Topological Quantum Dimers emerging from Kitaev Spin Liquid Bilayer: Anyon Condensation Transition

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KOREA INSTITUTE FOR ADVANCED STUDY

Quantum Spin Liquid

- No broken symmetry
- Highly quantum entangled
- Anyon quasiparticles

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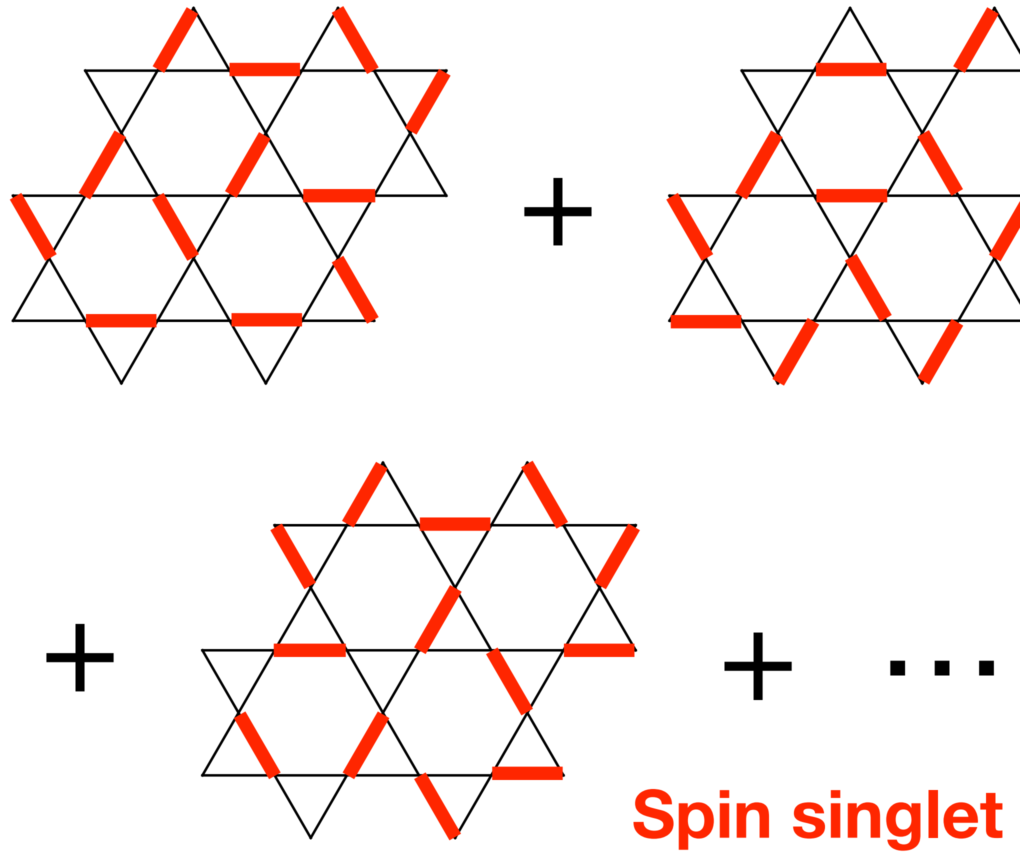
RVB spin liquid



Kagome lattice Heisenberg model

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

Herbertsmithite $ZnCu_3(OH)_6Cl_2$

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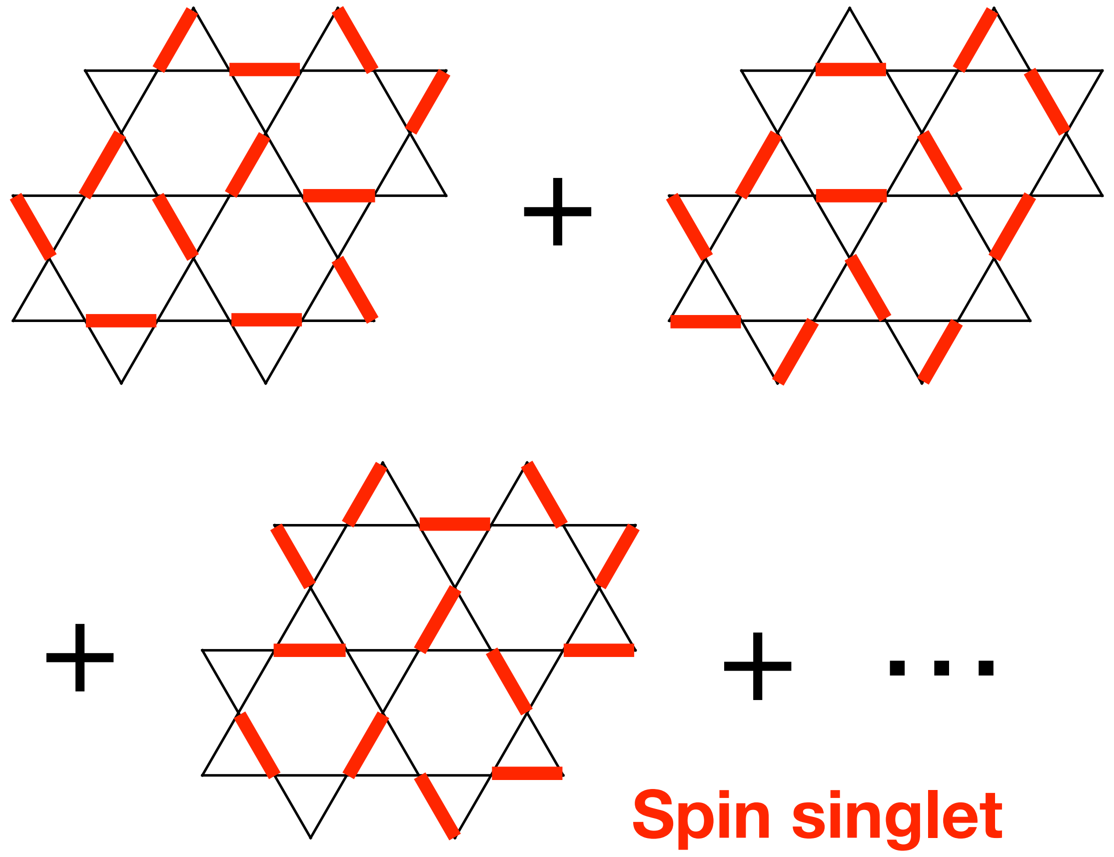
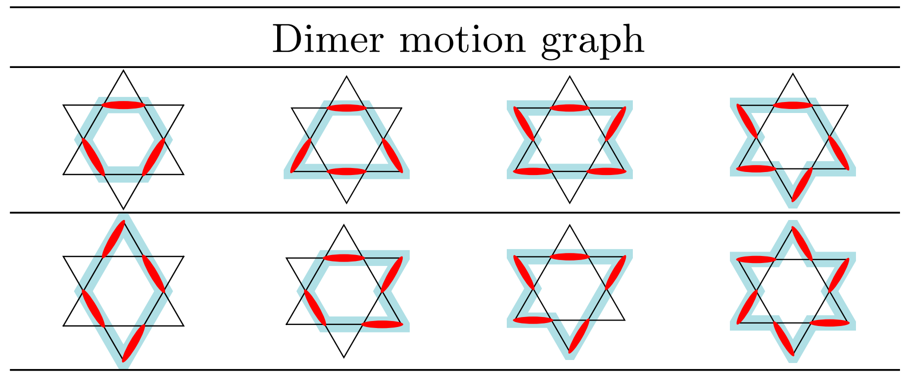
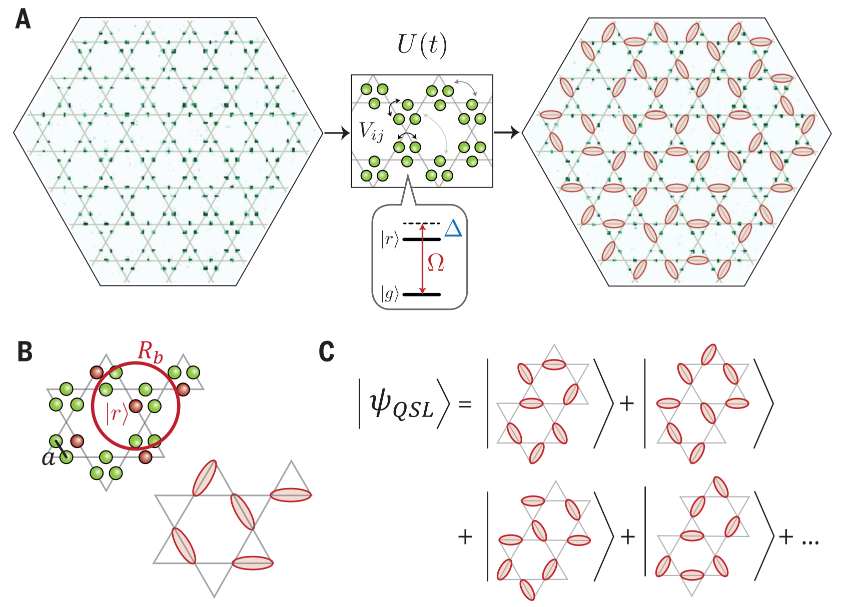


Quantum dimer model

$$H = \sum_{\mathcal{D}} |\overline{\mathcal{D}}\rangle\langle\mathcal{D}| + \mathrm{H.c.}$$



Rydberg atom array



Quantum Spin Liquid

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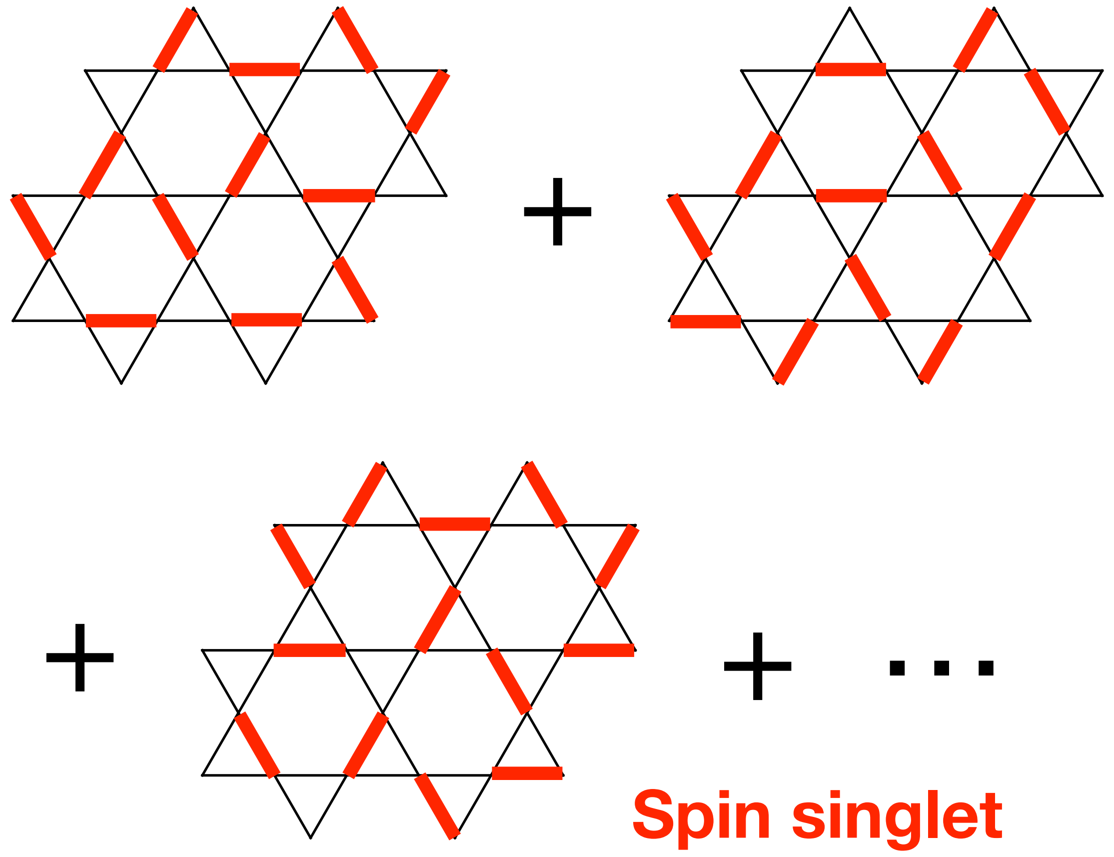
RVB spin liquid



Spin singlet

Quantum dimer model

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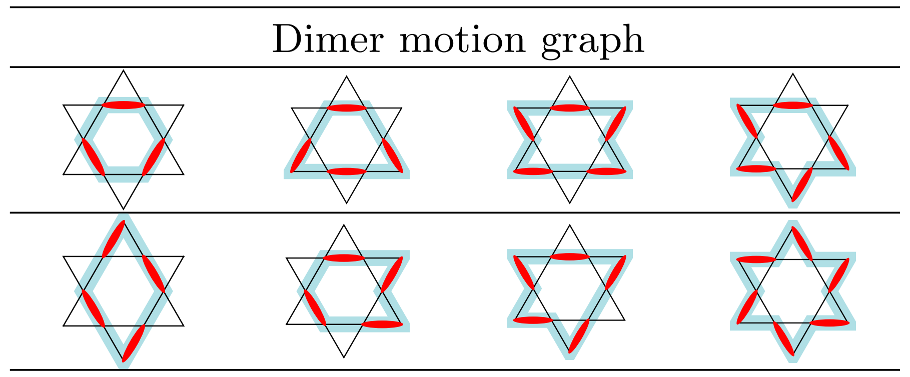

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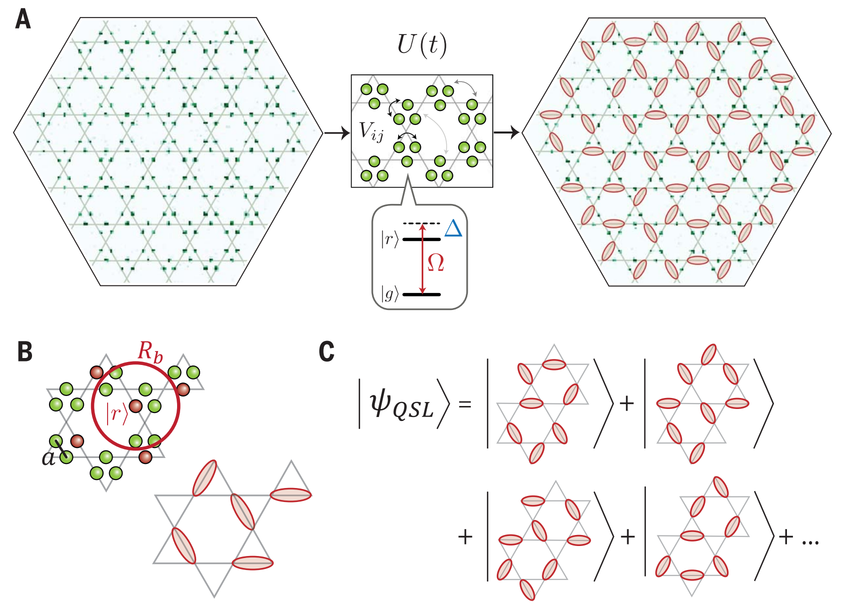
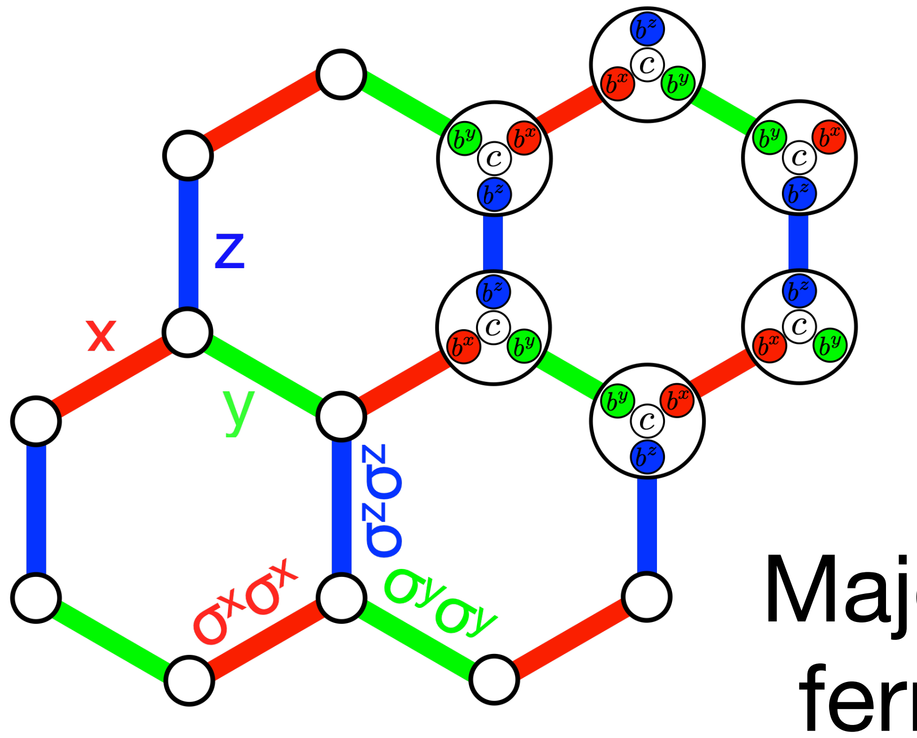


