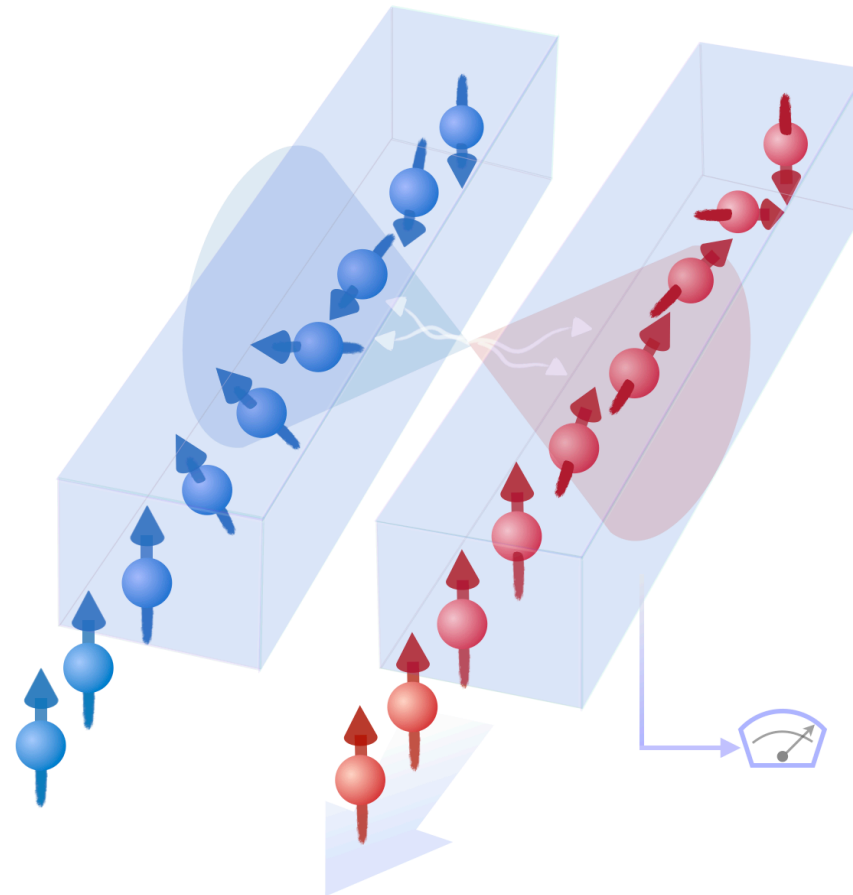


Quantum computing on racetracks with flying domain wall qubits



[arXiv:2212.12019](https://arxiv.org/abs/2212.12019)

Ji Zou (U of Basel)

w/ Stefano Bosco, Jelena Klinovaja, Daniel Loss
and Banabir Pal, Stuart Parkin (Halle, MPI)

Outline

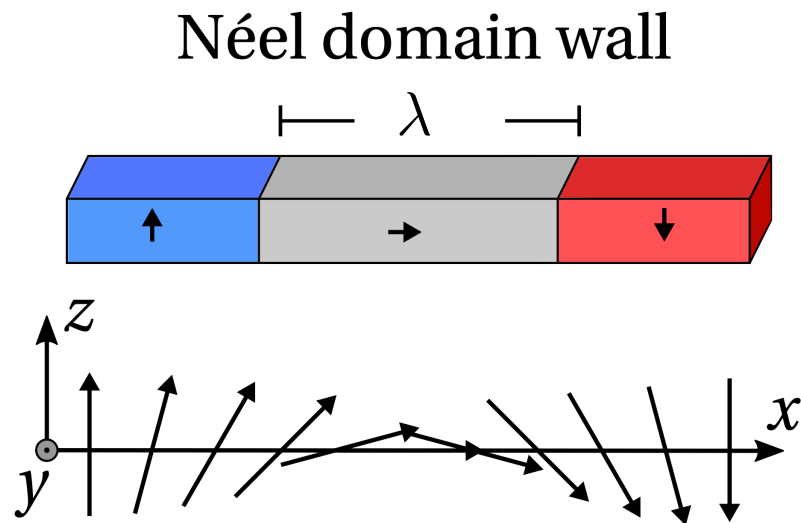
- Background
- Define the domain wall qubit
- Shuttling the domain wall qubits
- Single- and two-qubit gates
- Qubit readout and lifetime

Background

The main role today: domain wall

Parkin et al., Science 320, 190 (2008)

Bits=domains in tracks



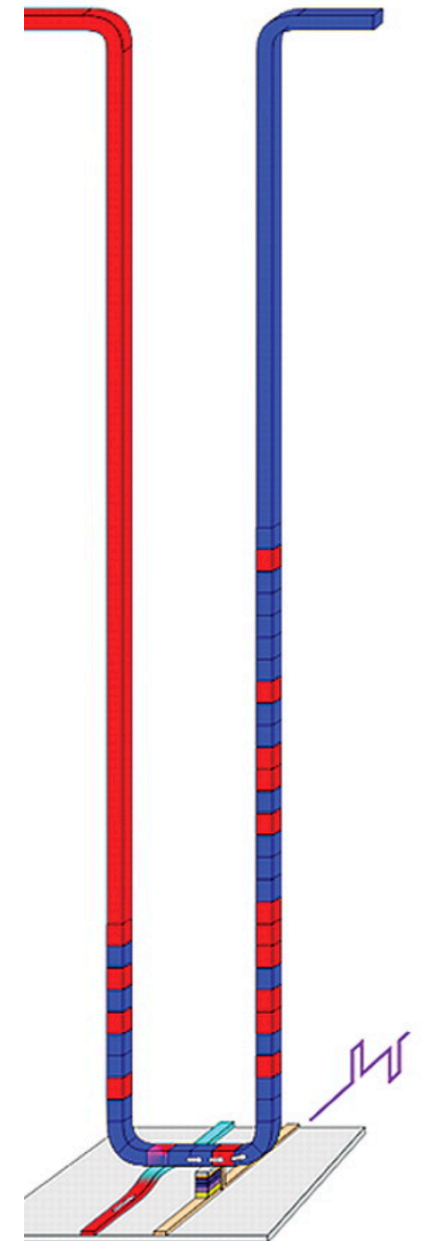
1. one important application (used as classical bits)
2. can be moved by magnetic fields, electric currents
3. Fast speed on antiferro- and ferrimagnetic tracks

$$v > 10^3 \text{ m/s}$$

Guan et al., Nature Commun. (2021)

Yang et al., Nature Phys. (2019)

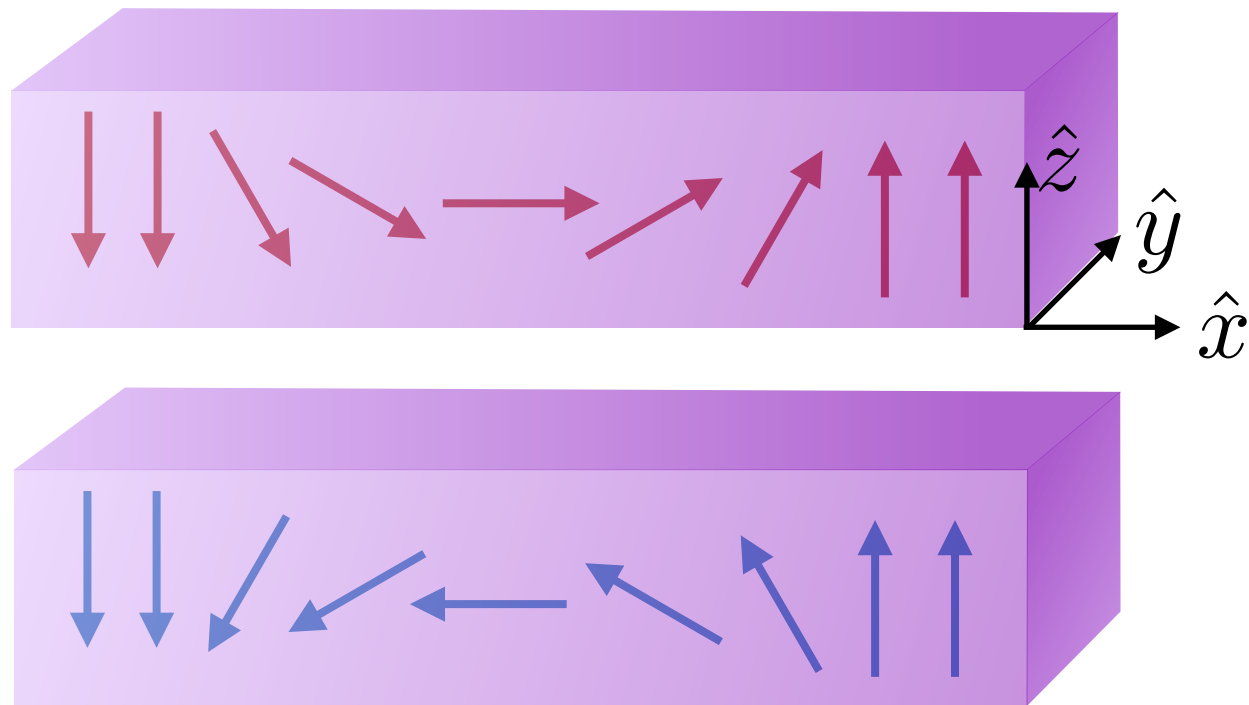
Parkin, Yang, Nature Nano. (2015)



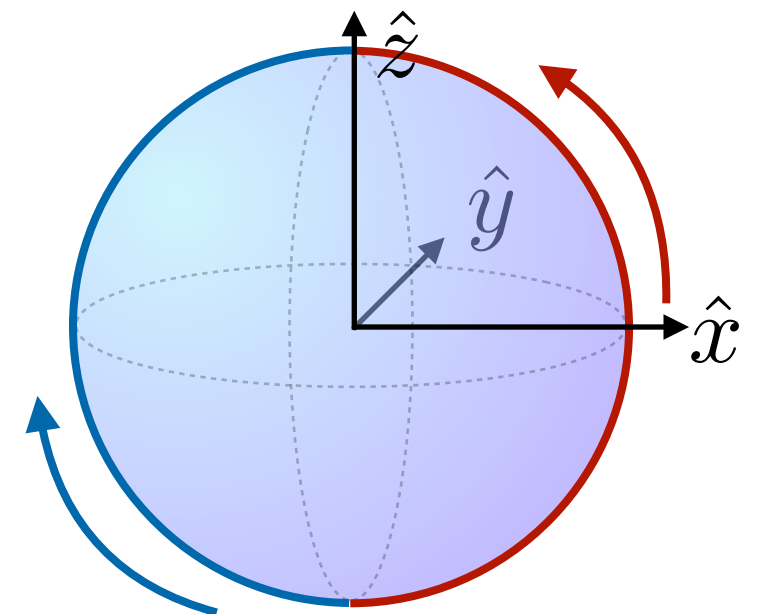
racetrack memory

Can we use it as a quantum bit?

