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Whispering gallery modes in deformed dielectric cavities via transformation optics

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Collaboration with
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Outline

- A longstanding issue in optical microcavity
  - Whispering gallery modes (WGM)
  - Deformed microcavity
  - Trade-off between high-Q factor and directional emission
- Transformation optics
- Transformation cavity
  - Conformal WGM (cWGM)
  - Directional emission from cWGM
  - Realization of transformation cavity

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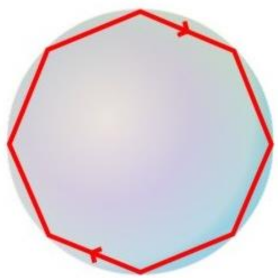
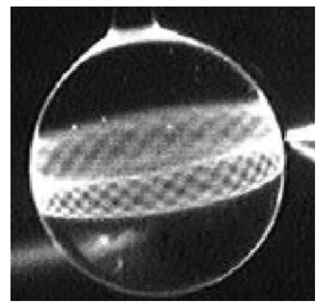
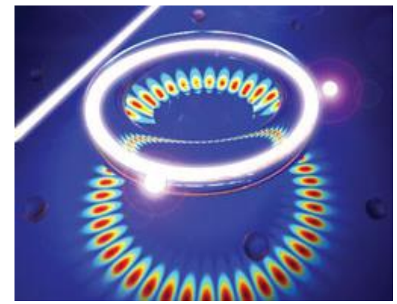
A longstanding issue in optical microcavity

- Whispering gallery modes (WGM) in dielectric cavity
  - extremely long lived modes supported by total internal reflection



Total internal reflection of light ray

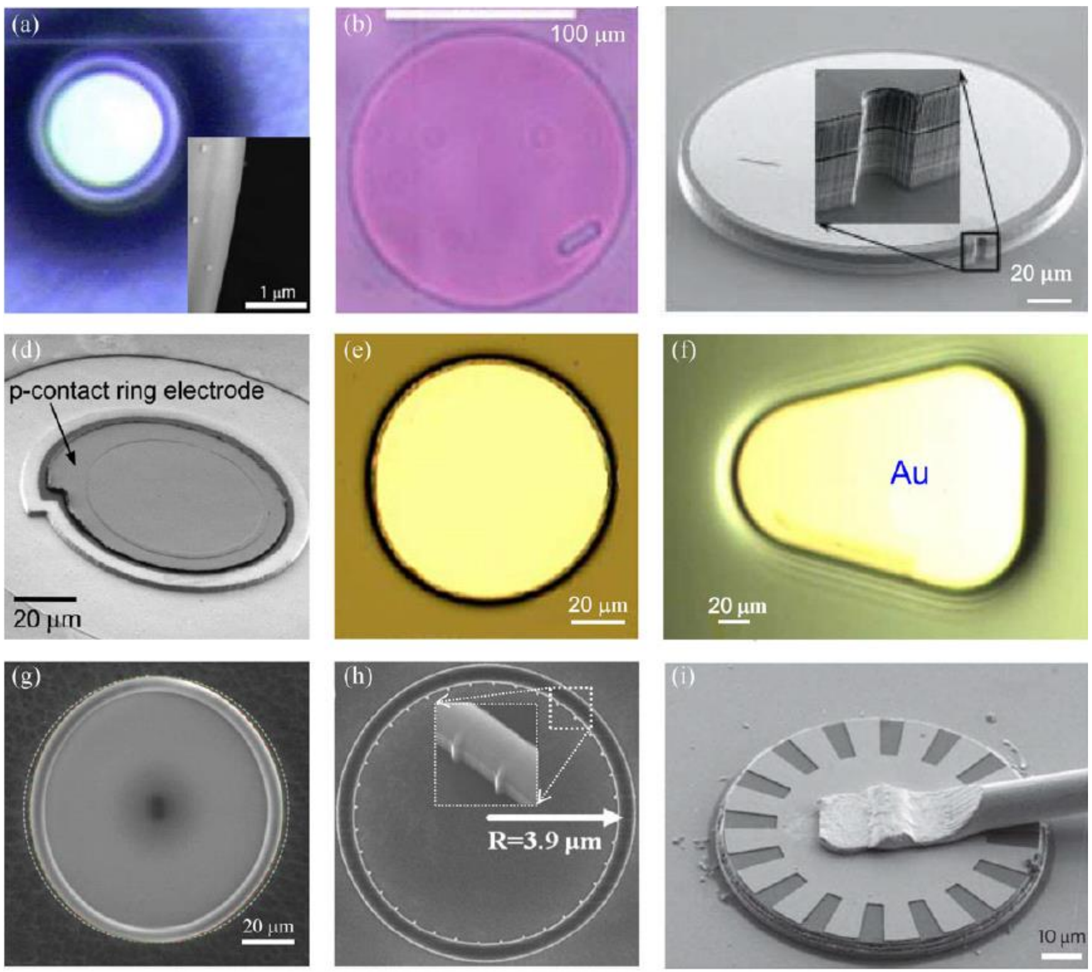




Optical whispering gallery mode in dielectric cavity

- **Ultra High Q-factor**
  - essential for low threshold laser, bio-sensor, and photonics applications for more sensitivity
- **Isotropic emission**
  - a serious drawback in applications requiring directional light sources and efficient free-space optical coupling
- Hence, how to obtain ultra-high Q mode in optical microcavities with a directional emission has been a long-standing issue for last 15 years

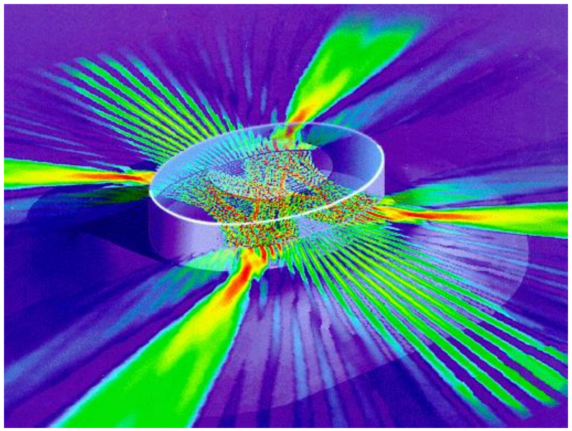
Optical microcavities with directional emissions



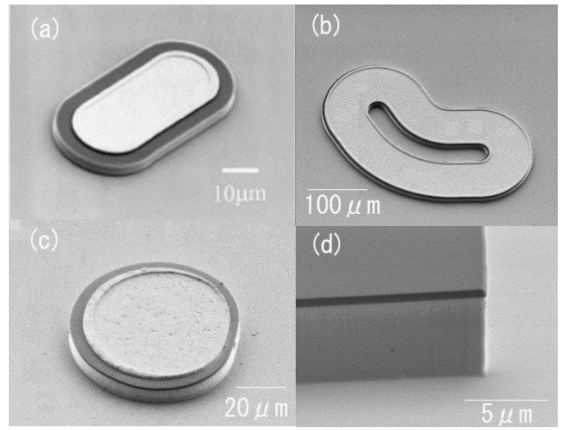
X.-F. Jiang *et al.*, *Laser Photonics Rev. (2016)*

