Floquet engineering and Topological Nonlinear Optics

IBS-APCTP Conference
2022/09/19

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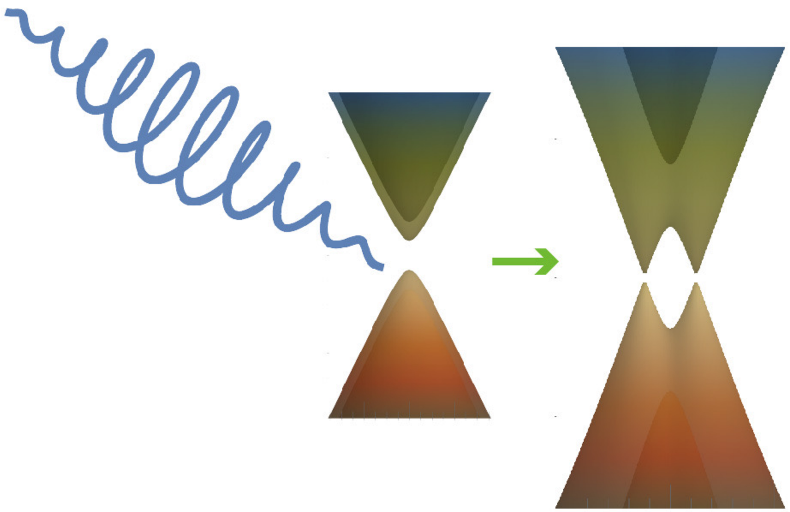
(Institute for Solid State Physics, The University of Tokyo）



picture around 2015

CREST “New developments in Topological Nonlinear Optics”

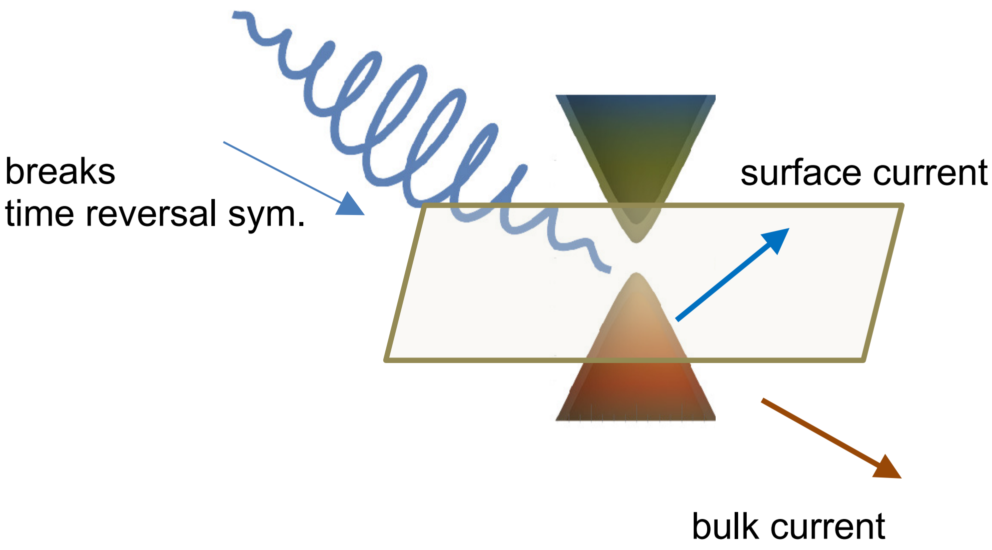
with R. Shimano (optics), M. Hayashi (spintronics), T. Morimoto (non-linear theory)

Introduction:

Quantum Materials + Laser

3D Dirac electron

circularly polarized laser (CPL)

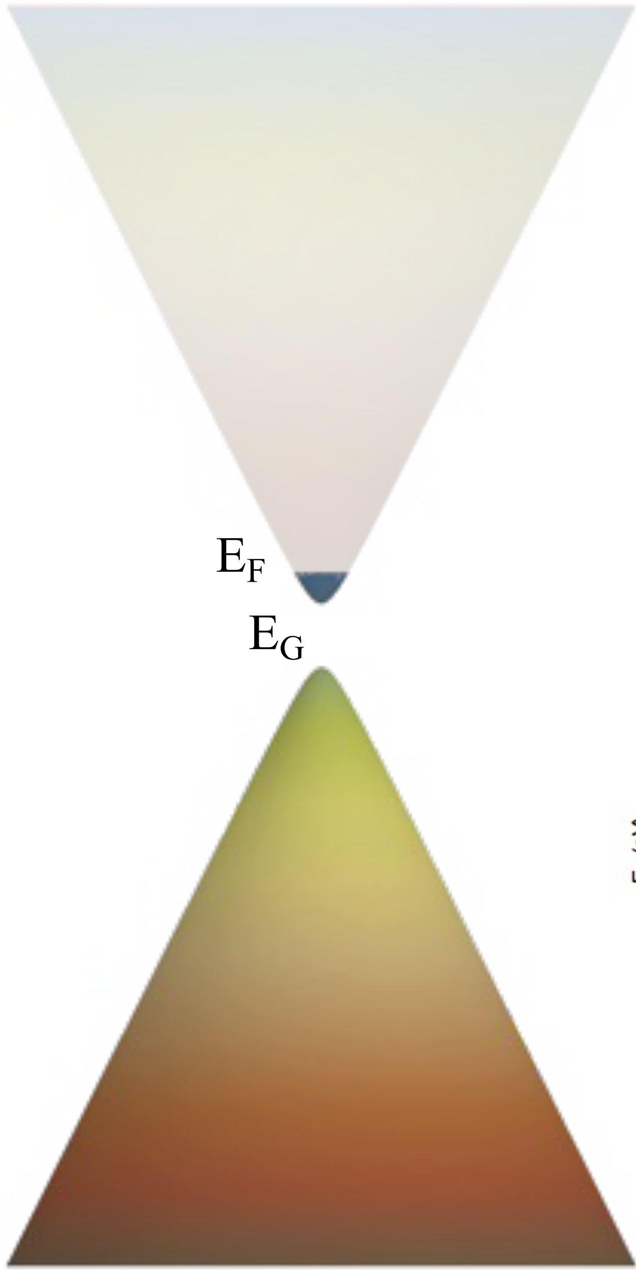


CPL-induced phenomena

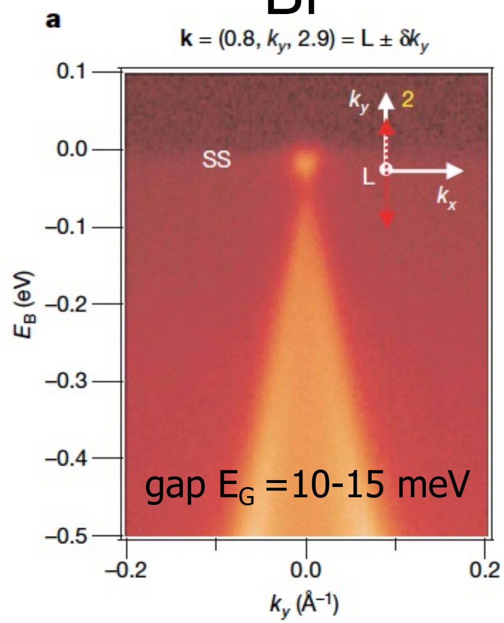
- Hall effect (Kerr rot.)
- bulk photo current
- surface current

⋮

3D Dirac electrons

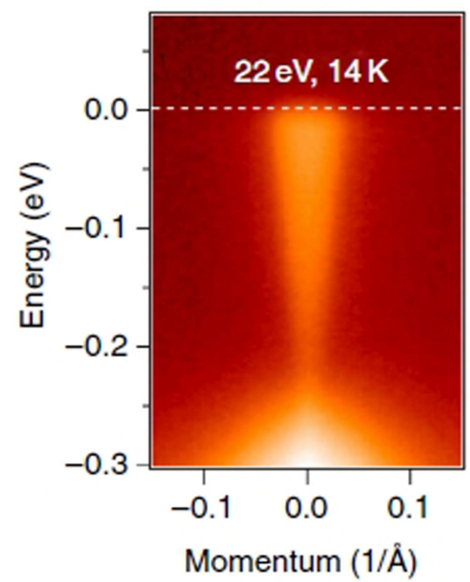


Bi



Hsieh, et al. Nature 2008

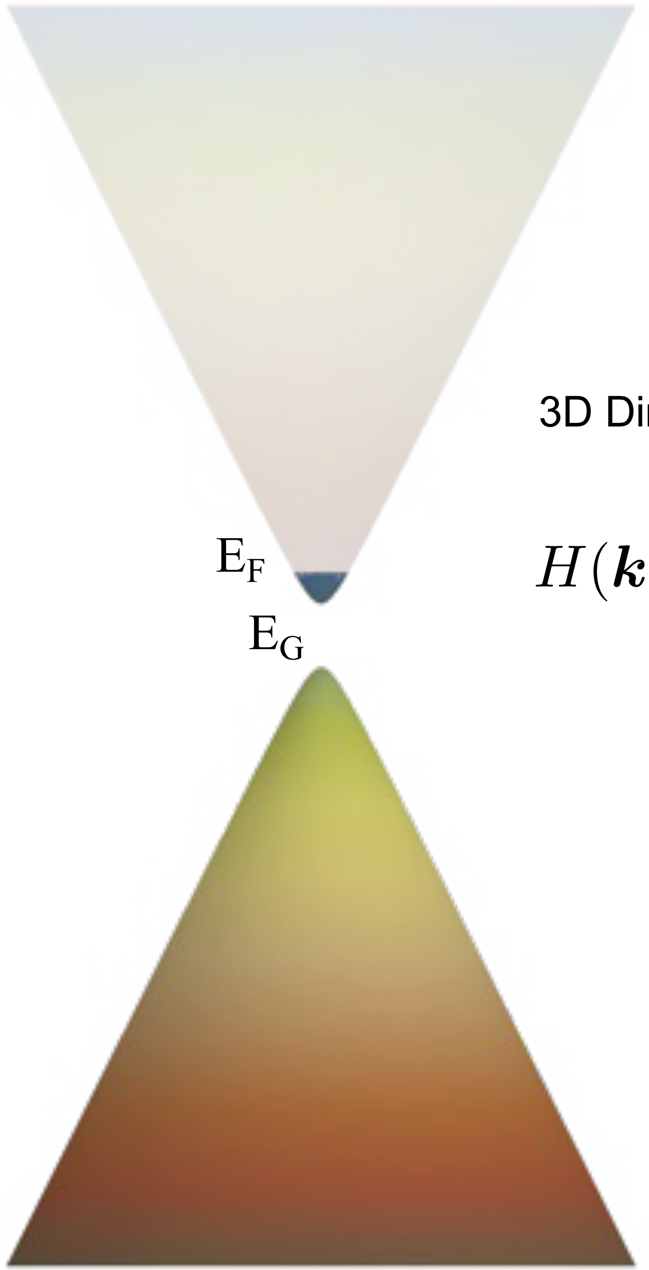
Cd_2As_3



Neupane, et al. Nat. Com. 2014

*Kane-Bodnar model

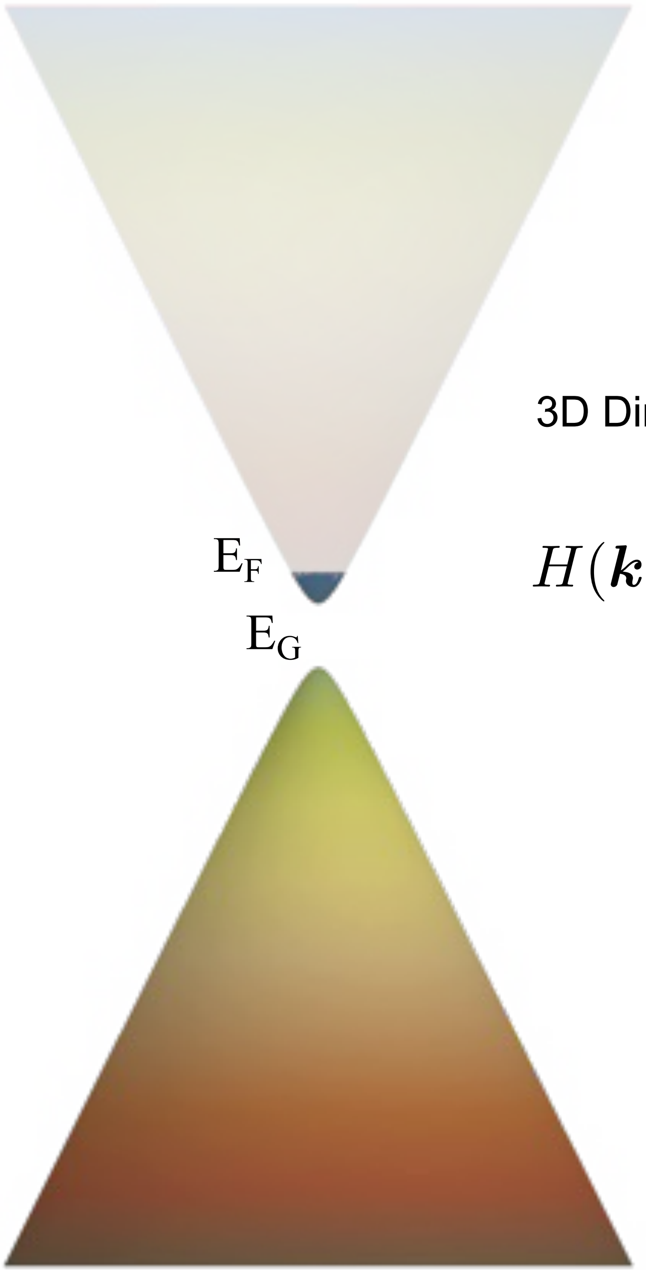
3D Dirac electrons



3D Dirac Hamiltonian (chiral basis)

$$H(\boldsymbol{k}) = \begin{pmatrix} -v\boldsymbol{\sigma}\cdot(\boldsymbol{k}-\boldsymbol{A}_5) & mI \\ mI & v\boldsymbol{\sigma}\cdot(\boldsymbol{k}+\boldsymbol{A}_5) \end{pmatrix}$$

3D Dirac electrons



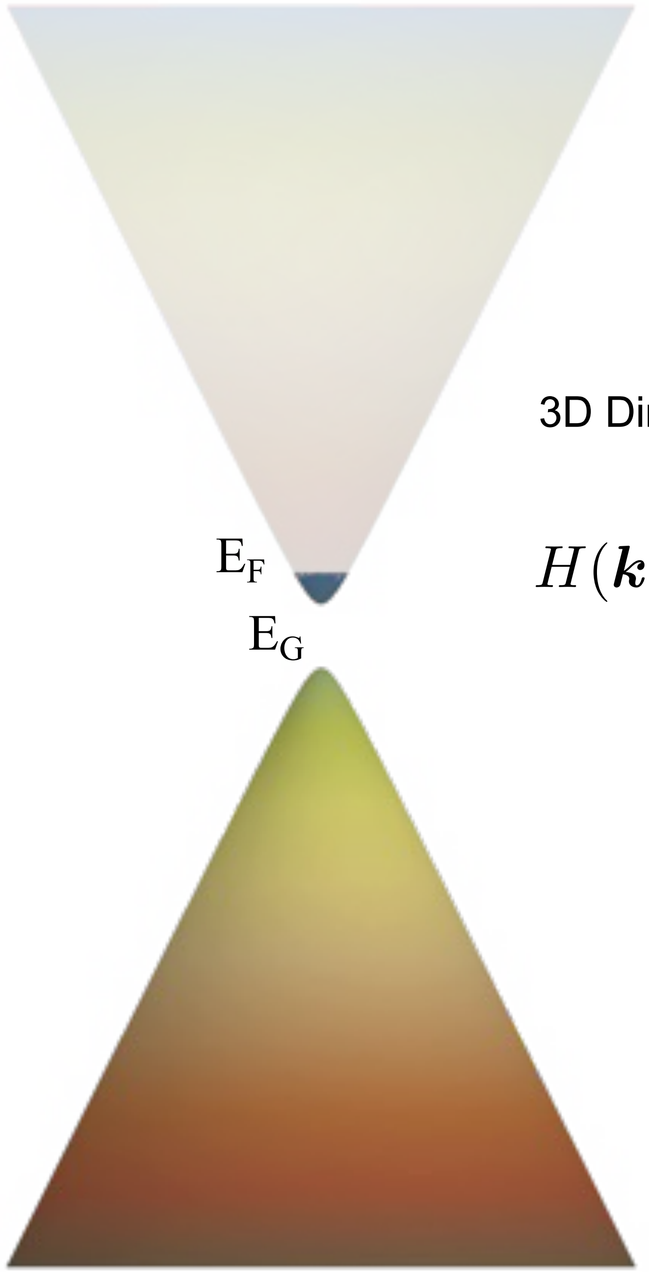
3D Dirac Hamiltonian (chiral basis)

Weyl electron (ξ=-)

$$H(\boldsymbol{k}) = \begin{pmatrix} -v\boldsymbol{\sigma}\cdot(\boldsymbol{k}-\boldsymbol{A}_5) & mI \\ mI & v\boldsymbol{\sigma}\cdot(\boldsymbol{k}+\boldsymbol{A}_5) \end{pmatrix}$$

Weyl electron (ξ=+)