Which degree of freedom we want to use to store quantum information?



order parameter space



One promising candidate: chirality

pseudo-spin $|\uparrow\rangle = |C = +1\rangle$ $|\downarrow\rangle = |C = -1\rangle$



DiVincenzo's criteria

From Wikipedia:

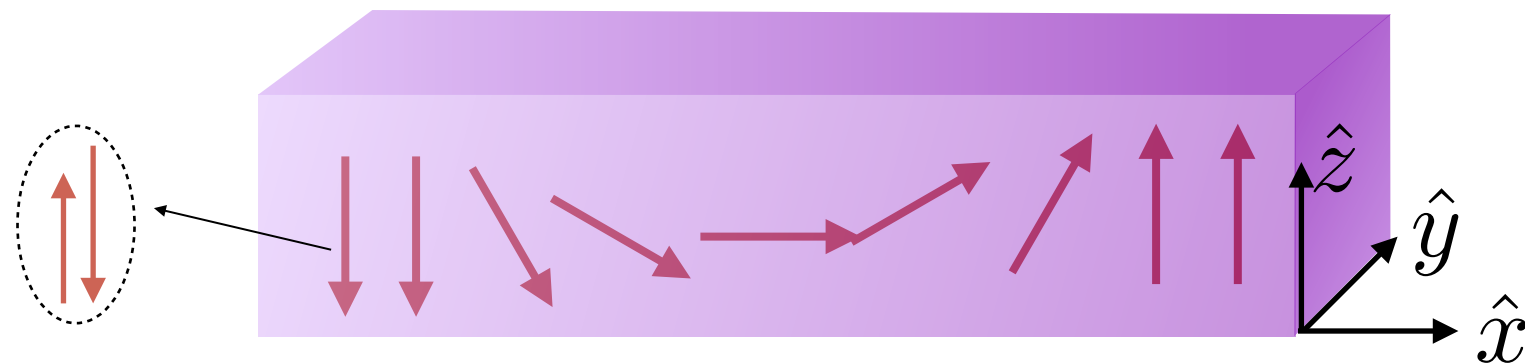
1. A scalable physical system with well-characterized **qubit**
2. The ability to initialize the state of the qubits to a simple fiducial state
3. Long relevant **decoherence times**
4. A "**universal**" set of **quantum gates**
5. A qubit-specific **measurement** capability

The remaining two are necessary for **quantum communication**:

1. The ability to interconvert stationary and flying qubits
2. The ability to faithfully transmit flying qubits between specified locations

Define the computational space

Consider a simple two-sublattice quasi-one-dimensional ferrimagnet:



Dynamics: Antiferromagnetic order + weak ferro order

Néel vector $\mathbf{n}(x, t)$

net spin per site S_e

Low-energy description:

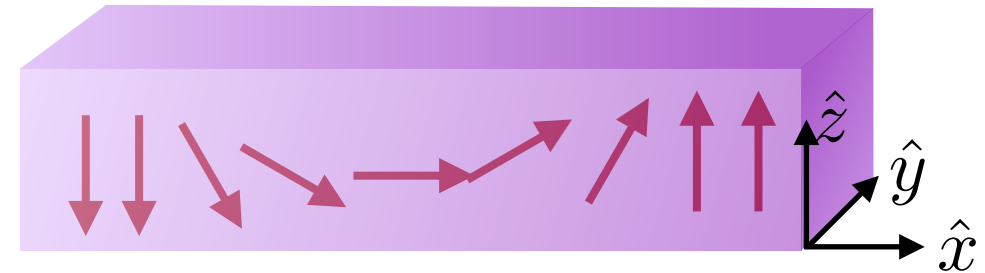
Kinetic energy: $K[\mathbf{n}(x, t)] = N \int dx \left[\frac{\hbar^2}{8J} (\dot{\mathbf{n}} - \mathbf{h} \times \mathbf{n})^2 + \hbar S_e \dot{\mathbf{n}} \cdot \mathbf{A} \right]$

number of spins within DW
antiferro
ferro

$\nabla_{\mathbf{n}} \times \mathbf{A} = -\mathbf{n}$

Define the computational space

Low-energy description:

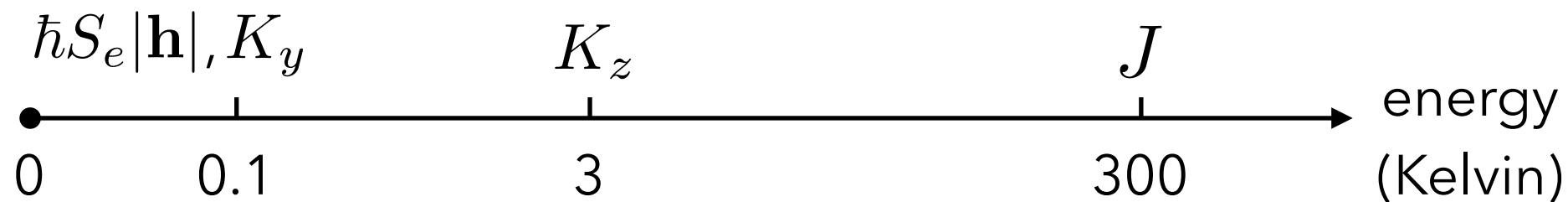


Potential energy: $U[\mathbf{n}(x, t)] = \frac{N K_z}{2} \int dx [(\partial_x \mathbf{n})^2 - n_z^2] + N \int dx (K_y n_y^2 - \hbar S_e \mathbf{n} \cdot \mathbf{h})$

$\nearrow$ number of spins within DW
 $\nearrow$ easy-z axis
 $\nearrow$ easy-xz plane

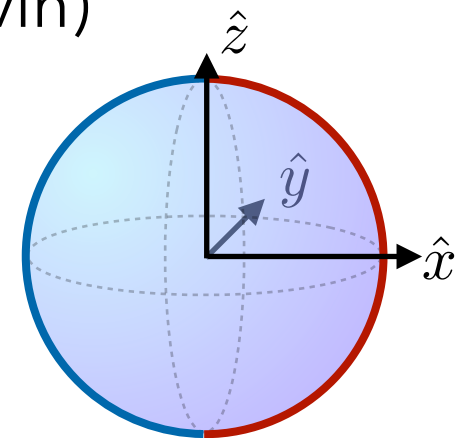
1. rescaled the spatial parameter x : $x \rightarrow \lambda x$ $\lambda = S a \sqrt{J/K_z}$

2. hierarchy of energy scales:



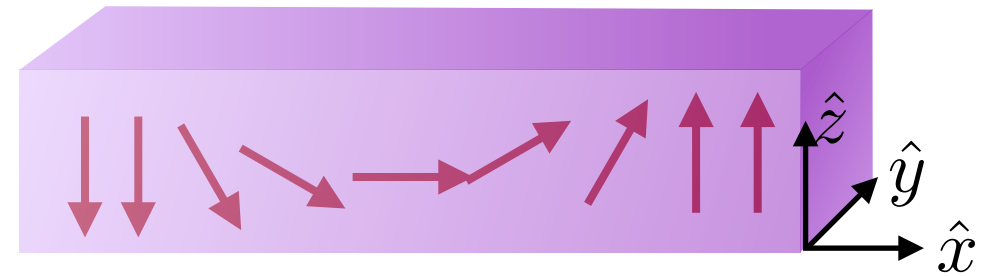
3. domain wall configuration: (two zero modes: X, Φ)

$$n_x + i n_y = e^{i\Phi} \operatorname{sech}(x - X), \quad n_z = \tanh(x - X)$$



Define the computational space

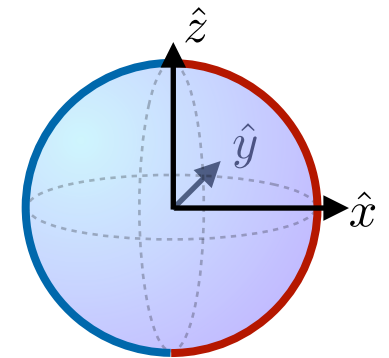
Low energy dynamics of the domain wall:



$$\begin{aligned}
 L(X, \Phi) = & \frac{M}{2} \dot{\Phi}^2 - N \left[2K_y \sin^2 \Phi + \frac{1}{4J} (\hbar h_x \cos \Phi + \hbar h_y \sin \Phi)^2 - \pi \hbar S_e (h_x \cos \Phi + h_y \sin \Phi) \right] \longrightarrow \text{Lagrangian for } \Phi \\
 & + \frac{M}{2} \dot{X}^2 - \frac{1}{2} M \omega_p^2 X^2 \longrightarrow \text{Lagrangian for } X \\
 & + \frac{\pi M}{2} (h_x \sin \Phi - h_y \cos \Phi) \dot{X} - 2N \hbar S_e X \dot{\Phi}, \longrightarrow \text{Couplings between } \Phi \text{ and } X
 \end{aligned}$$