- Microcavities with defect
- Deformed microcavities
- Coupled microcavities

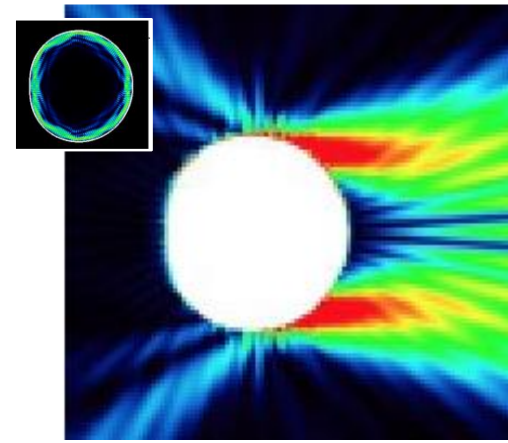
Deformed microcavity



C. Gmachl et al., Science (1998)



Deformed cavity micro lasers



J. Wiersig et al. PRL (2008)

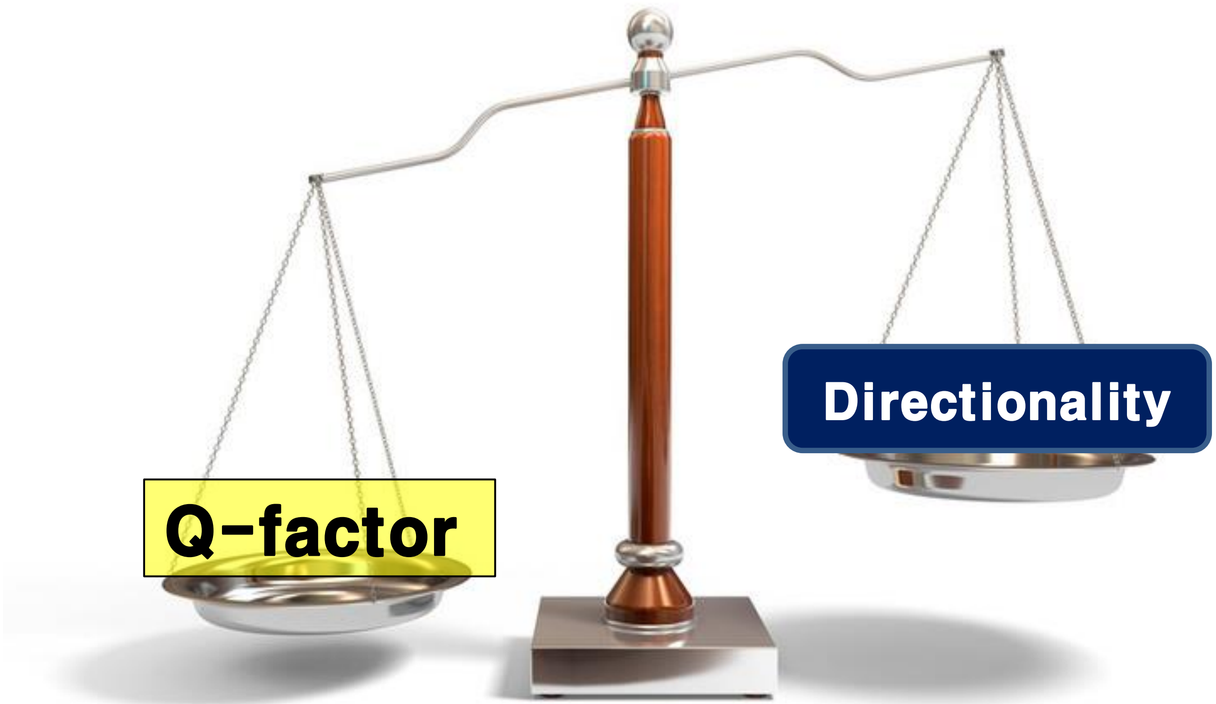
Attempts to obtain the directional emission are essentially based on breaking the rotational symmetry.

However, ultra high-Q WGM are based on the rotational symmetry.

Maintaining rotational symmetry → Ultra high-Q WGM

Breaking rotational symmetry → Directional emission

Trade-off between the cavity Q & emission directionality



Commonly believed knowledge

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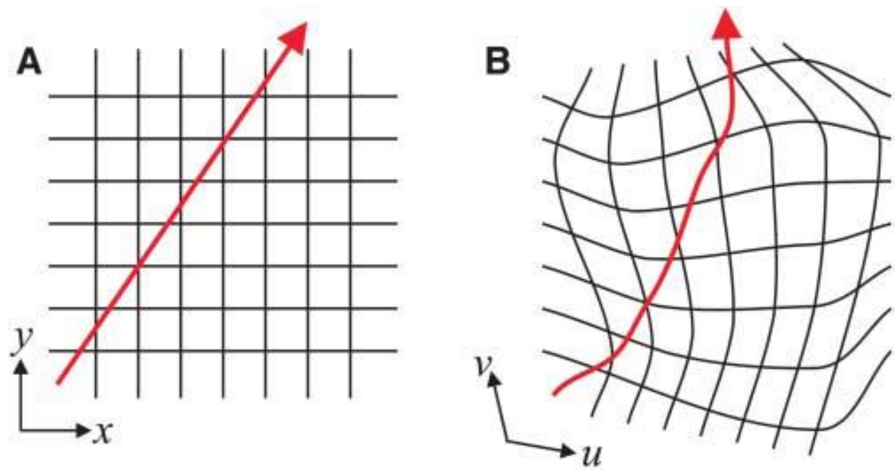
Transformation Optics

If we distort the coordinate system, $(x, y, z)\rightarrow(u,v,w)$, the ray trajectory of light should be distorted.
The transformed coordinate can be realized by the inhomogeneous distributions of refractive index.



The refractive index can be obtained from the coordinate transformation.

$$n' = n\, g(u,v,w)$$

U. Leonhardt, Science (2006)

J. B. Pendry et al., Science (2006)

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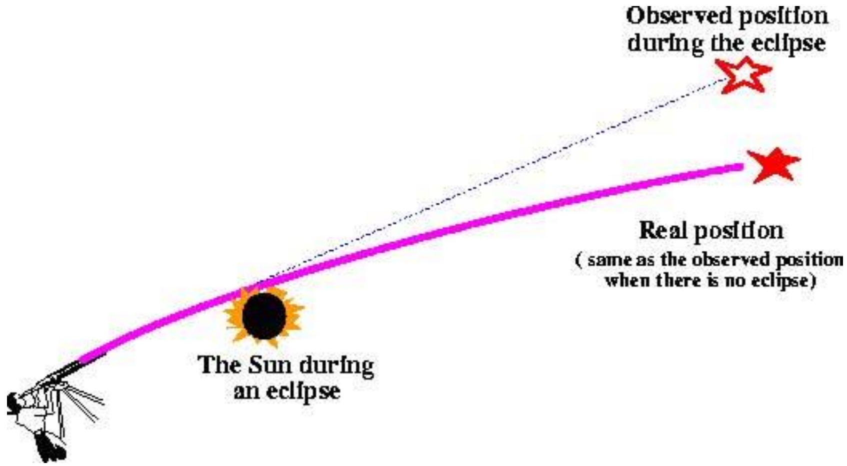
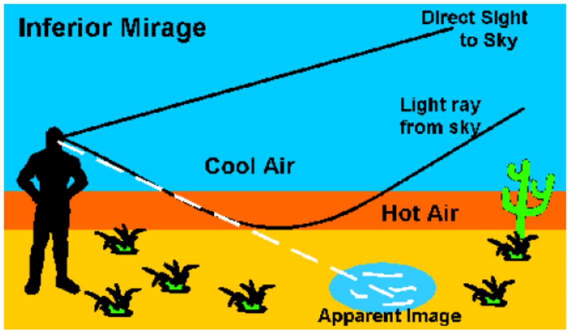
Transformation Optics

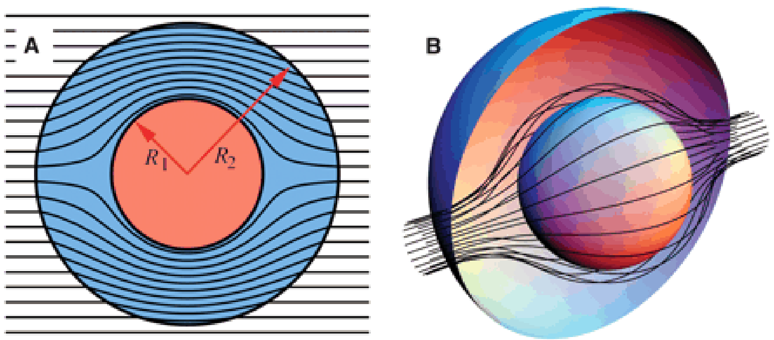
Einstein's general theory of relativity: gravity changes geometry. Therefore gravity should bend light.