Kitaev spin liquid

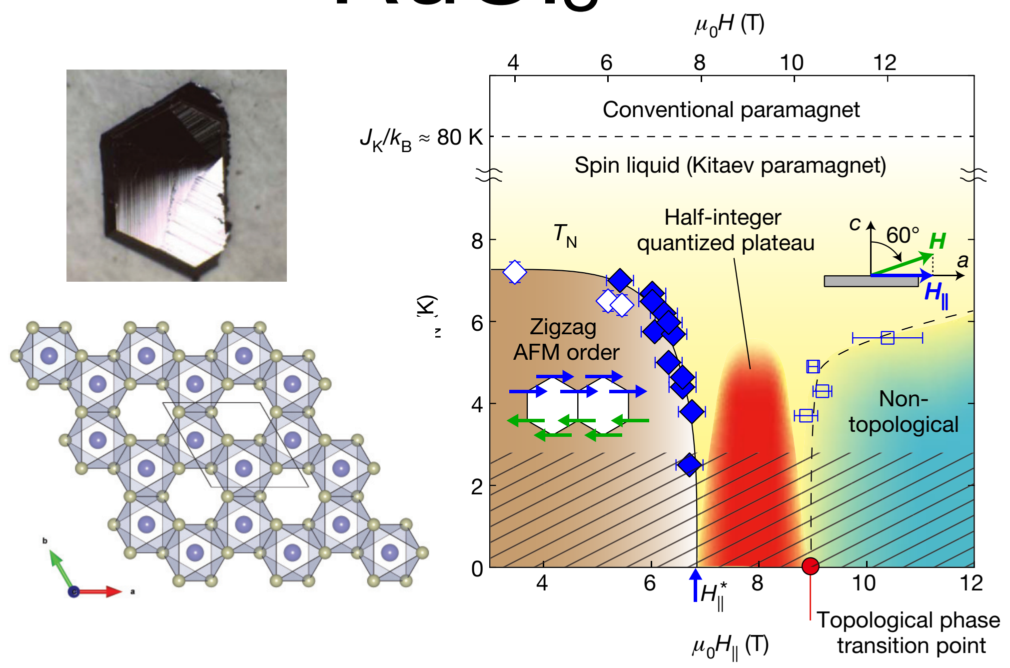


Majorana fermion

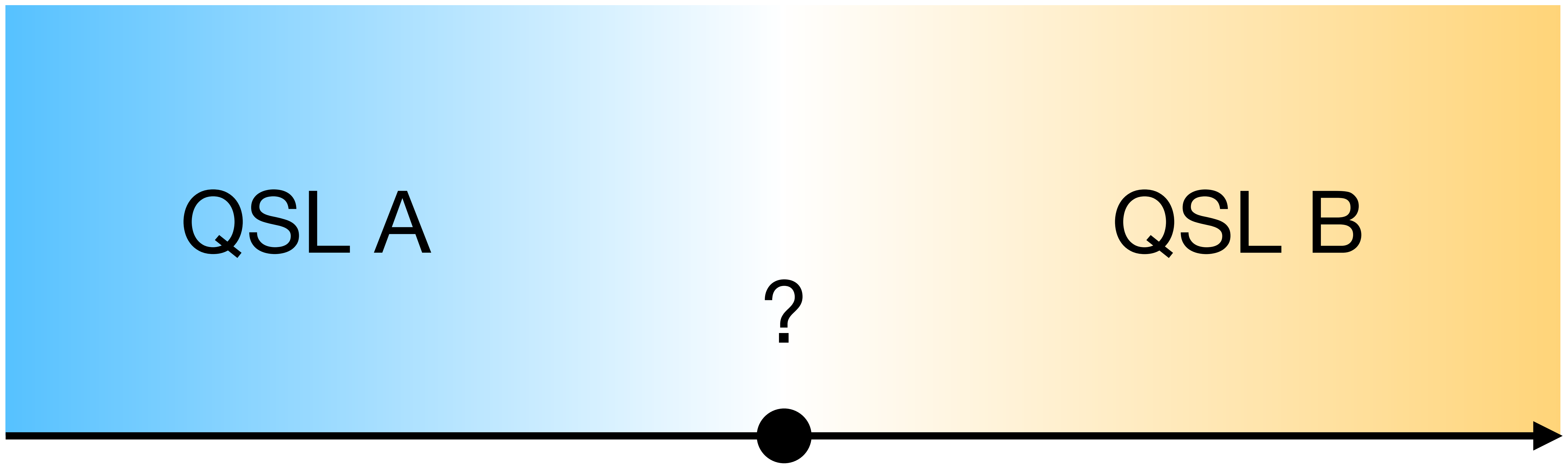
Kitaev honeycomb model

$$H = K \sum_{\langle ij\rangle_x} \sigma_i^x \sigma_j^x + K \sum_{\langle ij\rangle_y} \sigma_i^y \sigma_j^y + K \sum_{\langle ij\rangle_z} \sigma_i^z \sigma_j^z$$

Quantum magnet $RuCl_3$



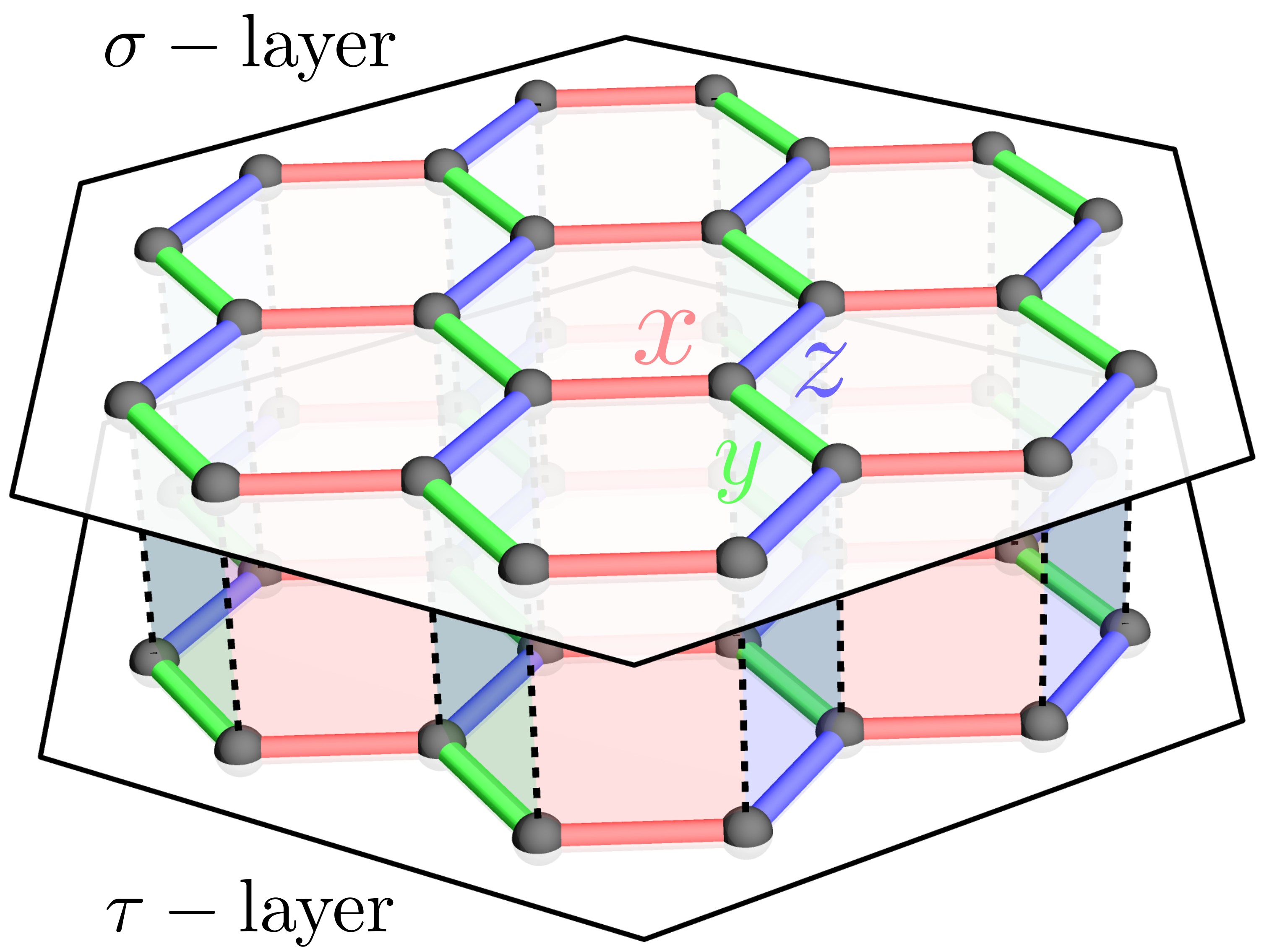
Motivation: Transition between QSLs

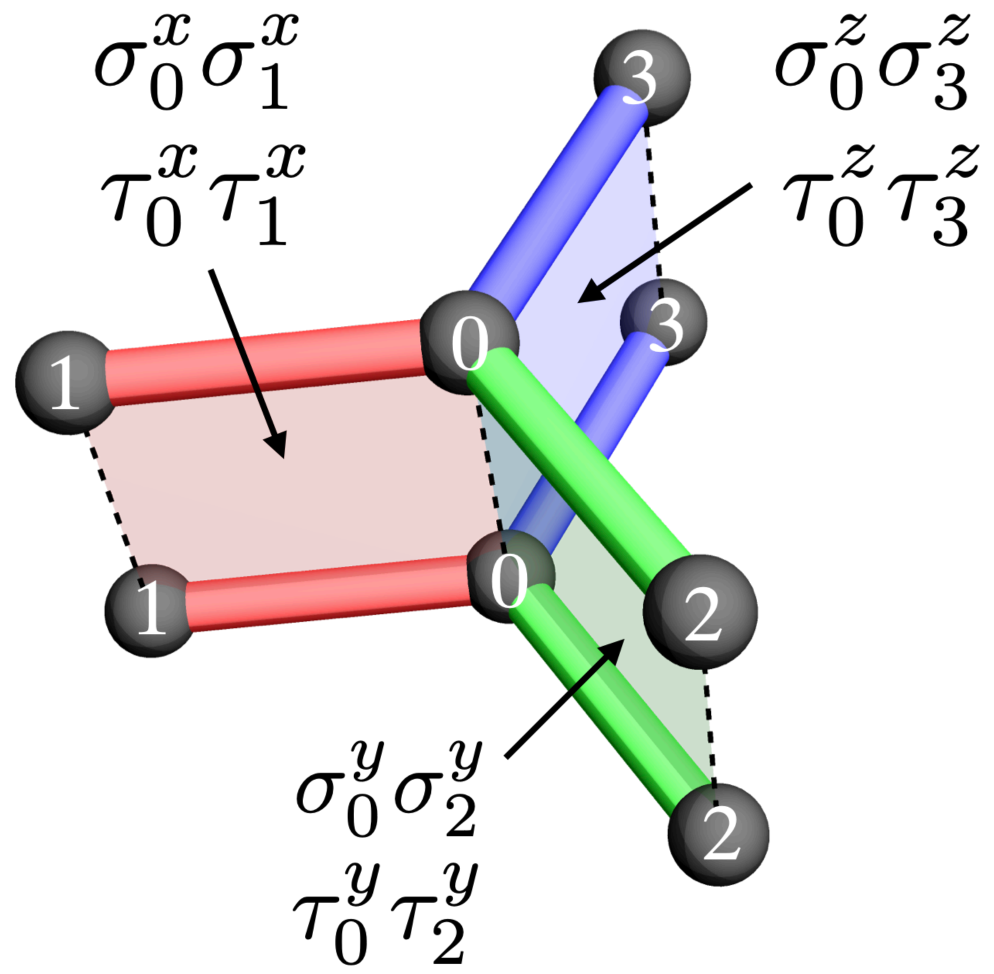


- No symmetry breaking
- Landau-Ginzburg-Wilson theory does not work
- How do we understand the transition?

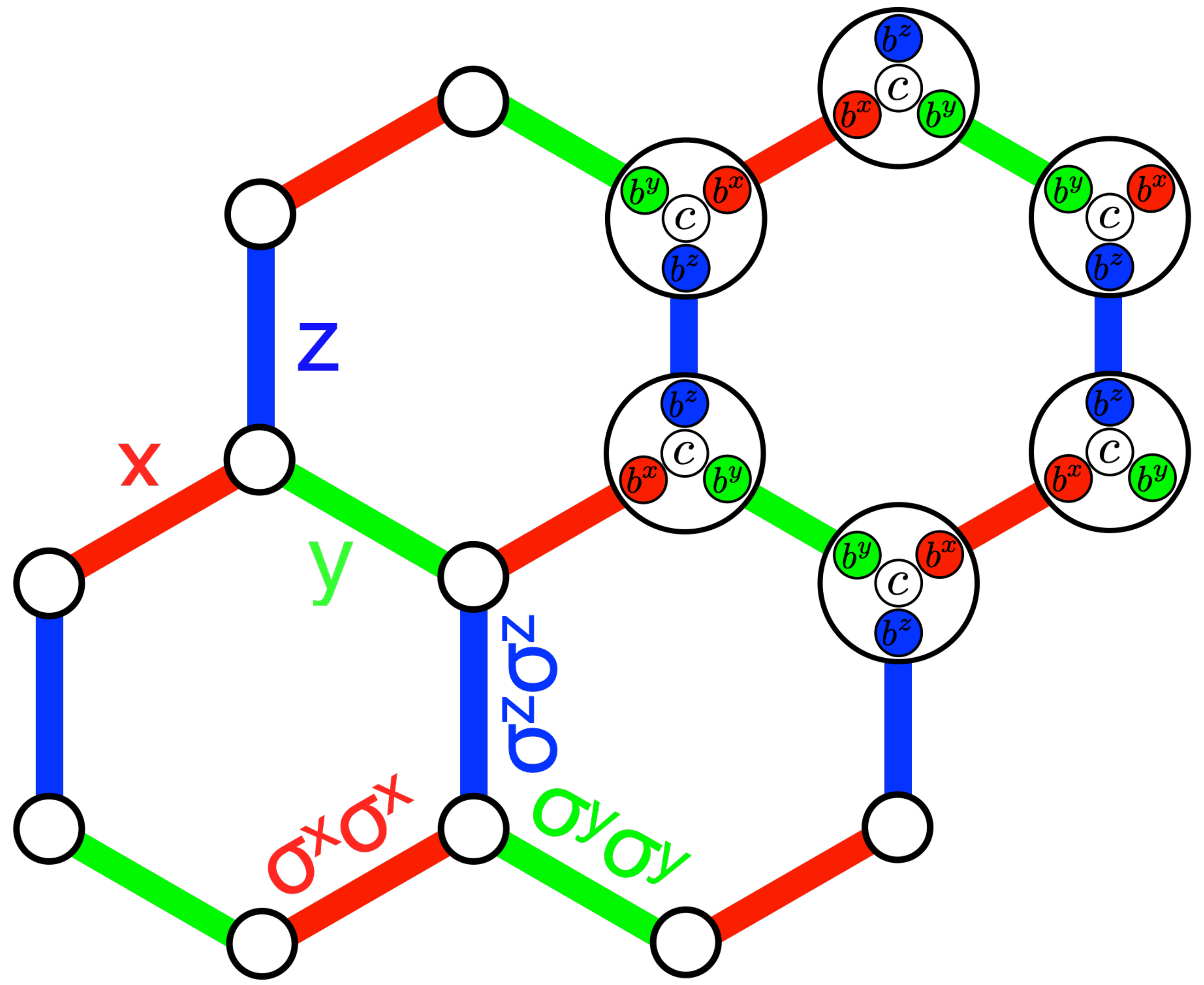
Bilayer Kitaev model

$$H = K_\sigma \sum_{\langle jk \rangle_\gamma}^{\text{upper layer}} \sigma_j^\gamma \sigma_k^\gamma + K_\tau \sum_{\langle jk \rangle_\gamma}^{\text{lower layer}} \tau_j^\gamma \tau_k^\gamma + G \sum_{\langle jk \rangle_\gamma}^{\text{inter-layer}} \sigma_j^\gamma \sigma_k^\gamma \tau_j^\gamma \tau_k^\gamma$$





