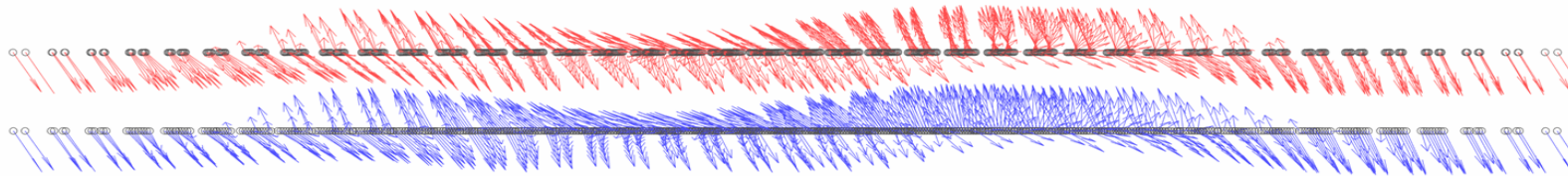


# Twisted Bilayer Magnet $\text{CrI}_3$



Moon Jip Park  
PCS-IBS

IBS-APCTP Conference  
09. 22.

# Acknowledgements

## Twisted Magnets $\text{CrI}_3$

Kyoung-Min Kim  
(PCS-IBS)



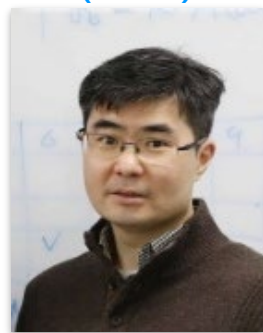
Grigory Bednik  
(PCS-IBS)



Do Hun Kim  
(KAIST)



Myung Joon Han  
(KAIST)

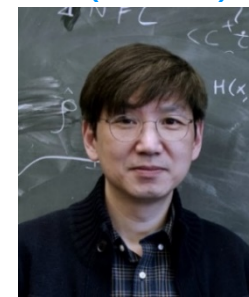


## 3D Twisted Superconductivity & Quasicrystal

SungBin Lee  
(KAIST)



Yong Baek Kim  
(Toronto)



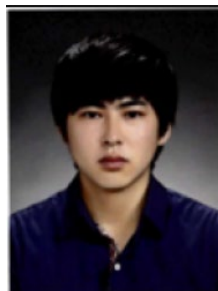
Ref: K.-M. Kim, D. H. Kiem, G. Bednik, M. J. Han, MJP, arXiv:2206.05264 (2022)

## Twisted Bilayer Graphene

Youngkuk Kim  
(SKKU)



Sunam Jeon  
(SKKU)

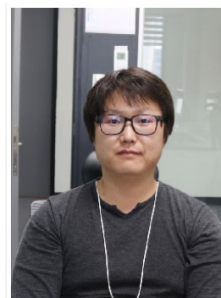


Gil Young Cho  
(Postech)

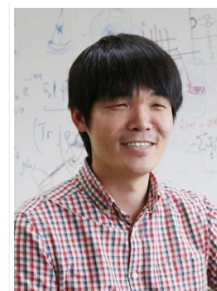


## Moire Photonics

ChangHwan Yi  
(PCS-IBS)



Hee Chul Park  
(PCS-IBS)



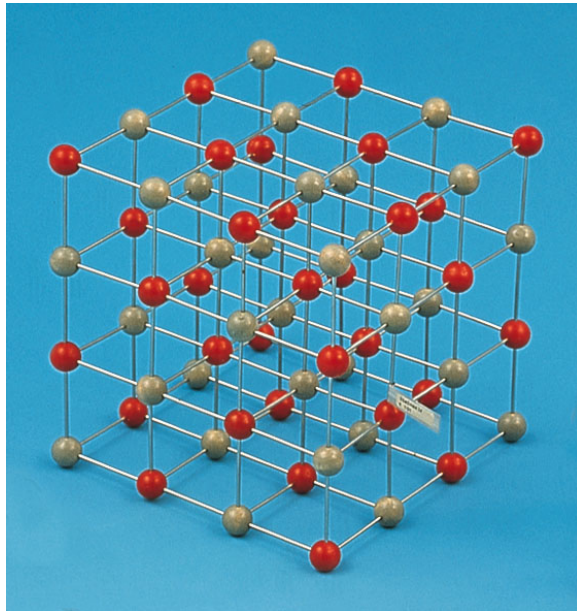
## Hofstadter Moire Replica HOTI

Sun-Woo Kim  
(KAIST/SKKU BRL→Cambridge)



# Physics of Length Scale

## Solid state lattice



## Magnetic Domains

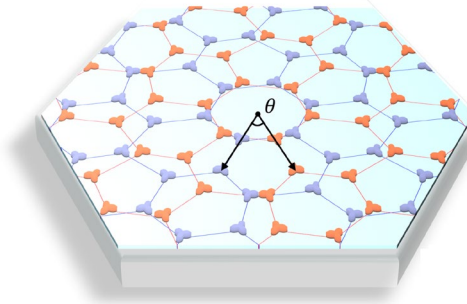
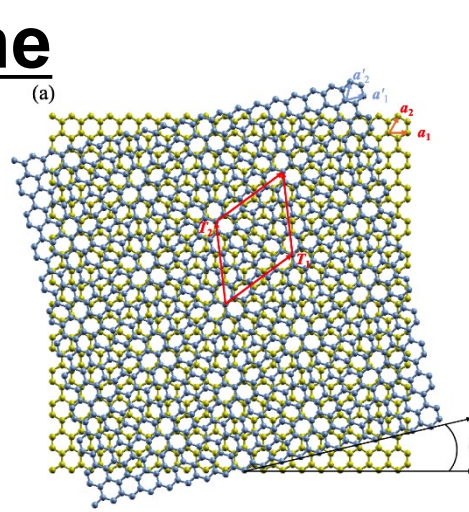
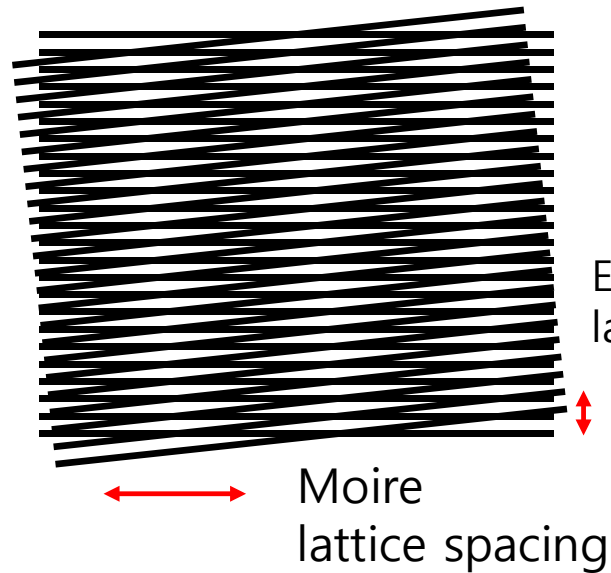


## Complex Network

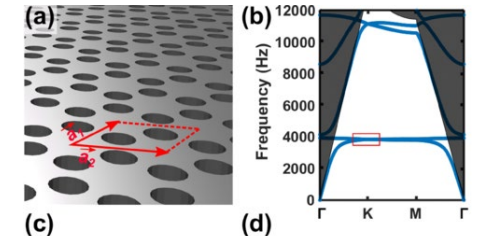
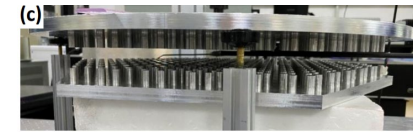


# Length Scale in solid state

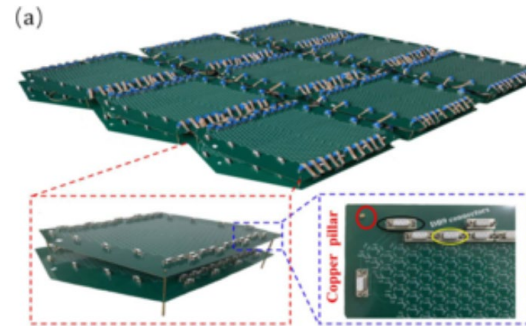
## Graphene



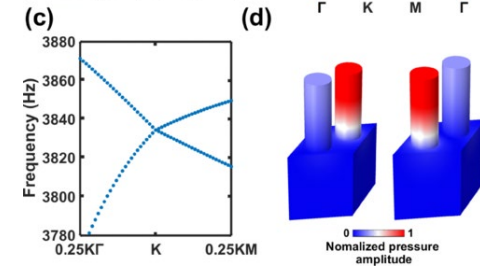
## Photonic Crystals



## Acoustic materials



Xiangdong Zhang group



Yun Jing group

In this talk, we generalize moire materials to spin systems,  
"twisted bilayer magnetism"

# Experimental progress

Letter | [Published: 29 November 2021](#)

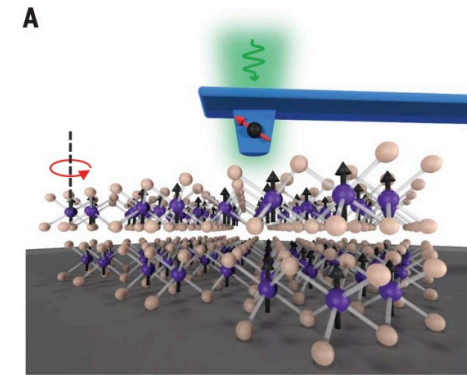
## Coexisting ferromagnetic–antiferromagnetic state in twisted bilayer CrI<sub>3</sub>

[Yang Xu](#), [Ariana Ray](#), [Yu-Tsun Shao](#), [Shengwei Jiang](#), [Kihong Lee](#), [Daniel Weber](#), [Joshua E. Goldberger](#), [Kenji Watanabe](#), [Takashi Taniguchi](#), [David A. Muller](#), [Kin Fai Mak](#) & [Jie Shan](#)

[Nature Nanotechnology](#) **17**, 143–147 (2022) | [Cite this article](#)

6584 Accesses | 1 Citations | 12 Altmetric | [Metrics](#)

Local (AFM) measurement



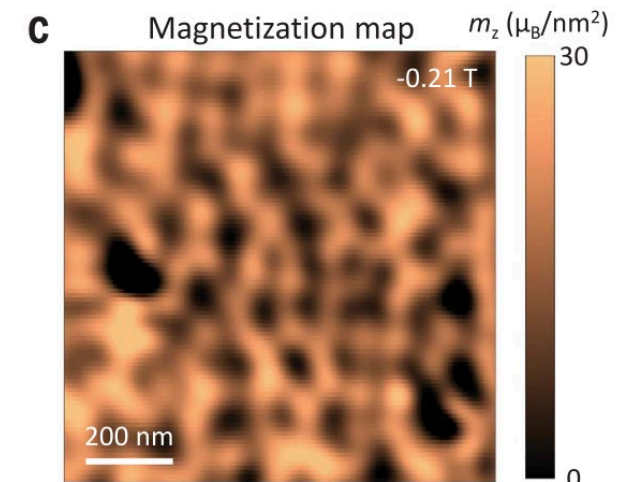
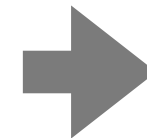
REPORT | MAGNETISM



## Direct visualization of magnetic domains and moiré magnetism in twisted 2D magnets

[TIANCHENG SONG](#), [QI-CHAO SUN](#), [ERIC ANDERSON](#), [CHONG WANG](#), [JIMIN QIAN](#), [TAKASHI TANIGUCHI](#), [KENJI WATANABE](#), [MICHAEL A. MCGUIRE](#), [RAINER STÖHR](#), [...] [XIAODONG XU](#) +4 authors [Authors Info & Affiliations](#)

*SCIENCE* • 25 Nov 2021 • Vol 374, Issue 6571 • pp. 1140-1144 • DOI: 10.1126/science.abj7478

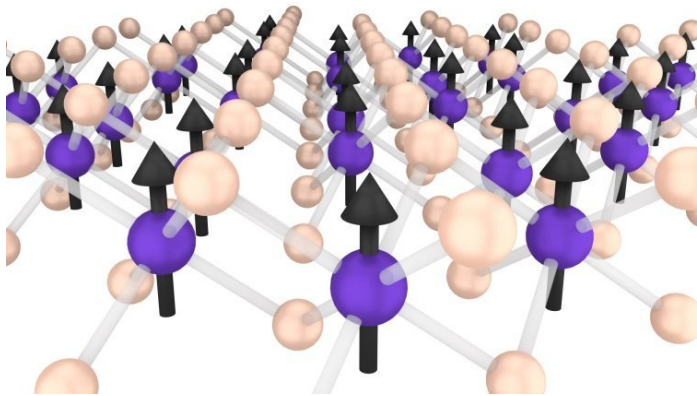


*Song et al. Science (2021)*

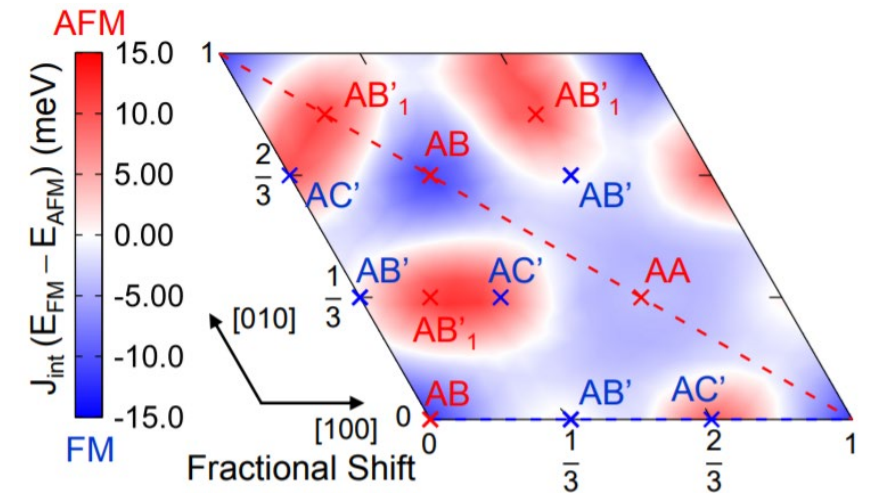
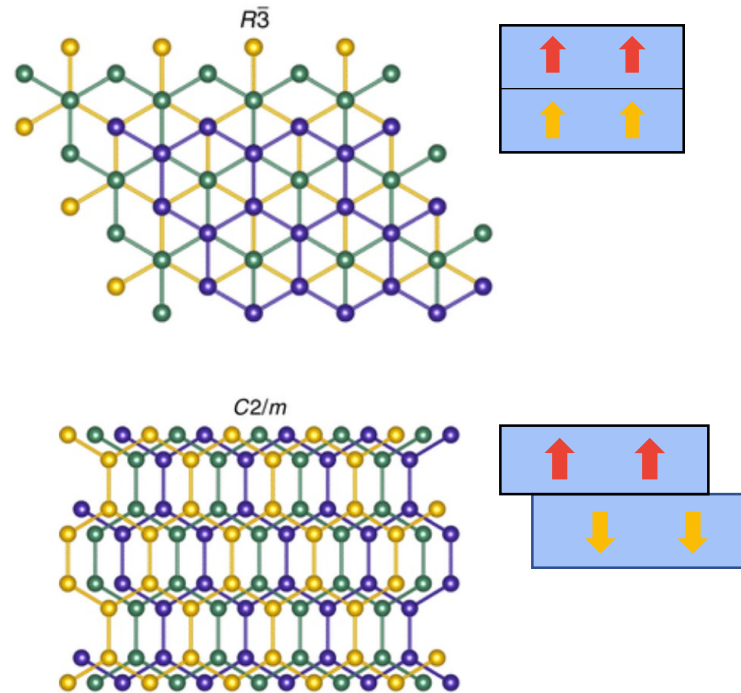
# Twisted bilayer magnet

