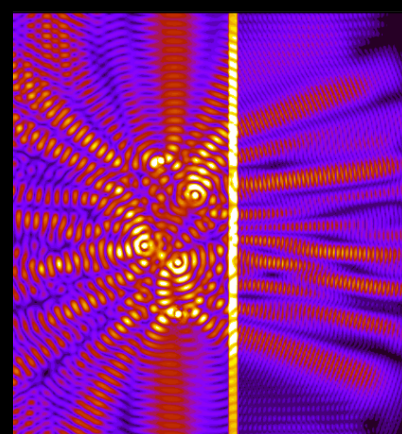
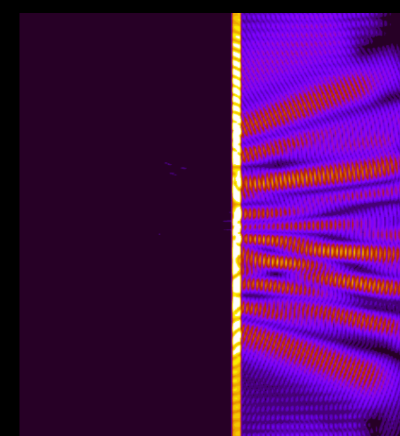
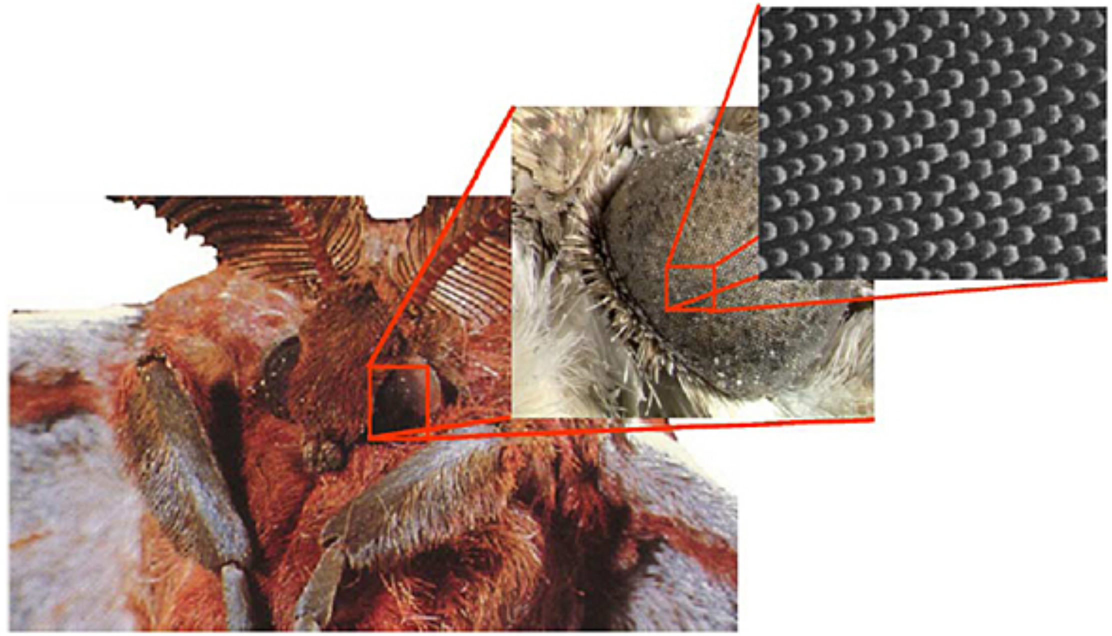
Univeral Impedance Matching
and non-local metamaterials

Q-Han Park
Korea Univ.





The story begins



- ***Bernhard***, ***C. G.***, "Structural and functional adaptation in a visual system", ***Endeavour 26***, ***79***–84 (***1967***). Some years ago, it was found that the cornea of nocturnal insects was covered with conical protuberance--nipples. They presumably affected the transmission of light,
- **P. Clapham** and **M. Hustley**, Reduction of Lens Reflexion by the "Moth eye" Principle, Naure 244, 281 (**1973**)

Gradual index change

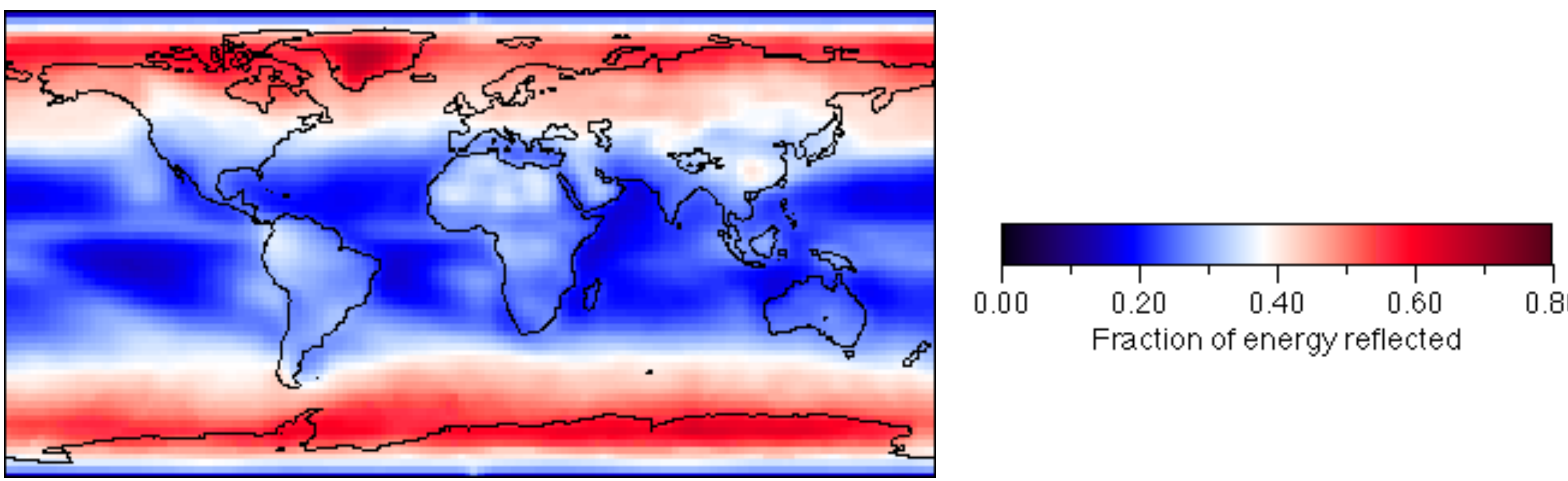
On Reflection of Vibrations at the Confines of two Media between which the Transition is Gradual. By Lord RAYLEIGH, F.R.S., Professor of Experimental Physics in the University of Cambridge.

[*Read February 12th*, 1880.]

$$\frac{d^2y}{dx^2}+n^2x^{-2}y=0 \qquad y=Ax^{\frac{1}{2}+im}+Bx^{\frac{1}{2}-im}.$$

$$m^2=n^2-\tfrac{1}{4}\ldots\ldots$$

Reflectivity of the Earth's surface (1987, PhysicalGeography.net)



Tarnished glass

L. Rayleigh, Proc. R. Soc. Lond. 41, 275–294 (1886).

IV. "On the Intensity of Light reflected from certain Surfaces at nearly Perpendicular Incidence." By Lord Rayleigh, M.A., D.C.L., Sec. R.S. Received October 6, 1886.

Prism (II), before Repolishing.

	Lord Rayleigh.		Mrs. Sidgwick.		Mr. Gordon.
Aug. 26. .	0·0349	. . . .	—	. . . .	0·0344
„ 27. .	0·0342	. . . .	0·0350	. . . .	—

Mean. . 0·0346.

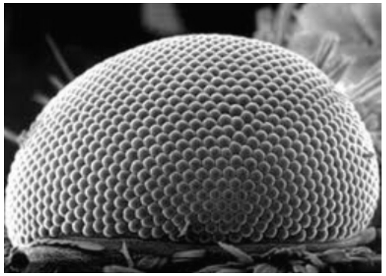
Prism (II), after Repolishing.

	Mrs. Sidgwick.		Lord Rayleigh.		Mr. Gordon.
Aug. 28, morn. . .	0·0491	. . . .	0·0488	. . . .	—
„ 28, aft.	—	. . . .	0·0479	. . . .	0·0473
„ 30, morn. . .	—	. . . .	0·0484	. . . .	0·0481

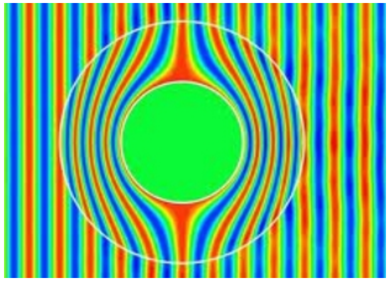
Mean. . 0·0483.

