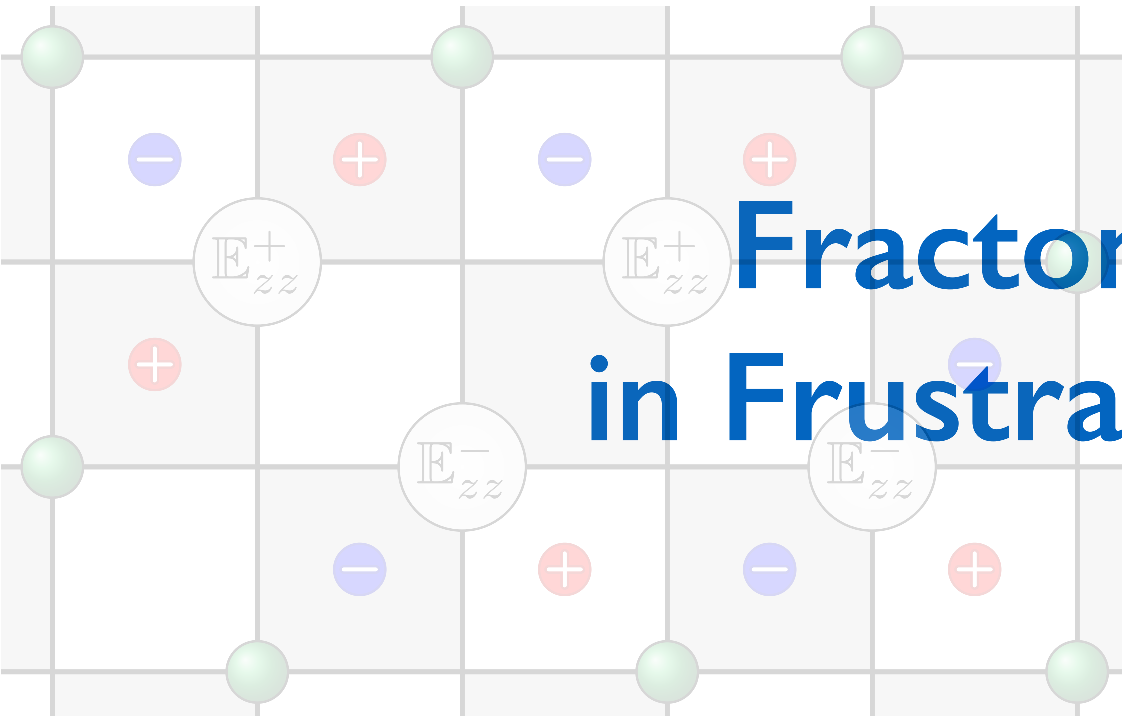
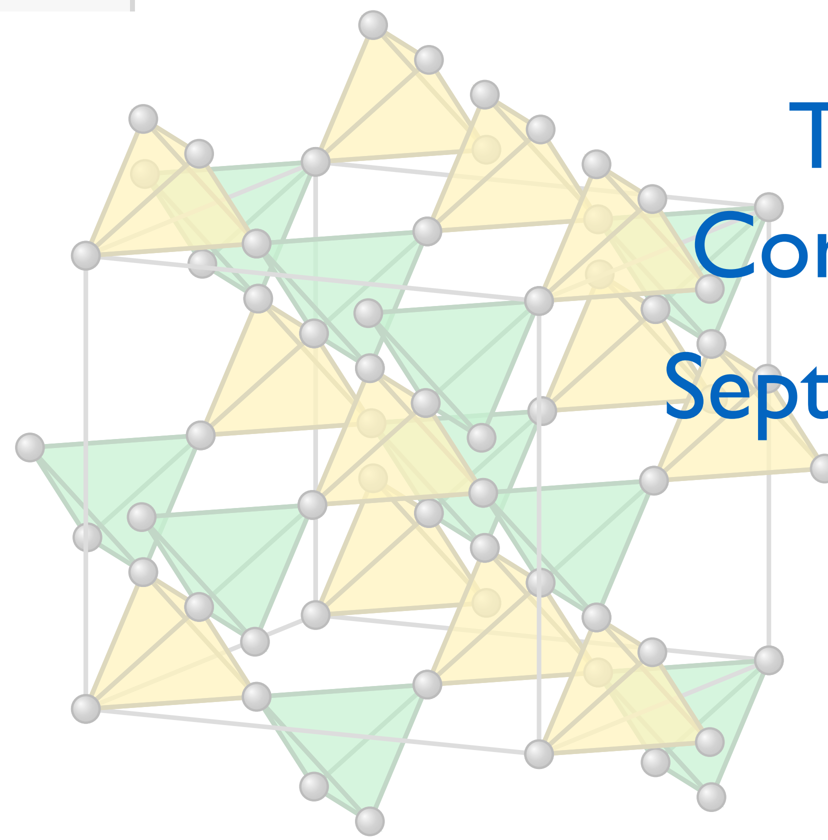


Fractonic Phases in Frustrated Magnets

Yong Baek Kim
University of Toronto



IBS-APCTP

Topological & Correlated Matter

September 22, 2022

NSERC CRSNG
SIMONS FOUNDATION
John Simon Guggenheim Memorial Foundation

Expanding Landscape of “Topological” Phases of Matter

SPT
(Symmetry protected topology)

Haldane Phase

Topological Insulators

Topological Superconductors

Topological Semi-metals (Dirac and Weyl)

band topology

Long-Range Entanglement

Fractional Quantum Hall States

Quantum Spin Liquids

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↓

Fracton Phases

New kid in town !

Topology and Geometry

Error correcting codes

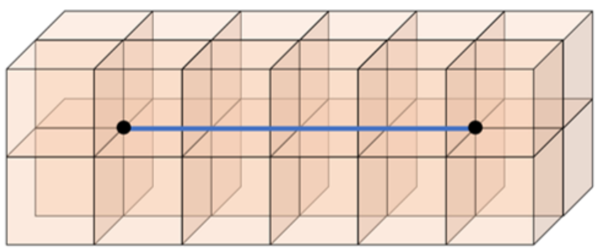
New quantum field theory (UV-IR)

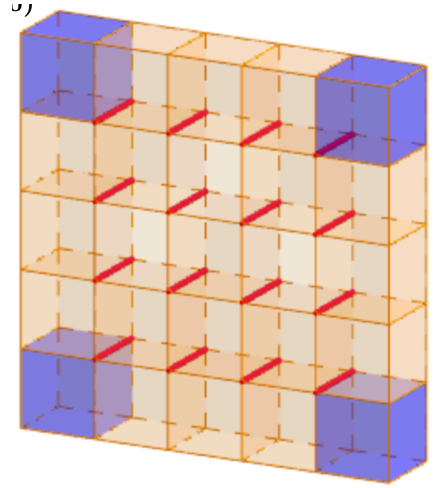
Quantum Glassiness

Fracton Order

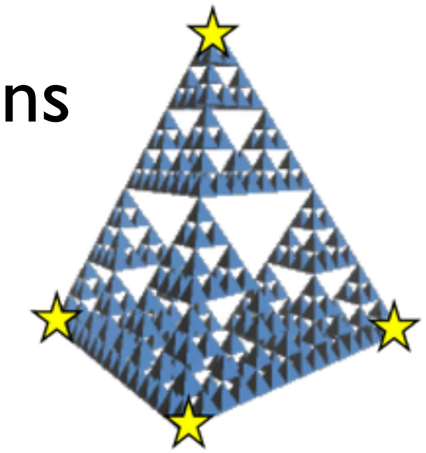
Quasiparticles with restricted mobility

lineons





fractons



Exactly solvable commuting projector models (X-Cube, Haah's codes, Chamon's code)

Vijay, Haah, Fu '15

Haah '11 Chamon '05

Fracton Order

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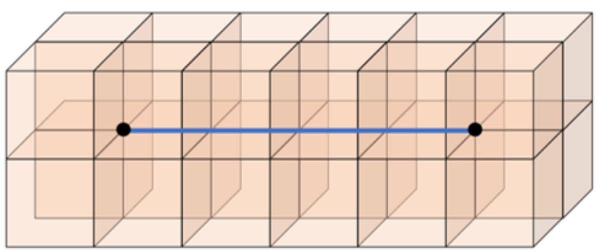

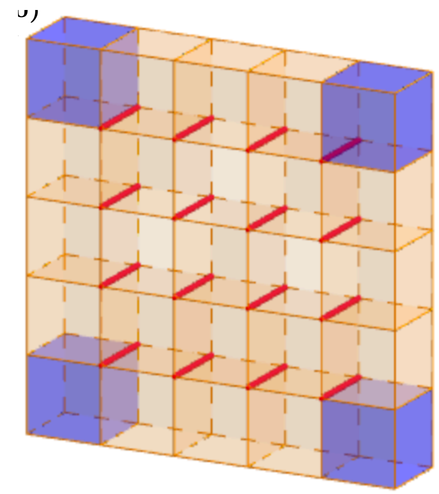

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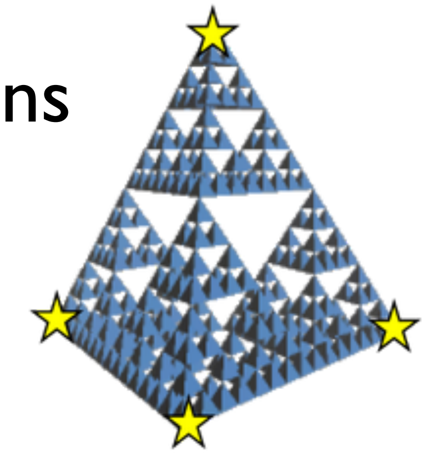
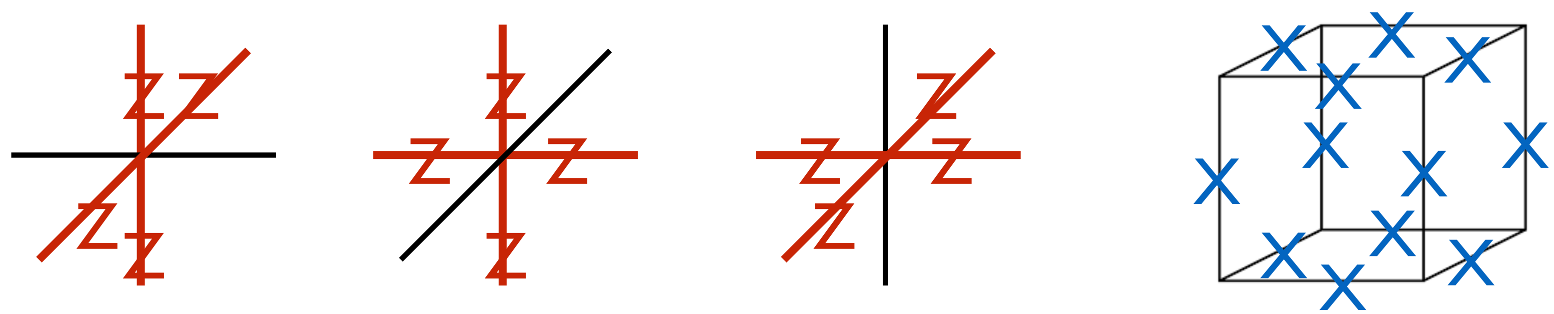


Exactly solvable commuting projector models — Vijay, Haah, Fu '15

(X-Cube, Haah's codes, Chamon's code) — Haah '11 Chamon '05

$$H = -\sum_v (A_v^x + A_v^y + A_v^z) - \sum_c B_c$$

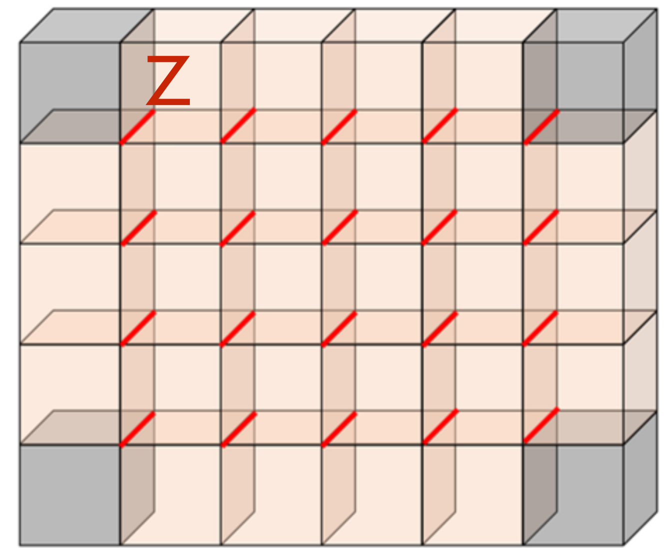
Ground state $A_v = B_c = +1$



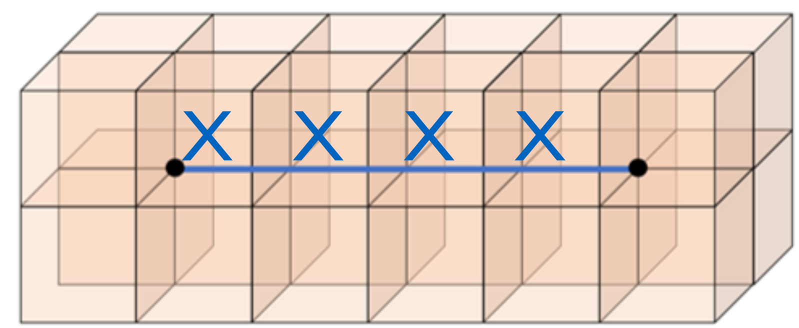
A_v^x A_v^y A_v^z B_c

Excitations with mobility restrictions

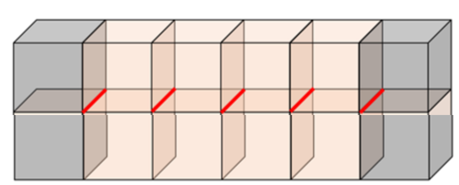
Four fractons at the corner of the membrane geometry



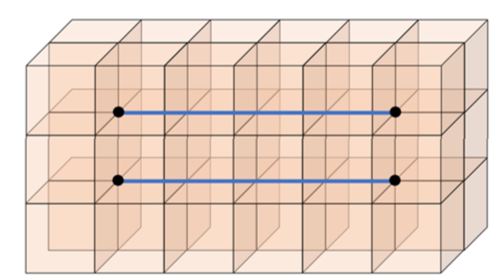
Lineons at the edge of the string

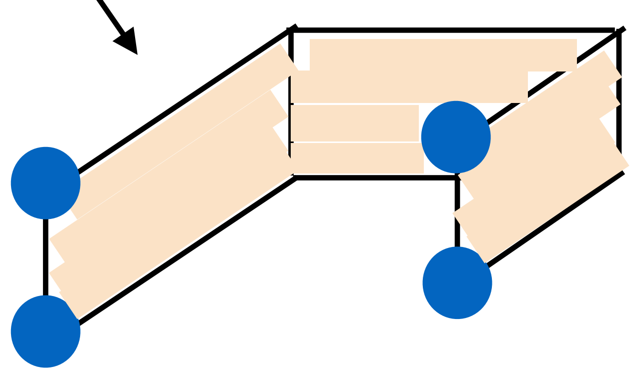


Fracton dipoles



Lineon dipoles





Planeons

Vijay, Haah, Fu ‘15

Fracton Order

Sub-extensive ground state degeneracy depends on geometry

e.g. X-Cube model GSD = $2^{2(L_x+L_y+L_z)-3}$

Fracton quantum field theory — Slagle, YBK, Shirley, X.Chen, Z.Wang, Hemele, Barkeshili, Blumash, Y.You, Burnell, Prem, M.Cheng, Williamson, Aasen, Pretko, Gromov, XGWen, J.Wang, Seiberg, Shao, …