effective mass: $M = N \hbar^2 / 2J$

The action is proportional to the domain wall size: N



$$L = L_{\Phi} + L_X + L_{\Phi X}$$

define the qubit

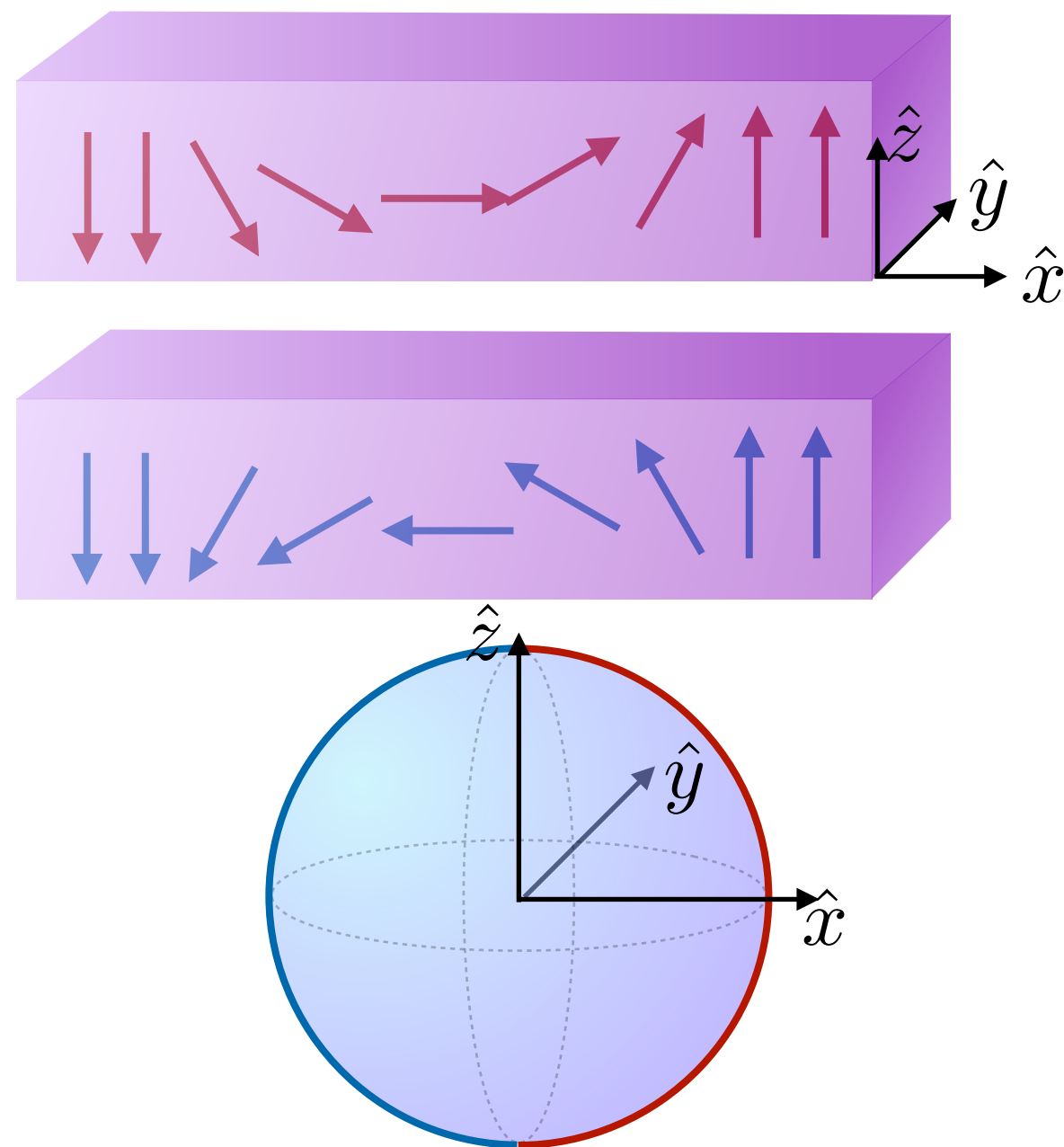
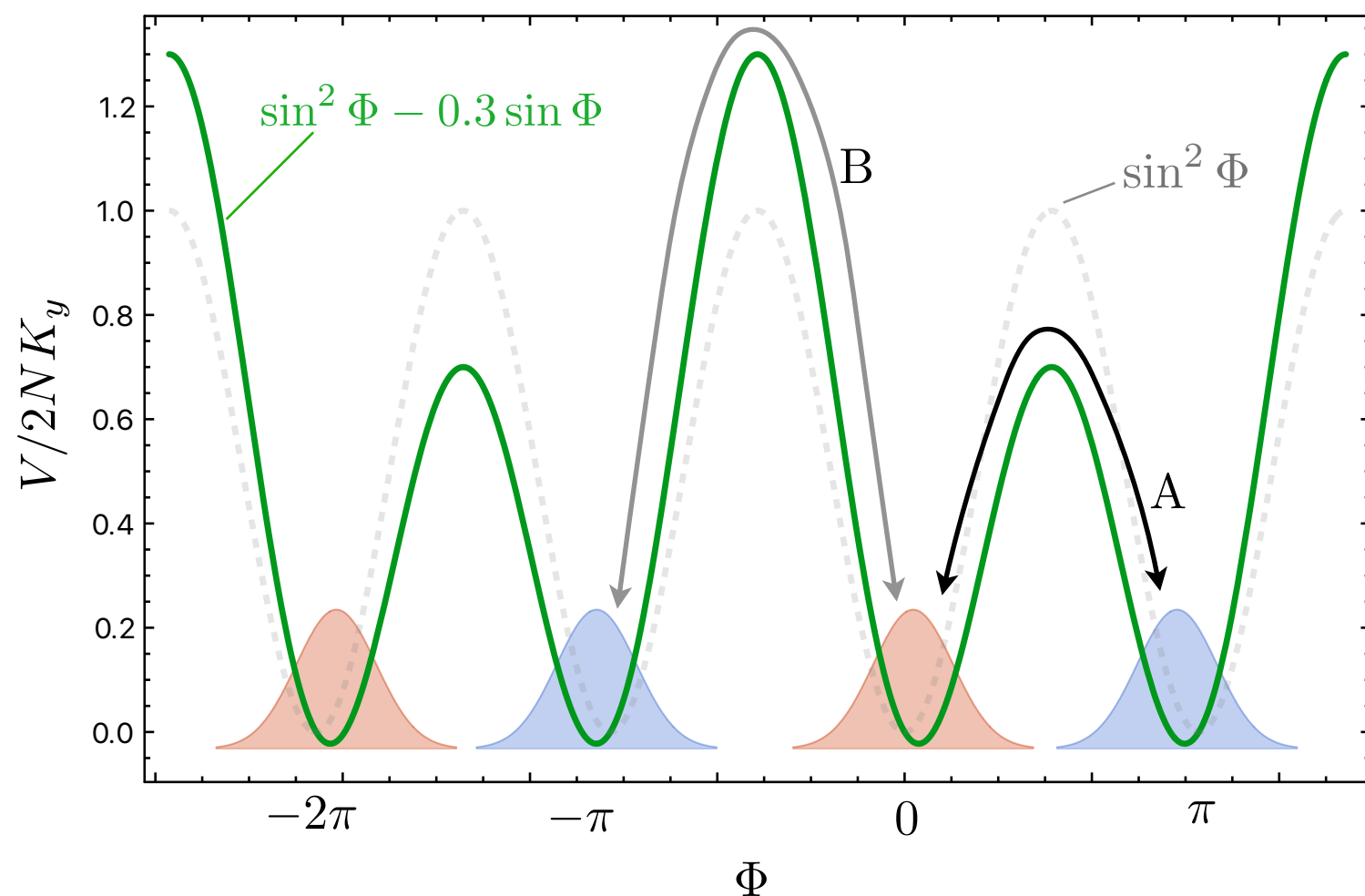
implement single- and two-qubit gates

Define the computational space

Potential energy: $V(\Phi) = 2NK_y(\sin^2 \Phi - 2b_x \cos \Phi - 2b_y \sin \Phi)$

dimensionless magnetic fields: $b_i \equiv \pi \hbar S_e h_i / 4K_y$

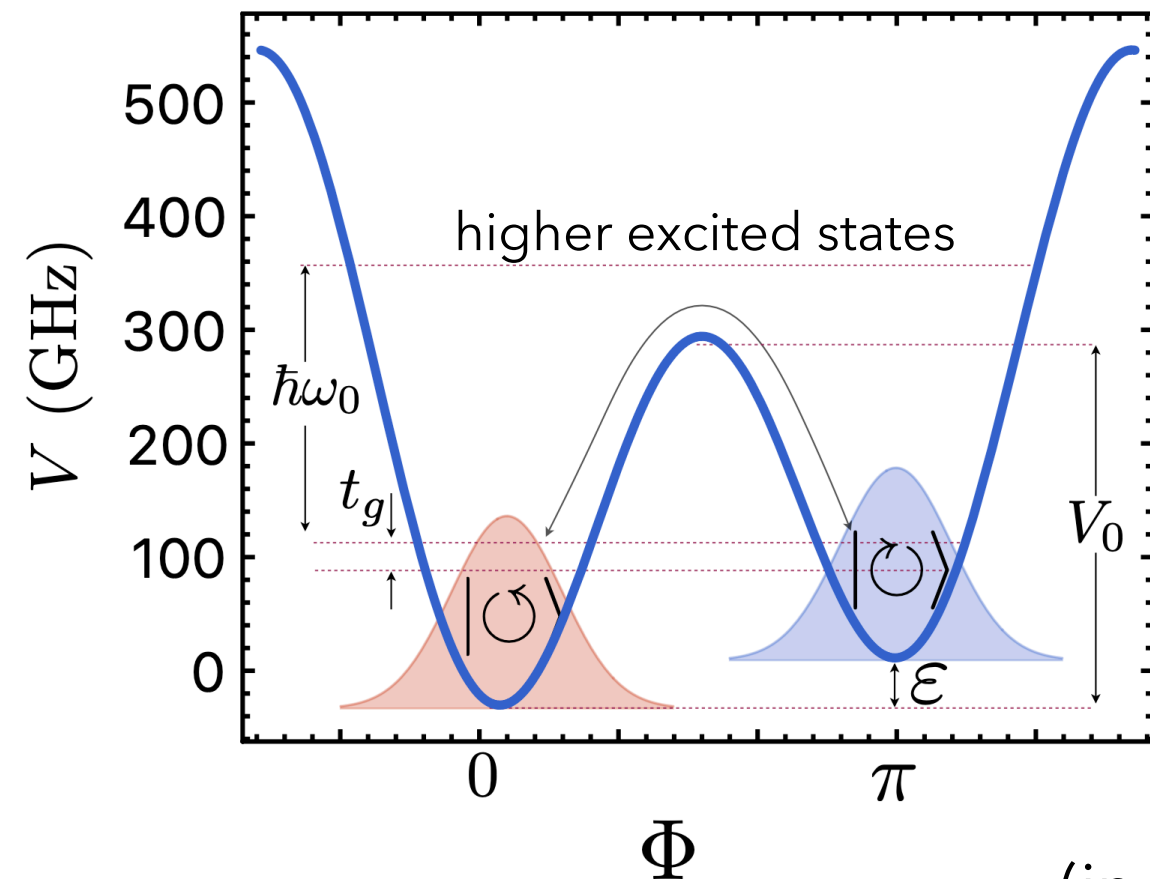
$$b_y = 0.15 \text{ (1 T)} \quad b_x = 0$$



Define the computational space

Potential energy: $V(\Phi) = 2NK_y(\sin^2 \Phi - 2b_y \sin \Phi) - 2NK_y \times 2b_x \cos \Phi$