- 1886 Rayleigh
  - *discovered tarnished glass less reflective than new glass.*
- 1935 Alexander Smakula (Carl Zeiss)
  - *patented AR (T Transparenz) coating,*
  - *a military secret until about 1940*

- 1973 P. Clapham and M. Hustley, Naure
  - *Reduction of Lens Reflexion by the "Moth eye" Principle*



- 2006 J.B. Pendry, D.Schurig, D.R. Smith
  - *Invisibility cloaking*



- 2009 *high school science project*

Reduce unwanted reflections

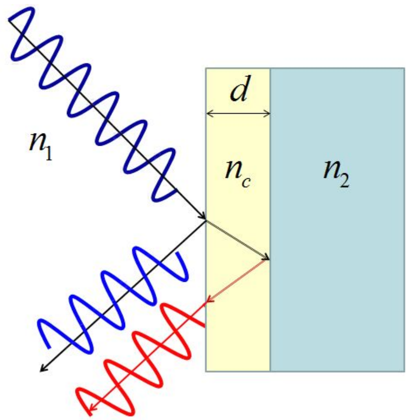
Antireflection coating

Fresnel, Rayleigh

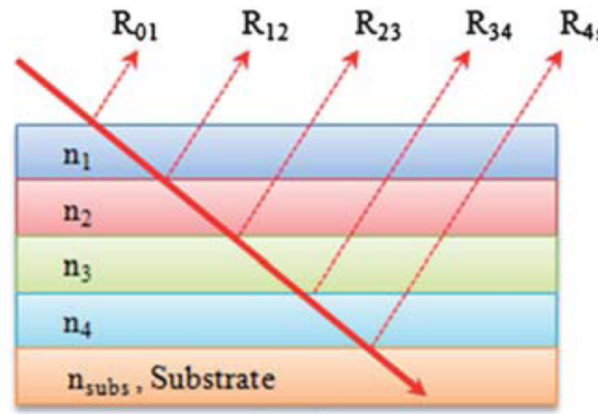
$$n_c = \sqrt{n_1 n_2}$$

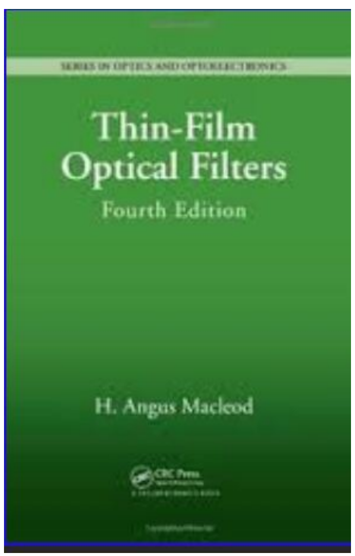
$$d = \lambda / 4n_c$$

monochromatic AR



broadband AR

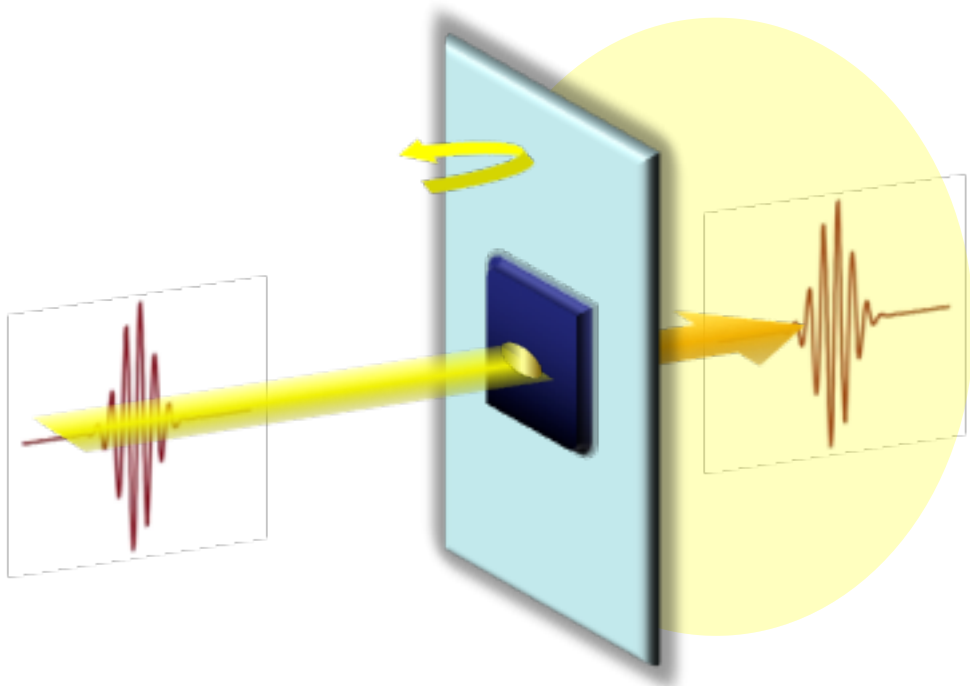




H. A. Macleod “Thin-film optical filters”

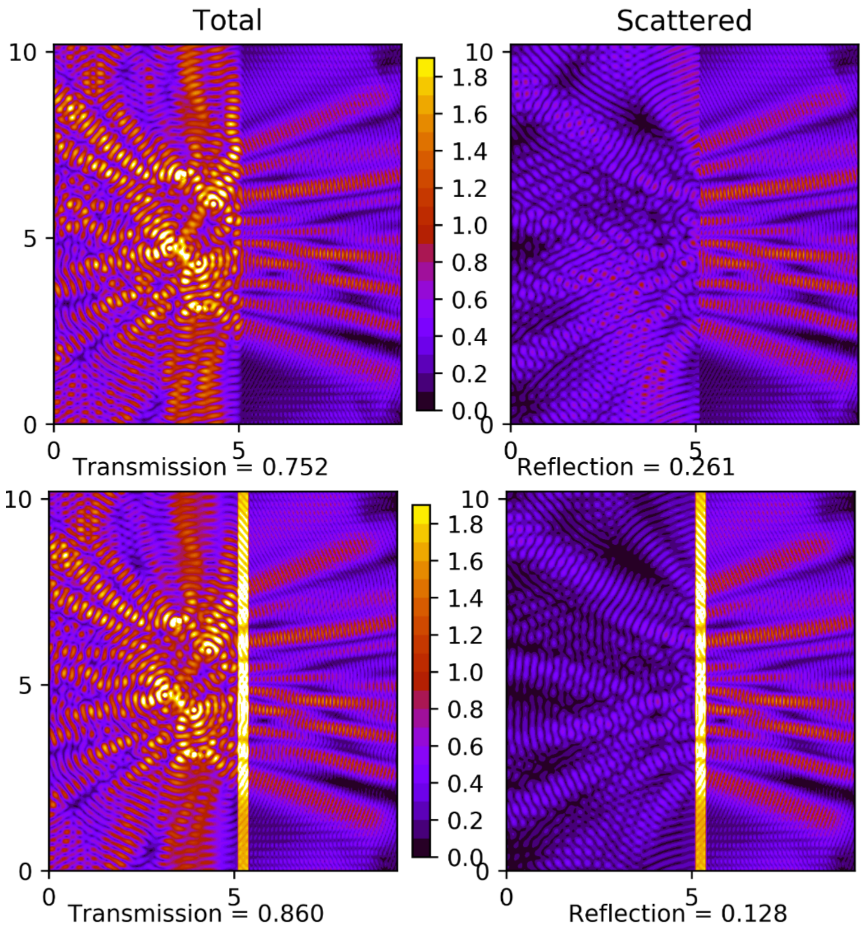
-- There is **no systematic method** for the design of antireflection coatings. **Trial and error**, assisted by approximate techniques backed up by accurate computer calculation, is frequently used.

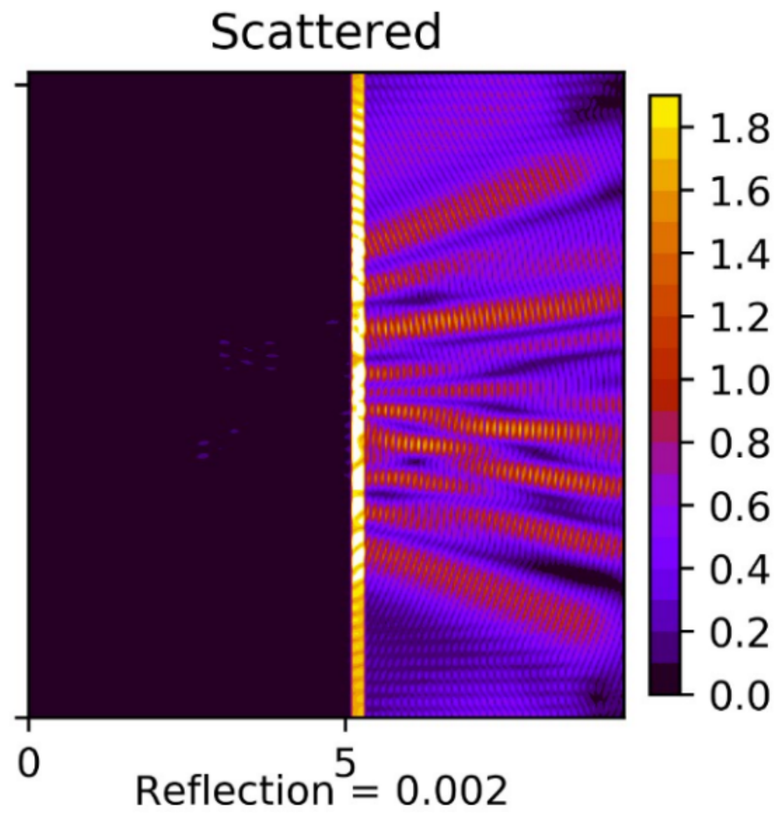
Principle of perfect anti-reflection ?