Relation to elasticity theory — Radzihovsky, Pretko '18-19

Rank-2 tensor gauge theory — Pretko '17

Higher-rank Gauge Theory

Gauss's law

Rank-1 U(1) gauge theory

$$\partial_i E_i = 0$$

$$\partial_i E^i = \rho \neq 0$$

$$A_i(x) \rightarrow A_i(x) + \partial_i \lambda(x)$$

Higher-rank Gauge Theory

	Gauss's law	
Rank-1 U(1) gauge theory	$\partial_i E_i = 0$ $\partial_i E^i = \rho \neq 0$	$A_i(x) \rightarrow A_i(x) + \partial_i \lambda(x)$
Rank-2 U(1) gauge theory (Scalar charge)	$\partial_i \partial_j E_{ij} = 0$ $\partial_i \partial_j E^{ij} = \rho \neq 0$	$A_{ij} \rightarrow A_{ij} + \partial_i \partial_j \phi(x)$
Rank-2 U(1) gauge theory (Vector charge)	$\partial_i E_{ij} = 0$ $\partial_i E_{ij} = \rho_j \neq 0$	$A_{ij} \rightarrow A_{ij} + \partial_i \lambda_j(x) + \partial_j \lambda_i(x)$

Pretko '17

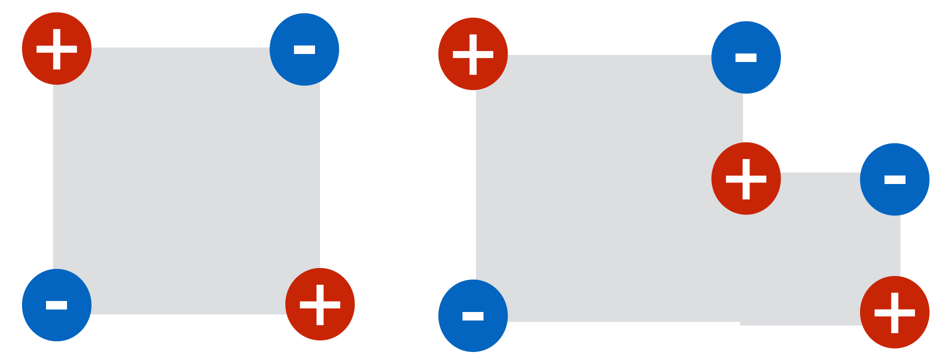
Higher-rank Gauge Theory

Rank-2 U(1)
gauge theory
(Scalar charge)

$$\partial_i \partial_j E_{ij} = 0 \qquad \int \rho = 0 \qquad \int \vec{x}\rho = 0$$

Both charge and dipole moments
are conserved

Quadrupolar charge configurations



Pretko '17

Higher-rank Gauge Theory

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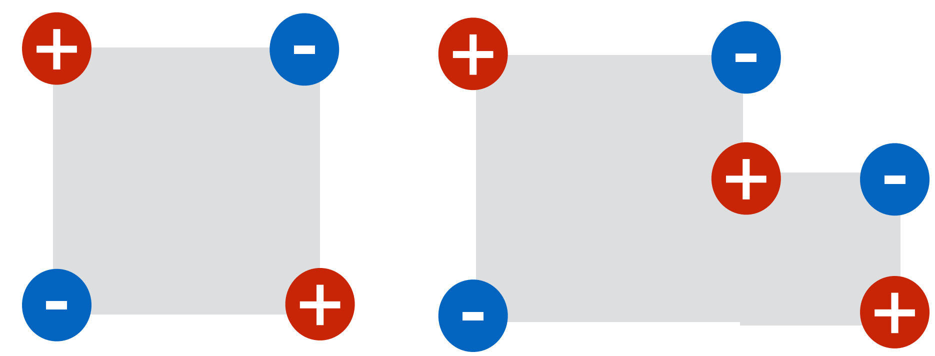


Rank-2 U(1)
gauge theory
(Vector charge)

$$\partial_i E_{ij} = 0 \qquad \int \vec{\rho} = 0 \qquad \int \vec{x} \times \vec{\rho} = 0$$

Both "momentum" and "angular momentum" are conserved

Charges restricted to move along the charge vector directions

Pretko '17

Outline

1. Quantum spin ice (a 3D quantum spin liquid) as a U(1) gauge theory (review)

2. A realistic spin model for a rank-2 U(1) gauge theory in breathing pyrochlore lattice (Classical)

   Yan, Benton, Jaubert, Shannon 2020

3. A realistic spin model for fractonic phases in breathing pyrochlore lattice (Quantum):

   SangEun Han, Adarsh Patri, YBK, arXiv:2109.03835

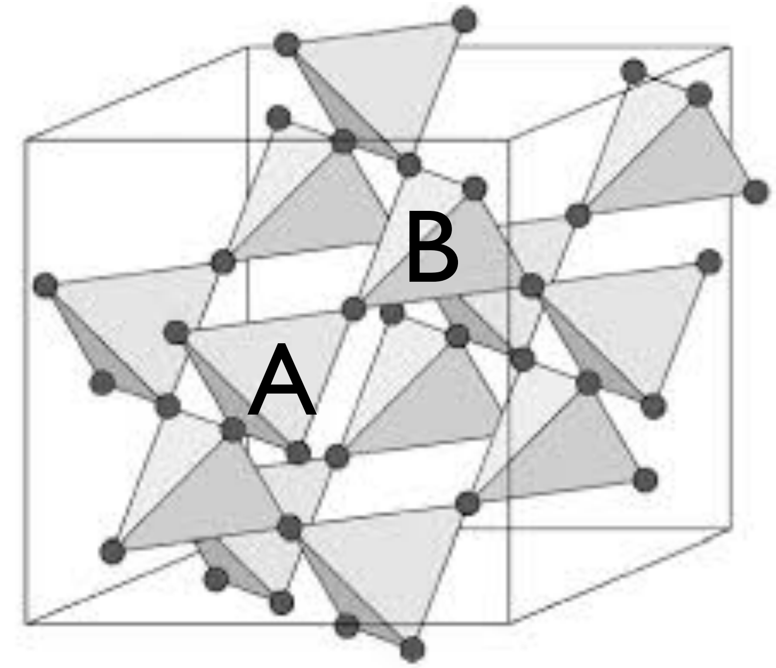
Ising Model: Classical Spin Ice

$$\mathcal{H}_I = J_z \sum_{\langle ij \rangle} S_i^z S_j^z$$

$$\mathcal{H}_I = \frac{J_z}{2} \sum_{\triangleleft} (S_{\triangleleft}^z)^2$$

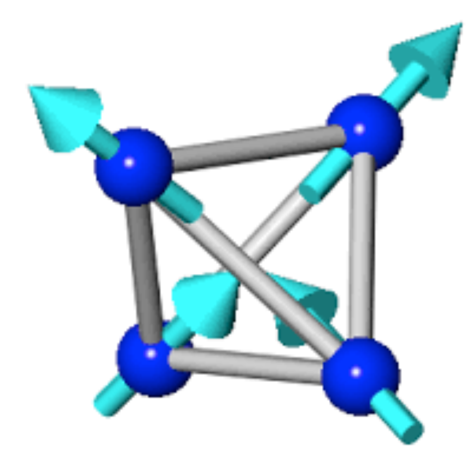
+ constant

$$S_{\triangleleft}^z = \sum_{i \in \triangleleft} S_i^z$$



$$S_{\triangleleft}^z = \sum_{i \in \triangleleft} S_i^z = 0$$

2-in/2-out: Classical Spin Ice



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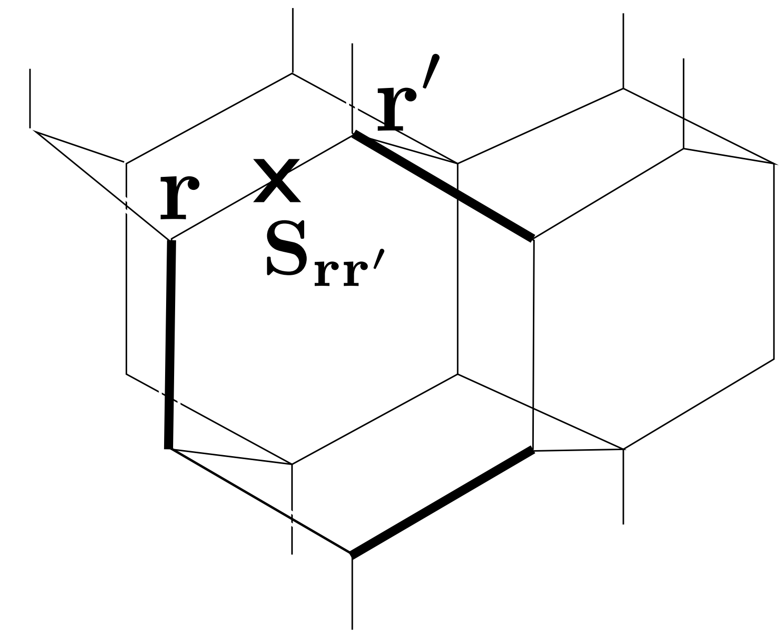


$\mathbf{S}_i = \mathbf{S}_{\mathbf{rr}'}$

$\mathbf{r}$ (dual) diamond lattice

$\mathbf{rr}'$ link connecting two diamond lattice sites

$E_{\mathbf{rr}'} = \pm S_{\mathbf{rr}'}^z$ dual diamond lattice

$$(\nabla \cdot E)_{\mathbf{r}} = \sum_{\mathbf{r}' \leftarrow \mathbf{r}} E_{\mathbf{rr}'} = \pm S_{\triangleleft}^z$$

$(\nabla \cdot E)_{\mathbf{r}} = 0$

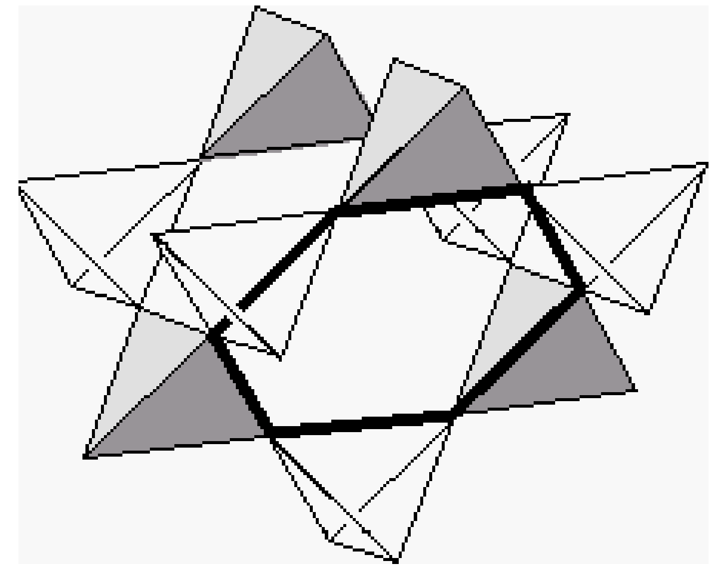
Gauss's law

Classical spin ice ground state manifold

Moessner, Sondhi '04-05

Quantum Spin Ice

$$\mathcal{H} = \mathcal{H}_I + \mathcal{H}'$$



$$\mathcal{H}_I = \frac{J_z}{2} \sum_{\triangleleft} (S^z_{\triangleleft})^2 \qquad S^z_{\triangleleft} = \sum_{i \in \triangleleft} S^z_i$$

$$\mathcal{H}' = -\frac{J_\perp}{2} \sum_{\langle ij \rangle} (S^+_i S^-_j + h.c.)$$

$J_z \gg J_\perp$ degenerate perturbation theory

$$\mathcal{H}_{eff} = -J_{ring} \sum_{\hexagon} (S^+_1 S^-_2 S^+_3 S^-_4 S^+_5 S^-_6 + h.c.)$$

$$J_{ring} = 12 J^3_\perp / J^2_z$$

Hermele, Balents, Fisher ’03

Banerjee, Isakov, Damle, YBK ’08

Quantum Electrodynamics

$$S^z_{\mathbf{rr}'} = \pm E_{\mathbf{rr}'} \qquad S^+_{\mathbf{rr}'} = e^{\pm i A_{\mathbf{rr}'}} \qquad \pm \ \mathbf{r} \in \mathrm{A/B} \qquad [A_{\mathbf{rr}'}, E_{\mathbf{rr}'}] = i$$

Quantum Electrodynamics

$$S^z_{\mathbf{rr'}} = \pm E_{\mathbf{rr'}} \qquad S^+_{\mathbf{rr'}} = e^{\pm i A_{\mathbf{rr'}}} \qquad \pm\ \mathbf{r} \in \mathrm{A/B} \qquad [A_{\mathbf{rr'}}, E_{\mathbf{rr'}}] = i$$

$$\mathcal{H}_{eff} = -J_{ring} \sum_{\hexagon} (S_1^+ S_2^- S_3^+ S_4^- S_5^+ S_6^- + h.c.)$$

$$\longrightarrow \sum_{\hexagon} e^{i(A_{12} - A_{23} + A_{34} - A_{45} + A_{56} - A_{61})} + h.c. = \sum_{\hexagon} 2\cos(\nabla \times A)_{\hexagon}$$

Quantum Electrodynamics

$$S^z_{\mathbf{rr'}} = \pm E_{\mathbf{rr'}} \qquad S^+_{\mathbf{rr'}} = e^{\pm i A_{\mathbf{rr'}}} \qquad \pm\ \mathbf{r} \in \mathrm{A/B} \qquad [A_{\mathbf{rr'}}, E_{\mathbf{rr'}}] = i$$

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$$\longrightarrow \sum_{\hexagon} e^{i(A_{12} - A_{23} + A_{34} - A_{45} + A_{56} - A_{61})} + h.c. = \sum_{\hexagon} 2\cos(\nabla \times A)_{\hexagon}$$

$$\mathcal{H} = \frac{U}{2} \sum_{\langle \mathbf{rr'} \rangle} \left(E^2_{\mathbf{rr'}} - \frac{1}{4} \right) - K \sum_{\hexagon} \cos(\nabla \times A)_{\hexagon}$$

large U

$K \sim J_{ring}$

$$(\nabla \times A)_{\hexagon} = \sum_{\mathbf{rr'} \in \hexagon}^{\circlearrowleft} A_{\mathbf{rr'}} = A_{12} - A_{23} + A_{34} - A_{45} + A_{56} - A_{61}$$

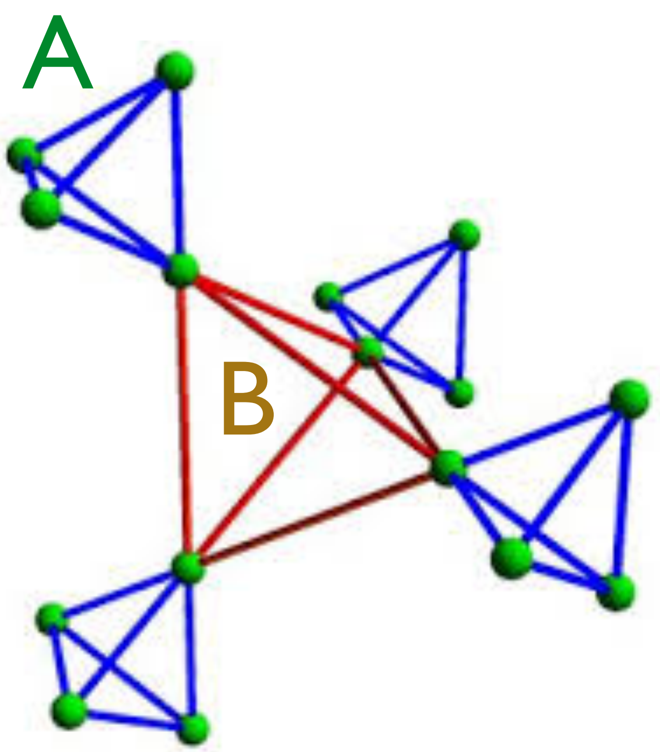
$$(\nabla \cdot E)_{\mathbf{r}} = \sum_{\mathbf{r'} \leftarrow \mathbf{r}} E_{\mathbf{rr'}} = \pm S^z_{\triangle}$$

