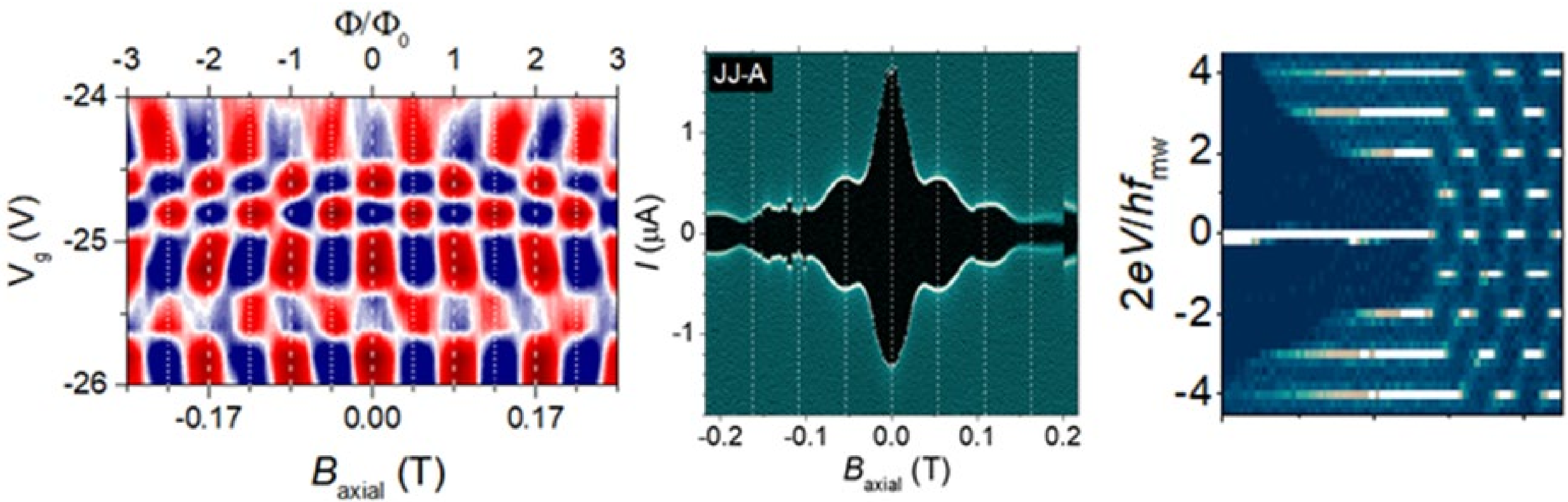
# Fractional Josephson effects in Dirac materials

**Yong-Joo Doh**
Department of Physics & Photon Science,
Gwangju Institute of Science and Technology (GIST)




Gwangju Institute of Science and Technology

June 28, 2023

# Content & keywords



# Motivation

# Josephson effect in topological insulator nanoribbons

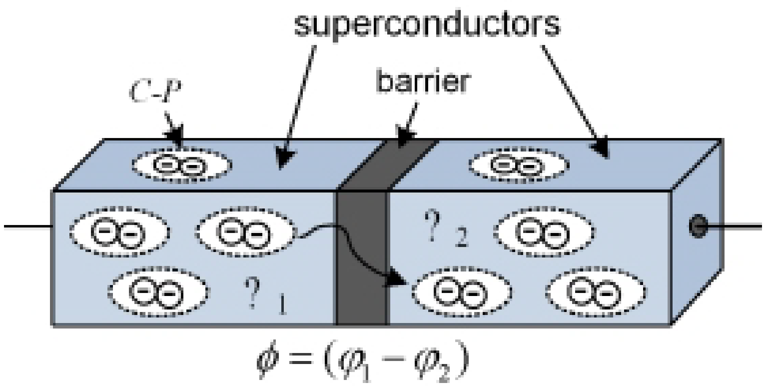
# 2DEG-based Josephson junctions

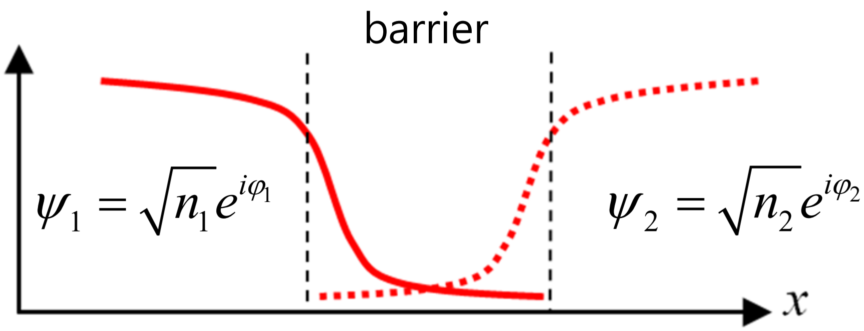
# Josephson effect in Dirac semi-metal nanowires

# Conclusion

# Josephson effect





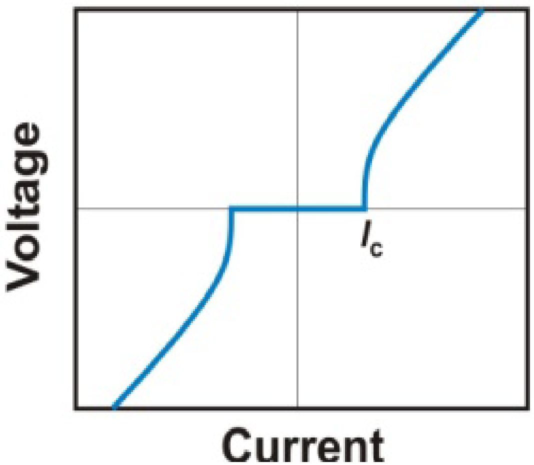

$\psi_1\,(\psi_2)$: probability amplitude to find a Cooper pair in the left (right) electrode

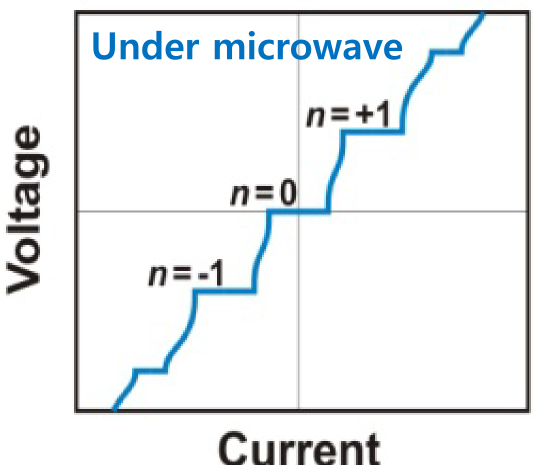
B. D. Josephson

Nobel prize (1973)

$$I_S = I_C \sin \Delta\varphi$$

$$\frac{d(\Delta\varphi)}{dt} = \frac{2eV}{\hbar}$$

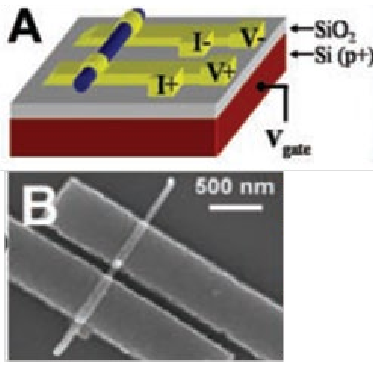


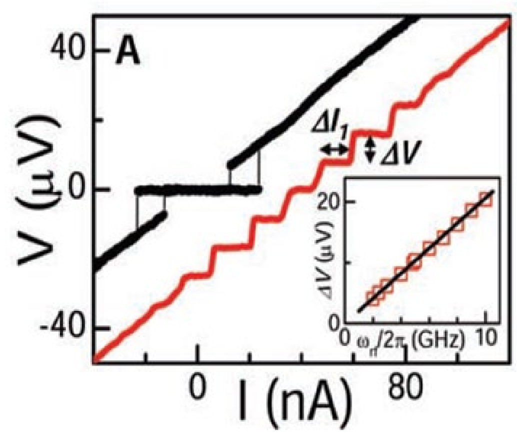




Shapiro steps ~ $V_n = n\frac{hf_{MW}}{2e}$, $n$ = Integer

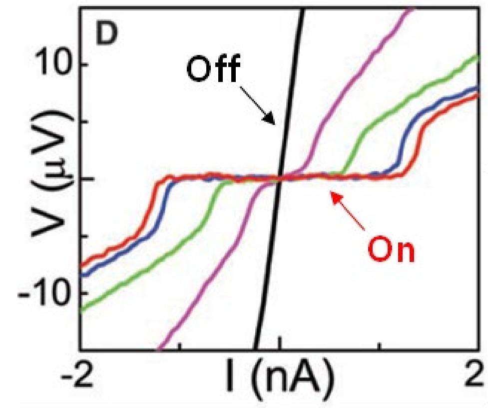
# Nano-hybrid Josephson junctions

## InAs nanowire



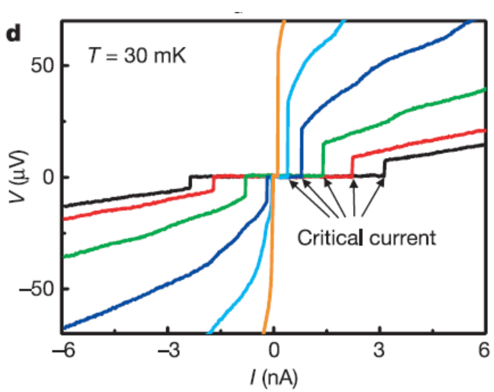




"supercurrent transistor"