? AR for white light

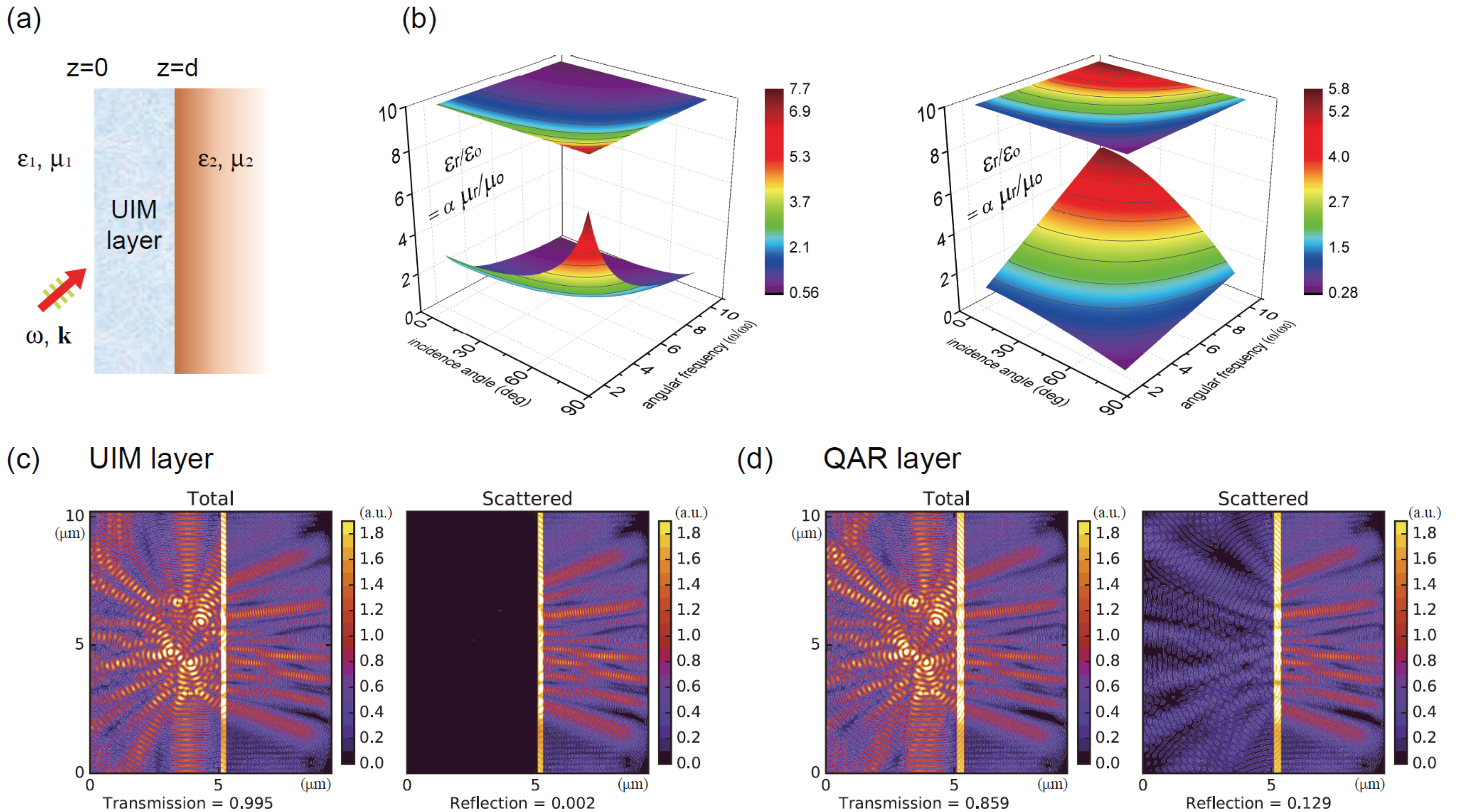
with random polarization, incidence angle, frequency





Universal impedance matching and the perfect transmission of white light

Ku Im, Ji-Hun Kang & Q-Han Park, Nature Photonics vol.12, 143 (2018)



Uniaxial & graded matching layer

$$\varepsilon_{xx} = \varepsilon_{yy} \equiv \varepsilon_r(z) \neq \varepsilon_{zz} \equiv \varepsilon_z(z), \ \mu_{xx} = \mu_{yy} \equiv \mu_r(z) \neq \mu_{zz} \equiv \mu_z(z).$$

EM fields

$$\vec{\mathrm{E}} = e^{ik_x x + ik_y y - i\omega t}\vec{E}(z), \ \vec{\mathrm{H}} = e^{ik_x x + ik_y y - i\omega t}\vec{H}(z)$$

Maxwell equation

$$\partial_z E_x = ik_x E_z + i\omega\mu_r H_y, \ \partial_z E_y = ik_y E_z - i\omega\mu_r H_x, \ ik_x E_y - ik_y E_x = i\omega\mu_z H_z,$$

$$\partial_z H_x = ik_x H_z - i\omega\varepsilon_r E_y, \ \partial_z H_y = ik_y H_z + i\omega\varepsilon_r E_x, \ ik_x H_y - ik_y H_x = -i\omega\varepsilon_z E_z.$$

New variables; potential, admittance/impedance functions

TM $$E_x = -k_x\Phi_E, \ E_y = -k_y\Phi_E, \ E_z = (k^2/\omega\varepsilon_z)Y\Phi_E, \ H_x = k_y Y\Phi_E, \ H_y = -k_x Y\Phi_E$$

TE $$E_x = k_y Z\Phi_H, \ E_y = -k_x Z\Phi_H, \ H_x = k_x\Phi_H, \ H_y = k_y\Phi_H, \ H_z = -(k^2/\omega\mu_z)Z\Phi_H$$

Inverse scattering; reduced Maxwell eq.

$$(a) \quad i\omega\varepsilon_r = \partial_z Y + (i\omega\mu_r - ik^2/\omega\varepsilon_z)Y^2 \qquad (b) \quad \partial_z \ln\Phi_E = (i\omega\mu_r - ik^2/\omega\varepsilon_z)Y$$

$$(a) \quad \omega\mu_r = \partial_z Z + (i\omega\varepsilon_r - ik^2/\omega\mu_z)Z^2 \qquad (b) \quad \partial_z \ln\Phi_H = (i\omega\varepsilon_r - ik^2/\omega\mu_z)Z$$

$$E_x^I = \frac{k_y}{k}\left(e^{ik_{1z}z} + re^{-ik_{1z}z}\right), \qquad E_y^I = -\frac{k_x}{k}\left(e^{ik_{1z}z} + re^{-ik_{1z}z}\right), \qquad E_z^I = 0,$$

$$E_x^{II} = k_y\left(AZ\Phi + B\bar{Z}\bar{\Phi}\right), \qquad E_y^{II} = -k_x\left(AZ\Phi + B\bar{Z}\bar{\Phi}\right), \qquad E_z^{II} = 0,$$

$$E_x^{III} = \frac{k_y}{k} te^{ik_{2z}z}, \qquad E_y^{III} = -\frac{k_x}{k} te^{ik_{2z}z}, \qquad E_z^{III} = 0,$$

$$r = \frac{\bar{\Phi}(0)\Phi(d)[Y(d) - Y_2][\bar{Y}(0) + Y_1] - \bar{\Phi}(\mathrm{d})\Phi(0)[Y(0) - Y_1][\bar{Y}(\mathrm{d}) + Y_2]}{\bar{\Phi}(0)\Phi(d)[Y(d) - Y_2][\bar{Y}(0) - Y_1] - \bar{\Phi}(\mathrm{d})\Phi(0)[Y(0) + Y_1][\bar{Y}(\mathrm{d}) + Y_2]}$$