Fracton Phases on Breathing Pyrochlore Lattice and rank-2 Gauge Theory

SangEun Han   Adarsh Patri

arXiv:2109.03835

Breathing Pyrochlore Lattice



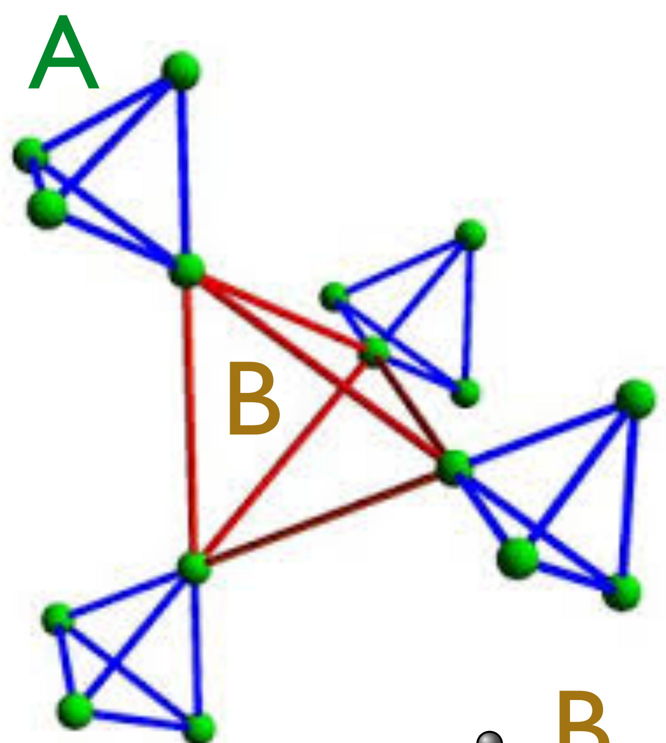
$Ba_3Yb_2Zn_5O_{11}$

Spin exchange interactions are different on A and B tetrahedra

J_B is order of magnitude smaller than J_A

Kimura, Nakatsuji, Kimura '14

Breathing Pyrochlore Lattice



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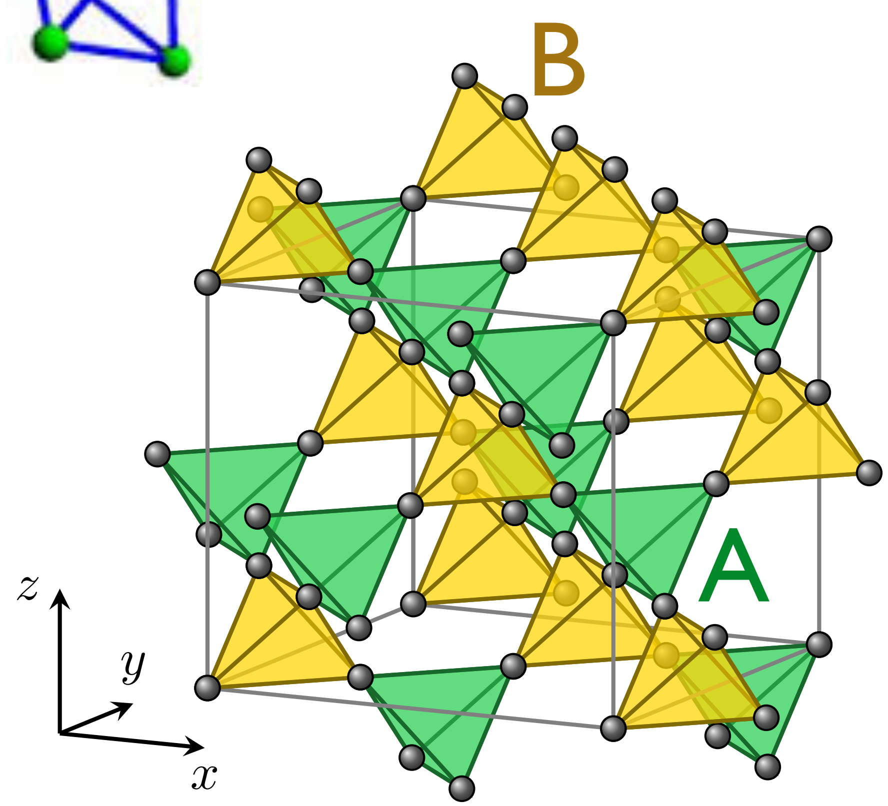
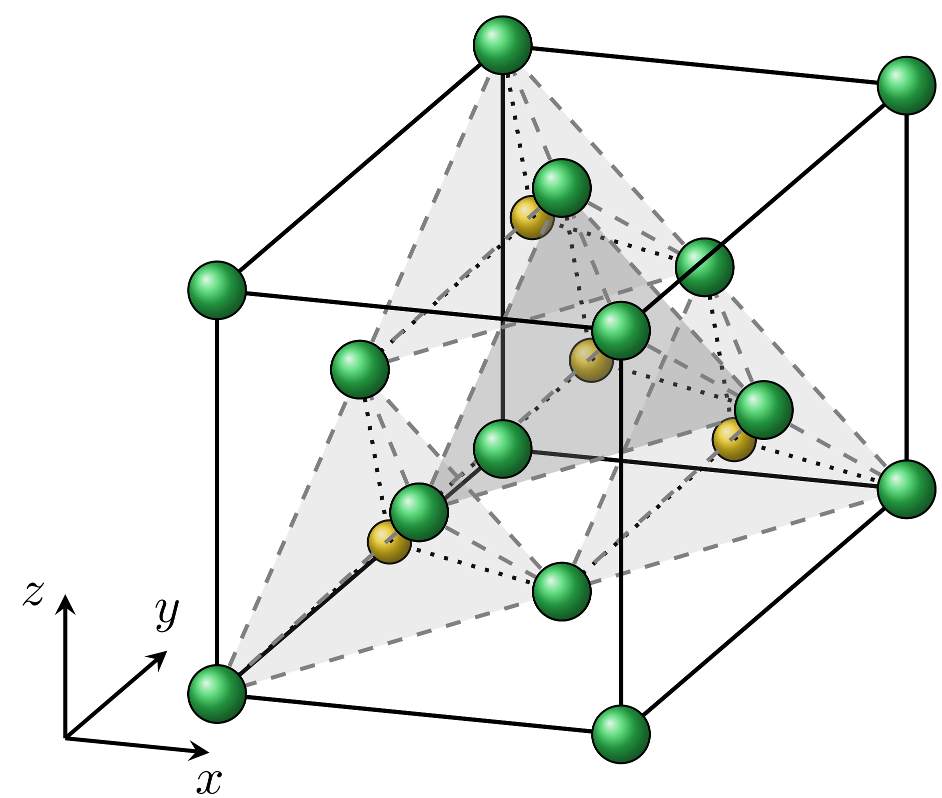




A and B sub-lattices make FCC lattices, respectively

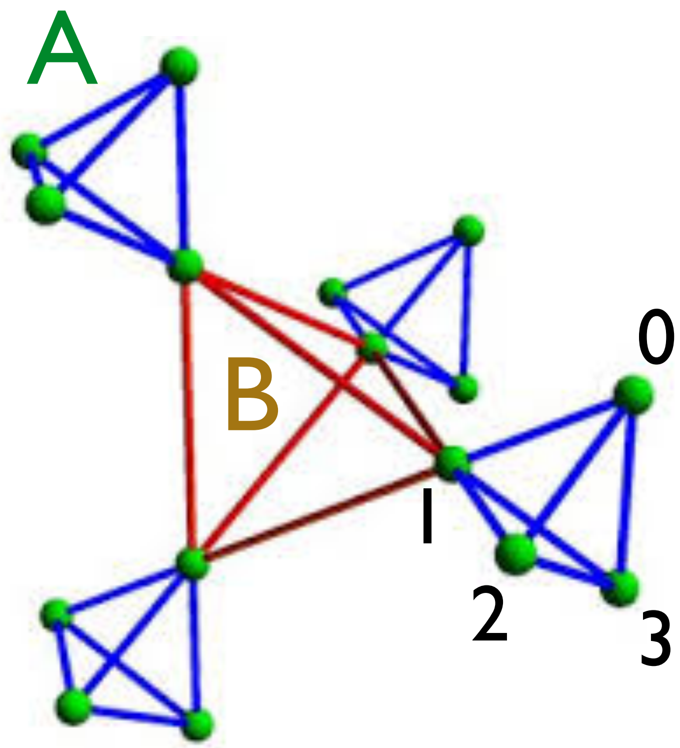
Most generic spin model

$$H = \sum_{\langle ij \rangle \in \mathrm{A}} \left[J_{\mathrm{A}} \mathbf{S}_i \cdot \mathbf{S}_j + D_{\mathrm{A}} \hat{\mathbf{d}}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j) + K^{\alpha}_{\mathrm{A},ij} S^{\alpha}_i S^{\alpha}_j + \Gamma^{\gamma\delta}_{\mathrm{A},ij} (S^{\gamma}_i S^{\delta}_j + S^{\delta}_i S^{\gamma}_j) + E_{\mathrm{A},0} \right]$$

$$+ \sum_{\langle ij \rangle \in \mathrm{B}} \left[J_{\mathrm{B}} \mathbf{S}_i \cdot \mathbf{S}_j + D_{\mathrm{B}} \hat{\mathbf{d}}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j) + K^{\alpha}_{\mathrm{B},ij} S^{\alpha}_i S^{\alpha}_j + \Gamma^{\gamma\delta}_{\mathrm{B},ij} (S^{\gamma}_i S^{\delta}_j + S^{\delta}_i S^{\gamma}_j) + E_{\mathrm{B},0} \right]$$

$J_A, J_B > 0$ Heisenberg — K_A, K_B Kitaev

D_A, D_B Dzyaloshinski-Moriya — Γ_A, Γ_B Symmetric anisotropic exchange



Yan, Benton, Jaubert, Shannon '17

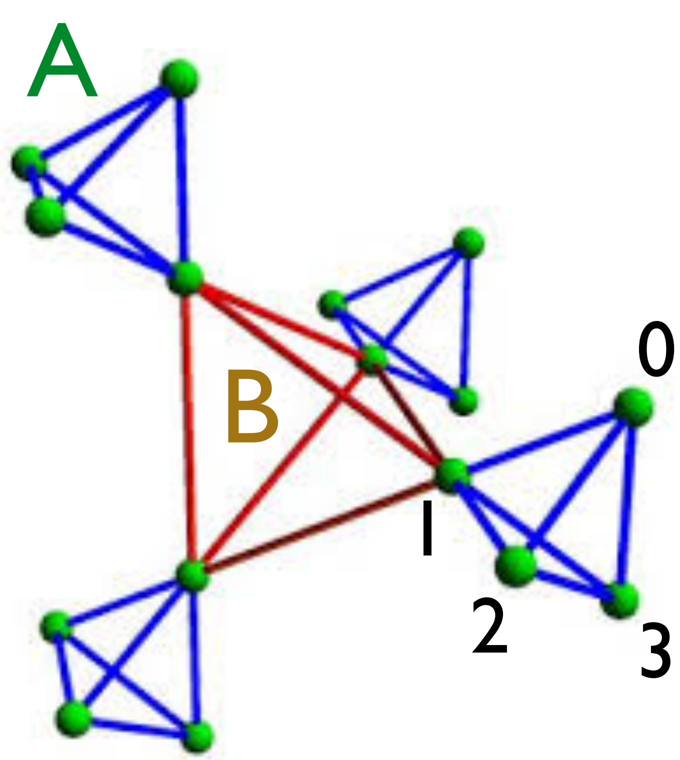
Most generic spin model

$$H = \sum_{\langle ij \rangle \in \mathrm{A}} \left[J_{\mathrm{A}} \mathbf{S}_i \cdot \mathbf{S}_j + D_{\mathrm{A}} \hat{\mathbf{d}}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j) + K^{\alpha}_{\mathrm{A},ij} S^{\alpha}_i S^{\alpha}_j + \Gamma^{\gamma\delta}_{\mathrm{A},ij} (S^{\gamma}_i S^{\delta}_j + S^{\delta}_i S^{\gamma}_j) + E_{\mathrm{A},0} \right]$$
$$+ \sum_{\langle ij \rangle \in \mathrm{B}} \left[J_{\mathrm{B}} \mathbf{S}_i \cdot \mathbf{S}_j + D_{\mathrm{B}} \hat{\mathbf{d}}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j) + K^{\alpha}_{\mathrm{B},ij} S^{\alpha}_i S^{\alpha}_j + \Gamma^{\gamma\delta}_{\mathrm{B},ij} (S^{\gamma}_i S^{\delta}_j + S^{\delta}_i S^{\gamma}_j) + E_{\mathrm{B},0} \right]$$

$J_A, J_B > 0$	Heisenberg	K_A, K_B	Kitaev
D_A, D_B	Dzyaloshinski-Moriya	Γ_A, Γ_B	Symmetric anisotropic exchange

Interactions on B much smaller than those on A

Heisenberg dominates over other anisotropic interactions



Yan, Benton, Jaubert, Shannon '17

Spin model via normal mode representation

$$H = \frac{1}{2} \sum_{\mathrm{A},\Gamma} a_{\mathrm{A},\Gamma} m_{\mathrm{A},\Gamma}^2 + \frac{1}{2} \sum_{\mathrm{B},\Gamma} a_{\mathrm{B},\Gamma} m_{\mathrm{B},\Gamma}^2$$

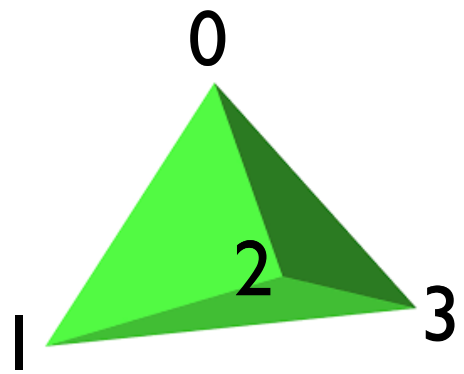
$$\Gamma = \{\mathsf{A}_2, \mathsf{E}, \mathsf{T}_2, \mathsf{T}_{1+}, \mathsf{T}_{1-}\}$$

Irreducible representations of T_d

$a_{\mathrm{A/B},\Gamma}$

Interaction coefficients

"mass"



Spin model via normal mode representation

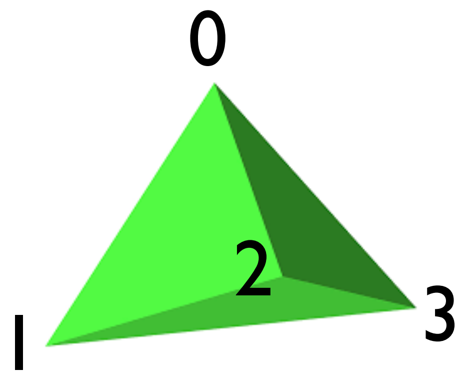
$$H = \frac{1}{2}\sum_{\mathrm{A},\Gamma} a_{\mathrm{A},\Gamma} m^2_{\mathrm{A},\Gamma} + \frac{1}{2}\sum_{\mathrm{B},\Gamma} a_{\mathrm{B},\Gamma} m^2_{\mathrm{B},\Gamma}$$

$$\Gamma \;=\; \{\mathsf{A}_2, \mathsf{E}, \mathsf{T}_2, \mathsf{T}_{1+}, \mathsf{T}_{1-}\}$$

Irreducible representations of T_d

$a_{\mathrm{A/B},\Gamma}$

Interaction coefficients "mass"



$$a_{\mathsf{A}_2} = \frac{2E_0}{3} - J_\mathrm{A} - \frac{4D_\mathrm{A}}{\sqrt{2}} + K_\mathrm{A} - 4\Gamma_\mathrm{A},$$

$$a_{\mathsf{E}} = \frac{2E_0}{3} - J_\mathrm{A} + \frac{2D_\mathrm{A}}{\sqrt{2}} + K_\mathrm{A} + 2\Gamma_\mathrm{A},$$

$$a_{\mathsf{T}_{1-}} = \frac{2E_0}{3} - J_\mathrm{A} + \frac{2D_\mathrm{A}}{\sqrt{2}} - K_\mathrm{A} - 2\Gamma_\mathrm{A},$$