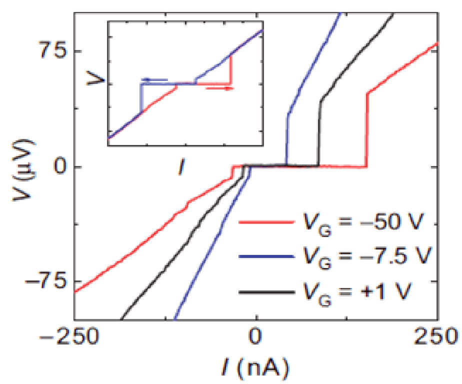

Y.-J. Doh et al.,
Science (2005)

## Carbon nanotube



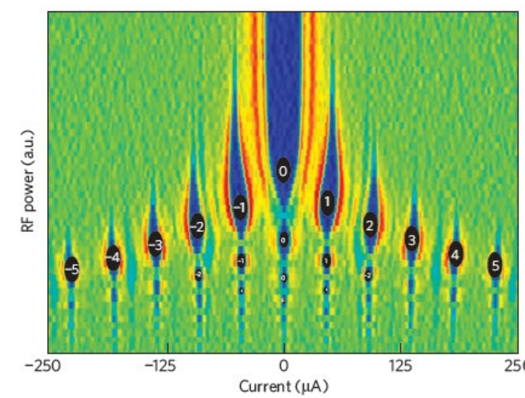

Jarillo-Herrero et al.,
Nature (2006)

## Graphene



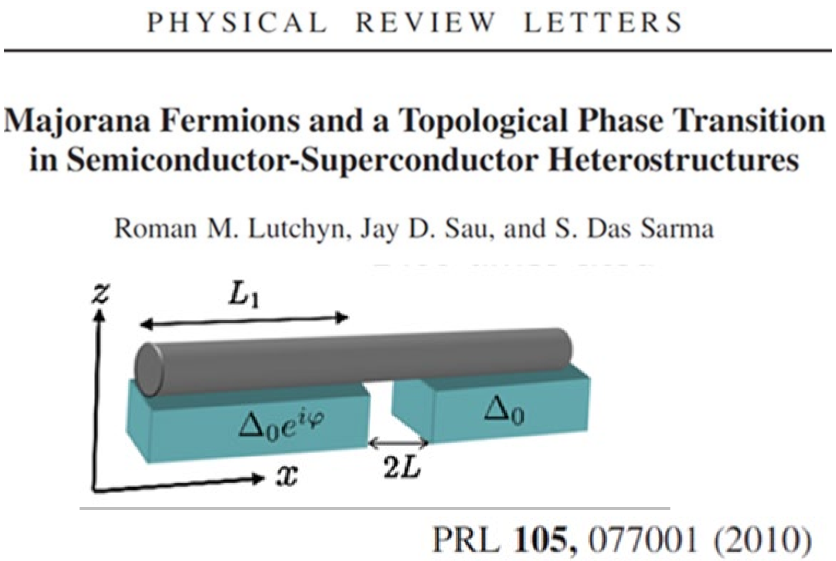

H. B. Heersche et al.,
Nature (2007)

## $Bi_2Te_3$ topological insulator




Brinkman group (Twente)
Nature Materials (2012)

# Majorana Bound States


PHYSICAL REVIEW LETTERS

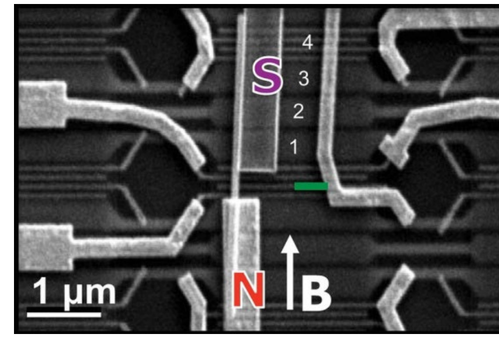
**Majorana Fermions and a Topological Phase Transition in Semiconductor-Superconductor Heterostructures**

Roman M. Lutchyn, Jay D. Sau, and S. Das Sarma



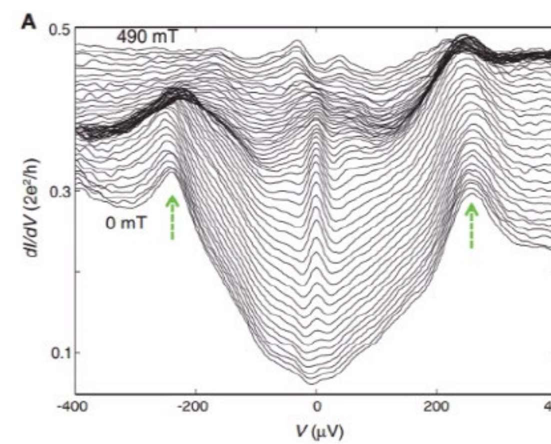
PRL **105**, 077001 (2010)

Ettore Majorana (1906-1938)




**InSb NW with NbTi contacts**

Kouwenhoven group at TU Delft




Mourik et al., Science (2012)

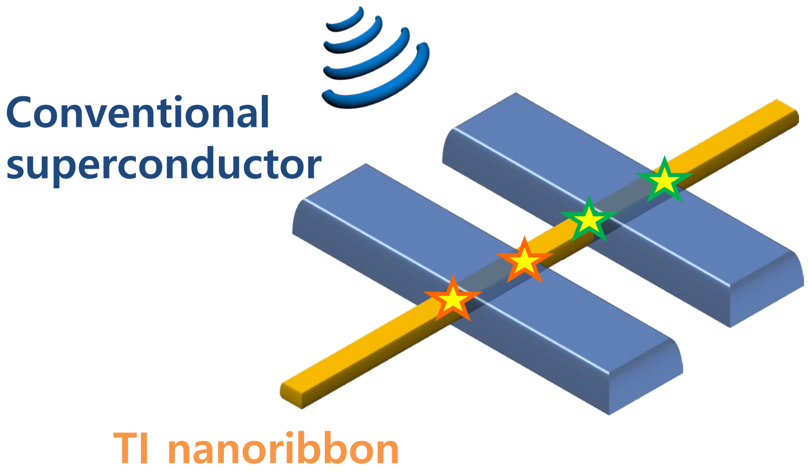
Das et al., Nat. Phys. (2012)

- A building block for topological quantum computation
- Majorana fermion: "a particle that is equal to its own anti-particle"
- Majorana bound states in a spinless topological superconductor
  1. III-V nanowire or 2DEG contacted with conventional superconductor
  2. Ferromagnetic atomic chain on top of *s*-wave superconductor
  3. Topological insulator with superconducting contact
- Issue : non-locality of zero-bias conductance peak

"Quantized Majorana conductance" Nature (2018) ~ RETRACTED (2021)