Review: Kitaev honeycomb model

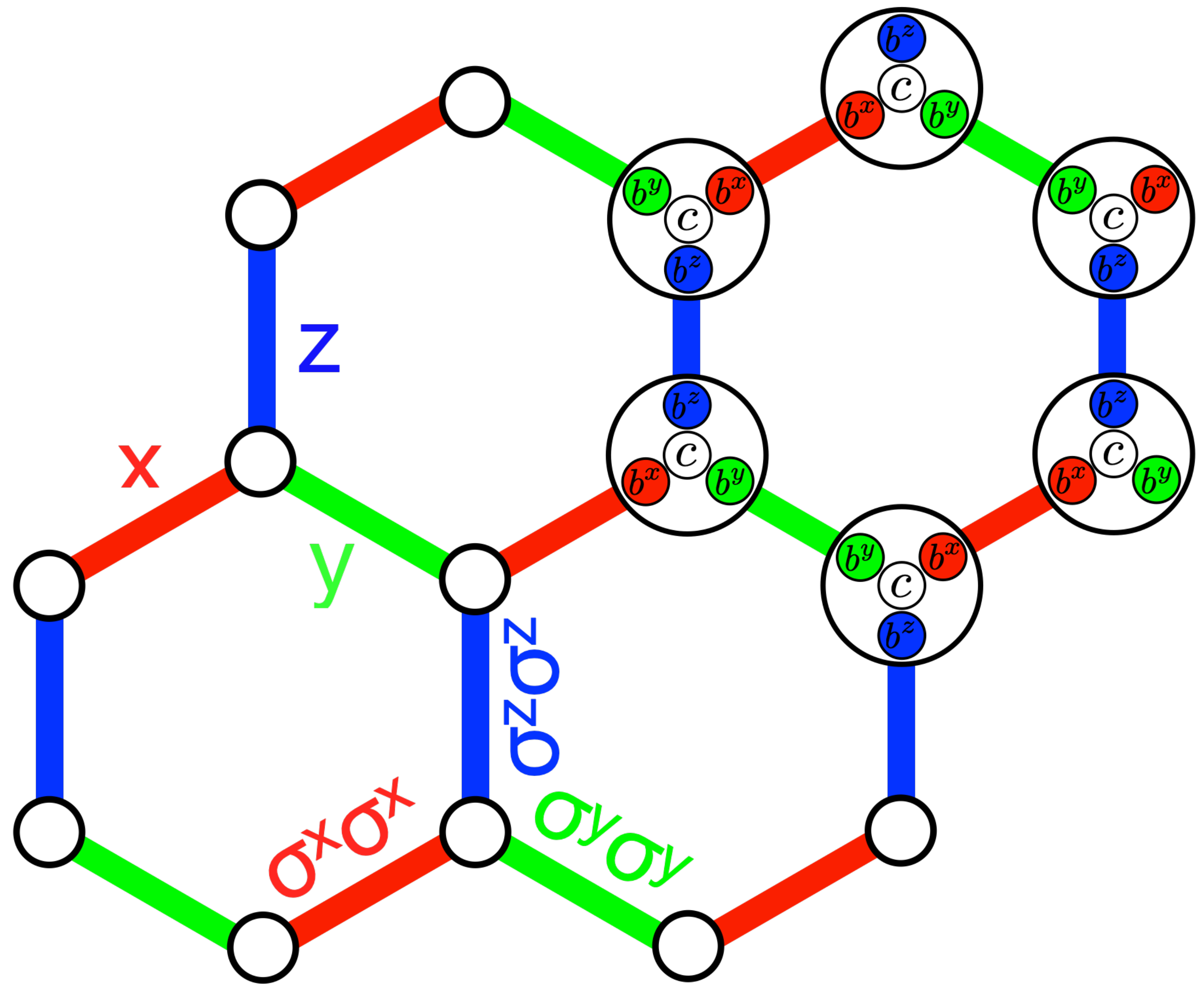


bond-dep. Ising terms

$$H = K\sum_{\langle ij\rangle_x}\sigma_i^x\sigma_j^x + K\sum_{\langle ij\rangle_y}\sigma_i^y\sigma_j^y + K\sum_{\langle ij\rangle_z}\sigma_i^z\sigma_j^z$$

- Exact solved by parton theory
- Majorana fermions

Review: Kitaev honeycomb model



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Majorana fermion rep.

$$\sigma^\alpha = i b^\alpha c$$

$$H_{\mathrm{eff}} = -iK \sum_{\langle ij \rangle_\alpha} c_i c_j \cdot u_{ij}$$

c-Majorana fermions

Z_2 gauge fields

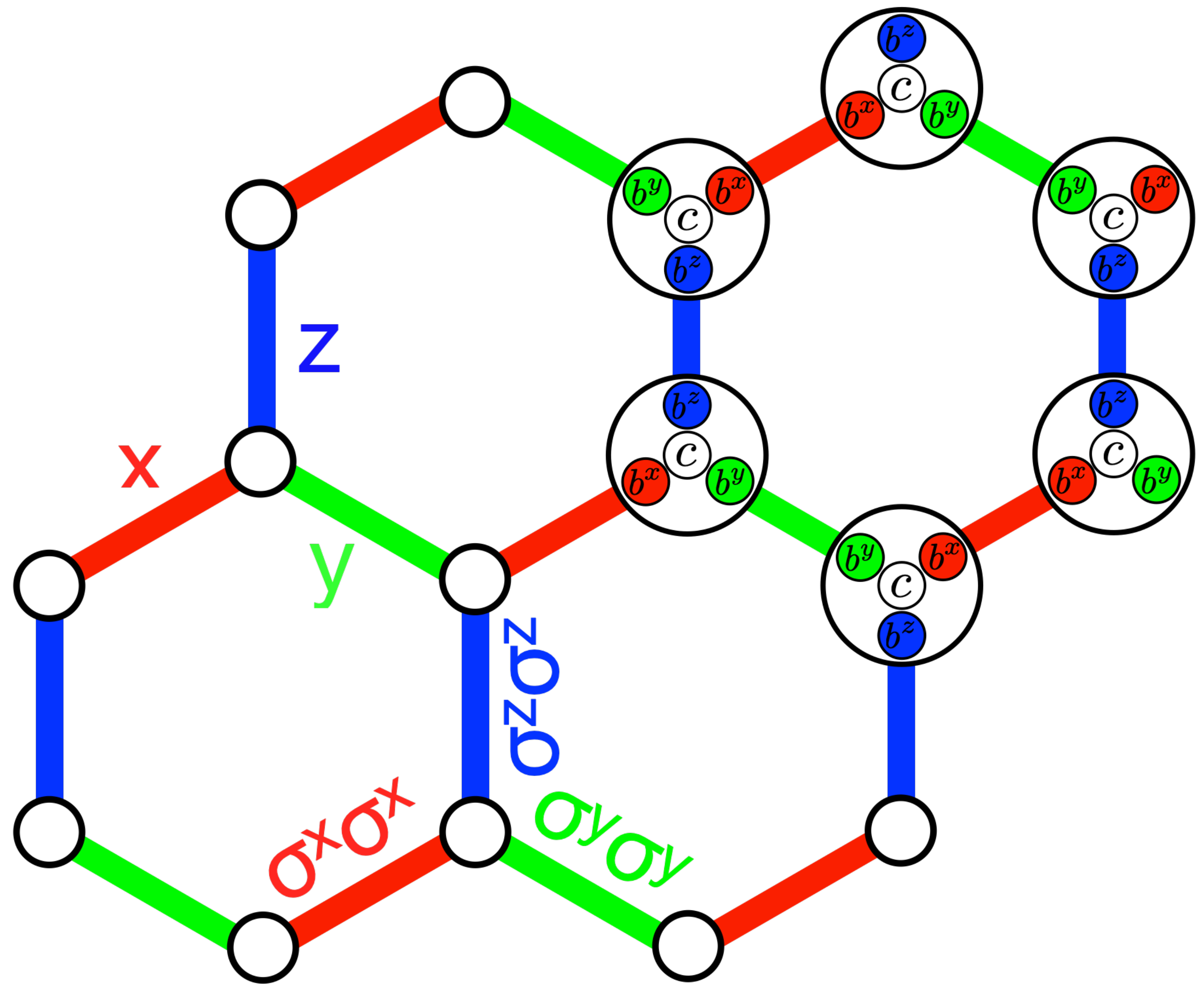
$$u_{ij} = i b_i^\alpha b_j^\alpha = \pm 1$$

$$b^\dagger = b$$
$$c^\dagger = c$$

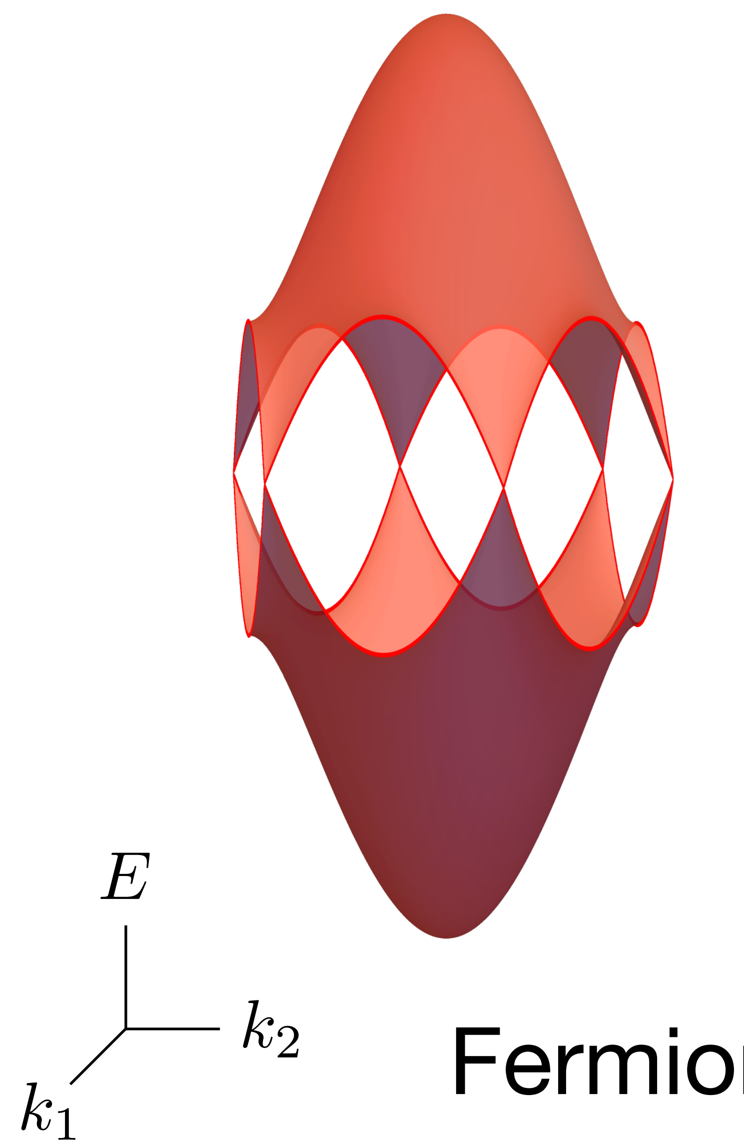
Review: Kitaev honeycomb model

bond-dep. Ising terms

$$H = K \sum_{\langle ij \rangle_x} \sigma_i^x \sigma_j^x + K \sum_{\langle ij \rangle_y} \sigma_i^y \sigma_j^y + K \sum_{\langle ij \rangle_z} \sigma_i^z \sigma_j^z$$



$h = 0$

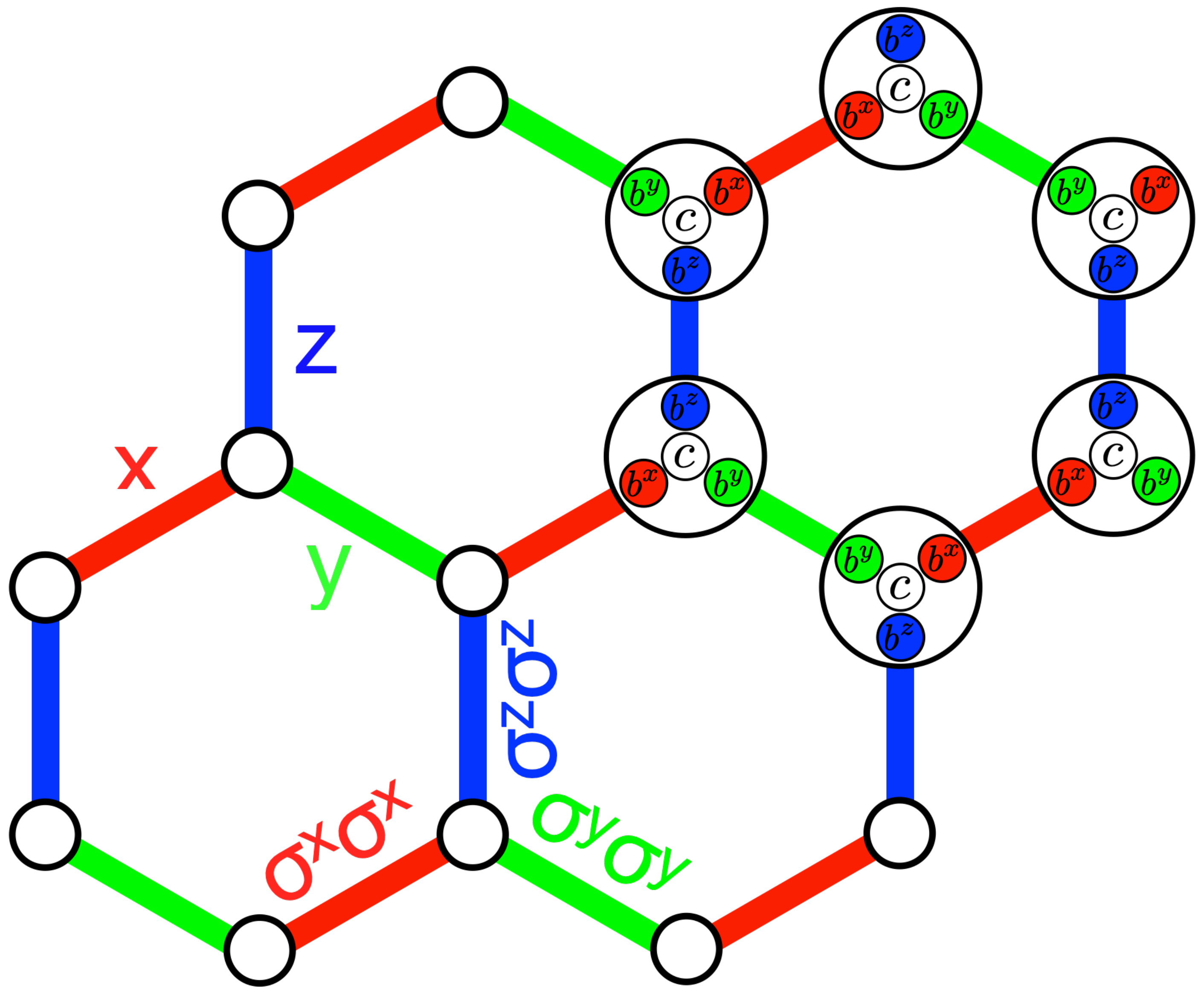


Fermion excitations

Review: Kitaev honeycomb model

bond-dep. Ising terms

$$H = K \sum_{\langle ij \rangle_x} \sigma_i^x \sigma_j^x + K \sum_{\langle ij \rangle_y} \sigma_i^y \sigma_j^y + K \sum_{\langle ij \rangle_z} \sigma_i^z \sigma_j^z$$