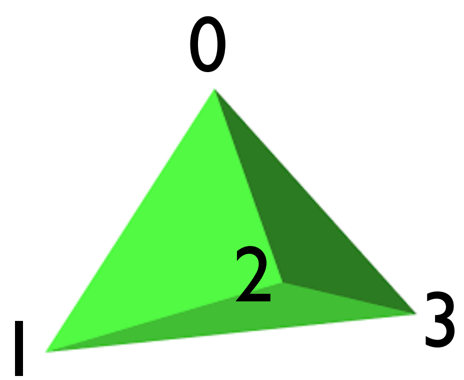
$$a_{\mathsf{T}_2} = \frac{2E_0}{3} - J_\mathrm{A} - \frac{2D_\mathrm{A}}{\sqrt{2}} - K_\mathrm{A} + 2\Gamma_\mathrm{A},$$

$$a_{\mathsf{T}_{1+}} = \frac{2E_0}{3} + 3J_\mathrm{A} + K_\mathrm{A}.$$

similarly for B sub-lattices

Spin model via normal mode representation

$$H = \frac{1}{2}\sum_{\mathrm{A},\Gamma} a_{\mathrm{A},\Gamma} m^2_{\mathrm{A},\Gamma} + \frac{1}{2}\sum_{\mathrm{B},\Gamma} a_{\mathrm{B},\Gamma} m^2_{\mathrm{B},\Gamma}$$



$$\Gamma = \{\mathsf{A}_2, \mathsf{E}, \mathsf{T}_2, \mathsf{T}_{1+}, \mathsf{T}_{1-}\}$$

Irreducible representations of T_d

$a_{\mathrm{A/B},\Gamma}$

Interaction coefficients "mass"

$$a_{\mathsf{A}_2} = \frac{2E_0}{3} - J_\mathrm{A} - \frac{4D_\mathrm{A}}{\sqrt{2}} + K_\mathrm{A} - 4\Gamma_\mathrm{A},$$

$$a_{\mathsf{E}} = \frac{2E_0}{3} - J_\mathrm{A} + \frac{2D_\mathrm{A}}{\sqrt{2}} + K_\mathrm{A} + 2\Gamma_\mathrm{A},$$

$$a_{\mathsf{T}_{1-}} = \frac{2E_0}{3} - J_\mathrm{A} + \frac{2D_\mathrm{A}}{\sqrt{2}} - K_\mathrm{A} - 2\Gamma_\mathrm{A},$$

$$a_{\mathsf{T}_2} = \frac{2E_0}{3} - J_\mathrm{A} - \frac{2D_\mathrm{A}}{\sqrt{2}} - K_\mathrm{A} + 2\Gamma_\mathrm{A},$$

$$a_{\mathsf{T}_{1+}} = \frac{2E_0}{3} + 3J_\mathrm{A} + K_\mathrm{A}.$$

similarly for B sub-lattices

Assume $J_A, J_B > 0$

$$J \gg |D|, |K|, |\Gamma|$$

Heaviest mode

$$a_{(\mathrm{A},\mathrm{B}),\mathsf{T}_{1+}} > 0$$

We can set

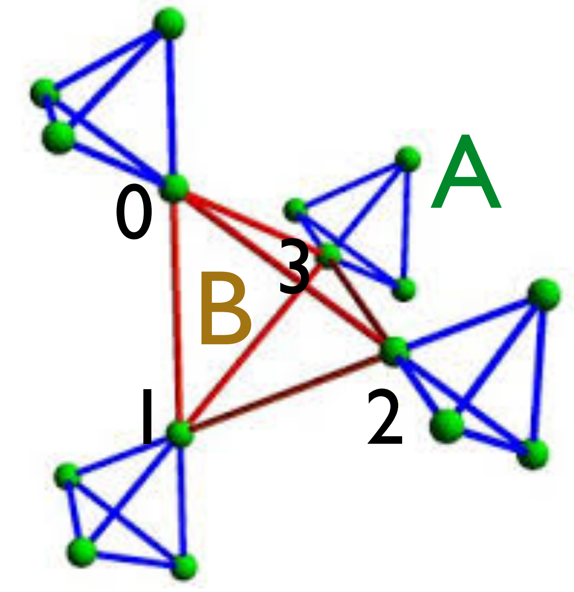
$$\mathbf{m}_{\mathrm{A},\mathsf{T}_{1+}} = 0 \qquad \mathbf{m}_{\mathrm{B},\mathsf{T}_{1+}} = 0$$

Emergent constraints

Yan, Benton, Jaubert, Shannon ‘20

$$\mathbf{m}_{\mathrm{B},\mathsf{T}_{1+}} = 0$$

generates the constraints on the normal modes of the A tetrahedra



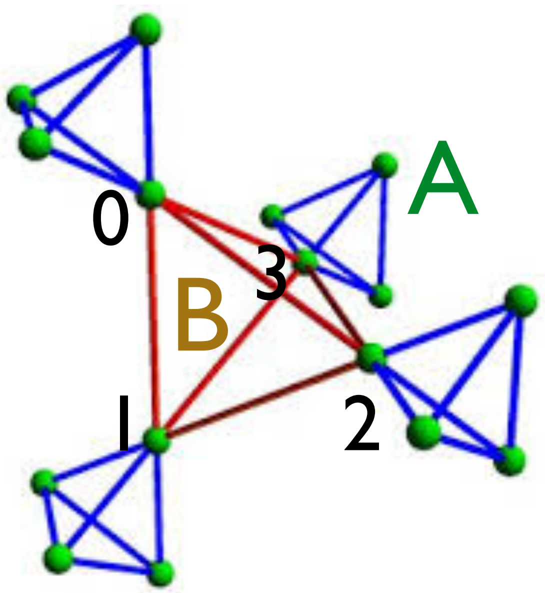
Emergent constraints

$\mathbf{m}_{\mathrm{B},\mathsf{T}_{1+}} = 0$

Yan, Benton, Jaubert, Shannon '20

generates the constraints on the normal modes of the A tetrahedra

$$\frac{2}{\sqrt{3}}\begin{pmatrix} \partial_x m^1_{\mathrm{A},\mathsf{E}} \\ -\frac{1}{2}\partial_y m^1_{\mathrm{A},\mathsf{E}} + \frac{\sqrt{3}}{2}\partial_y m^2_{\mathrm{A},\mathsf{E}} \\ -\frac{1}{2}\partial_z m^1_{\mathrm{A},\mathsf{E}} - \frac{\sqrt{3}}{2}\partial_z m^2_{\mathrm{A},\mathsf{E}} \end{pmatrix} + \begin{pmatrix} \partial_y m^z_{\mathrm{A},\mathsf{T}_{1-}} + \partial_z m^y_{\mathrm{A},\mathsf{T}_{1-}} \\ \partial_x m^z_{\mathrm{A},\mathsf{T}_{1-}} + \partial_z m^x_{\mathrm{A},\mathsf{T}_{1-}} \\ \partial_x m^y_{\mathrm{A},\mathsf{T}_{1-}} + \partial_y m^x_{\mathrm{A},\mathsf{T}_{1-}} \end{pmatrix}$$
$$-\sqrt{\frac{2}{3}}\boldsymbol{\nabla} m_{\mathrm{A},\mathsf{A}_2} - \nabla \times \mathbf{m}_{\mathrm{A},\mathsf{T}_2} = 0.$$



Emergent constraints

$\mathbf{m}_{\mathrm{B},\mathsf{T}_{1+}} = 0$

Yan, Benton, Jaubert, Shannon '20

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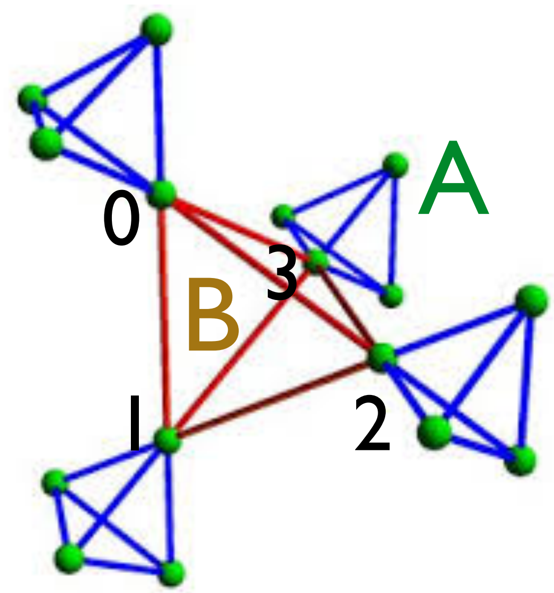
$$-\sqrt{\frac{2}{3}}\boldsymbol{\nabla} m_{\mathrm{A},\mathsf{A}_2} - \nabla \times \mathbf{m}_{\mathrm{A},\mathsf{T}_2} = 0.$$

$$= \nabla \cdot (\mathbf{E}^{\mathrm{trace}}_{\mathrm{A}} + \mathbf{E}^{\mathrm{sym}}_{\mathrm{A}} + \mathbf{E}^{\mathrm{antisym}}_{\mathrm{A}}) = 0$$

Gauss's law !



Emergent constraints

Yan, Benton, Jaubert, Shannon ‘20

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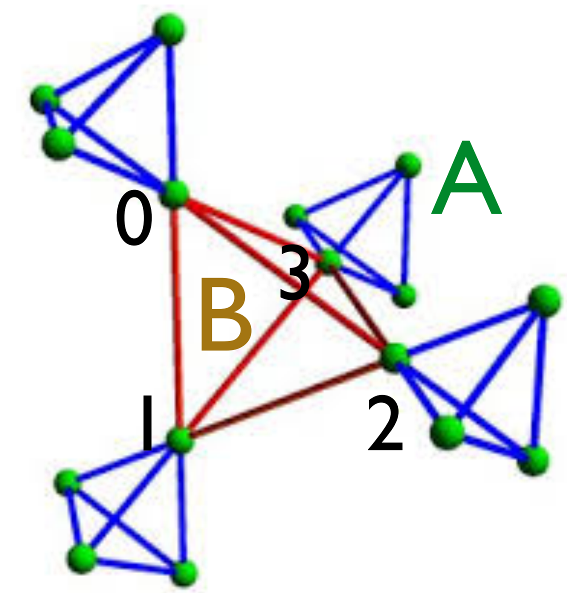
Gauss’s law !



$$\mathbf{E}^{\mathrm{sym}}_{\mathrm{A}} = \begin{pmatrix} \frac{2}{\sqrt{3}} m^1_{\mathrm{A},\mathsf{E}} & m^z_{\mathrm{A},\mathsf{T}_{1-}} & m^y_{\mathrm{A},\mathsf{T}_{1-}} \\ m^z_{\mathrm{A},\mathsf{T}_{1-}} & -\frac{1}{\sqrt{3}} m^1_{\mathrm{A},\mathsf{E}} + m^2_{\mathrm{A},\mathsf{E}} & m^x_{\mathrm{A},\mathsf{T}_{1-}} \\ m^y_{\mathrm{A},\mathsf{T}_{1-}} & m^x_{\mathrm{A},\mathsf{T}_{1-}} & -\frac{1}{\sqrt{3}} m^1_{\mathrm{A},\mathsf{E}} - m^2_{\mathrm{A},\mathsf{E}} \end{pmatrix}$$

Rank-2 electric fields

$$(\mathbf{E}^{\mathrm{trace}}_{\mathrm{A}})_{ij} = -\sqrt{\tfrac{2}{3}} m_{\mathrm{A},\mathsf{A}_2} \delta_{ij} \qquad (\mathbf{E}^{\mathrm{antisym}}_{\mathrm{A}})_{ij} = -\epsilon_{ijk} m^k_{\mathrm{A},\mathsf{T}_2}$$

Rank-2 gauge theory (Classical)

Yan, Benton, Jaubert, Shannon ‘20

Choose

$$a_{\mathrm{E}} = a_{\mathrm{T}_{1-}} < a_{\mathrm{A}_2}, a_{\mathrm{T}_2}, a_{\mathrm{T}_{1+}}$$

Then $\mathbf{m}_{\mathsf{T}_{1+}} = \mathbf{m}_{\mathsf{T}_2} = \mathbf{0},\ m_{\mathsf{A}_2} = 0$ in the low energy limit

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Then $\mathbf{m}_{\mathsf{T}_{1+}} = \mathbf{m}_{\mathsf{T}_2} = \mathbf{0}, \; m_{\mathsf{A}_2} = 0$ in the low energy limit

The remaining electric fields are symmetric and traceless

$$\mathbf{E}^{\mathrm{sym}}_{\mathrm{A}} = \begin{pmatrix} \frac{2}{\sqrt{3}} m^1_{\mathrm{A},\mathsf{E}} & m^z_{\mathrm{A},\mathsf{T}_{1-}} & m^y_{\mathrm{A},\mathsf{T}_{1-}} \\ m^z_{\mathrm{A},\mathsf{T}_{1-}} & -\frac{1}{\sqrt{3}} m^1_{\mathrm{A},\mathsf{E}} + m^2_{\mathrm{A},\mathsf{E}} & m^x_{\mathrm{A},\mathsf{T}_{1-}} \\ m^y_{\mathrm{A},\mathsf{T}_{1-}} & m^x_{\mathrm{A},\mathsf{T}_{1-}} & -\frac{1}{\sqrt{3}} m^1_{\mathrm{A},\mathsf{E}} - m^2_{\mathrm{A},\mathsf{E}} \end{pmatrix}$$

$$H = \frac{1}{2} \sum_{\mathrm{A},\Gamma} a_{\mathrm{A},\Gamma} m^2_{\mathrm{A},\Gamma} \quad \longrightarrow \quad H = \frac{1}{2} \int d^3x \; E_{ij} E^{ij}$$

$$\Gamma = \mathrm{E}, \mathrm{T}_{1-} \qquad\qquad \partial_i E_{ij} = 0$$

Traceless and symmetric tensor gauge theory

Rank-2 gauge theory (Classical)

Yan, Benton, Jaubert, Shannon '20

Choose

$$a_{\mathrm{E}} = a_{\mathrm{T}_{1-}} < a_{\mathrm{A}_2}, a_{\mathrm{T}_2}, a_{\mathrm{T}_{1+}}$$

Then $\mathbf{m}_{\mathsf{T}_{1+}} = \mathbf{m}_{\mathsf{T}_2} = \mathbf{0},\ m_{\mathsf{A}_2} = 0$ in the low energy limit

We could achieve this by taking

$$a_{\mathsf{A}_2} = - J_{\mathrm{A}} - \frac{4D_{\mathrm{A}}}{\sqrt{2}},$$

$$a_{\mathsf{E}} = a_{\mathsf{T}_{1-}} = -J_{\mathrm{A}} + \frac{2D_{\mathrm{A}}}{\sqrt{2}},$$