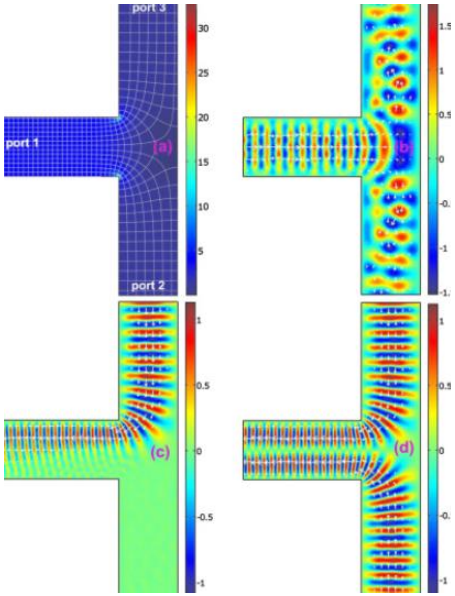


Transformation Optics

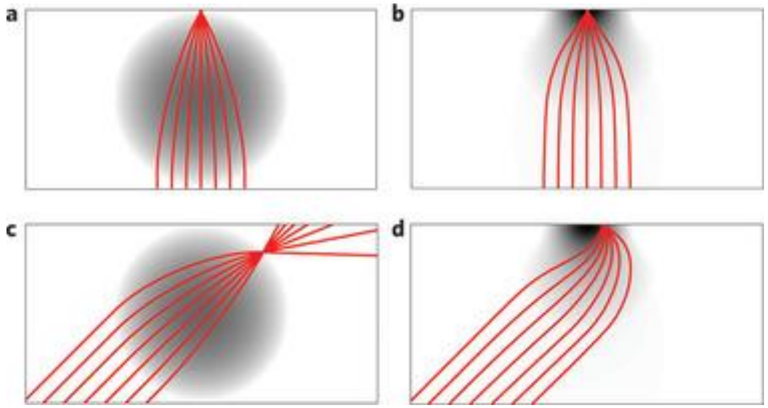
Optical cloaking - *J. B. Pendry et al., Science (2006)*



Waveguides

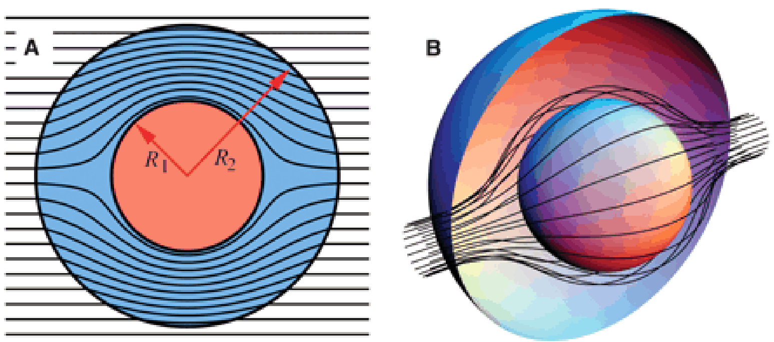


Flat luneburg lens - *N. Kundtz et al., Nat. Mater. (2010)*

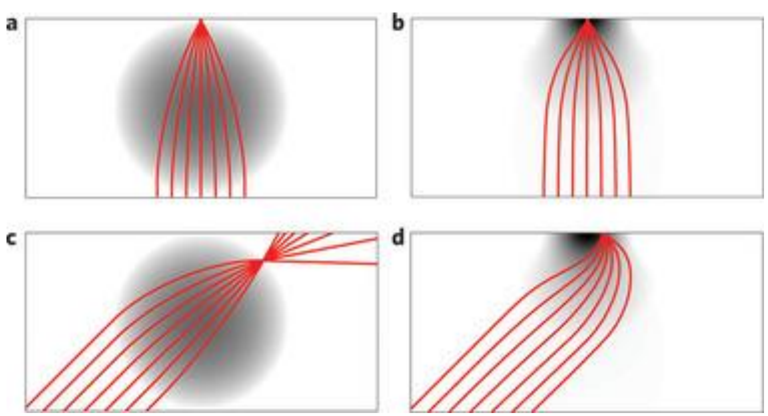


Transformation Optics

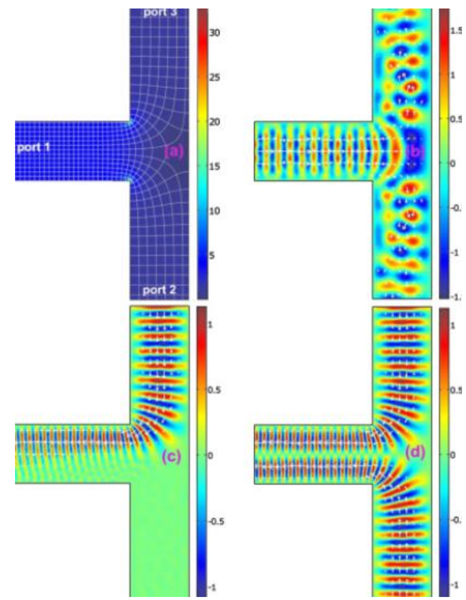
Optical cloaking - *J. B. Pendry et al., Science (2006)*



Flat luneburg lens - *N. Kundtz et al., Nat. Mater. (2010)*



Waveguides



Applications of TO have focused on the control of light propagation path.

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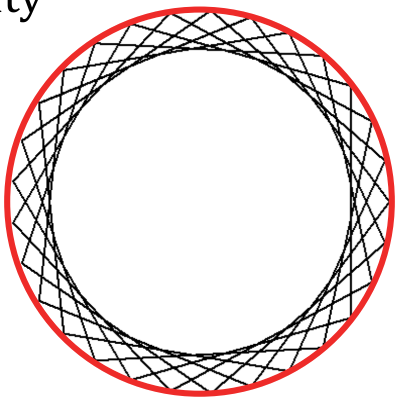
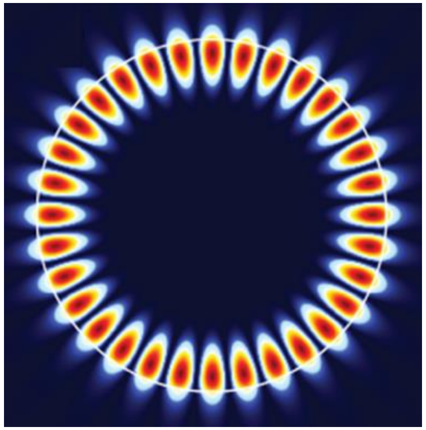
Deformed microcavity

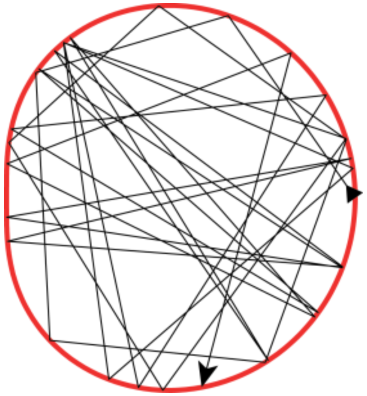
Disk cavity

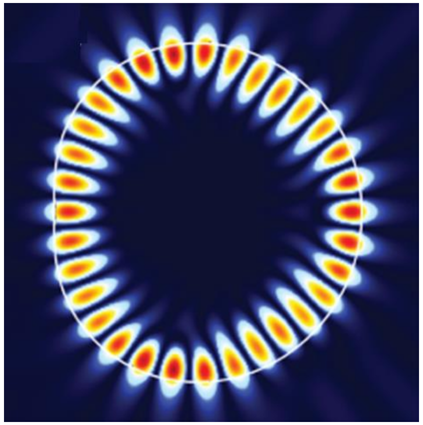




Whispering Gallery Mode (WGM)
supported by
Total internal reflection

Deformed cavity





Q-spoiling due to the breaking of rotational symmetry
- High-Q mode supported by localization