# Transition metal trihalides

## ➤ Honeycomb magnet CrI<sub>3</sub>



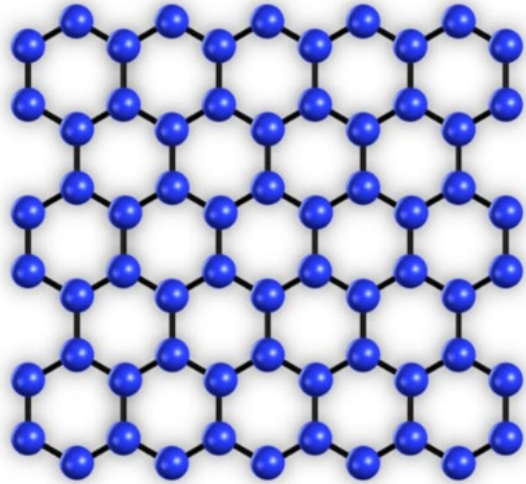
## ➤ Stacking dependent magnetism



$$H = \sum_{\langle i,j \rangle} JS_i \cdot S_j + \sum_{z_j=z_i+d} J_{ij}^{\perp} S_i \cdot S_j + D_z \sum_i (S_i^z)^2$$

# Symmetry of Twisted $\text{CrI}_3$

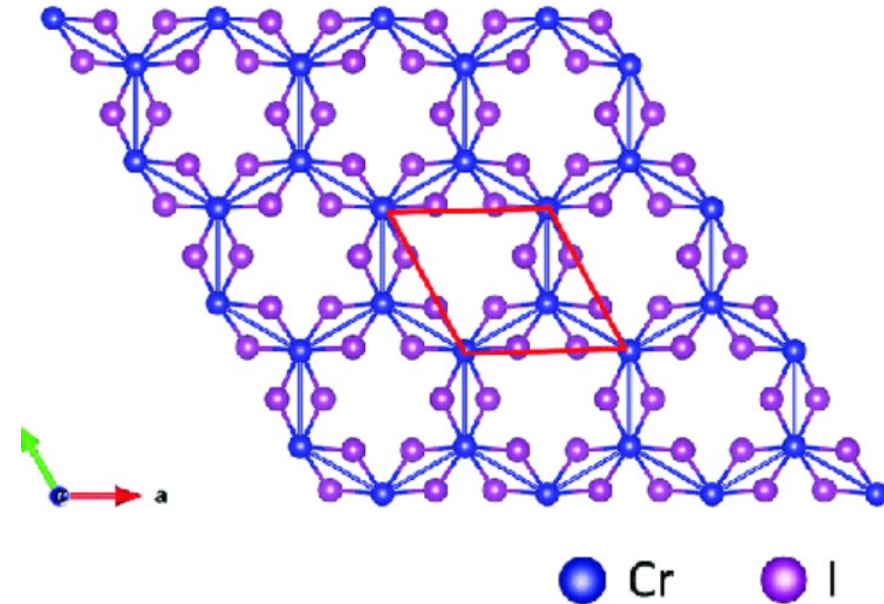
## ➤ Graphene



● Carbon atom

- Monolayer preserves  $\mathbf{C}_{2z}$  and  $\mathbf{P}$  symmetry
- $\mathbf{C}_{2z}$  is preserved in twisted bilayer
- Point group  $D_6$

## ➤ $\text{CrI}_3$



● Cr ● I

- Non-magnetic I atoms break  $\mathbf{C}_{2z}$
- Twisted bilayer breaks both  $\mathbf{C}_{2z}$  and  $\mathbf{P}$  symmetry
- Point group  $D_3$



# Ab-initio model construction

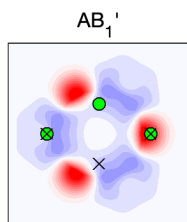
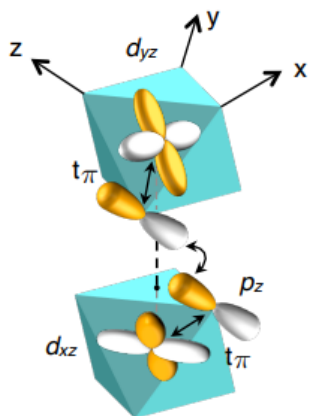
Atomic-level calculations

Ab-initio DFT calculations

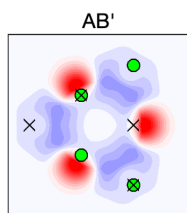
Effective spin model approach

$$H = \sum I_{ij} S_i \cdot S_j$$

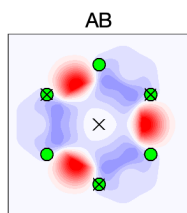
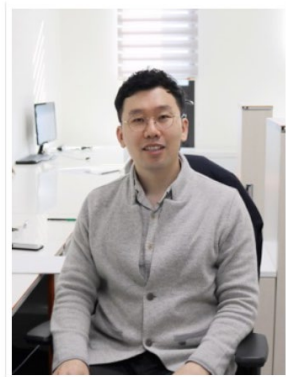
Analytical methods



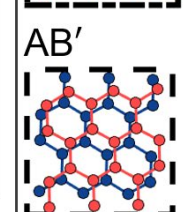
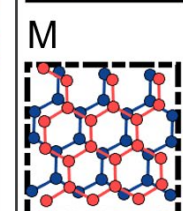
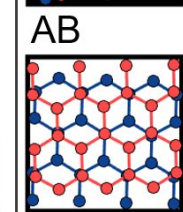
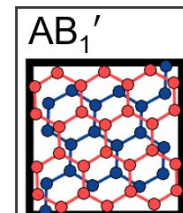
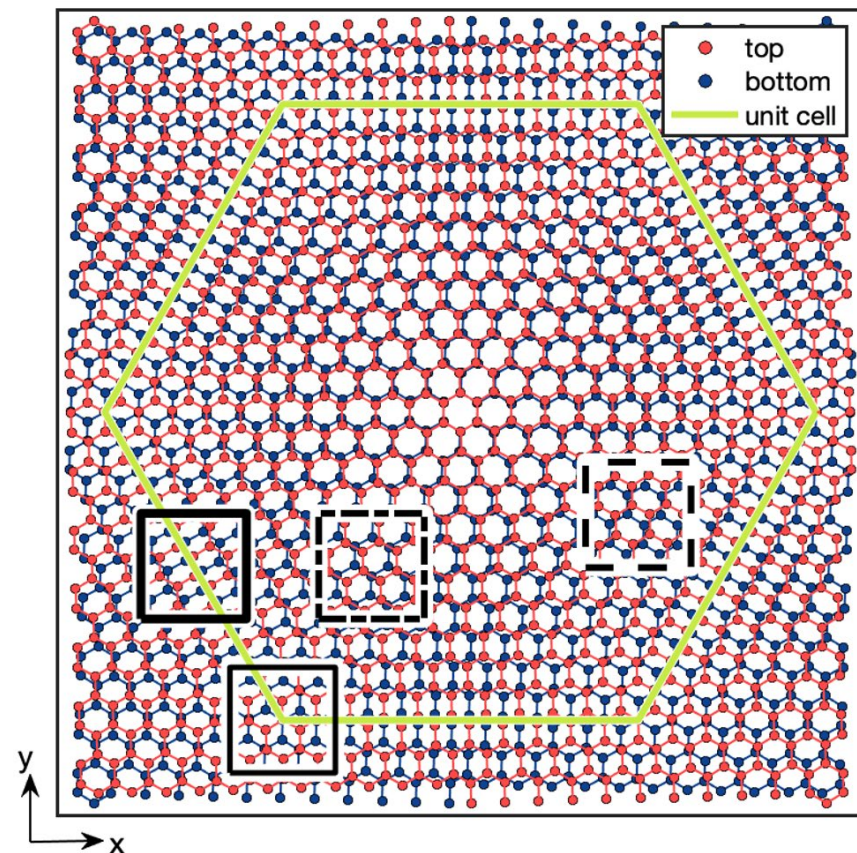
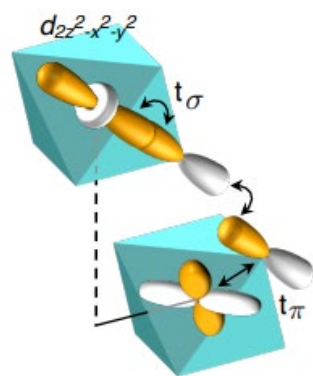
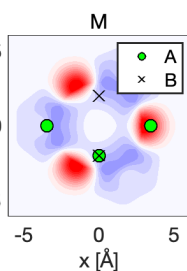
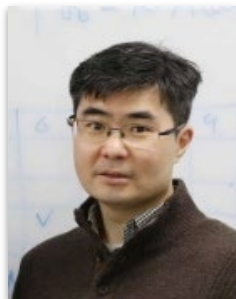
Do Hun Kim  
(KAIST)



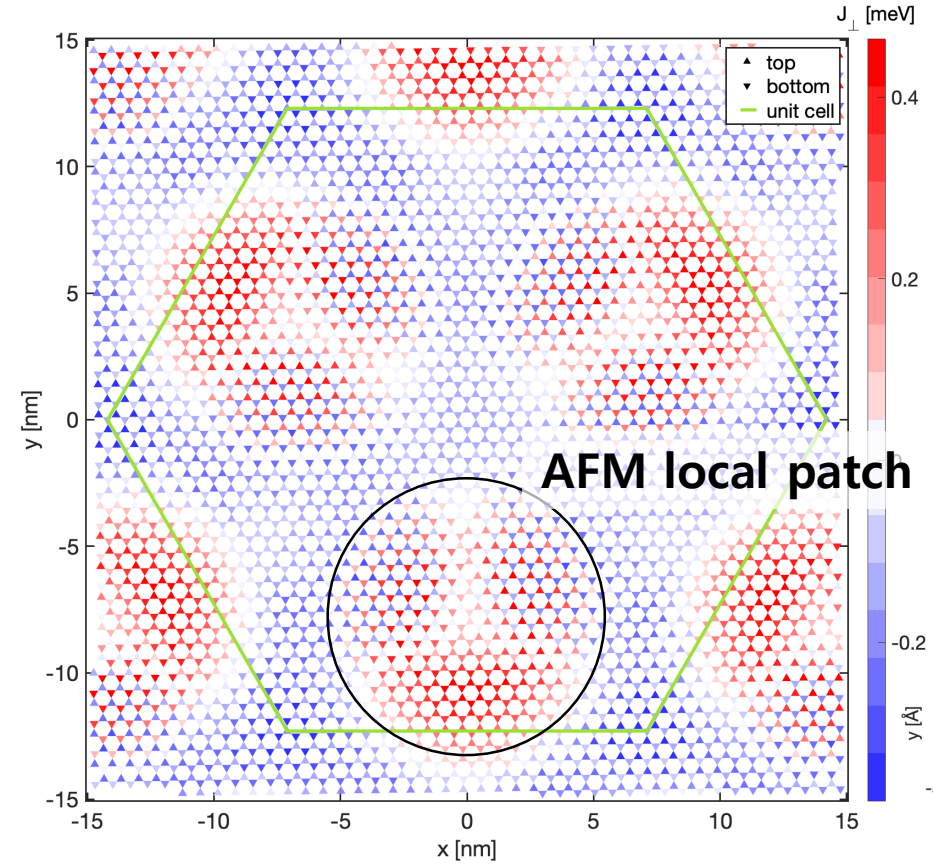
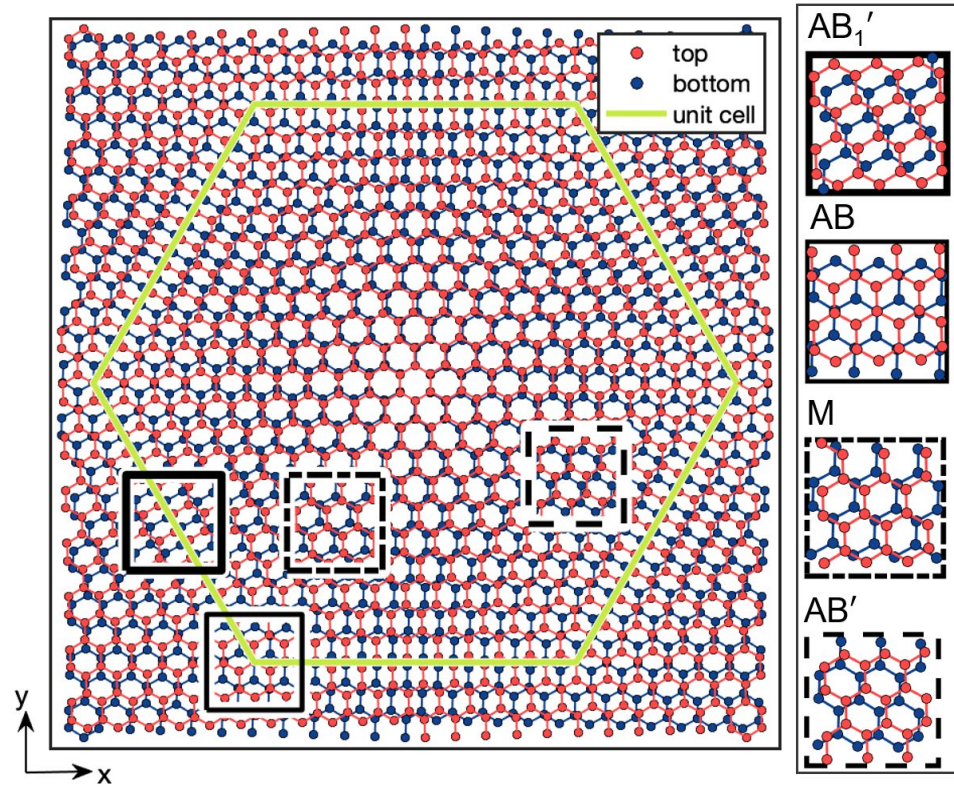
Kyoung-Min Kim  
(PCS-IBS)