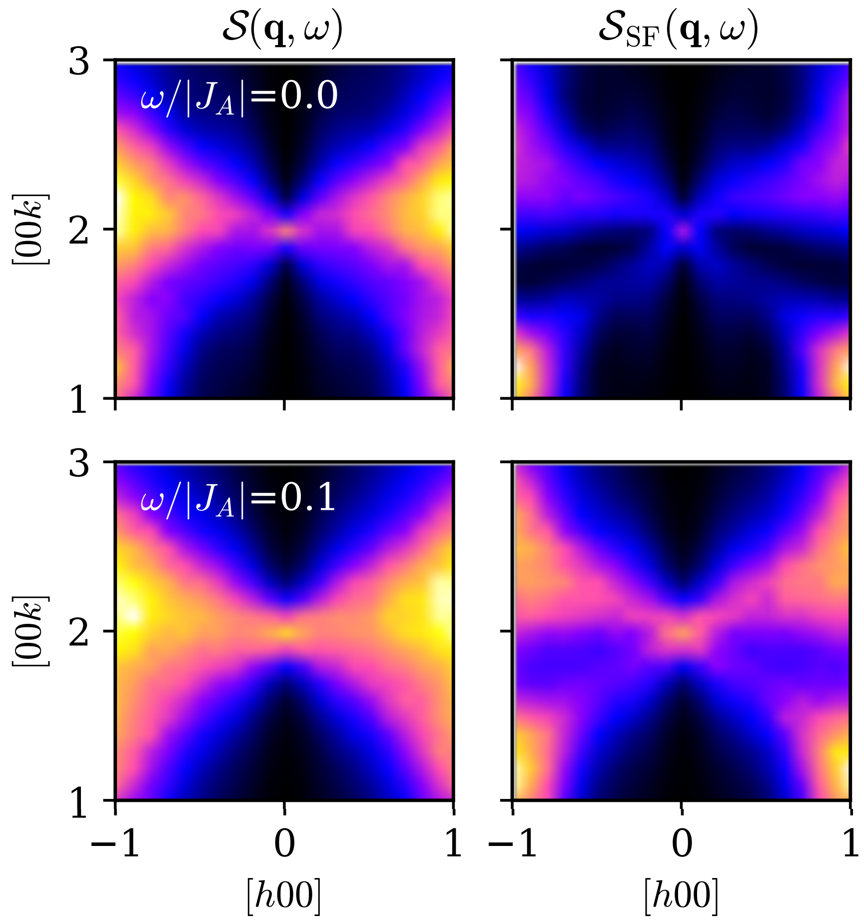
$$a_{\mathsf{T}_2} = - J_{\mathrm{A}} - \frac{2D_{\mathrm{A}}}{\sqrt{2}}$$

$$a_{\mathsf{T}_{1+}} = 3J_{\mathrm{A}}.$$

K, Γ and E_0 to zero

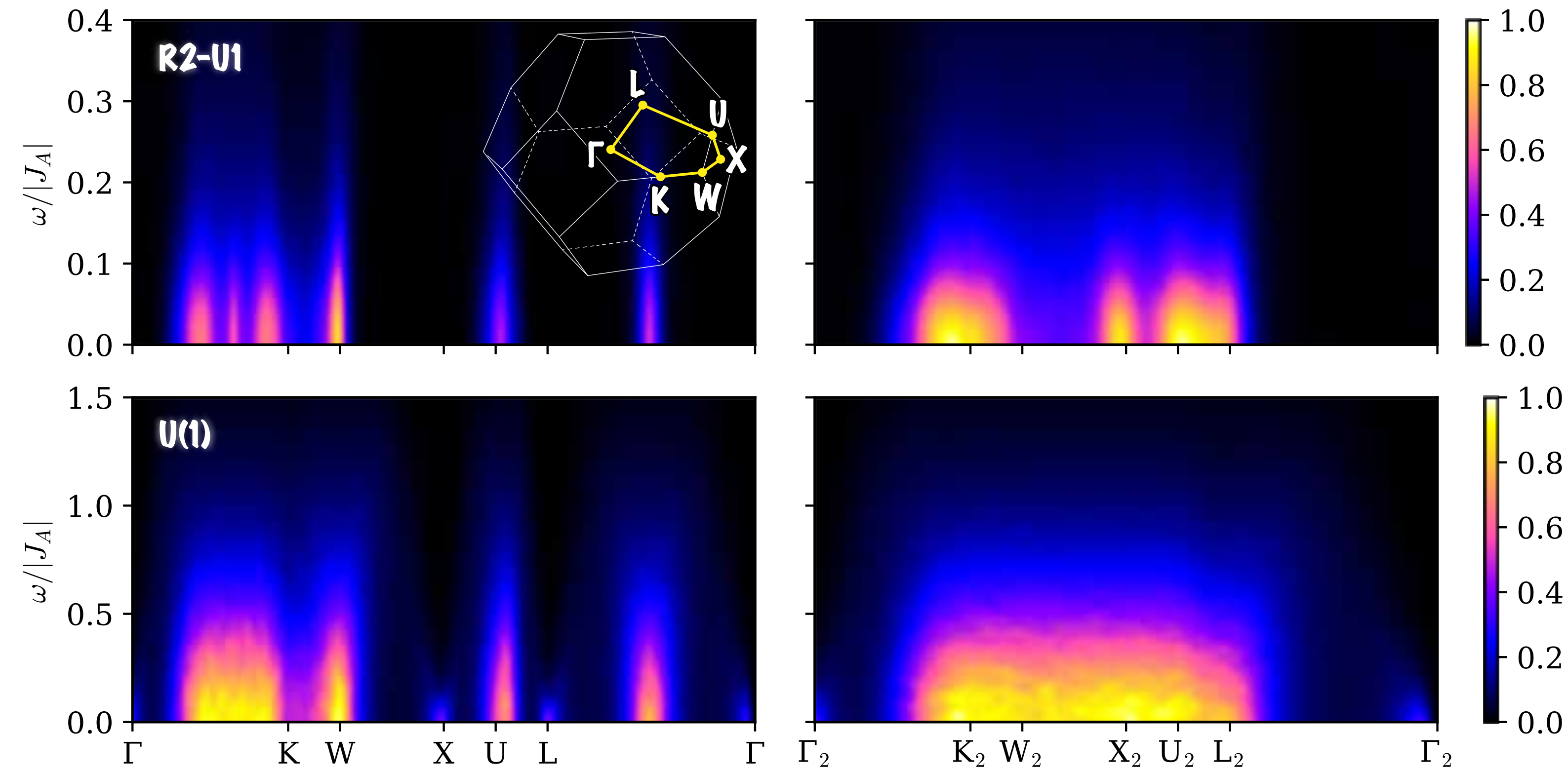
Dynamical signatures in the neutron scattering

E. Z. Zhang, F. L. Buessen, YBK, arXiv:2110.10180



Unpolarized

Spin flip scattering (Polarized)



Spin flip scattering (Polarized)

4-fold pinch point

c.f. Equaltime correlator
Yan, Benton, Jaubert, Shannon ‘20

Quantum Theory

$\mathbf{m}_{A,\Gamma}$ are essentially spin variables

E_{ij} are non-commuting fields

$\longrightarrow$ Non-commutative quantum field theory

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Look at a different limit

$$a_{\mathrm{A}_2} = a_{\mathrm{E}} < a_{\mathrm{T}_{1-}} < a_{\mathrm{T}_2} < a_{\mathrm{T}_{1+}}$$

Then $m_{\mathrm{A},\mathrm{A}_2}, \mathbf{m}_{\mathrm{A},\mathrm{E}} \neq 0$ in the low energy limit

Quantum Theory

$\mathbf{m}_{A,\Gamma}$ are essentially spin variables

E_{ij} are non-commuting fields

→ Non-commutative quantum field theory

Look at a different limit

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Then $m_{\mathrm{A},\mathsf{A}_2}, \mathbf{m}_{\mathrm{A},\mathsf{E}} \neq 0$ in the low energy limit

Recall

$$\mathbf{E}_{\mathrm{A}}^{\mathrm{sym}} = \begin{pmatrix} \frac{2}{\sqrt{3}} m^1_{\mathrm{A},\mathsf{E}} & m^z_{\mathrm{A},\mathsf{T}_{1-}} & m^y_{\mathrm{A},\mathsf{T}_{1-}} \\ m^z_{\mathrm{A},\mathsf{T}_{1-}} & -\frac{1}{\sqrt{3}} m^1_{\mathrm{A},\mathsf{E}} + m^2_{\mathrm{A},\mathsf{E}} & m^x_{\mathrm{A},\mathsf{T}_{1-}} \\ m^y_{\mathrm{A},\mathsf{T}_{1-}} & m^x_{\mathrm{A},\mathsf{T}_{1-}} & -\frac{1}{\sqrt{3}} m^1_{\mathrm{A},\mathsf{E}} - m^2_{\mathrm{A},\mathsf{E}} \end{pmatrix}$$

$$(\mathbf{E}_{\mathrm{A}}^{\mathrm{trace}})_{ij} = -\sqrt{\frac{2}{3}} m_{\mathrm{A},\mathsf{A}_2} \delta_{ij} \qquad (\mathbf{E}_{\mathrm{A}}^{\mathrm{antisym}})_{ij} = -\epsilon_{ijk} m^k_{\mathrm{A},\mathsf{T}_2}$$

E_{ij} becomes symmetric, diagonal and traceful

Quantum Theory

$\mathbf{m}_{A,\Gamma}$ are essentially spin variables

E_{ij} are non-commuting fields

$\longrightarrow$ Non-commutative quantum field theory

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This can be achieved by

$$a_{\mathrm{A},\mathsf{A}_2} = a_{\mathsf{E}} = -J_{\mathrm{A}} - |K_{\mathrm{A}}|,$$

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$$a_{\mathrm{A},\mathsf{T}_2} = -J_{\mathrm{A}} + \frac{4|D_{\mathrm{A}}|}{\sqrt{2}} + |K_{\mathrm{A}}|,$$

$$a_{\mathrm{A},\mathsf{T}_{1+}} = 3J_{\mathrm{A}} - |K_{\mathrm{A}}|,$$

$D_A < 0$

$\Gamma_{\mathrm{A}} = |D_{\mathrm{A}}|/\sqrt{2} > 0$

$K_A < 0$

$J_A > 0$

Quantum Theory

We can work with

$$(\mathbb{E}_{\mathrm{A}})_{ij} = \sqrt{2}(\mathbf{E}_{\mathrm{A}}^{\mathrm{sym}} + \mathbf{E}_{\mathrm{A}}^{\mathrm{trace}})_{ij}$$

$$\partial_i (\mathbb{E}_{\mathrm{A}})_{ij} = \mathbf{0} \ \forall i \in \{x, y, z\}$$

Gauss's law constraint

Quantum Theory

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$$H = H_0 + H'$$

$$a_{\rm A,\mathsf{A_2}} = a_{\rm A,\mathsf{E}} = -8|a_{\rm A}|$$

$$H_0 = -4|a_{\rm A}| \sum_{\rm A} \left(\mathbf{m}_{\rm A,\mathsf{E}}^2 + m_{\rm A,\mathsf{A_2}}^2 \right)$$

$$H' = \frac{1}{2} \sum_{\substack{\rm B,\Gamma \\ \Gamma \neq T_{1+}}} a_{\rm B,\Gamma} m_{\rm B,\Gamma}^2$$

perturbation

Quantum Theory

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$$H_0 = -|a_{\mathrm{A}}| \sum_{\mathrm{A}} \left(\mathbb{E}_{\mathrm{A},xx}^2 + \mathbb{E}_{\mathrm{A},yy}^2 + \mathbb{E}_{\mathrm{A},zz}^2 \right)$$

$$= -|a_{\mathrm{A}}| \sum_{\mathrm{A}} \vec{\mathbb{E}}_{\mathrm{A}}^2 .$$

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$$H' = \frac{1}{2} \sum_{\substack{\mathrm{B},\Gamma \\ \Gamma \neq \mathrm{T}_{1+}}} a_{\mathrm{B},\Gamma} m^2_{\mathrm{B},\Gamma}$$

perturbation

$$[\mathbb{E}_{\mathrm{A},i}, \mathbb{E}_{\mathrm{A}',j}] = i\delta_{\mathrm{A},\mathrm{A}'} \epsilon_{ijk} \mathbb{E}_{\mathrm{A},k}$$

$$\vec{\mathbb{E}}_A \equiv (\mathbb{E}_{\mathrm{A},xx}, \mathbb{E}_{\mathrm{A},yy}, \mathbb{E}_{\mathrm{A},zz})$$

$$\{i, j, k\} \in \{xx, yy, zz\}$$

Quantum Theory

$[\vec{\mathbb{E}}_A^2, \mathbb{E}_{A,zz}] = 0$ This is just like $\vec{S}^2$ and S^z

$\vec{\mathbb{E}}_A^2 = S(S+1)$ $\mathbb{E}_{A,zz} = S^z$

For each tetrahedron, the ground state is five-fold degenerate with

$\mathbb{E}_{A,zz} = -2, -1, 0, 1, 2$ (S=2 states)

Quantum Theory

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Ground state manifold of the network of A-tetrahedra is described by the S=2 multiplet, satisfying the Gauss's law constraint

$$\partial_i (\mathbb{E}_\mathrm{A})_{ij} = \mathbf{0} \ \forall i \in \{x, y, z\}$$

Massive degeneracy

Spinor charges

Relaxing the Gauss's law constraint, the electric charges are located at the B-tetrahedra center

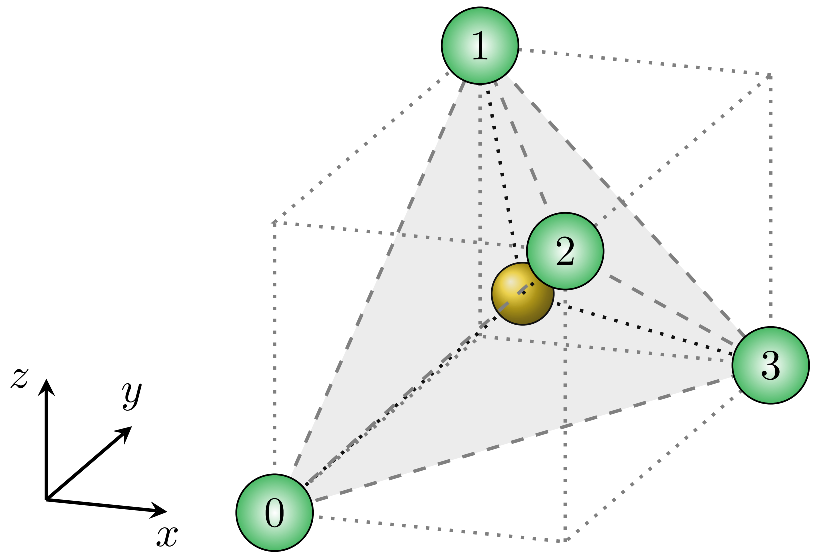
$$\rho_B^x = -\mathbb{E}_{xx}^0 - \mathbb{E}_{xx}^1 + \mathbb{E}_{xx}^2 - \mathbb{E}_{xx}^3$$

$$\rho_B^y = -\mathbb{E}_{xx}^0 + \mathbb{E}_{xx}^1 - \mathbb{E}_{xx}^2 + \mathbb{E}_{xx}^3$$

$$\rho_B^z = -\mathbb{E}_{xx}^0 + \mathbb{E}_{xx}^1 + \mathbb{E}_{xx}^2 - \mathbb{E}_{xx}^3$$

$$[\rho_{\mathrm{B}}^i, \rho_{\mathrm{B}}^j] = i\epsilon_{ij}^k \rho_{\mathrm{B}}^k$$

k = x, y, z



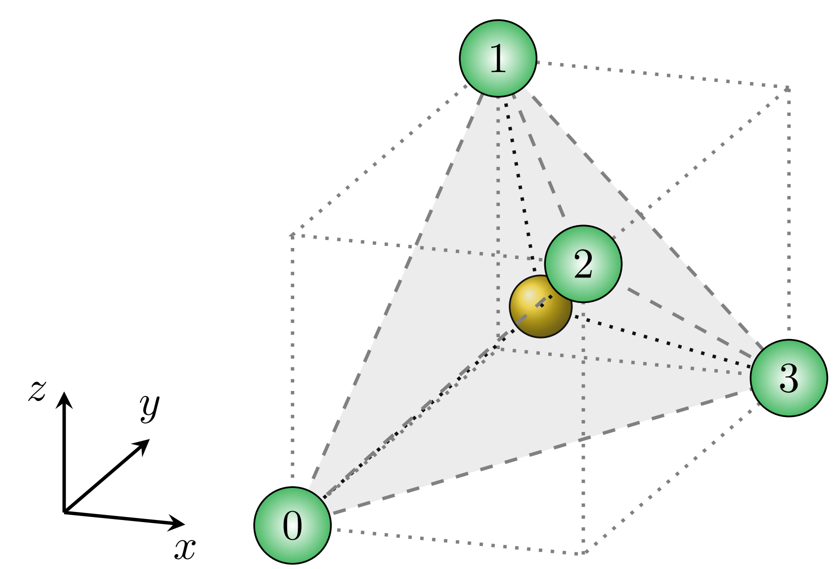
Quantum fluctuations

Note the non-commuting nature of the electric fields

$$\mathbb{E}^{\pm}_{A,zz} = (\mathbb{E}_{A,xx} \pm i\mathbb{E}_{A,yy})/2 \qquad [\mathbb{E}_{A,zz}, \mathbb{E}^{\pm}_{A,zz}] = \pm\mathbb{E}^{\pm}_{A,zz}$$

$$[\rho^z_B, \mathbb{E}^{\pm}_{0,zz}] = \mp\mathbb{E}^{\pm}_{0,zz} \qquad [\rho^z_B, \mathbb{E}^{\pm}_{2,zz}] = \pm\mathbb{E}^{\pm}_{2,zz}$$

$$[\rho^z_B, \mathbb{E}^{\pm}_{1,zz}] = \pm\mathbb{E}^{\pm}_{1,zz} \qquad [\rho^z_B, \mathbb{E}^{\pm}_{3,zz}] = \mp\mathbb{E}^{\pm}_{3,zz}$$



