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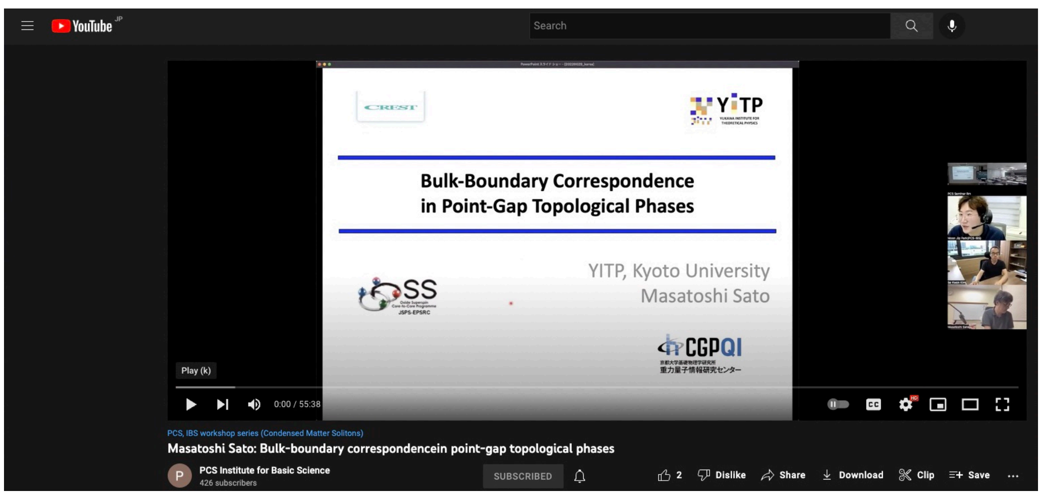
Majorana Fermions and Pairing Symmetries in Topological Superconductors



https://www.youtube.com/watch?v=0OomCbnTD4o

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In collaboration with

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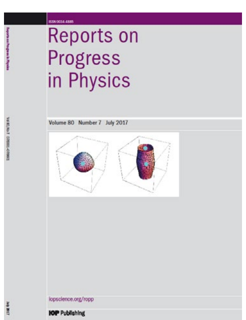
- Yuki Yamazaki (Nagoya Univ.)
- Yukio Tanaka (Nagoya Univ.)

S.Kobayashi

- S. Kobayashi, A. Yamakage, Y. Tanaka, MS, “Majorana multipole response in topological superconductors”, Phys. Rev. Lett. (2019)
- S. Kobayashi, Y. Yamazaki, A. Yamakage, MS, “Majorana multipole response: General theory and application to wallpaper groups” Phys. Rev. B (2021)

+ recent progress..



A review paper on TSCs

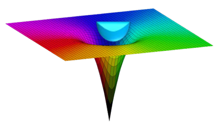
MS, Ando, Rep. Prog. Phys. 80, 076501 (17)

Motivation

Topological superconductivity sheds a new light on the investigation of superconductors

Majorana Fermions

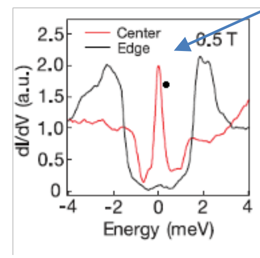
iron-based SC Fe(Se, Te) [Z. Wang et al. PRB (15), **Hong Ding's HP talk**]

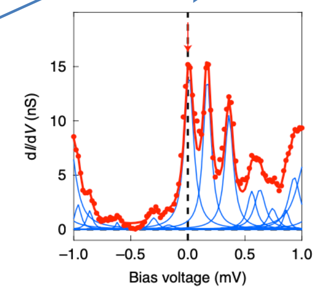


MF in a vortex

Evidence of MF

[D.Wang et al Science (18)]

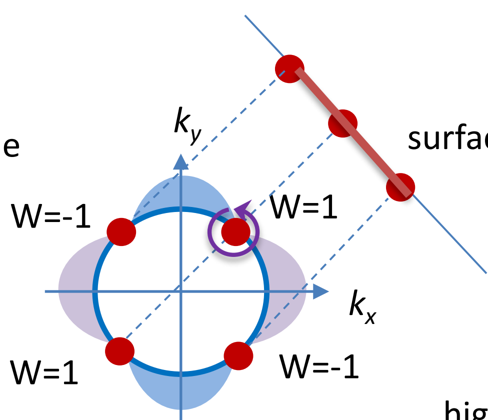




[T. Machida et al Nat. Mat. (18)]

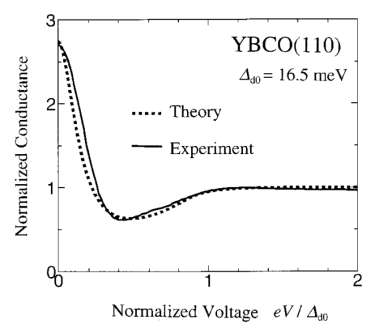
Topological Nodal Structures

line node



surface flat band

high-T_c cuprate



[Kashiwaya and Tanaka (00)]

The most distinct feature of TSCs is possible realization of emergent MFs in various SCs. However, we should note that there are two different types of MFs.

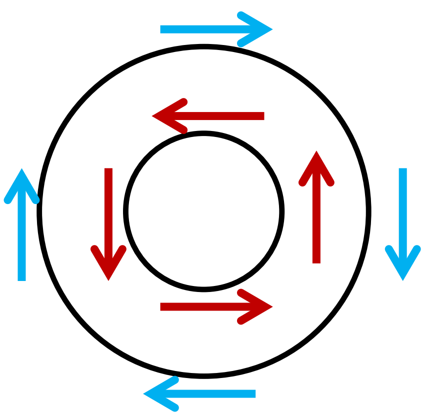
- **spinless MFs**
- **spinful MFs**

As I explain below, they have different properties, so they have probably different roles in studies of TSCs.

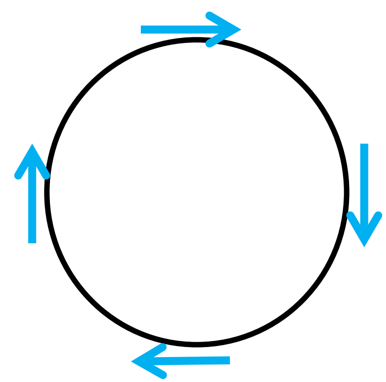
Spinless MFs

Spinless MFs have only PHS (= charge conjugation sym.), so they require no particular pairing symmetry except for superconducting gap. Indeed, only the Fermi surface topology in the normal state determines the presence or absence of a spinless MF.

> A spinless MF is realized if and only if the corresponding SC has an odd number of non-spin-degenerate Fermi surfaces.