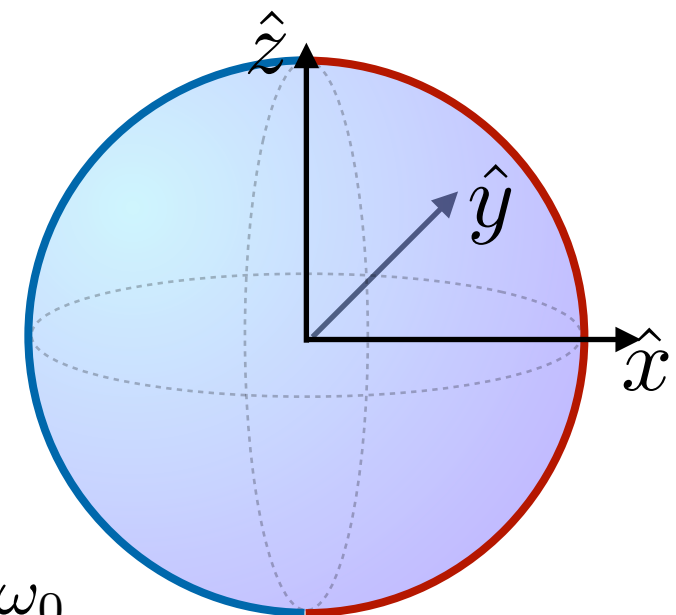
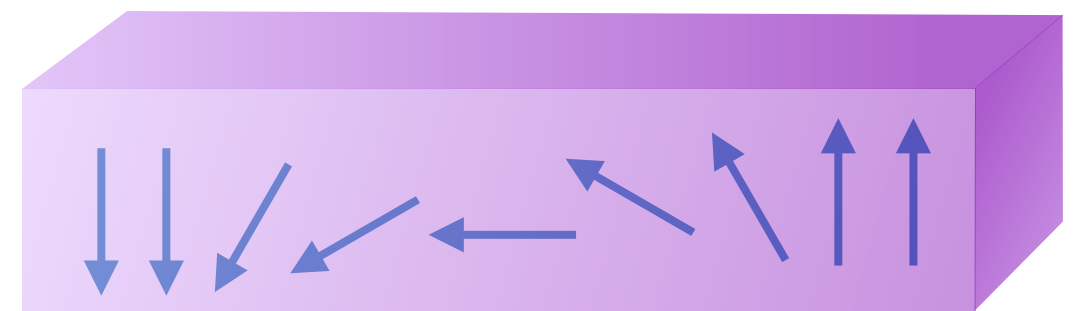
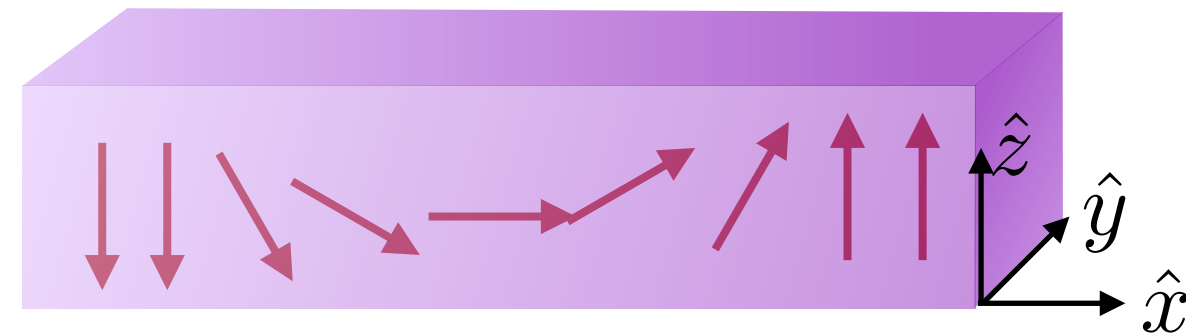
$$b_x = 0.02 \ll b_y = 0.15$$



(in chirality basis)

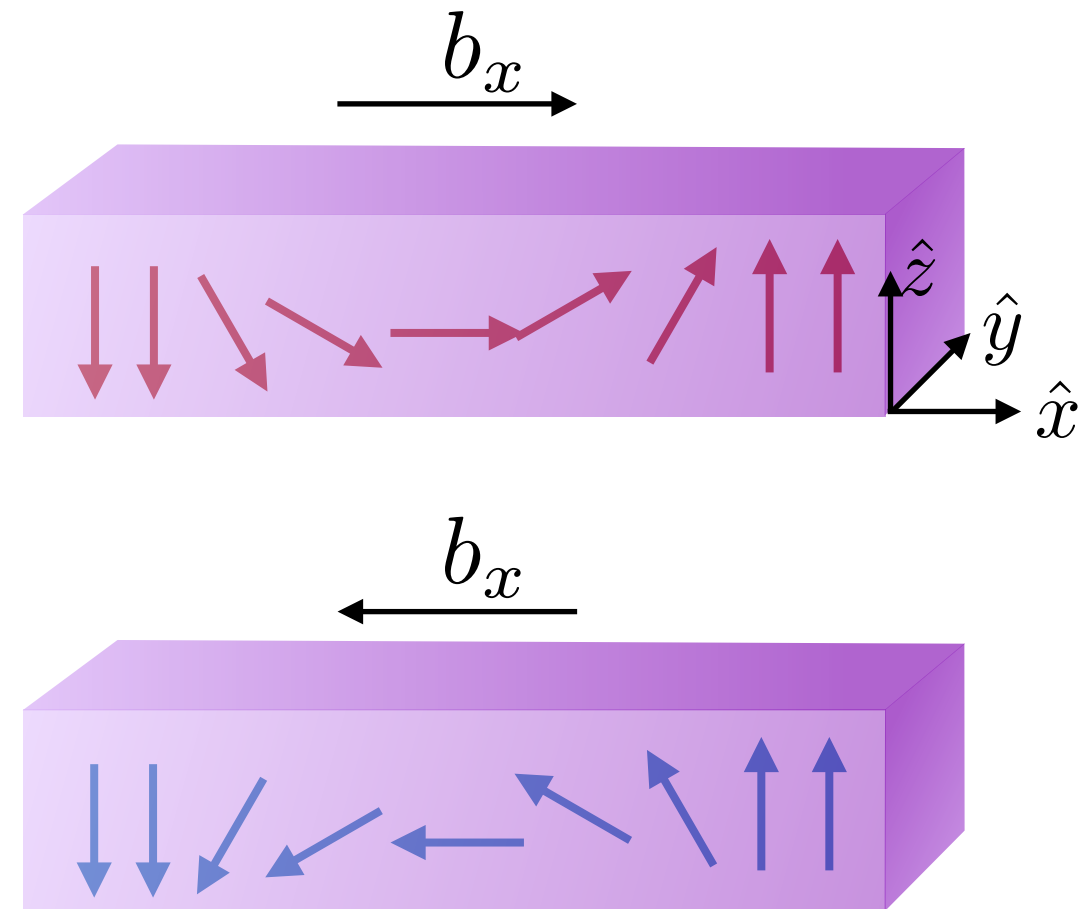
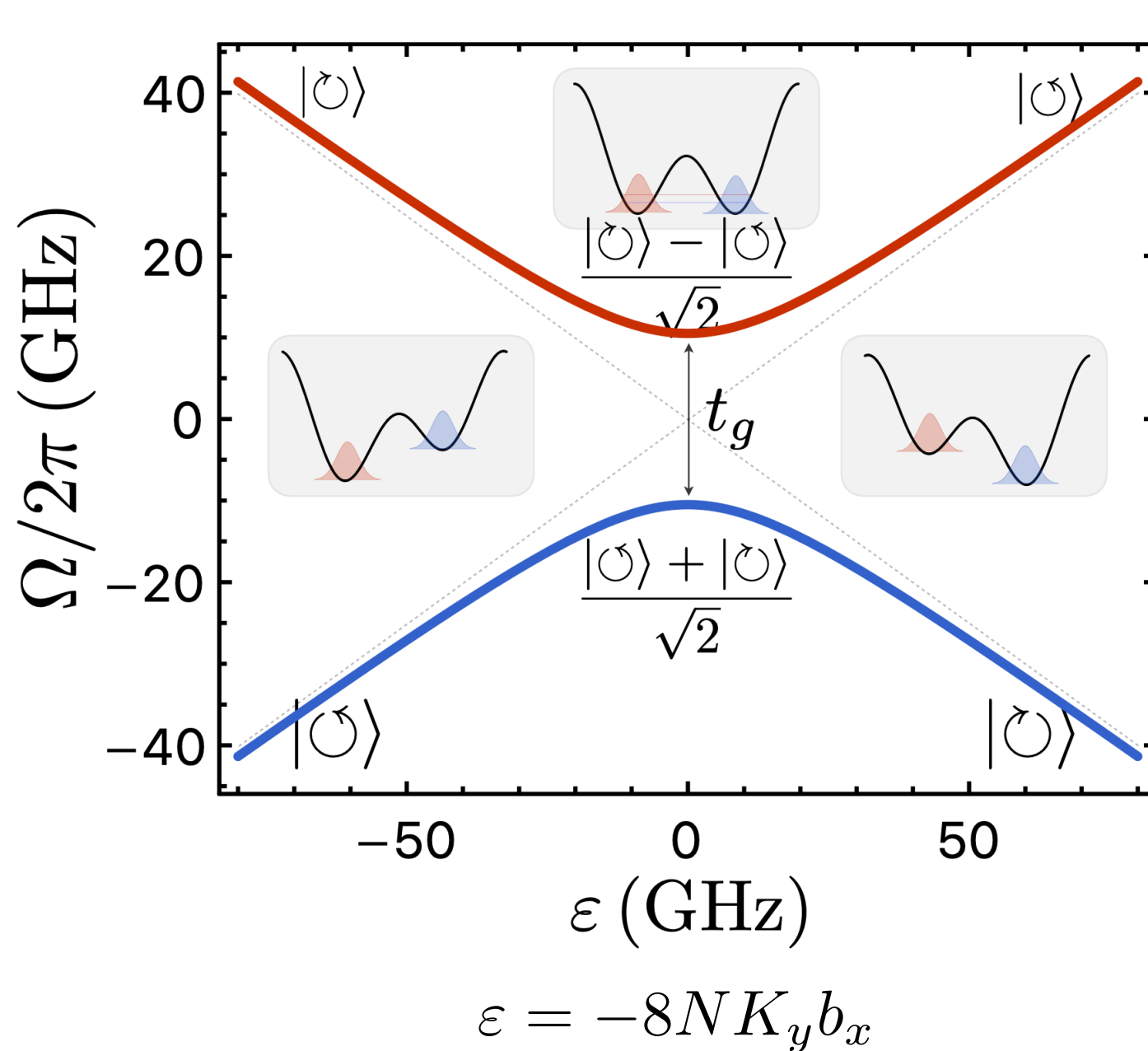
$$H = \frac{\epsilon}{2}\sigma_z - \frac{t_g}{2}\sigma_x$$

$$\epsilon = -8NK_y b_x \quad t_g \approx \hbar\omega_0 \sqrt{S_{\text{inst}}/\hbar} e^{-S_{\text{inst}}/\hbar}$$



$$S_{\text{inst}} \approx 4V_0/\omega_0$$

Define the computational space



$$H = \frac{\varepsilon}{2}\sigma_z - \frac{t_g}{2}\sigma_x \quad \Omega_{\pm} = \pm \frac{1}{2}\sqrt{t_g^2 + \varepsilon^2} \quad \text{with} \quad t_g \approx 20 \text{ GHz}$$