3D Dirac electrons



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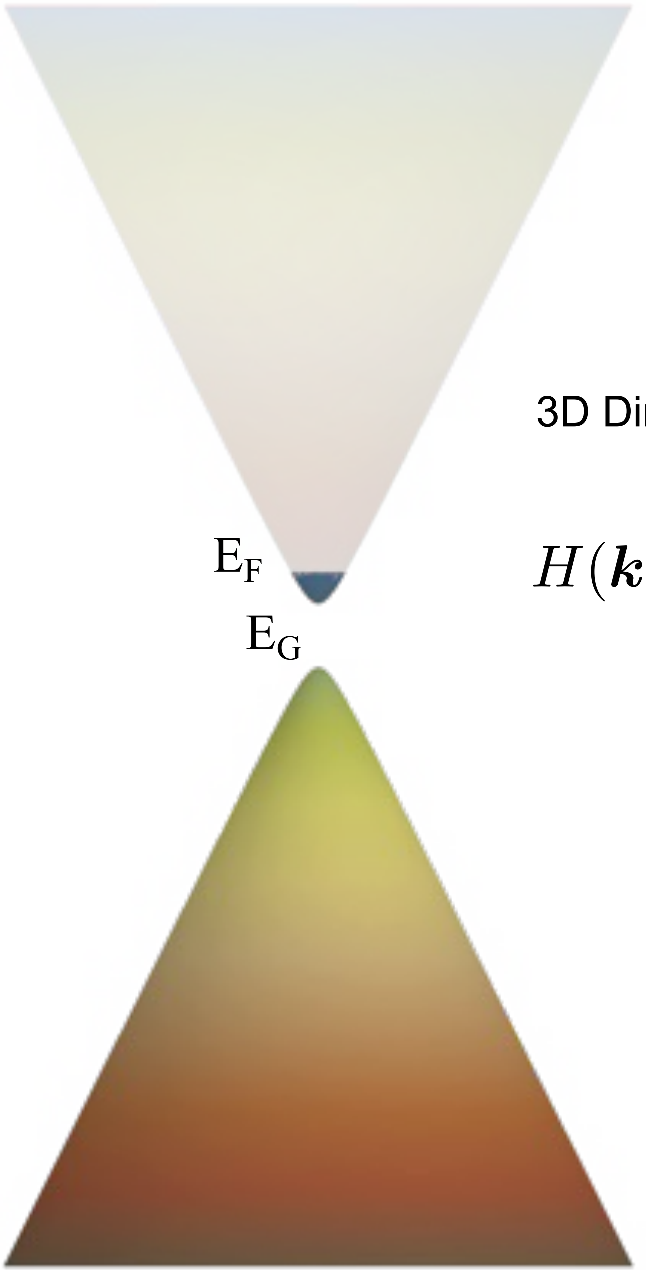
Dirac mass $E_G=2m$

3D Dirac electrons



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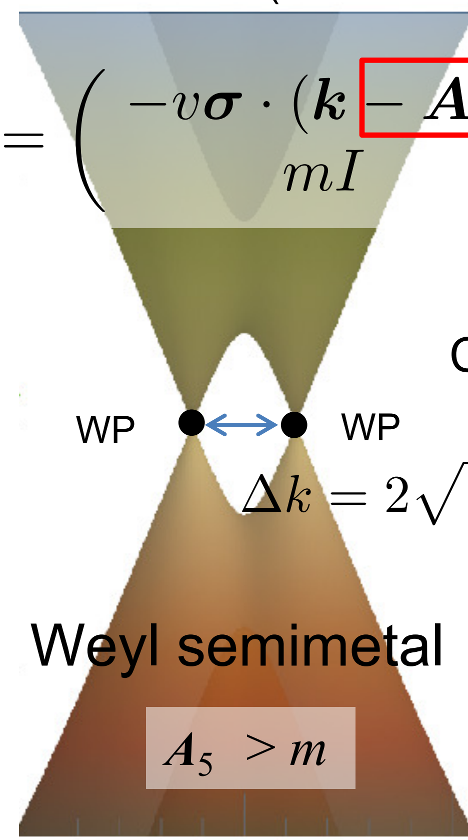
$$H(\boldsymbol{k}) = \begin{pmatrix} -v\boldsymbol{\sigma}\cdot(\boldsymbol{k} - \boldsymbol{A}_5) & mI \\ mI & v\boldsymbol{\sigma}\cdot(\boldsymbol{k} + \boldsymbol{A}_5) \end{pmatrix}$$



Chiral gauge field $\boldsymbol{A}_5$ $(>m)$

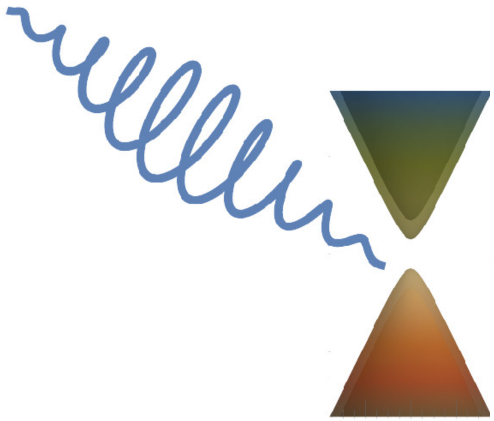
$$\Delta k = 2\sqrt{|\boldsymbol{A}_5|^2 - m^2}$$

Introduction:

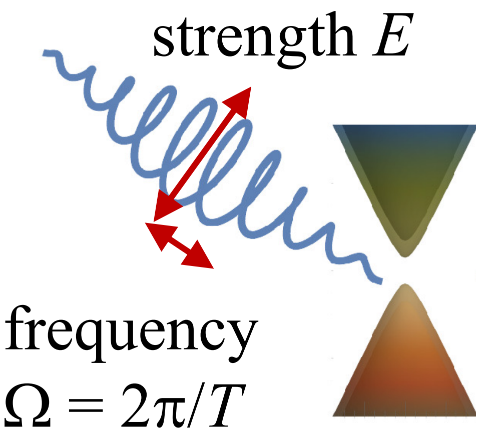
Quantum Materials + Laser

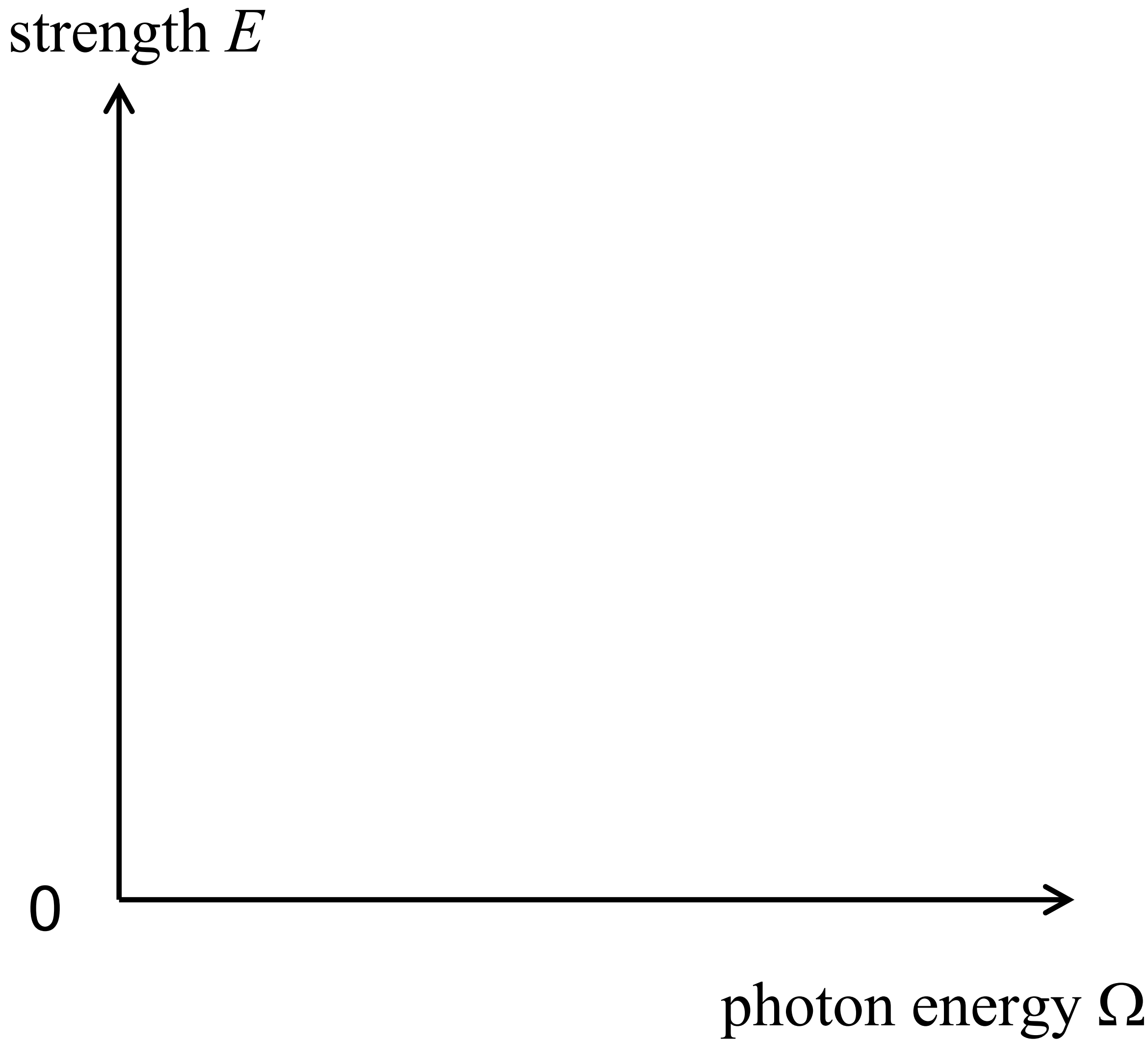
3D Dirac electron

circularly polarized laser

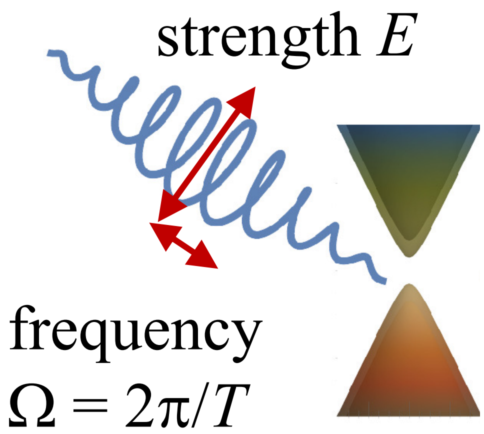


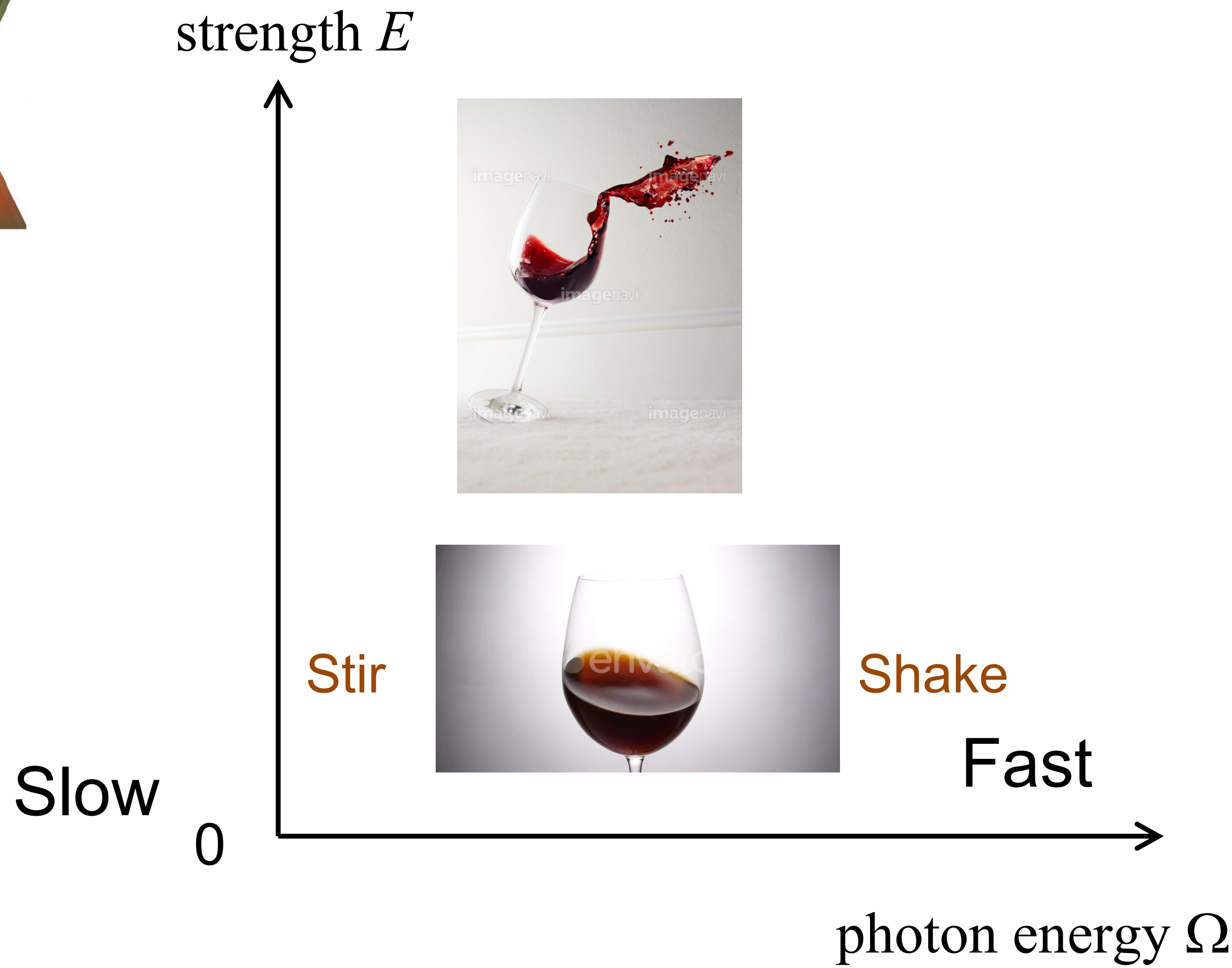
Laser and several limits



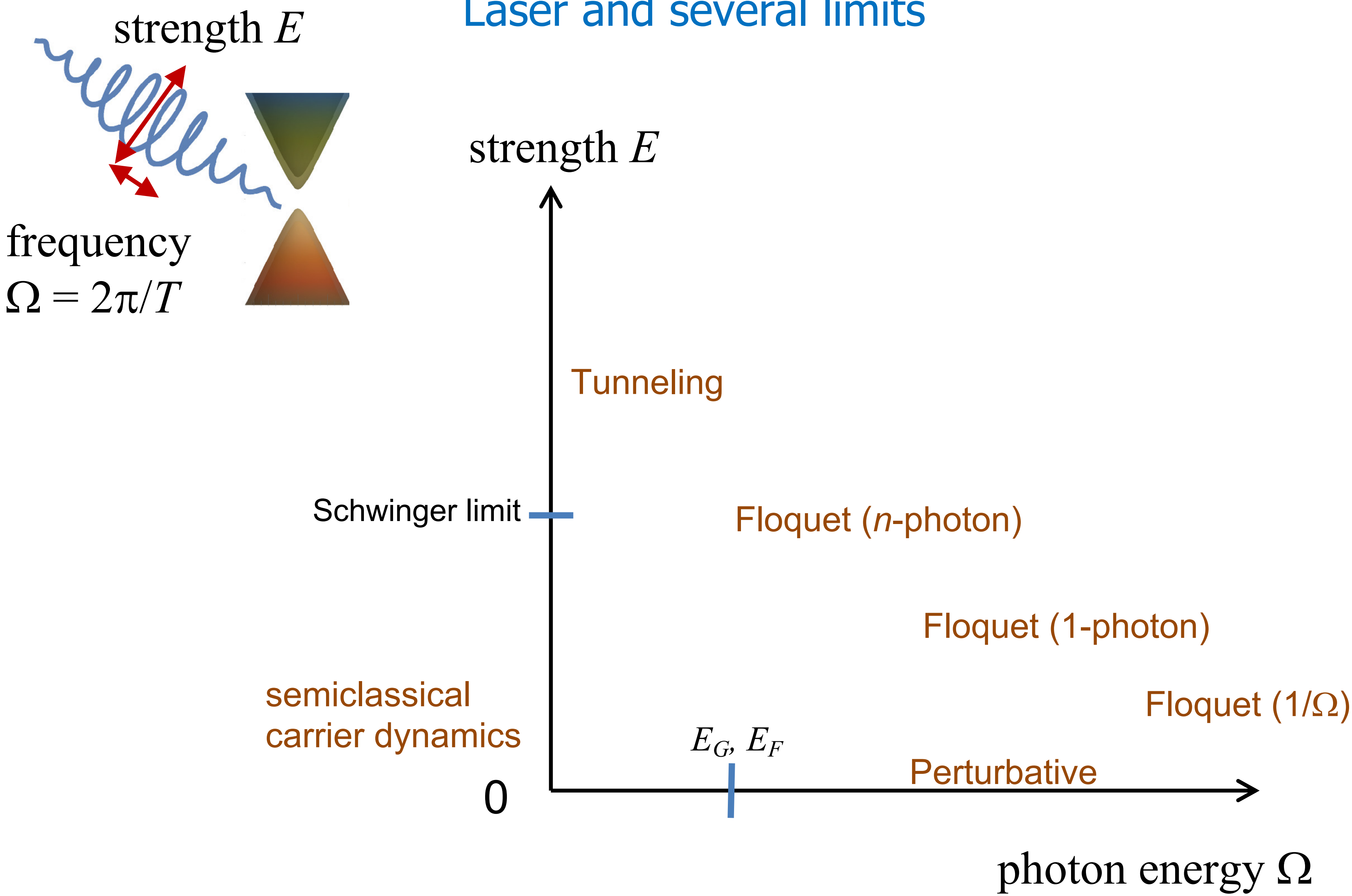


Laser and several limits





Laser and several limits



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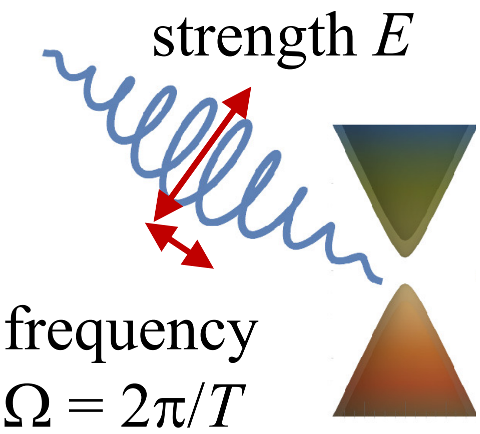
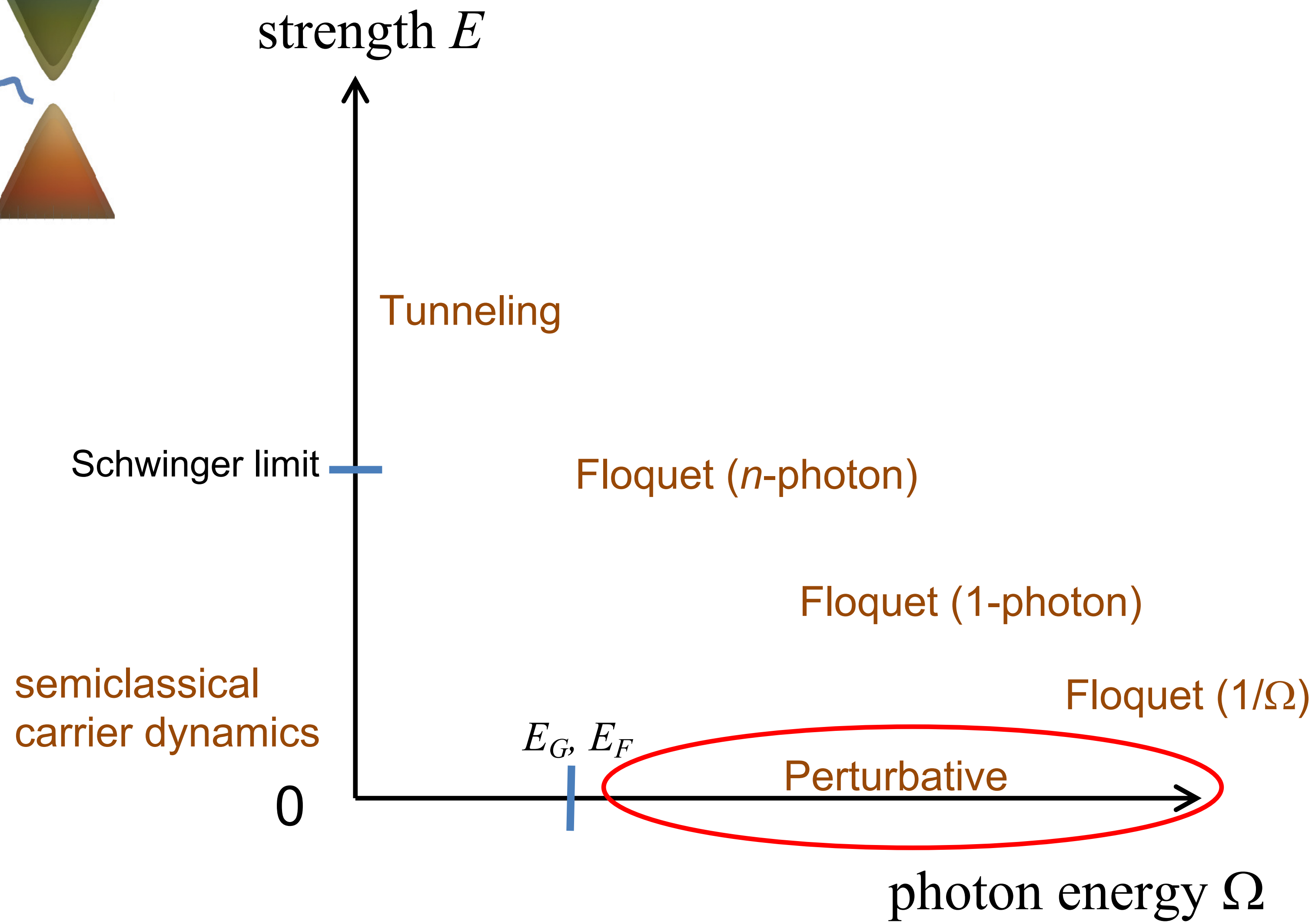




Perturbative Non-linear optics

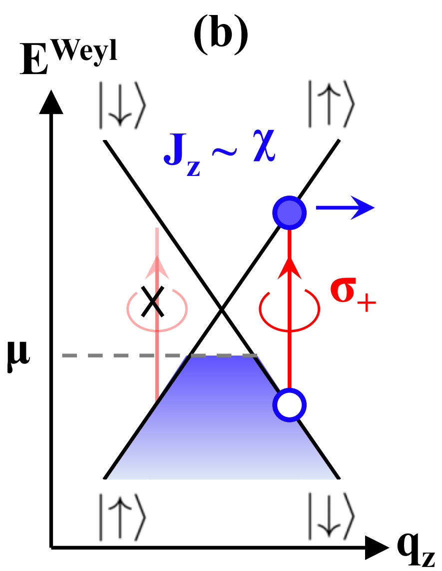


Photo-current

grp. velocity   Transition dipole

$$\bar{J}_i(\omega) = 4\int d^3\left(\frac{v_F q}{\omega}\right)\frac{\partial[\Delta E(\vec{q})/\hbar]}{\partial(v_F q_i)}\left|\langle q_+|\frac{V_+}{\hbar v_F A}|q_-\rangle\right|^2$$
$$\times\,\delta\left(\frac{\Delta E(\vec{q})}{\hbar\omega}-1\right)[n^0_-(\vec{q})-n^0_+(\vec{q})],$$

Optical selection rule = transition dipole matrix
= Berry connection

$$\mathcal{A}_{mn}(\boldsymbol{k}) = \langle\psi_m(\boldsymbol{k})|i\partial_{\boldsymbol{k}}|\psi_n(\boldsymbol{k})\rangle$$

Chan, Lindner, Rafael, Lee, PRB '17

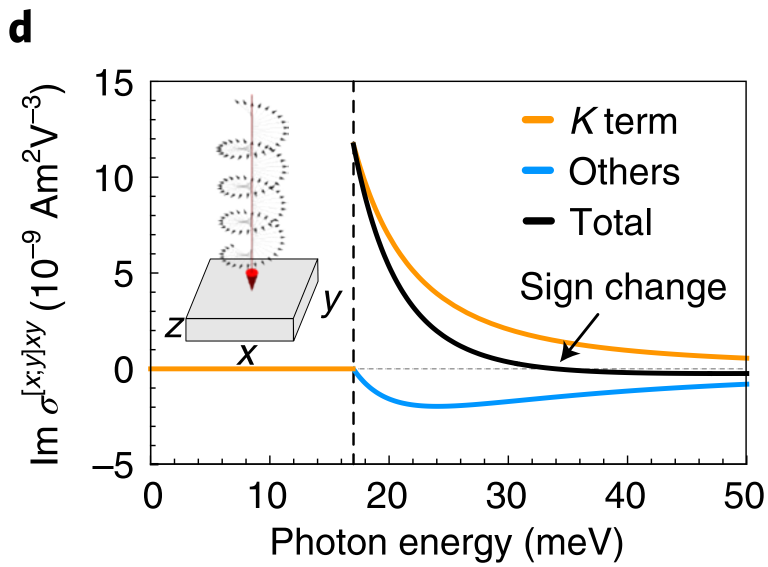
Exp. Ma, Gedik et al. Nat. Phys. '17

Perturbative Non-linear optics

Hall effect ~ $\sigma_{\text{inj}}^{d;abc} = \frac{\pi e^4}{6\Gamma \hbar^3} \sum_{m,n} \int_{\mathbf{k}} \delta(\omega - \omega_{mn}) f_{nm} i K_{cbad}^{mn} + \ldots$, × $|E_{pump}|^2$

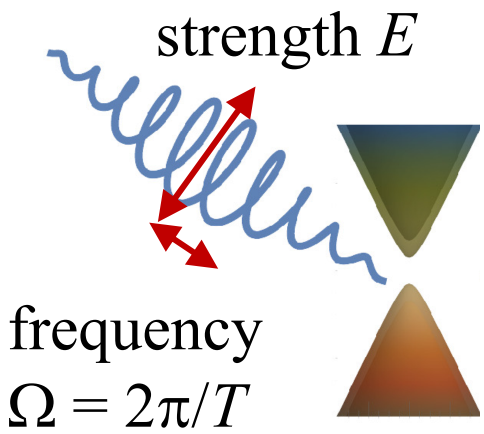
$$K_{badc}^{mn} \equiv (\hat{e}_b^{mn}, (\nabla_d \nabla_c - \nabla_c \nabla_d) \hat{e}_a^{mn})$$

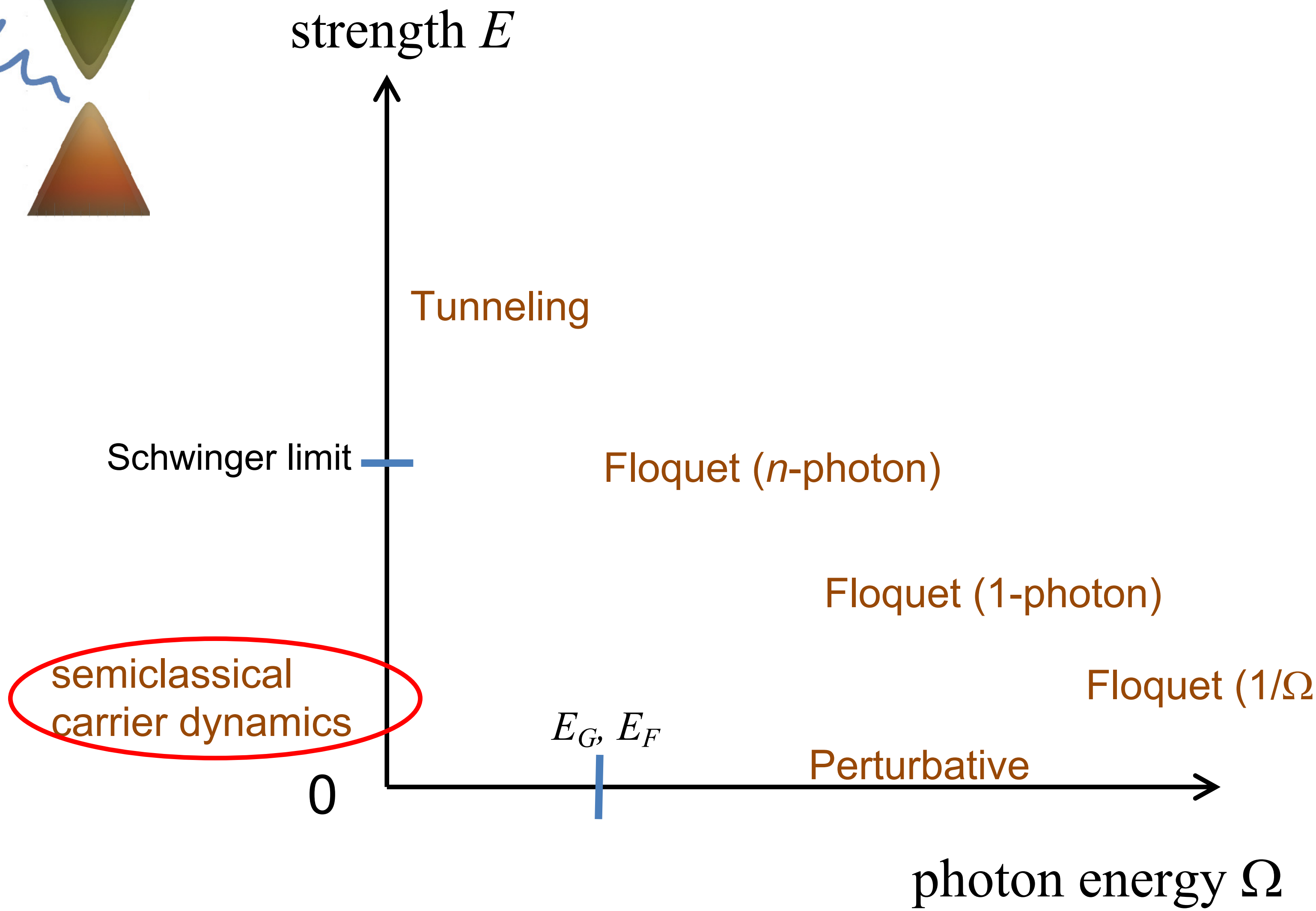
Hermitian curvature (gauge inv.)



Ahn, Guo, Nagaosa, Vishwanath, Nat. Phys.'22

Laser and several limits





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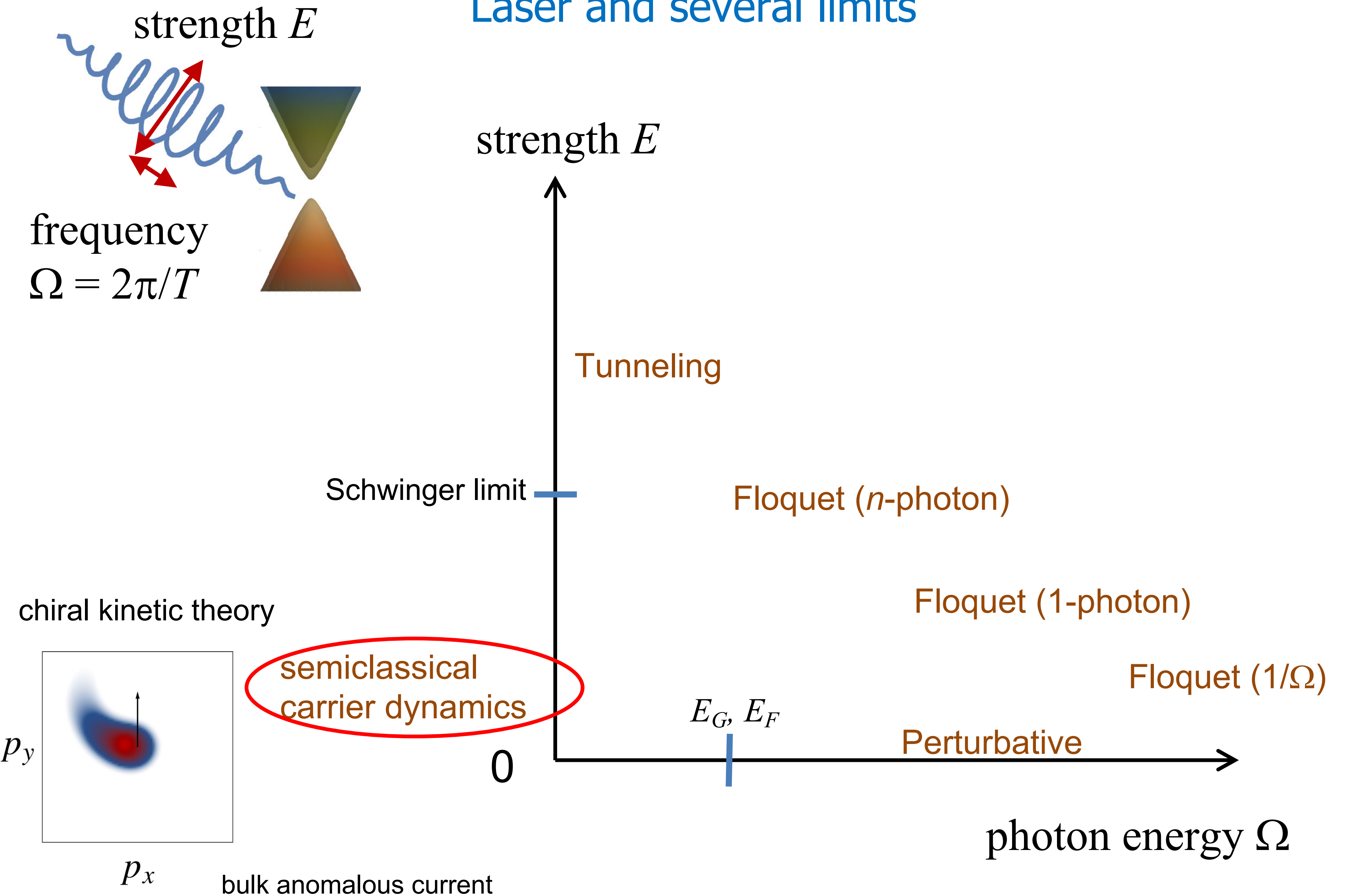
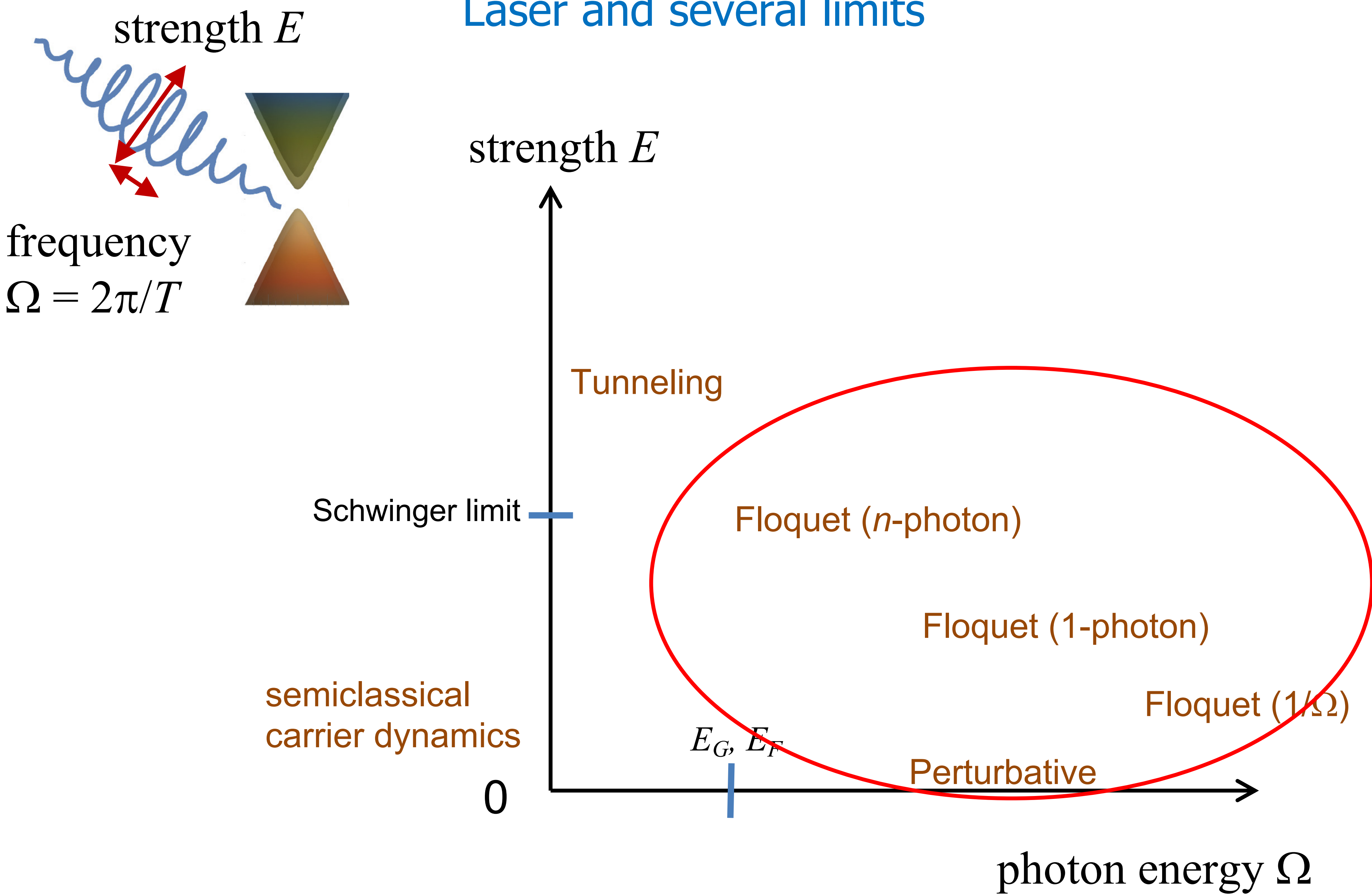