Even

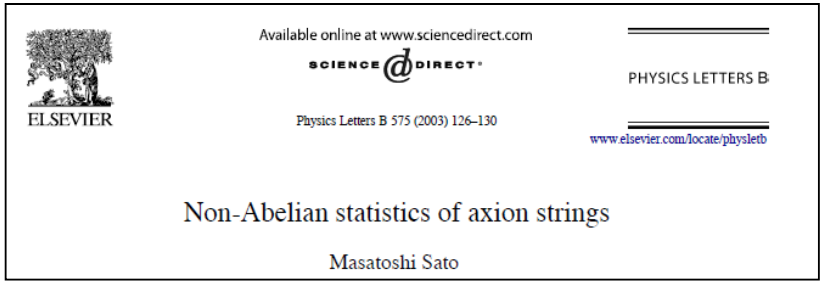


Odd

✕ No spinless MF

○ spinless MF

2d Dirac femion+s-wave condensate MS (03), Fu-Kane(08)



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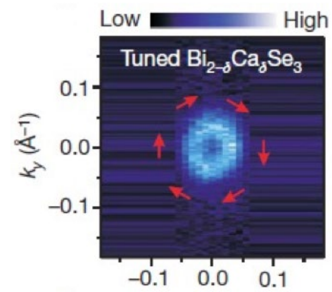
Non-Abelian statistics of axion strings

Masatoshi Sato

$$\mathcal{H} = \begin{pmatrix} -i\sigma_i\partial_i & \Phi \\ \Phi^* & i\sigma_i\partial_i \end{pmatrix}$$

MS (03)

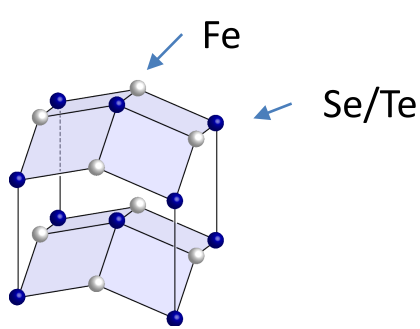


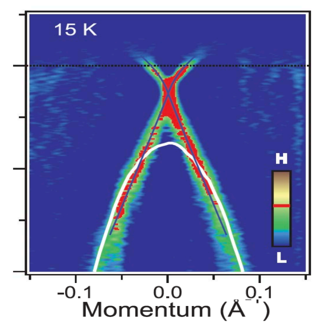
Hsieh et al (09)

Single non-spin-degenerate FS

Spinless MF in iron based SCs

Fe(Se,Te)





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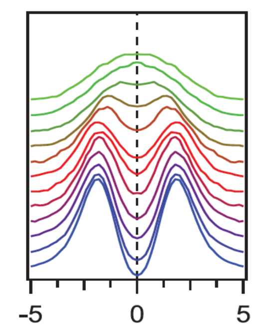


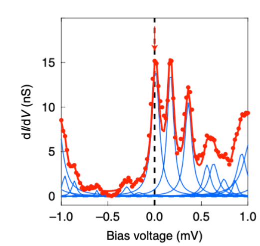
[P. Zhang et al Science (18)]

Surface Dirac fermion in the normal state

SC gap in Dirac fermion

Surface TSC

G. Xu et al PRL (16)

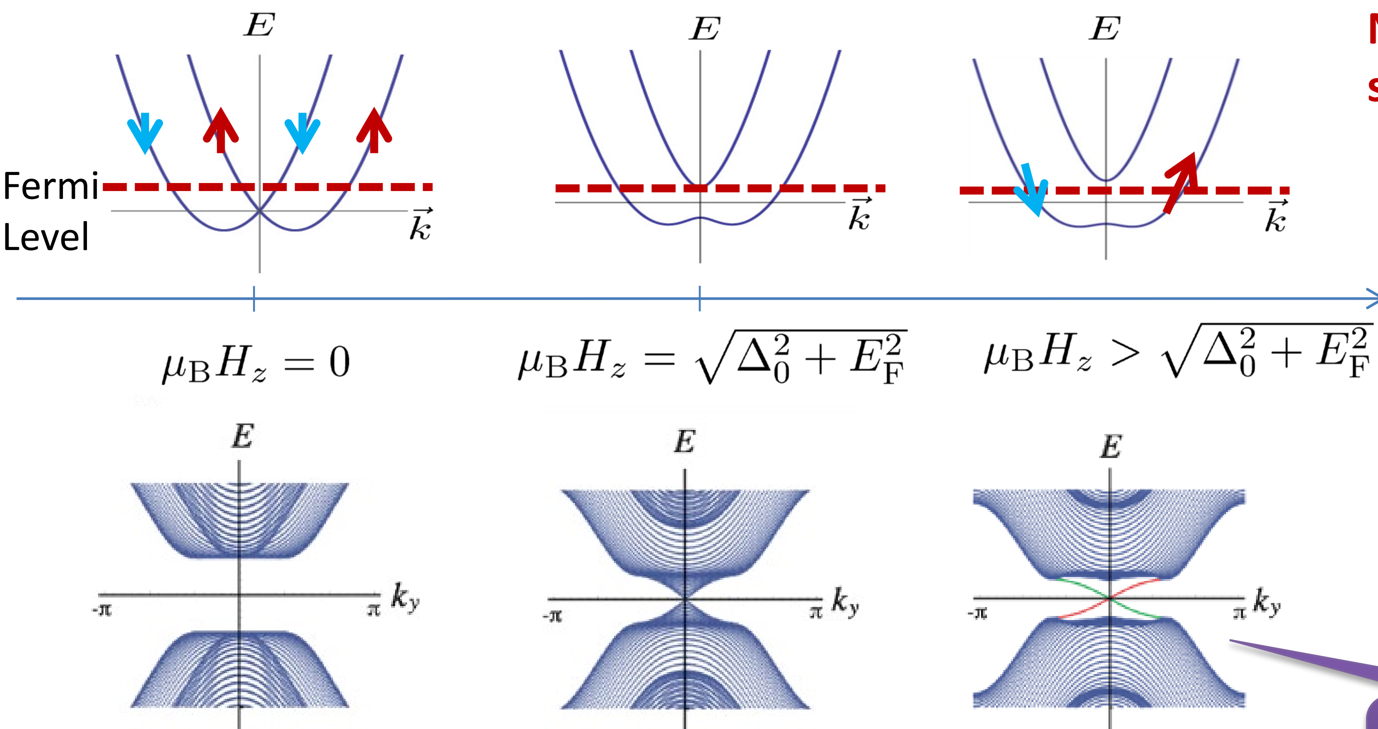


[T. Machida et al (18)]

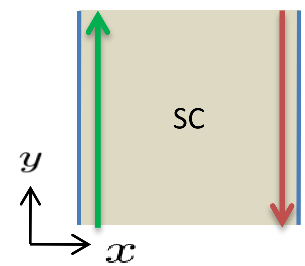
S-wave SC with Rashba SO + Zeeman field

MS-Takahashi-Fujimoto (09), J. Sau et al (10)