Shuttling domain wall qubits

Project coupled dynamics onto the qubit space:

$$L = L_{\Phi} + L_X + L_{\Phi X}$$

$$\downarrow b_x = 0$$

$$H = \frac{(\hat{P} + \beta_z \sigma_z)^2}{2M} + \frac{M\omega_p^2}{2} [\hat{X} - X_0(t)]^2 - \frac{t_g}{2} \sigma_x$$

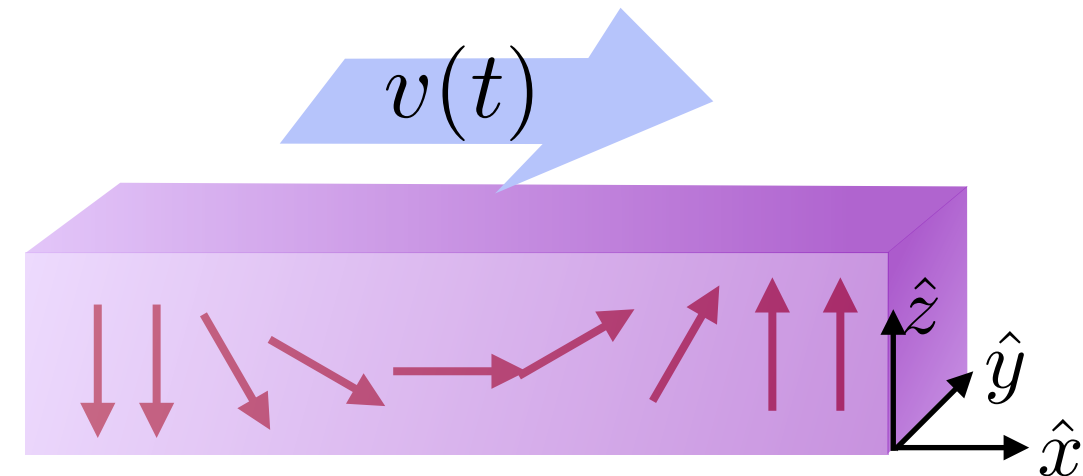
$$\downarrow \hat{T} = \exp[-i\hat{P}X_0(t)/\hbar]$$

$$H_m = \frac{(\hat{P} + \beta_z \sigma_z)^2}{2M} + \frac{M\omega_p^2}{2} \hat{X}^2 - \frac{t_g}{2} \sigma_x - \dot{X}_0(t) \hat{P}$$

$$\downarrow \hat{\mathcal{G}} = \exp(-i\hat{X}\sigma_z/l_{\text{so}})$$

$$\mathcal{H}_m = \frac{\tilde{\varepsilon}(t)}{2} \sigma_z - \frac{\tilde{t}_g}{2} \sigma_x$$

Adiabatic condition: $\dot{X}_0(t) < 100 \text{ m/s}$



effective spin-orbital coupling:

$$\beta_z = \pi N \hbar S_e + \pi M \hbar y / 2$$

spin-orbital length:

$$l_{\text{so}} = \hbar / \beta_z \sim 1.5 \text{ nm}$$

harmonic length:

$$l_p = \sqrt{\hbar / M \omega_p} \sim 2.25 \text{ nm}$$

Galilean term: $-i\hbar \hat{T}^\dagger \partial_t \hat{T}$

$$\tilde{\varepsilon}(t) = 2\hbar \dot{X}_0(t) / l_{\text{so}}$$

$$\tilde{t}_g = t_g \exp(-l_p^2 / l_{\text{so}}^2) \sim 3 \text{ GHz}$$

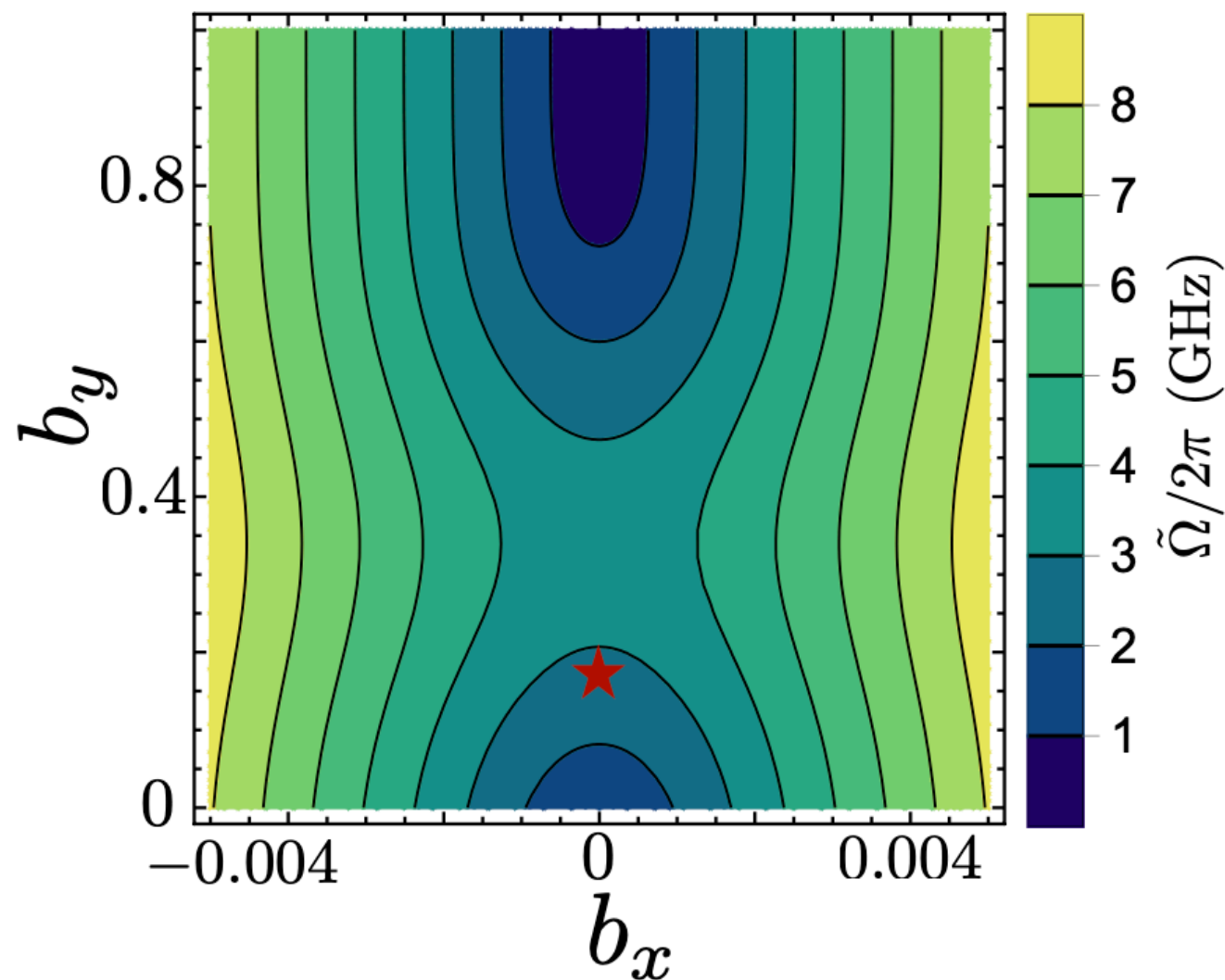
Shuttling domain wall qubits

Physical qubit splitting:

$$\mathcal{H}_m = \frac{\tilde{\varepsilon}(t)}{2}\sigma_z - \frac{\tilde{t}_g}{2}\sigma_x$$

$$\tilde{t}_g = t_g \exp(-l_p^2/l_{\text{so}}^2) \quad \tilde{\varepsilon}(t) = 2\hbar\dot{X}_0(t)/l_{\text{so}} - 8NK_y b_x$$

$$\hbar\tilde{\Omega} = \sqrt{\tilde{\varepsilon}^2 + \tilde{t}_g^2}$$



splitting increases first when b_y increases since **the barrier is reduced**.

splitting decreases when b_y further increases since **the suppression of soc**.

Single-qubit gates

Shuttling with constant speed:

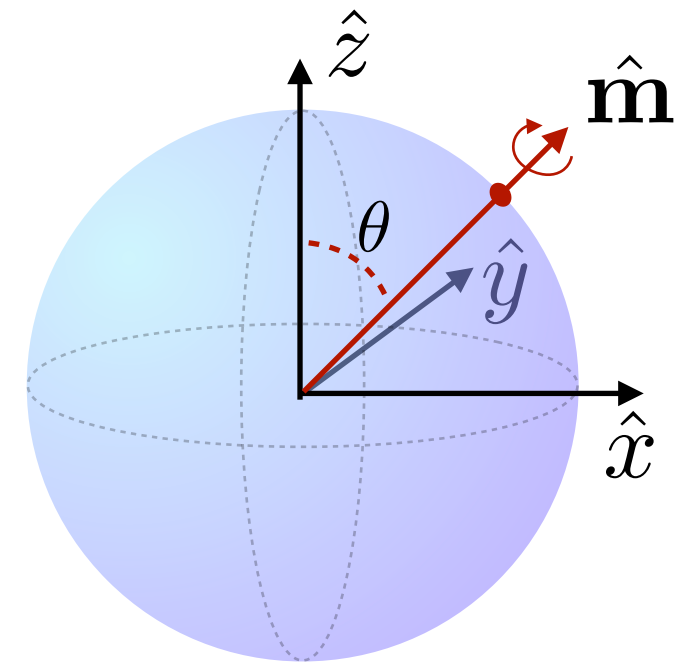
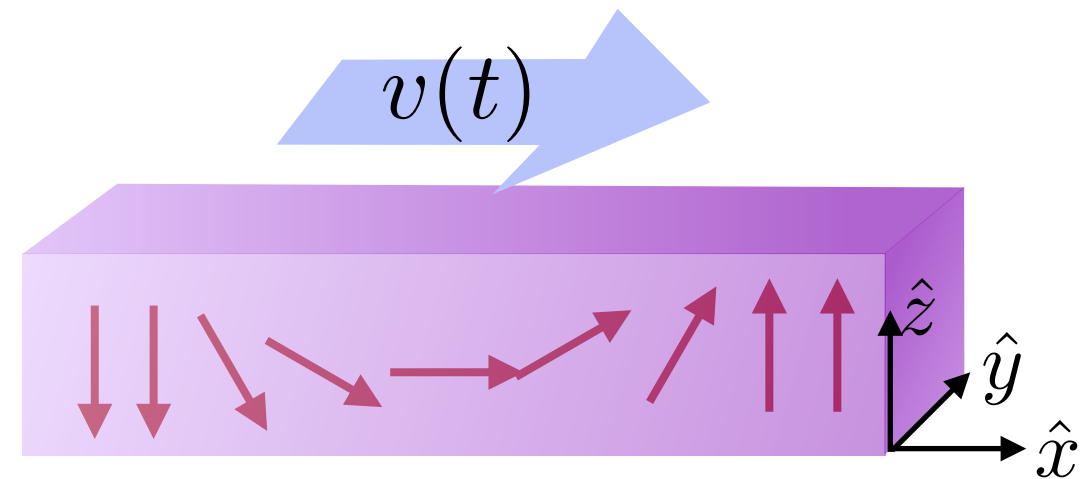
$$\mathcal{H}_m = \frac{\tilde{\varepsilon}(t)}{2} \sigma_z - \frac{\tilde{t}_g}{2} \sigma_x$$



$$U(t) = \exp(-i\tilde{\Omega}t\hat{\mathbf{m}} \cdot \boldsymbol{\sigma}/2)$$

$$\hat{\mathbf{m}} = (\sin \theta, 0, \cos \theta) \quad [\tan \theta = -\tilde{t}_g/\tilde{\varepsilon}]$$

b_y



spin rotations around $\hat{\mathbf{m}}$

- We can also implement single-qubit gates by **shaking the qubit**.