Dantes, Wang, Surówka, TO, PRB 2021

Laser and several limits



2. Floquet theory

$$H(t+T) = H(t)$$

“weird helicopter” (youtube)

Stroboscopic motion

$t = 0, T, 2T, \ldots$

Effective Floquet Hamiltonian

Micromotion

$t: 0 \rightarrow T$

Floquet state

2. Floquet theory

$$H(t+T) = H(t)$$

“weird helicopter” (youtube)

Stroboscopic motion

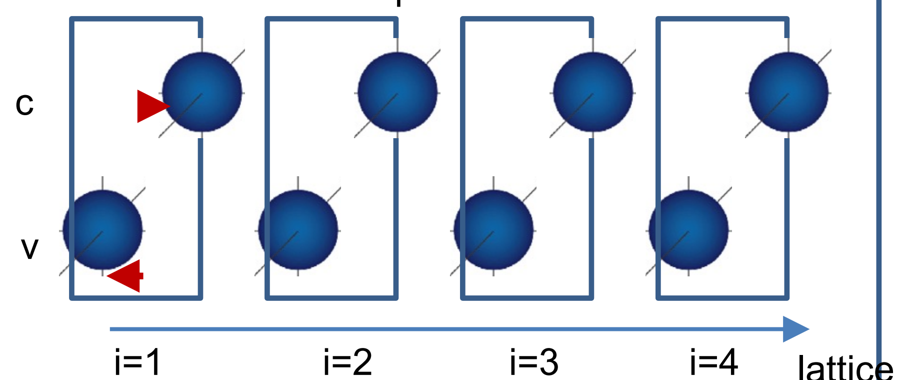
$t = 0, T, 2T, \ldots$

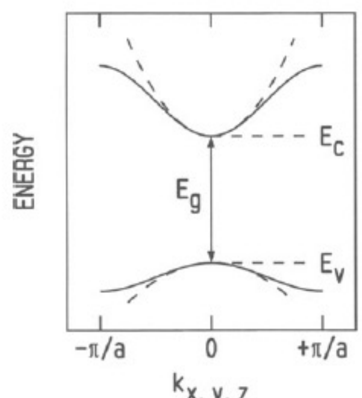
Effective Floquet Hamiltonian

Micromotion

$t: 0 \rightarrow T$

Bloch state ~ Floquet state





Dispersion ~ Effective Hamiltonian

Aim of Floquet "engineering"

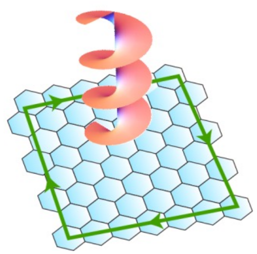
(1) Start from a trivial system
(2) Apply a time periodic external field

$$H(t) = H_0 + \delta H(t)$$

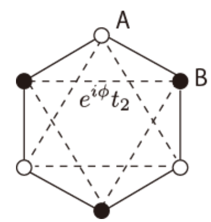
(3) Realize a state with an interesting H_F, $U(T)$, $V(t)$

Example: Floquet topological insulator

$H(t)=$



$H_F =$



graphene +
circularly polarized laser

Quantum Hall state
(Haldane model)

TO, Aoki, PRB'09
Kitagawa TO, et al. '11

- How do we obtain the Floquet states?

Floquet space-time picture
(Sambe picture)

- How can we construct H_F?

$1/\Omega$ expansions

Floquet Space-Time picture 1

Sambe 1973

Treat "time" as an extra space coordinate

time dependent problem

$$i\partial_t \psi = H(t)\psi$$

$$H(t) = H(t+T)$$

$$\Omega = 2\pi/T$$

$$\psi(t) = e^{-i\varepsilon t}\phi(t)$$

$$\phi(t+T) = \phi(t)$$

Floquet state

eigenvalue problem

$$\mathcal{H}\phi_\alpha = \varepsilon_\alpha \phi_\alpha$$

$$\mathcal{H} = H(t) - i\partial_t$$

ε: Floquet quasi-energy

Fourier transformation

Floquet Hamiltonian

$$\sum_{m=-\infty}^{\infty} \mathcal{H}^{mn}\phi_\alpha^m = \varepsilon_\alpha \phi_\alpha^n \qquad \phi(t) = \sum_m \phi^m e^{-im\Omega t}$$

$$(\mathcal{H})^{mn} = \frac{1}{T}\int_0^T dt H(t) e^{i(m-n)\Omega t} + m\delta_{mn}\Omega I$$

$$H_m = \mathcal{H}^{m0}$$

~ absorption of m "photons"

Floquet Space-Time picture 2

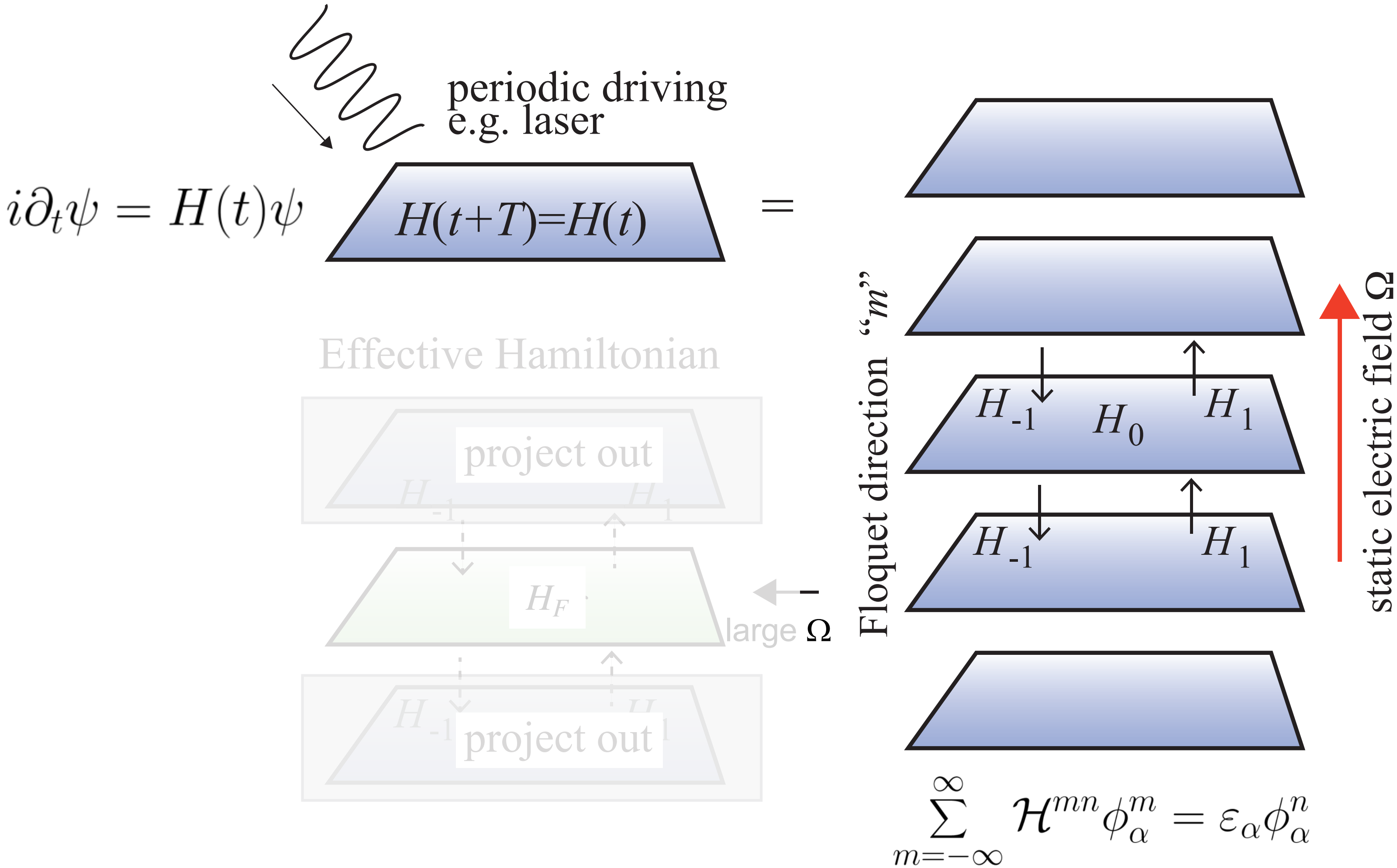
$$\sum_{m=-\infty}^{\infty} \mathcal{H}^{mn} \phi_\alpha^m = \varepsilon_\alpha \phi_\alpha^n$$

$$\begin{pmatrix} \ddots & & & & & & \\ & H_0 - 2\Omega & H_{+1} & 0 & 0 & 0 & \\ & H_{-1} & H_0 - \Omega & H_{+1} & 0 & 0 & \\ & 0 & H_{-1} & H_0 & H_{+1} & 0 & \\ & 0 & 0 & H_{-1} & H_0 + \Omega & H_{+1} & \\ & 0 & 0 & 0 & H_{-1} & H_0 + 2\Omega & \\ & & & & & & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ |\Phi^2\rangle \\ |\Phi^1\rangle \\ |\Phi^0\rangle \\ |\Phi^{-1}\rangle \\ |\Phi^{-2}\rangle \\ \vdots \end{pmatrix} = \epsilon \begin{pmatrix} \vdots \\ |\Phi^2\rangle \\ |\Phi^1\rangle \\ |\Phi^0\rangle \\ |\Phi^{-1}\rangle \\ |\Phi^{-2}\rangle \\ \vdots \end{pmatrix}$$

$$H_m = \frac{1}{T} \int_0^T H(t) e^{im\Omega t} dt$$

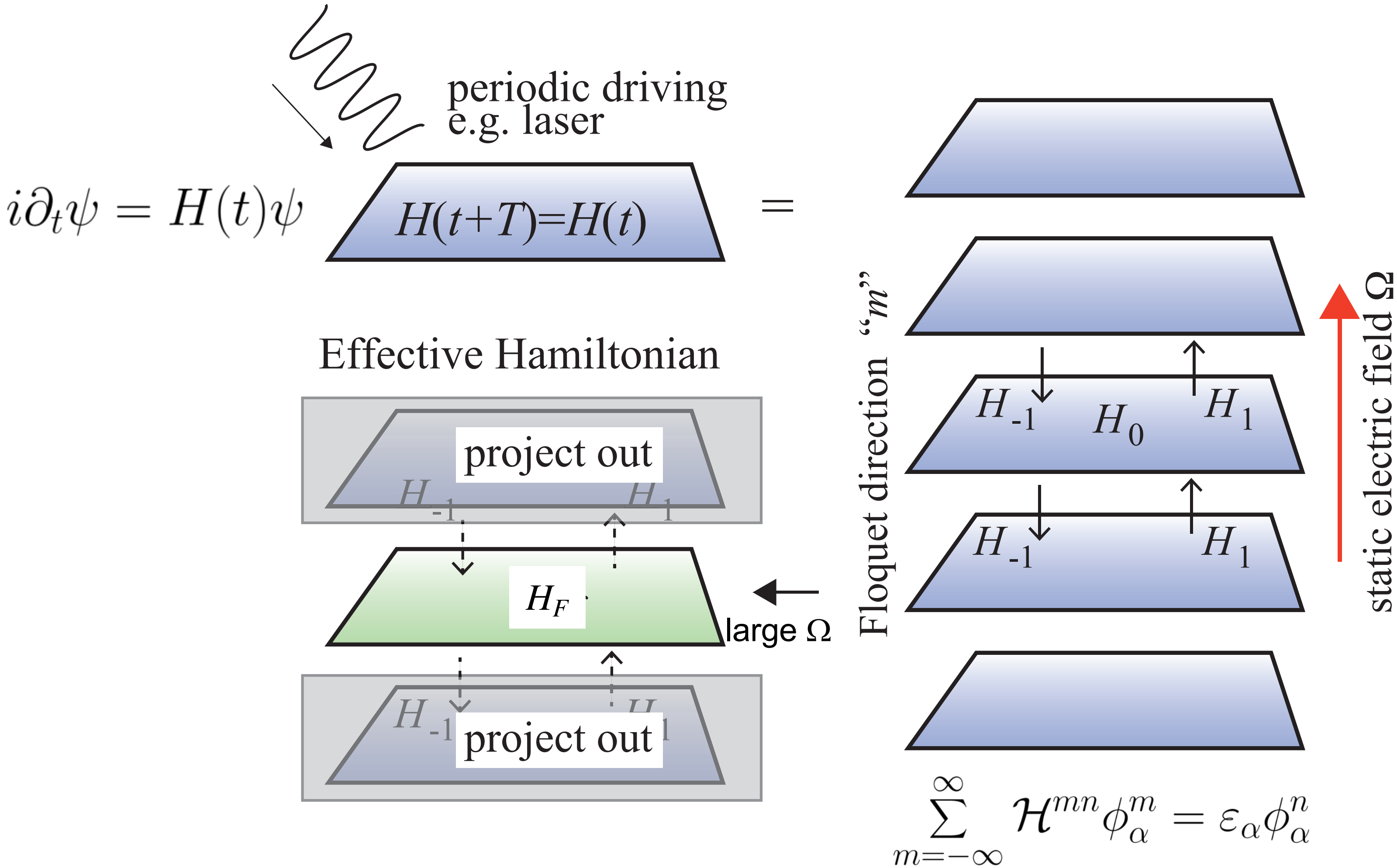
$H_{\pm 2},\ H_{\pm 3}$ not displayed

Floquet Space-Time picture 3



TO, Kitamura, Annual Review of CMP '19

Floquet Space-Time picture 3



TO, Kitamura, Annual Review of CMP '19

High frequency expansion

Floquet-Magnus expansion (captures stroboscopic dynamics)

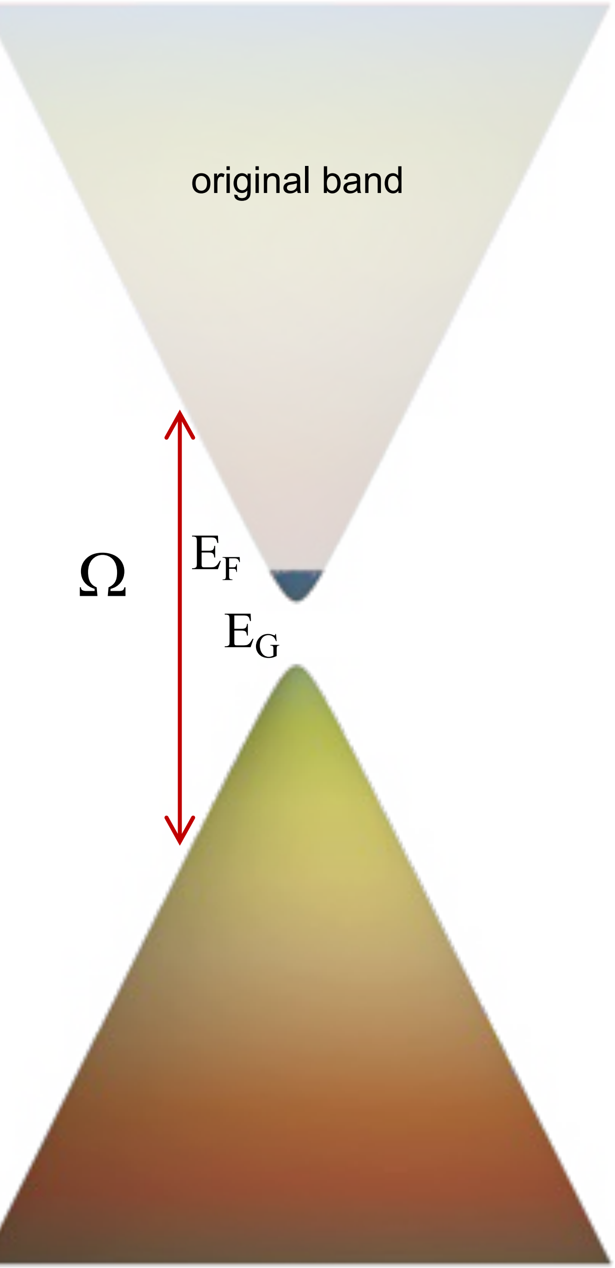
$$H_F = \frac{i}{T} \ln \hat{T} e^{-i \int_0^T H(s) ds}$$

$$H_F = H_0 + \sum_{m>0} \frac{[H_{-m}, H_m]}{m\Omega} + \frac{1}{3} \sum_{m,n \neq 0} \frac{[H_{-m}, [H_{m-n}, H_n]]}{nm\Omega^2} + \frac{1}{2} \sum_{m,n \neq 0} \frac{[H_m, [H_0, H_{-m}]]}{m^2\Omega^2} + \dots$$

Note:

1. Log(exp(i θ)) is not well-defined (monodromy)
2. This expansion is divergent in many-body systems
3. Initial time dependence is dropped