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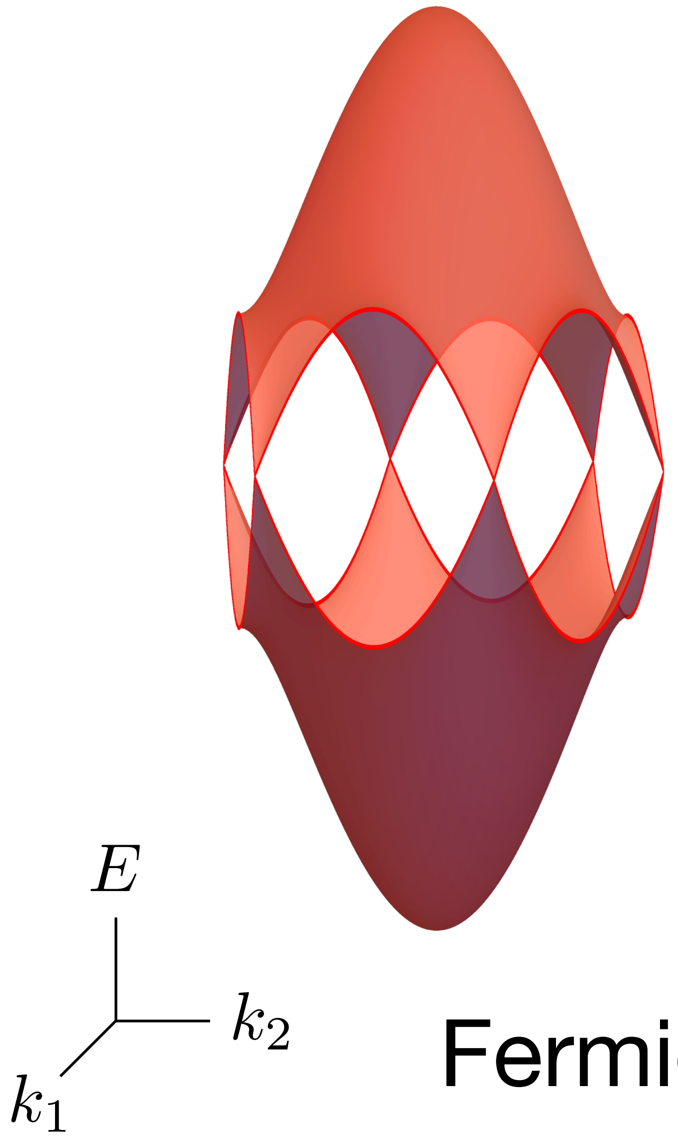
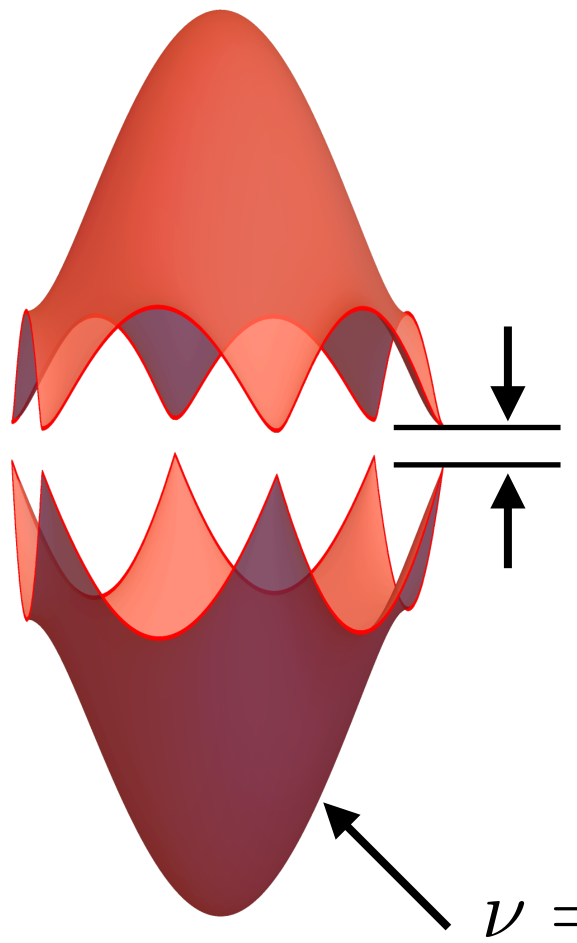


Fermion excitations

$$- \mathbf{h} \cdot \sum_i \boldsymbol{\sigma}_i$$

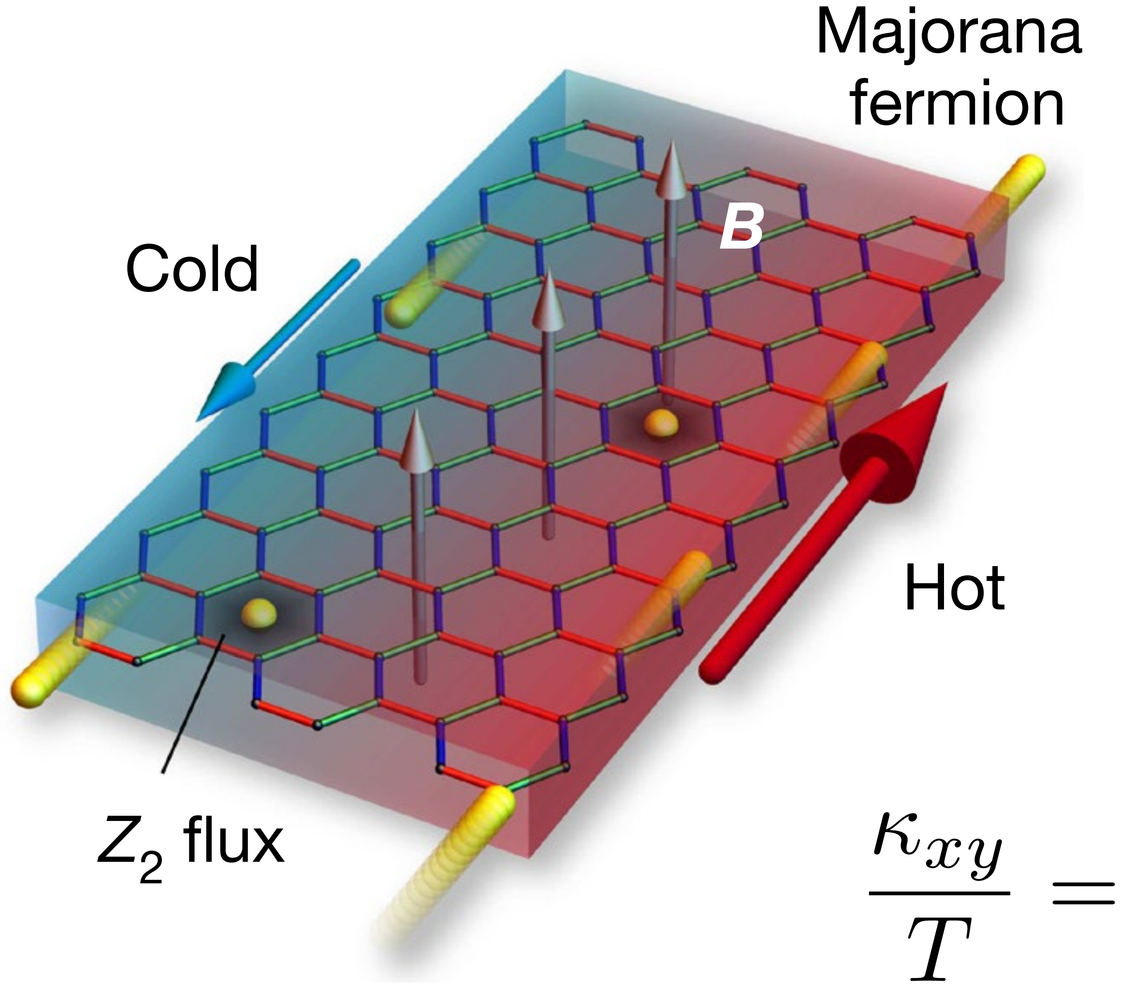
$h \neq 0$



$$\Delta_M \propto \left| \frac{h_x h_y h_z}{\Delta_{\text{flux}}^2} \right|$$

$$\nu = \text{sgn}\left(\frac{h_x h_y h_z}{\Delta_{\text{flux}}^2} \right) = \pm 1$$

Chern number

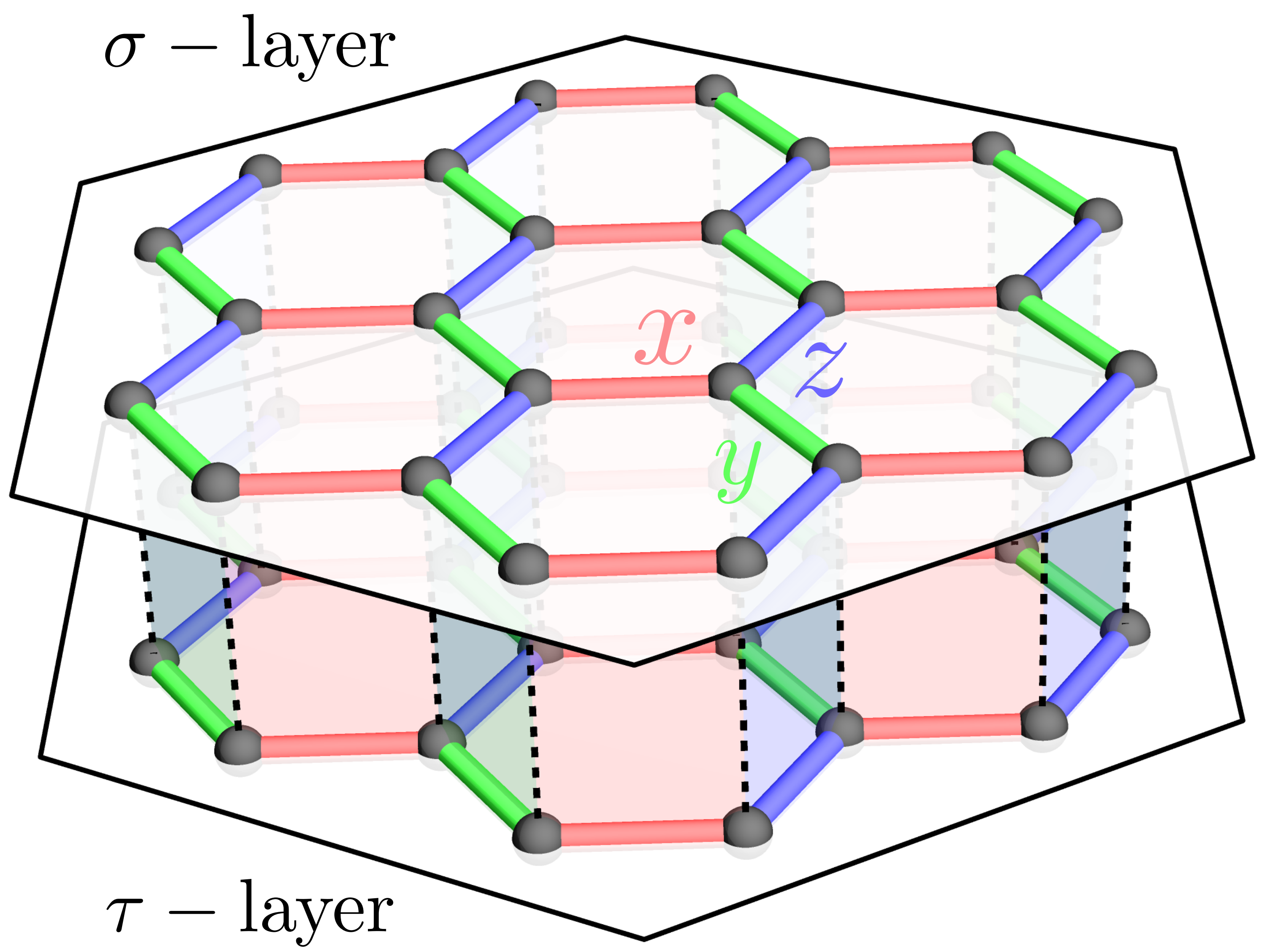


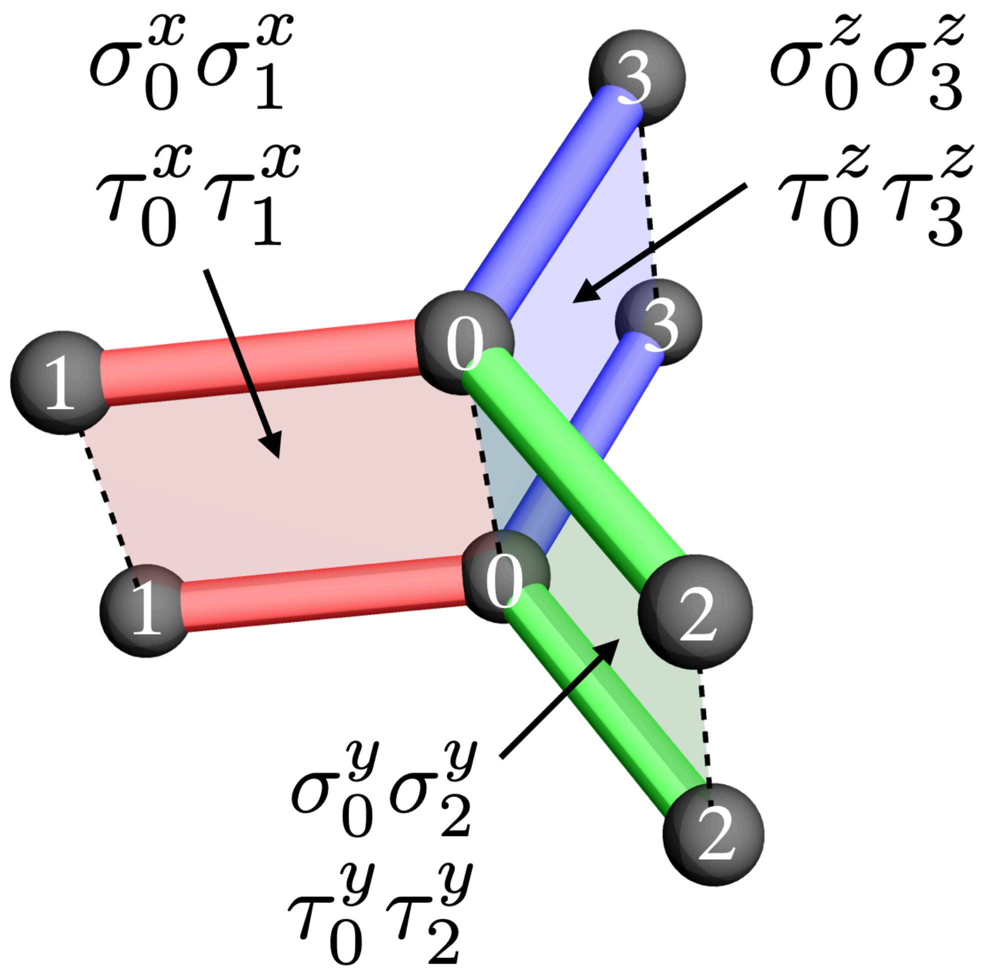
$$\frac{\kappa_{xy}}{T} = \nu \frac{\pi}{12} \frac{k_B^2}{\hbar}$$