Single-qubit gates

Shaking the qubit on the racetrack:

$$\mathcal{H}_m = \frac{\tilde{\varepsilon}(t)}{2} \sigma_z - \frac{\tilde{t}_g}{2} \sigma_x \quad \tilde{\varepsilon}(t) \propto v(t) = v_0 s(t) \cos(\omega_d t + \phi_0)$$

envelope function

In rotating frame and at resonance: $\omega_d = \tilde{\Omega}$

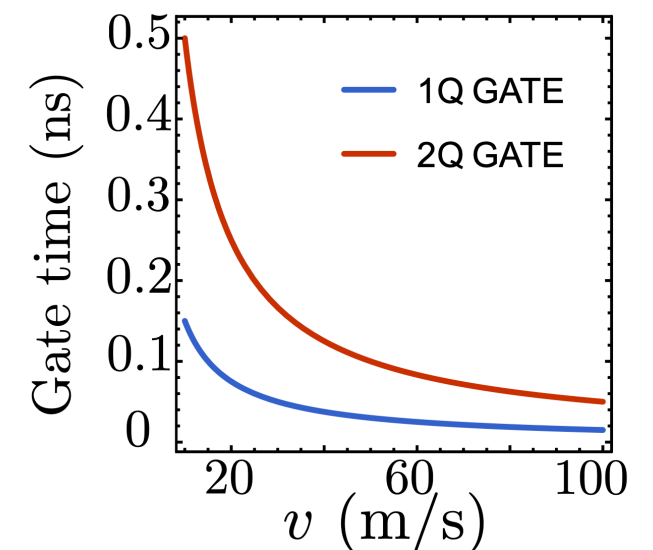
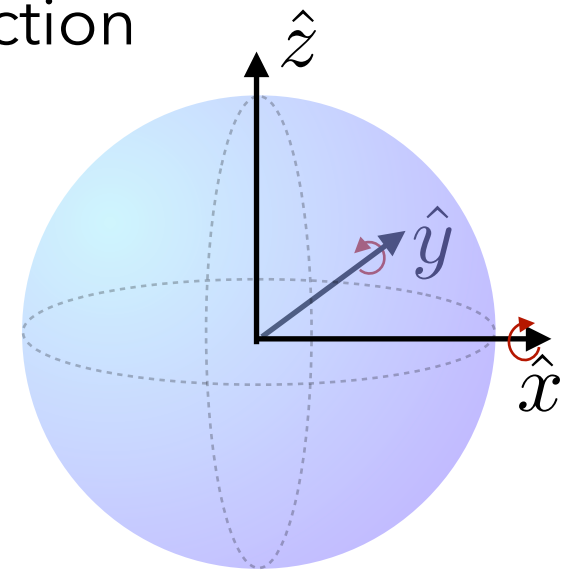
$$\mathcal{H}_r = \frac{\hbar \Omega_R s(t)}{2} (\cos \phi_0 \hat{\sigma}_x + \sin \phi_0 \hat{\sigma}_y), \quad \Omega_R = v_0 / l_{\text{so}}$$

1. In-phase pulse: $\phi_0 = 0$

$$\hat{R}_x^{\phi_0=0} = \exp \left\{ -i \Omega_R \int_0^t d\tau s(\tau) \frac{\hat{\sigma}_x}{2} \right\}$$

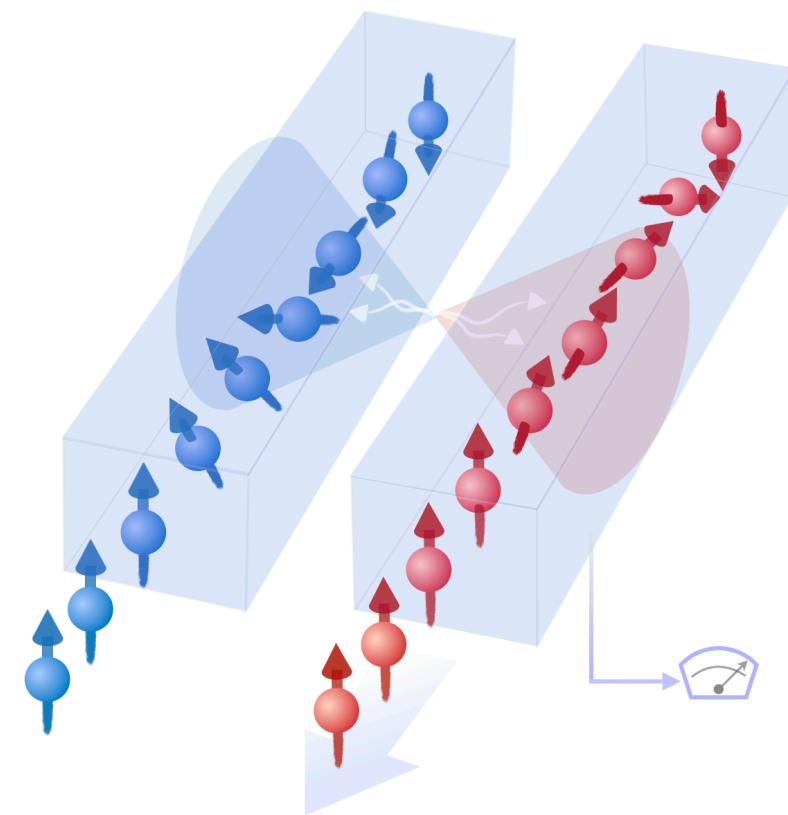
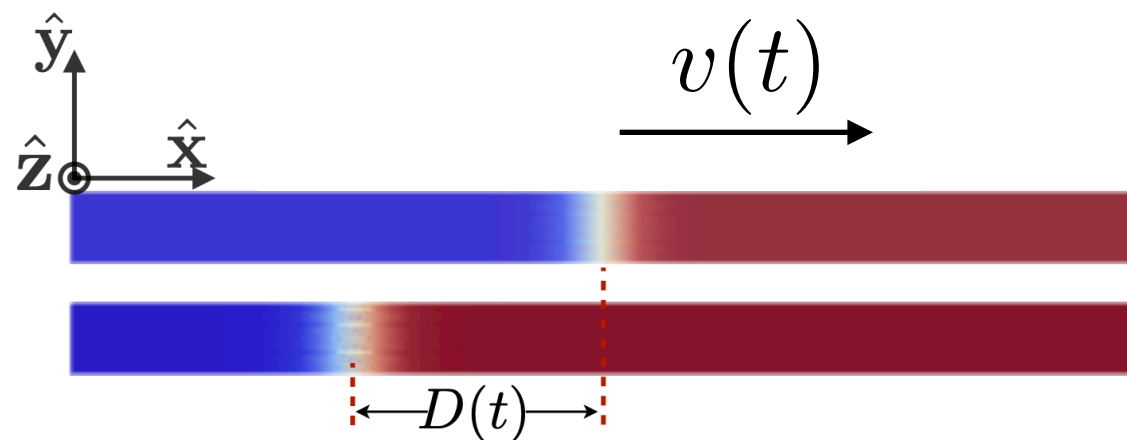
2. out-of-phase pulse: $\phi_0 = \frac{\pi}{2}$

$$\hat{R}_y^{\phi_0=\pi/2} = \exp \left\{ -i \Omega_R \int_0^t d\tau s(\tau) \frac{\hat{\sigma}_y}{2} \right\}$$



Two-qubit gates

two domain wall qubits on two racetracks



$$H^{(2)} = N^2 \mathcal{J}_{\alpha\beta} \int dx n_1^\alpha(x) n_2^\beta(x)$$

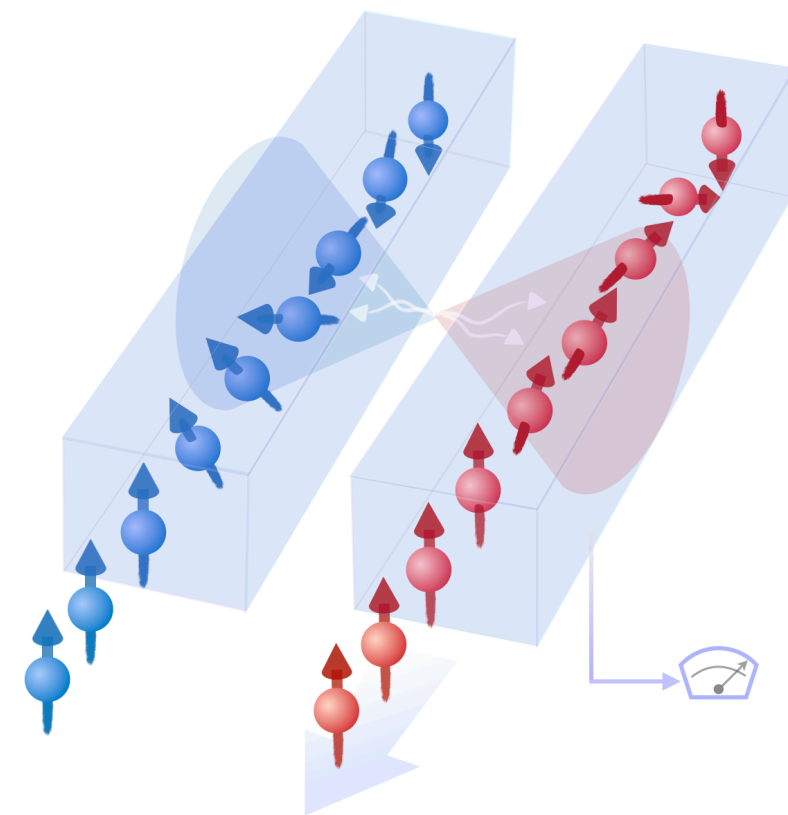
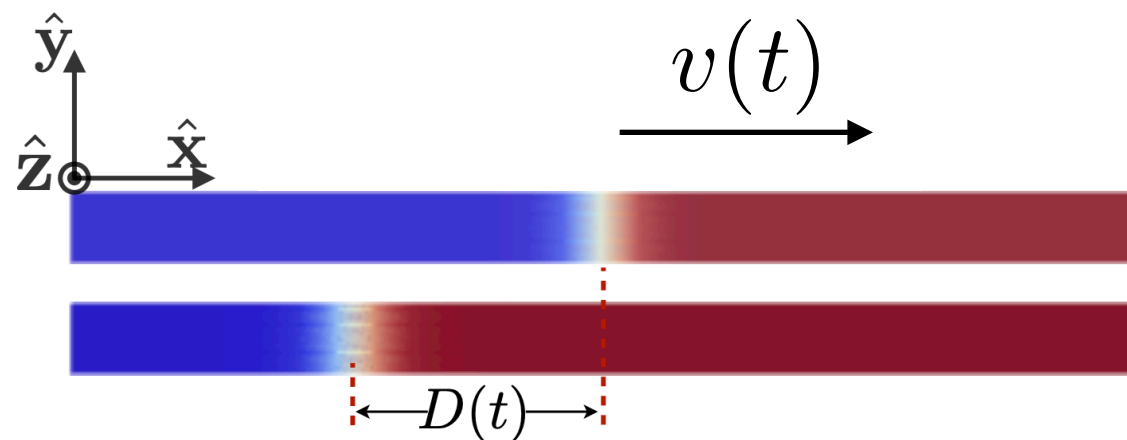
$$\mathbf{n}_1 = \mathbf{n}_0(\Phi_1, X_1) \quad \mathbf{n}_2 = \mathbf{n}_0(\Phi_2, X_2 = X_1 + D)$$

