Long-lived solitons and anomalous dynamics in the classical Heisenberg chain

ct.qmat
Complexity and Topology in Quantum Matter

Julius-Maximilians-UNIVERSITÄT WÜRZBURG

TECHNISCHE UNIVERSITÄT DRESDEN

Adam McRoberts[1], Thomas Bilitewski[1,2], Masudul Haque[1,3,4], Roderich Moessner[1]

[1]**Max Planck Institute for the Physics of Complex Systems**, Nöthnitzer Strasse 38, 01187 Dresden, Germany
[2]Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078, USA
[3]Department of Theoretical Physics, Maynooth University, County Kildare, Ireland
[4]Institut für Theoretische Physik, Technische Universität Dresden, 01062 Dresden, Germany

Abstract

The search for departures from standard hydrodynamics in many-body systems has yielded a number of promising leads, especially in low dimensions. Here, we study one of the simplest classical interacting lattice models, the nearest-neighbour Heisenberg chain, and find a long-lived regime of Kardar-Parisi-Zhang (KPZ) scaling at low temperatures in the ferromagnet. Motivated by this, we investigate the role of solitonic excitations in this model. We find that the Heisenberg chain, although well known to be non-integrable, supports a two-parameter family of long-lived solitons. We connect these to the exact soliton solutions of the integrable Ishimori chain with $\log(1 + S_i \cdot S_j)$ interactions. We provide an adiabatic construction procedure, which shows that Ishimori solitons have a long-lived Heisenberg counterpart when they are not too narrow and not too fast-moving. Finally, we demonstrate their presence in thermal states of the Heisenberg chain, even when the typical soliton width is larger than the spin correlation length, and argue that these excitations likely underlie the KPZ scaling [1, 2].

Classical spin dynamics

Heisenberg chain	Ishimori chain (integrable)	Equations of motion
$\mathcal{H} = -J\sum_i \boldsymbol{S}_i\cdot\boldsymbol{S}_{i+1}$	$\mathcal{H} = -2J\sum_i \log\left(\frac{1+\boldsymbol{S}_i\cdot\boldsymbol{S}_{i+1}}{2}\right)$	$\dot{\boldsymbol{S}}_j = \frac{\partial H}{\partial \boldsymbol{S}_j}\times\boldsymbol{S}_j$

We consider the classical Heisenberg chain, and, by comparison, the integrable Ishimori chain. We transform smoothly between the chains with the interpolating Hamiltonian,

$$\mathcal{H} = -2J\gamma^{-1}\sum_i \log\left(1+\gamma\frac{\boldsymbol{S}_i\cdot\boldsymbol{S}_{i+1}-1}{2}\right)$$

with the classical (Landau-Lifshitz) equations of motion:

$$\dot{\boldsymbol{S}}_i = 2\boldsymbol{S}_i\times\left(\frac{\boldsymbol{S}_{i-1}}{2-\gamma+\gamma\boldsymbol{S}_i\cdot\boldsymbol{S}_{i-1}}+\frac{\boldsymbol{S}_{i+1}}{2-\gamma+\gamma\boldsymbol{S}_i\cdot\boldsymbol{S}_{i+1}}\right)$$

KPZ scaling at low temperature

We start by considering the dynamics of the Heisenberg chain in thermal equilibrium. We use combined Monte-Carlo and molecular dynamics simulations to evaluate the connected correlators of the conserved quantities,

$$\mathcal{C}^S(j,t) = \langle \boldsymbol{S}_j(t)\cdot\boldsymbol{S}_0(0)\rangle \qquad \mathcal{C}^E(j,t) = \langle E_j(t)E_0(0)\rangle - \langle\mathcal{E}^2\rangle, \quad E_j = -J\boldsymbol{S}_j\cdot\boldsymbol{S}_{j+1}$$

The generic expectation is that these correlators follow a diffusive scaling form,

$$\mathcal{C}(x,t) = \frac{\kappa}{(Dt)^\alpha}\exp\left[-\left(\frac{x}{(Dt)^\alpha}\right)^2\right], \quad \alpha = 1/2$$

though, exceptions are possible, and, more generally,

$$\mathcal{C}(x,t)\sim t^{-\alpha}\mathcal{F}(t^{-\alpha}x)$$

The most well-known anomalous scaling is the KPZ universality class, with $\alpha = 2/3$

We find that the energy correlations are diffusive, but, at low temperatures in the ferromagnet, the spin correlations follow the KPZ scaling form (Fig. 1).