$$Y_1 = \frac{\varepsilon_1\omega}{k_{1z}} = \frac{\varepsilon_1\omega}{\sqrt{\varepsilon_1\mu_1\omega^2 - k^2}}, \quad Y_2 = \frac{\varepsilon_2\omega}{k_{2z}} = \frac{\varepsilon_2\omega}{\sqrt{\varepsilon_2\mu_2\omega^2 - k^2}}.$$

$$r = \frac{\bar{\Phi}(0)\Phi(d)[\mathrm{Z}(d) - Z_2][\bar{\mathrm{Z}}(0) + Z_1] - \bar{\Phi}(\mathrm{d})\Phi(0)[\mathrm{Z}(0) - Z_1][\bar{\mathrm{Z}}(\mathrm{d}) + Z_2]}{\bar{\Phi}(0)\Phi(d)[\mathrm{Z}(d) - Z_2][\bar{\mathrm{Z}}(0) - Z_1] - \bar{\Phi}(\mathrm{d})\Phi(0)[\mathrm{Z}(0) + Z_1][\bar{\mathrm{Z}}(\mathrm{d}) + Z_2]}$$

$$\mathrm{Z}(0) = Z_1 = \frac{\mu_1\omega}{k_{1z}} = \frac{\mu_1\omega}{\sqrt{\varepsilon_1\mu_1\omega^2 - k^2}}, \quad \mathrm{Z}(\mathrm{d}) = Z_2 = \frac{\mu_2\omega}{k_{2z}} = \frac{\mu_2\omega}{\sqrt{\varepsilon_2\mu_2\omega^2 - k^2}}.$$

Inverse Scattering:

$$\varepsilon_r = \frac{1}{\omega}\partial_z Y_I^M + \frac{\partial_z Y_R^M[(Y_R^M)^2 - (Y_I^M)^2]}{2\omega Y_R^M Y_I^M}, \qquad \varepsilon_z = \frac{2k^2}{\omega}\left[\frac{\partial_z Y_R^E}{Y_R^E Y_I^E} - \frac{\partial_z Y_R^M}{Y_R^M Y_I^M}\right]^{-1},$$

$$\mu_r = \frac{\partial_z Y_R^E}{2\omega Y_R^E Y_I^E}, \qquad \mu_z = \frac{k^2}{\omega^2}\left[\varepsilon_r - \frac{1}{\omega}\partial_z Y_I^E - \frac{\partial_z Y_R^E[(Y_R^E)^2 - (Y_I^E)^2]}{2\omega Y_R^E Y_I^E}\right]^{-1}.$$

Boundary Conditions:

i) $$Y_R^M(0) = \frac{1}{R_1}\sqrt{\frac{\varepsilon_1}{\mu_1}},\ Y_R^M(d) = \frac{1}{R_2}\sqrt{\frac{\varepsilon_2}{\mu_2}},\ Y_R^E(0) = R_1\sqrt{\frac{\varepsilon_1}{\mu_1}},\ Y_R^E(d) = R_2\sqrt{\frac{\varepsilon_2}{\mu_2}},$$

$$R_1 \equiv \sqrt{1 - k^2/\varepsilon_1\mu_1\omega^2},\ R_2 \equiv \sqrt{1 - k^2/\varepsilon_2\mu_2\omega^2}$$

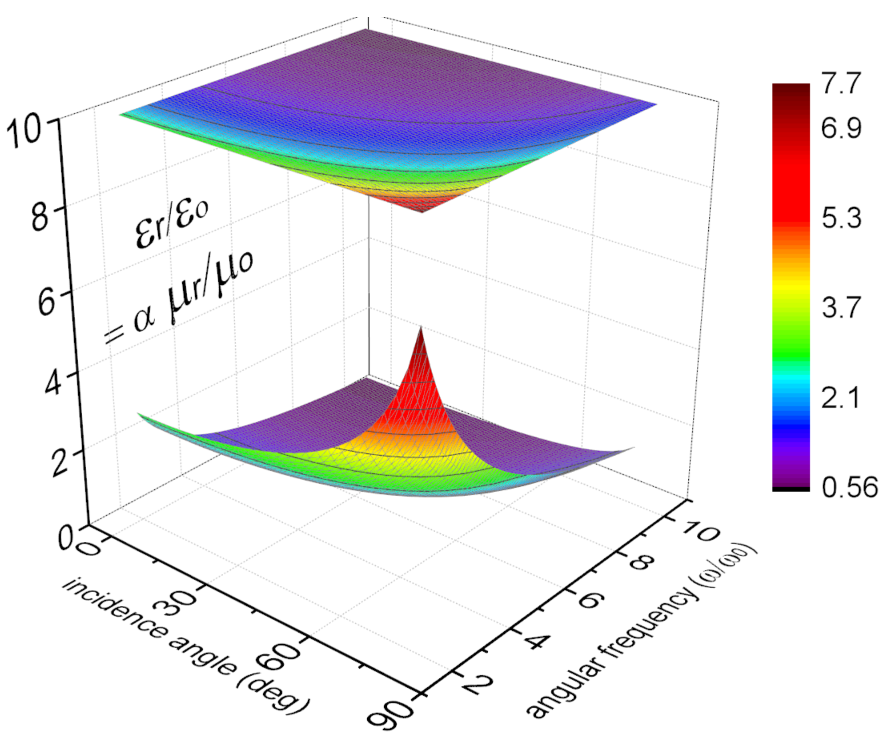
ii) $$Y_I^M(0) = Y_I^M(d) = Y_I^E(0) = Y_I^E(d) = 0$$

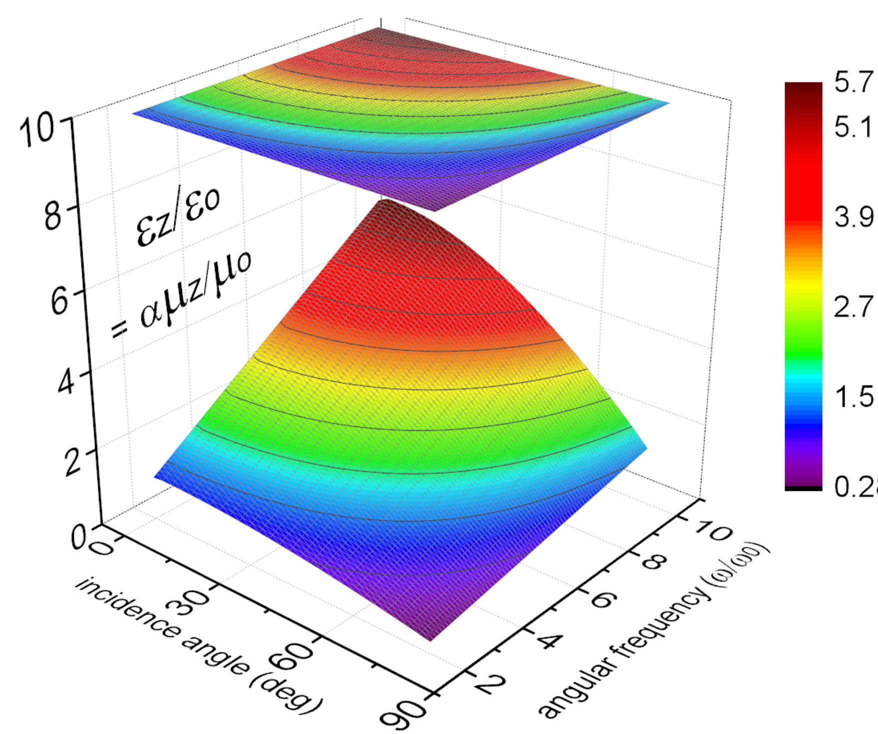
iii) $$Y^M\big|_{\vec{k}=0} = Y^E\big|_{\vec{k}=0}.$$

Universal Impedance Matching layer

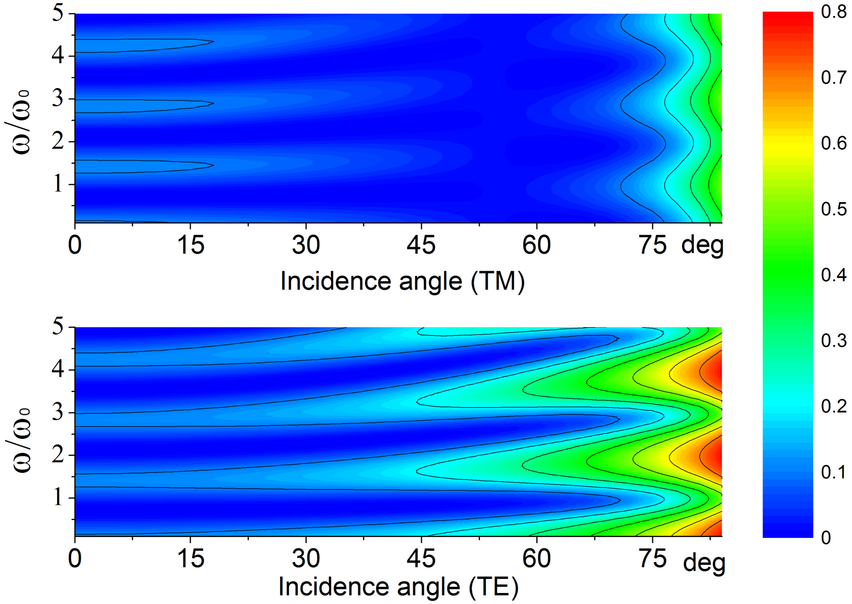
$$\varepsilon_r = \frac{\pi\langle Y\rangle}{2\omega d\langle R\rangle},\quad \mu_r = \frac{\pi}{2\omega d\langle Y\rangle\langle R\rangle},\quad \frac{k^2}{\omega^2\varepsilon_z} = \frac{\pi}{2\omega d\langle Y\rangle}\left(\frac{1}{\langle R\rangle}-\langle R\rangle\right),\quad \frac{k^2}{\omega^2\mu_z} = \frac{\pi\langle Y\rangle}{2\omega d}\left(\frac{1}{\langle R\rangle}-\langle R\rangle\right),$$