$$\mathcal{H}^{(2)} = \frac{2N^2 \mathcal{J}_{xx} D}{\sinh D} \hat{\sigma}_1^x \hat{\sigma}_2^x$$

interactions decreases exponentially
when two DWs are far away

Two-qubit gates

two domain wall qubits on two racetracks



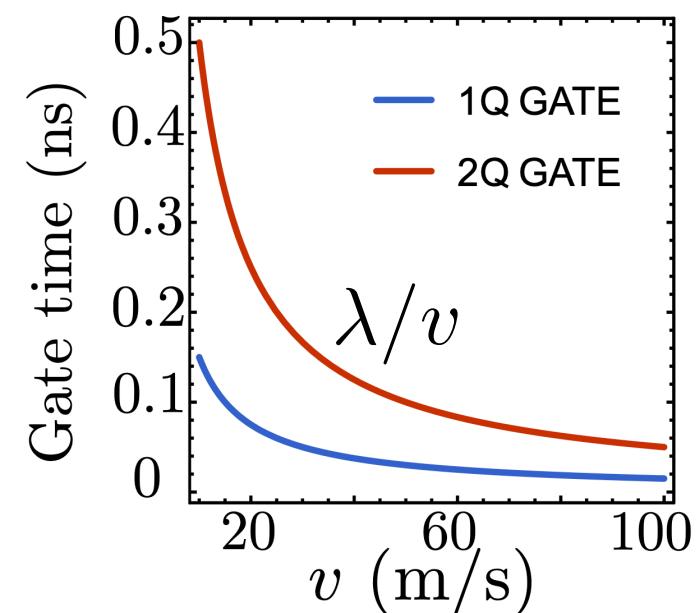
$$\mathcal{H}^{(2)} = \frac{2N^2 \mathcal{J}_{xx} D}{\sinh D} \hat{\sigma}_1^x \hat{\sigma}_2^x$$

$$\downarrow v(t) = v$$

$\pi/4$ CNOT gate

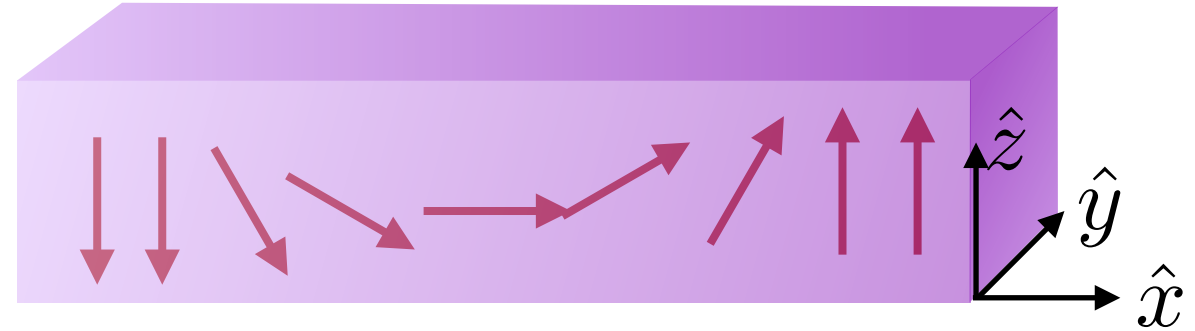
$$\hat{\mathcal{U}}^{(2)} = \exp \left\{ - \frac{i\pi^2 N^2 \mathcal{J}_{xx}}{\hbar v} \hat{\sigma}_1^x \otimes \hat{\sigma}_2^x \right\}$$

$$N^2 \mathcal{J}_{xx} \sim 50 \text{ MHz} \quad v = 20 \text{ m/s} \quad T_G \approx 0.2 \text{ ns}$$



Qubit lifetime

1. Both Φ and X would fluctuate due to random kicks from the environment



2. Fluctuations in the potential:

$$U(\Phi, X) \rightarrow U(\Phi, X) + \xi(t)\Phi + \xi(t)X$$

$$\langle \xi(t) \rangle = 0, \langle \xi(t)\xi(t') \rangle = S(t - t')$$

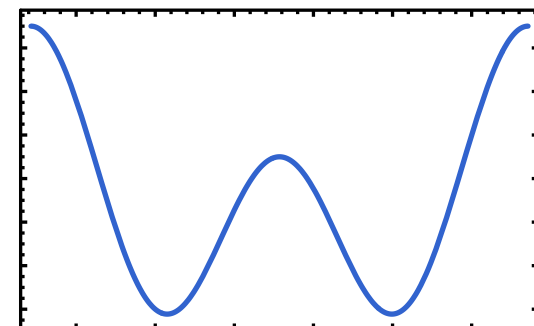
$$S_\xi(\omega) = \tilde{\alpha} \hbar \omega \coth \frac{\hbar \omega}{2k_B T}$$

$$\tilde{\alpha} = \alpha N \hbar S$$

↓
Gilbert damping

3. Fluctuating terms in the qubit Hamiltonian:

$$\delta \mathcal{H}_m = \frac{\xi(t)}{2} \sigma_z + \partial_{V_0} \tilde{t}_g \frac{\xi(t)}{2} \sigma_x + \frac{l_p^2 \dot{\xi}(t)}{\omega_p l_{\text{so}}} \sigma_z$$



Qubit lifetime

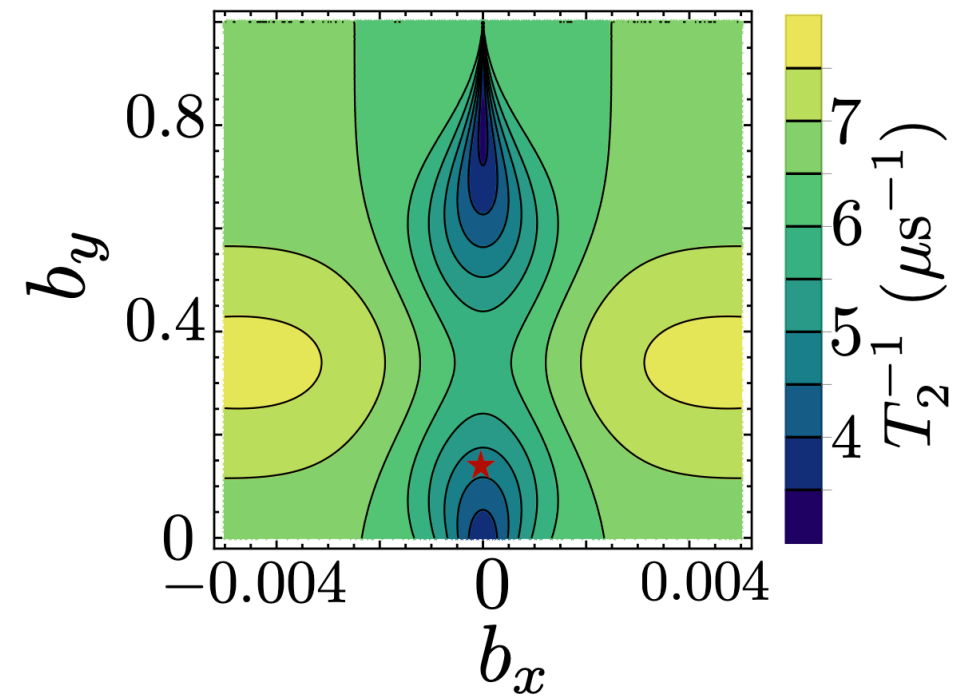
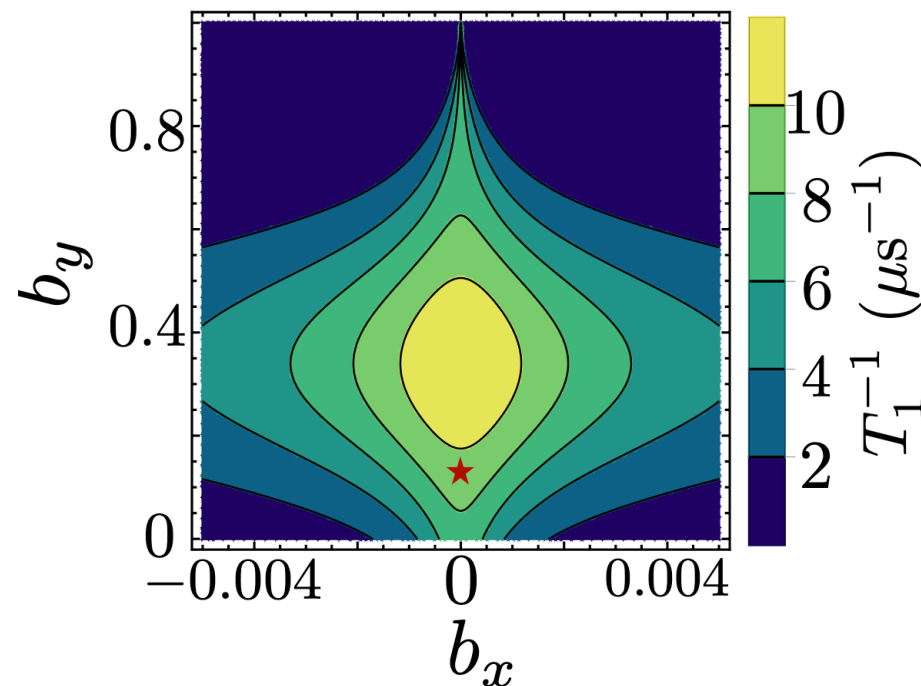
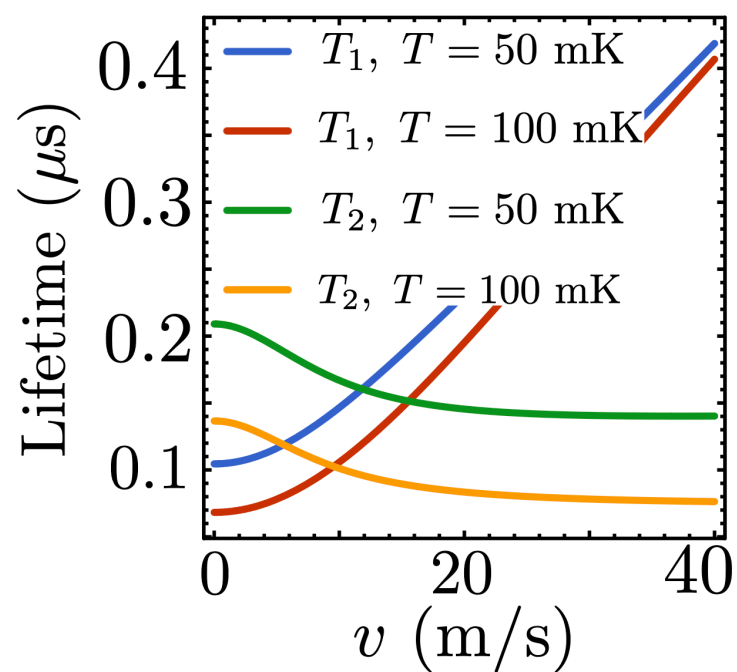
Relaxation rate: $\Gamma_1 = T_1^{-1} = \frac{\sin^2 \theta S_\xi(\tilde{\Omega})}{2\hbar^2} \left(1 + \frac{4l_p^4 \tilde{\Omega}^2}{\omega_p^2 l_{so}^2} \right)$