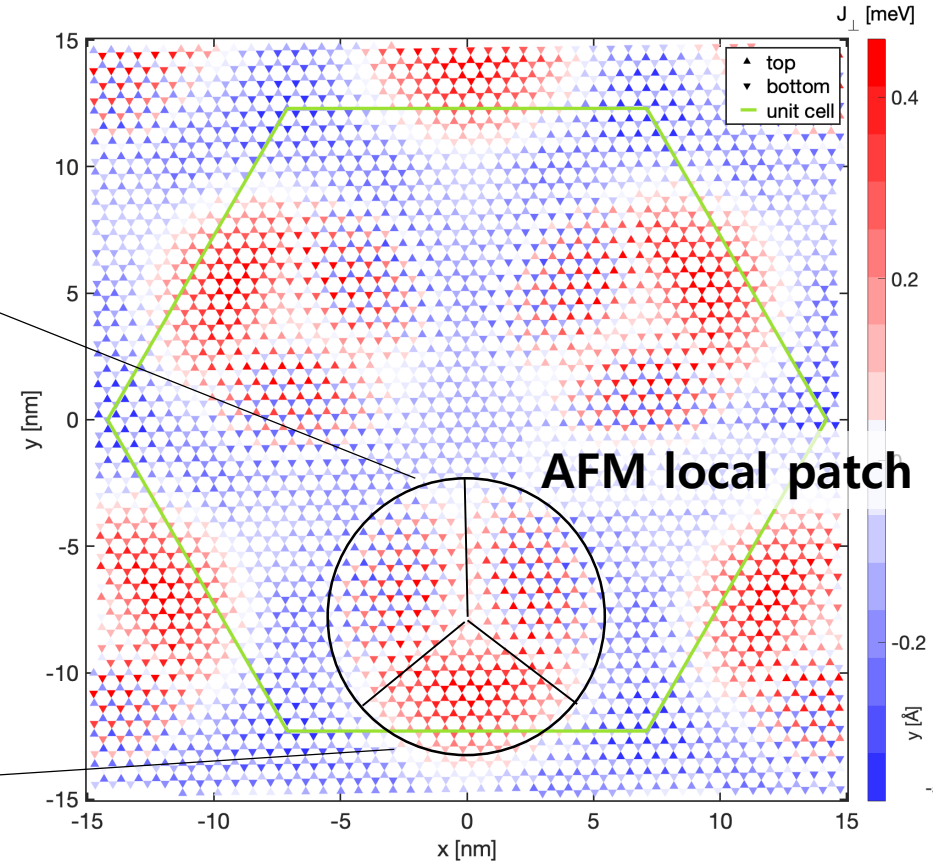
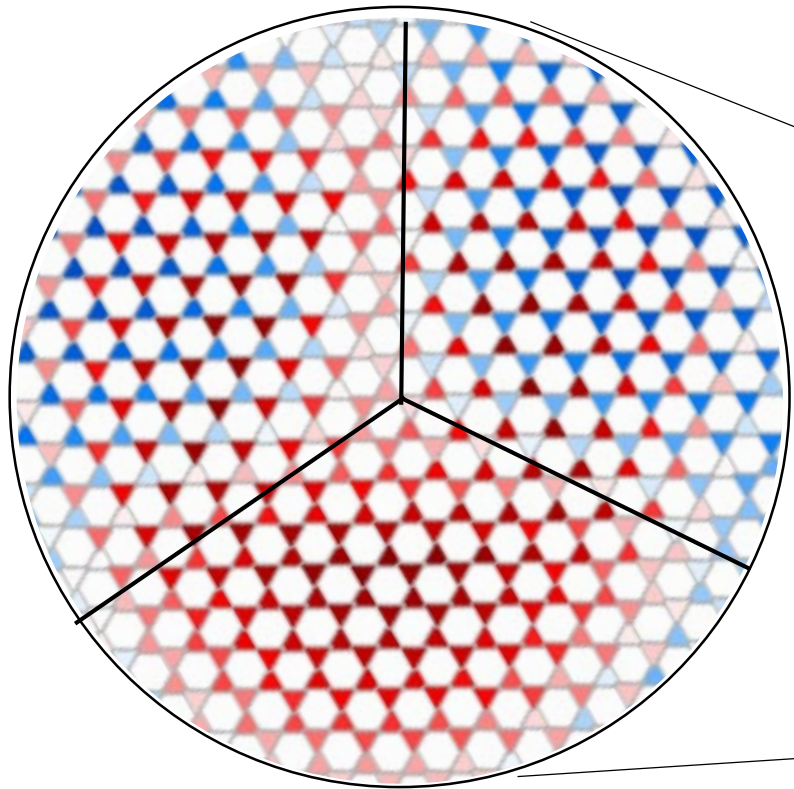
Myung Joon Han  
(KAIST)



# Local stacking structure



# Local stacking structure

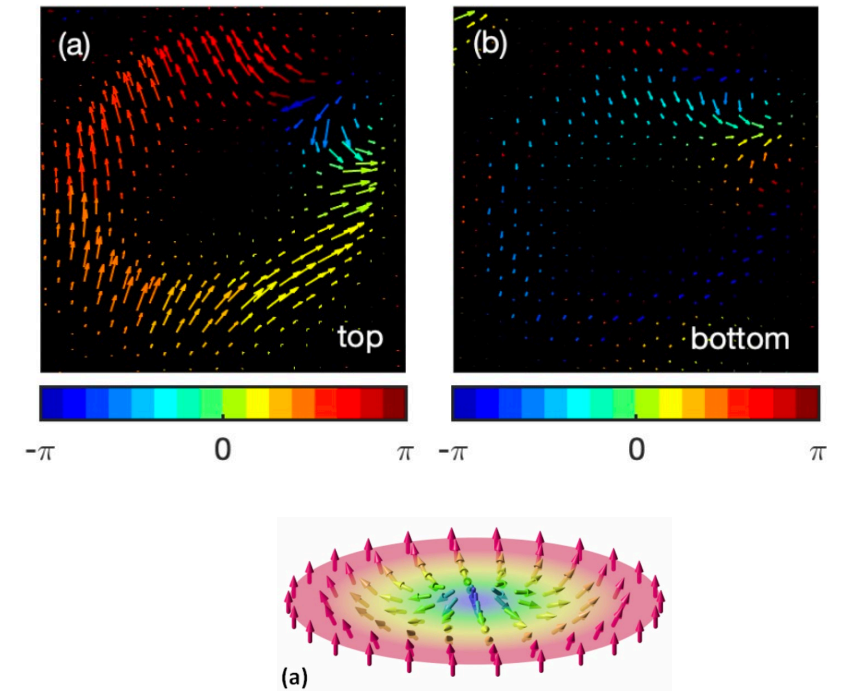
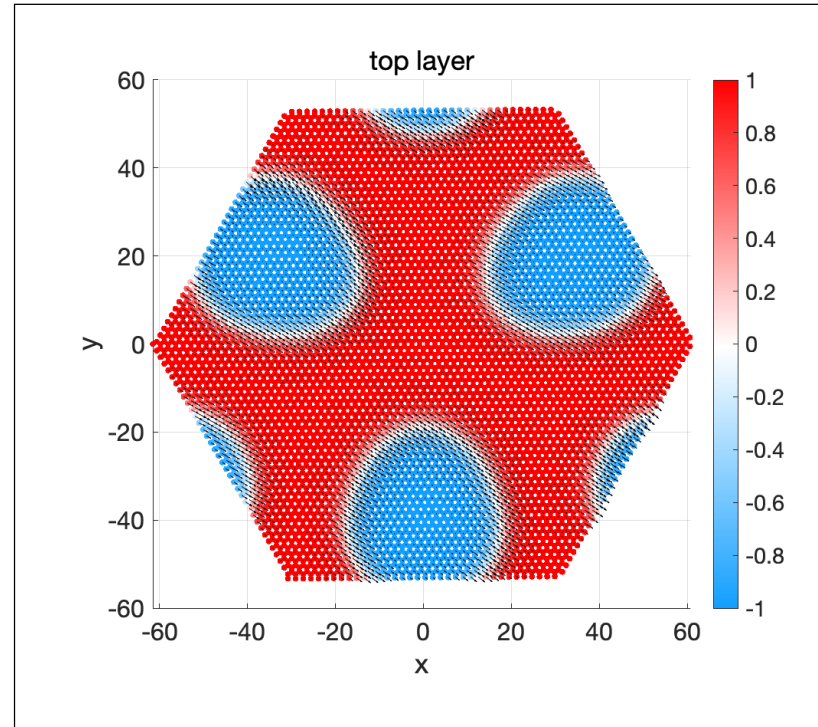
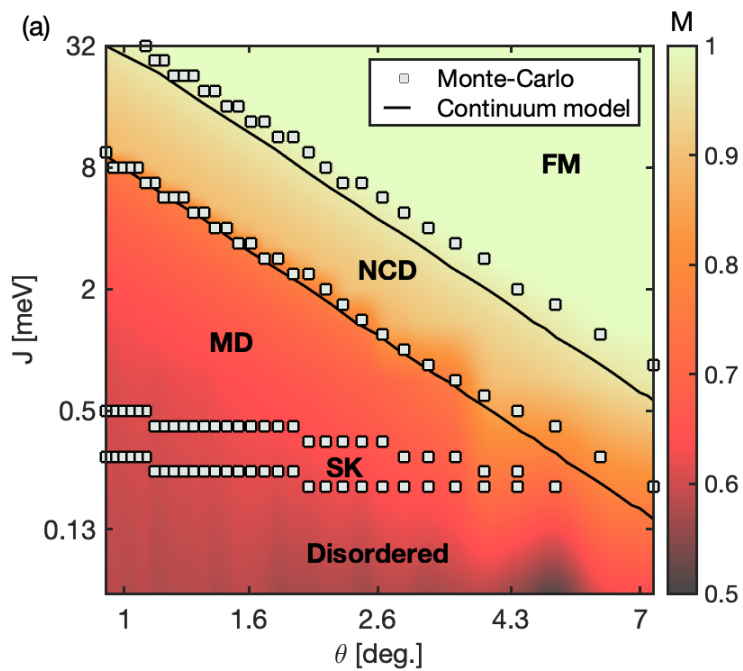


- Local FM and AFM interlayer coupling coexists.
- AB sublattice symmetry breaking.



# Skyrmion without DMI

## ➤ Skyrmions in moire superlattice



### Standard recipe for skyrmion :

Exchange + DMI + Magnetic field

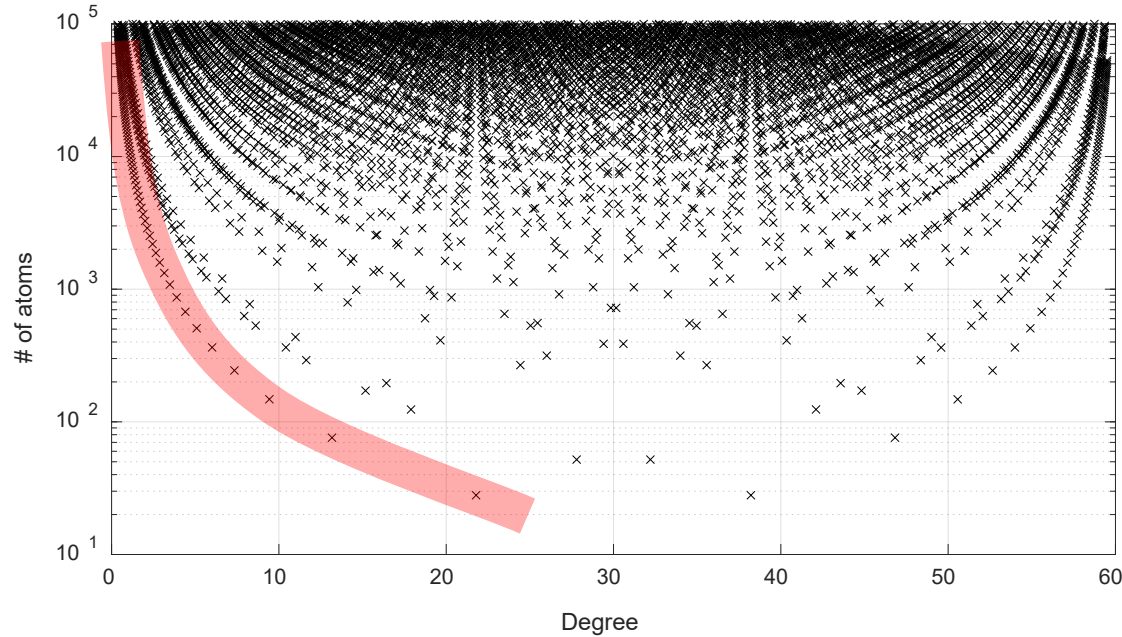
### In twisted bilayer magnets :