Half-quantized thermal Hall effect

Bilayer Kitaev model

$$H = K_\sigma \overset{\text{upper layer}}{\sum_{\langle jk\rangle_\gamma}} \sigma_j^\gamma \sigma_k^\gamma + K_\tau \overset{\text{lower layer}}{\sum_{\langle jk\rangle_\gamma}} \tau_j^\gamma \tau_k^\gamma + G \overset{\text{inter-layer}}{\sum_{\langle jk\rangle_\gamma}} \sigma_j^\gamma \sigma_k^\gamma \tau_j^\gamma \tau_k^\gamma$$





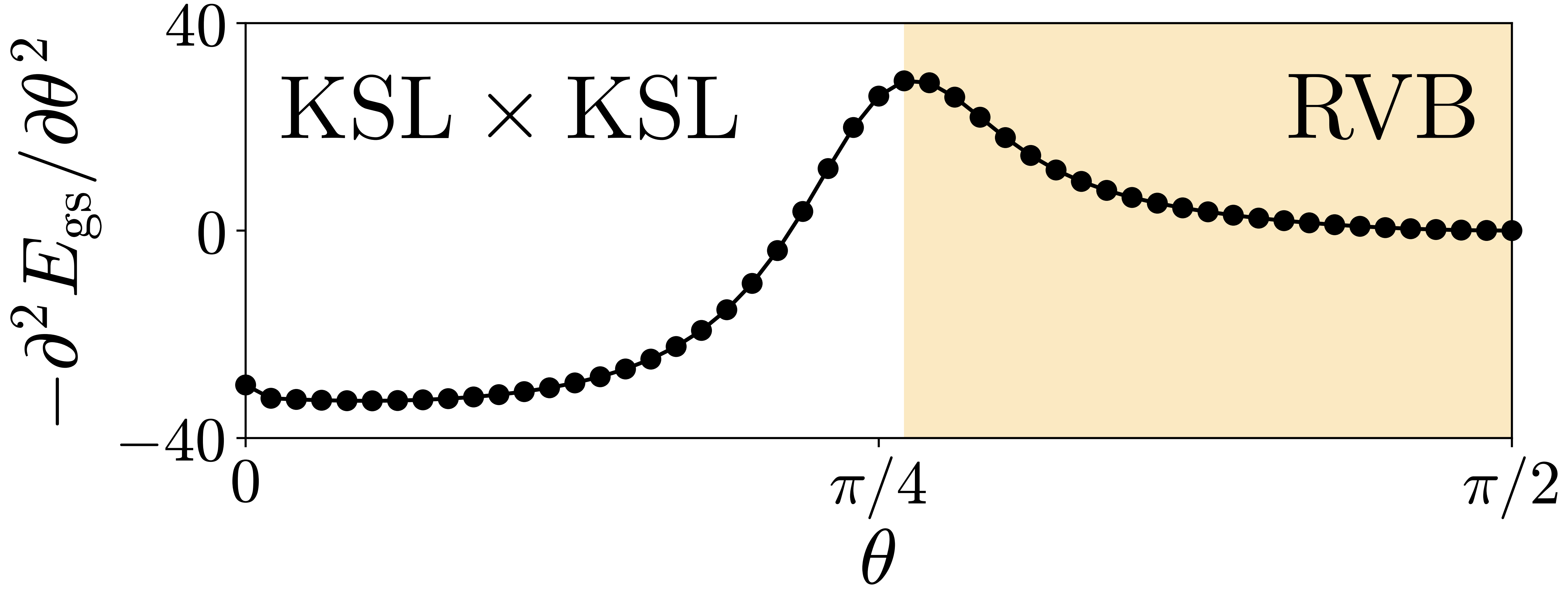
- (24+24)-site ED
- Effective theories

Phase diagram

(24+24)-site bilayer cluster

$$K_\sigma = -K_\tau = \cos\theta$$

$$G = \sin\theta$$



$K \gg G$ ←

→ $K \ll G$

Majorana mean-field theory

$\nu_{\mathrm{upper}} = +1 \quad \nu_{\mathrm{lower}} = -1$

$$\{1_1, \sigma_1, \psi_1\} \times \{1_2, \sigma_2, \psi_2\}$$

Quantum dimer model

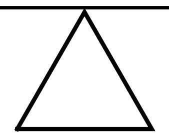
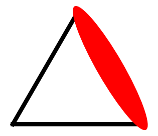
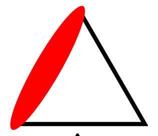
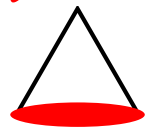
(dual kagome lattice)

$$\{1, e, m, \epsilon\}$$

Large G limit: RVB spin liquid

<u>G-only model:</u>

- Exactly solved
- Hilbert space for dimer model

$\sigma^z \otimes \tau^z$ basis	$\lvert\phi^x, \phi^y, \phi^z\rangle$ basis	Dimer rep.
$\lvert s\rangle = \frac{1}{\sqrt{2}}(\lvert\uparrow\downarrow\rangle - \lvert\downarrow\uparrow\rangle)$	$\lvert -1, -1, -1\rangle$	
$\lvert t_x\rangle = \frac{-1}{\sqrt{2}}(\lvert\uparrow\uparrow\rangle - \lvert\downarrow\downarrow\rangle)$	$\lvert -1, +1, +1\rangle$	
$\lvert t_y\rangle = \frac{i}{\sqrt{2}}(\lvert\uparrow\uparrow\rangle + \lvert\downarrow\downarrow\rangle)$	$\lvert +1, -1, +1\rangle$	
$\lvert t_z\rangle = \frac{1}{\sqrt{2}}(\lvert\uparrow\downarrow\rangle + \lvert\downarrow\uparrow\rangle)$	$\lvert +1, +1, -1\rangle$	

$$H_0 = G \sum_{\langle jk\rangle_\gamma} \phi_j^\gamma \phi_k^\gamma$$

$$\phi_j^\gamma \phi_k^\gamma = -1 \quad (G > 0)$$

Ground state condition

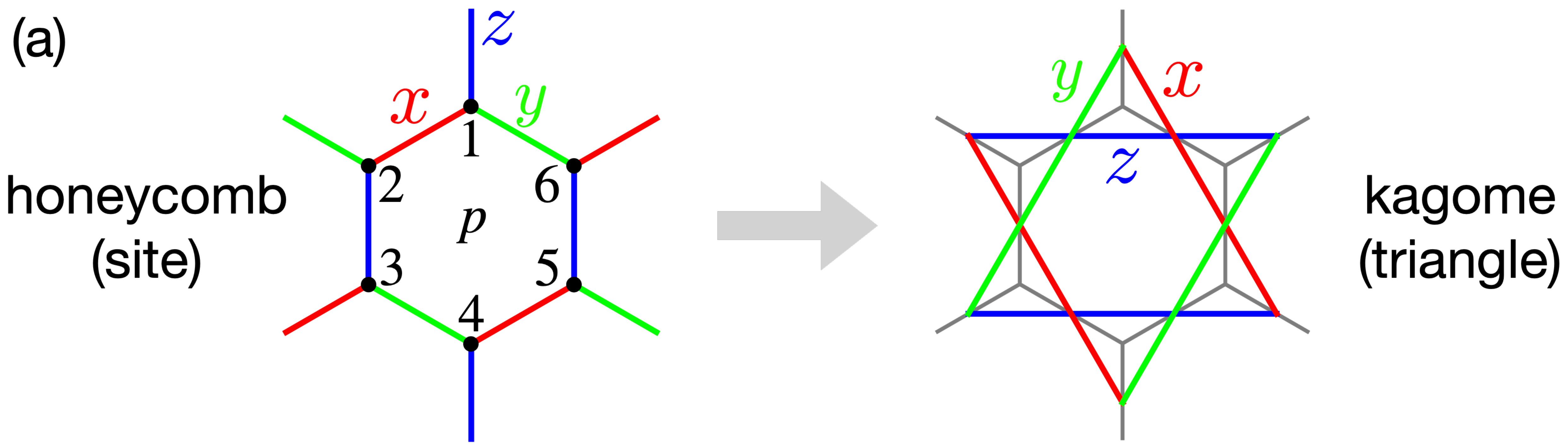
$$\phi_j^\gamma = \sigma_j^\gamma \tau_j^\gamma$$

$$[\phi_j^\gamma, \phi_k^\lambda] = 0 \qquad \phi_j^x \phi_j^y \phi_j^z = -1$$

Large G limit: RVB spin liquid

Dual mapping

(a)



$\sigma^z \otimes \tau^z$ basis	$\lvert\phi^x, \phi^y, \phi^z\rangle$ basis	Dimer rep.
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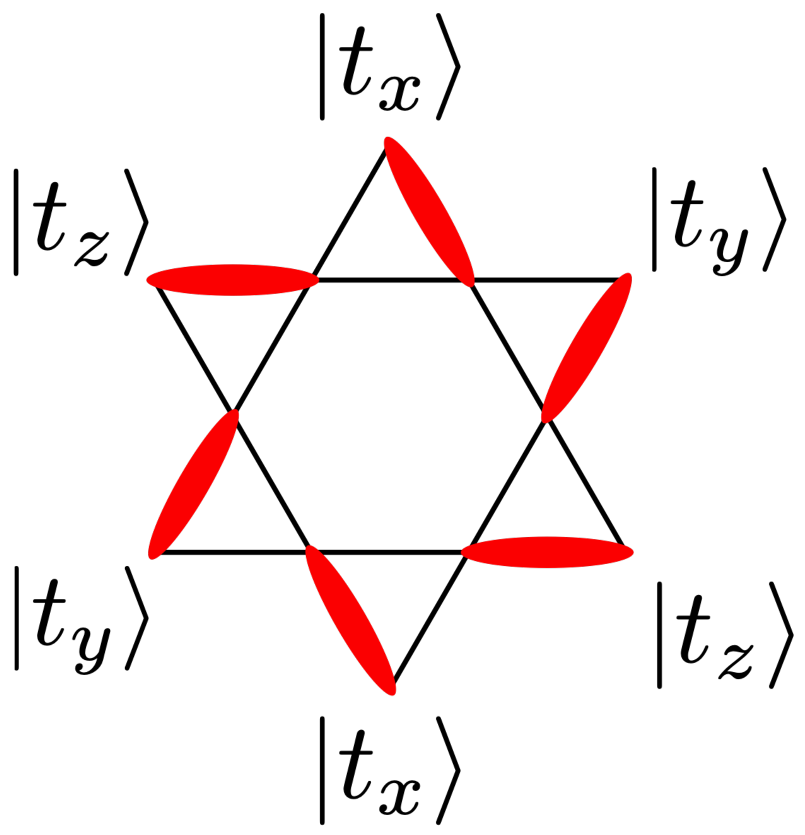
$\lvert t_x\rangle = \frac{-1}{\sqrt{2}}(\lvert\uparrow\uparrow\rangle - \lvert\downarrow\downarrow\rangle)$	$\lvert -1, +1, +1\rangle$	
$\lvert t_y\rangle = \frac{i}{\sqrt{2}}(\lvert\uparrow\uparrow\rangle + \lvert\downarrow\downarrow\rangle)$	$\lvert +1, -1, +1\rangle$	
$\lvert t_z\rangle = \frac{1}{\sqrt{2}}(\lvert\uparrow\downarrow\rangle + \lvert\downarrow\uparrow\rangle)$	$\lvert +1, +1, -1\rangle$	

$$\phi_j^\gamma \phi_k^\gamma = -1 \quad (G > 0)$$

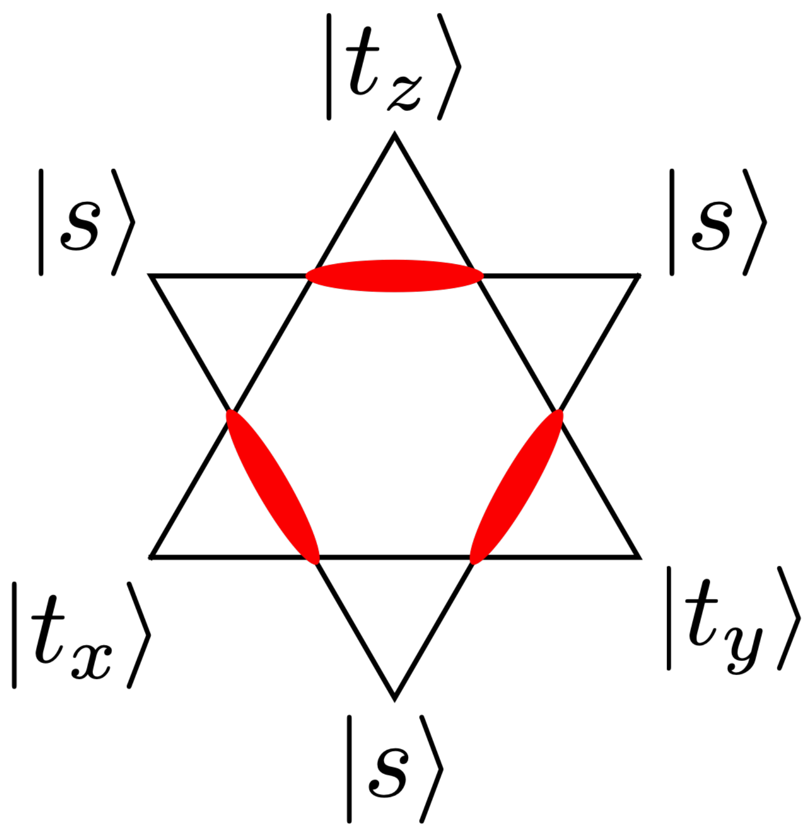
Ground state condition

Hardcore dimer configurations on the dual kagome

(b)