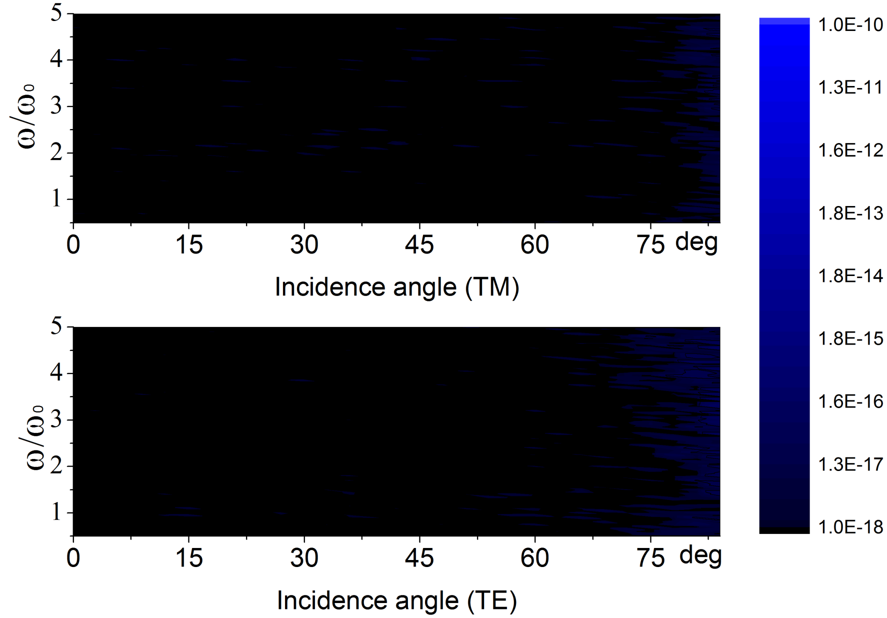
$$\langle Y\rangle \equiv \sqrt{Y_1(0)Y_2(0)},\ \langle R\rangle \equiv \sqrt{\cos\theta_1\cos\theta_2},\quad Y_n(0) = \sqrt{\frac{\varepsilon_n}{\mu_n}},\ \cos\theta_n = \sqrt{1-\frac{k^2}{\varepsilon_n\mu_n\omega^2}};\ n=1,2.$$