Non spin-degenerate single Fermi surface



Zeeman field



Spinless MF

nanowire

spinless MF

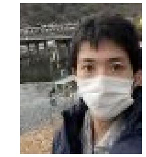
Zeeman field

Lutchyn et al (10), Oreg et al (10)

Interestingly, a similar TSC with a spinless MF can be obtained by using Rashba like spin-dependent dissipation

Non-Hermitian Rashba nanowire

[Okuma-MS PRL (19)]

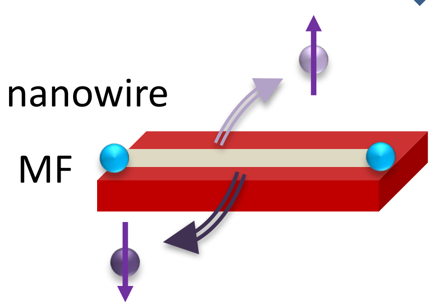


non-Hermitian SOI

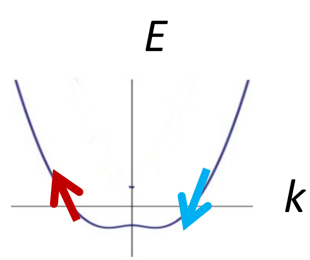
Infinitesimal Zeeman field

$$H = \sum_{k,\sigma_z=\pm} \left(-2t\cos k - \frac{i\Gamma}{2}(1+\sin k\sigma_z) \right) a^\dagger_{k,\sigma_z} a_{k,\sigma_z} + \sum_k (\Delta a^\dagger_{k,\uparrow} a^\dagger_{-k,\downarrow} + \mathrm{h.c}) + \sum_k (\delta h a^\dagger_{k,\uparrow} a_{k,\downarrow} + \mathrm{h.c}) \quad (\Gamma > \Delta)$$

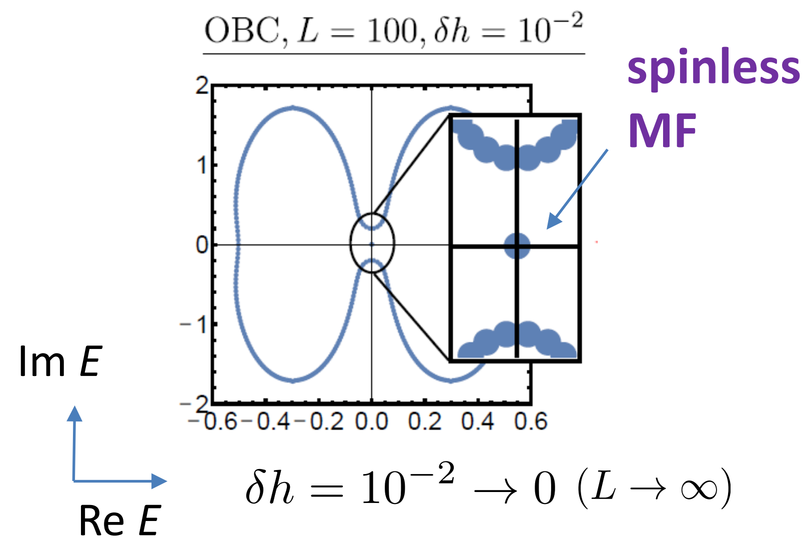
s-wave pairing int



$\approx$



Non spin-degenerate single FS by dissipation



$\delta h = 10^{-2} \to 0 \ (L \to \infty)$

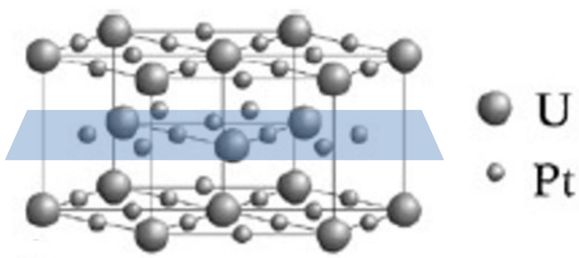
To obtain spinless MFs, only the information of FS topology is necessary, and no particular pairing symmetry is required.

Spinful MFs

Most SCs have spin-degenerate FSs, so they can only support spinful MFs. In contrast to spinless MFs, symmetry plays a crucial role in spinful MFs.

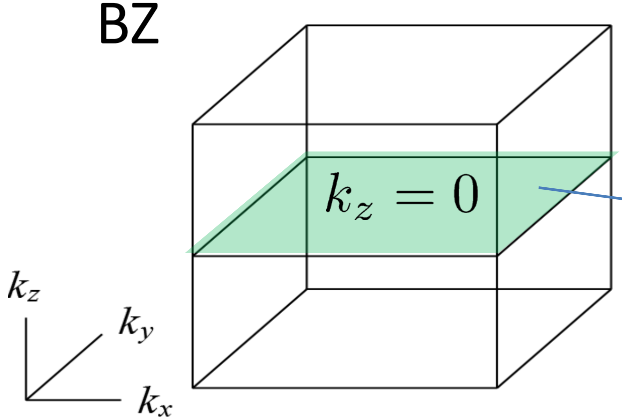
mirror reflection symmetry $\mathcal{M}_{xy}$ [Ueno-Yamakage-Tanaka-MS (13)]