Exchange + Modulating interlayer coupling(sublattice breaking)

$$N = \int \vec{n} \cdot \left( \frac{\partial \vec{n}}{\partial x} \times \frac{\partial \vec{n}}{\partial y} \right) dA$$

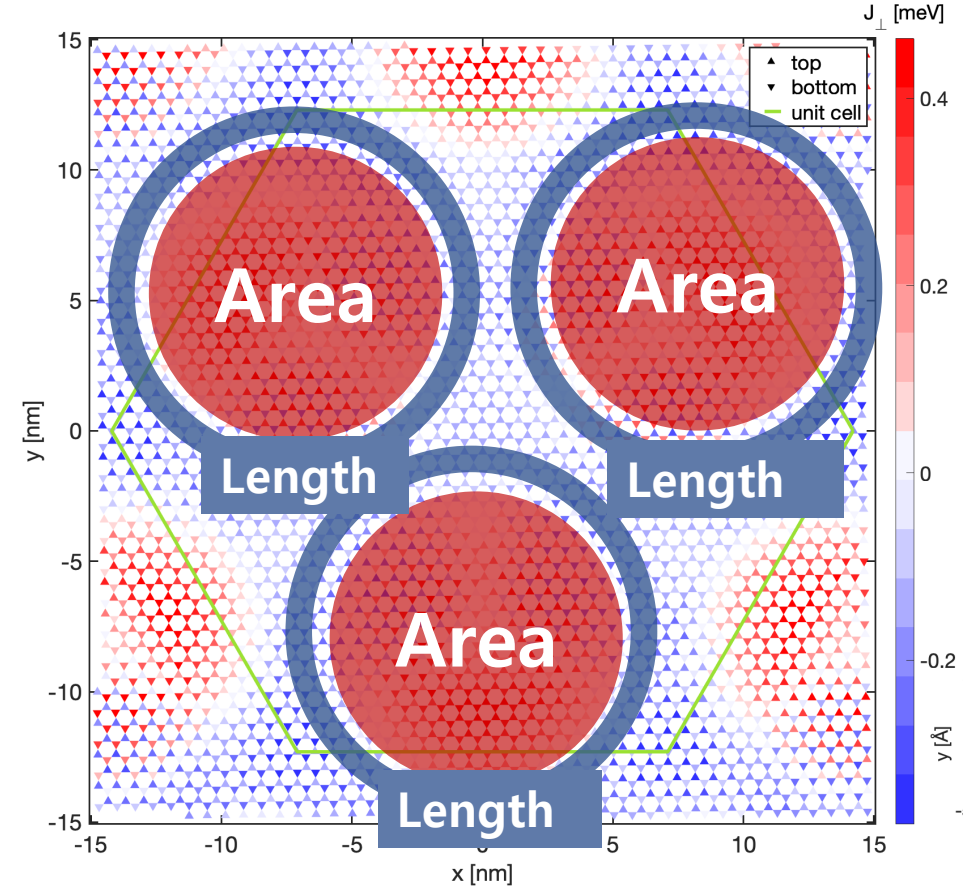
# Small-angle limit

## Moire size as a function of angle



Central observation of moire pattern

“as  $\theta \rightarrow 0$ , moire size diverges”



(Interlayer coupling) X (Area)  
~Spin gradient

VS

(Intralayer coupling) X (Length)  
~Collinear order

# Magnetic phase transition

## ➤ Landau theoretical description

Free energy functional:

$$F[\mathbf{n}_t, \mathbf{n}_b] = \sum_{l=t,b} \int d^2\mathbf{x} \left\{ \frac{3a_0^2}{2} J [\nabla_{\mathbf{x}} \mathbf{n}_l(\mathbf{x})]^2 - D_z [n_l^z(\mathbf{n}_l)]^2 \right\} + \int d^2\mathbf{x} \bar{J}_{\perp} \mathbf{n}_t(\mathbf{x}) \cdot \mathbf{n}_b(\mathbf{x}),$$

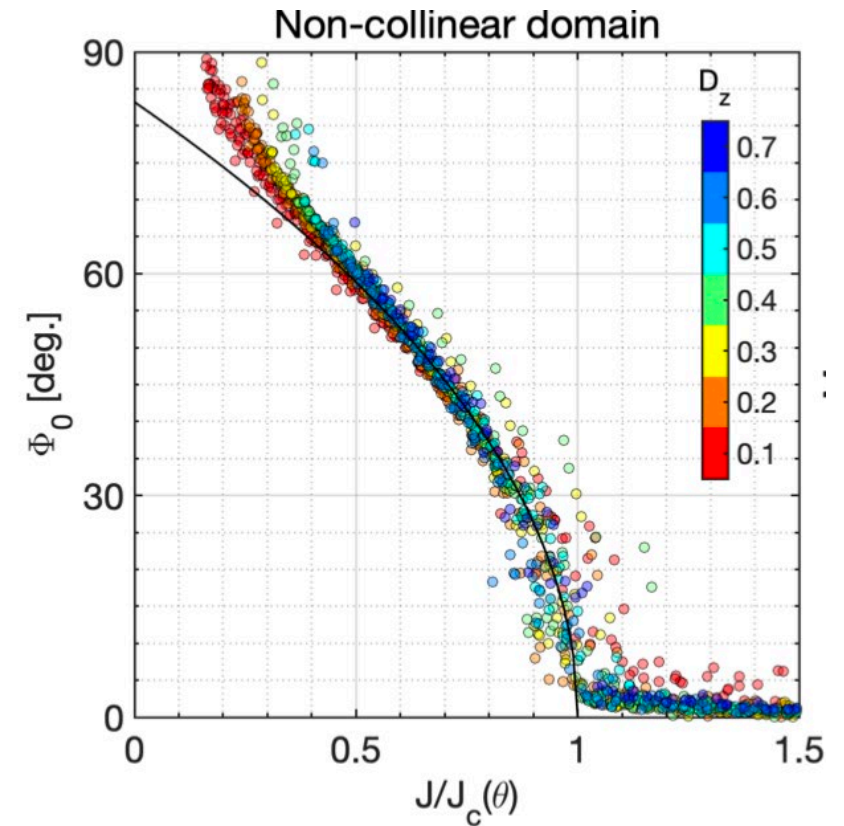
Continuum Ansatz:

$$\begin{aligned} \mathbf{n}_t &= (\sin \Phi_t, 0, \cos \Phi_t), \\ \mathbf{n}_b &= (-\sin \Phi_t, 0, \cos \Phi_t), \end{aligned}$$

Expansion:

$$F[\Phi_0] = N_{\text{ncd}}(\theta) (\bar{J}_{\perp} - 2D_z) + \frac{a}{2} [J - J_c(\theta)] \Phi_0^2 + \frac{b}{4} J_c(\theta) \Phi_0^4 + \mathcal{O}(\Phi_0^6)$$

$$\Phi_0 = \pm \sqrt{(a/b) [1 - J/J_c(\theta)]}$$



Conventional second order phase transitions as a function of tilt angle

# Magnetic phase transition II

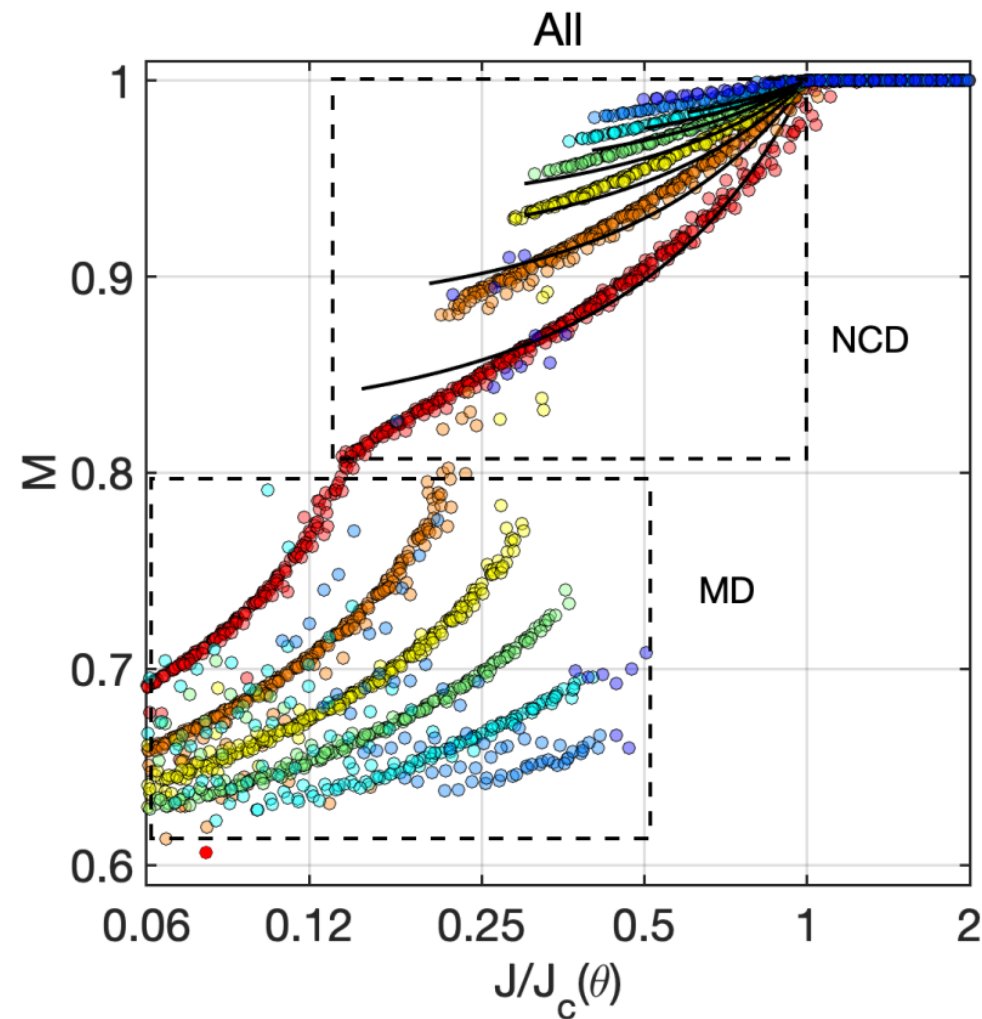
## ➤ Landau theoretical description

Continuum Ansatz:

$$\begin{aligned} \mathbf{n}_t &= (\sin \Phi_t, 0, \cos \Phi_t), & \mathbf{n}_t &= (\sin \Phi_t, 0, \cos \Phi_t), \\ \mathbf{n}_b &= (-\sin \Phi_t, 0, \cos \Phi_t), & \mathbf{n}_b &= (0, 0, 0), \end{aligned}$$

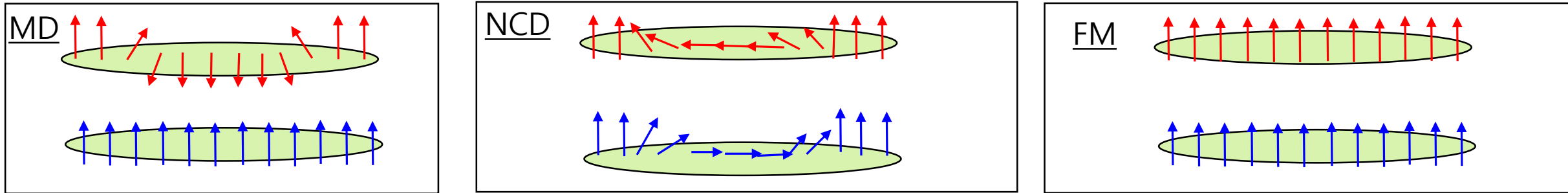
Expansion:

$$\begin{aligned} F[\Phi_0] &= N_{\text{ncd}}(\theta)(\bar{J}_\perp - 2D_z) \\ &\quad + \frac{a}{2}[J - J_c(\theta)]\Phi_0^2 + \frac{b}{4}J_c(\theta)\Phi_0^4 + \mathcal{O}(\Phi_0^6) \\ F[l] &= \frac{aJ\pi^2}{4} \left( \frac{2R}{l} - 1 \right) - D_z N_{\text{md}}(\theta) \left[ 1 + \frac{1}{2} \left( 1 - \frac{l}{R} \right)^2 \right] \\ &\quad - \bar{J}_\perp N_{\text{md}}(\theta) \left[ \left( 1 - \frac{l}{R} \right)^2 - \frac{4}{\pi^2} \left( \frac{l}{R} \right)^2 \right], \end{aligned} \quad (4.6)$$





# Competing scales of moire magnet



MD

NCD

Collinear FM



Second order phase transition

First order phase transition

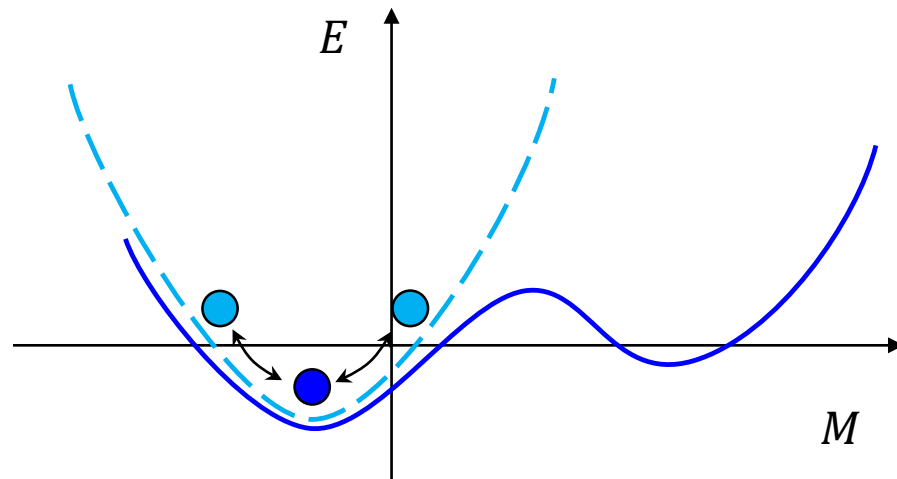
(Interlayer coupling) X (Area) ~Spin gradient VS (Intralayer coupling) X (Length) ~Collinear order