# Fractional Josephson effect ~ First Shapiro Step Missing (FSSM) 

**Microwave (MW)**




- Topological supercurrent

$$I_s = I_c \sin(\varphi/2) \sim 4\pi \text{ periodic}$$

- Shapiro steps occur at

$$V_n = 2n \cdot \frac{hf_{MW}}{2e} = 2nV_J \text{ with } n = \text{integer}$$

**Conventional JJ**

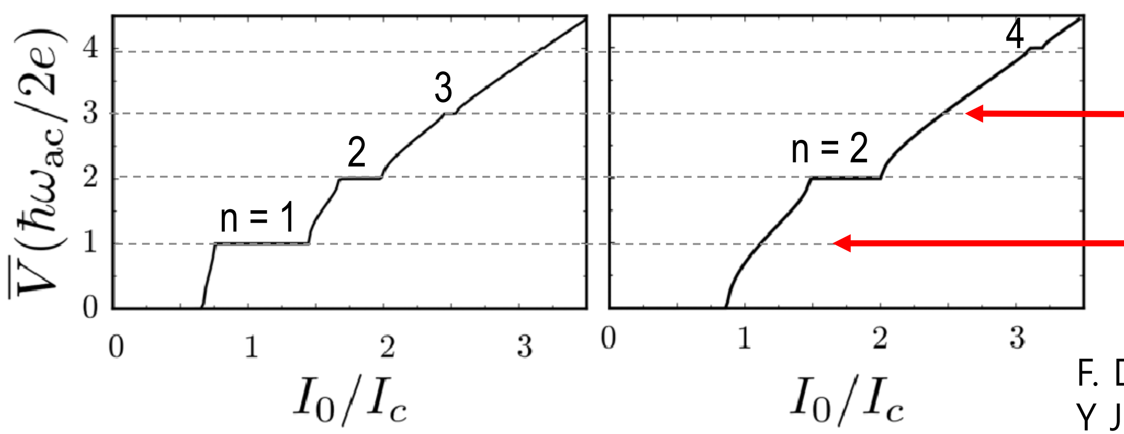
$$I_s = I_{c,2\pi} \sin(\varphi)$$

$$V_n = nV_J$$

**Topological JJ**

$$I_s = I_{c,4\pi} \sin(\varphi/2)$$

$$V_n = 2nV_J$$




Odd steps are missing

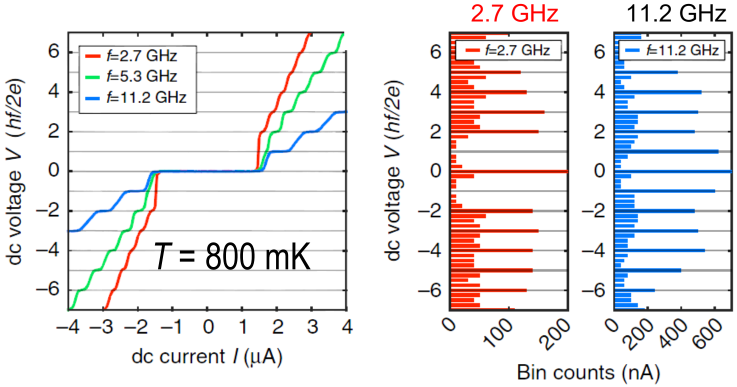
F. Dominguez, PRB (2017)
Y Jang & YJD, JKPS (2021)

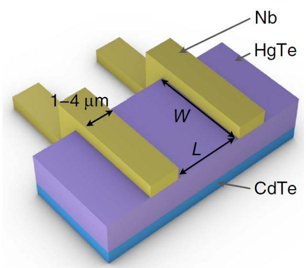
# Previous Works : FSSM in HgTe System

Received 15 Sep 2015 | Accepted 27 Nov 2015 | Published 21 Jan 2016 DOI: 10.1038/ncomms10303 OPEN

## 4π-periodic Josephson supercurrent in HgTe-based topological Josephson junctions

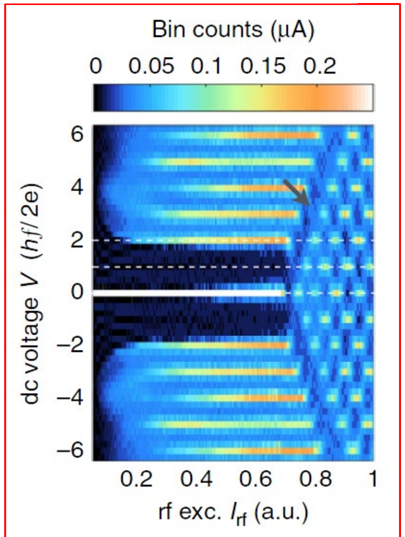
J. Wiedenmann[1,*], E. Bocquillon[1,*], R.S. Deacon[2,3,*], S. Hartinger[1], O. Herrmann[1], T.M. Klapwijk[4,5], L. Maier[1], C. Ames[1], C. Brüne[1], C. Gould[1], A. Oiwa[6], K. Ishibashi[2,3], S. Tarucha[3,7], H. Buhmann[1] & L.W. Molenkamp[1]




Nb-HgTe-Nb JJ
(3-dim TI)

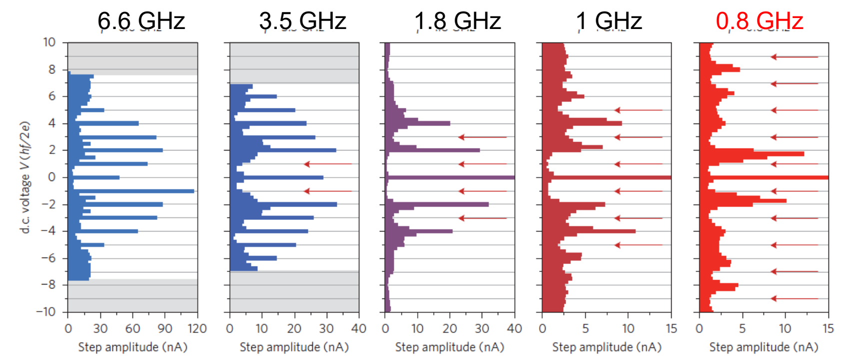
Wiedenmann et al,
Nat. Com. (2016)

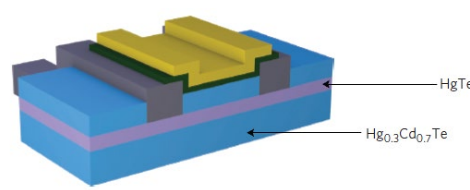



nature nanotechnology ARTICLES
PUBLISHED ONLINE: 29 AUGUST 2016 | DOI: 10.1038/NNANO.2016.159

## Gapless Andreev bound states in the quantum spin Hall insulator HgTe

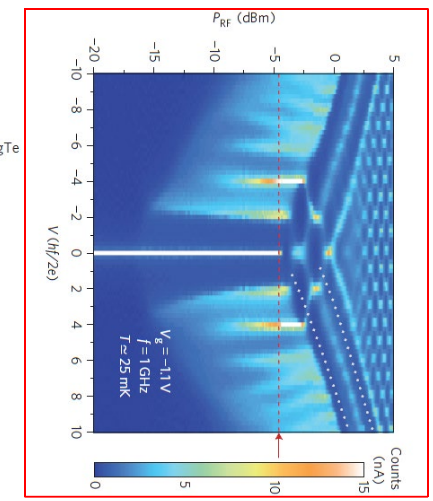
Erwann Bocquillon[1*†], Russell S. Deacon[2,3†], Jonas Wiedenmann[1†], Philipp Leubner[1], Teunis M. Klapwijk[4], Christoph Brüne[1], Koji Ishibashi[2,3], Hartmut Buhmann[1] and Laurens W. Molenkamp[1]



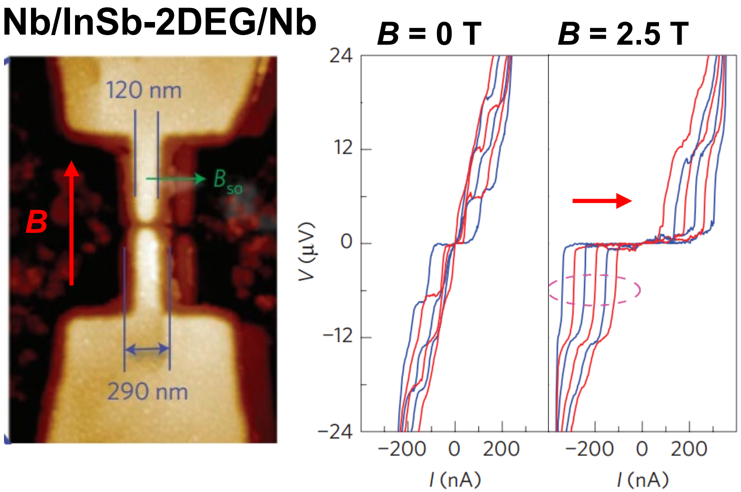

Al-HgTe-Al JJ
(QSH insulator)

Bocquillon et al,
Nat. Nanotech. (2016)

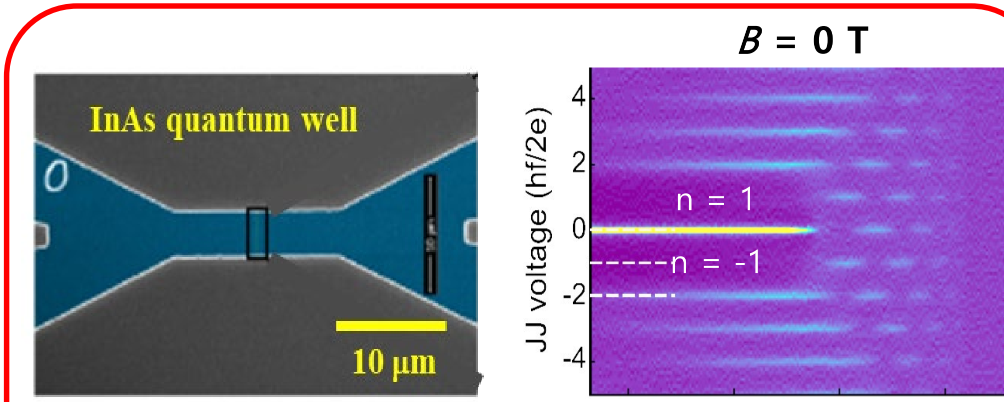



FSSM behavior has not been clearly observed in other TI materials yet.