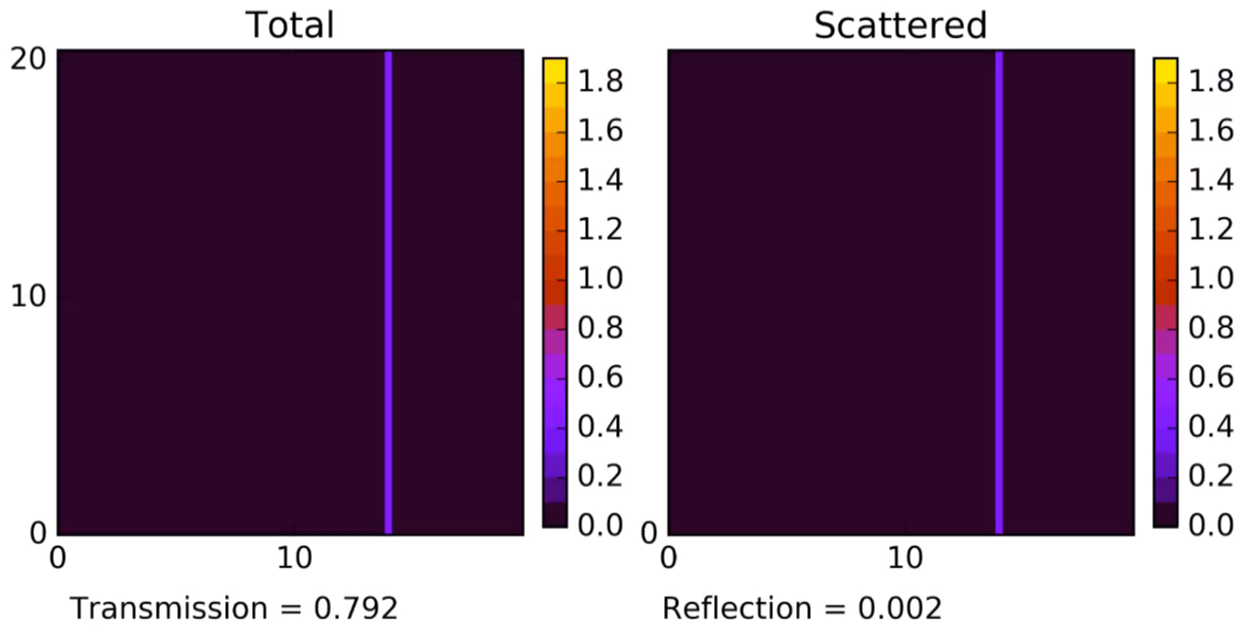


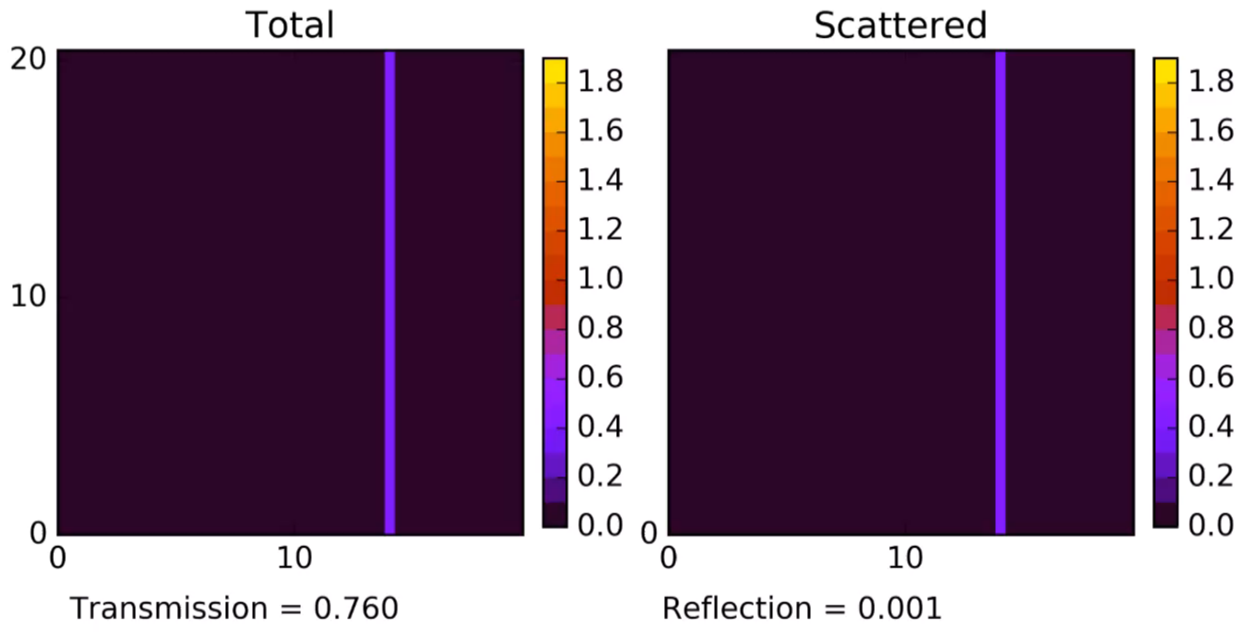


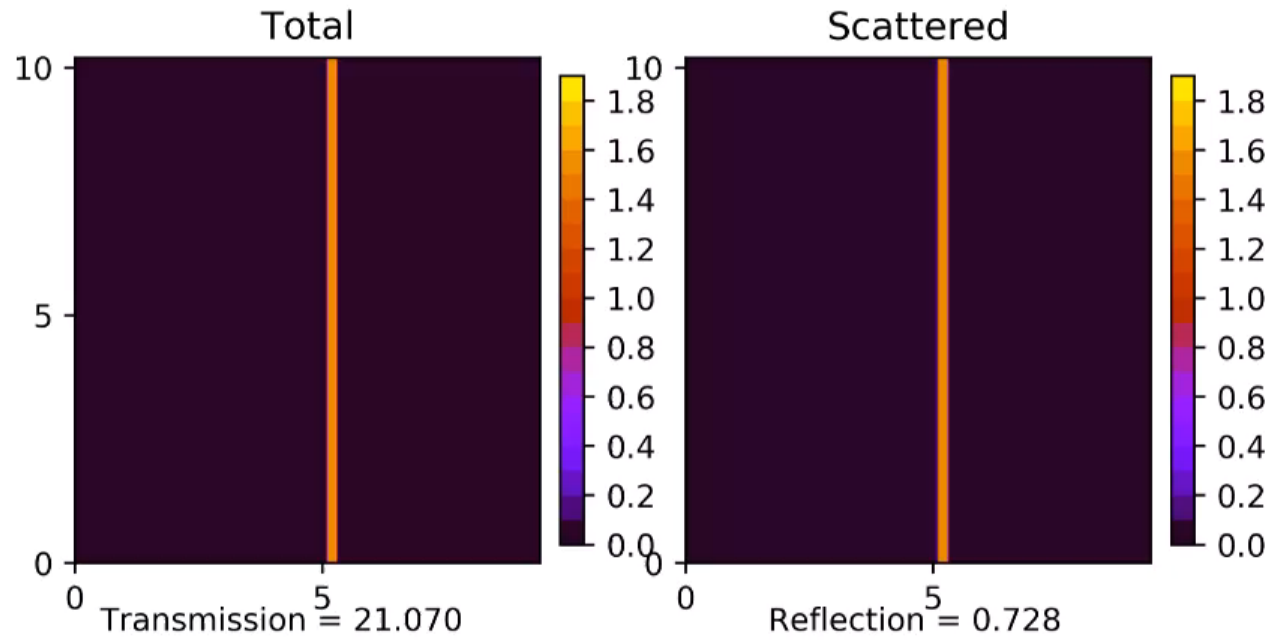
Reflectance (QAR vs. UIM)






Total
Scattered
20
10
0
0
10
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Transmission = 0.792
Reflection = 0.002


Total
Scattered
20
10
0
0
10
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Transmission = 0.760
Reflection = 0.001

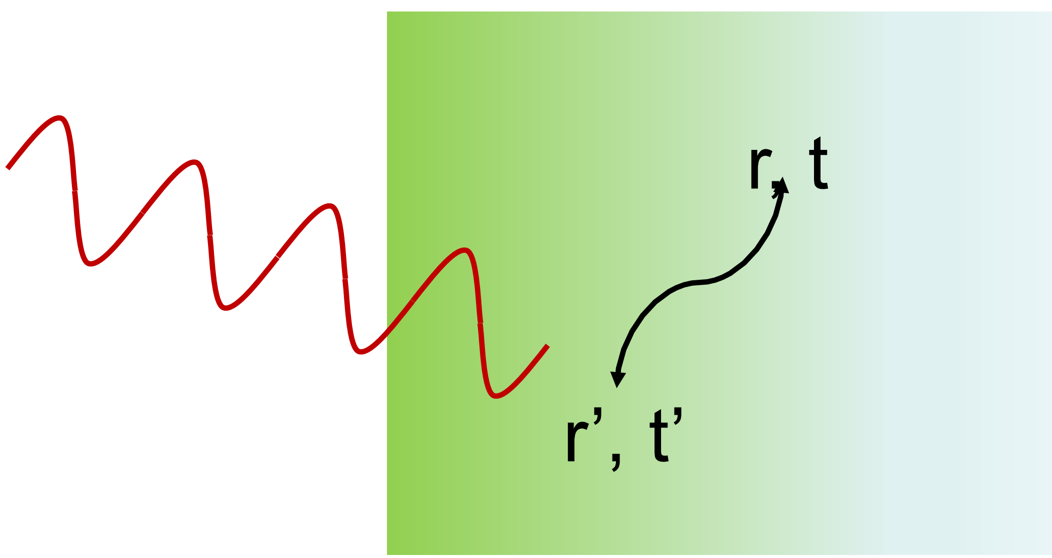

Total
Scattered
10
5
0
0
5
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Transmission = 21.070
Reflection = 0.728

UIML requires spatial & temporal dispersion

$$P_i\left(\vec{k},\omega\right)=\left[\varepsilon_{ij}\left(\vec{k},\omega\right)-\varepsilon_0\delta_{ij}\right]E_j\left(\vec{k},\omega\right)$$

⇨ Non-local in space and time

$$P_i\left(\vec{r},t\right)=\iint\chi_{ij}\left(\vec{r}-\vec{r}',t-t'\right)E_j(\vec{r}',t')d^3r'dt'$$



UIML

$$\varepsilon(k,\omega),\ \ \mu(\mathrm{k},\omega)$$

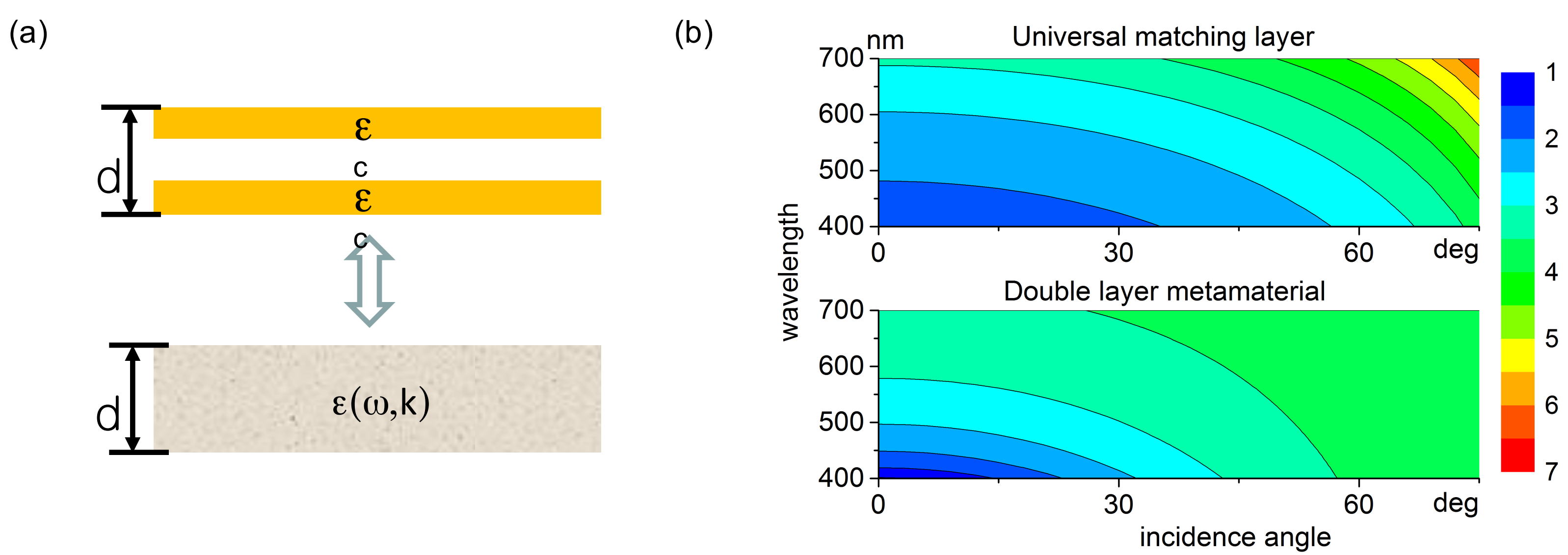
Can we make it ?

All metamaterials are non-local !