(Interlayer coupling) X (Area) ~NC2 VS (Single ion anisotropy) X (Area) ~NC1

- Magnetic phases
- Phase transitions
- Excitations

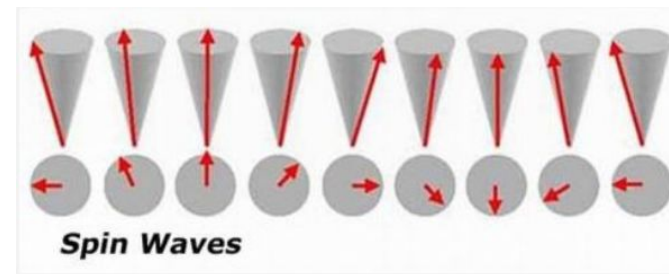
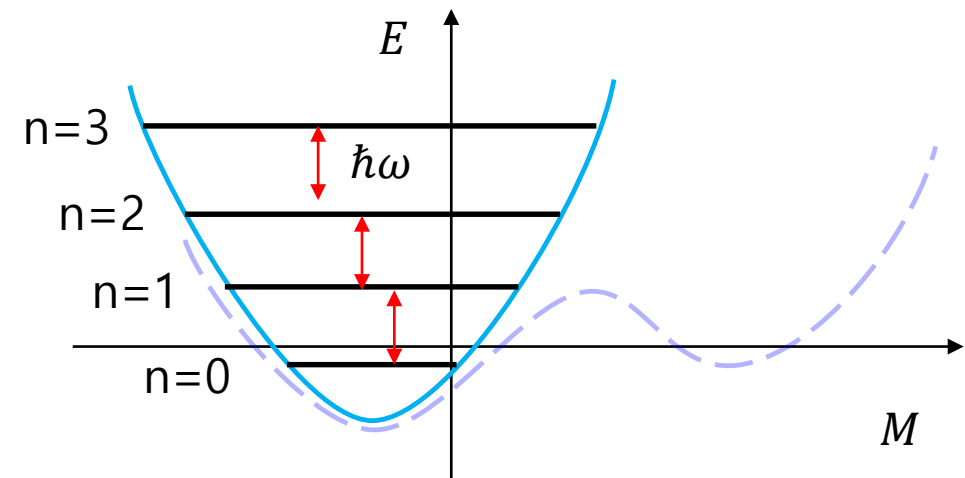
# Holestein-Primakoff Boson

## ➤ Global magnetic ground state

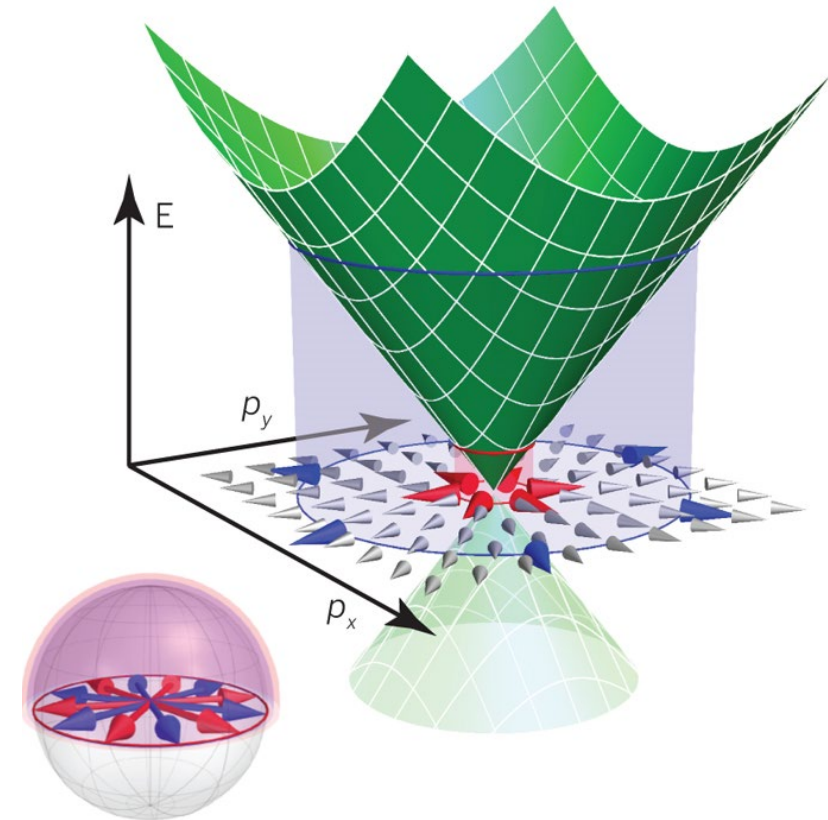
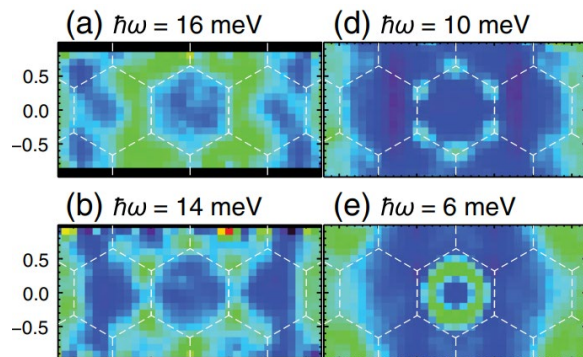
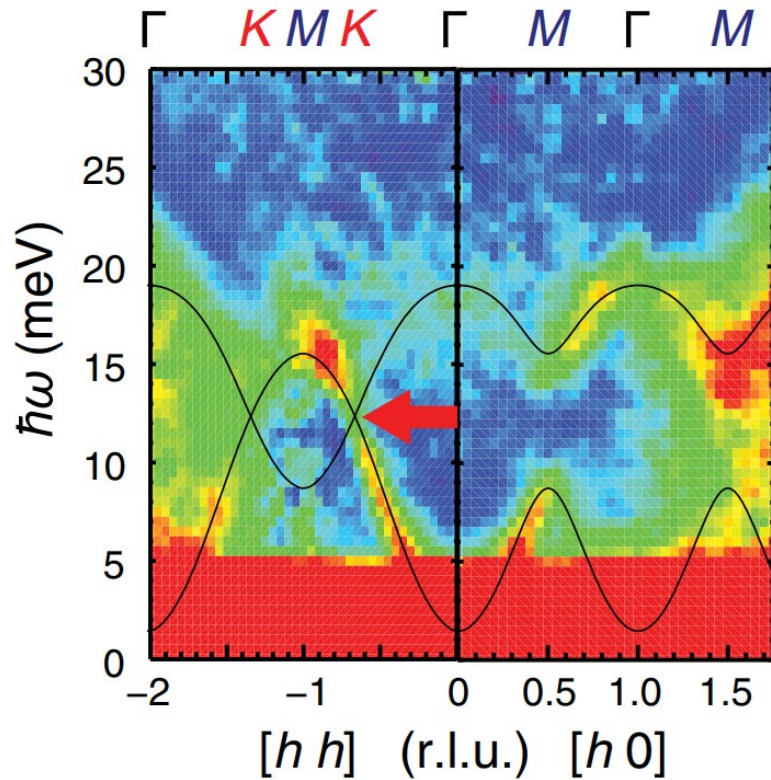


$$S^+ = \hbar\sqrt{2S}a^+\sqrt{1 - \frac{a^+a}{2S}},$$
$$S^- = \hbar\sqrt{2S}\sqrt{1 - \frac{a^+a}{2S}}a,$$
$$S^z = \hbar(a^+a - S),$$

## ➤ Local harmonic oscillator



# Dirac magnons

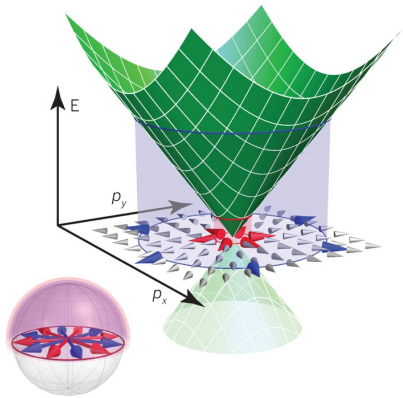


Dirac magnons are protected by coexistence of the following three symmetries.

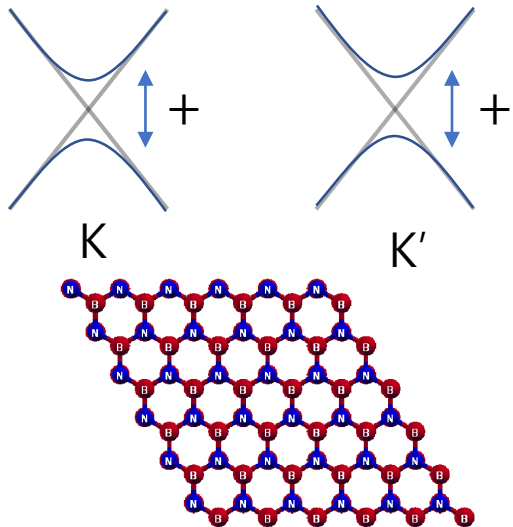
- **U(1)<sub>c</sub> symmetry**  
(Collinearity)
- **U(1)<sub>v</sub> symmetry**  
(Valley decoupling)
- **C<sub>2z</sub> symmetry**  
(Lateral shift)

# Topological magnons

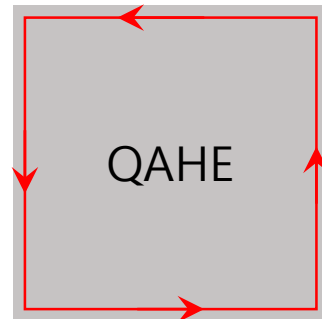
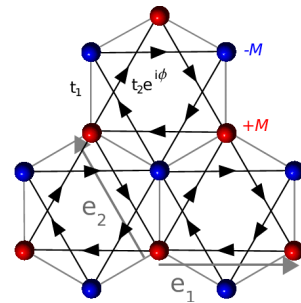
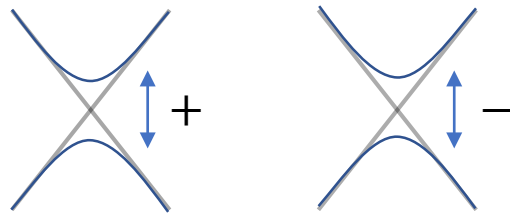
➤  $\{U(1)_C, U(1)_V, C_{2Z}\}$  symmetry



➤  $C_{2Z}$ -breaking

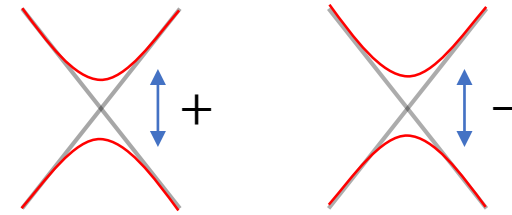


➤  $SO(3)$  breaking Spin-space group (DMI)

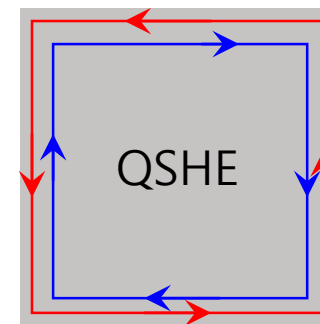
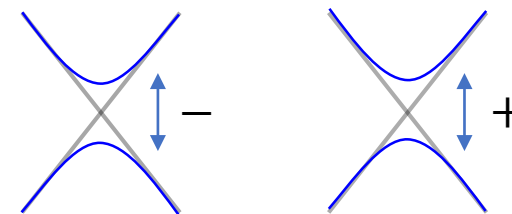


➤ T-preserving (Spin-orbit coupling)

Spin-up

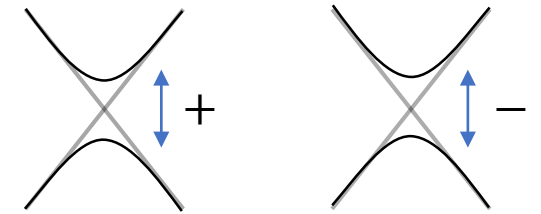


Spin-down

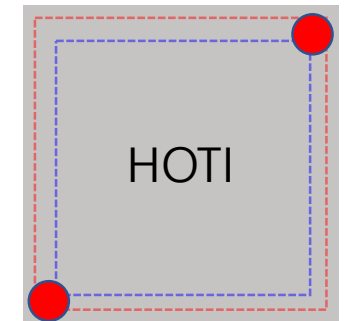
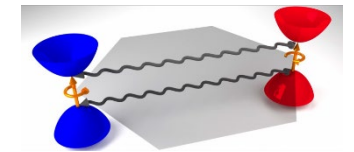
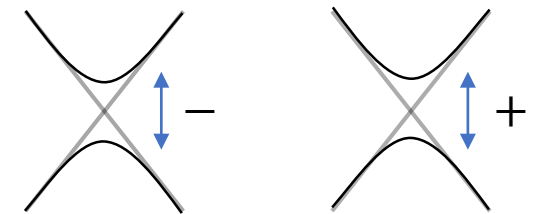


➤  $U(1)_V$ -breaking (Interlayer coupling)