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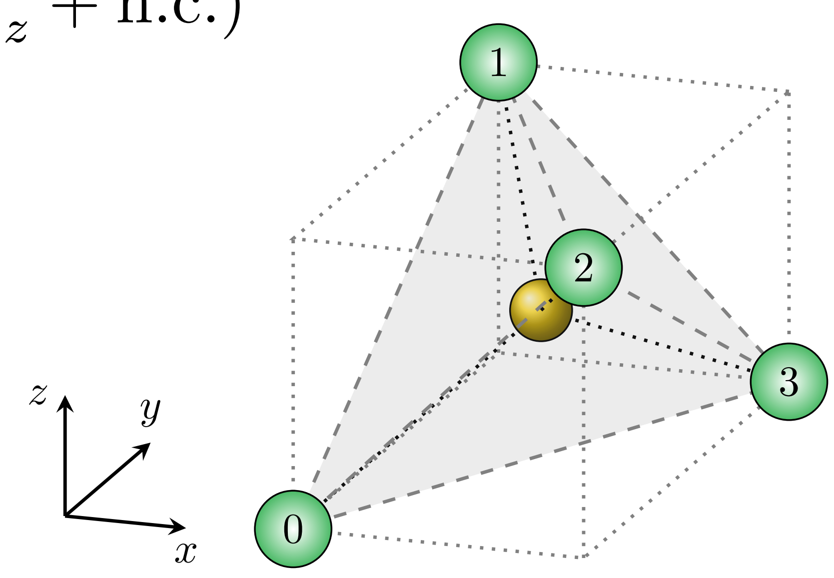
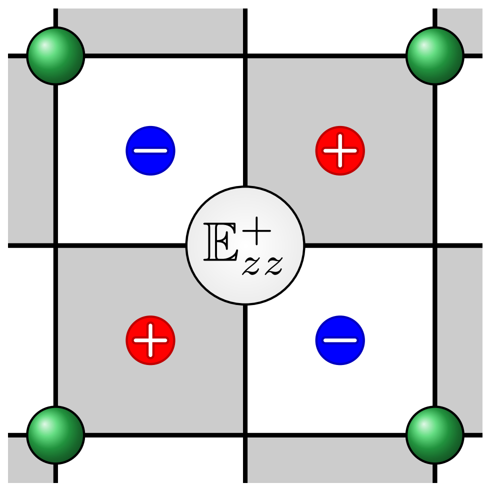
Rewriting $H' = \frac{1}{2}\sum_{\substack{\mathrm{B},\Gamma \\ \Gamma \neq \mathrm{T}_{1+}}} a_{\mathrm{B},\Gamma} m^2_{\mathrm{B},\Gamma}$ as a perturbation

$$H' = \sum_{A,A'} a_{AA'}\mathbb{E}_{A,zz}\mathbb{E}_{A',zz} + \sum_{A,A'} (b_{AA'}\mathbb{E}^{+}_{A,zz}\mathbb{E}^{-}_{A',zz} + \mathrm{h.c.})$$
$$+ \sum_{A,A'} (c_{AA'}\mathbb{E}_{A,zz}\mathbb{E}^{+}_{A',zz} + \mathrm{h.c.})$$
$$+ \sum_{A,A'} (d_{AA'}\mathbb{E}^{+}_{A,zz}\mathbb{E}^{+}_{A',zz} + \mathrm{h.c.})$$

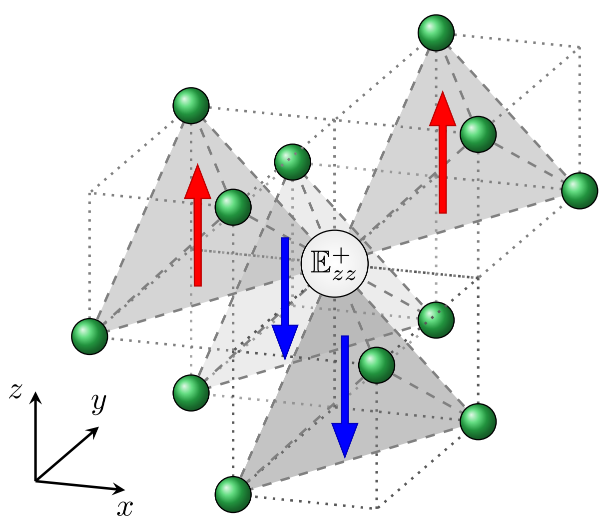
$$A, A' = 0, 1, 2, 3$$



Quantum fluctuations

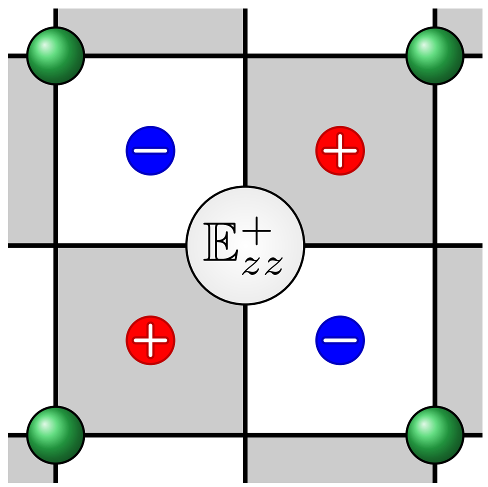


top view for the xy plane

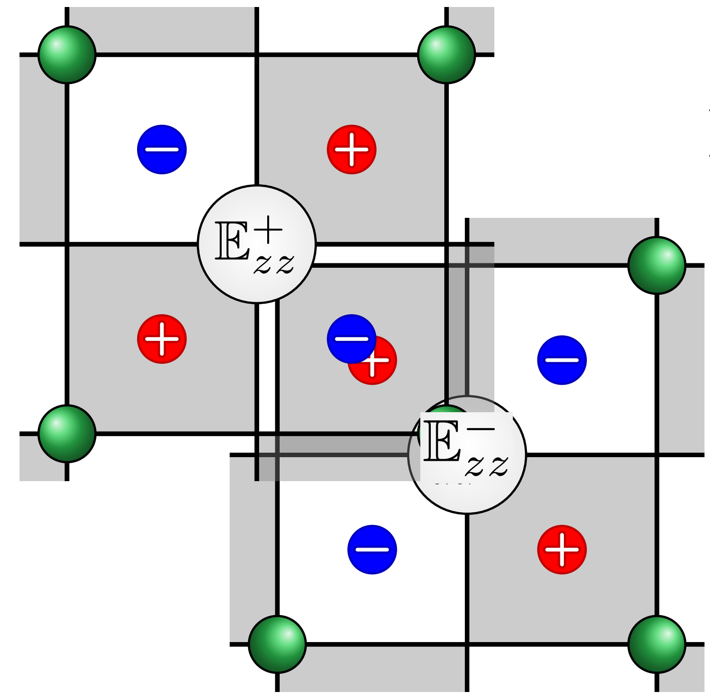


Start from some background charge configuration, e.g. a uniform background

Quantum fluctuations

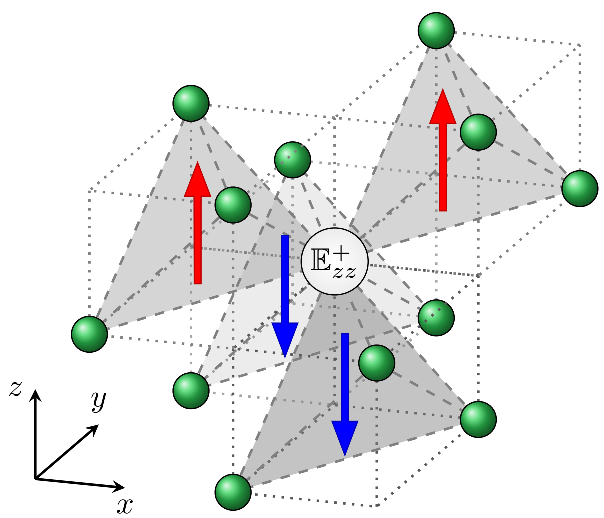


top view for the xy plane



Apply

$\mathbb{E}^{\pm}_{A,zz}\mathbb{E}^{\mp}_{A',zz}$



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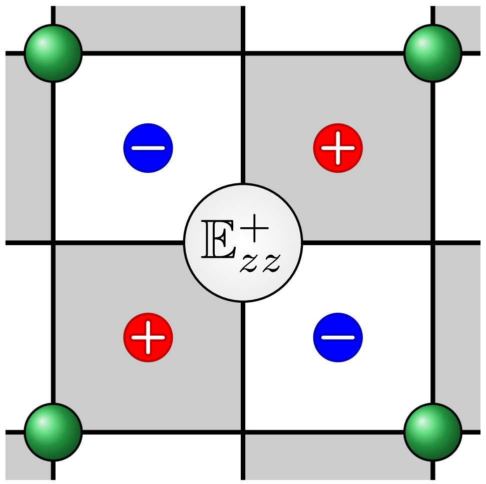
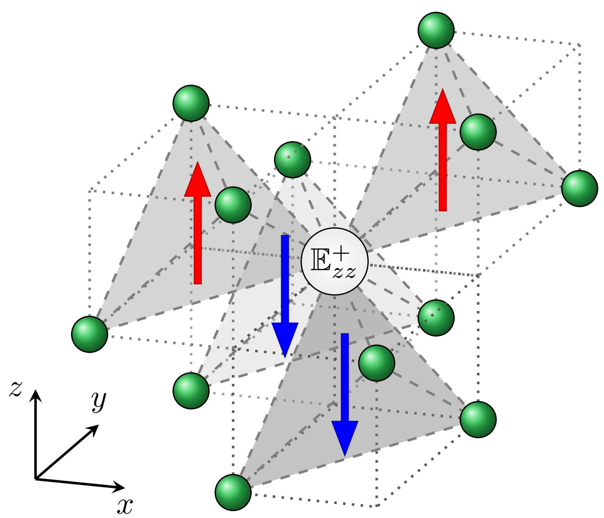
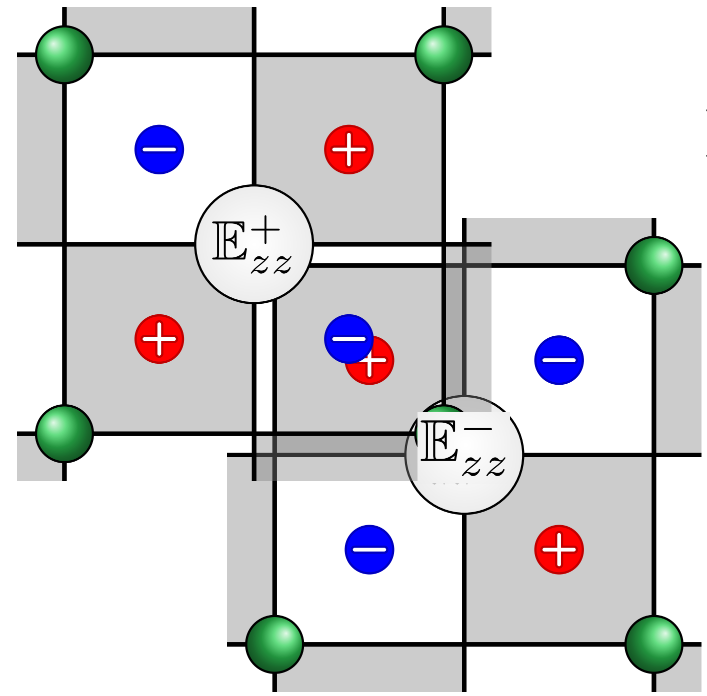


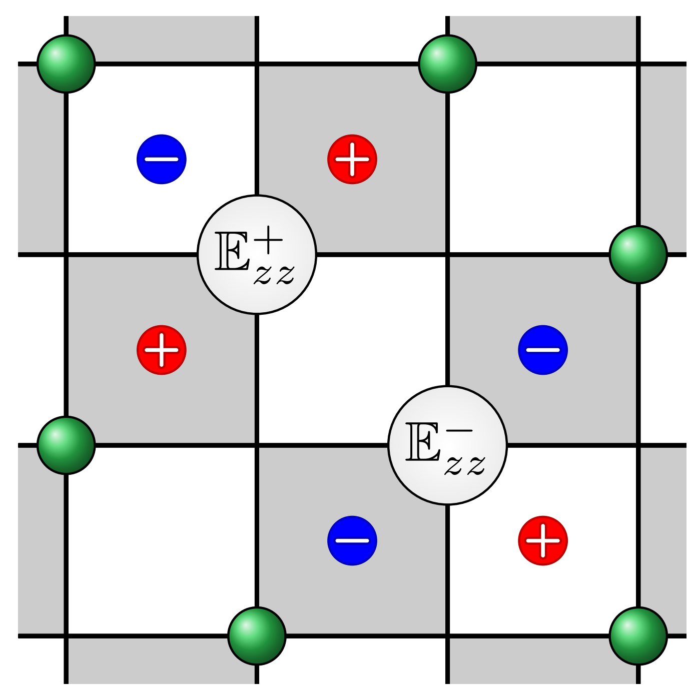
top view for the xy plane



Start from some background charge configuration, e.g. a uniform background

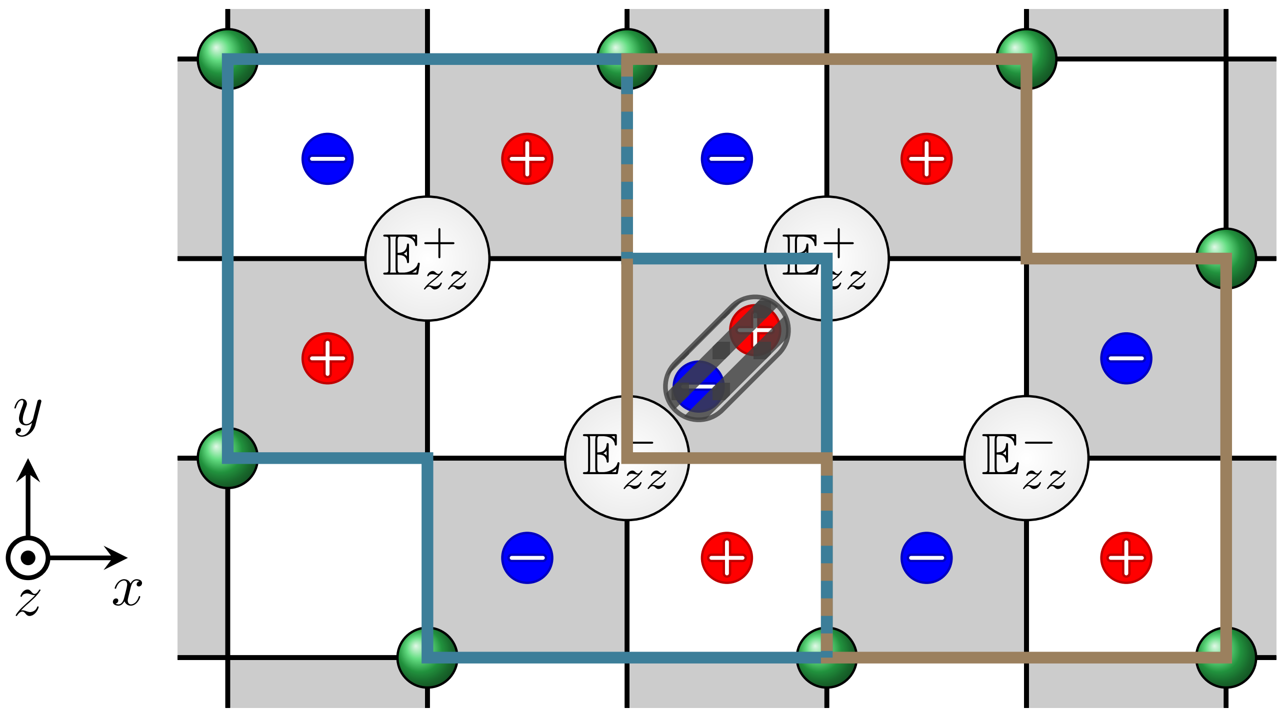
Apply $\mathbb{E}^{\pm}_{A,zz}\mathbb{E}^{\mp}_{A',zz}$