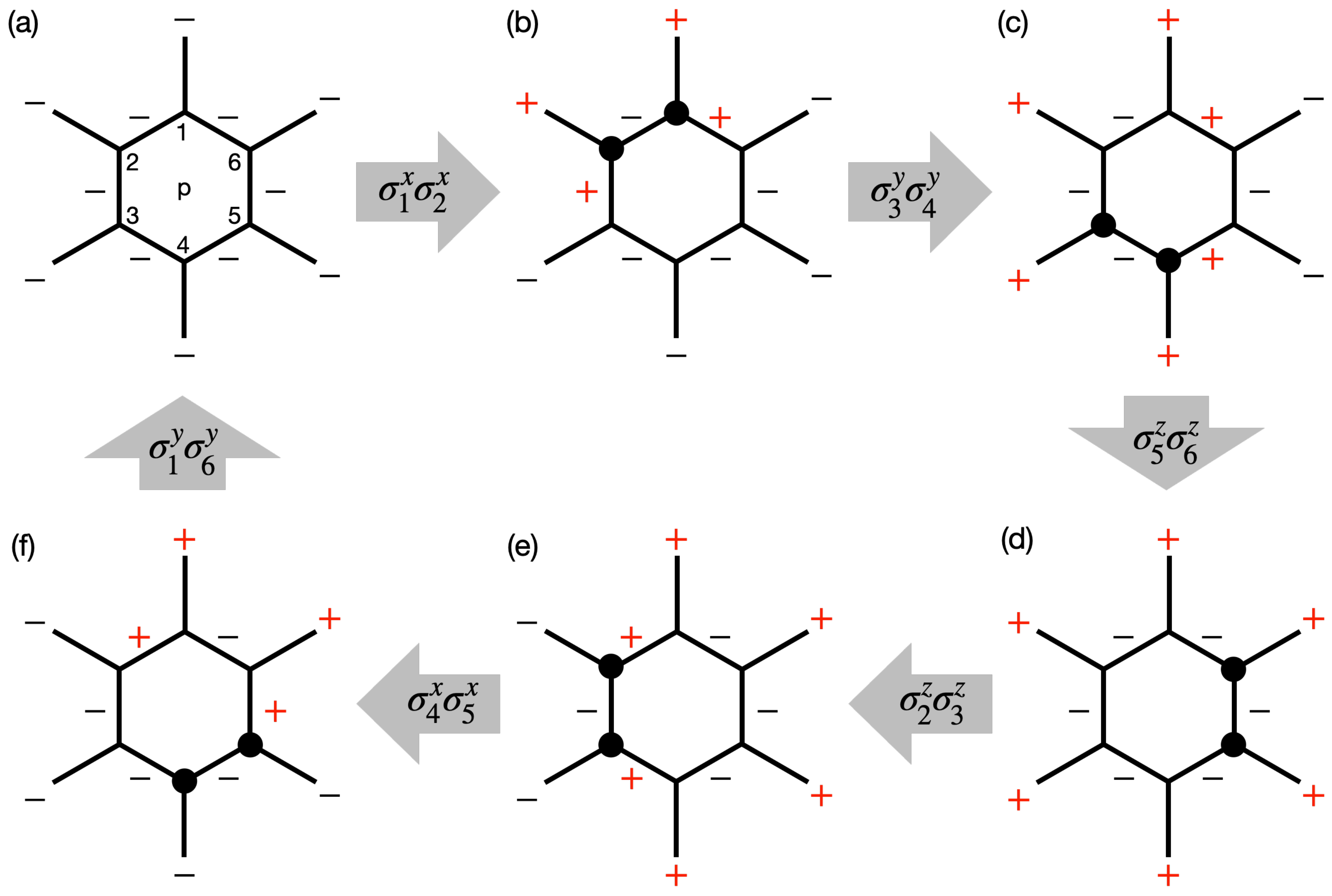
(c)



Large G limit: RVB spin liquid

Sixth order perturbation

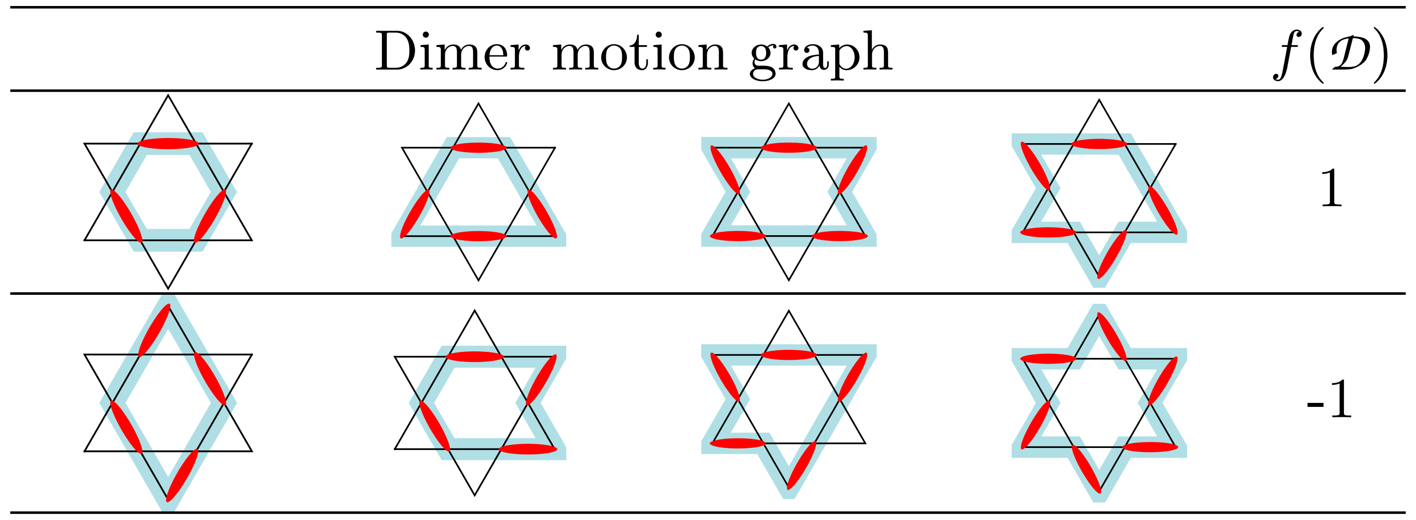
$$\mathcal{H}_{\text{eff}}(G \gg K) = -\lambda \sum_p \hat{W}_p$$



$$\lambda \propto K_\sigma^6/G^5$$

Quantum dimer model

$$\hat{W}_p = \sum_{\mathcal{D}} f(\mathcal{D})|\mathcal{D}\rangle\langle\bar{\mathcal{D}}| + \text{H.c.}$$



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Quantum Dimer Model on the Kagome Lattice: Solvable Dimer-Liquid and Ising Gauge Theory

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(Received 19 April 2002; published 6 September 2002)

Large G limit: RVB spin liquid

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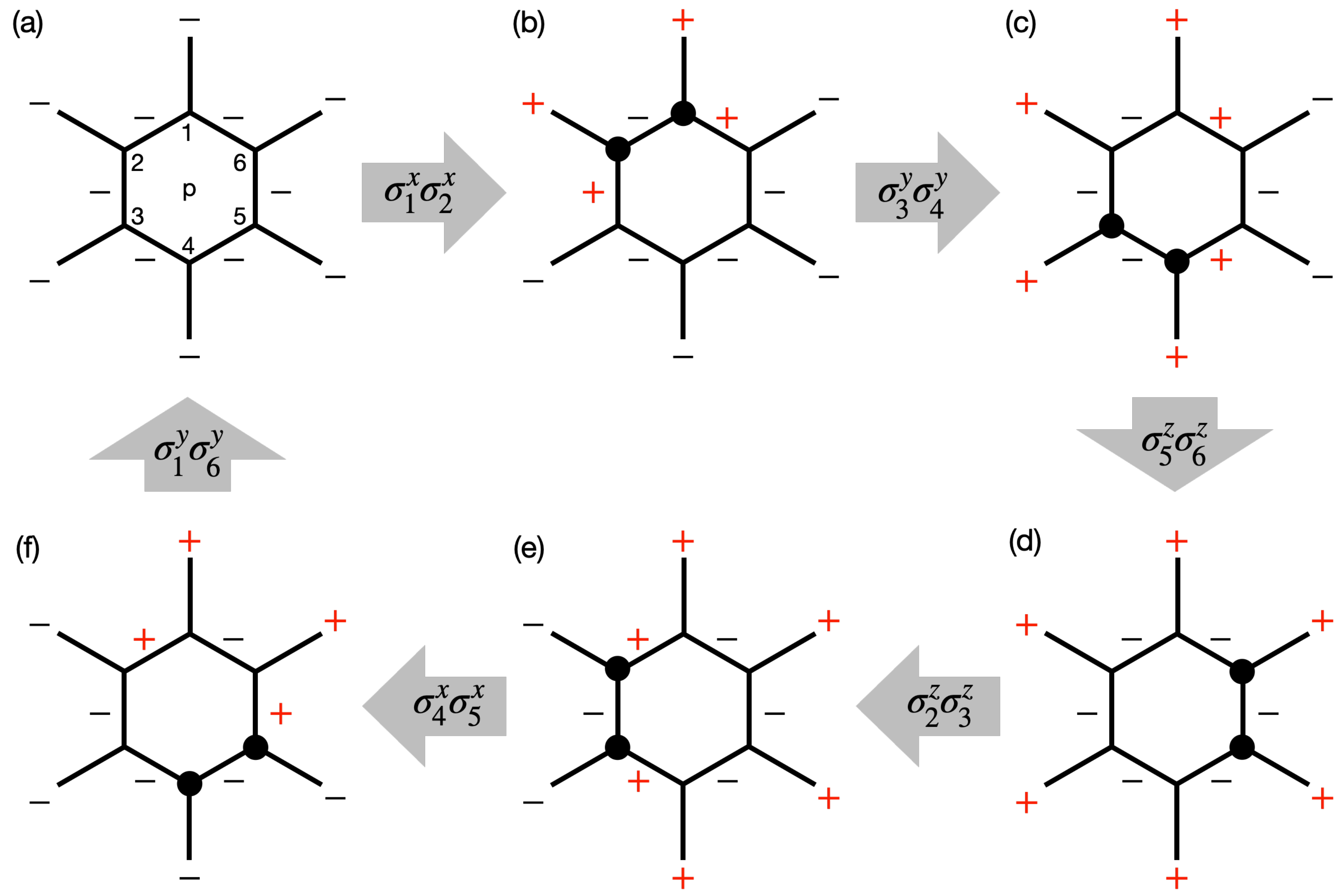

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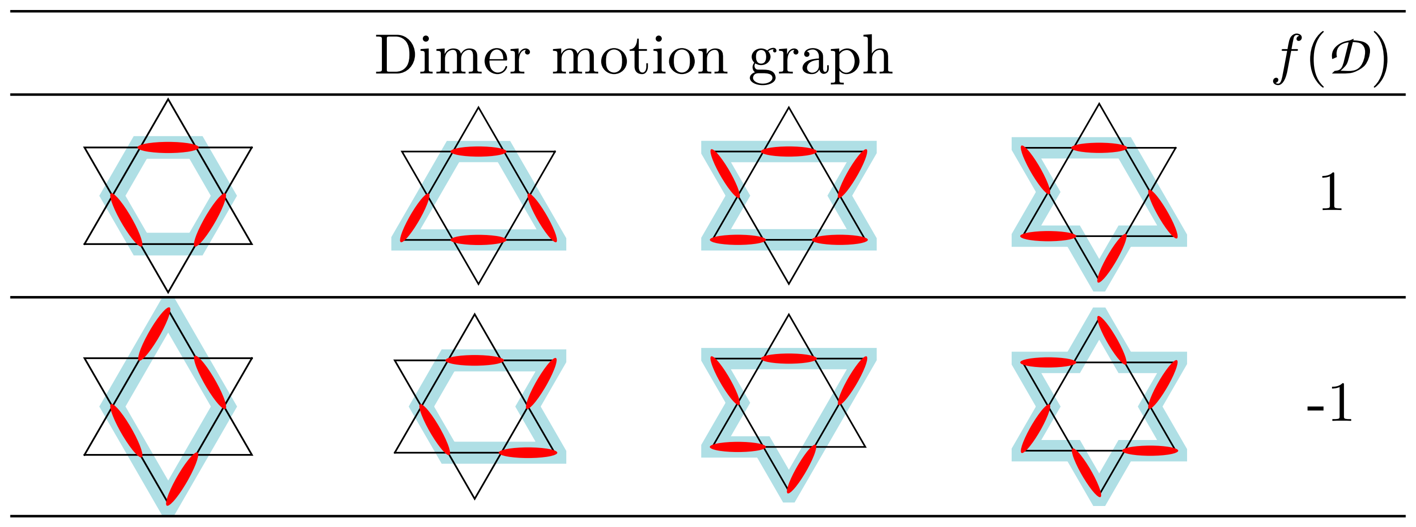

Ground state: RVB spin liquid!!!

$$|\Psi\rangle = \mathcal{N} \prod_p \frac{1+\hat{W}_p}{2} |\Phi\rangle$$

Large G limit: RVB spin liquid

Sixth order perturbation

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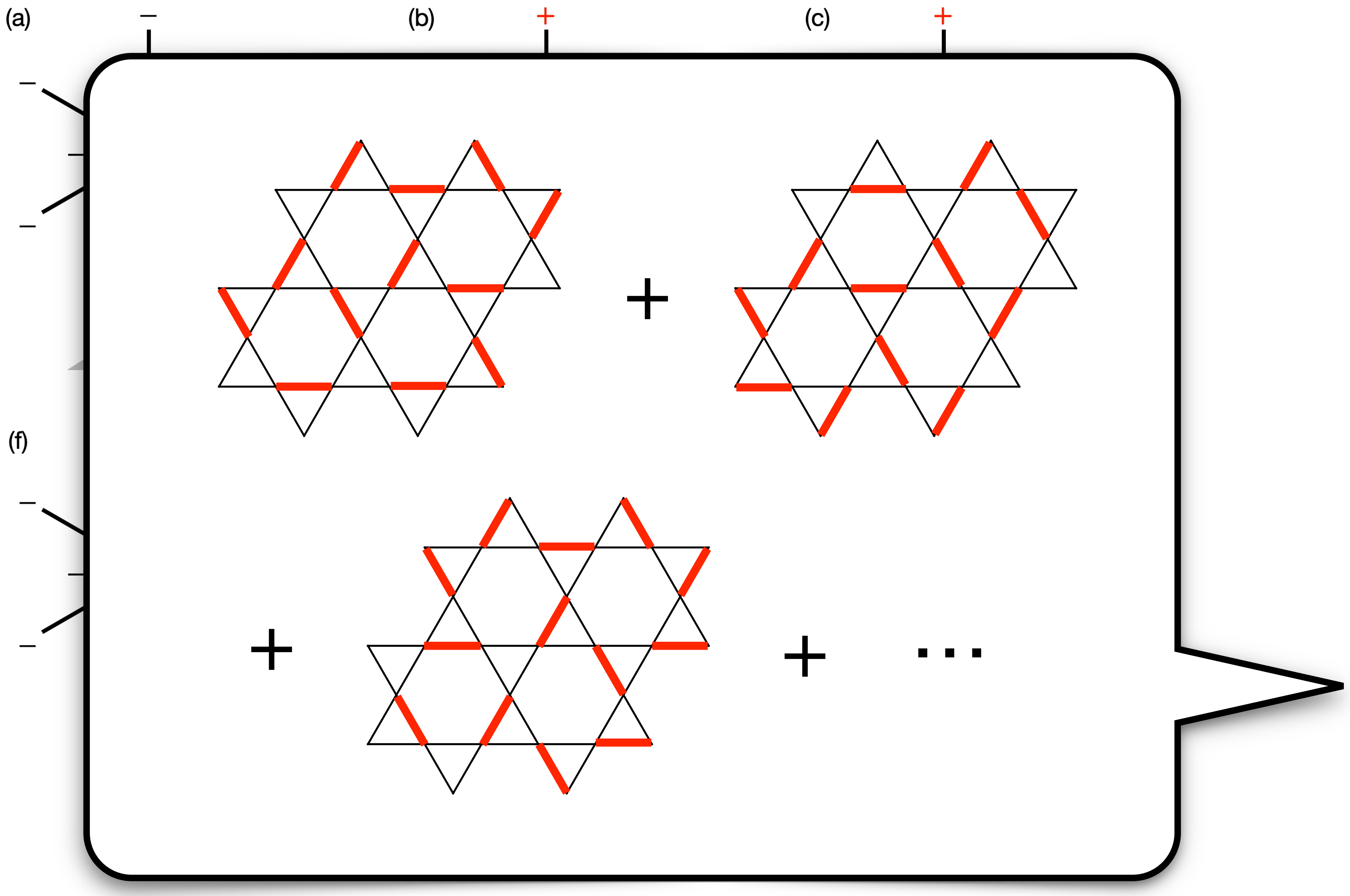