Top-layer

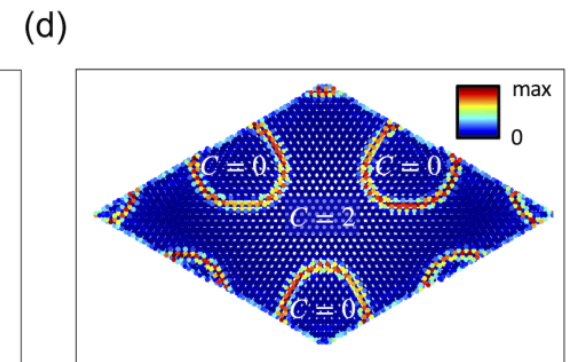
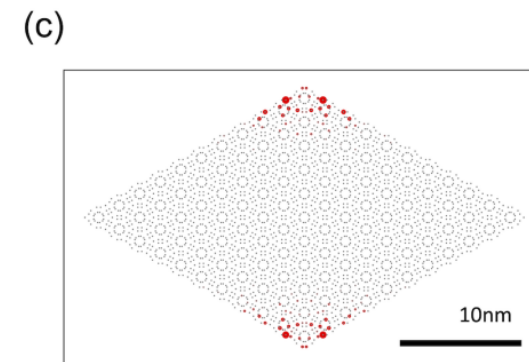
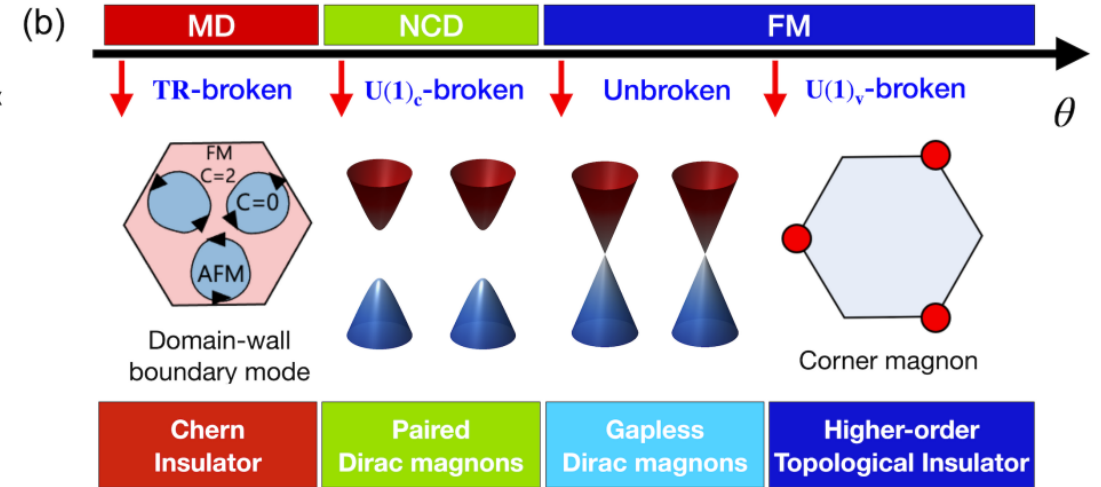
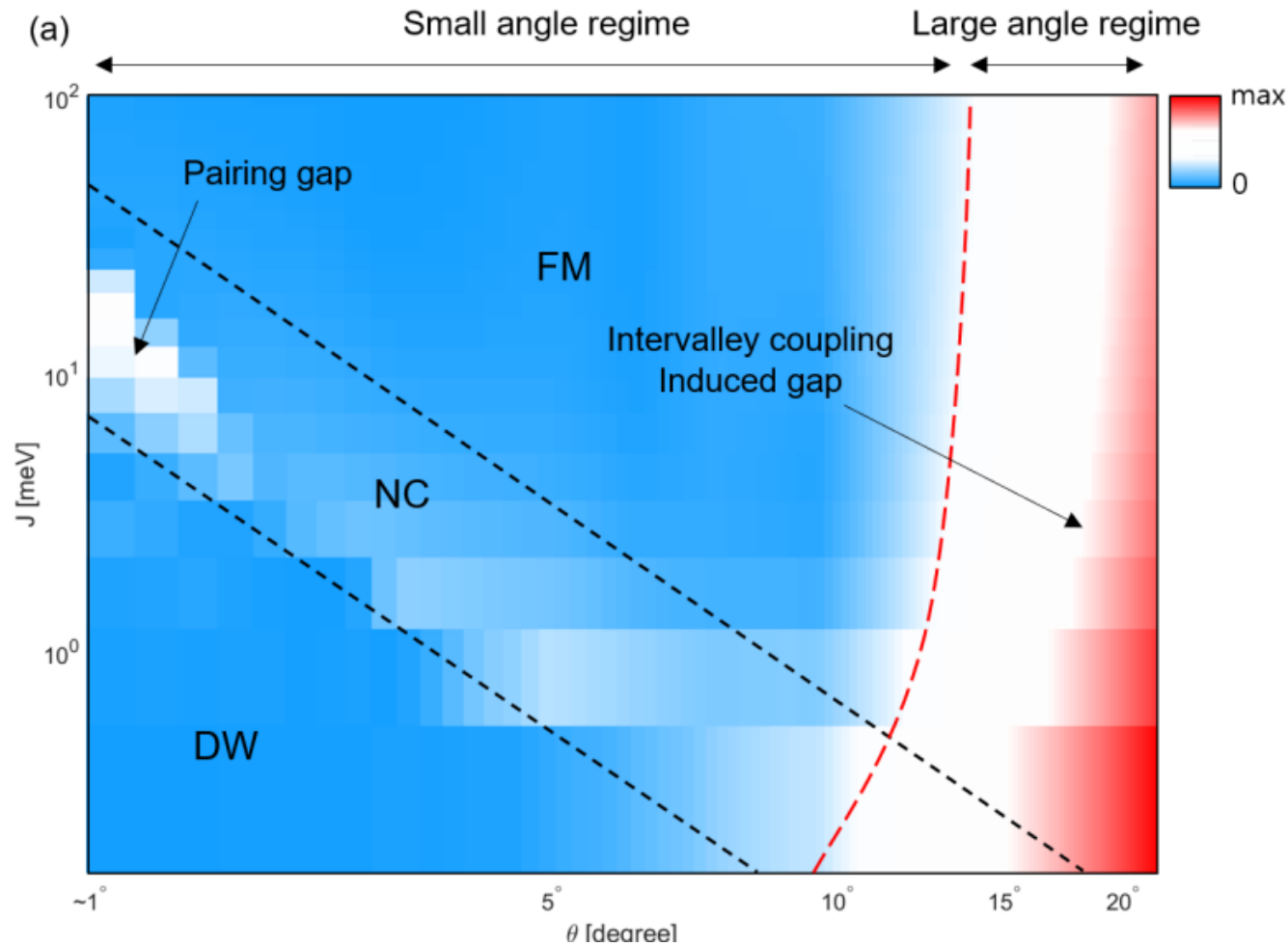


Bottom-layer



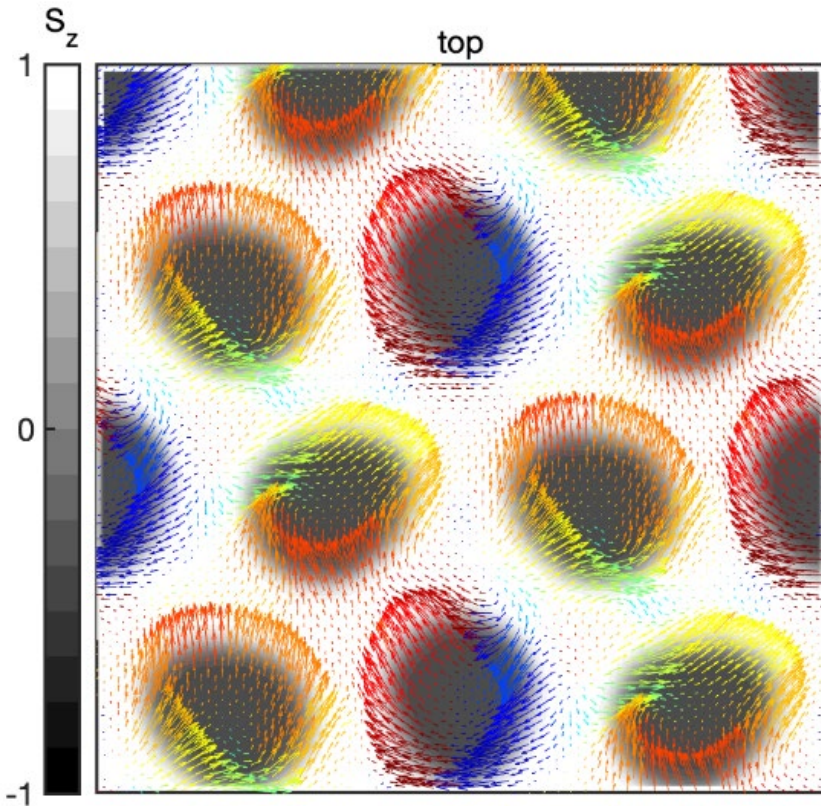
# Magnon phase diagram

Different magnon gaps are realized as a function of twist angles.

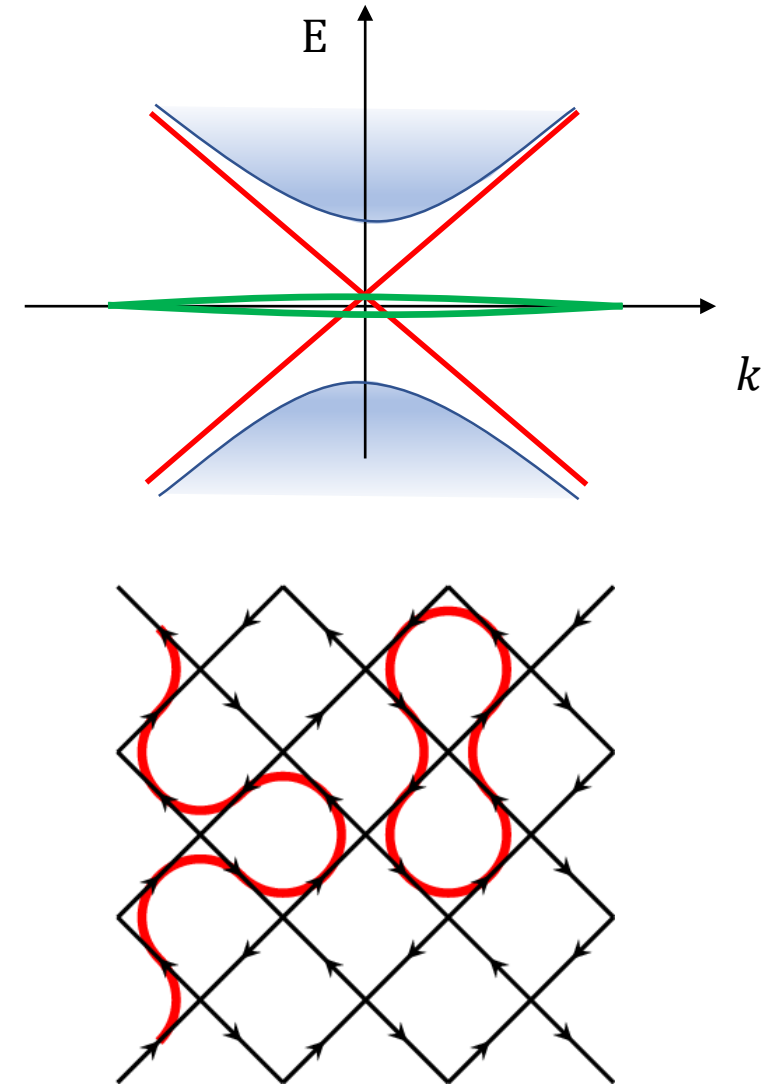
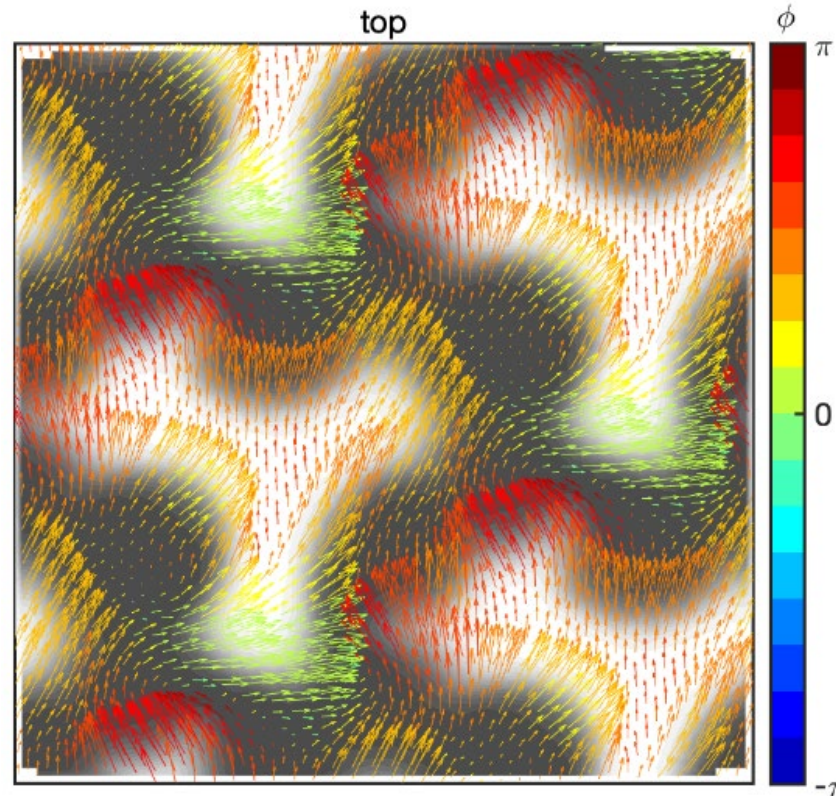


# Chalker-Coddington network

➤ Localized edge  
(Large angle)

