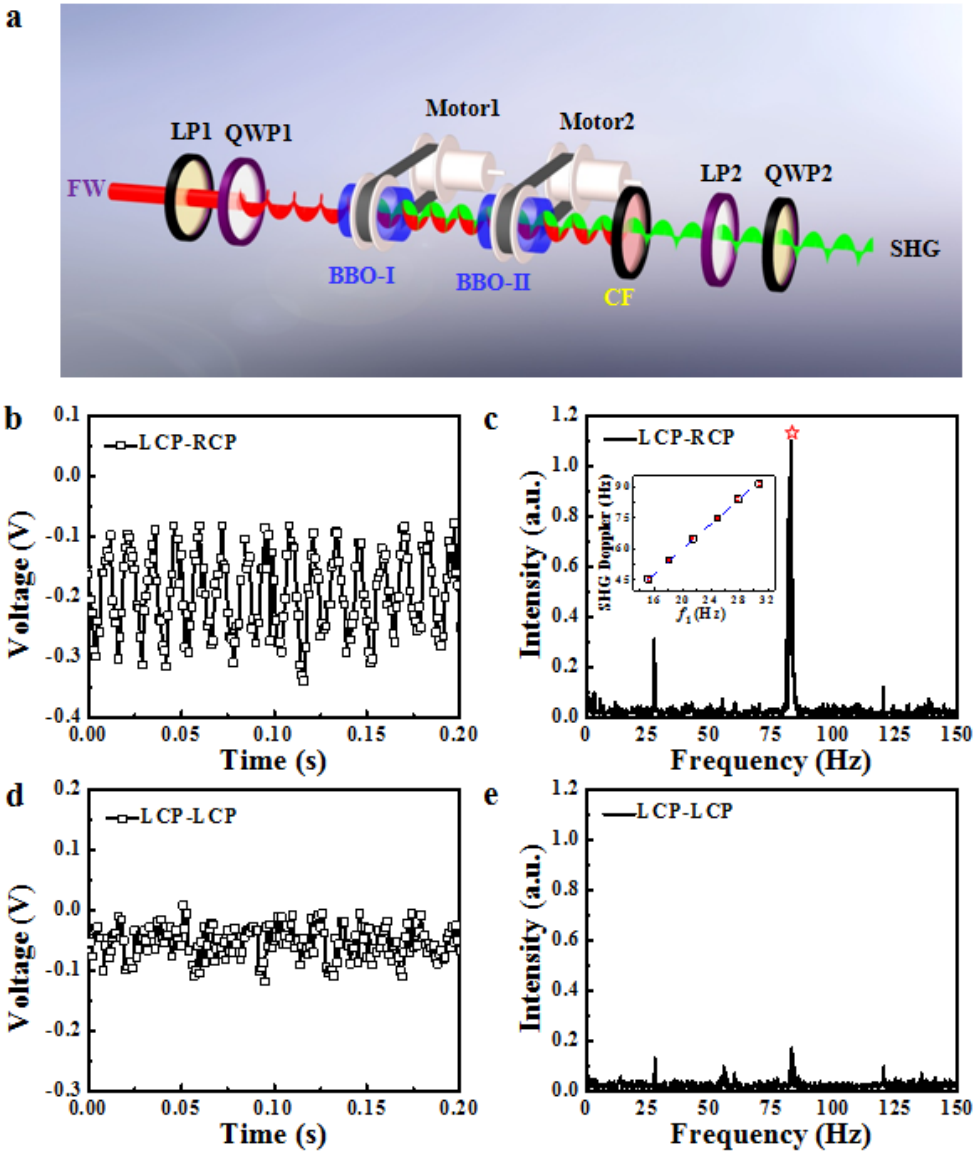
Beta-BBO along C-axis

C3 Crystal Symmetry



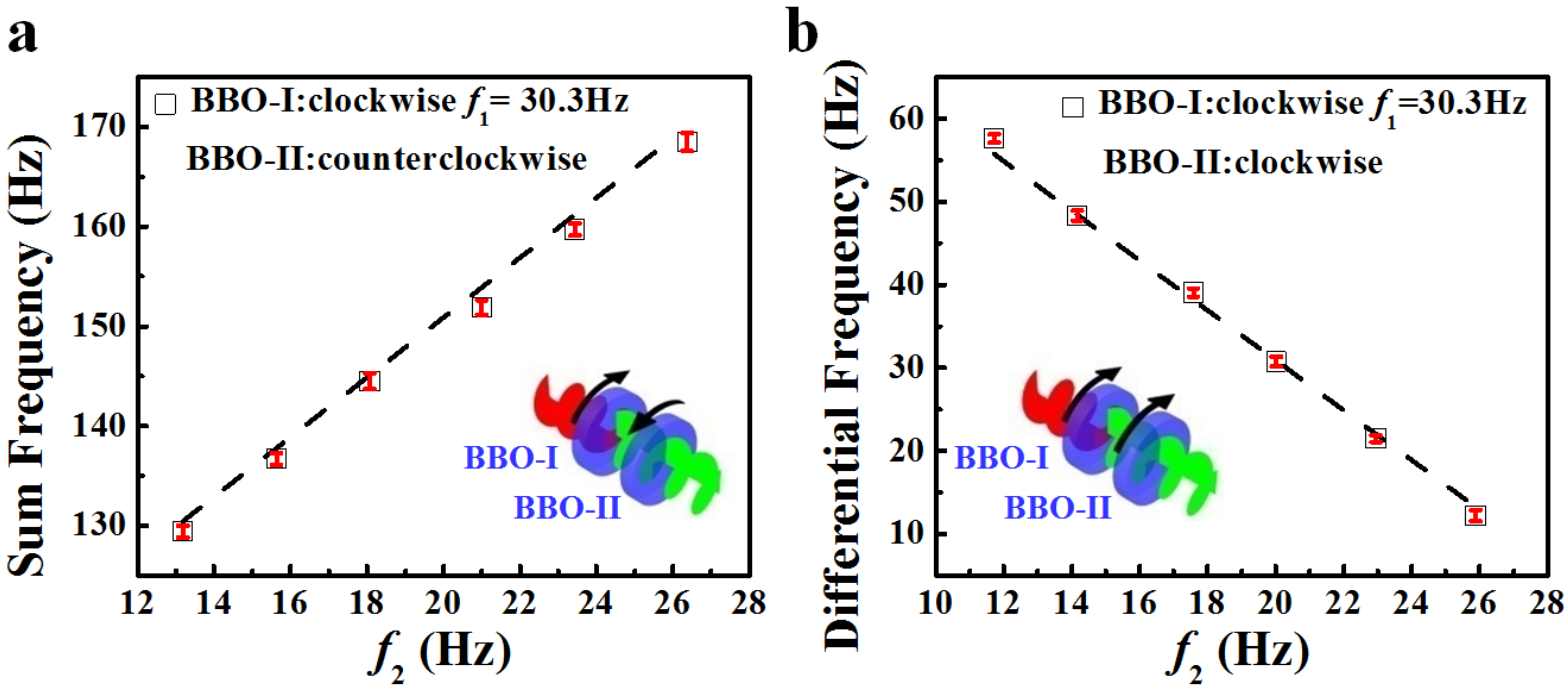


SHG Rotational Doppler Effect



$$\left|\Delta\omega_{SHG}\right| = 3\Omega$$

Sum and Differential Frequencies



$3f_1 + 3f_2$

$3f_1 - 3f_2$

G. X. Li, T. Zentgraf, and S. Zhang, "Rotational Doppler Effect in Nonlinear Optics", **Nat. Phys.** 12, 736-740 (2016).

Nonlinear Phase in Space & Time Domains

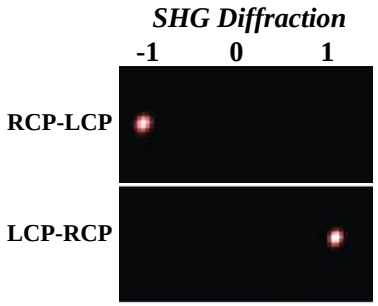
Nonlinear Spin-Orbit Interaction

Gradient of orientation angle in *space*

➡ ***Momentum deflection***

$$k_x = \pm 3\frac{d\theta}{dx}$$

Spin dependent SHG deflection