UPt$_3$



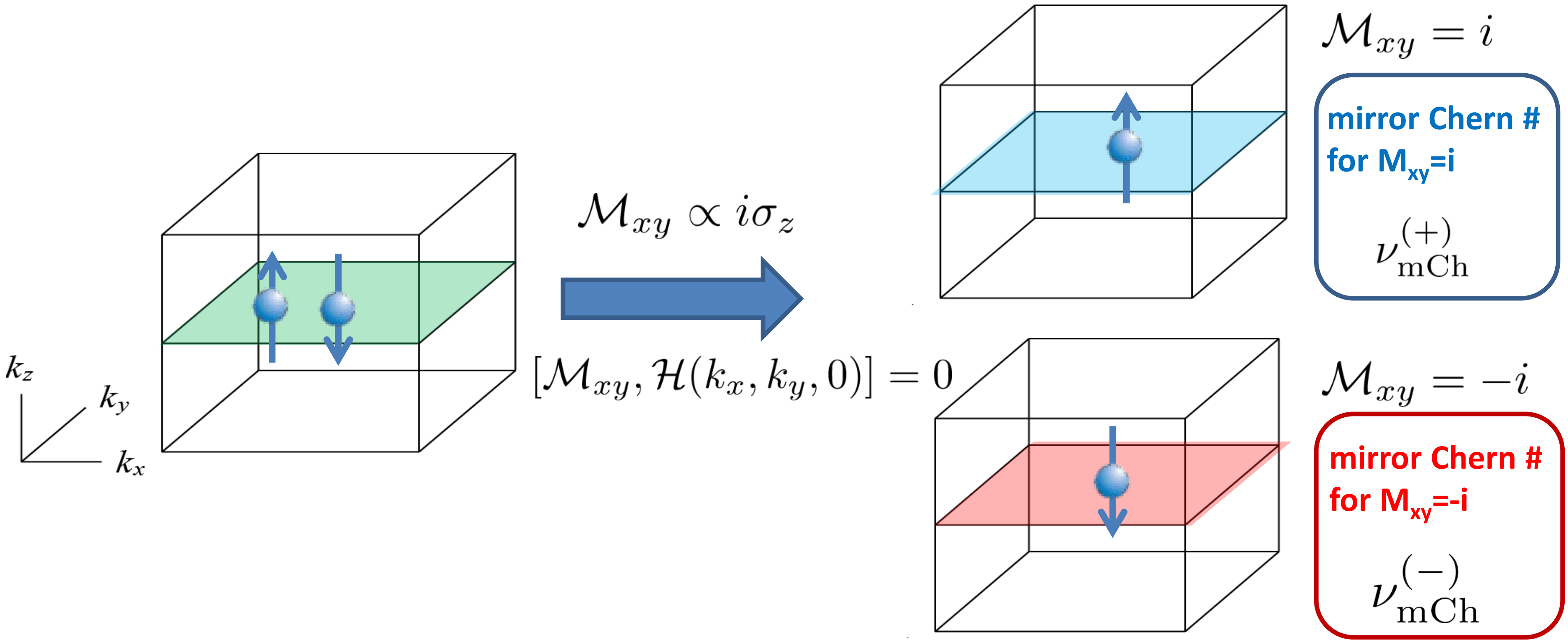
$$\mathcal{M}_{xy}\mathcal{H}(k_x,k_y,k_z)\mathcal{M}_{xy}^{-1}=\mathcal{H}(k_x,k_y,-k_z)$$

BZ



$$[\mathcal{M}_{xy},\mathcal{H}(k_x,k_y,0)]=0$$

Like topological crystalline insulators, k_z=0 plane can be separated into two mirror subsectors

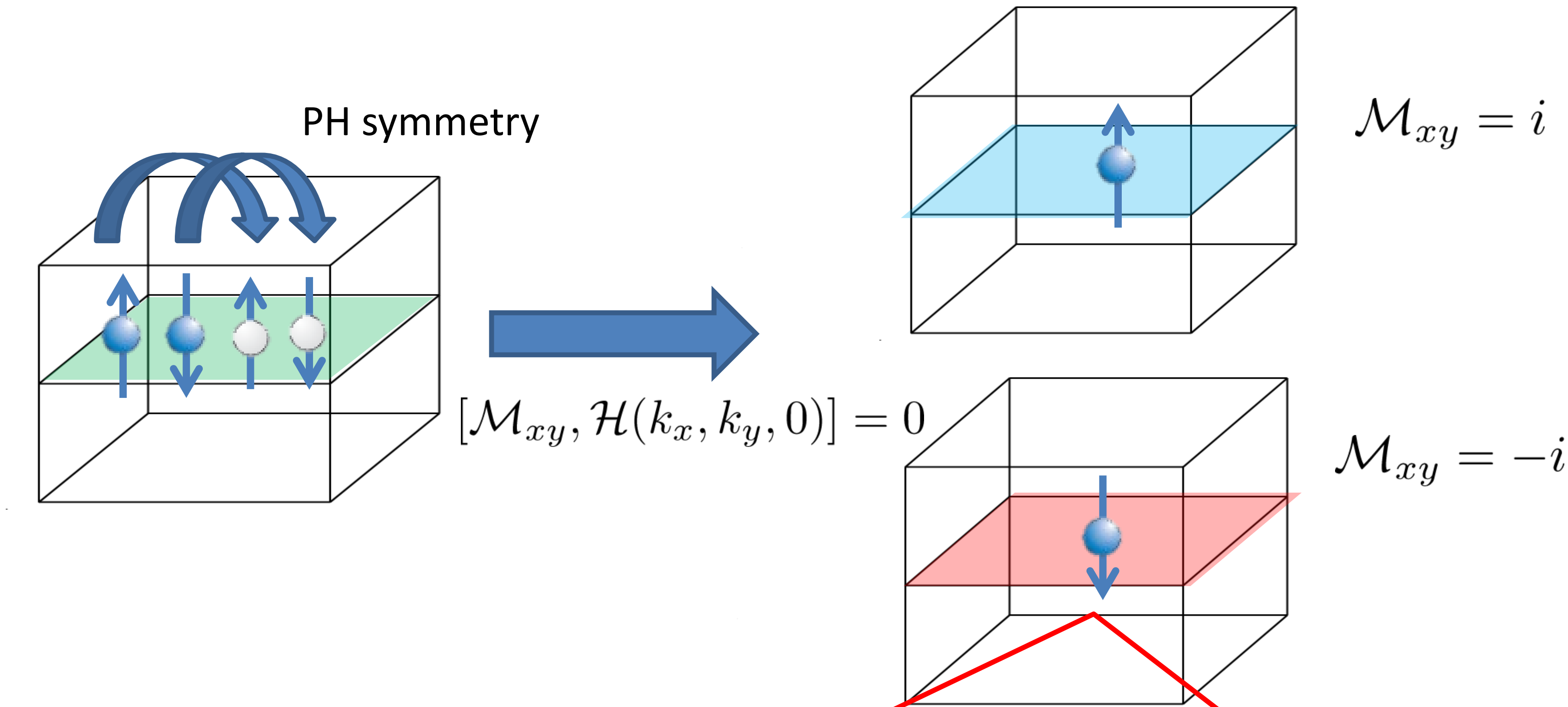


When the mirror Chern numbers are nonzero, we have gapless surface states

However, we have additional condition to obtain MFs.

Particle-hole symmetry = Majorana condition

$$\mathcal{C}\mathcal{H}(\boldsymbol{k})\mathcal{C}^{\dagger} = -\mathcal{H}^{*}(-\boldsymbol{k}) \qquad \Psi = \mathcal{C}\Psi^{\dagger}$$

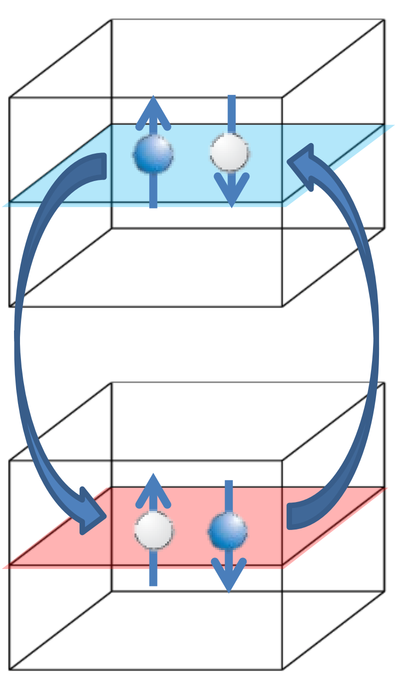


The problem is how the particle-hole symmetry is realized in the mirror subsectors.