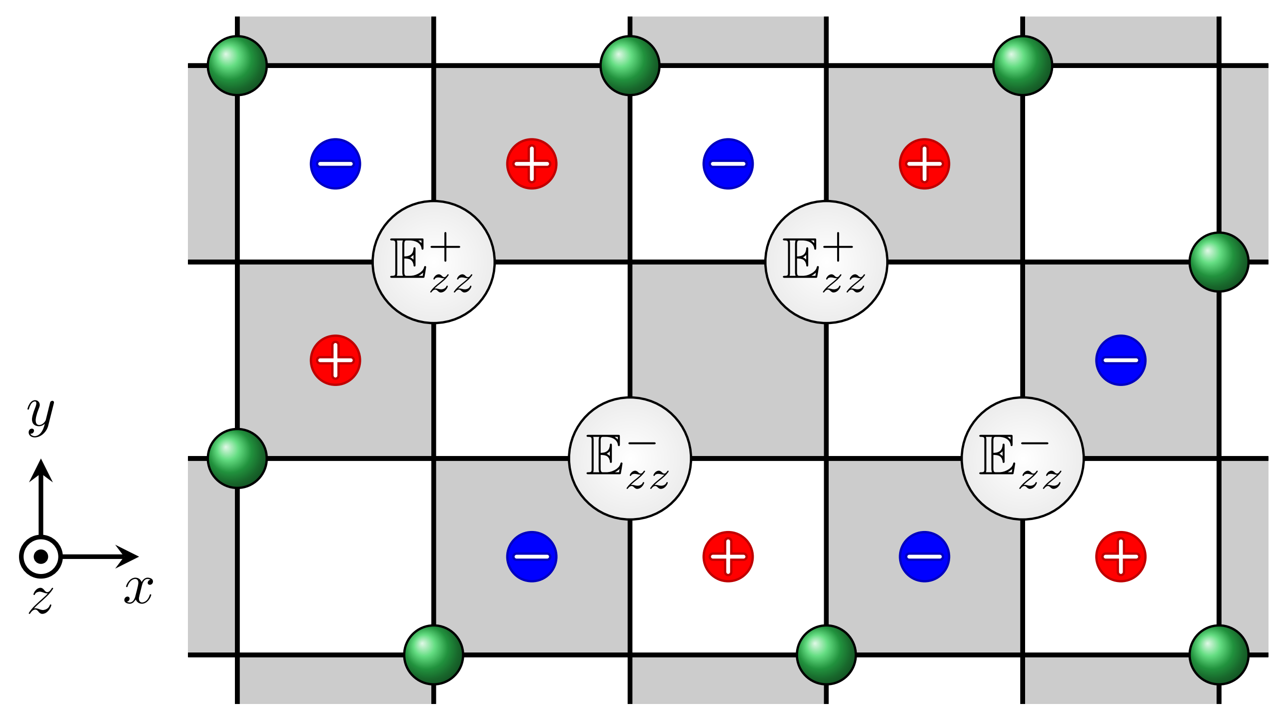
Cancelled charges in the bulk

Quantum fluctuations

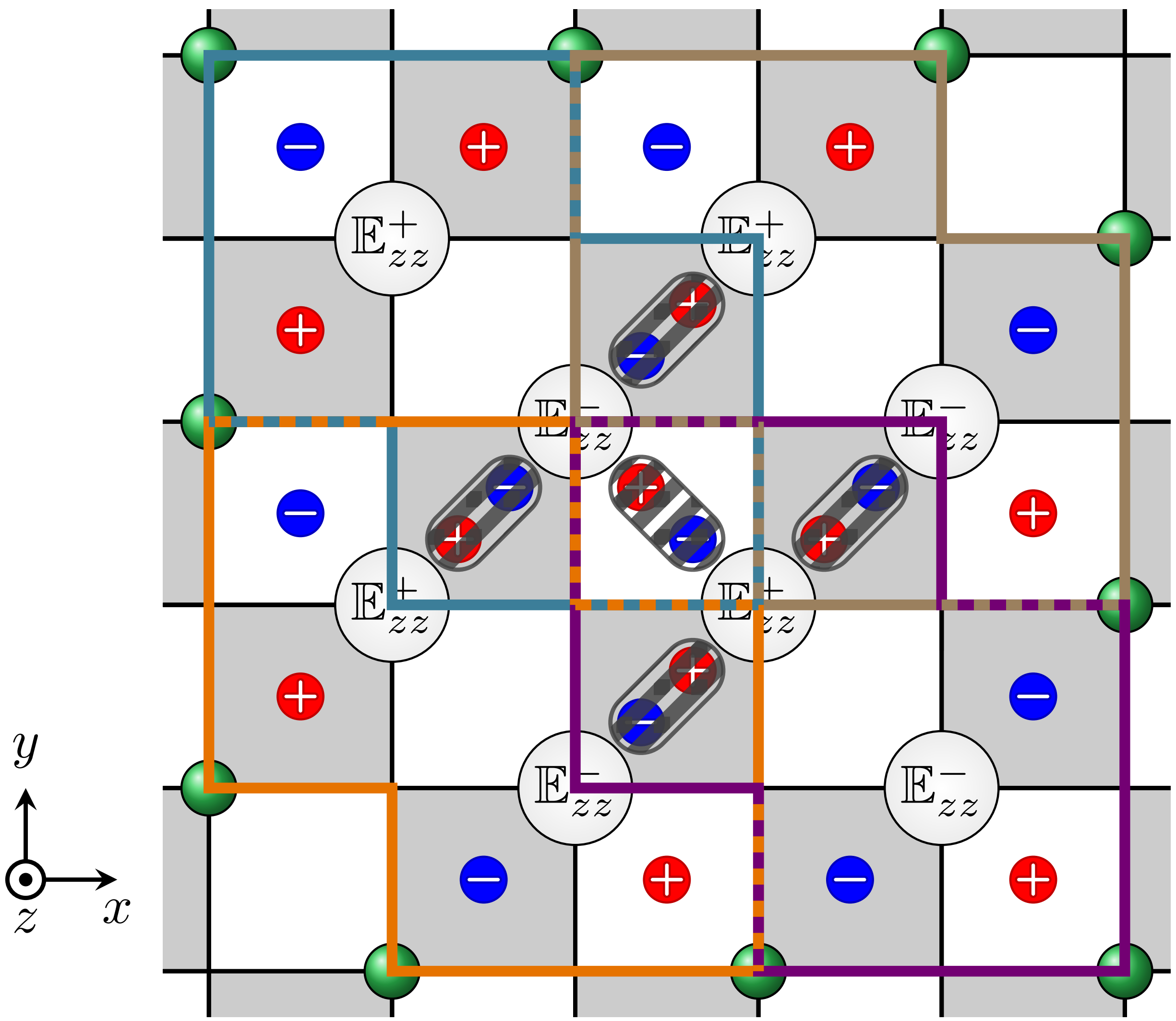


Cancelled charges in the bulk

Charges pushed to the boundary

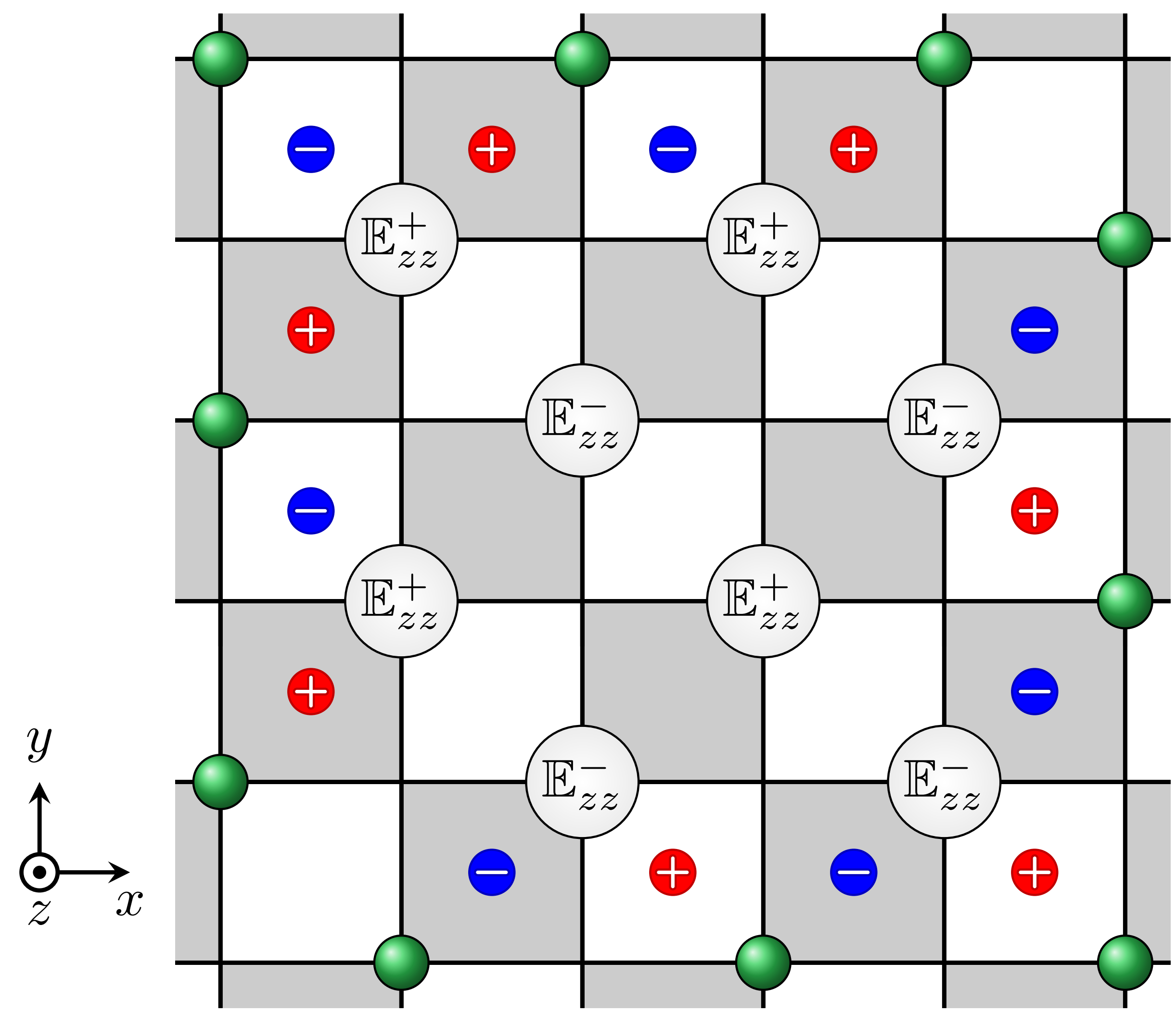


Quantum fluctuations

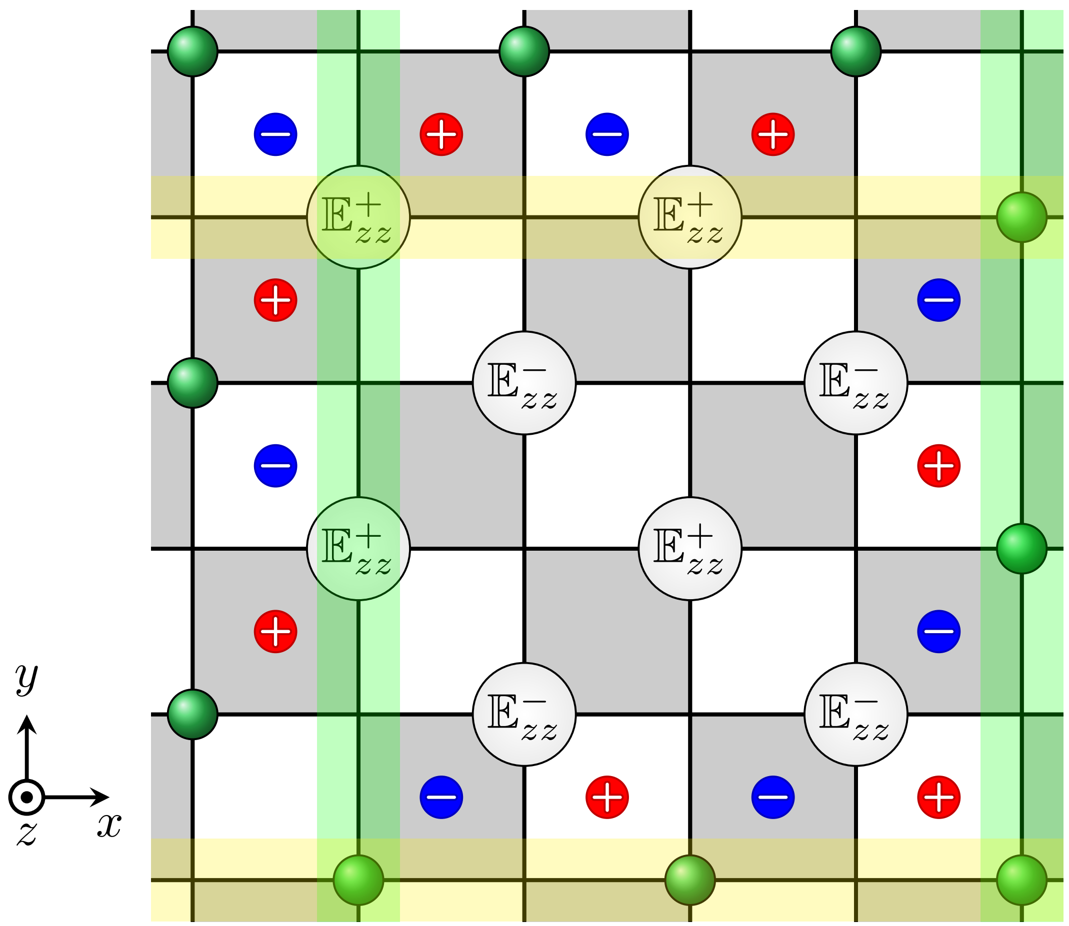


Membrane operators

This is happening
in the xy plane



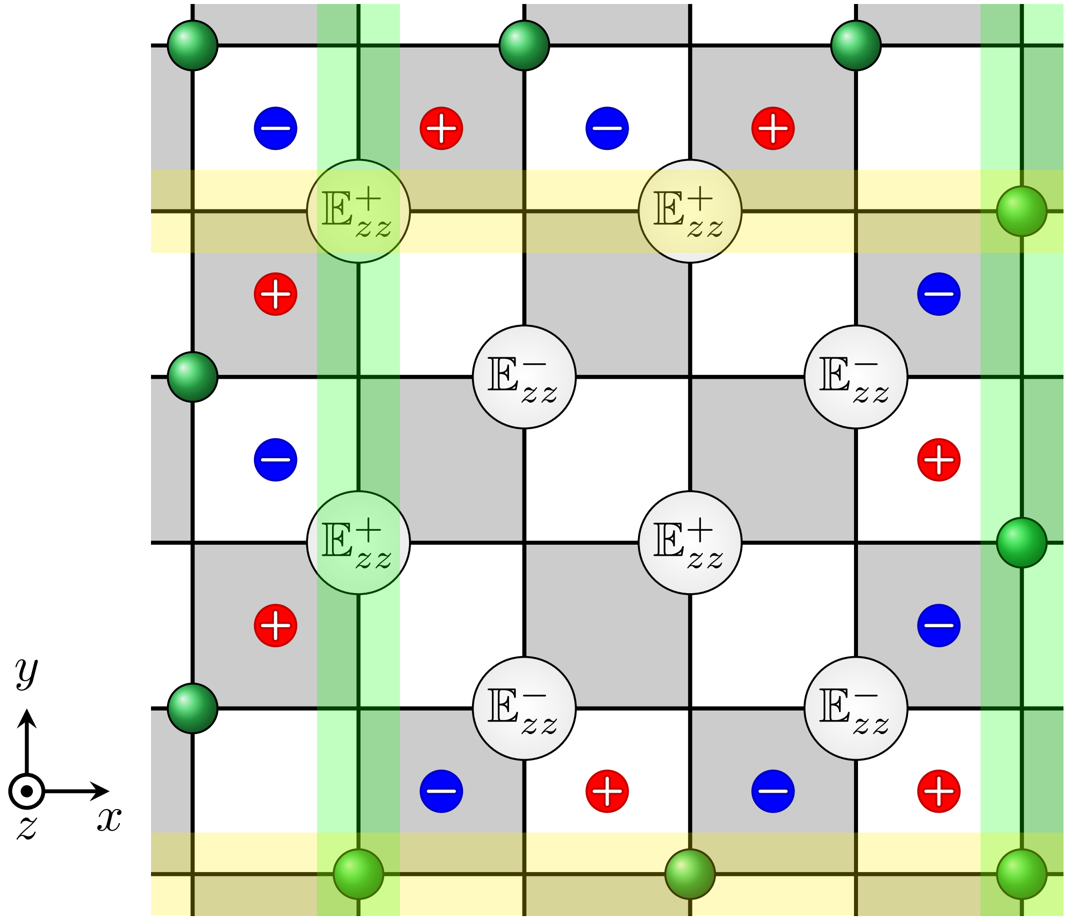
Quantum fluctuations



Periodic boundary condition will cancel the charges at the boundaries

One can show that this leads to another ground state that satisfies the constraint.

Quantum fluctuations



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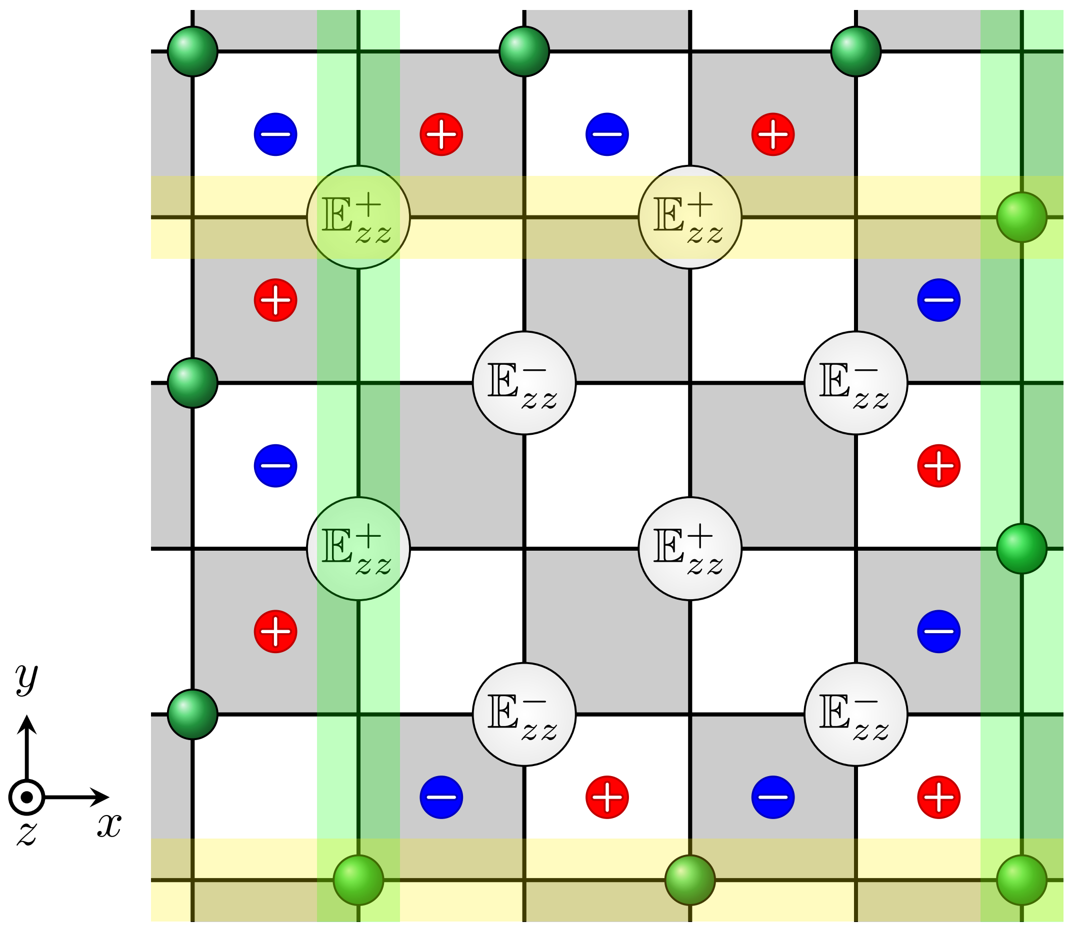
We can keep doing this and generate all the degenerate ground states

Quantum fluctuations



Periodic boundary condition will cancel the charges at the boundaries

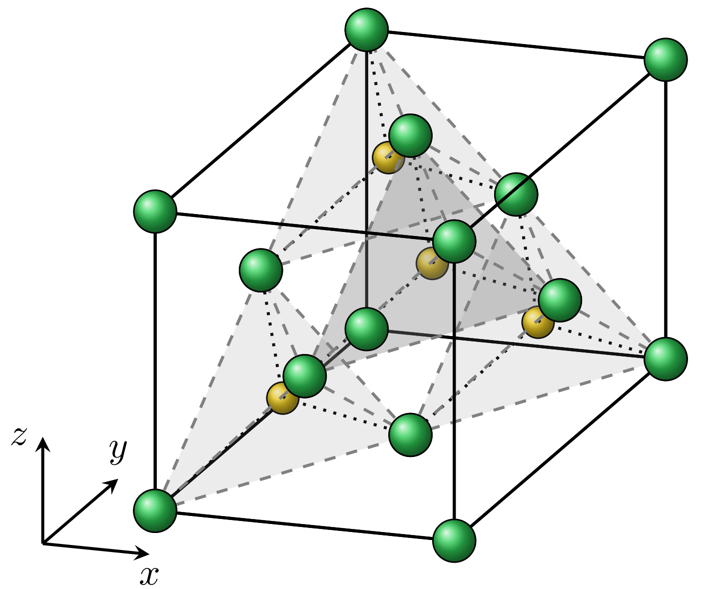
One can show that this leads to another ground state that satisfies the constraint.

This is similar to quantum Hall states, where moving quasiparticles and annihilating them leads to a degenerate ground state

We can keep doing this and generate all the degenerate ground states

Ground state degeneracy

L_x	L_y	L_z	volume $L_x L_y L_z$	perimeter $L_x + L_y + L_z$	GSD	constraints
1	1	1	1	3	85	1
2	1	1	2	4	1,333	3
3	1	1	3	5	25,405	5
4	1	1	4	6	535,333	7
5	1	1	5	7	11,982,925	9
6	1	1	6	8	278,766,133	11
2	2	1	4	5	10,213	16
3	2	1	6	6	116,653	24
4	2	1	8	7	1,664,533	32
3	3	1	9	7	889,525	36
5	2	1	10	8	27,510,973	40
4	3	1	12	8	9,103,453	48
2	2	2	8	6	49,541	32
3	2	2	12	7	392,365	48
4	2	2	16	8	4,201,589	64
3	3	2	18	8	2,258,486	72
5	2	2	20	9	55,306,813	80
4	3	2	24	9	18,470,173	96
3	3	3	27	9	9,912,253	108



$$(L_x, L_y, L_z) = (1,1,1)$$

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GSD is different for the same volume or perimeter

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For a fixed perimeter, a larger volume gives smaller GSD