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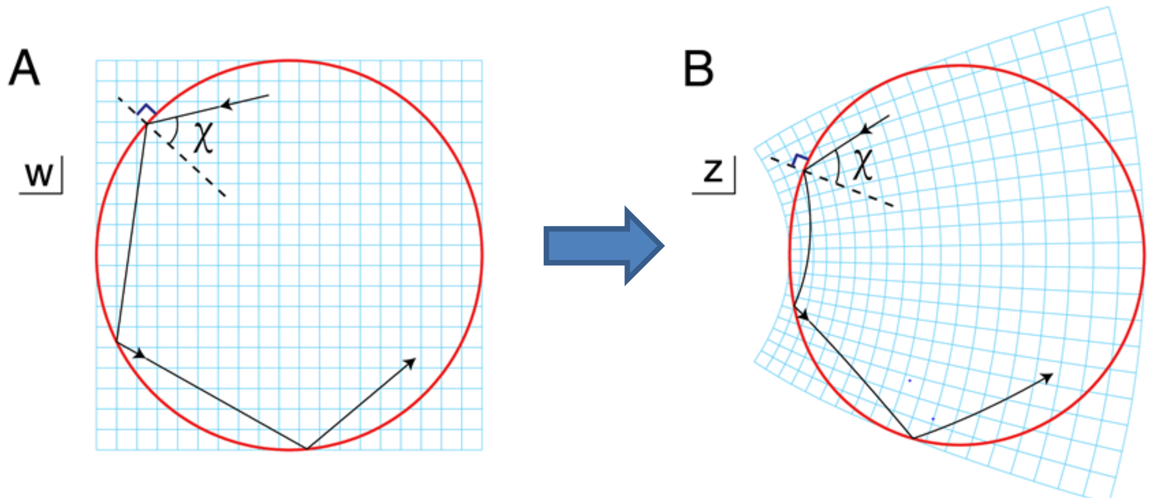
Conformal mapping

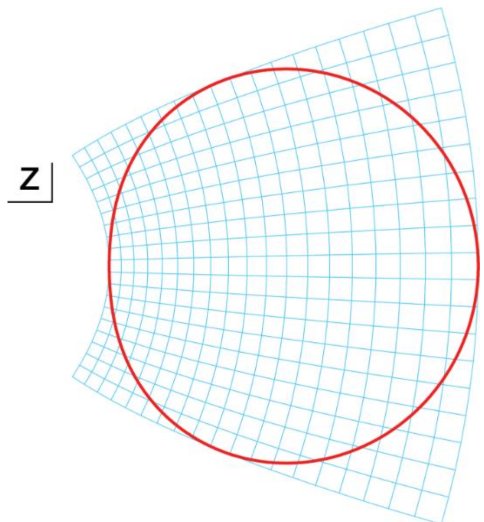


The corresponding conformal transformation which maps the unit circle to the limaçon boundary is given by a function of complex variable,

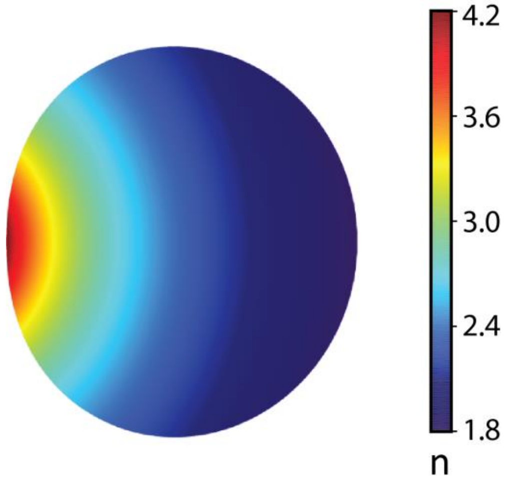
$$z = \beta(w + \alpha w^2) \qquad \text{where} \ \beta = 1/\sqrt{1 + 4\alpha(1 + \alpha)}$$

- **Transformation is conformal mapping**
- **Keeping total internal reflection condition at cavity boundary**

Refractive index profile obtained from transformation optics theory



Transformed coordinate & cavity boundary



Refractive index profile

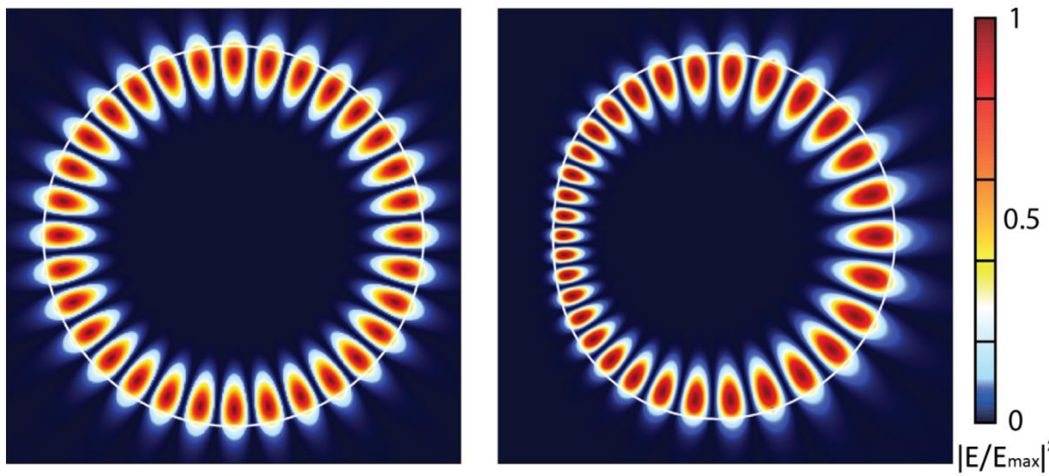
The transformed refractive index :

$$\frac{n(z)}{n_0} = \left|\frac{dw}{dz}\right| = \frac{1}{\beta\left|\sqrt{1+4\alpha z/\beta}\right|}$$

, where n_0=1.8

Set refractive index $n_{out} = 1$ outside the cavity

Restored High-Q WGM mode in the transformation cavity



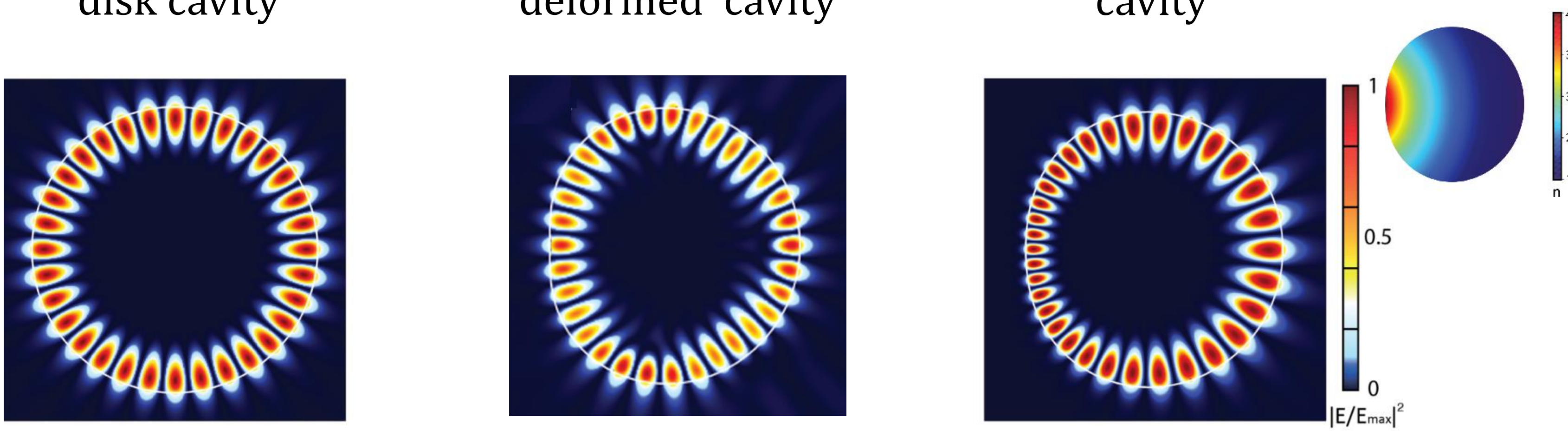
WGM in disk WGM in the transformation cavity