Actually, we have two different realizations of PHS, depending on pairing sym

mirror even paring sym

$$\mathcal{M}_{xy}\Delta(\boldsymbol{k})\mathcal{M}_{xy}^{t} = \Delta(k_x, k_y, -k_z)$$



$\mathcal{M}_{xy} = i$

no PHS

QH state

$\mathcal{M}_{xy} = -i$

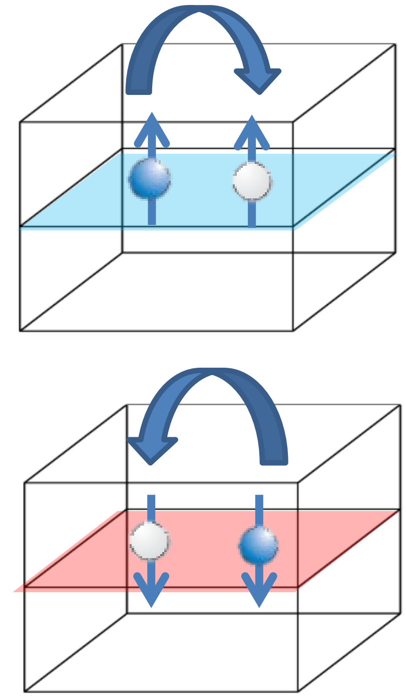
no PHS

QH state

Dirac fermion

mirror odd pairing sym

$$\mathcal{M}_{xy}\Delta(\boldsymbol{k})\mathcal{M}_{xy}^{t} = -\Delta(k_x, k_y, -k_z)$$



$\mathcal{M}_{xy} = i$

PHS

TSC

$\mathcal{M}_{xy} = -i$

PHS

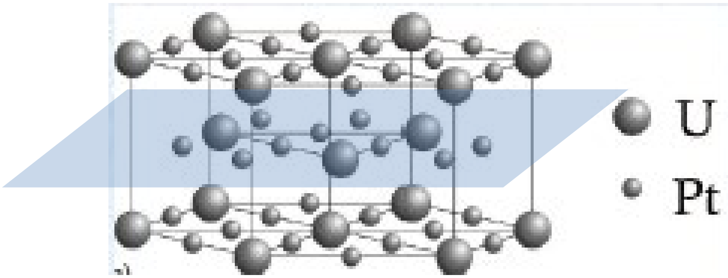
TSC

Majorana fermion

Thus, we have MFs only for mirror odd SCs

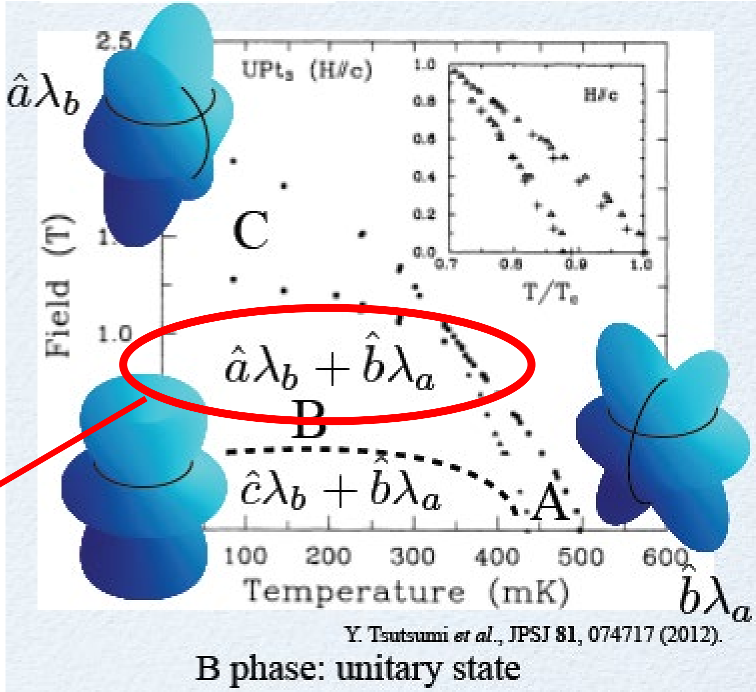
UPt_3 [Tsutsumi-Yamamoto-Mizushima-MS-Ichioka-Machida(13)]

D_{6h}: hexagonal



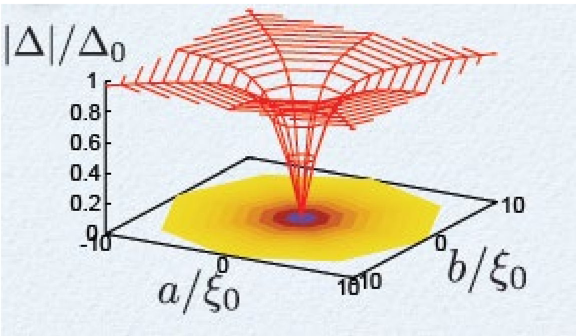
Mirror odd SC

$$\mathcal{M}_{xy}\Delta(\boldsymbol{k})\mathcal{M}_{xy}^{t} = -\Delta(k_x, k_y, -k_z)$$