# Issue: Landau-Zener Transition (LZT)

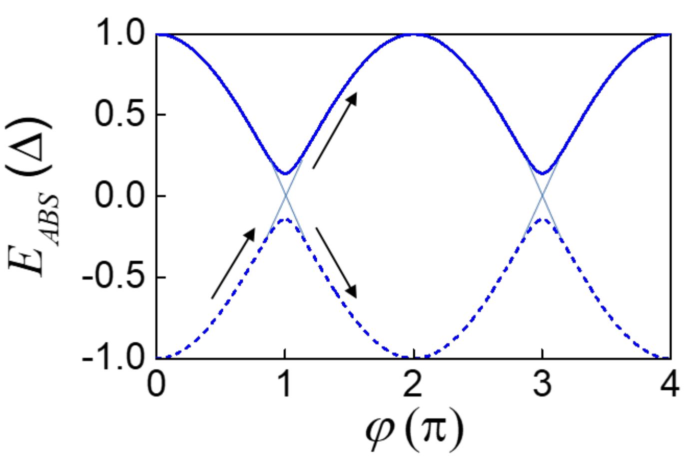



Rokhinson et al., Nat. Phys. (2012)




- FSSM observed in topologically trivial JJ's at zero B field

Dartiailh et al., Nat. Com. (2021)



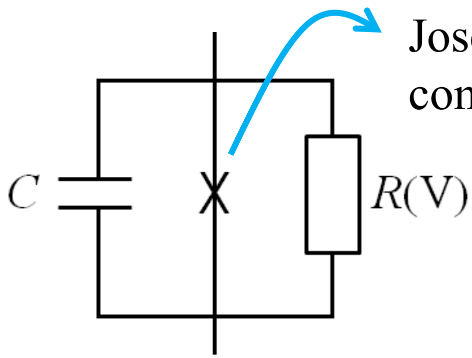

- 2π-periodic Andreev bound states are formed in a SNS junction, as shown in blue solid/dashed lines in the figure
- When the N-S contacts are highly transparent and the SNS junction is irradiated with high-frequency microwave, non-adiabatic transition can occur, resulting in non-topological 4π-periodic Andreev bound states (or supercurrent)

Averin & Bardas, PRL (1995)
Sau and Seiawan, PRB (2017)

# Question: How to distinguish topological and non-topological FSSM ?

# Josephson Junction - RCSJ Model (1)

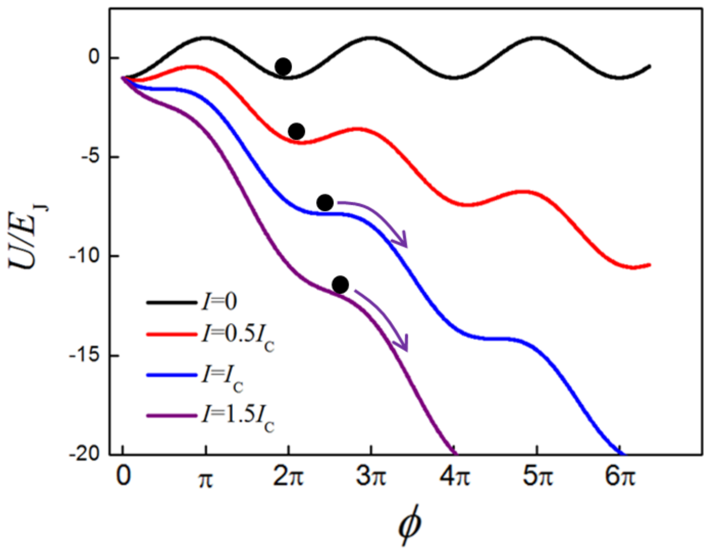





"Resistively and Capacitively Shunted Junction"

$$I = I_c \sin\phi + \frac{V}{R} + C\frac{dV}{dt}$$

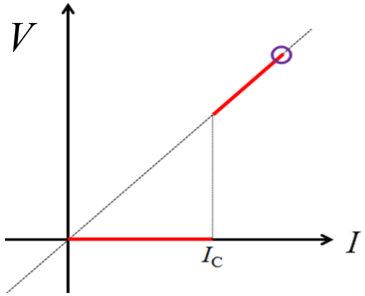
$$I_S = I_C \sin(\Delta\varphi)$$
$$\frac{d(\Delta\varphi)}{dt} = \frac{2eV}{\hbar}$$




$$\left(\frac{\hbar}{2e}\right)^2 C\frac{d^2\phi}{dt^2} = -\frac{dU}{d\phi} - \left(\frac{\hbar}{2e}\right)^2 \frac{1}{R}\frac{d\phi}{dt}$$

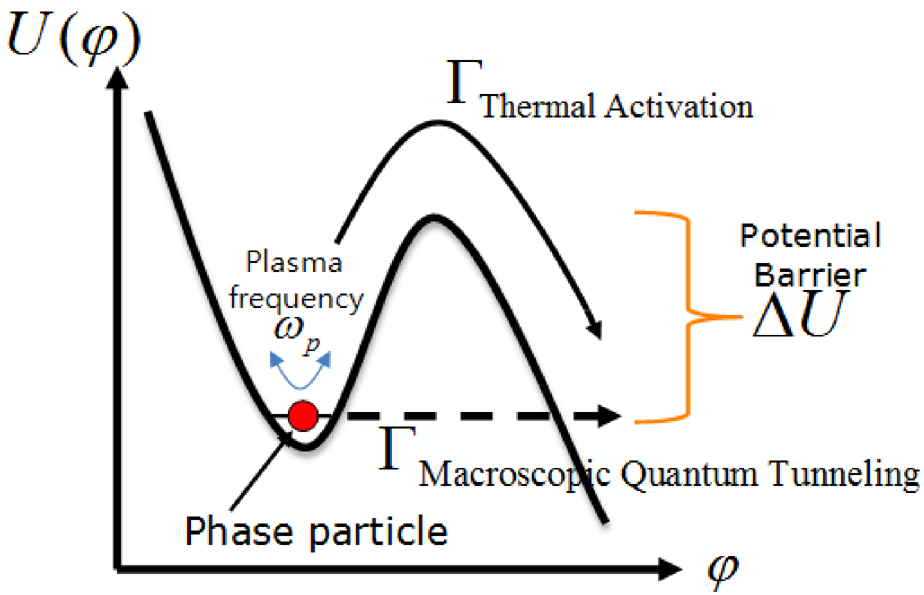
$$U(\phi) = -E_J\cos\phi - \frac{\hbar I}{2e}\phi$$ ~ tilted washboard potential

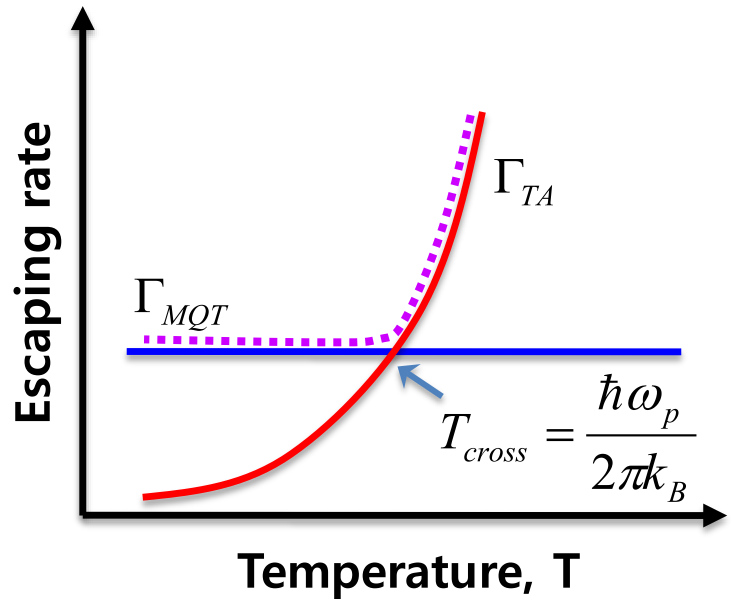
$$E_J = \frac{\hbar I_C}{2e}$$ ~ Josephson coupling energy




**The escaping motion of the phase particle corresponds to the $I_C$ switching event**

# Macroscopic Quantum Tunneling of a Josephson Phase Particle




$$\Gamma_{TA}(I) = \frac{\omega_p}{2\pi}\exp\left[-\frac{\Delta U}{k_B T_{escape}}\right]$$ **: T-dependent**

$$\omega_{p0} = (2eI_{c0}/\hbar C)^{1/2}$$

$$\Gamma_{MQT}(I) = 12\omega_p\left(\frac{3\Delta U}{2\pi\hbar\omega_p}\right)^{1/2}\exp\left[-\frac{36\Delta U}{5\hbar\omega_p}\right]$$ **: T-independent**

$$P(I) = \frac{\Gamma(I)}{dI/dt}\,exp\left[-\frac{1}{dI/dt}\int_0^I \Gamma(I')dI'\right]$$ ~ **Switching probability density ∝ SCD's**

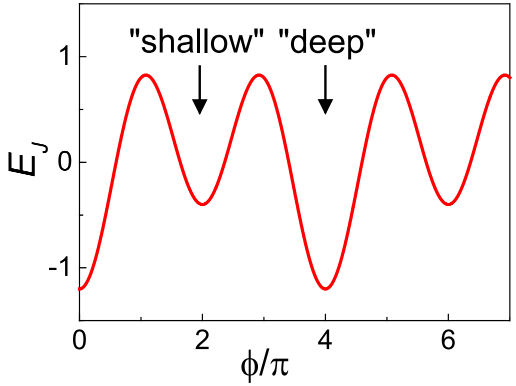
A.Garg, Phys. Rev. B (1995)