$$\cos \theta = \frac{\tilde{\varepsilon}}{\sqrt{\tilde{\varepsilon}^2 + \tilde{t}_g^2}}$$

Dephasing rate: $\Gamma_2 = T_2^{-1} = \frac{\Gamma_1}{2} + \Gamma_\varphi$ with $\Gamma_\varphi = \frac{\cos^2 \theta S_\xi(0)}{2\hbar^2}$

$$b_x = \hbar v / (l_{so} N K_y)$$

sweet spot

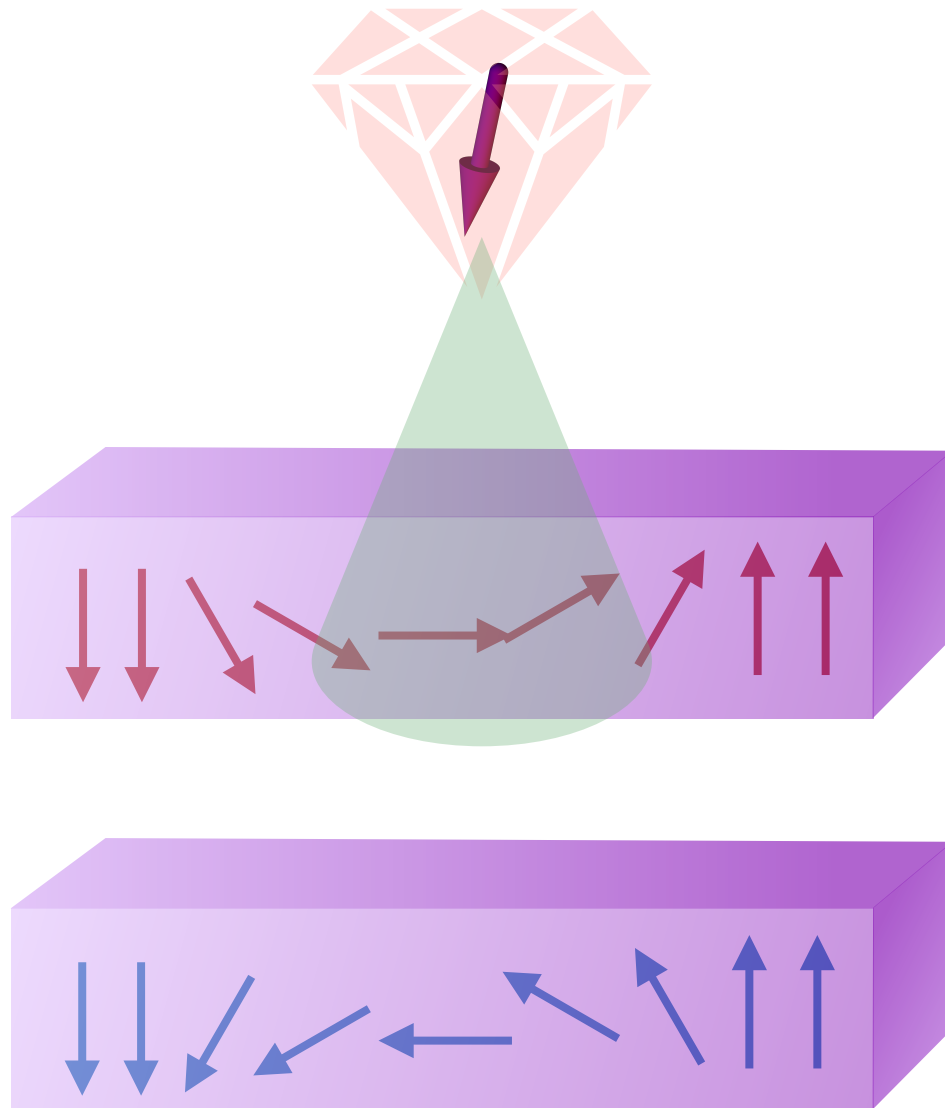


$v = 20$ m/s $T = 50$ mK: $T_1 = 0.23 \mu s$ $T_2 = 0.15 \mu s$

Coherent shuttling distance: $3 \mu m$ Quality factor: $\Omega_R T_2 \sim 10^3$

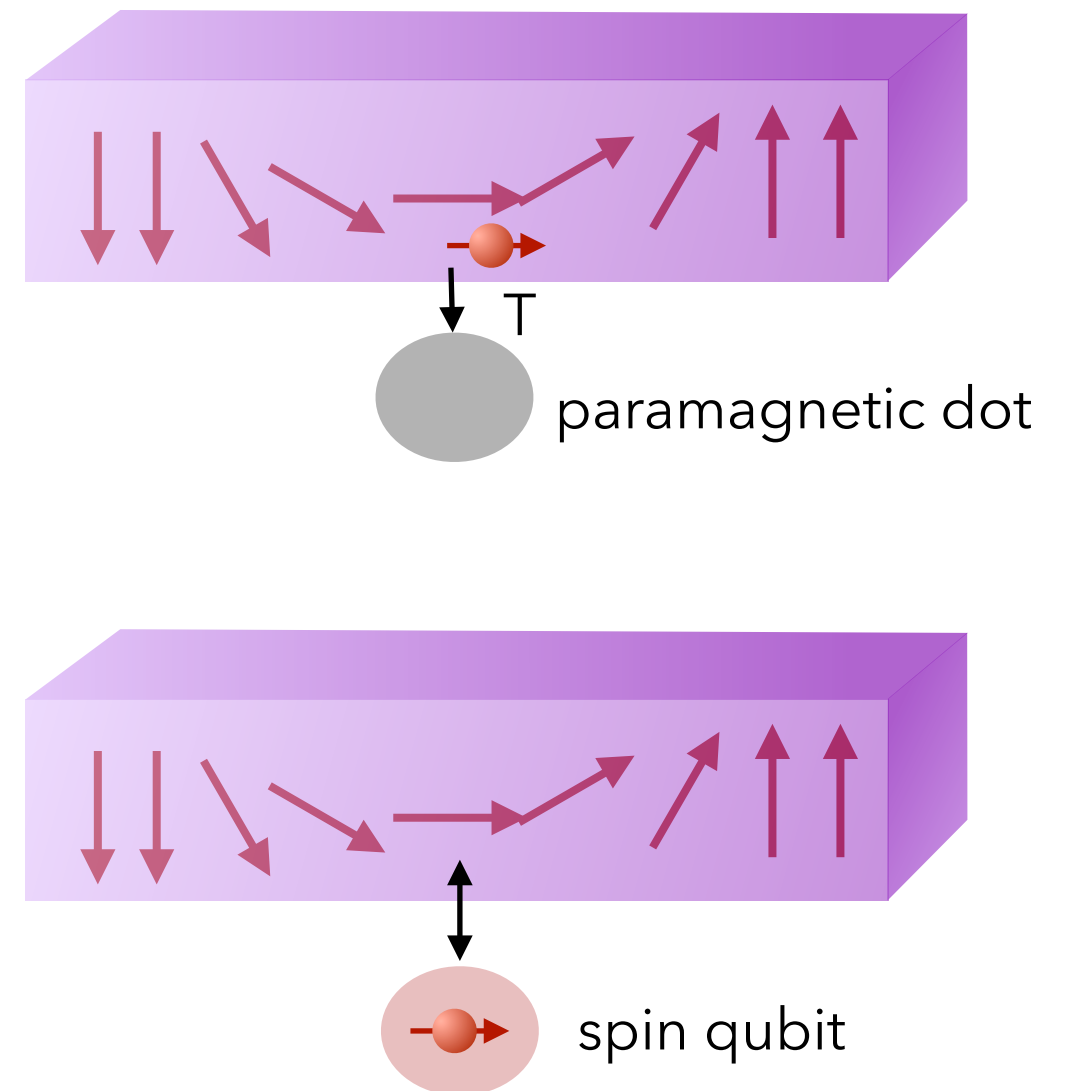
Qubit readout

use NV center to image the texture



Song, et al., Science 374, 1140 (2021)
Finco, et al., Nature Com. 12, 767 (2021)

use readout for spin qubits



Elzerman, et al., Nature 430, 431 (2004)
Pla, et al., Nature 496, 334 (2013)

What we have achieved

Our setup is perfect for both quantum computation and communication

1. A scalable physical system with well-characterized **qubit**
2. The ability to initialize the state of the qubits to a simple fiducial state
3. Long relevant **decoherence times**
4. A "**universal**" set of **quantum gates**
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Thank you !

[arXiv:2212.12019](https://arxiv.org/abs/2212.12019)



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Daniel Loss