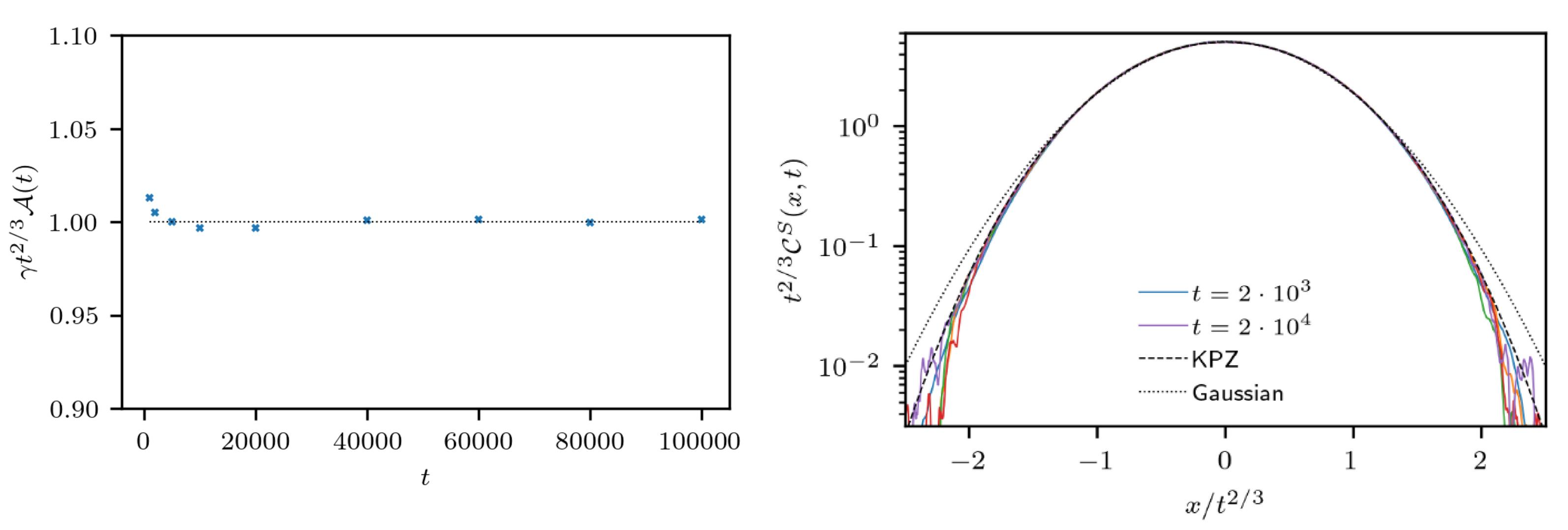


Fig. 1. KPZ scaling at low temperatures (T = 0.2 J) in the classical Heisenberg ferromagnet. In the left-hand panel we show that the autocorrelation function (A(t) = C(0, t)) is well-converged to the KPZ exponent at least up to t = 10^5/J. In the right-hand panel we show that the correlations at different times collapse onto the KPZ scaling function (and do not follow a Gaussian scaling). The system size is L = 8192, and the correlations are averaged over a thermal ensemble of 20,000 initial states.

In integrable spin chains, KPZ scaling arises because a complete set of quasiparticles (the solitons – wave-packets that do not dissipate) are protected by the extensive number of conserved quantities.