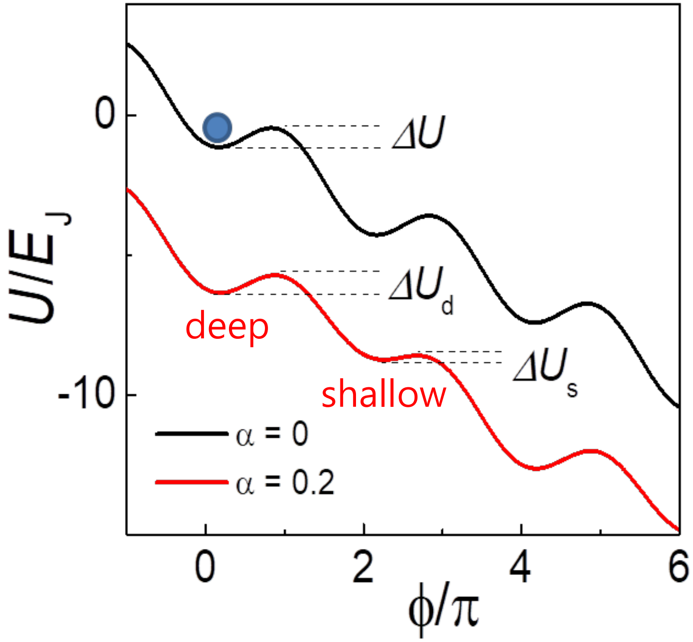
# Josephson Potential Well of Mixed Supercurrents

## TI NR JJ's ~ 2π- and 4π-periodic mixed supercurrents

$$I_{sc} = I_{c,2\pi} \sin(\phi) + I_{c,4\pi} \sin(\phi/2)$$

$$U(\phi) = -E_{J,2\pi} \cos\phi - E_{J,4\pi} \cos(\phi/2),\ E_J \equiv \frac{\hbar I_c}{2e}$$

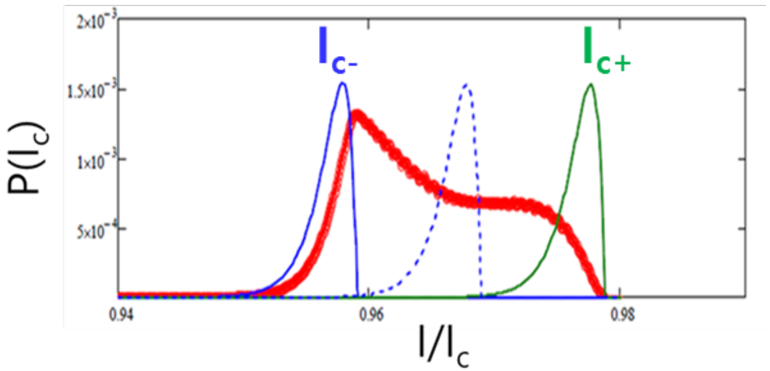



## Bimodal switching current distributions

Two $I_c$ peaks at $I_{c+}$ and $I_{c-}$, where

$$\left.\begin{aligned} I_{c+} &\approx I_{c,2\pi} + I_{c,4\pi}/2 \\ I_{c-} &\approx I_{c,2\pi} - I_{c,4\pi}/2 \end{aligned}\right\} \quad \begin{aligned} I_{c+} - I_{c-} &\approx I_{c,4\pi} \\ I_{c+} + I_{c-} &\approx I_{c,2\pi} \end{aligned}$$



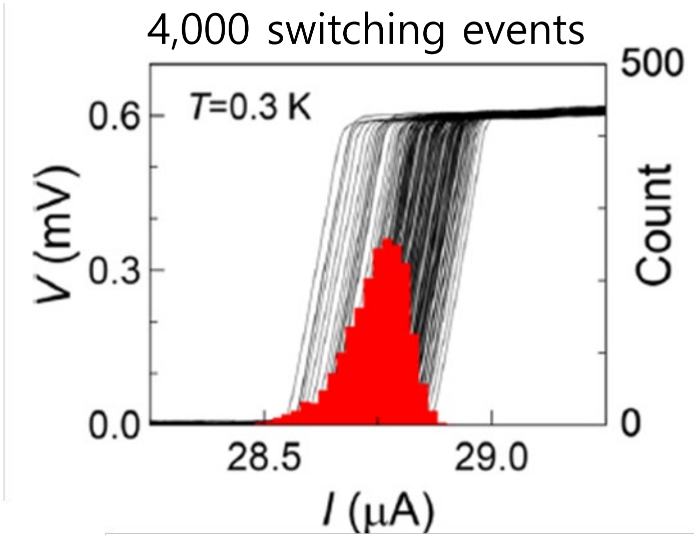

N. Abboud, D. Van Harlingen et al., PRB (2022)

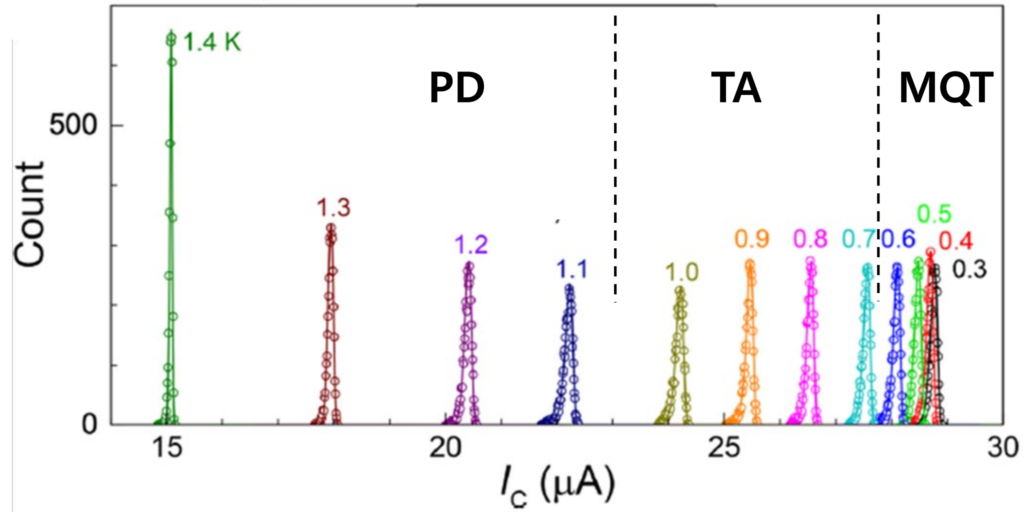
# Switching Current Distributions of Conventional JJ’s



## $Ag_2Se$ & PbS nanowire JJ’s :

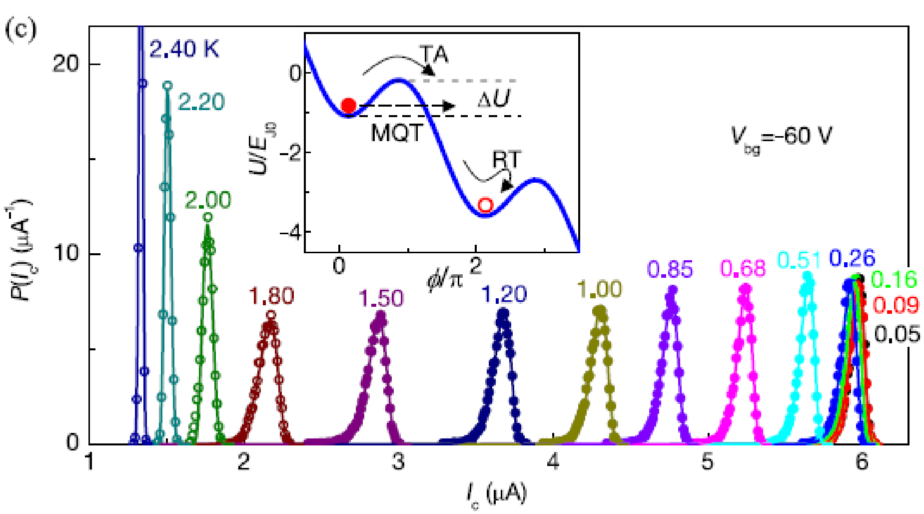
J. Kim, YJD* et al., Nano Lett. (2017)
B.-K. Kim, YJD* et al., ACS Nano (2017)