3D Dirac electrons



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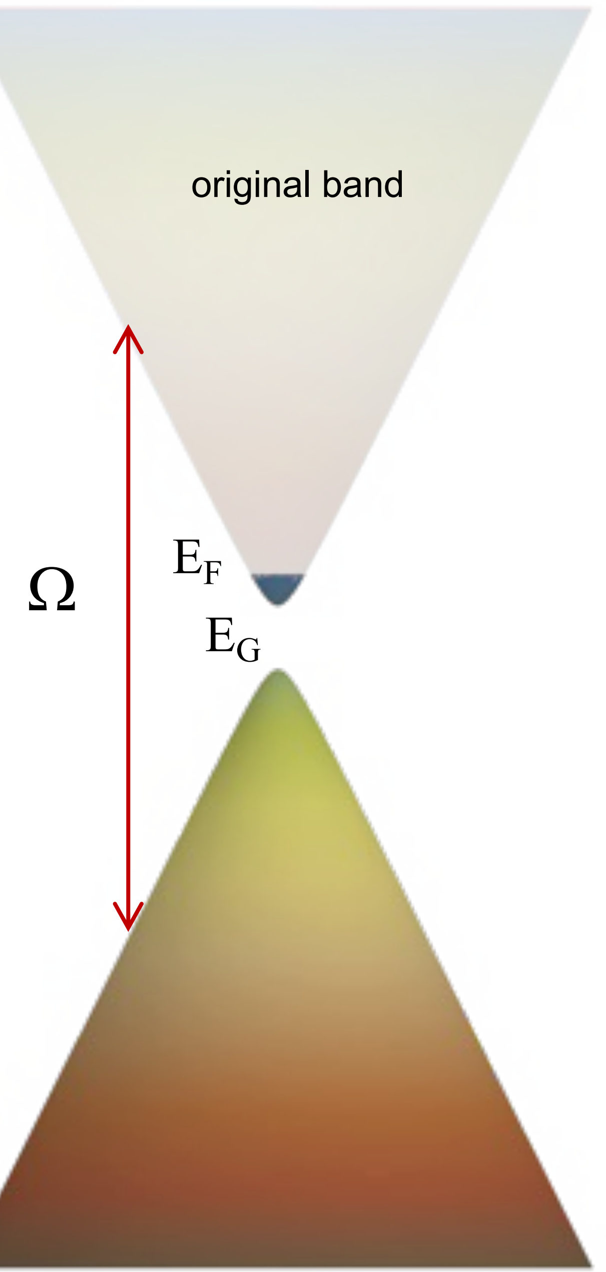
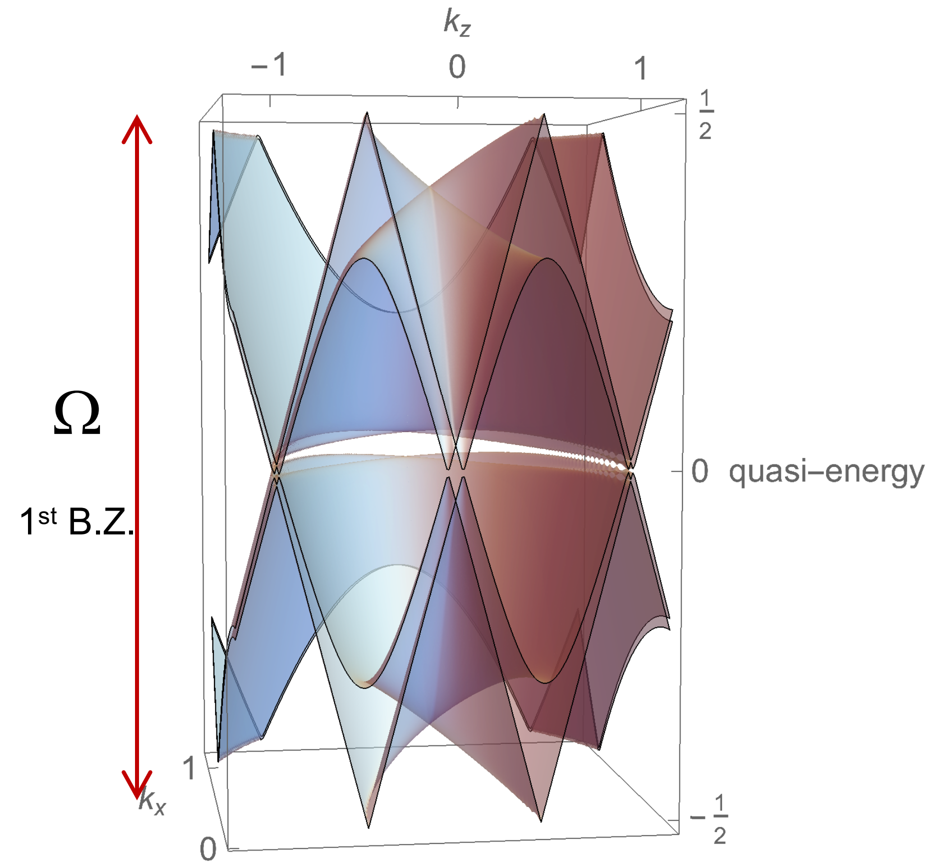
$\boldsymbol{k} \rightarrow \boldsymbol{k} + \boldsymbol{A}$ minimum coupling

$$\boldsymbol{A} = A(\cos\Omega t, \sin\Omega t, 0)$$

$\boldsymbol{A}_5 = 0$ Start from Dirac

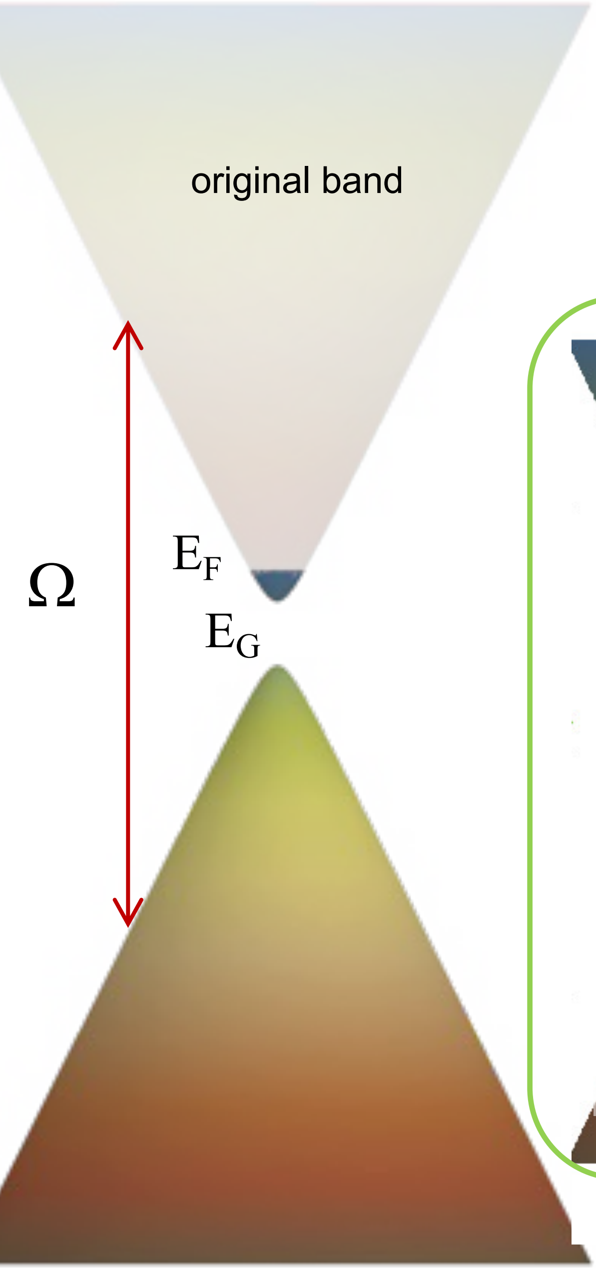
3D Dirac electrons (bulk Floquet bands)

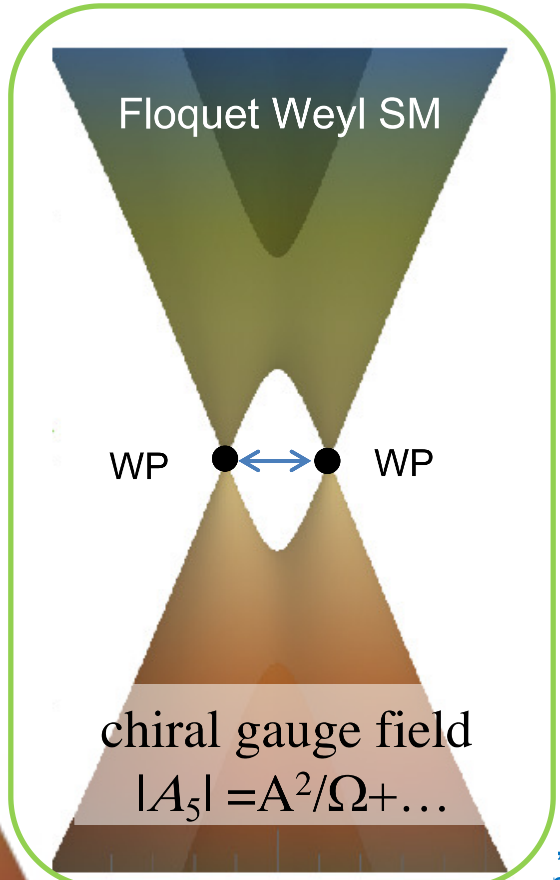


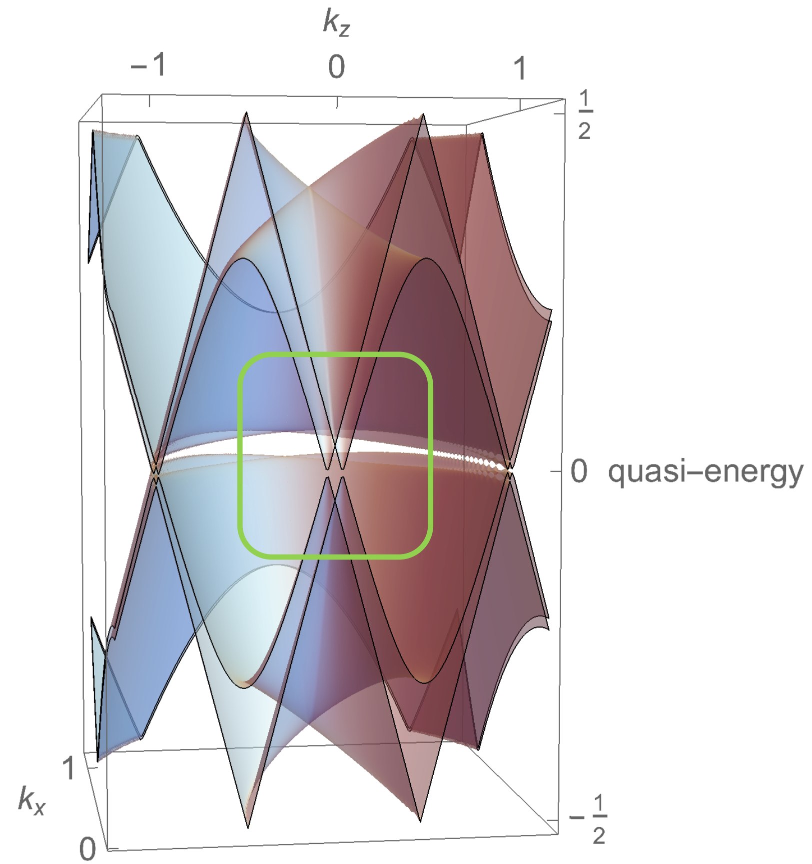


Bucciantini, Roy, Kitamura, Oka, ‘16 (Floquet spectrum)
Wang, Wang, Sheng, Sheng, Xing, EPL’14 ($1/\Omega$ spectrum)
Ebihara, Fukushima, Oka, PRB ‘16, ($1/\Omega$, Chiral pumping effect)
Hübener, Sentef, de Giovannini, Kemper, Rubio, Nat.Com.’16

3D Dirac electrons (1/Ω spectrum)







, Roy, Kitamura, Oka, '16 (Floquet spectrum)
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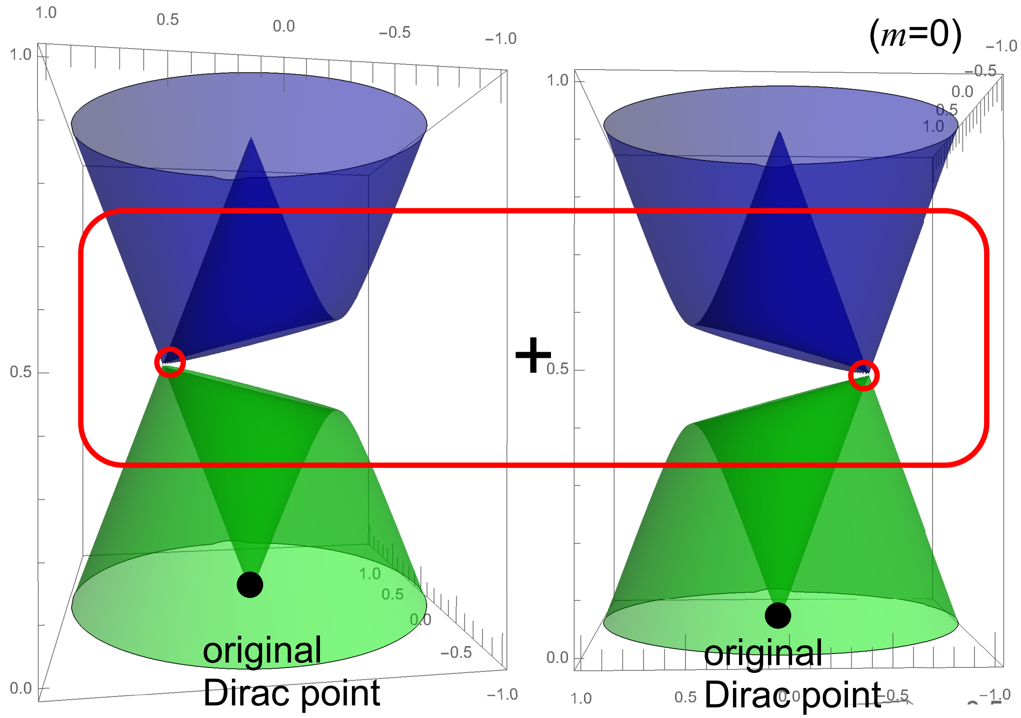
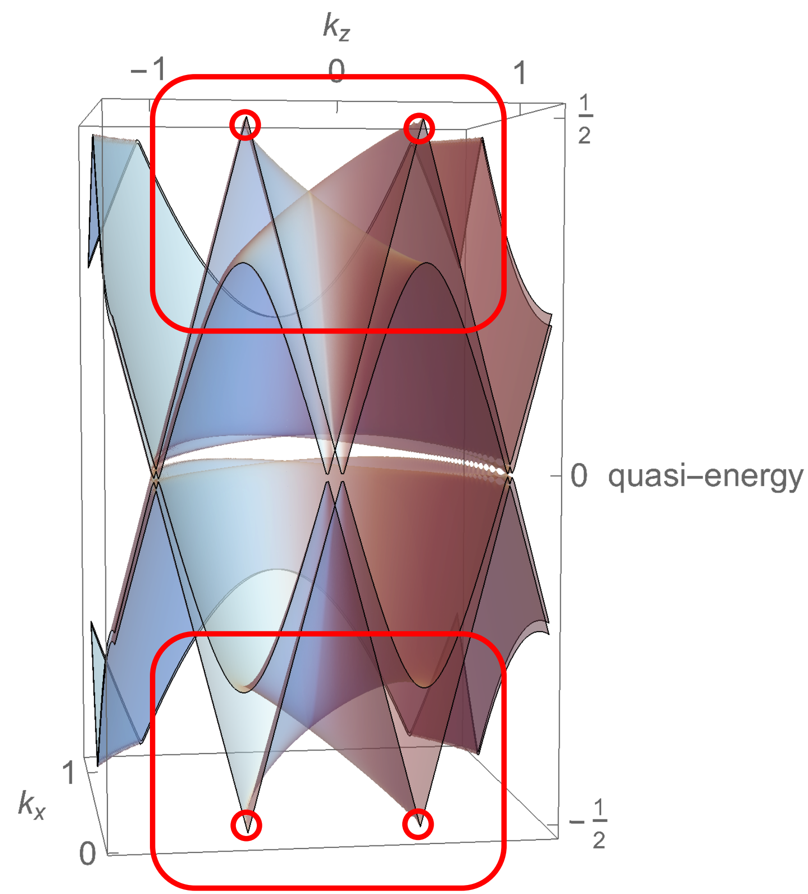
Hübener, Sentef, de Giovannini, Kemper, Rubio, Nat.Com.'16

3D Dirac electrons (1-photon resonance)



Weyl component (ξ=1)　　Weyl component (ξ=-1)