This, *a priori*, does not apply to the Heisenberg chain – so, why do we observe KPZ scaling?

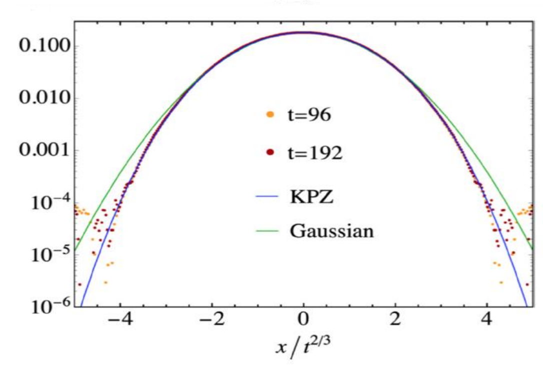


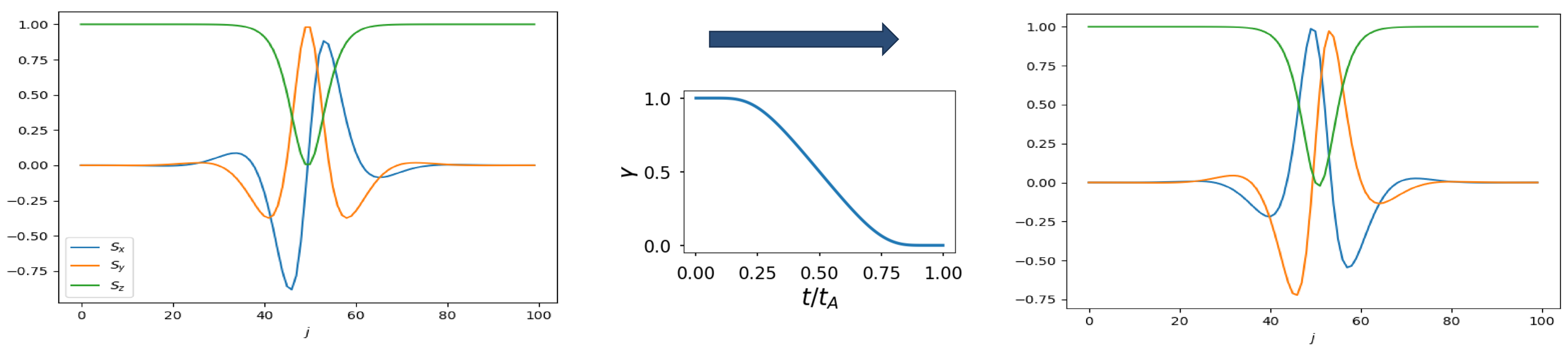
Fig. 2. From [3]. KPZ scaling in the Ishimori chain.

Key questions

I. Are there solitons in the classical Heisenberg chain?
II. Are these solitons stable to collisions?
III. Are the (low-temperature) thermal states comprised mostly of solitons?

Adiabatic time-evolution

Ishimori solitons are parameterised by an inverse-width R and wave-number k; we transform these to solitons of the Heisenberg chain adiabatically.



Ishimori soliton

Fig. 3. Adiabatic transformation.