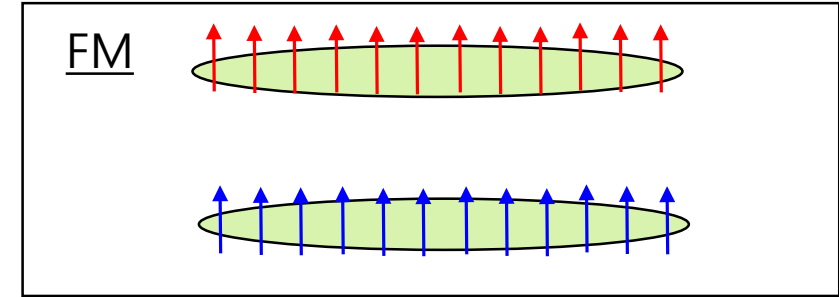
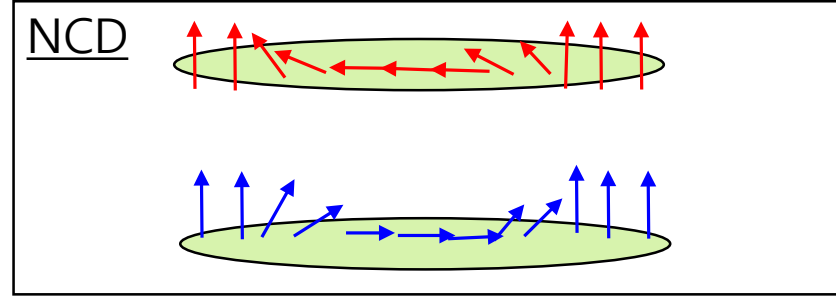
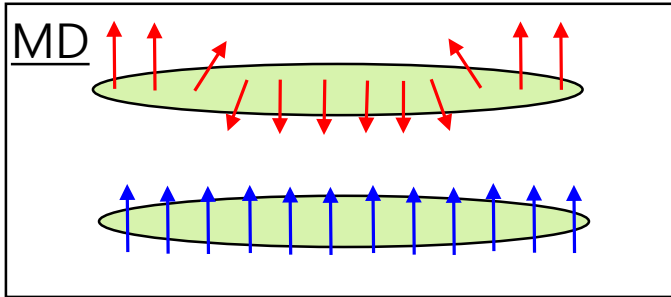


➤ Delocalized edge  
(Small angle)



# Overall Structure of moire magnets

## Magnetic phases



## Phase transitions

First order phase transition

(Interlayer coupling)  $\times$  (Area)  $\sim$  NC2 VS (Single ion anisotropy)  $\times$  (Area)  $\sim$  NC1

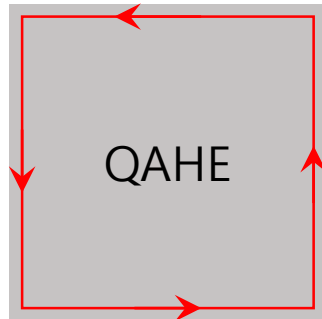
Second order phase transition

(Interlayer coupling)  $\times$  (Area)  $\sim$  Spin gradient VS (Intralayer coupling)  $\times$  (Length)  $\sim$  Collinear order

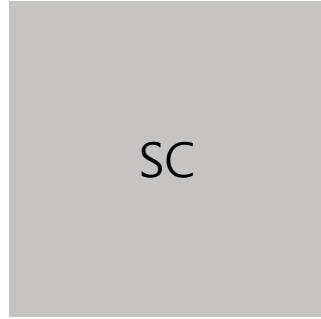


## Excitations

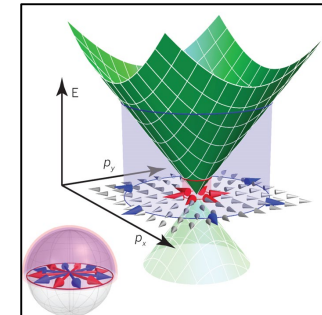
T-broken



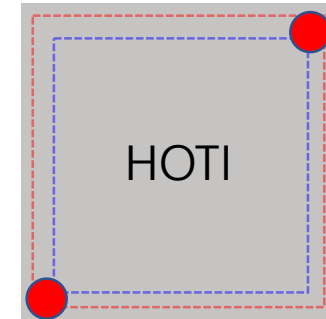
U(1)c broken



Three-symmetry preserved



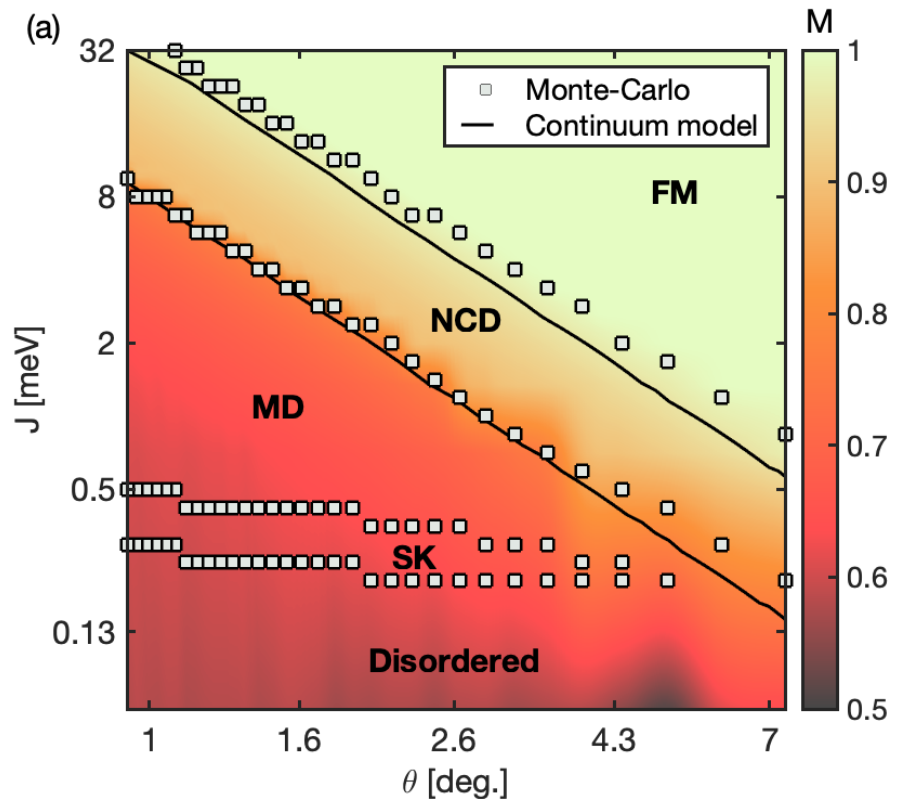
U(1)v broken





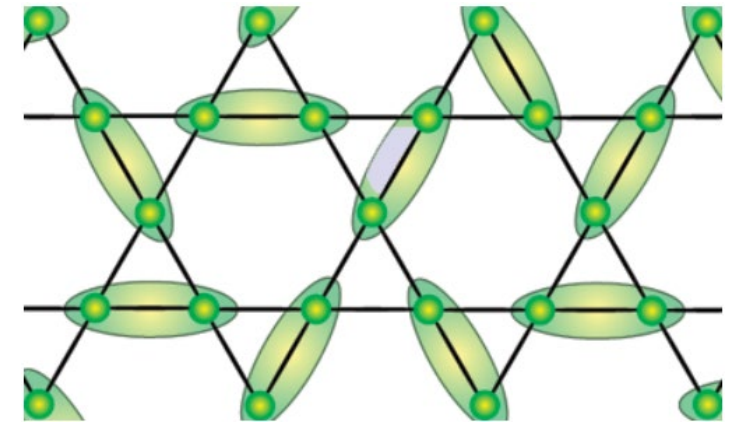
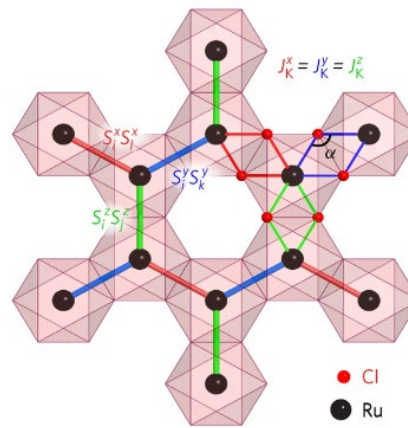
We extend theory of moire magnetism to various magnetic materials

➤ Development of Extensive Monte-Carlo methods

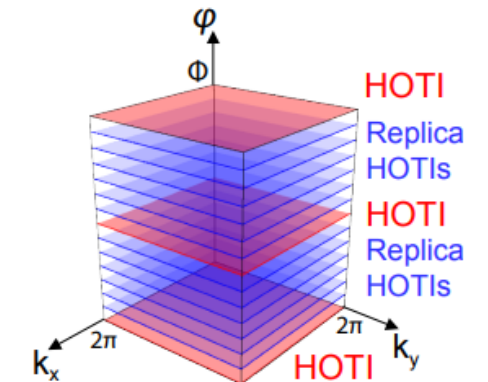
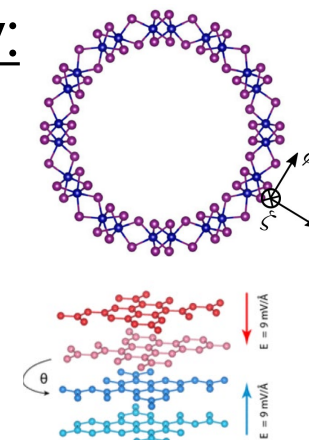
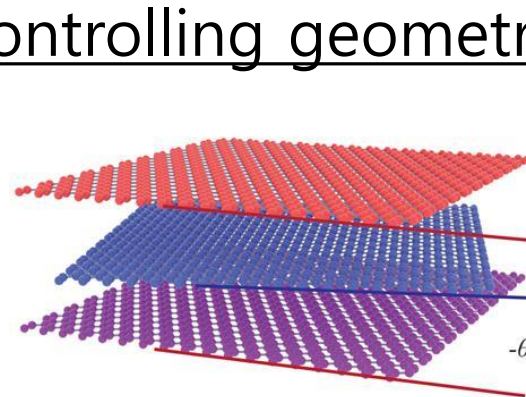


arXiv:2206.05264 (2022)

➤ Various magnetic materials :  
Spin liquid  $\alpha$ -RuCl<sub>3</sub> Magnetic TMDCs

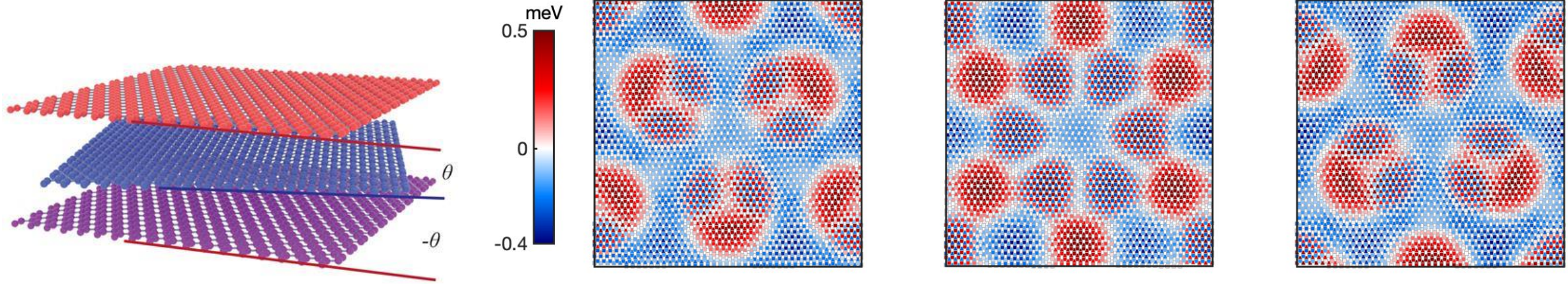


➤ Controlling geometry:

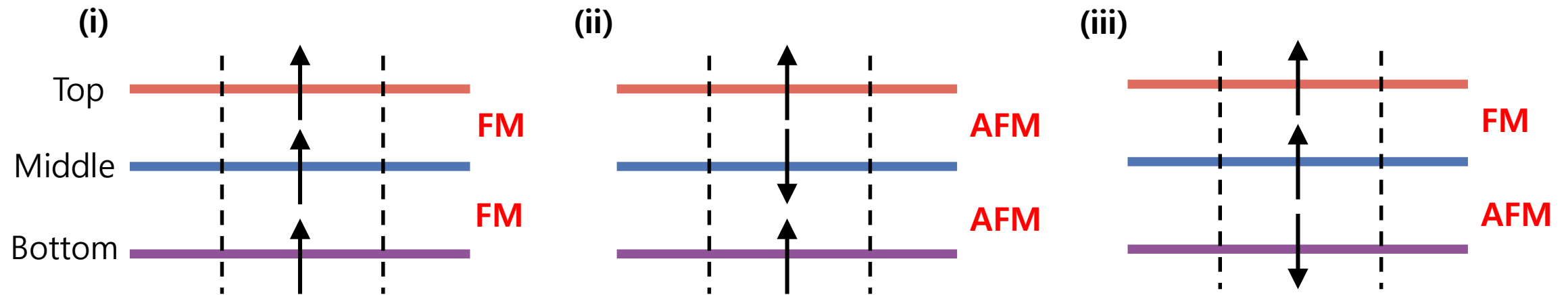


# Twisted trilayer Magnet

## ➤ Twisted trilayer CrI<sub>3</sub>:



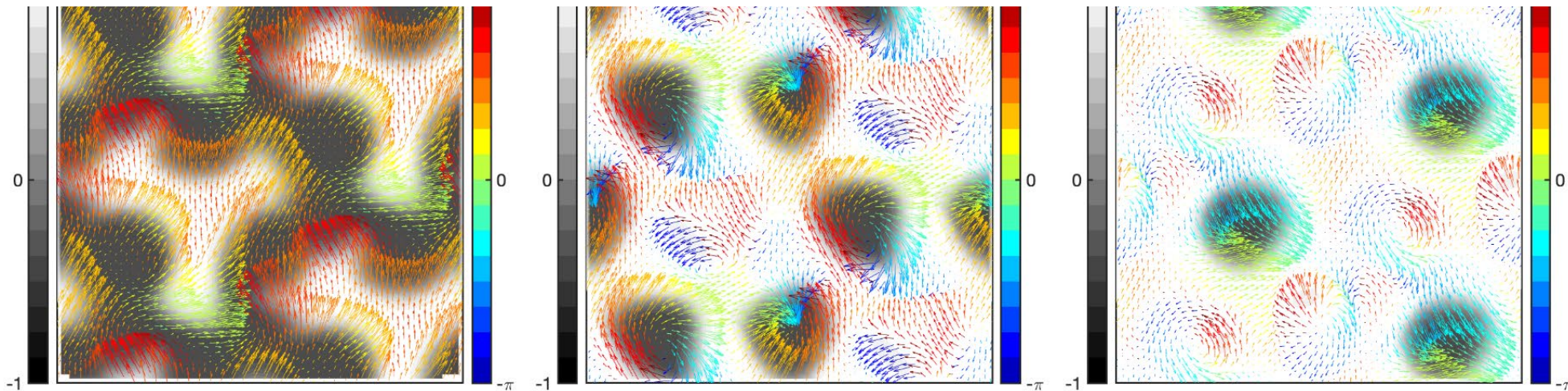
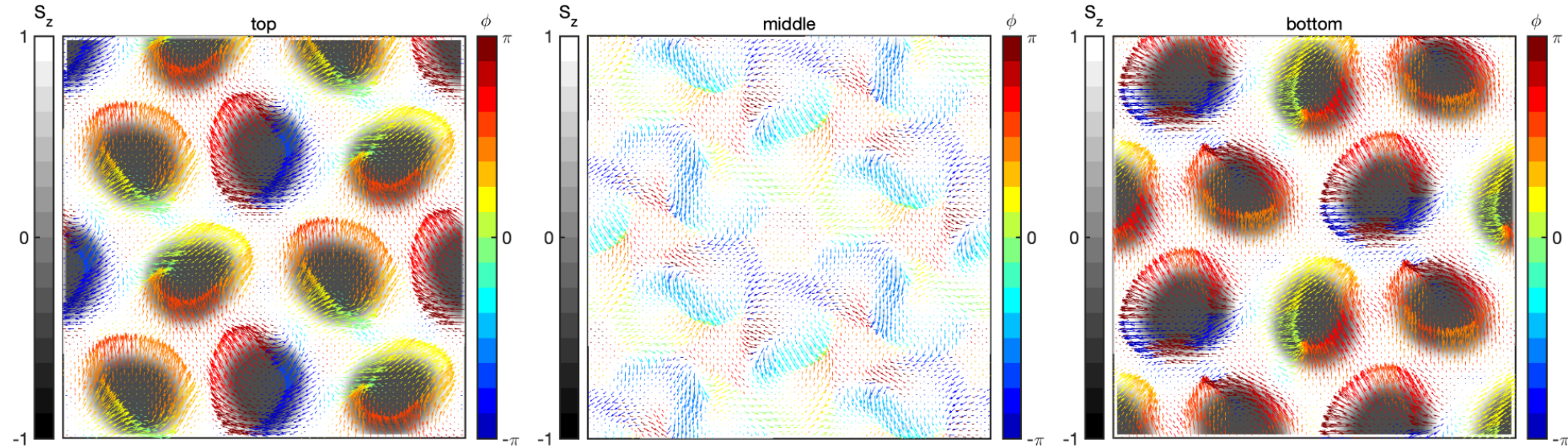
## ➤ Stacking dependent couplings



(In preparation)

# Twisted triple layer

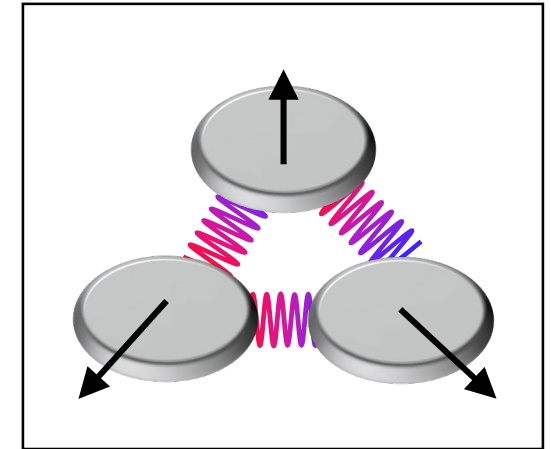
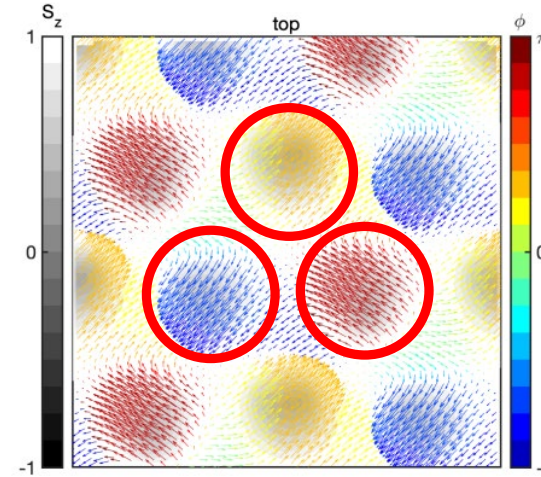
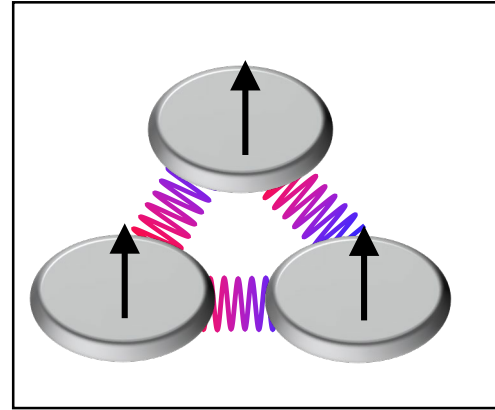
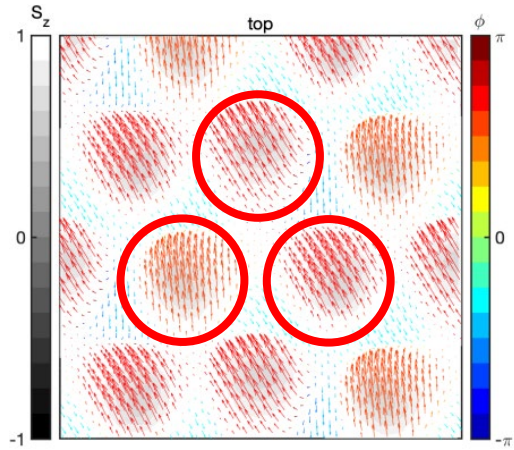
➤ Controlling geometry:



(In preparation)

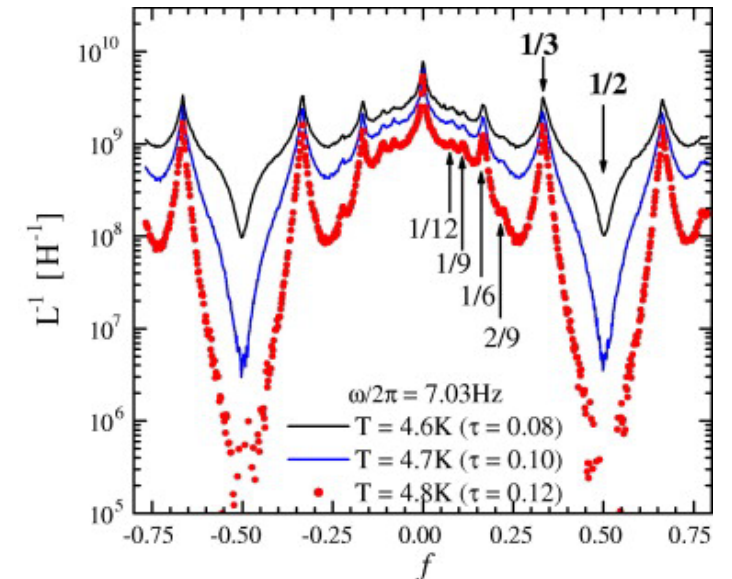
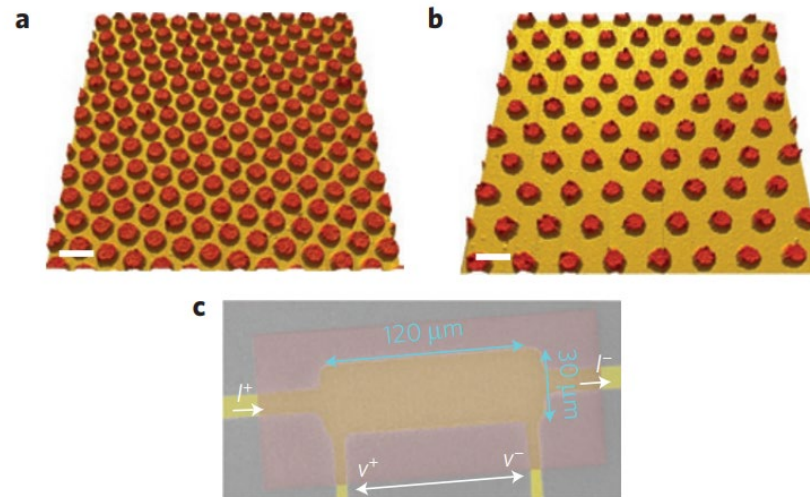
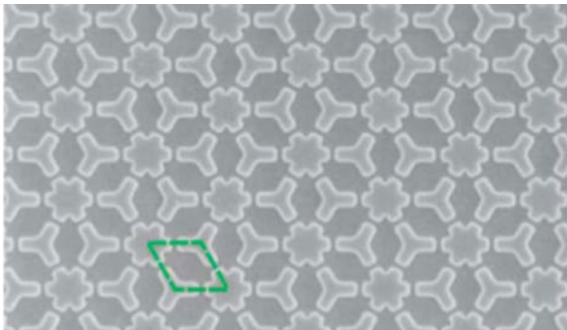
# Twisted triple layer

➤ C3 symmetry breaking order:



➤ Analogy with Josephson junction network:

$$H = -J \sum_P \sum_{\langle ij \rangle \in P} \cos(\theta_i - \theta_j)$$



Eley et al. Nature Physics (2011)

# Summary

## First order phase transition

(Interlayer coupling)  
~NC2

VS

(Single ion anisotropy)  
~NC1

## Second order phase transition

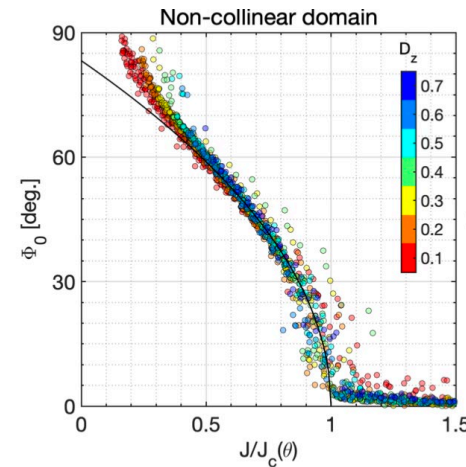
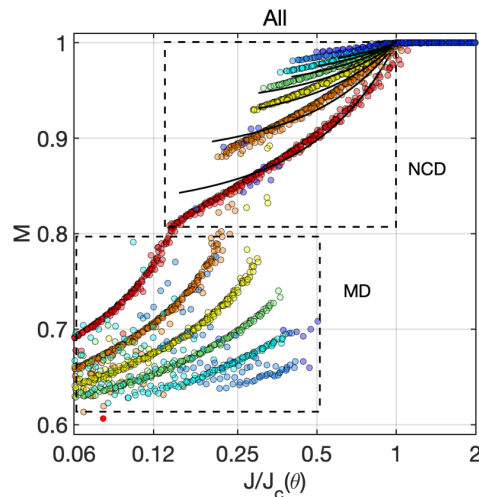
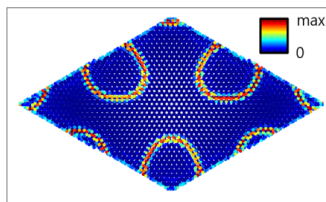
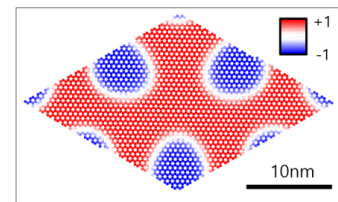
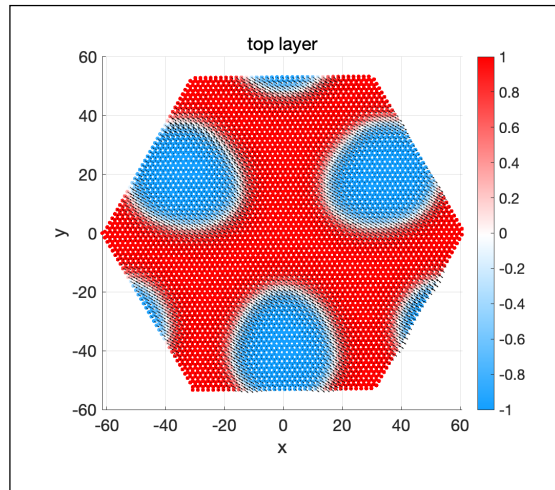
(Interlayer coupling) X (Area)  
~Spin gradient

VS

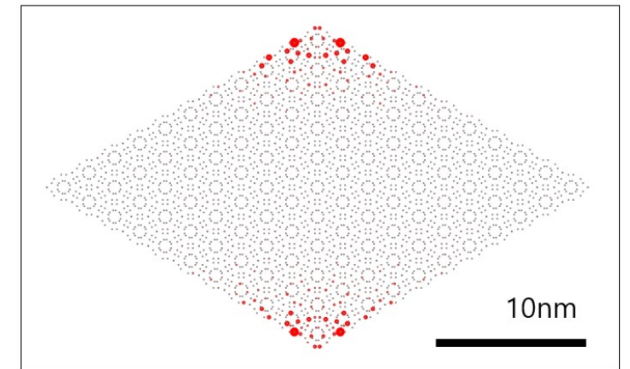
(Intralayer coupling) X (Length)  
~Collinear order



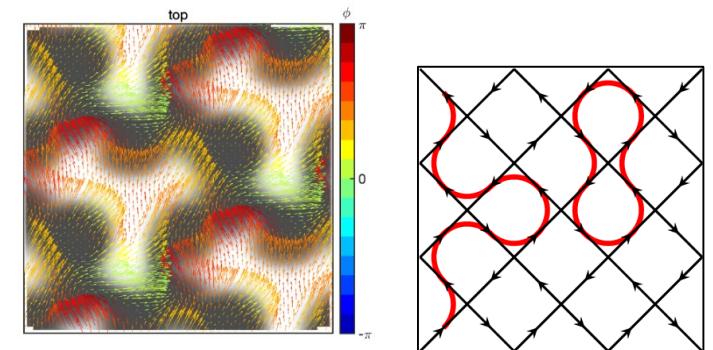
### Magnetic domains



### Higher-order topological magnons



### Topological magnon network



### Skyrmions