Floquet Double Weyl point

$$H_{\text{eff}} = \boldsymbol{b}\cdot\boldsymbol{\sigma}$$

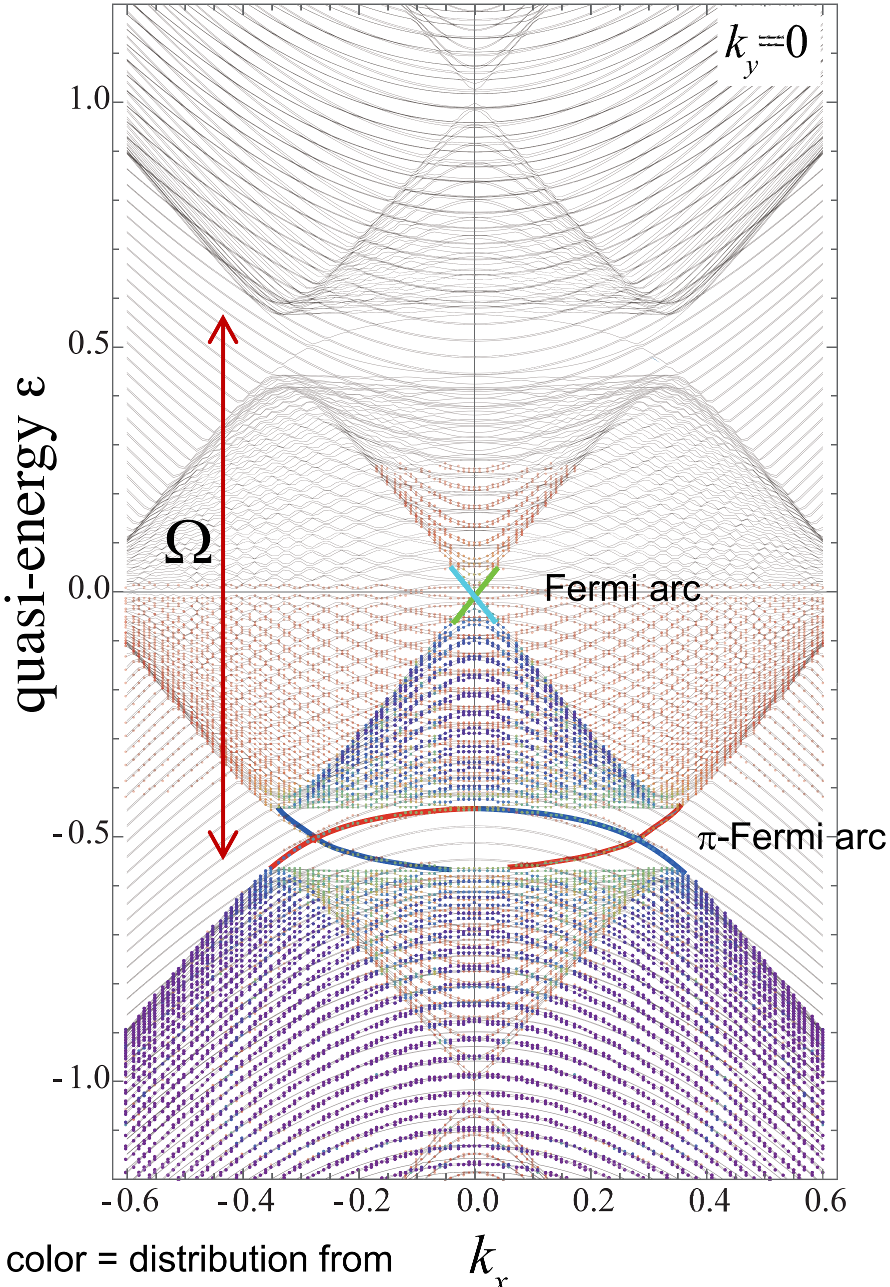
$$b_+ = -\frac{A}{A^2+\Omega^2}k^2$$

$$b_z = -k_z - \frac{\Omega}{A^2+\Omega^2}|k|^2$$

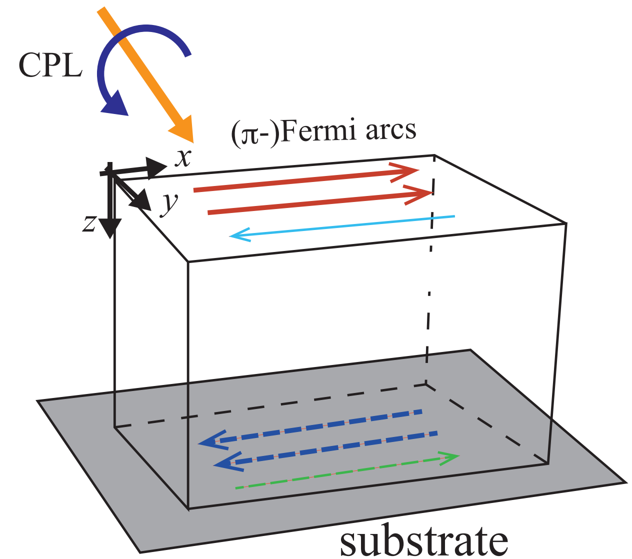
→ Hall effect

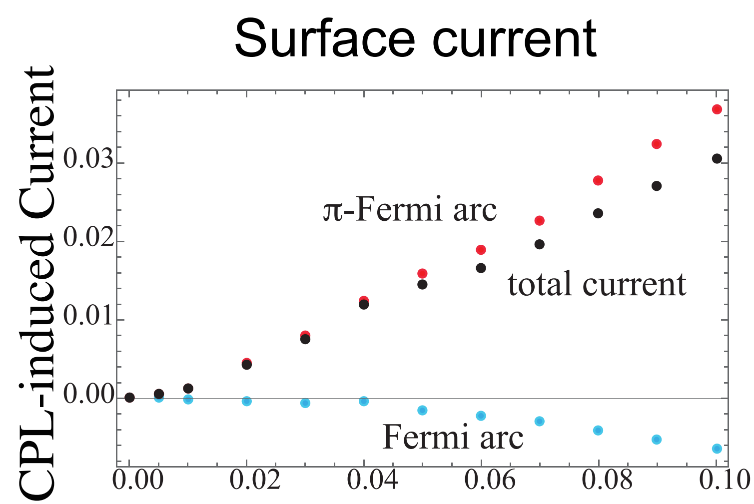
TO, Hirai, Okumura, Yoshioka, Shimano, *in progress*

3D Dirac electrons (Floquet spectrum with edges)



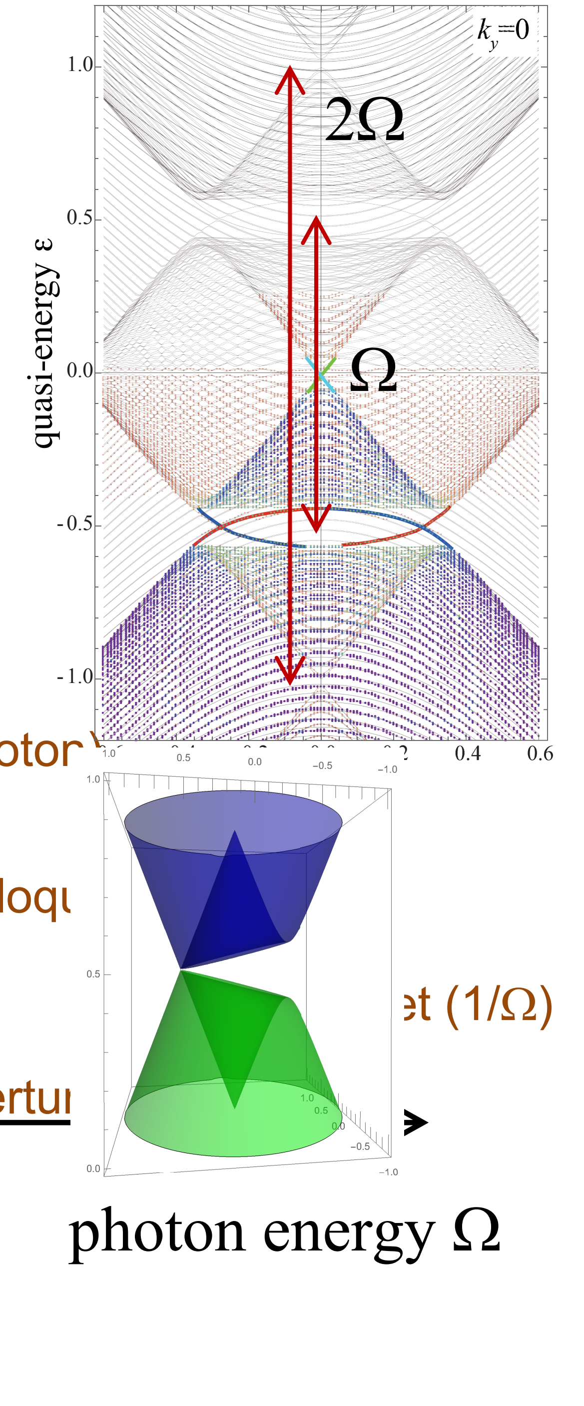
color = distribution from local electron bath

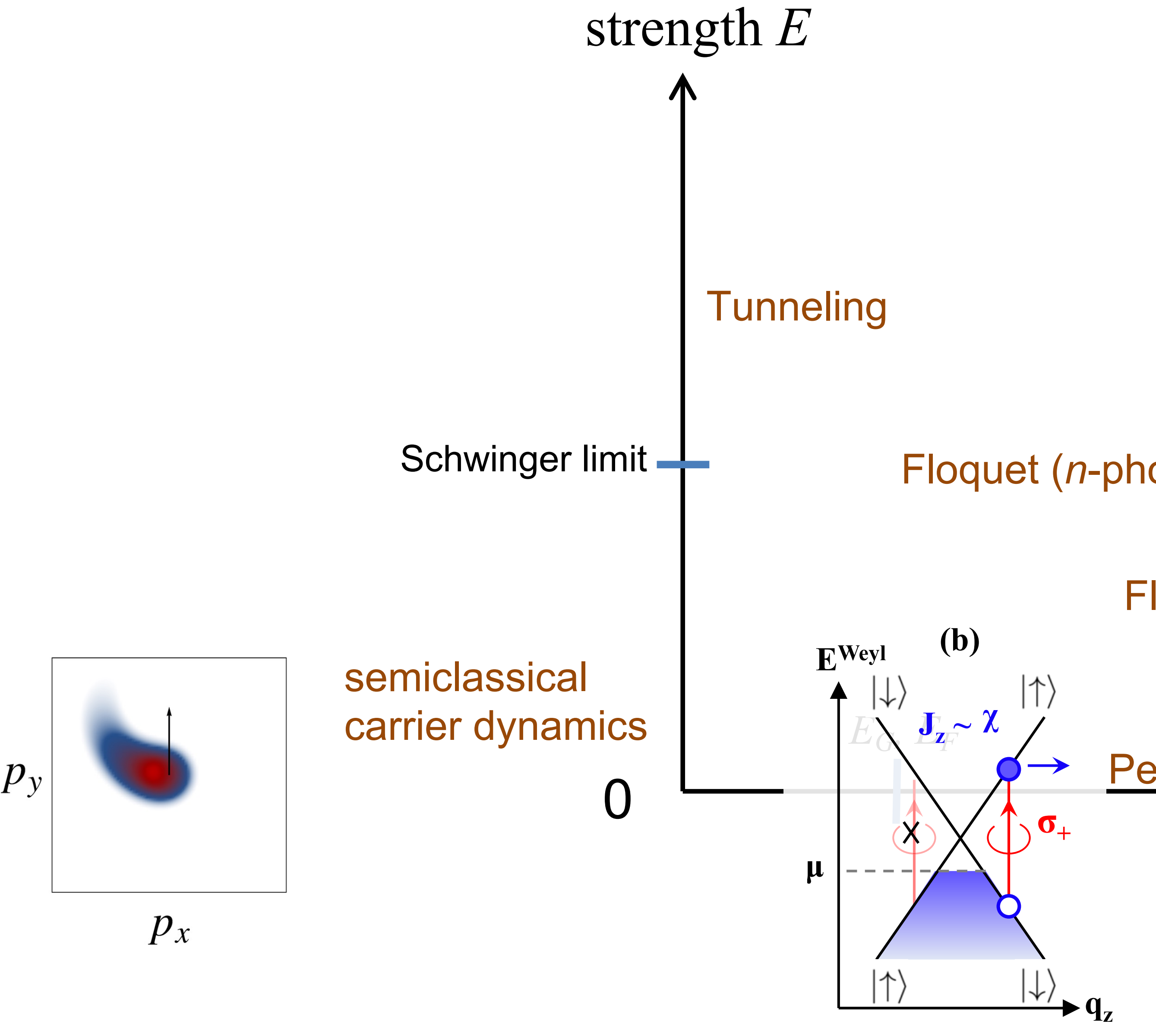


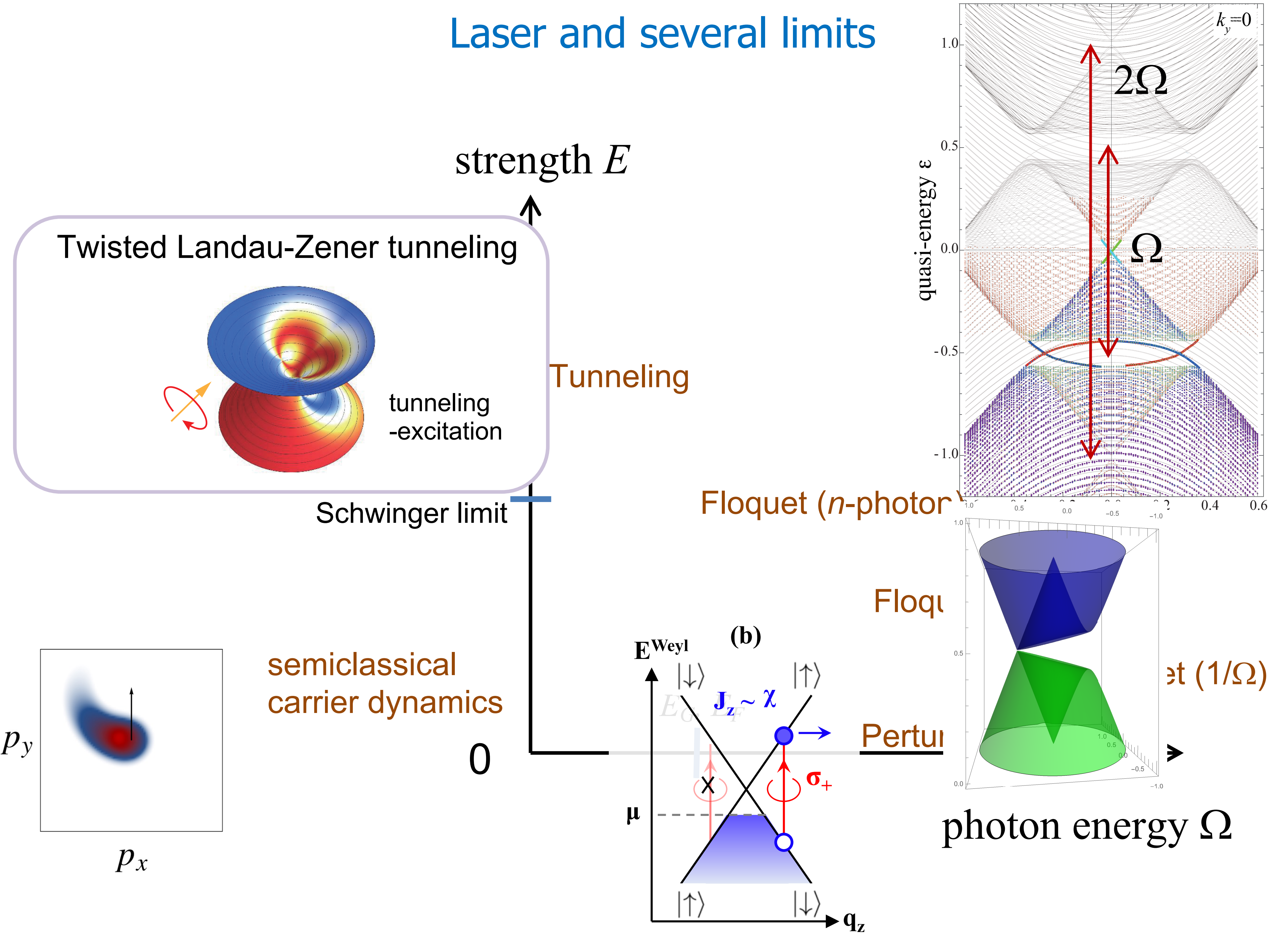


TO, Hirai, Okumura, Yoshioka, Shimano, *in progress*

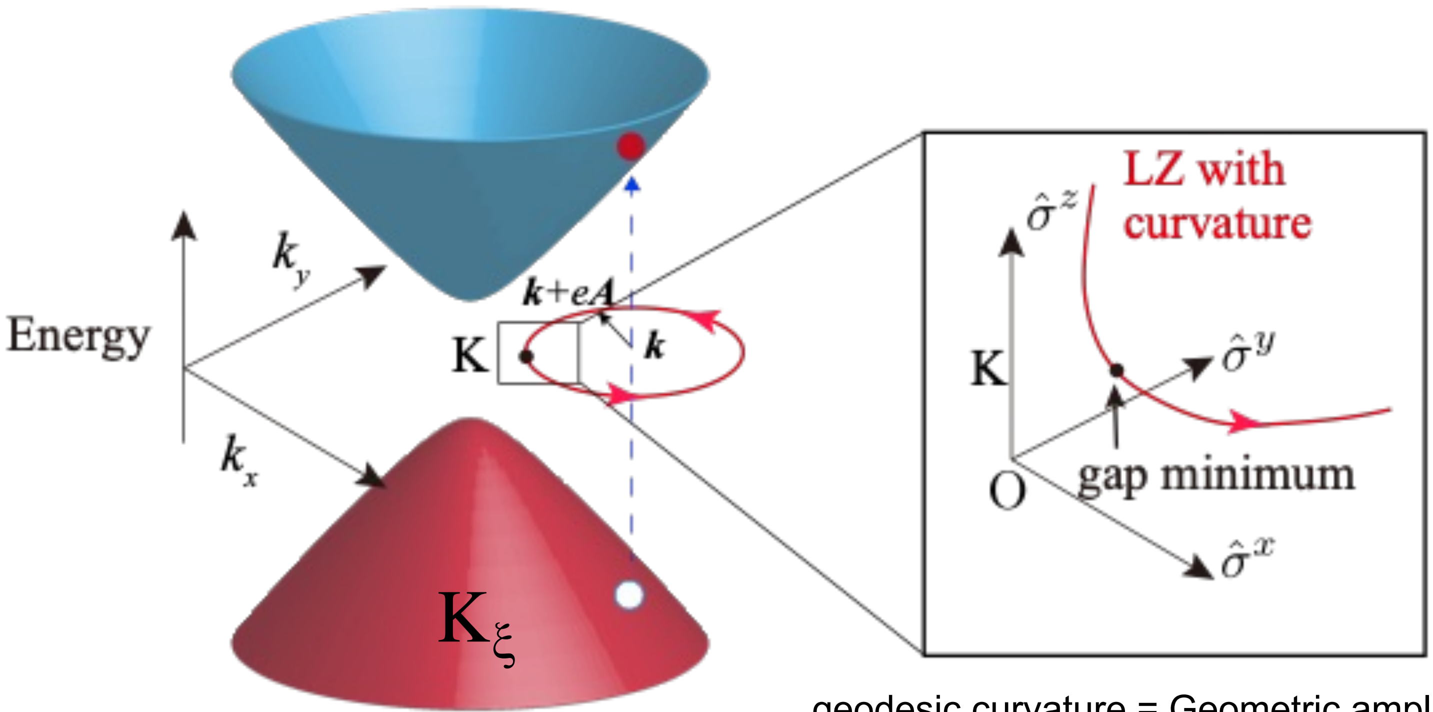
Laser and several limits






Laser and several limits
strength E
Twisted Landau-Zener tunneling
tunneling
-excitation
Tunneling
Schwinger limit
Floquet (n-photon)
Floqu
semiclassical
carrier dynamics
et (1/Ω)
Pertur
0
photon energy Ω
p_y
p_x
(b)
E^{Weyl}
$J_z \sim \chi$
σ_+
μ
q_z
quasi-energy ε
k_y=0
2Ω
Ω