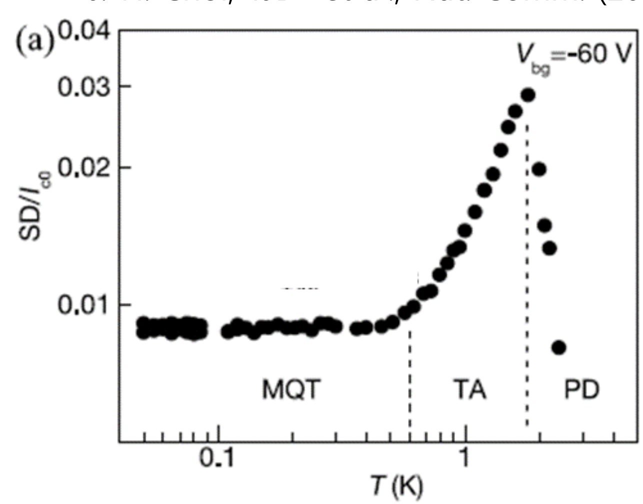



## Graphene JJ’s :

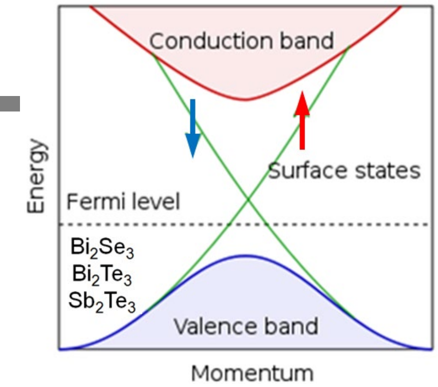
G.-H. Lee, YJD* et al., Phys. Rev. Lett. (2011)
J.-H. Choi, YJD* et al., Nat. Comm. (2013)



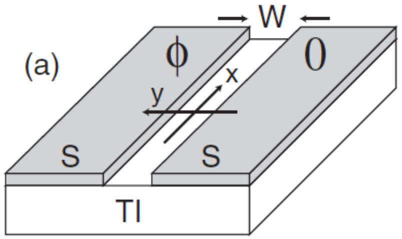



# Topological insulator nanoribbon (3-dim)



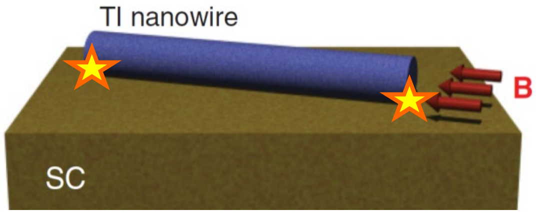



• **Topological insulator** (TI) **:** A bulk insulator with metallic surface states, which are topologically protected with spin-momentum locking



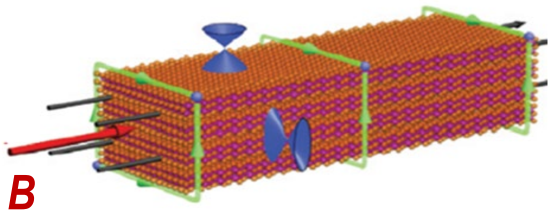

• **Superconducting proximity effect** at the surface of a topological insulator supports Majorana bound states

Fu & Kane, Phys. Rev. Lett. (2008)




• **TI NR-superconductor junction :** Majorana zero modes are localized at both ends of TI NW, proximity-coupled to an $s$-wave superconductor with axial magnetic field of $\Phi = 0.5\Phi_0 = h/2e$

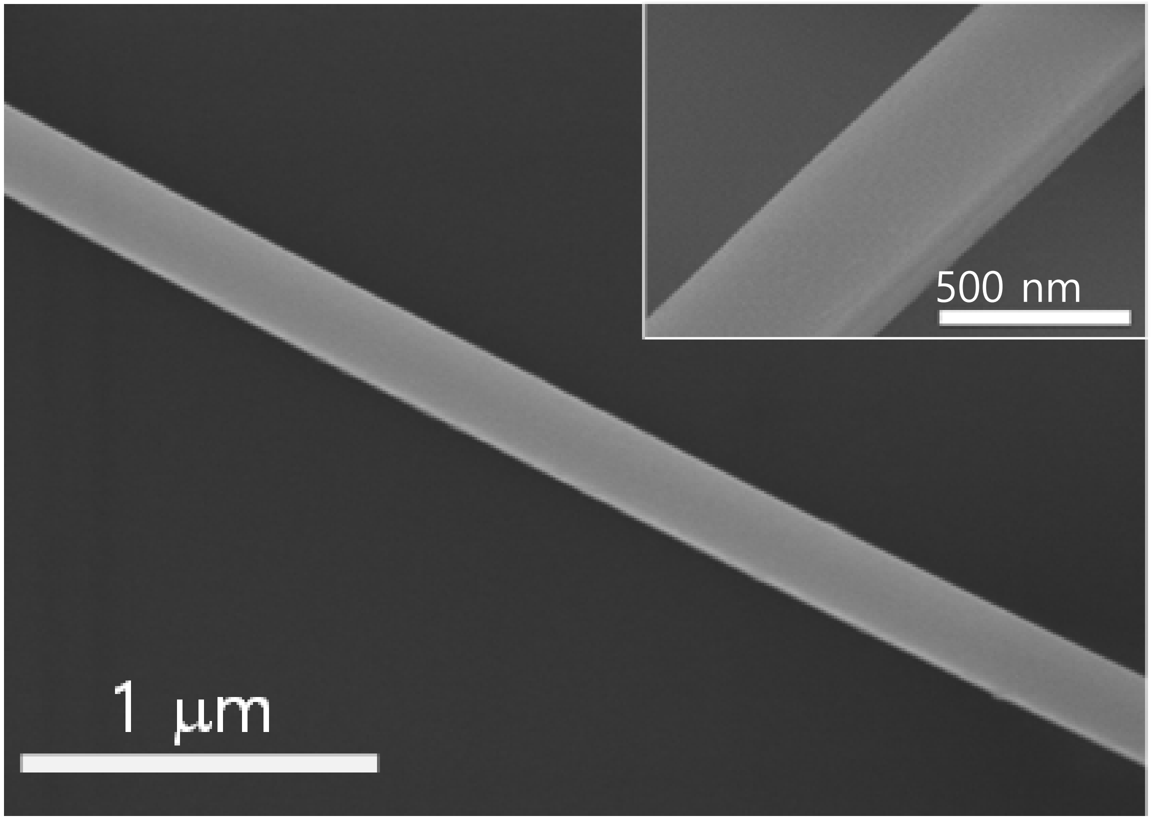
Cook & Franz, Phys. Rev. B (2011)
Bardarson, Phys. Rev. Lett. (2014)

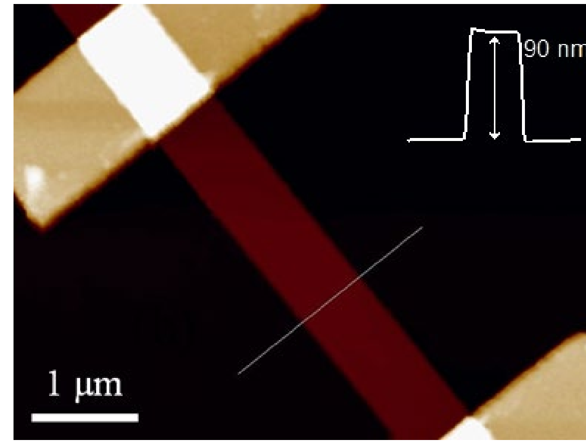



• **TI nanoribbon (NR) :** Periodic boundary condition along the circumference results in 1D subbands including Aharonov-Bohm phase ("$h/e$ periodic")