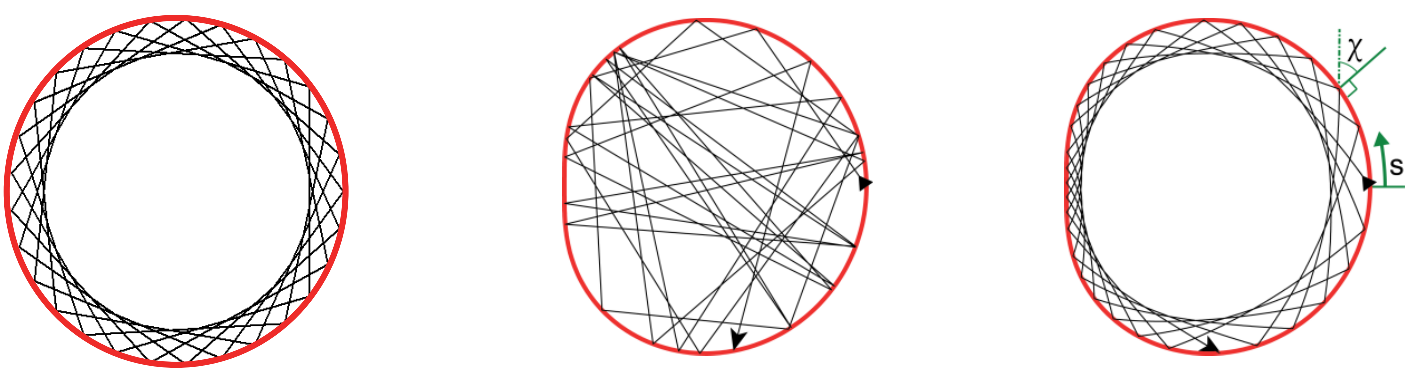
WGM in original disk and transformation limaçon shaped cavity :
- Resonance mode with azimuthal mode number m=16

In transformation limaçon cavity,
- approximately WGM transforms conformally (cWGM: conformal WGM)
- distance between adjacent nodes varies according to the refractive index profile

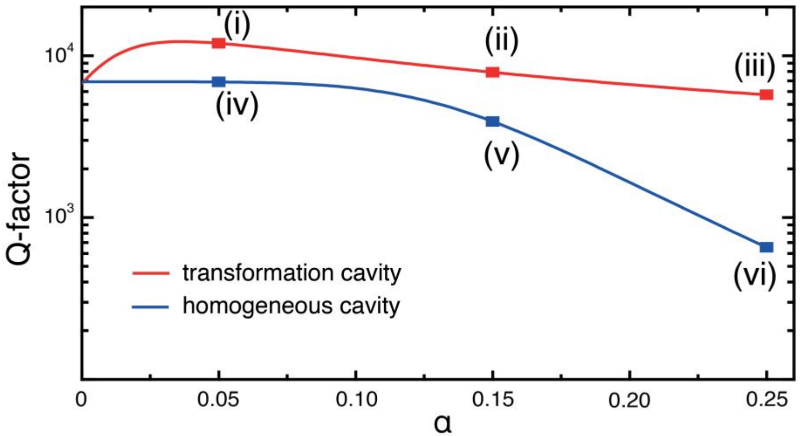
Restored High-Q WGM mode in the transformation cavity

Homogeneous disk cavity | Homogeneous deformed cavity | Transformation cavity



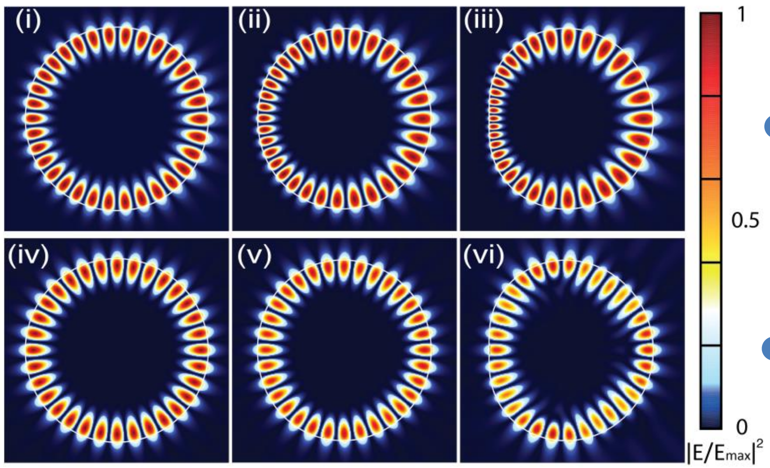


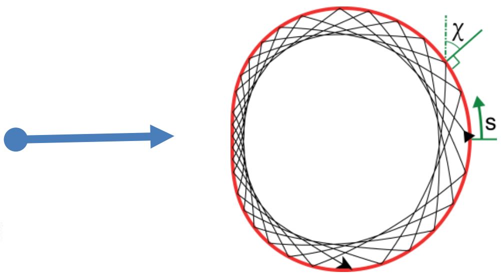
Q-factor variation with cavity deformation



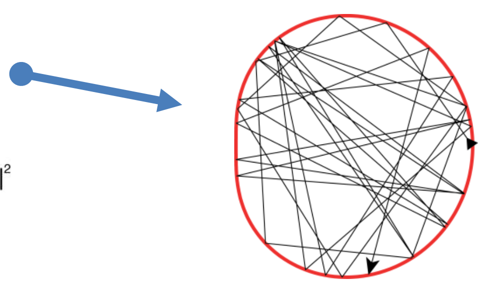
- Q factor of cWGM is comparable to that of WGM
- Q factor of homogeneous cavity decreases exponentially

- cWGM maintains WGM feature even in high deformation (α=0.25)
- Homogeneous cavity have polygon shaped mode in hig h deformation (α=0.25)





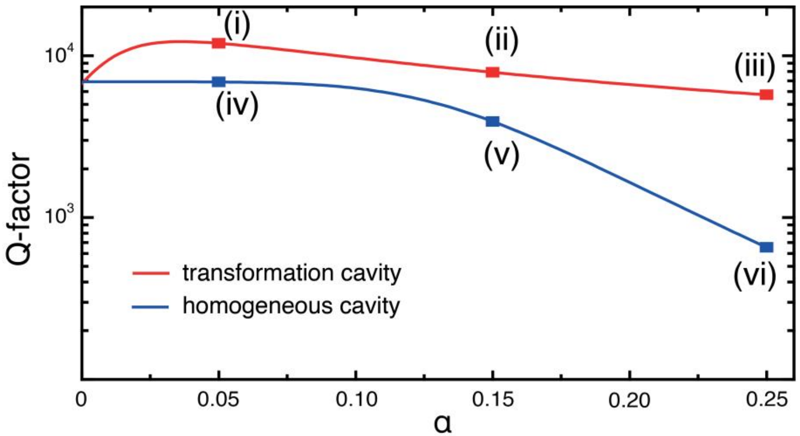
Transformation cavity

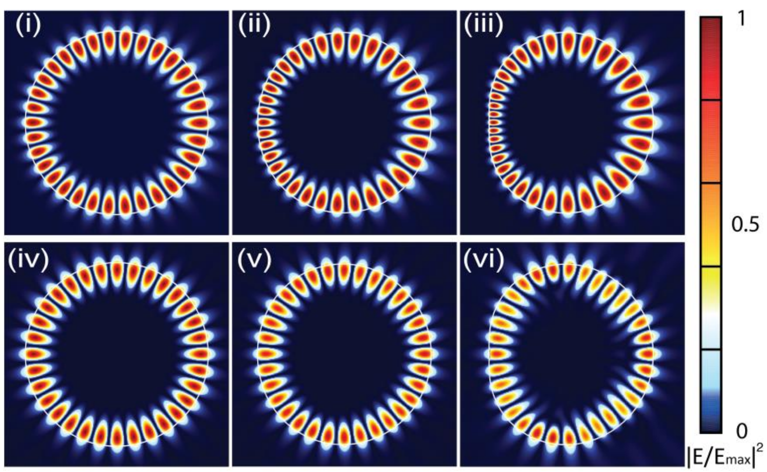


Homogeneous cavity

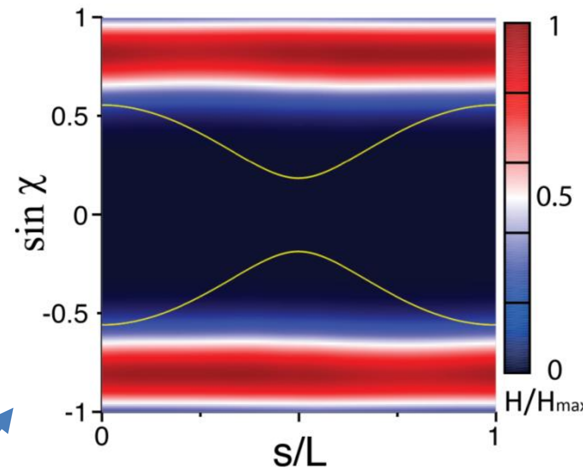
Q-factor as a function of and corresponding resonance modes