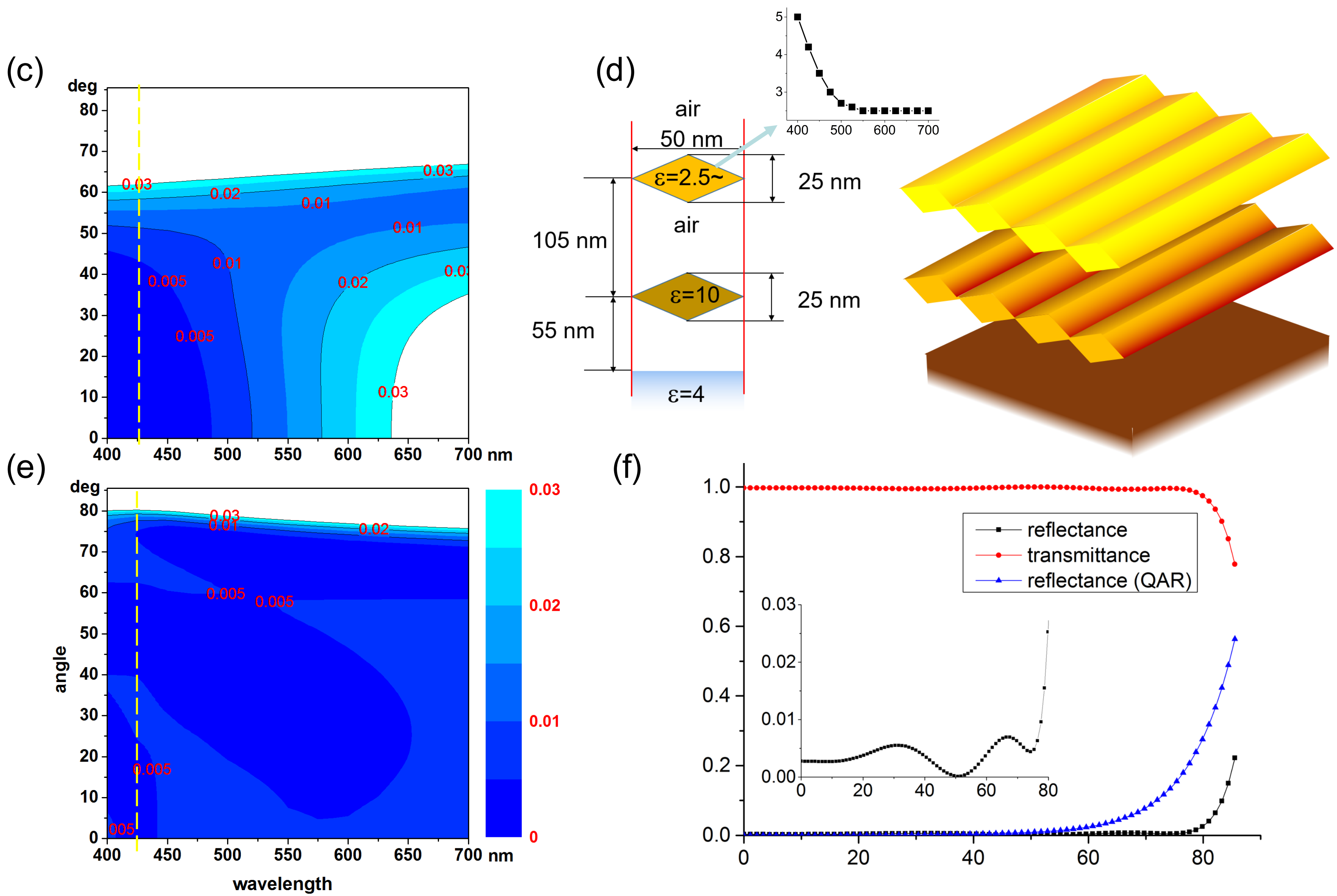
NRW parameter retrieval

$$\varepsilon_r = -\frac{k_z \varepsilon_0 (G+1)}{k_{0z}(G-1)}, \quad k_z = \sqrt{\varepsilon_r \mu_r \omega^2 - \frac{\varepsilon_r}{\varepsilon_z} k^2} = -\frac{i}{d} \ln\left[\frac{S_{11}+S_{21}-G}{1-G(S_{11}+S_{21})}\right]$$

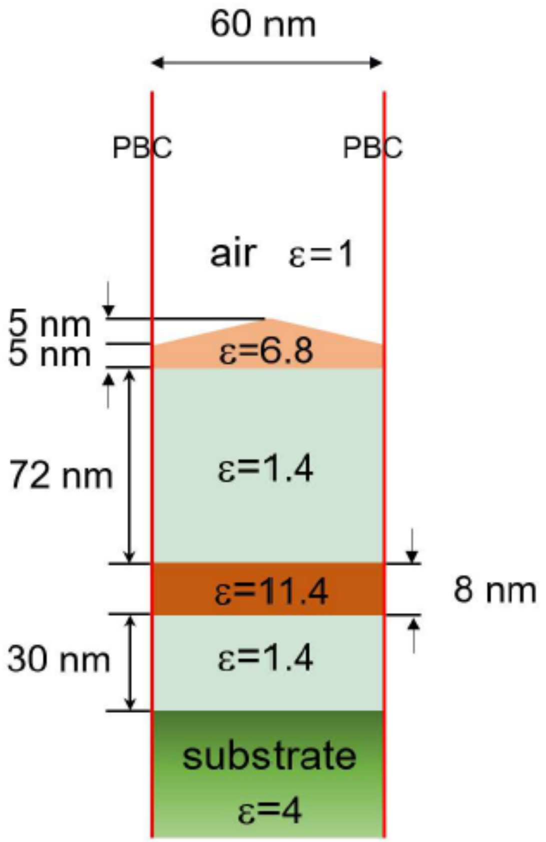
$$k_{0z} = \sqrt{\varepsilon_0 \mu_0 \omega^2 - k^2}, \; G = K \pm \sqrt{K^2-1}, \; K = \frac{\left(S_{11}^2 - S_{21}^2 + 1\right)}{2S_{11}}, \; S_{11} = R, \; S_{21} = Te^{ik_{0z}d}$$