# $(Bi_{0.81}Sb_{0.19})_2Se_3$ nanoribbons (NRs)






Width w ~ 400 nm

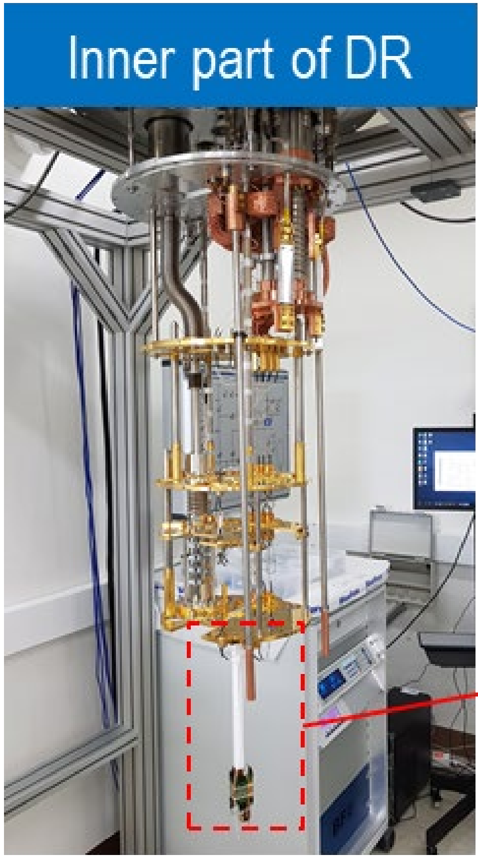
Thickness t ~ 100 nm

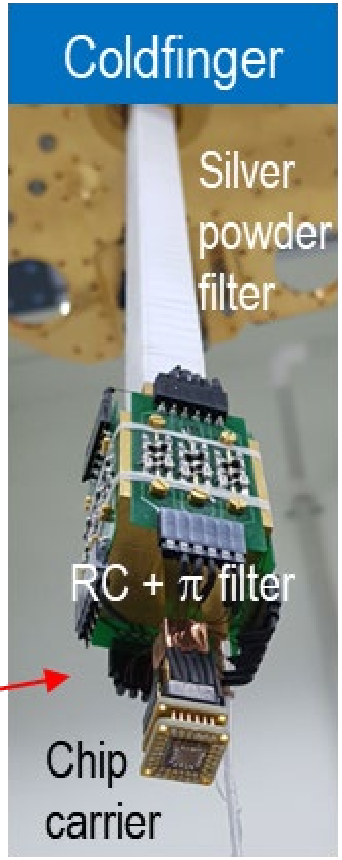
Perimeter $L_p$ ~ 1 µm

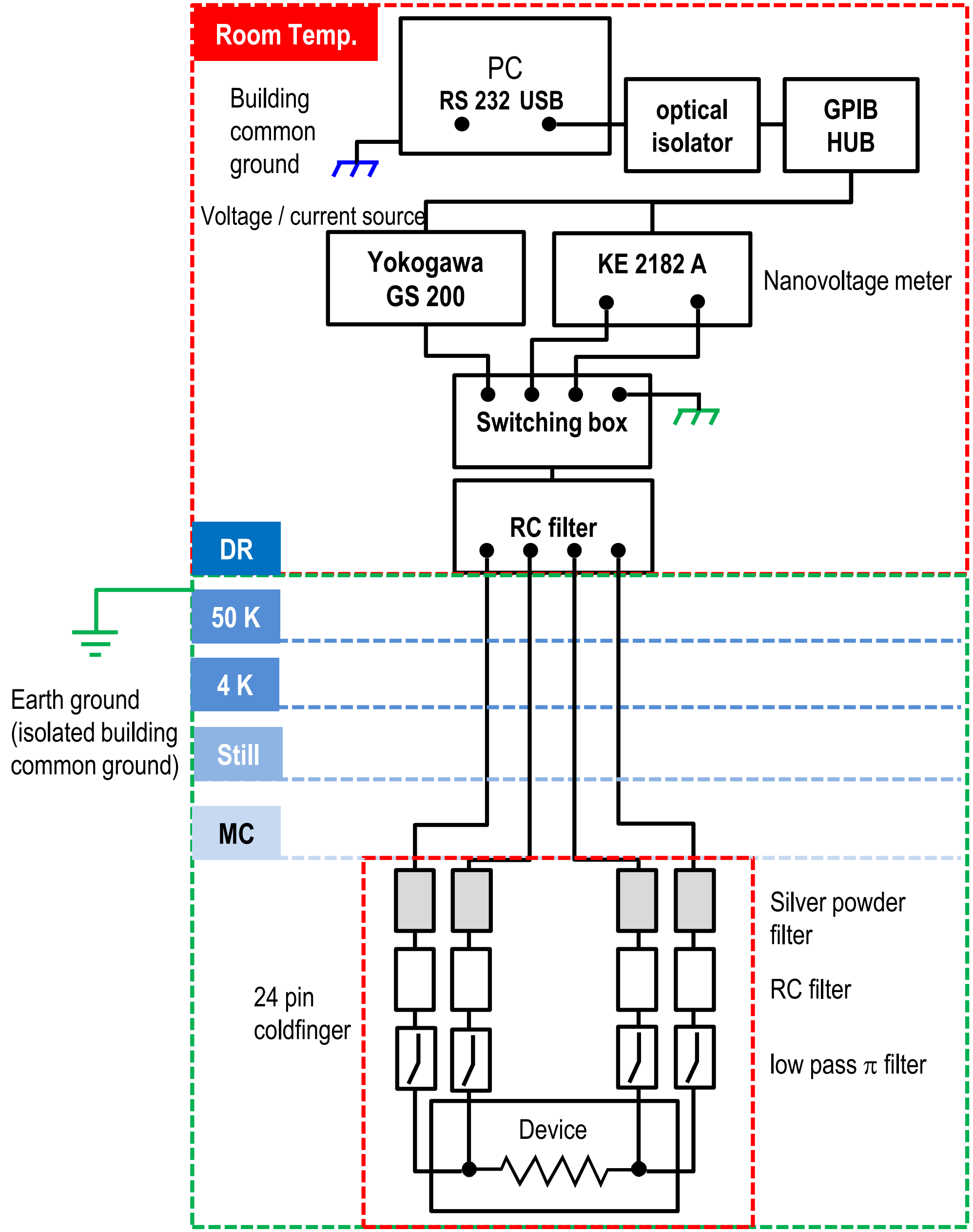
**Growth of nanoribbons by prof. Dong Yu at UC Davis**