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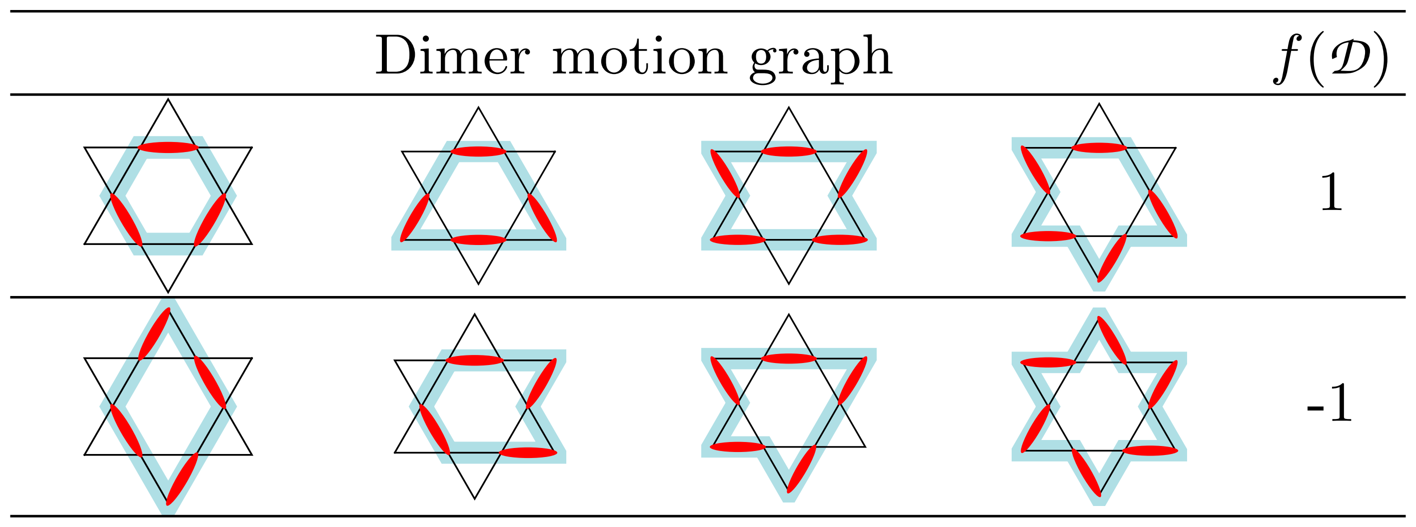

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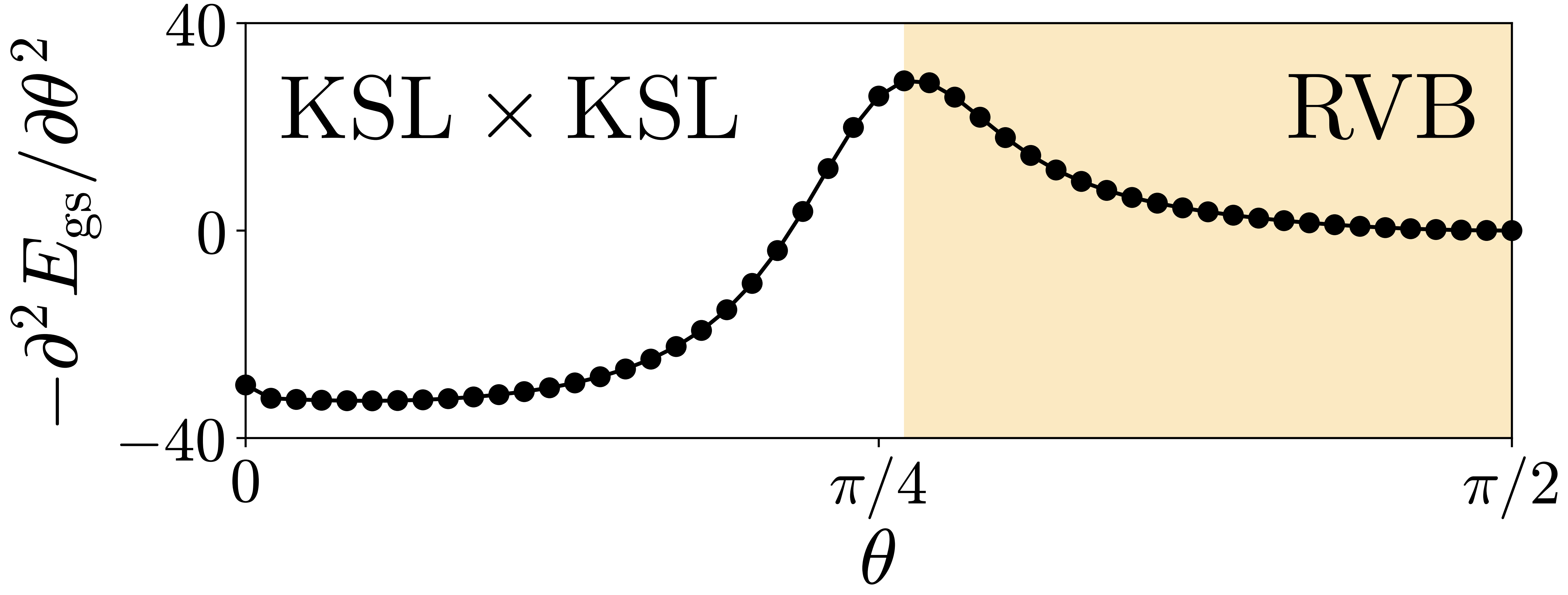
Phase diagram

(24+24)-site bilayer cluster

$$K_\sigma = -K_\tau = \cos\theta$$
$$G = \sin\theta$$



$K \gg G$ ⟵ ⟶ $K \ll G$

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$\nu_{\rm upper} = +1 \qquad \nu_{\rm lower} = -1$

$$\{1_1, \sigma_1, \psi_1\} \times \{1_2, \sigma_2, \psi_2\}$$

Quantum dimer model

(dual kagome lattice)

$$\{1, e, m, \epsilon\}$$

1. Entanglement entropy

(24+24)-site bilayer cluster

$$K_\sigma = -K_\tau = \cos\theta$$

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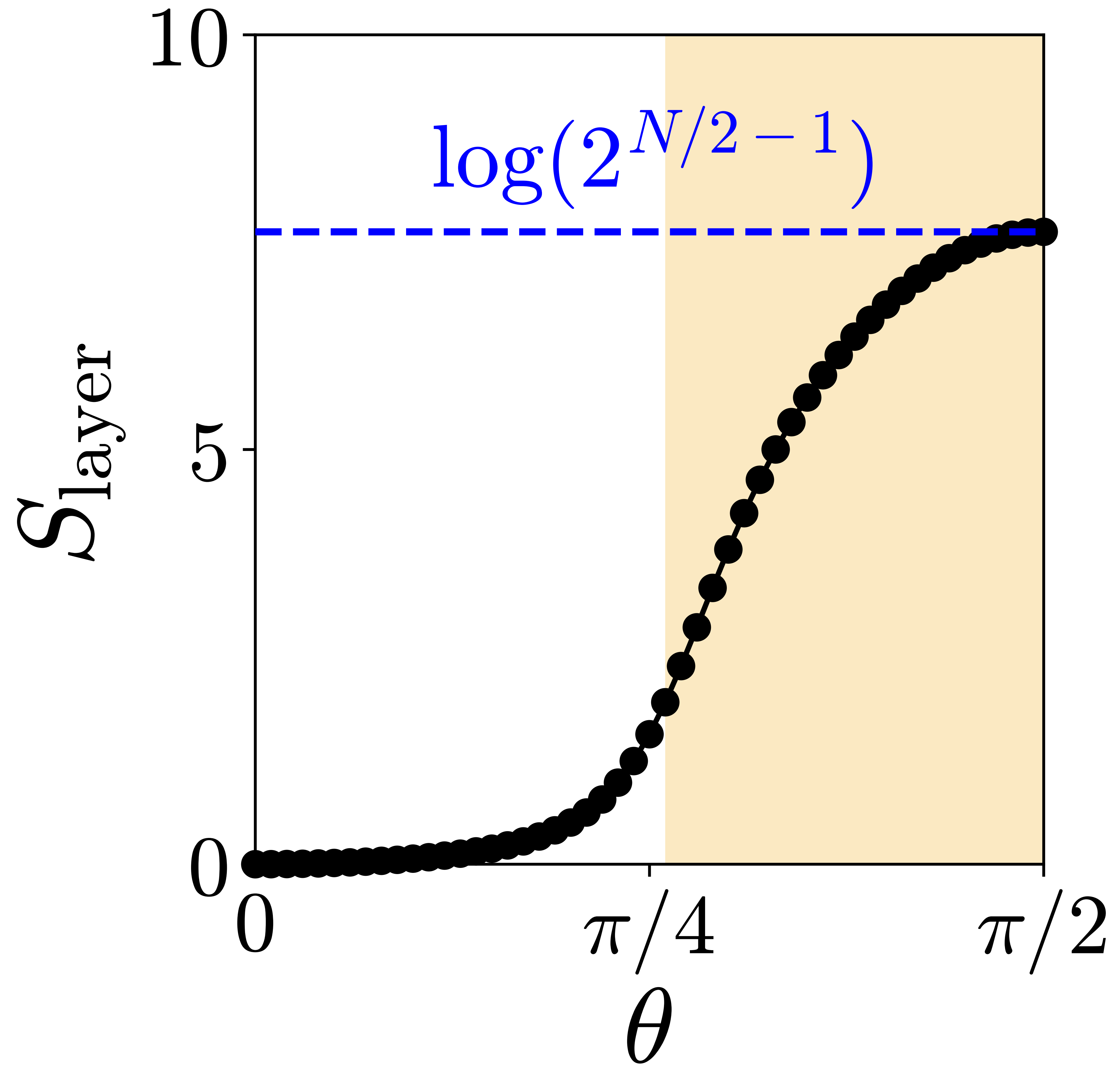


KSL × KSL

$S_{\text{layer}} \approx 0$ Bilayer product state

RVB

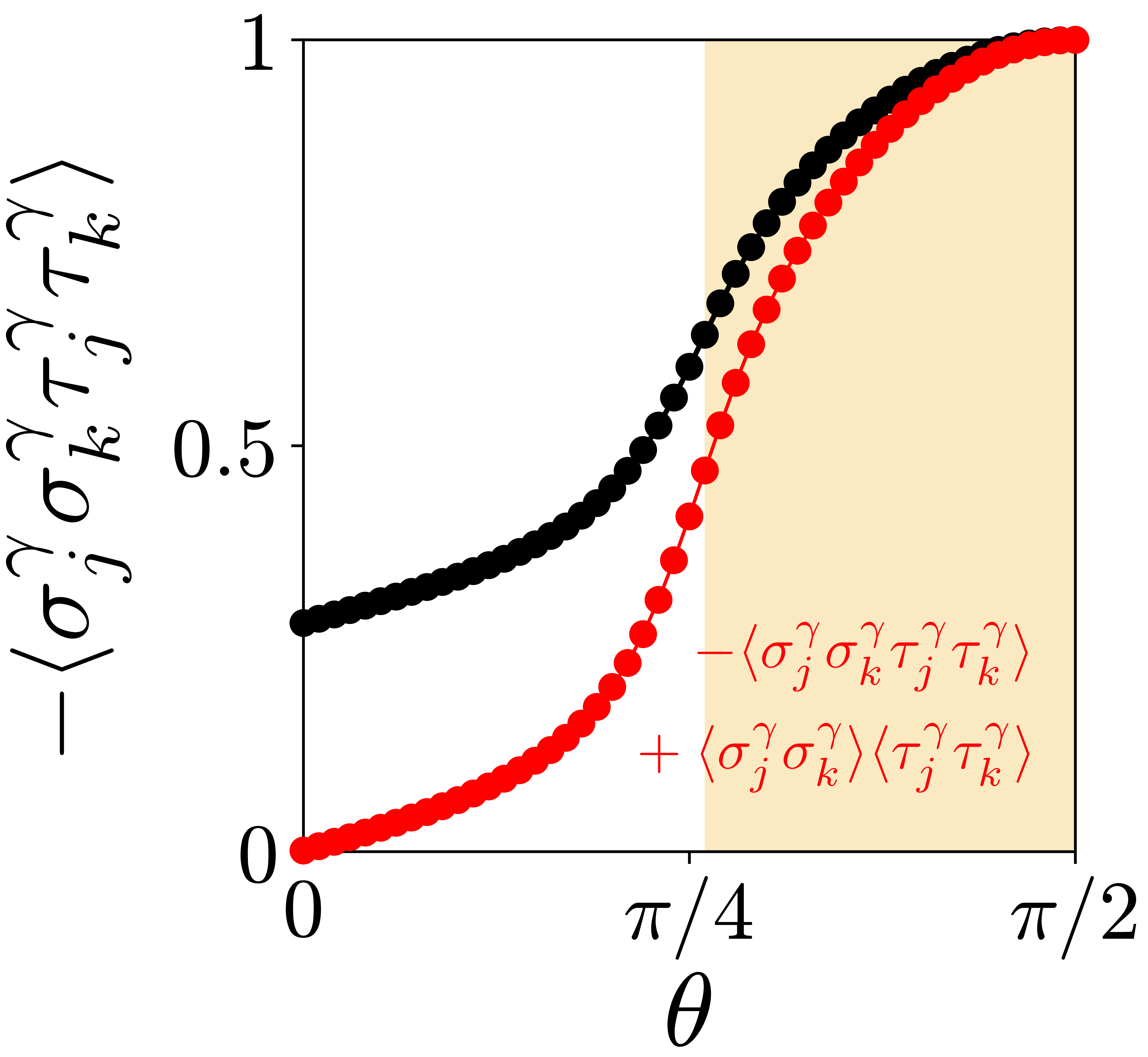
$S_{\text{layer}} \approx \log(2^{N/2-1})$ Dimer Hilbert space

$S_{\text{layer}} = -\log(\mathrm{Tr}_{\{\sigma\}}\rho^2_{\text{layer}})$ $\rho_{\text{layer}} = \mathrm{Tr}_{\{\tau\}}|\psi_{\text{gs}}\rangle\langle\psi_{\text{gs}}|$

2. Spin correlation

(24+24)-site bilayer cluster

$$K_\sigma = -K_\tau = \cos\theta$$
$$G = \sin\theta$$



<u>KSL × KSL</u>

$$\langle \sigma_j^\gamma \sigma_k^\gamma \tau_j^\gamma \tau_k^\gamma \rangle \approx \langle \sigma_j^\gamma \sigma_k^\gamma \rangle \langle \tau_j^\gamma \tau_k^\gamma \rangle$$

<u>RVB</u>

$$\langle \sigma_j^\gamma \sigma_k^\gamma \tau_j^\gamma \tau_k^\gamma \rangle \approx -1 \qquad \langle \sigma_j^\gamma \sigma_k^\gamma \rangle = -\langle \tau_j^\gamma \tau_k^\gamma \rangle \approx 0$$