Dirac electron in circularly polarized laser
= twisted LZ problem (1-pulse)



chirality $\xi = \pm$

geodesic curvature = Geometric amplitude factor

$$R^a_{nm}(k) = -\mathcal{A}^a_{nn}(k) + \mathcal{A}^a_{mm}(k) + \partial_{k_a} \arg \mathcal{A}_{nm}(k)$$

c.f. Berry curvature = solid angle

Twisted Landau Zener tunneling

Berry 1984,…

$$|\psi(t)\rangle \sim \sqrt{1-P}\,e^{i\gamma_{AA}}e^{i\theta}|1\rangle + \sqrt{P}e^{i\beta}|2\rangle$$

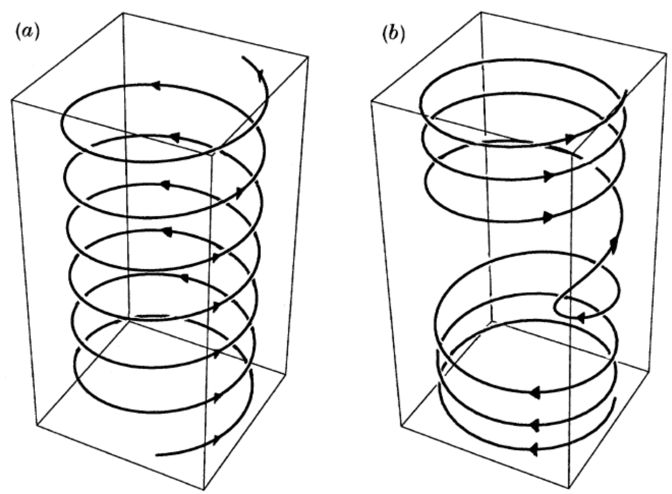
Berry 1990 Geometric effects also in P

Geometric amplitude factors in adiabatic quantum transitions

By M. V. Berry

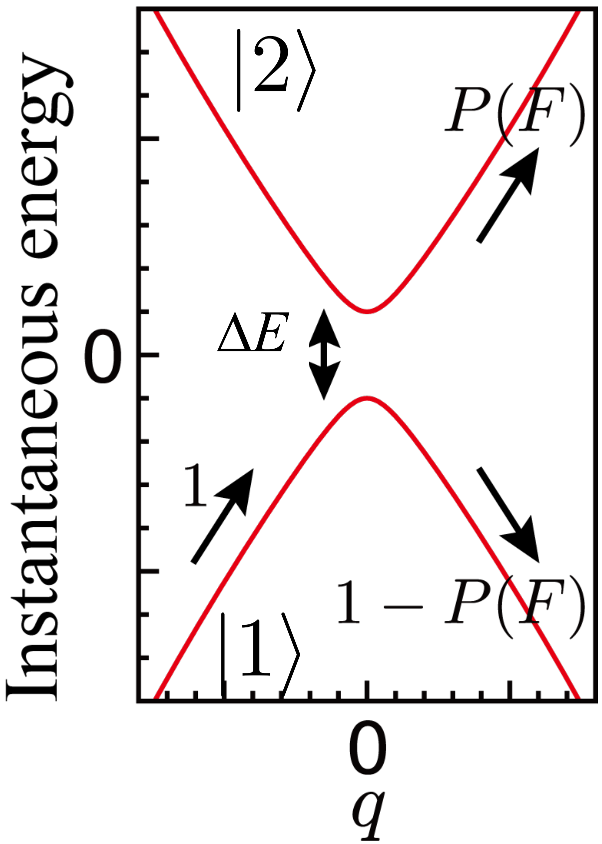
H. H. Wills Physics Laboratory, Tyndall Avenue, Bristol BS8 1TL, U.

Proc. Roy. Soc. London 430, 405 (1990)



$$\boldsymbol{H}(\tau) = (\Delta\cos\phi(\tau), \Delta\sin\phi(\tau), A\tau)$$

Twisted Landau Zener tunneling



Tunneling probability

$$P(F) = \exp\left[-\frac{\pi}{4v|F|}\left(\Delta E + \frac{FR_{+-}}{2}\right)^2\right]$$

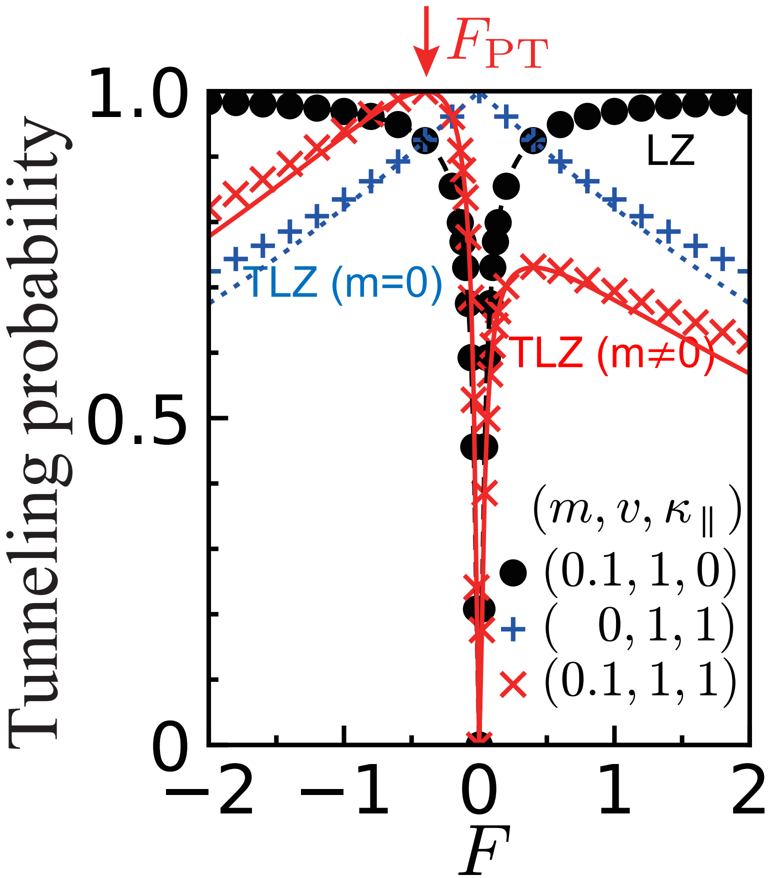
$$R^a_{nm}(k) = -\mathcal{A}^a_{nn}(k) + \mathcal{A}^a_{mm}(k) + \partial_{k_a} \arg \mathcal{A}_{nm}(k)$$

Geometric amplitude factor
(= Quamtum geometric potential, shift vector)

Takayoshi, Wu, TO, SciPost 2021

Twisted Landau Zener tunneling

$$P(F) = \exp\left[-\frac{\pi}{4v|F|}\left(\Delta E + \frac{FR_{+-}}{2}\right)^2 \right]$$



Rectification (non-reciprocal):

$$P(F) \neq P(-F)$$

Perfect tunneling (PT):

$$P(F) = 1 \quad \text{at} \quad F = -2\Delta E/R_{+-}$$

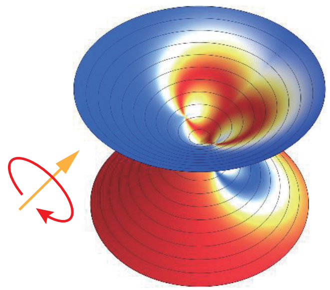
Counter diabaticity:

$P(F)$ decrease at large $|F|$

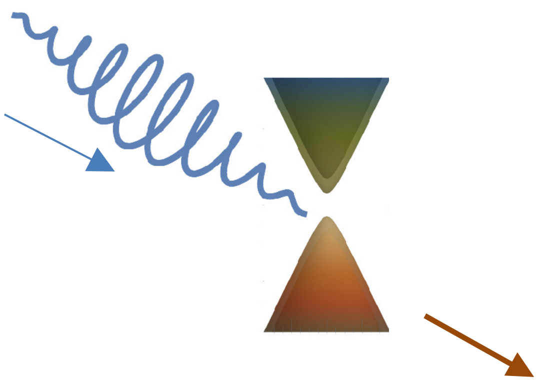
Takayoshi, Wu, TO, SciPost 2021

Bulk current by twisted LZ tunneling

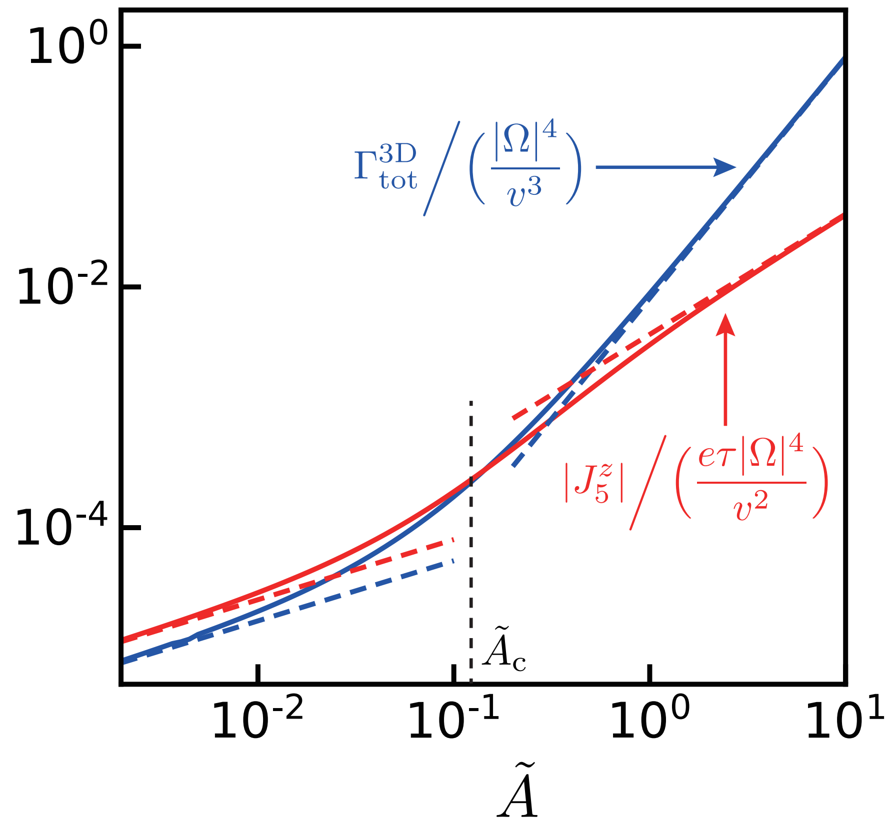
tunneling excitation



distribution
shifted in k_z



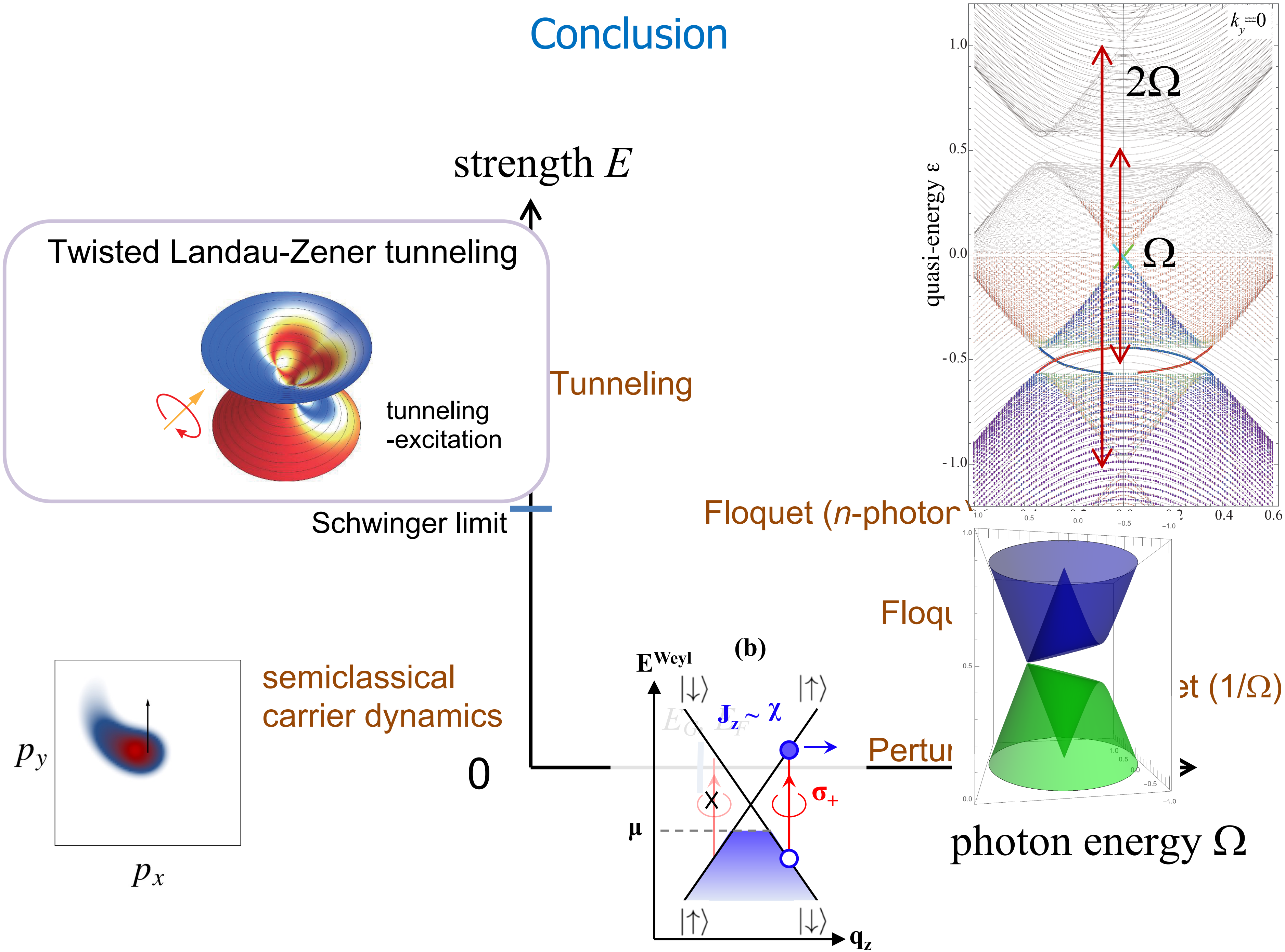
bulk current J_5



$$\tilde{A} = veA/|\Omega| = veE/\Omega^2$$

Takayoshi, Wu, TO, SciPost 2021

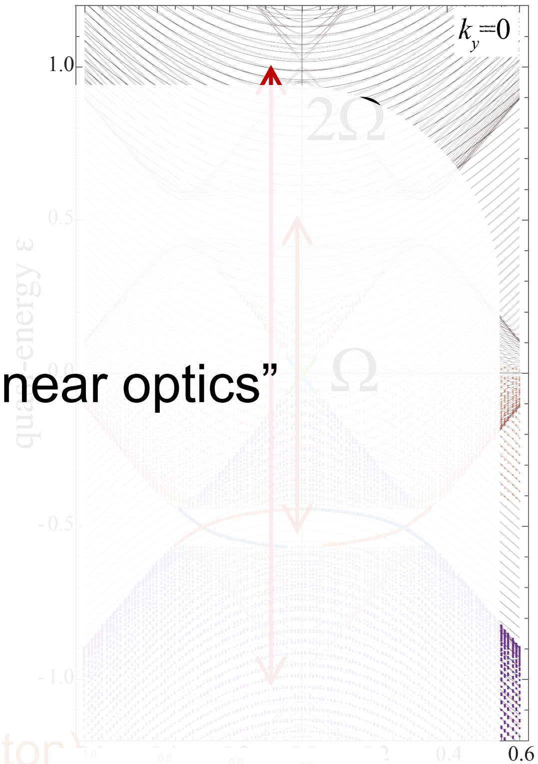
Conclusion



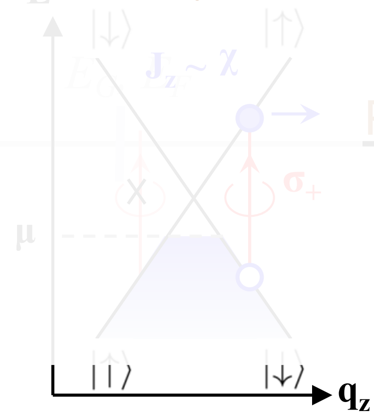
strength E
Twisted Landau-Zener tunneling
tunneling
-excitation
Tunneling
Schwinger limit
Floquet (n-photon)
Floqu
et (1/Ω)
semiclassical
carrier dynamics
Pertur
0
photon energy Ω
p_y
p_x
(b)
E^Weyl
J_z ~ χ
σ_+
μ
q_z
quasi-energy ε
k_y=0
2Ω
Ω

Conclusion

- Various processes in “Topological nonlinear optics”
- Important concepts:
  - Gauge invariance
  - Floquet theory
  - Twisted LZ tunneling
- Experiment on-going