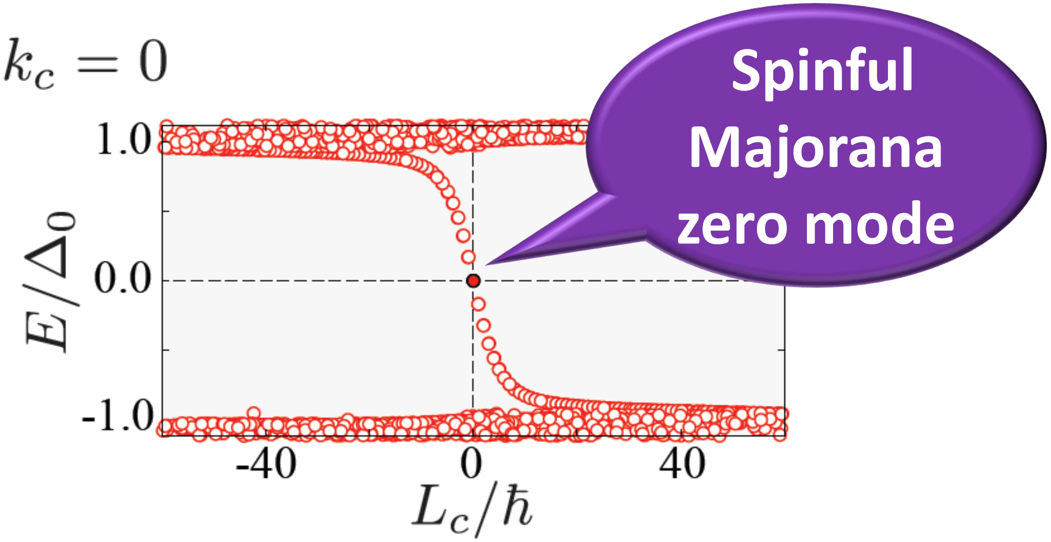


B phase: unitary state

Vortex bound state



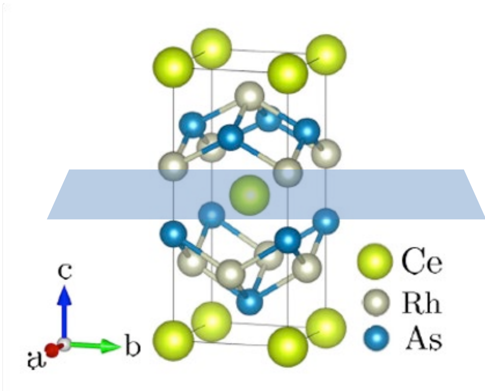
$k_c = 0$



Another candidate SC having this type of spinful MFs is $CeRh_2As_2$

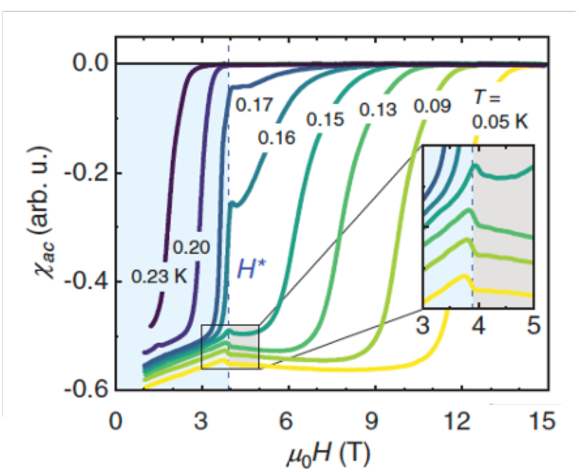
[Nogaki-Daido-Ishizuka-Yanase (21)]

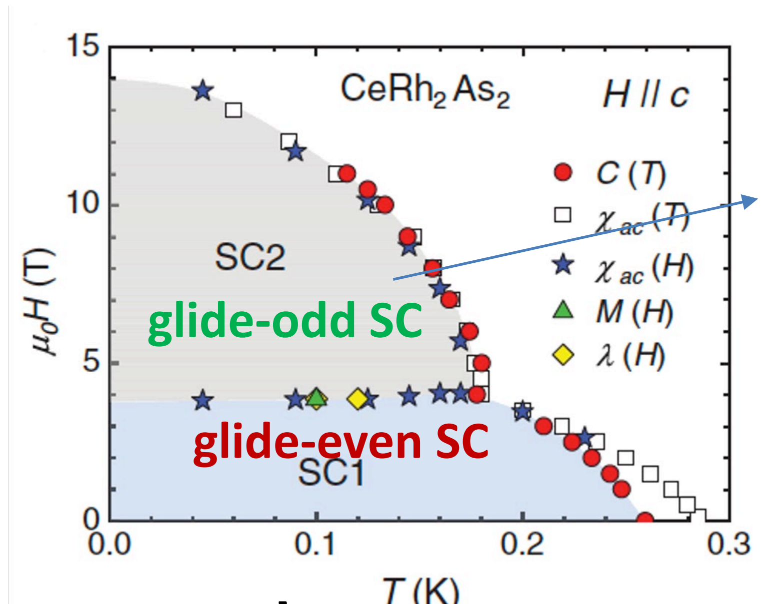
$CeRh_2As_2$

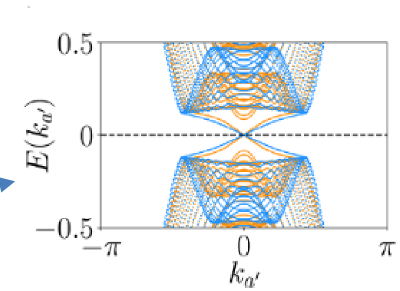


glide reflection symmetry

=a kind of mirror reflection symmetry







Majorana Fermion

[Nogaki-Daido-Ishizuka-Yanase (21)]

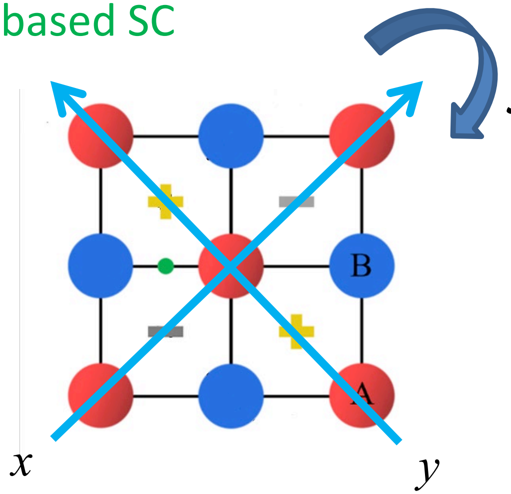
[S. Khim et al Science 373 (2021)]

[Yoshida-Sigrist-Yanase (12)]

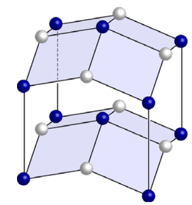
Very recently, a similar idea also has revealed possible MFs for s_{+-}-wave pairing state in iron-based SCs.

[Qin-Fang-Zang-Hu PRX (22)]

Iron-based SC



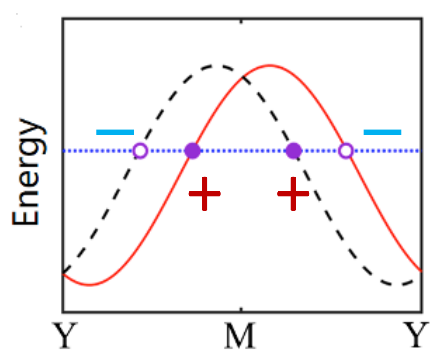
$\mathcal{M}_y \quad y \to -y$



P4/nmm

non-symmorphic

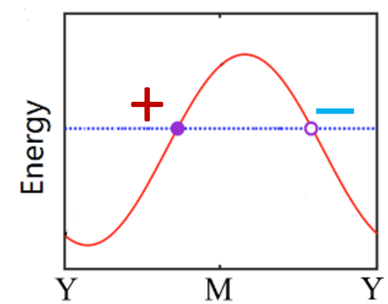
$k_y = \pi$



$k_x \longrightarrow$

s_{+-}-wave SC

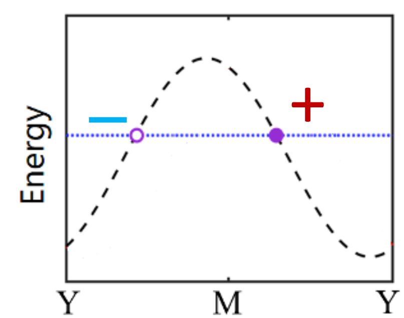
$[\mathcal{M}_y, \mathcal{H}(k_x, \pi)] = 0$