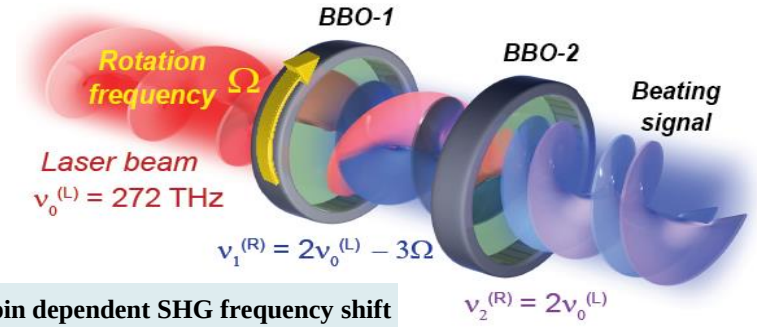


Nonlinear Rotational Doppler Effect

Gradient of orientation angle in *time*

➡ ***Energy (frequency) shift***

$$\Delta\omega = 3\frac{d\theta}{dt} = \pm 3\Omega$$



Spin dependent SHG frequency shift

Li, G., Zhang, S. & Zentgraf, T. Nonlinear photonic metasurfaces. ***Nat. Rev. Mater.*** **2**, 17010 (2017).

Thank You!

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- **Collaborators**
- Prof. Shuang Zhang (Birmingham, UK)
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- Prof. Kok-Wai Cheah (Hong Kong)
- Prof. Anatoly Zayats (KCL, UK)
- Prof. Yu Luo (NTU, Singapore)

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