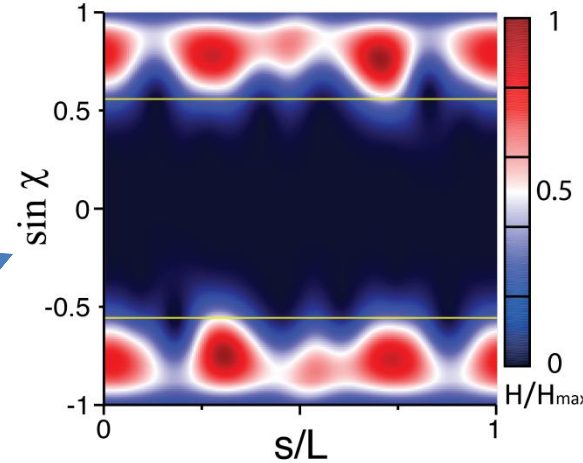
Husimi plot & emission route

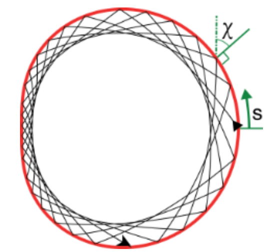


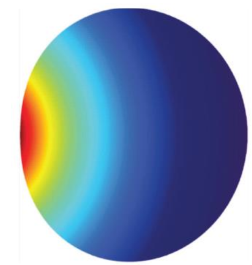


Q-factor as a function of α and corresponding resonance modes



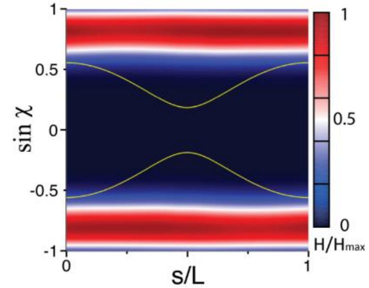


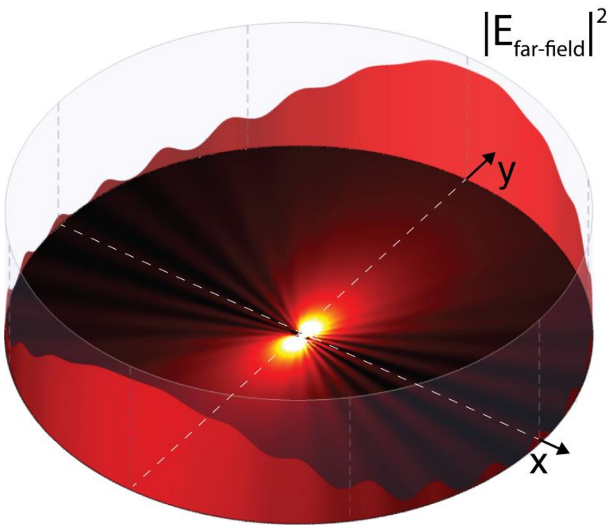




Phase space representation of cavity mode

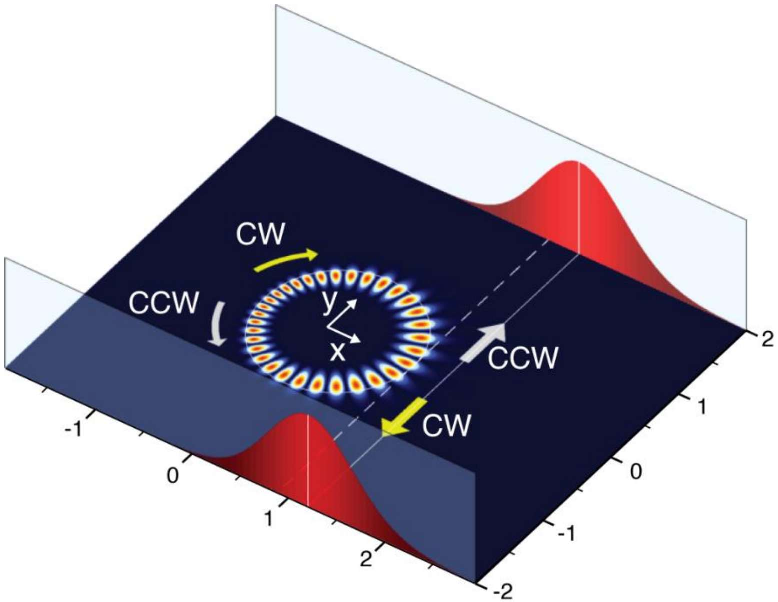
Far-field pattern of cWGM





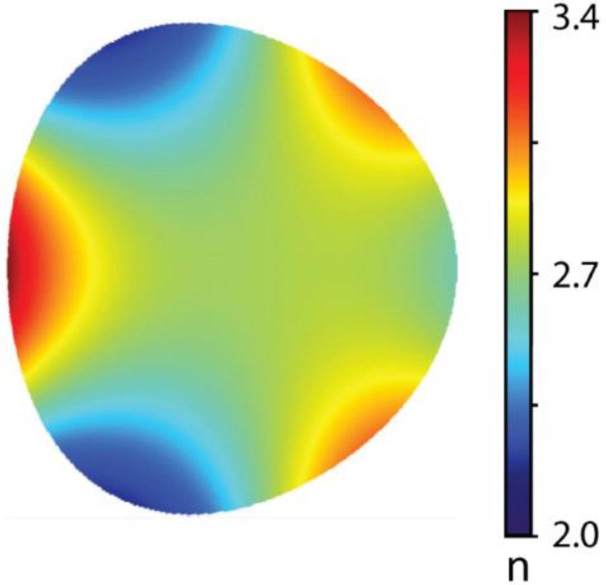
Far-field pattern:
- Bi-directional emission along y-axis

Near-field projection:
- Tunneling emission
(Peak from a point outside the cavity, not boundary)
- Two peak from CW and CCW components

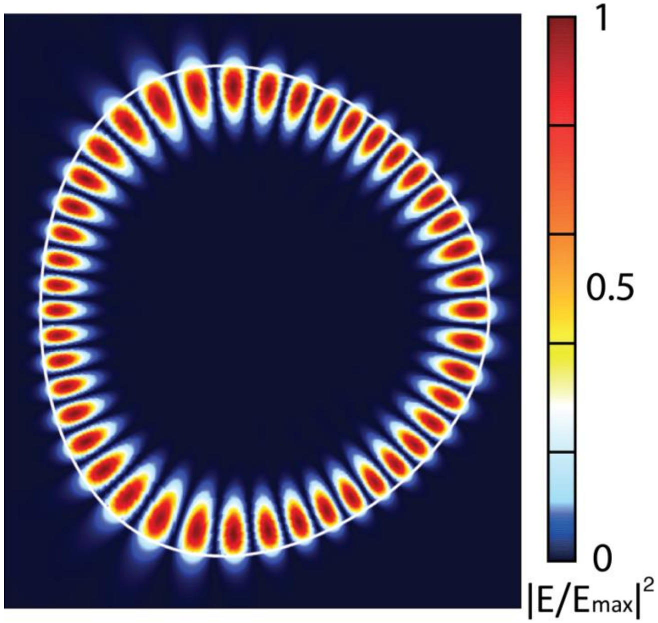


Field intensity on the projection plane perpendicular to far-field maximum directions