$\mathcal{M}_y = i$

p-wave SC

TSC

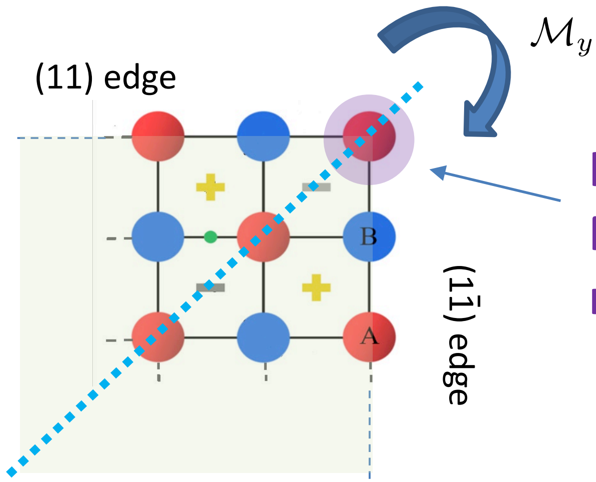


$\mathcal{M}_y = -i$

p-wave SC

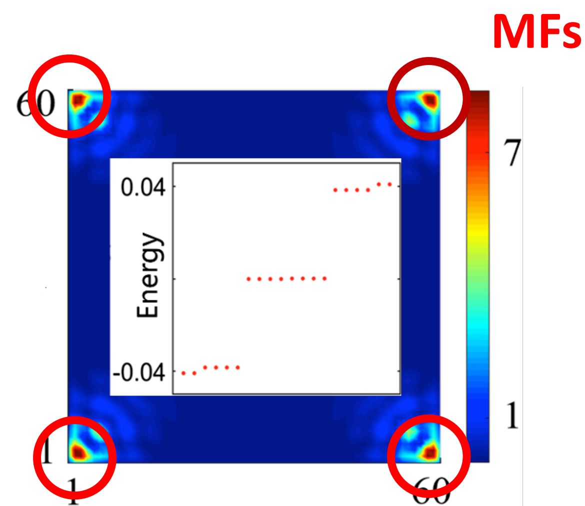
TSC

Interestingly, if one considers an interface b/w the (11) and $(1\bar{1})$ edges, the mirror sym protection works only for the corner, thus we have Majorana corner modes



Kramers pair of Majorana corner modes

[Qin-Fang-Zang-Hu PRX (22)]



[Qin-Fang-Zang-Hu PRX (22)]

Crystalline symmetry enables TSC having spinful MF even in TRI s_{+-} -wave SCs

Question

Spinless MFs can be topological qubits as non-Abelian anyons.
So do spinful MFs have a useful application in condensed matter physics?

Our answer

→ Yes.

They exhibit a unique electro-magnetic response, which may directly determine paring symmetries of various unconventional SCs.

Electromagnetic Response of spinful MFs

- S. Kobayashi, A. Yamakage, Y. Tanaka, MS, “Majorana multipole response in topological superconductors”, Phys. Rev. Lett. (2019)
- S. Kobayashi, Y. Yamazaki, A. Yamakage, MS, “Majorana multipole response: General theory and application to wallpaper groups” Phys. Rev. B (2021)

+ recent progress..

Pairing sym in top. classification

In top. classification, all the informations of symmetry are compactly encoded in the following three relations

Freed-Moore (12)

1) $U_g(\boldsymbol{k})H(\boldsymbol{k})U_g(\boldsymbol{k})^{-1} = c(g)H(g\boldsymbol{k}) \quad c(g) = \pm 1$

2) $U_g(\boldsymbol{k})i = \phi(g)iU_g(\boldsymbol{k}) \qquad \phi(g) = \pm 1$

3) $U_g(g'\boldsymbol{k})U_{g'}(\boldsymbol{k}) = \underbrace{e^{i\tau_{g,g'}(gg'\boldsymbol{k})}}_{\text{twist}} U_{gg'}(\boldsymbol{k})$

twist

Sym. or **Anti-sym.**

Unitary or **Anti-unitary**

Most systems have a trivial twist just determined by point groups, but some may have non-trivial one.

Shiozaki-MS-Gomi (18)

- **Unconventional SCs**
- **Nonsymmorphic crystals**

Actually, information of pairing sym is encoded in a twist between PHS and crystalline sym.

ex.) **odd-parity SC** $P\Delta(\boldsymbol{k})P^t = -\Delta(-\boldsymbol{k})$ $\boldsymbol{k} \xrightarrow{P} -\boldsymbol{k}$ inversion

For Nambu space

$$\tilde{P} = \begin{pmatrix} P & 0 \\ 0 & -P^* \end{pmatrix}$$

particle

hole

$\Rightarrow$ $\tilde{P}C = -C\tilde{P}$

Particle and hole behave in a different manner

twist

In general, we have

$$U_g(\boldsymbol{k})\Delta(\boldsymbol{k})U_g^T(-\boldsymbol{k}) = \eta_g\Delta(g\boldsymbol{k})$$

$$\Rightarrow \quad C\tilde{U}_g(\boldsymbol{k}) = \eta_g\tilde{U}_g(-\boldsymbol{k})C$$

information of pairing sym

Response of MFs

To evaluate possible response of MFs, we consider a general local operator of electrons

$$\hat{O} = \hat{c}^{\dagger}_{\sigma}(x) O_{\sigma,\sigma'} \hat{c}_{\sigma'}(x) = \frac{1}{2} \hat{\Psi}^{\dagger}(x) \mathcal{O} \hat{\Psi}(x)$$

hermitian

Nambu base

$$\mathcal{O} = \begin{pmatrix} O & 0 \\ 0 & -O^T \end{pmatrix} \quad \hat{\Psi} = \begin{pmatrix} \hat{c}_{\sigma} \\ \hat{c}^{\dagger}_{\sigma} \end{pmatrix}$$

To extract the contribution of MFs, we first use PHS of the Nambu spinor

$$\hat{O} = \frac{1}{2} \hat{\Psi}^{t}(x) \left[\tau_x \mathcal{O} \right] \hat{\Psi}(x)$$

$$\hat{\Psi}^{\dagger}(x) \tau_x = \hat{\Psi}^{t}(x)$$

Then, perform mode expansion of quantum field,

$$\Psi(x) = \sum_{a} \hat{\gamma}^{(a)} |u_0^{(a)}\rangle + \cdots ,$$

(gapless) MF

PHS partner

$$|Cu_0^{(a)}\rangle = \tau_x |u_0^{*(a)}\rangle$$

$$\hat{O}_{\mathrm{MF}} = -\frac{1}{8} \sum_{ab} [\hat{\gamma}^{(a)}, \hat{\gamma}^{(b)}] \mathrm{tr} \left[\mathcal{O} \rho^{(ab)}(\boldsymbol{x}) \right] \qquad \rho^{(ab)} = \left(|u_0^{(a)}\rangle\langle Cu_0^{(b)}| - |u_0^{(b)}\rangle\langle Cu_0^{(a)}| \right)$$

$\rho^{(ab)}$ determines possible local ops. of MFs

Interestingly, $\rho^{(ab)}$ transforms as a bi-product of electrons like Cooper pairs under crystalline symmetry.