Heisenberg soliton

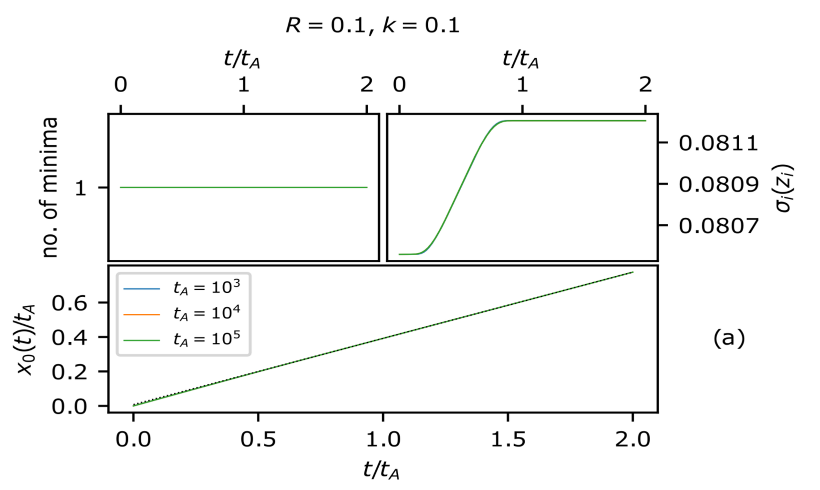


Fig. 4. Criteria for adiabatic continuity – no additional minima are produced, the profile does not spread after the transformation, and the soliton moves with a constant velocity. These conclusions are independent of the adiabatic time t_A.

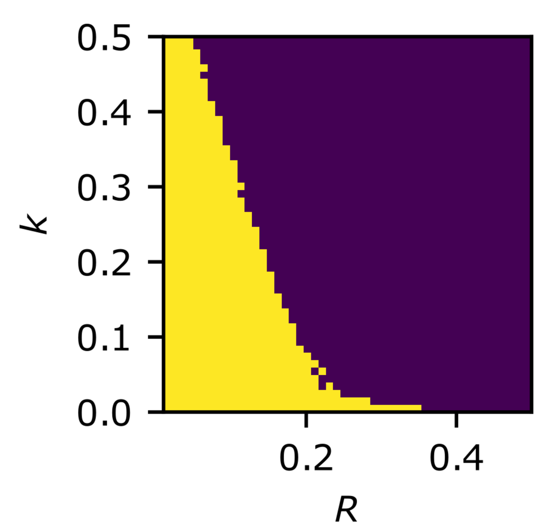
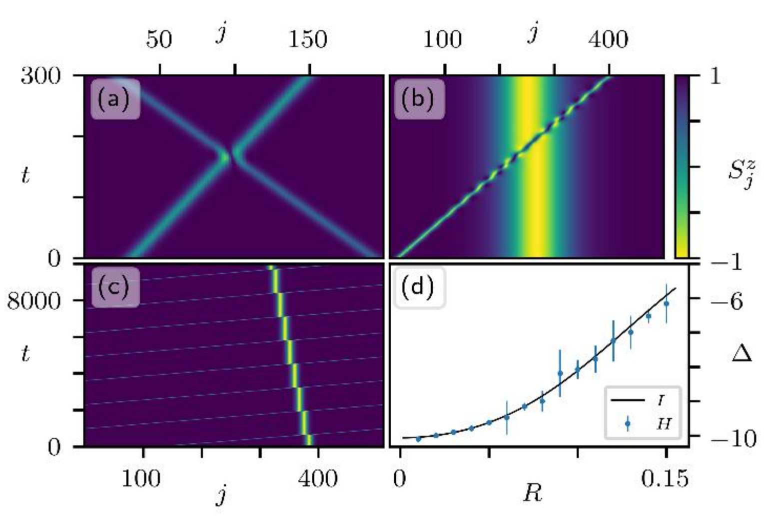


Fig. 5. Existence diagram of solitons in the Heisenberg chain – the soliton survives the transformation if it is not too narrow and not too fast-moving.

Scattering



We find the soliton scattering is broadly similar to the integrable case – the solitons are stable, and retain their velocity after collision, with a slightly shifted trajectory.

Fig. 6. Soliton scattering in the Heisenberg chain. Colour-scale is the same for (a)–(c). (a) Single scattering event between two solitons with parameters (*R, k*) = (0.1, 0.1) and (*R, k*) = (0.1,−0.15). (b) Screening of the magnetisation transported by a narrower soliton as it moves through a wider soliton. (c) Repeated scattering of two solitons [(*R, k*) = (0.1, 0.1) and (*R, k*) = (0.1, 0)] under periodic boundary conditions. (d) Comparison of the scattering shift (*R, k* = 0; 0.1, 0.1) in the Ishimori chain (solid line) and in the Heisenberg chain, for stationary target solitons.