A design for unidirectional emission



Refractive index profile



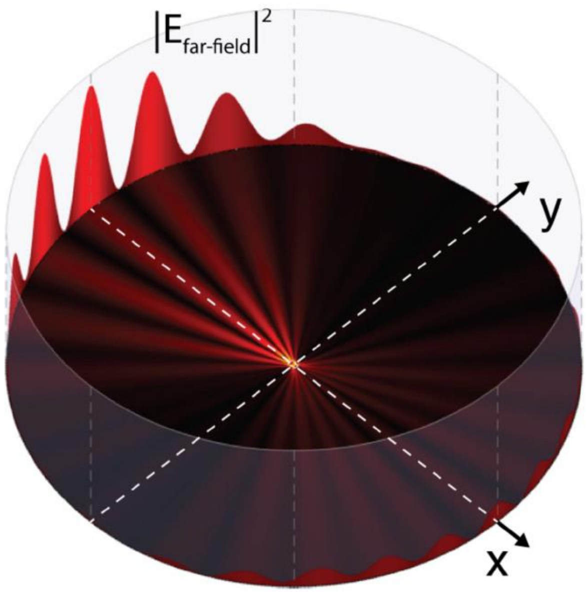
cWGM with unidirectional emission

Conformal mapping for triangular cavity with unidirectional emission:

$$z = f(w) = z_3 \circ z_2 \circ z_1(w)$$

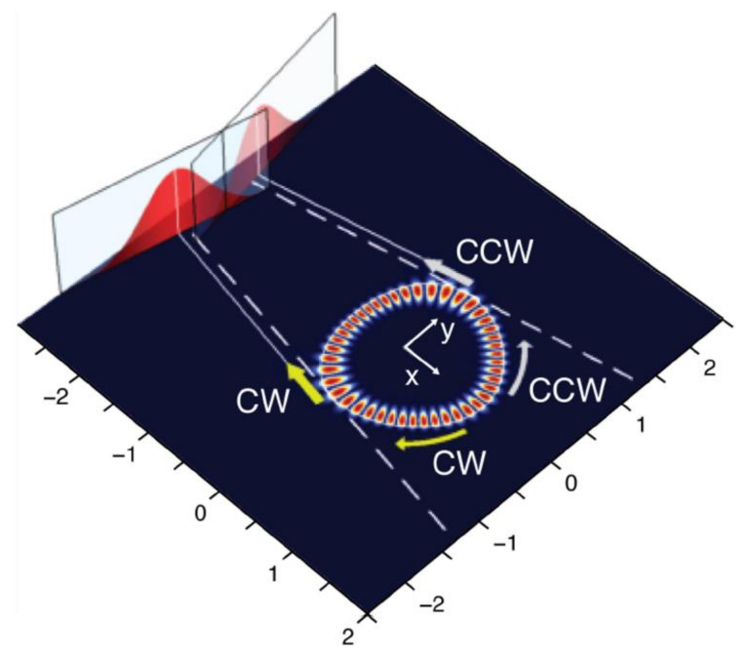
$$z_1(w) = \alpha\,\frac{w+\delta}{1+w\delta},\; z_2(w) = i\,\frac{1+w}{1-w},\; z_3(w) = \int_0^w e^{i\pi/6}(h+1)^{-2/3}(h-1)^{-2/3}dh$$

Far-field pattern of cWGM



Far-field pattern:
- Unidirectional emission along x-axis

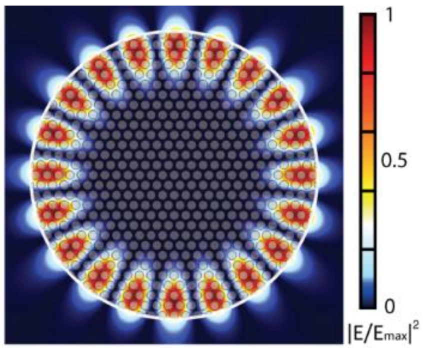
Near-field projection:
- Tunneling emission
(Peak from a point outside the cavity, not boundary)
- Two peak from CW and CCW components

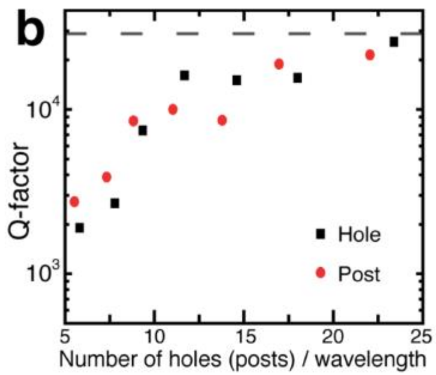


Field intensity on the projection plane perpendicular to far-field maximum directions

Feasibility for realization

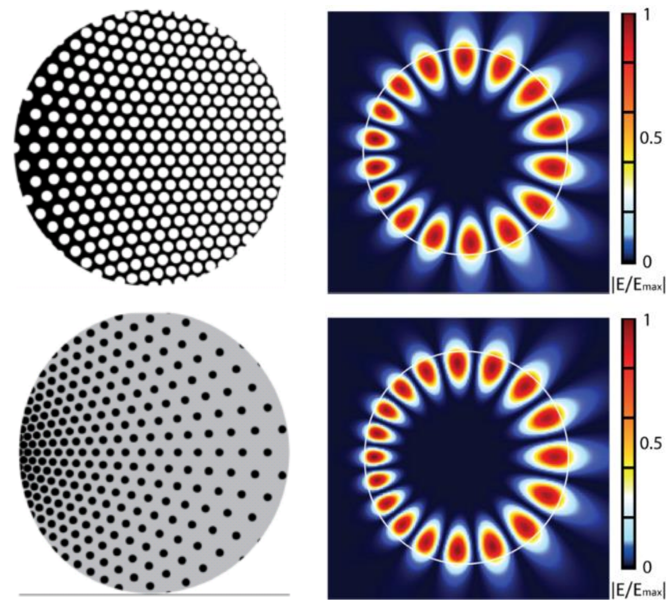
Q-factor convergence vs. Hole (Post) density





Q-factor in uniform hole (post) array (disk):
- Rapidly converges to the theoretical Q value
- Similar behavior for hole and post array

Conformal hole and post arrays (Limaçon):
- cWGM solutions are obtained
- Q is around 90% of continuous index cavity



Schematic view of hole and post arrays for transformation Limaçon cavity and its cWGM

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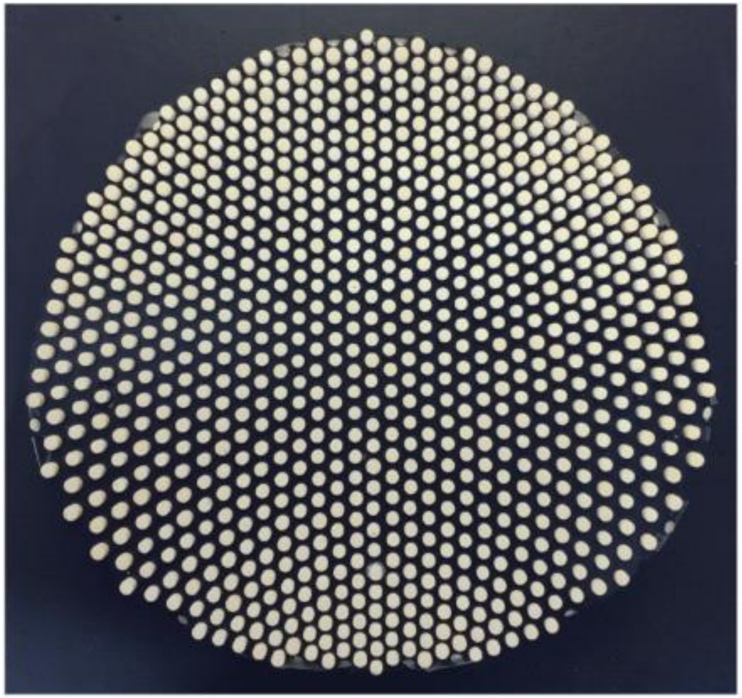
Experimental Demonstration

Sample

- triangular cavity with density controlled alumina posts
- alumina ($n \sim 3.1$): wide range of deformation
- posts are pinned on compressed PVC foam