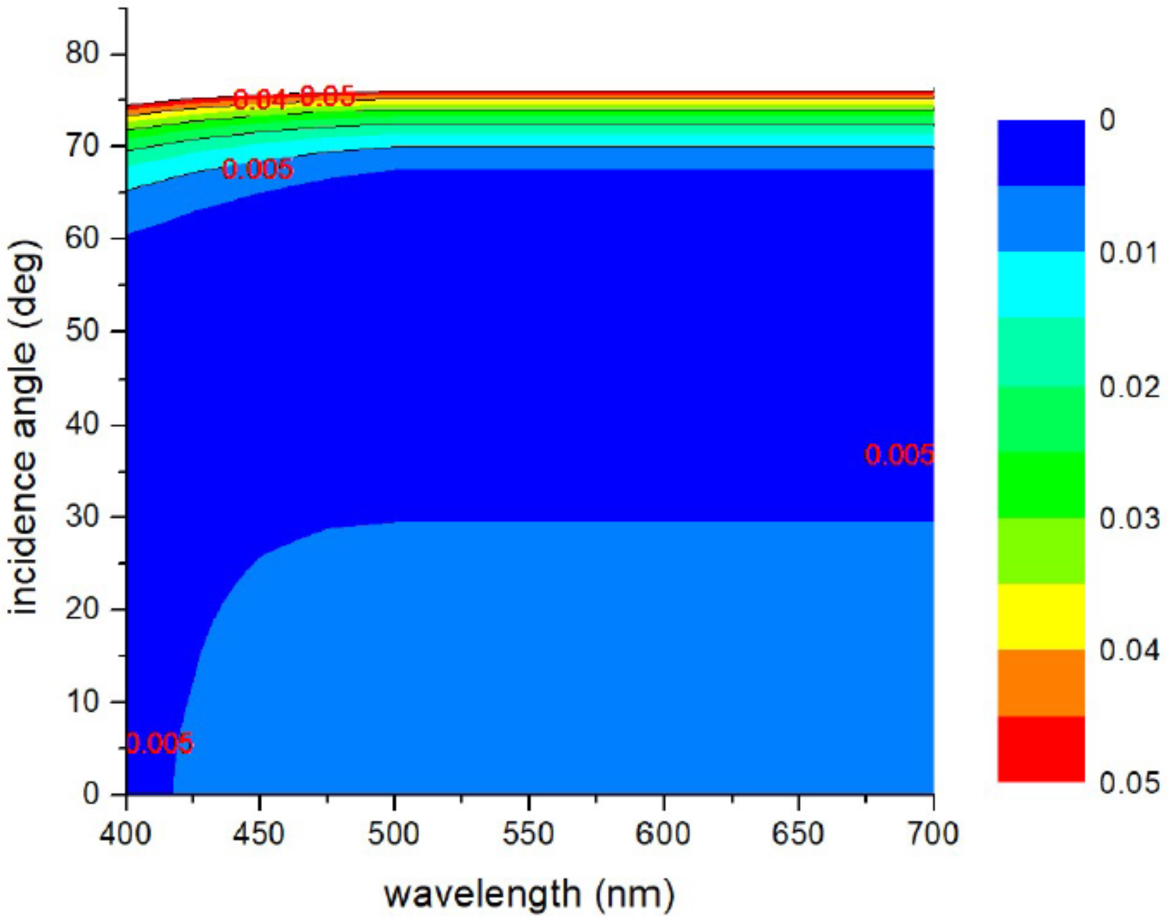


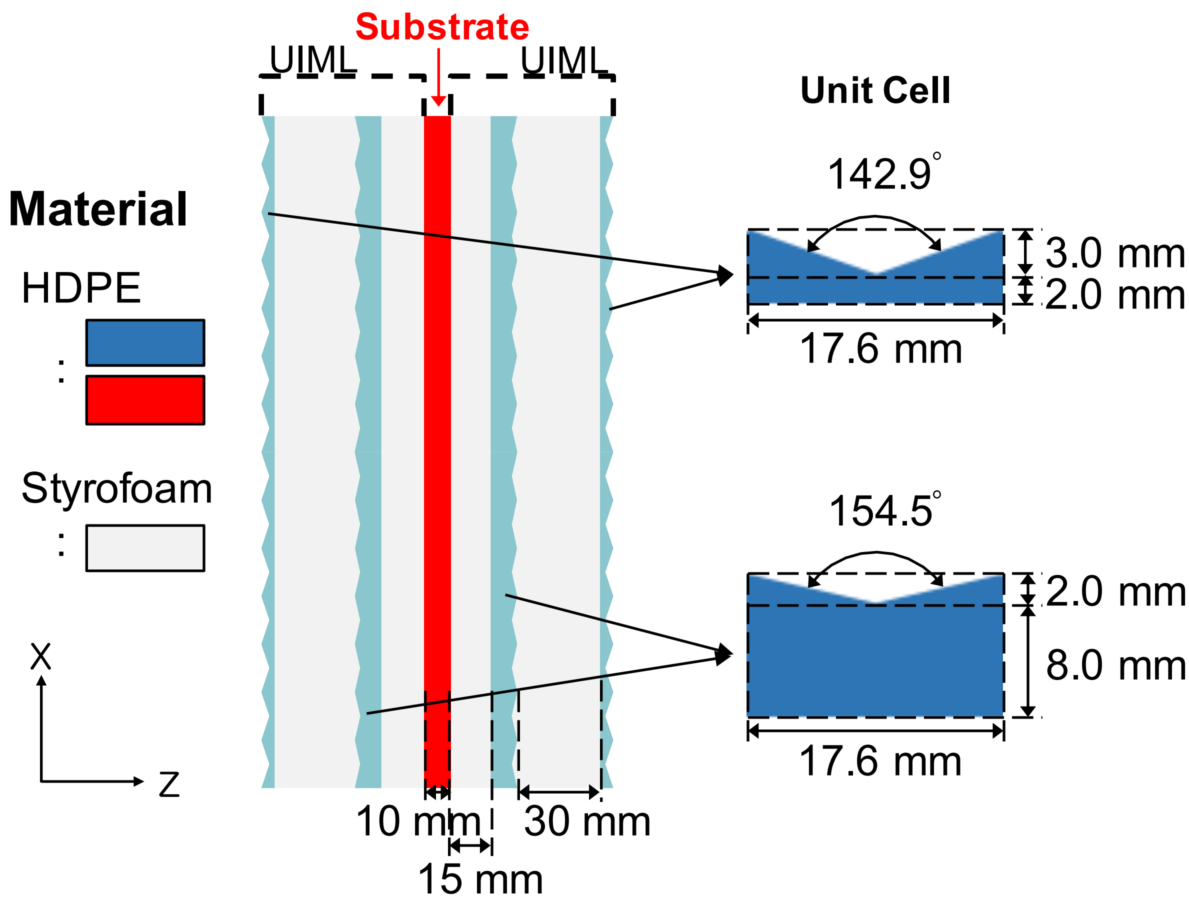
Perfect transmission of white light

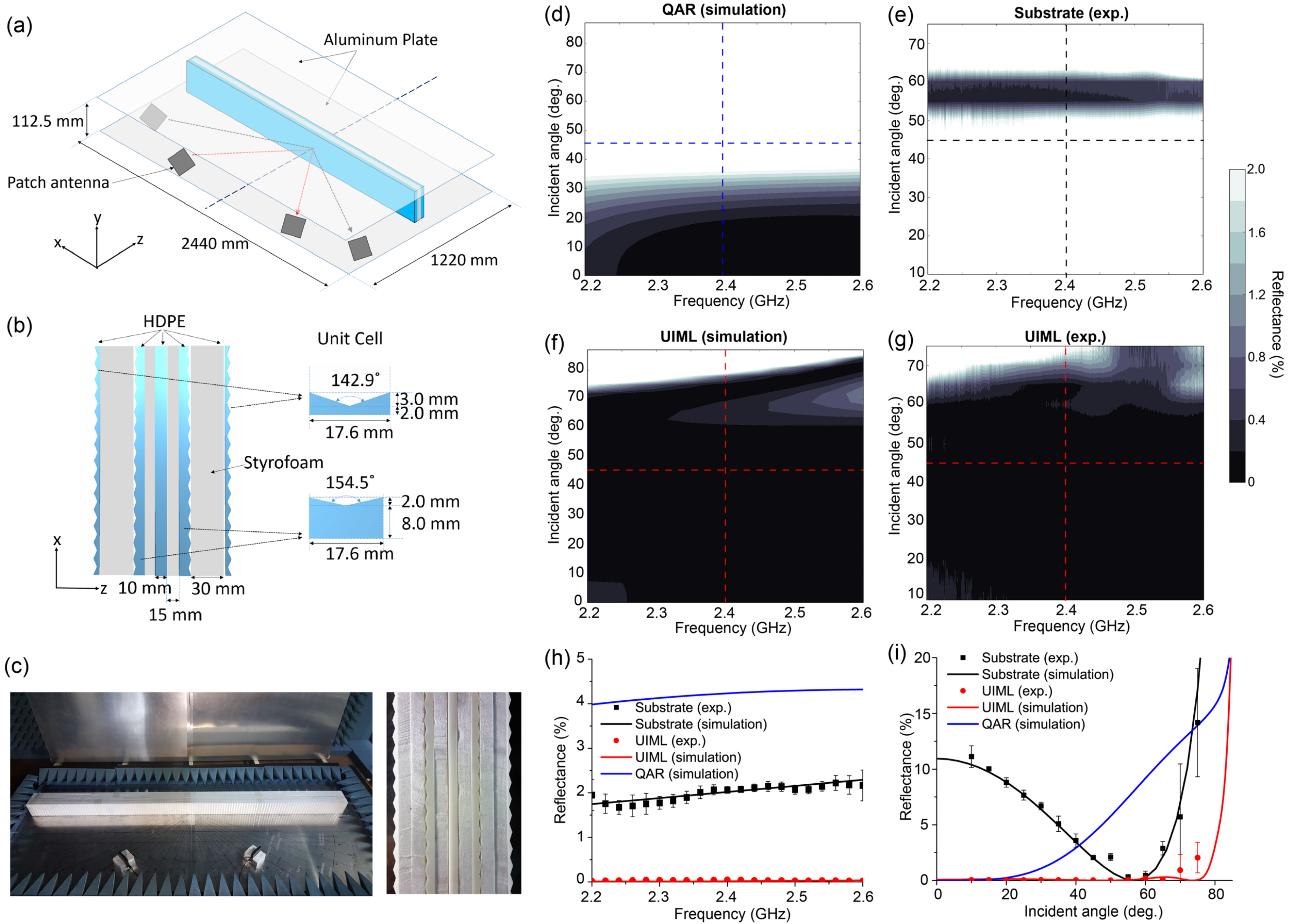


Perfect transmission of white light






Substrate
UIML
UIML
Unit Cell
Material
HDPE
:
:
Styrofoam
:
142.9°
3.0 mm
2.0 mm
17.6 mm
154.5°
2.0 mm
8.0 mm
17.6 mm
X
Z
10 mm
30 mm
15 mm


(a)
Aluminum Plate
112.5 mm
Patch antenna
2440 mm
1220 mm
x
y
z
(b)
HDPE
Unit Cell
142.9˚
3.0 mm
2.0 mm
17.6 mm
Styrofoam
154.5˚
2.0 mm
8.0 mm
17.6 mm
x
z
10 mm
30 mm
15 mm
(c)
(d)
QAR (simulation)
Incident angle (deg.)
Frequency (GHz)
(e)
Substrate (exp.)
Incident angle (deg.)
Frequency (GHz)
(f)
UIML (simulation)
Incident angle (deg.)
Frequency (GHz)
(g)
UIML (exp.)
Incident angle (deg.)
Frequency (GHz)
Reflectance (%)
(h)
Substrate (exp.)
Substrate (simulation)
UIML (exp.)
UIML (simulation)
QAR (simulation)
Reflectance (%)
Frequency (GHz)
(i)
Substrate (exp.)
Substrate (simulation)
UIML (exp.)
UIML (simulation)
QAR (simulation)
Reflectance (%)
Incident angle (deg.)

- ✓Perfectly Matched Layer in FDTD
- ✓Invisibility cloaking, transformation optics
- ✓Extensions to other fields
- ✓Inverse scattering & solitons
- ✓Solar cell, stealth, …

More light !

J.W. von Goethe