Under crystalline sym., a MF moves a MF.

$$\rho^{(ab)} = \left(|u_0^{(a)}\rangle\langle Cu_0^{(b)}| - |u_0^{(b)}\rangle\langle Cu_0^{(a)}|\right)$$

$$U_g|u^{(a)}(g\boldsymbol{x})\rangle = \sum_b |u^{(b)}(\boldsymbol{x})\rangle\mathcal{U}_{ba}$$

projective rep. of cry. sym. like "electron"

$$U_g|Cu^{(a)}(g\boldsymbol{x})\rangle = \eta_g C U_g|u^{(a)}(g\boldsymbol{x})\rangle$$

← $U_g C = \eta_g C U_g$

PHS partner

$$= \eta_g \sum_b |Cu^{(b)}(\boldsymbol{x})\rangle\mathcal{U}_{ba}^*$$

← anti-unitarity of PHS

projective rep. like "hole"

$$U_g \rho^{(ab)}(g\boldsymbol{x}) U_g^\dagger = \sum_{cd} \rho^{(cd)}(\boldsymbol{x})\underline{\eta_g^*}\mathcal{U}_{ca}\mathcal{U}_{db}$$

"bi-product of electrons"

pairing sym

$$U_g(\boldsymbol{k})\Delta(\boldsymbol{k})U_g^T(-\boldsymbol{k}) = \eta_g\Delta(g\boldsymbol{k})$$

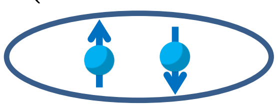
MFs know paring symmetry of Cooper pairs!

In particular, we have a direct relation b/w the MFs and pairing sym if we consider a single Majorana Kramers pair (MKP) protected by sym *g*.

In this case, ρ consists of a Kramers pair

TRS

$$\rho^{(ab)} = \left(|u_0^{(a)}\rangle\langle CTu_0^{(a)}| - |Tu_0^{(a)}\rangle\langle Cu_0^{(a)}| \right)$$

$\sim$  "spin-singlet"

Also, *g* preserves the position of MFs, so

$$U_g \rho^{(ab)}(g\boldsymbol{x}) U_g^\dagger = \sum_{cd} \rho^{(cd)}(\boldsymbol{x}) \eta_g^* \mathcal{U}_{ca} \mathcal{U}_{db}$$

"spin"

trivial

$$U_g \rho^{(ab)}(g\boldsymbol{x}) U_g^\dagger = \rho^{(ab)}(\boldsymbol{x}) \eta_g^*$$

same rep. as gap fn.

$$U_g(\boldsymbol{k}) \Delta(\boldsymbol{k}) U_g^T(-\boldsymbol{k}) = \eta_g \Delta(g\boldsymbol{k})$$

Under TRS, we can also show

$$T[\rho^{(ab)}]^\dagger T^{-1} = -\rho^{(ab)}$$

magnetic operator

Therefore

A single MKP protected by *g* has a magnetic structure with the same rep. as gap function.

Application to various SCs

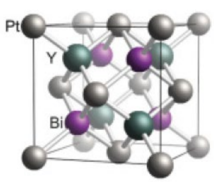
c.f.) classification:

Kobayashi-Yamazaki-Yamakage-MS (21)

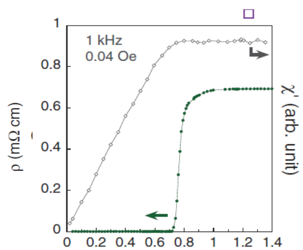
High-Spin TSC

YPtBi

J=3/2



experiment



[Butch et al (11)]

T_c=0.7K

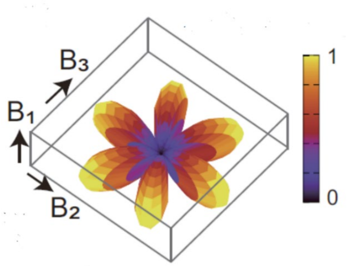
proposed gap fn.

[Brydon et al (16)]

$$\Delta(\boldsymbol{k}) \propto \left[\eta \mathbf{1}_{4\times 4} + \sum_i k_i (J_{i+1}J_iJ_{i+1} - J_{i+2}J_iJ_{i+2})\right] e^{-iJ_y\pi}$$

multipole structure due to higher spin

Mag resp of spinful MFs on (111)



$$\Delta E \propto |B_2|(B_2^2 - 3B_3^2)$$

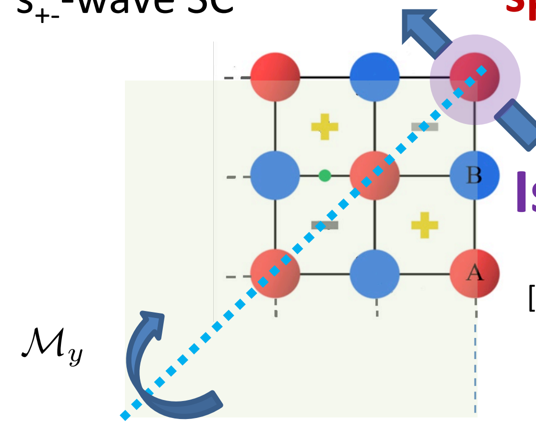
octupole response

[Kobayashi-Yamakage-Tanaka-MS (18)]

s_{+-}-wava TSC

s_{+-}-wave SC

spinful MF



Ising spin

[Kobayashi-MS (22)]

Mag. structure of MF should be trivial under the mirror reflection

Observation of the mag. structures (via STM) can establishe these exotic pairing states

Summary

1. We discuss relations between paring symmetry and spinful Majorana fermions.

2. We develop a systematic way to evaluate quantum response of spinful MFs and show that the magnetic response of spinful MFs can identify paring symmetry of Cooper pairs

3. Detection of magnetic response of spinful MFs gives a direct evidence of exotic TSCs such as high-spin TSC and s_{+-}-wave TSC

c.f) Y. Yamazaki, S. Kobayashi, A. Yamakage,
JPSJ 89, 0437003 (20); Phys. Rev. B103, 094508 (21); JPSJ 90, 073701 (21)