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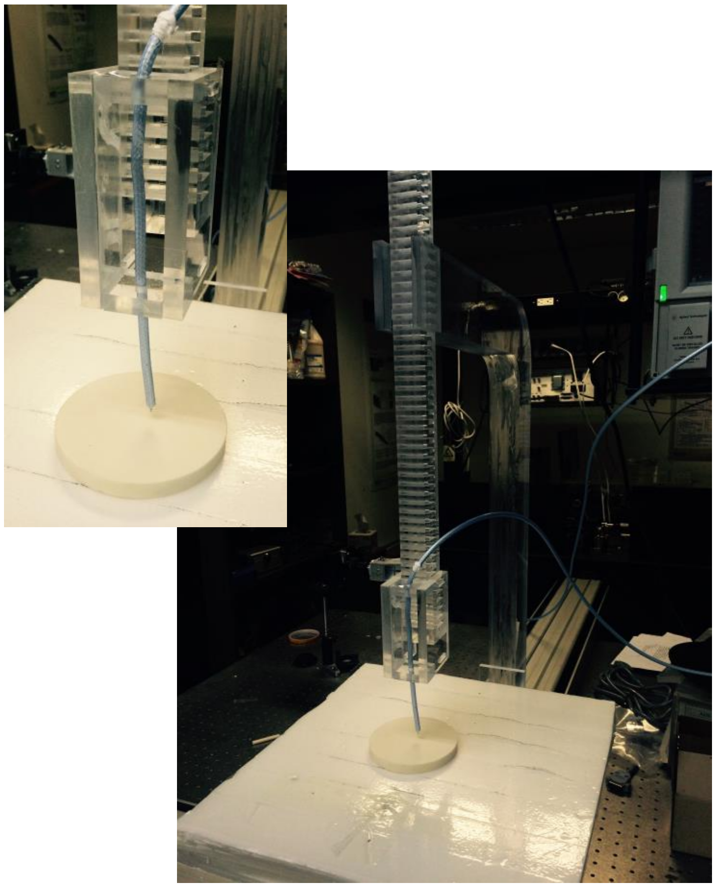
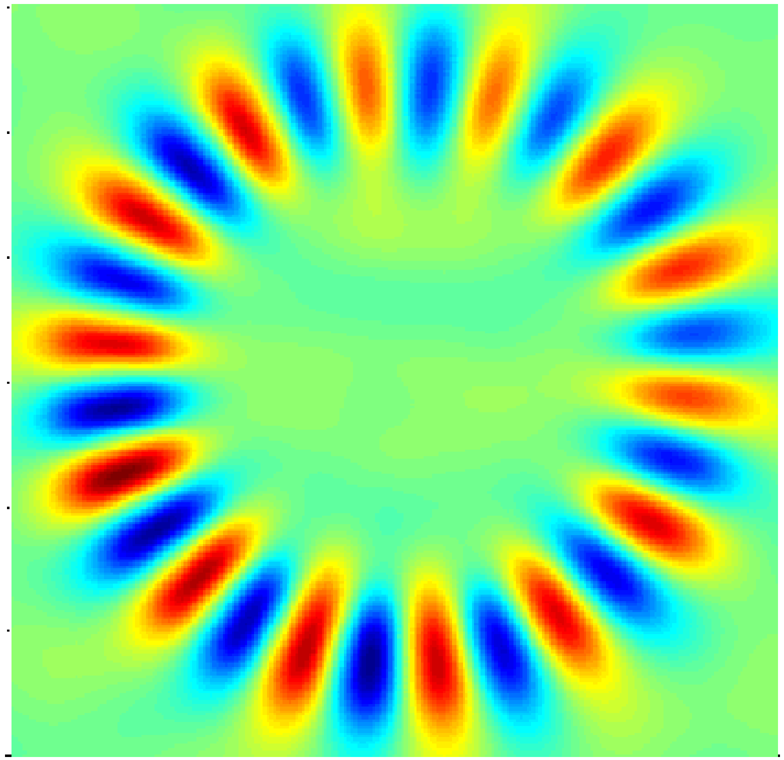
Experimental Demonstration



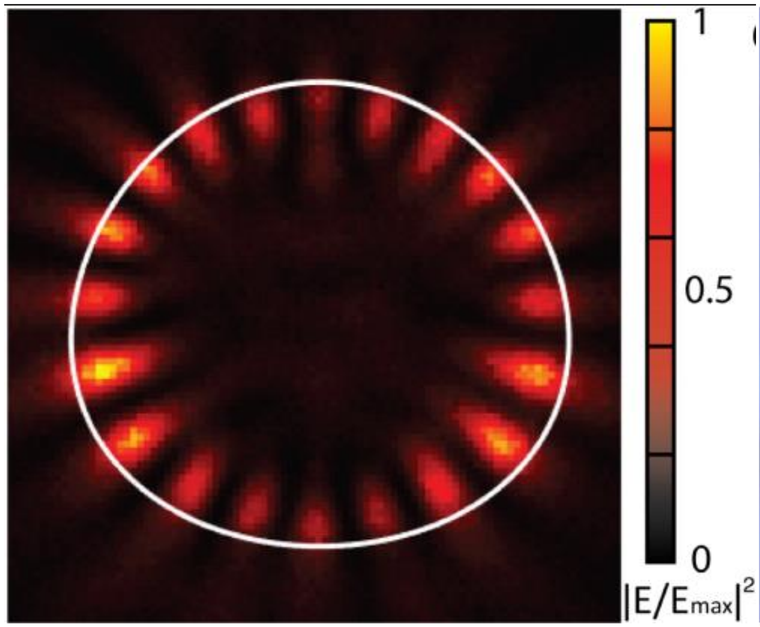


Microwave 2-d near-field scanning
- Direct mapping of 2-d mode pattern
- Monopole antenna as source and probe

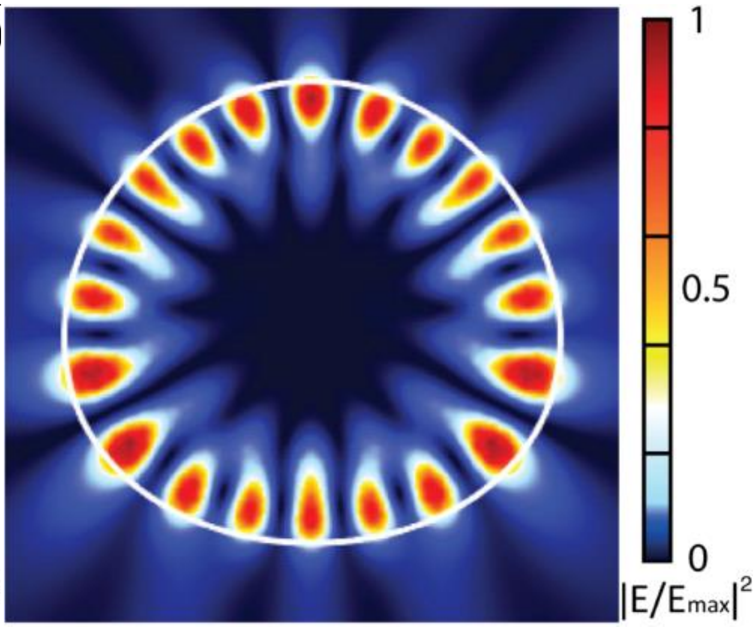
Measured WGM in alumina disc

Experimental Demonstration

Measured mode (2.648GHz)



Measured cWGM



Calculated cWGM

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Summary

- So far, most research efforts in transformation optics have been focused on the propagation path control of electromagnetic waves. In this work, we showed that it can be also useful in designing resonance mode properties.
- We proposed a new methodology based on transformation optics to manipul ate mode properties of dielectric cavity (Q-factor, emission directionality, chir ality of resonance modes, polarizations, mode couplings, etc.).
- Can be extended to control resonance modes of various waves (elastic wave, acoustic wave, etc.)
- Can be applicable to integrated optical circuits, bio-sensing devices with effective free-space optical coupling
- *Nature Photonics* **10**, 647 (2016)