Solitons in thermal states

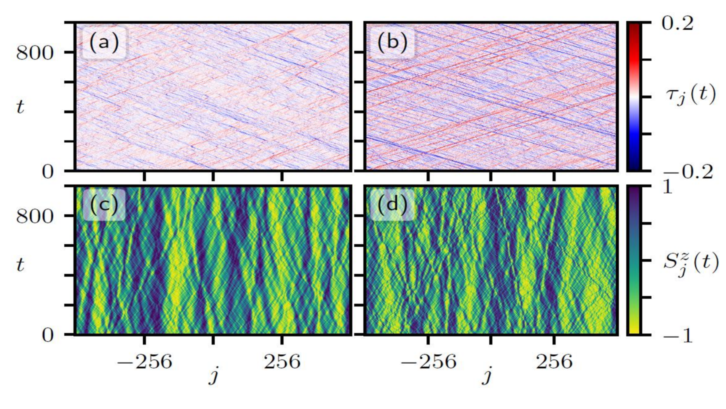


Fig. 7. Ballistic trajectories of solitons in thermal states for the Heisenberg (left) and Ishimori (right) chains. T = 0.1 J.

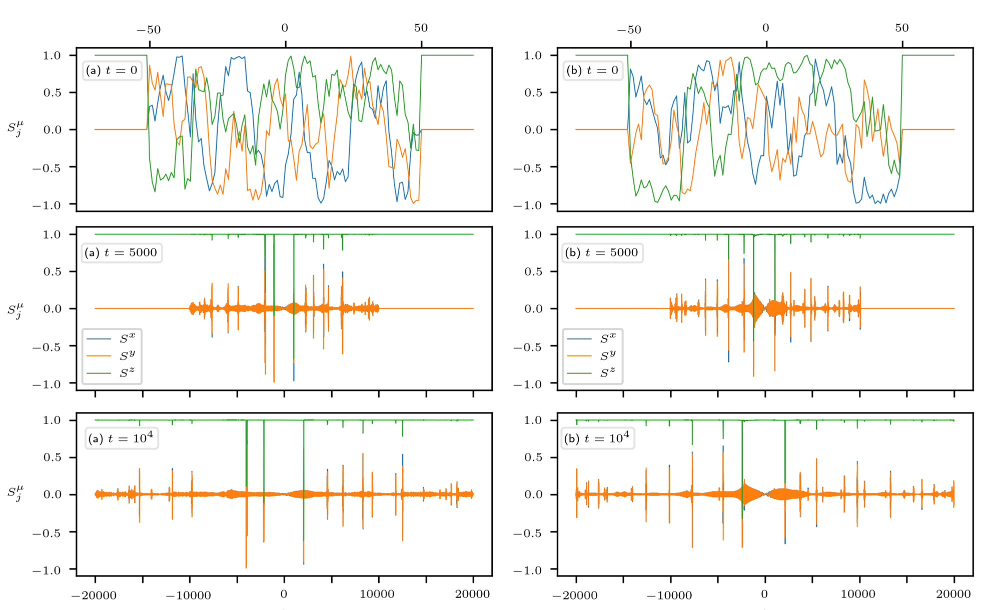


Fig. 8. Inverse-scattering – an initial thermal state separates into its constituent solitons when allowed to expand into a ferromagnetic vacuum.

We find, by measuring the first non-trivial conserved quantity of the Ishimori chain, that the thermal states contain significant soliton content.

Conclusions

- We find apparently stable solitons in a non-integrable lattice model, the classical Heisenberg chain.
- The low-temperature thermal states are comprised mainly of solitons.
- This underlies the remarkably long-lived regime of KPZ scaling in, arguably, the simplest model of magnetism.

mpipks

References
1. PRB 105, L100403 (2022)
2. PRE 106, L062202 (2022)
3. PRE 100, 042116 (2019)