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For a fixed volume, a larger perimeter gives larger GSD

Ground state degeneracy

(i) $L_i \geq 2$ and $L_j = L_k = 1$

(ii) $L_i, L_j \geq 2$ and $L_k = 1$

(iii) $L_i \geq 2$ for all $i = x, y, z$

Ground state degeneracy

(i) $L_i \geq 2$ and $L_j = L_k = 1$ GSD monotonically increases with L_i

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For FCC, there are $2L_i$ number of planes in each i direction

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GSD is non-extensive with volume and depends on the geometry

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Does quantum fluctuations generate
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This is like $H = \frac{\epsilon}{2}E^2 + \frac{1}{2\mu}B^2$

$1/\mu \propto (t/a_{\mathrm{B},\mathsf{T}_{1+}})^{L^2}$ Here $t \ll a_{\mathrm{B},\mathsf{T}_{1+}}$ is
the coefficient of
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$1/\mu \to 0$ in the thermodynamic limit

Quantum glassiness

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Here $t \ll a_{\mathrm{B},\mathsf{T}_{1+}}$ is the coefficient of the perturbation

"The speed of light"

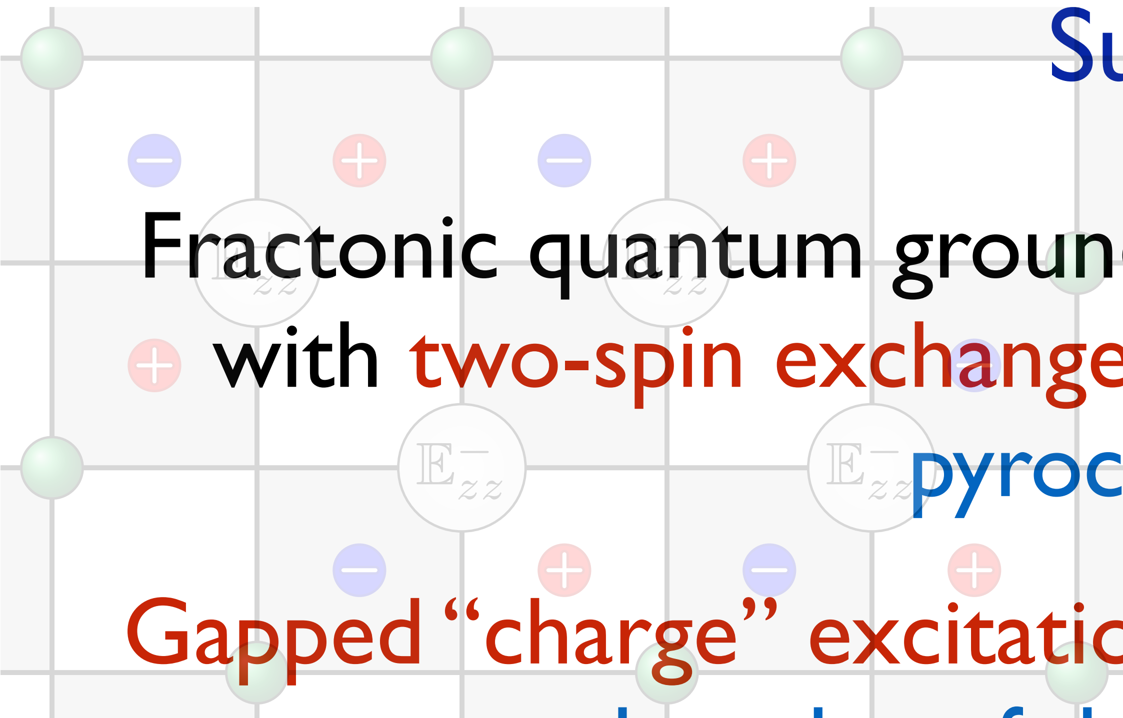
$c \sim 1/\sqrt{\mu} \propto (t/a_{\mathrm{B},\mathsf{T}_{1+}})^{L^2/2} \to 0$ in the thermodynamic limit

It will take a long time to tunnel between different ground states

$$t_{\mathrm{char}} \sim t_0 e^{(L^2/2)\ln(a_{\mathrm{B},\mathsf{T}_{1+}}/t)}$$

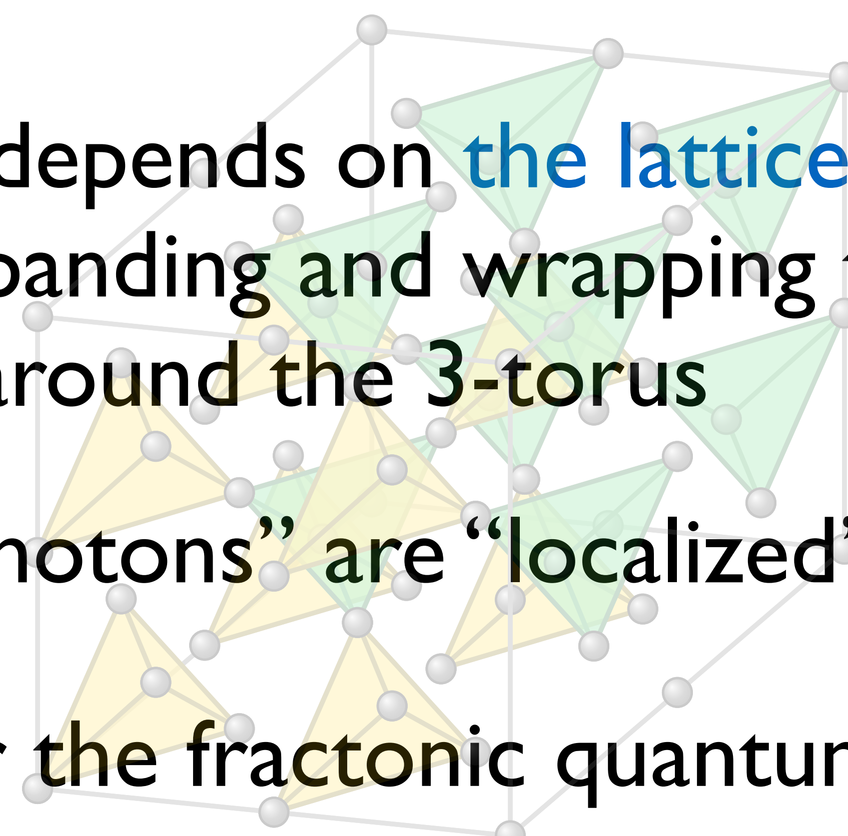
Similar to Chamon, Nankishore, ...

Summary



Fractonic quantum ground state in a quantum spin model with two-spin exchange interactions on the breathing pyrochlore lattice

Gapped "charge" excitations can only move as a cluster at the edge of the membrane objects



Sub-extensive GSD depends on the lattice geometry - can be generated by expanding and wrapping the membranes around the 3-torus

In this model, the "photons" are "localized"

A realistic model for the fractonic quantum phases !