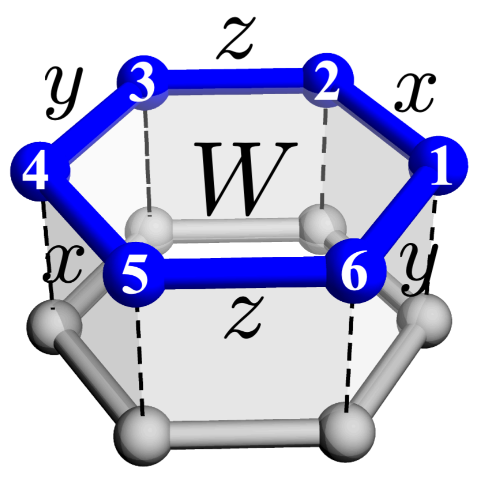
Hardcore dimer constraint

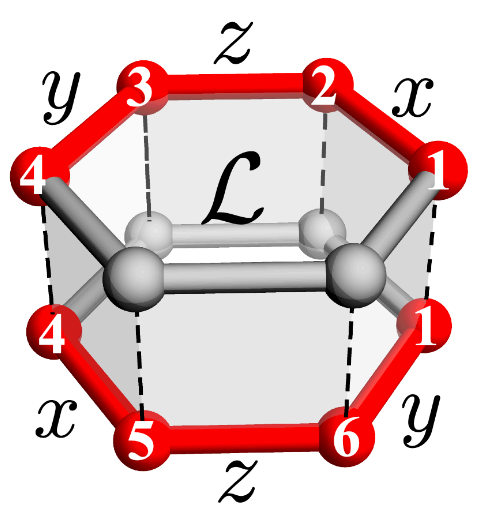
3. Fermion-pair condensation

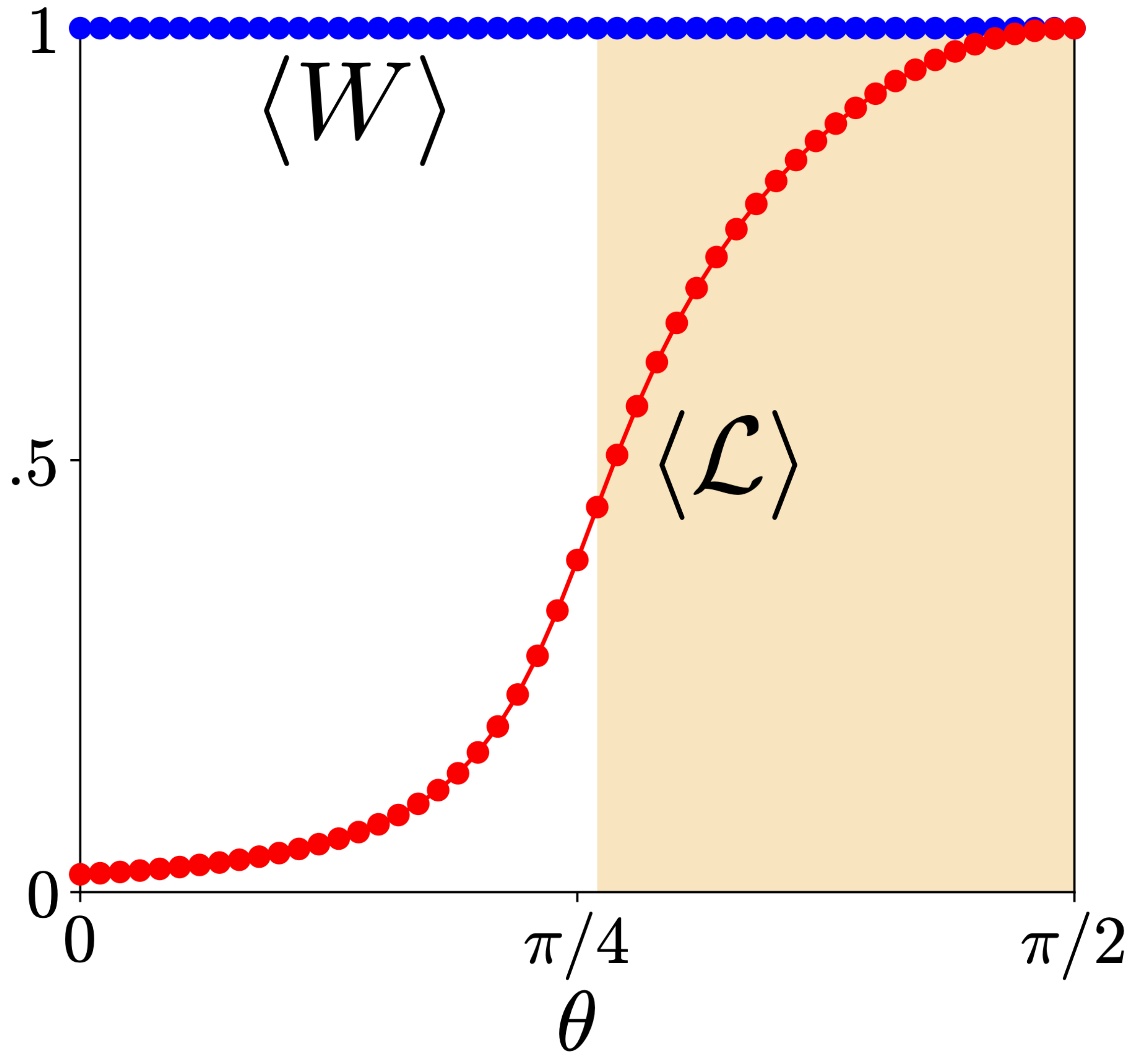
(24+24)-site bilayer cluster

$$K_\sigma = -K_\tau = \cos\theta$$

$$G = \sin\theta$$







<u>KSL × KSL</u>

$\langle \mathcal{L} \rangle \approx 0$ Fermion excitations **cannot** move across the layers

<u>RVB</u>

$\langle \mathcal{L} \rangle \approx 1$ Fermion excitations **can** move across the layers

Fermion-pair condensation! $\langle \psi_1 \psi_2 \rangle \neq 0$

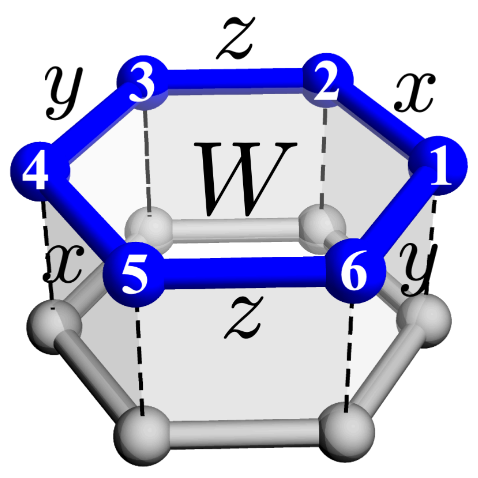
$W = (\sigma_1^y \sigma_6^y)(\sigma_6^z \sigma_5^z)(\sigma_5^x \sigma_4^x)(\sigma_4^y \sigma_3^y)(\sigma_3^z \sigma_2^z)(\sigma_2^x \sigma_1^x)$ $\mathcal{L} = -(\tau_1^y \tau_6^y)(\tau_6^z \tau_5^z)(\tau_5^x \tau_4^x)(\sigma_4^y \sigma_3^y)(\sigma_3^z \sigma_2^z)(\sigma_2^x \sigma_1^x)$ $\sigma_j^\gamma \sigma_k^\gamma = -i c_j c_k$

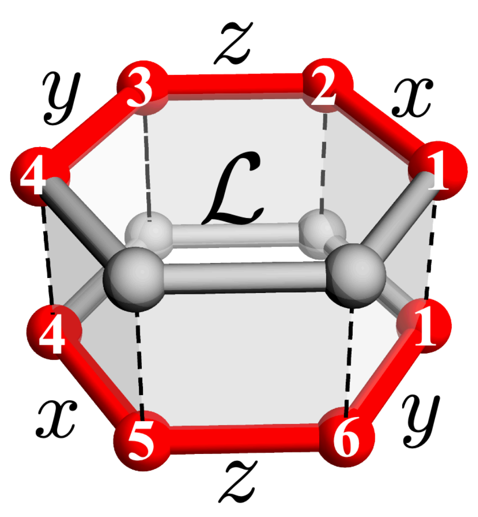
3. Fermion-pair condensation

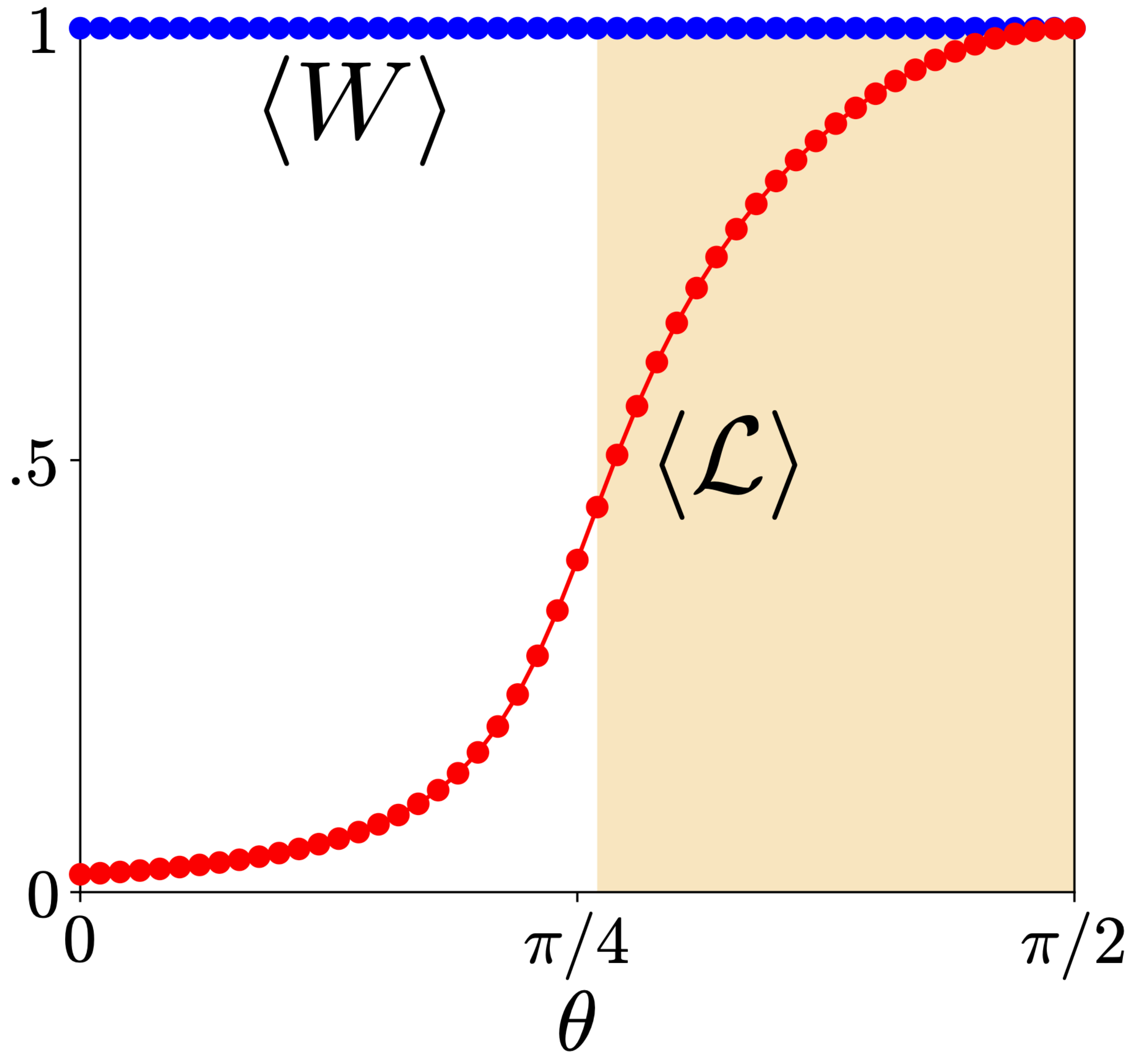
(24+24)-site bilayer cluster

$$K_\sigma = -K_\tau = \cos\theta$$

$$G = \sin\theta$$







$$\sigma_j^\gamma \sigma_k^\gamma \tau_j^\gamma \tau_k^\gamma (= \phi_j^\gamma \phi_k^\gamma) \approx -1$$

Hardcore dimer constraint

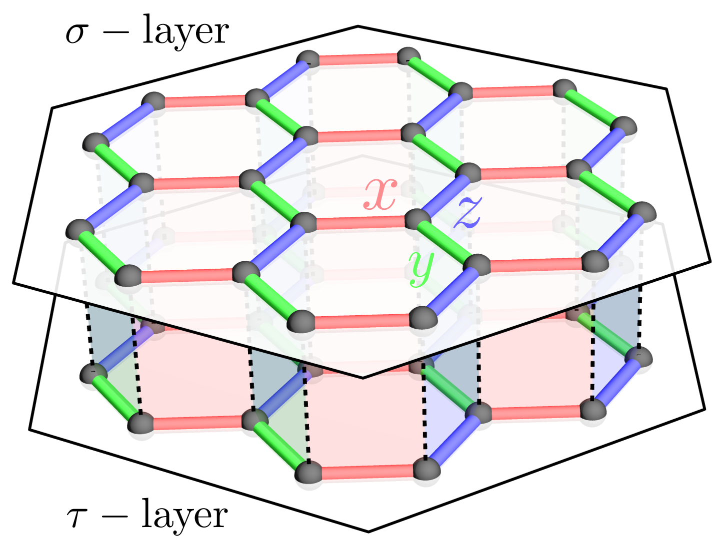
$$\mathcal{L} \approx W \approx Z$$

No distinction b/w ψ_1 & ψ_2

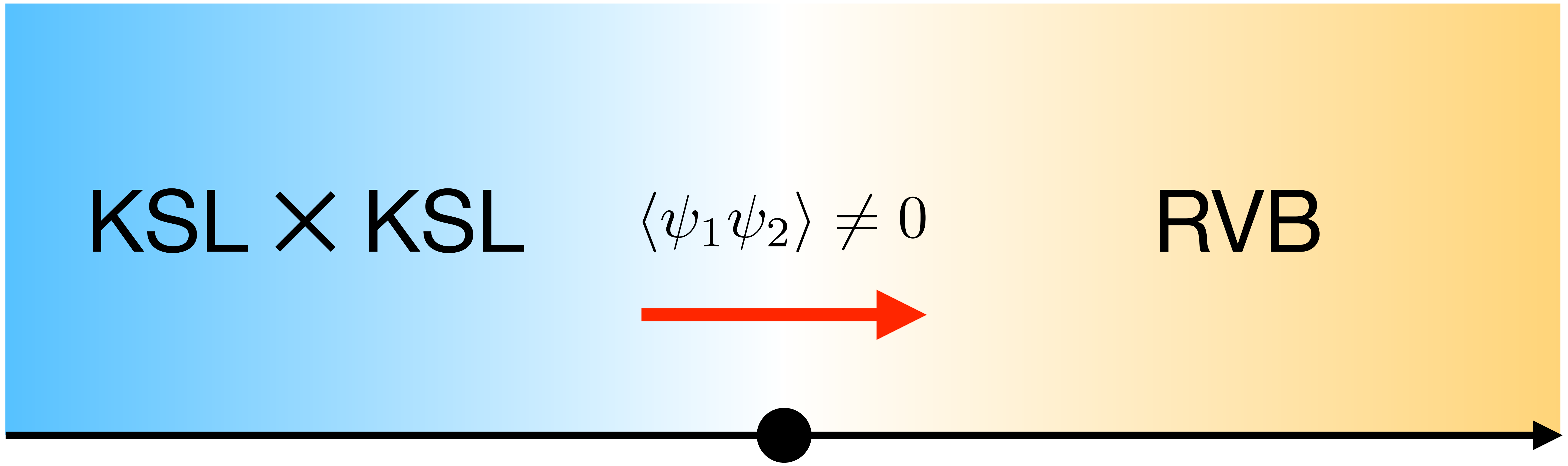
$W = (\sigma_1^y \sigma_6^y)(\sigma_6^z \sigma_5^z)(\sigma_5^x \sigma_4^x)(\sigma_4^y \sigma_3^y)(\sigma_3^z \sigma_2^z)(\sigma_2^x \sigma_1^x)$ $\mathcal{L} = -(\tau_1^y \tau_6^y)(\tau_6^z \tau_5^z)(\tau_5^x \tau_4^x)(\sigma_4^y \sigma_3^y)(\sigma_3^z \sigma_2^z)(\sigma_2^x \sigma_1^x)$ $\sigma_j^\gamma \sigma_k^\gamma = -ic_j c_k$

Anyon condensation topological transition

arXiv:2301.05721



$$H = K_\sigma \sum_{\langle jk \rangle_\gamma}^{\text{upper layer}} \sigma_j^\gamma \sigma_k^\gamma + K_\tau \sum_{\langle jk \rangle_\gamma}^{\text{lower layer}} \tau_j^\gamma \tau_k^\gamma + G \sum_{\langle jk \rangle_\gamma}^{\text{inter-layer}} \sigma_j^\gamma \sigma_k^\gamma \tau_j^\gamma \tau_k^\gamma$$



$\text{Ising} \times \overline{\text{Ising}}$ $\mathbb{Z}_2$ toric code

$\{1_1, \sigma_1, \psi_1\} \times \{1_2, \sigma_2, \psi_2\}$ $\{1, e, m, \epsilon\}$

Burnell, “Anyon Condensation and Its Applications,” Annu. Rev. Condens. Matter Phys. 9, 307–327 (2018).