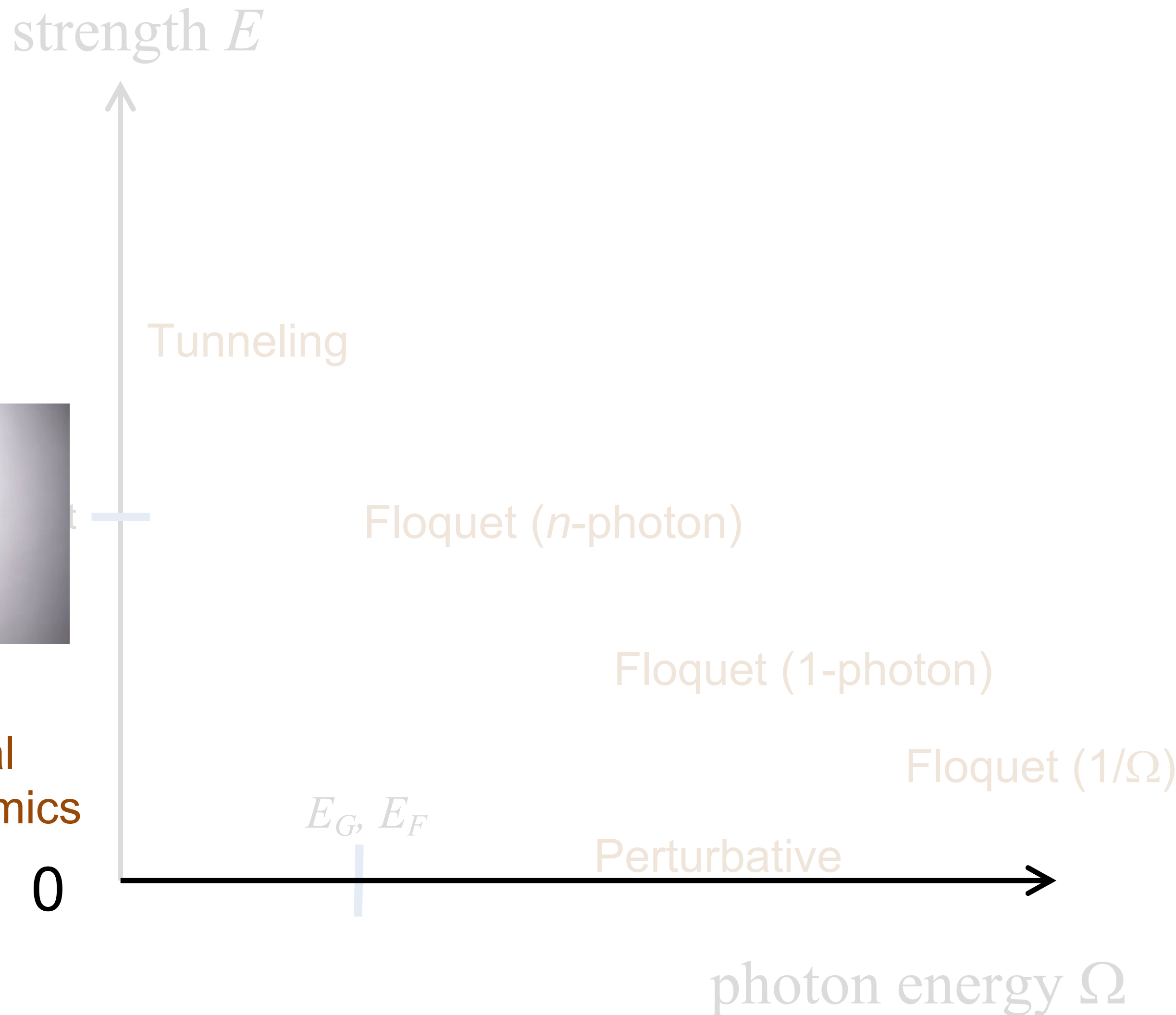


Thank you very much

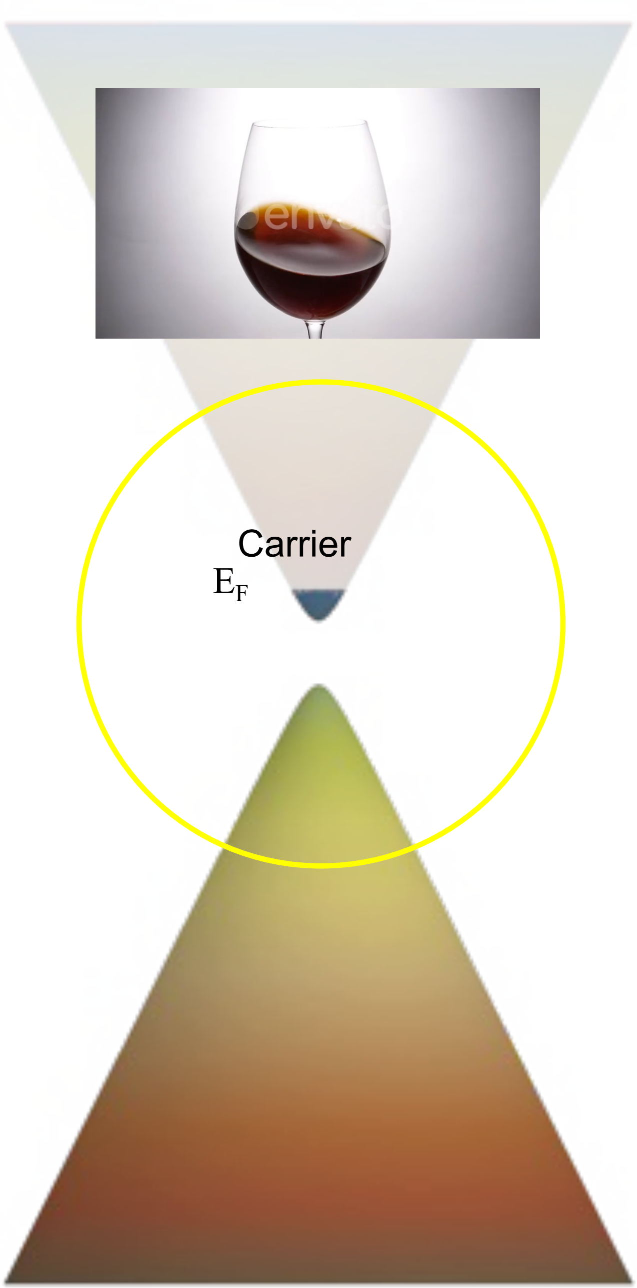


Laser and several limits

semiclassical
carrier dynamics



3.1 Semiclassical approach



Chiral kinetic equation (Boltzmann eq. with Berry curvature)

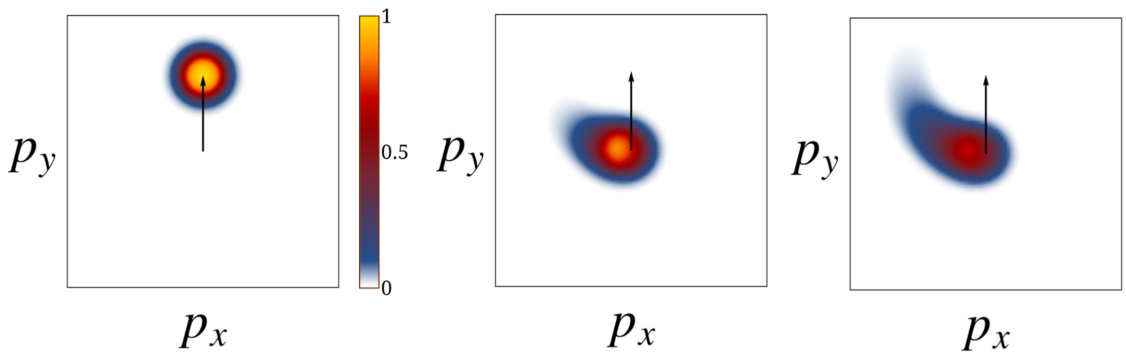
$$\partial_t f + \dot{\boldsymbol{r}} \cdot \nabla_{\boldsymbol{r}} f + \dot{\boldsymbol{p}} \cdot \nabla_{\boldsymbol{p}} f = \tau^{-1}(f_0 - f)$$

$$\dot{\boldsymbol{r}} = \nabla_{\boldsymbol{p}} \epsilon_{\boldsymbol{p}} - \hbar \dot{\boldsymbol{p}} \times \boldsymbol{\Omega}_{\boldsymbol{p}}, \qquad \dot{\boldsymbol{p}} = -e\boldsymbol{E} - e\dot{\boldsymbol{r}} \times \boldsymbol{B},$$

$$\boldsymbol{j} = -e \int_p \nabla_{\boldsymbol{p}} \epsilon_{\boldsymbol{p}} f - e^2 \hbar \boldsymbol{E}(t) \times \int_p \boldsymbol{\Omega}_{\boldsymbol{p}} f$$

total current   normal current   Anomalous current

$$\boldsymbol{\Omega}_{\boldsymbol{p}} = -\eta \frac{\hat{\boldsymbol{p}}}{2p^2}$$



Dantes, Wang, Surówka, TO, PRB 2021

HHG Experiment with linearly polarized field
Cheng, Kanda, (Matsunaga gr.), PRL 2020
Kovalev, Dantes, TO, (Wang gr.), Nat. Com. 2020

Theory of Floquet states (0-dim. case)

Let us solve the Schrödinger equation

$$i\partial_t \psi_k(t) = h(t)\psi_k(t) \; , \qquad h(t+T) = h(t)$$

$$\Rightarrow \quad \psi_k(t) = e^{-i\int_0^t h(s)ds}\psi_k(0)$$

$$= e^{-i\int_0^t h(s)ds + i\varepsilon_k t} e^{-i\varepsilon_k t}\psi_k(0)$$

$$= V(t)e^{-iH_F t}\psi_k(0)$$

Effective Floquet Hamiltonian

$$H_F = \varepsilon_k = \frac{1}{T}\int_0^T h(s)ds$$

"time average"

Micromotion

$$V(t)$$

$V(t+T) = V(t)$ is satisfied

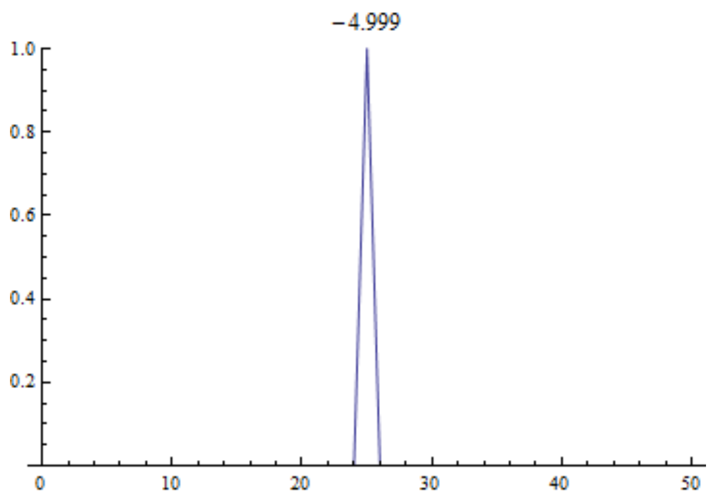
"Dynamic localization"

Dunlap and Kenkre '86

$$H(t) = -J\sum_i \left(e^{-i\theta(t)A\cos\Omega t} c^\dagger_{i+1} c_i + \text{h.c.}\right)$$

momentum space

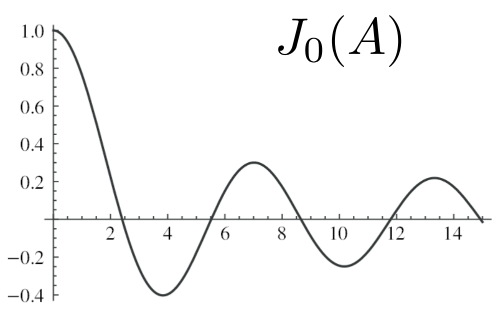
$$H(t) = \sum_k (-2J)\cos(k - A\cos\Omega t) c^\dagger_k c_k$$



Effective Floquet Hamiltonian

$$H_F = J_0(A)(-2J)\cos(k)$$ $\rightarrow 0$ at A=2.41…

0-th Bessel func.



Theory of Floquet states (general)

Time periodic systems $H(t+T) = H(t)$

$$U(t,0) = \hat{T}e^{-i\int_0^t H(s)ds} = V(t)e^{-iH_F t}$$

Effective Floquet Hamiltonian defined by

$$e^{-iH_F T} = U(T,0)$$

Stroboscopic motion

Micromotion

$$V(t)$$

Other realizations

"Bang-bang time evolution"

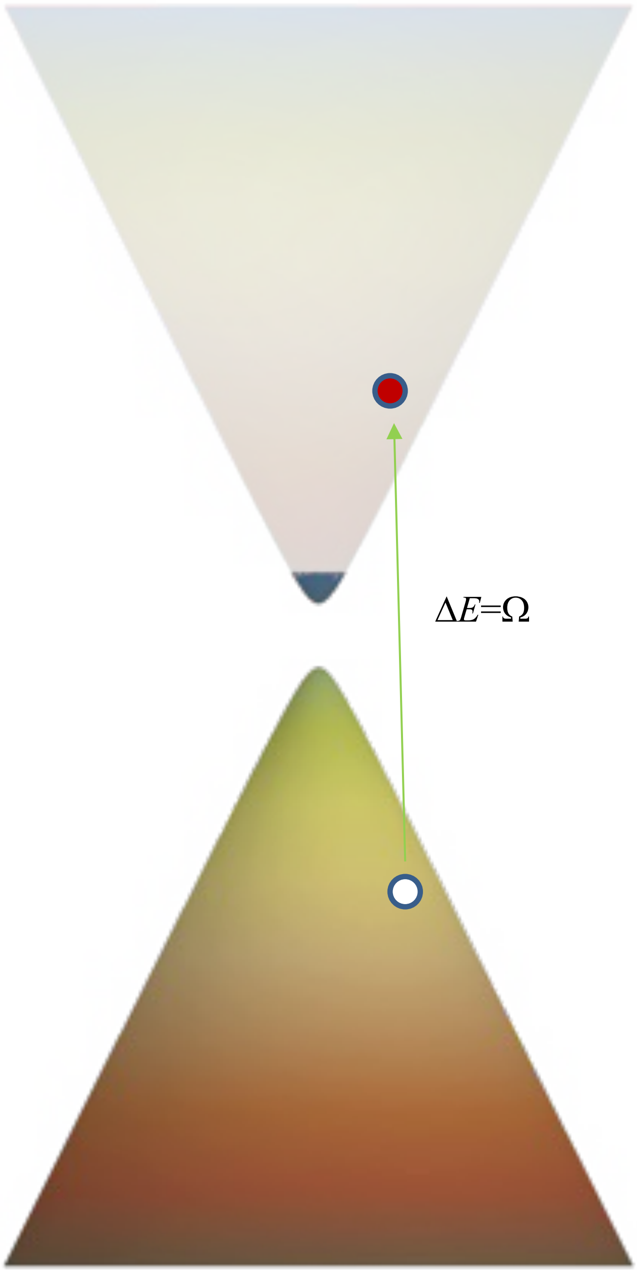
$$|\psi(nT)\rangle = (U_1U_2)^n|\psi(0)\rangle \Rightarrow e^{-iH_F T} = U_1U_2$$

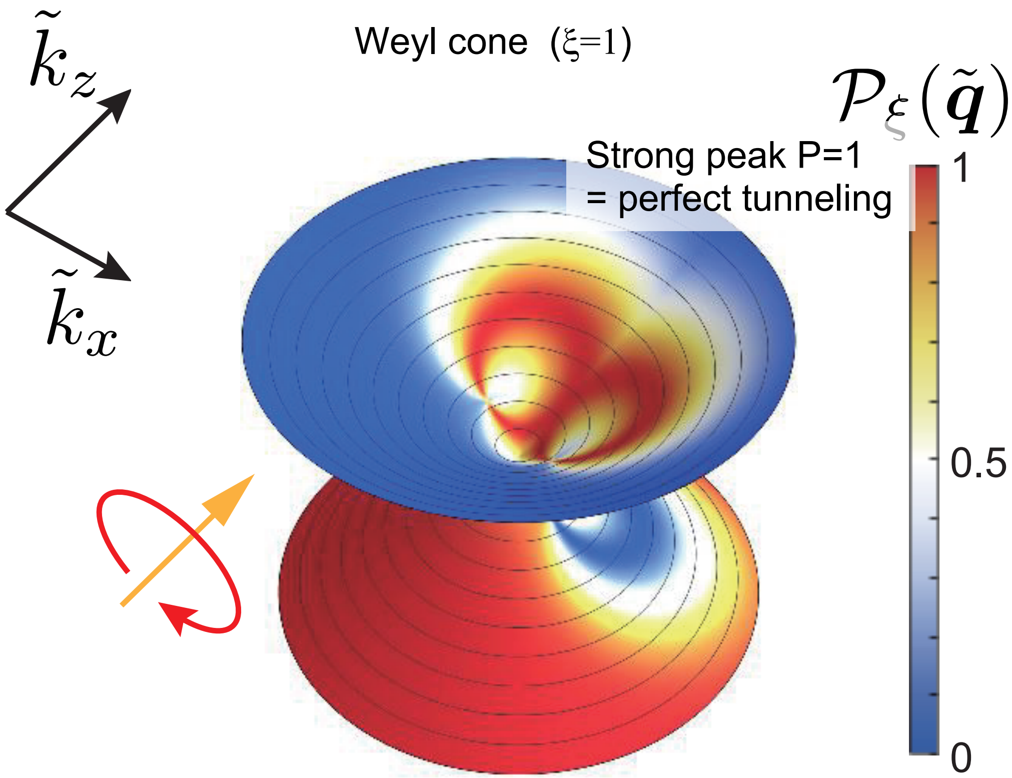
quantum walk, repeated quench,…

A. Eckardt, RMP '17

TO, Kitamura, Annual Review of CMP '19

3.4 Tunneling approach





- Strongly asymmetric
- Energy conservation ($\Delta E = \Omega$) violated
- Results in a (chiral) current

Takayoshi, Wu, Oka 2021

Twisted Landau Zener transition(excitation)

scaling parameter $\tilde{A} = veA/|\Omega| = veE/\Omega^2$