# Experimental setup for supercurrent measurement

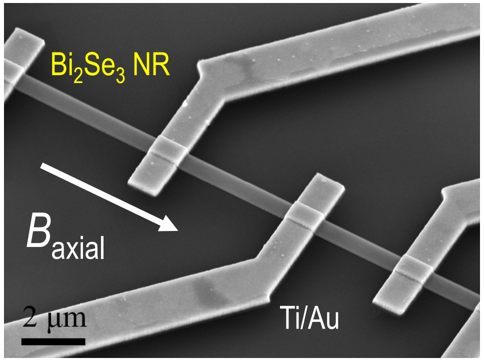


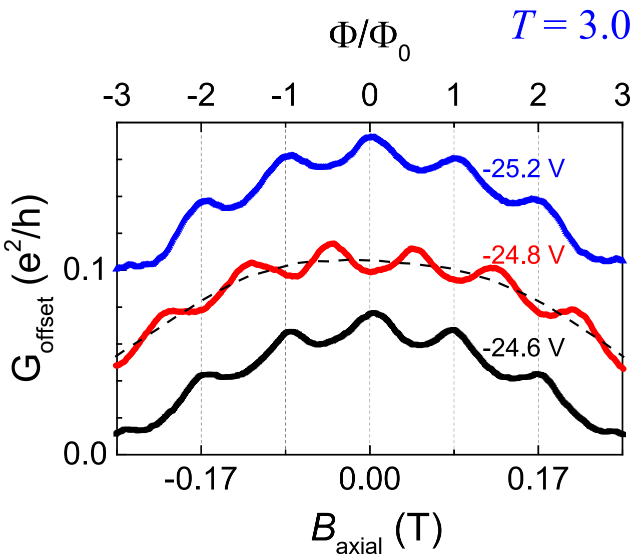


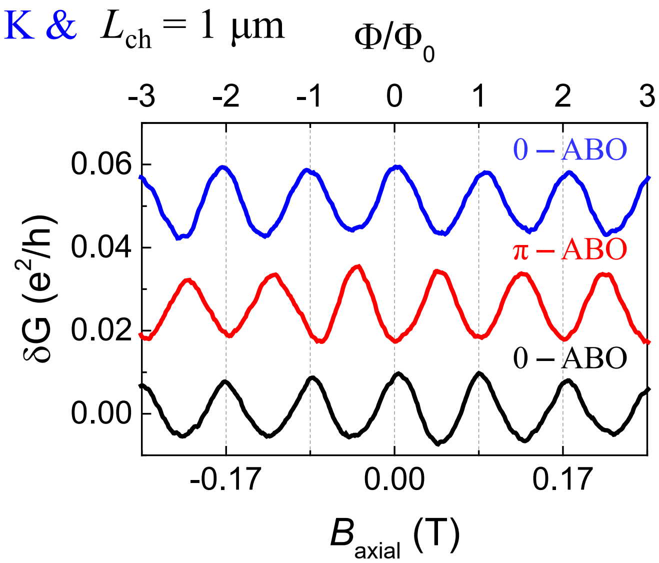



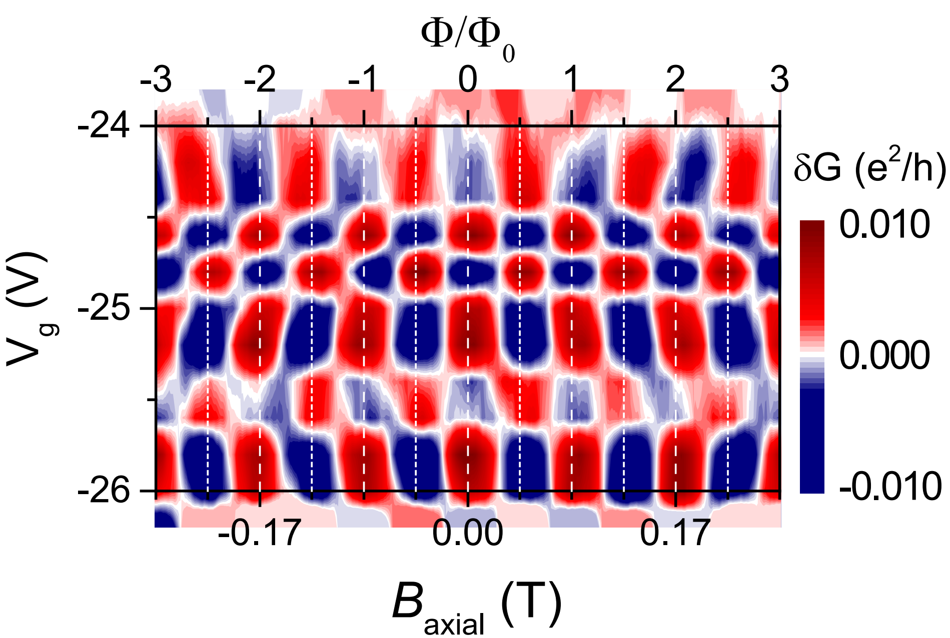



# Aharonov-Bohm oscillations (ABO's) in TI nanoribbon (NR)



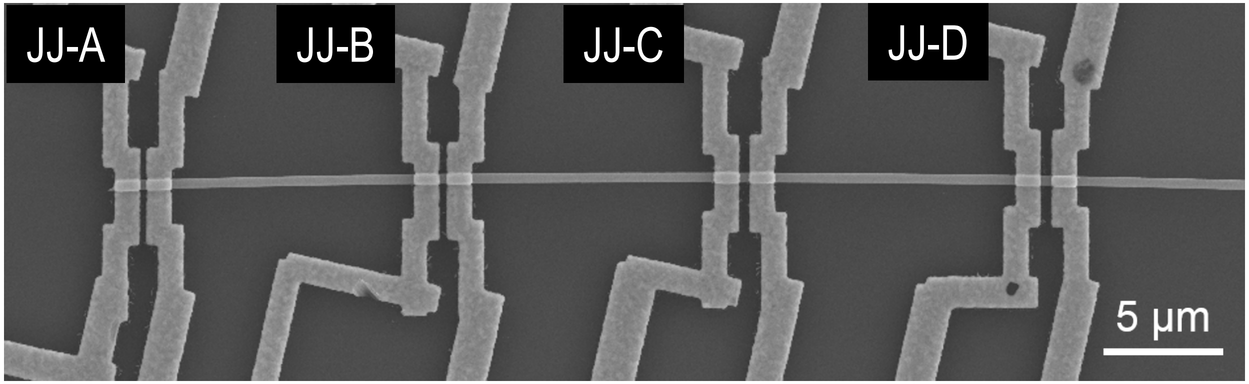

Checkerboard-like $V_g$ dependence:

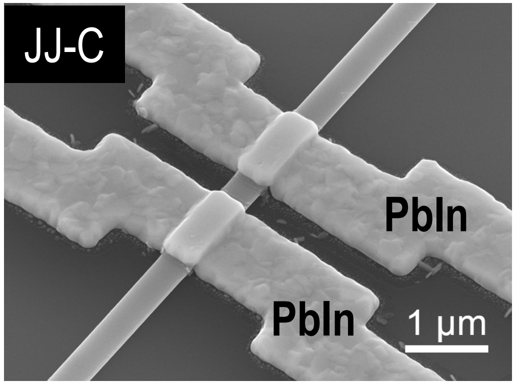
0- and π-phase alternation of ABO's

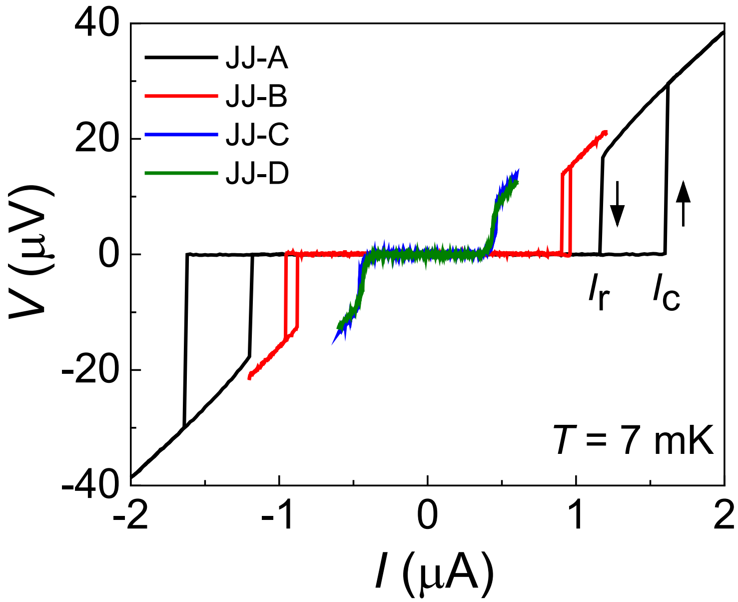
H.-S. Kim*, T.-H. Hwang*, YJD et al., ACS Nano (2020)

# Josephson junctions made of TI nanoribbon

PbIn – $(Bi_{0.81}Sb_{0.19})_2Se_3$ TI NR – PbIn



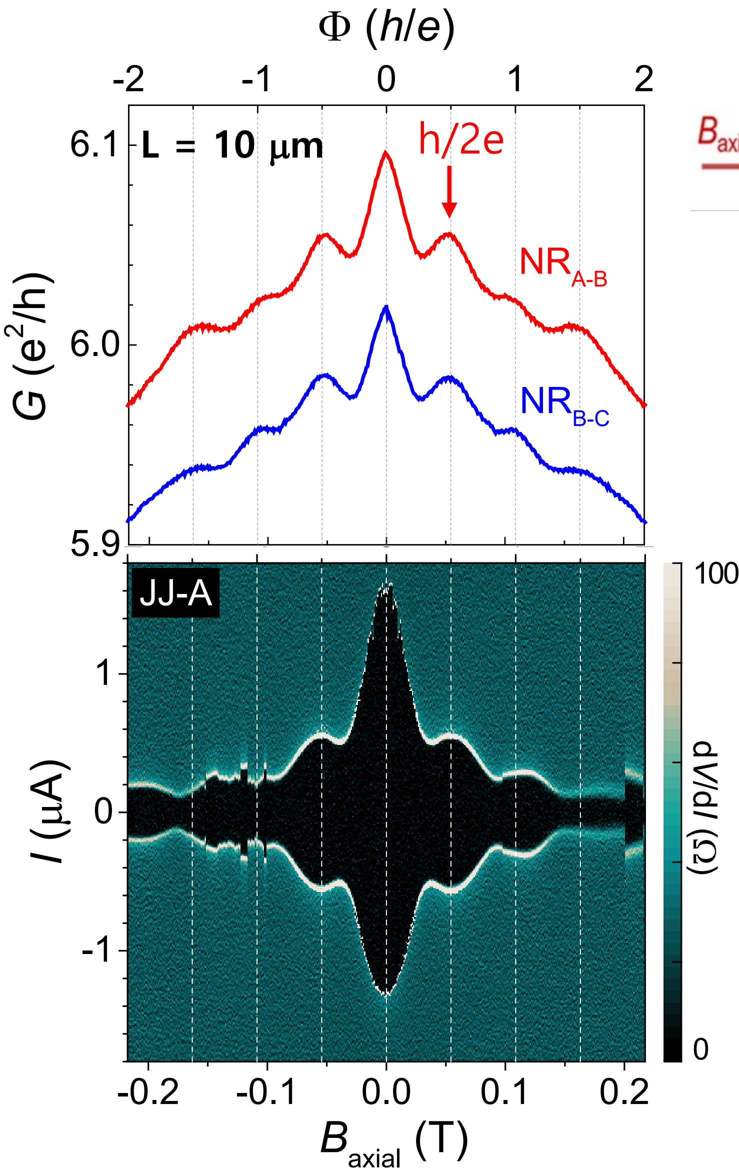

- PbIn superconducting electrodes with $T_c$ = 7 K
- TI NR with thickness = 113 nm & width = 420 nm

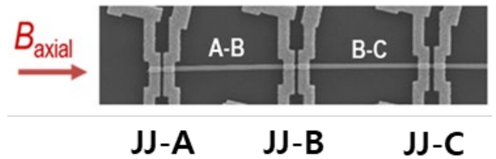
| | JJ-A | JJ-B | JJ-C | JJ-D |
|---|---|---|---|---|
| $L_{ch}$ (nm) | 240 | 280 | 380 | 430 |
| $I_c$ (μA) | 2.0 | 1.5 | 0.5 | 0.5 |

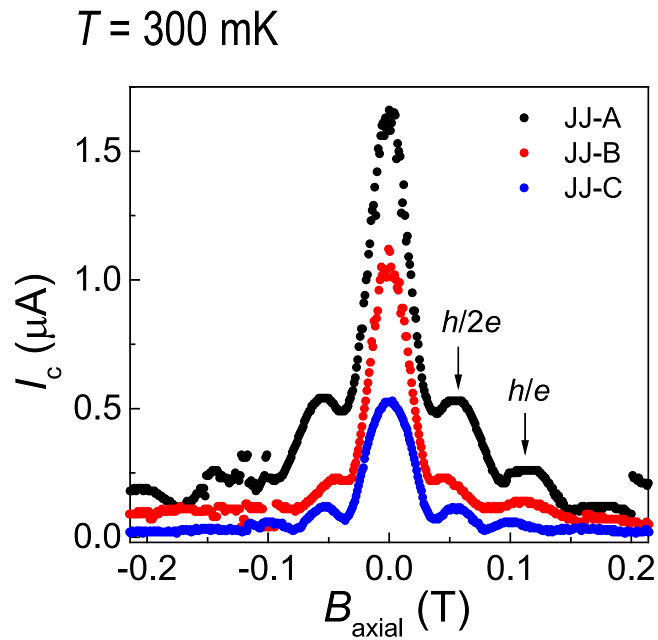
H.-S. Kim, YJD et al. (in preparation)

# Supercurrent ABO's with $B_{axial}$ field





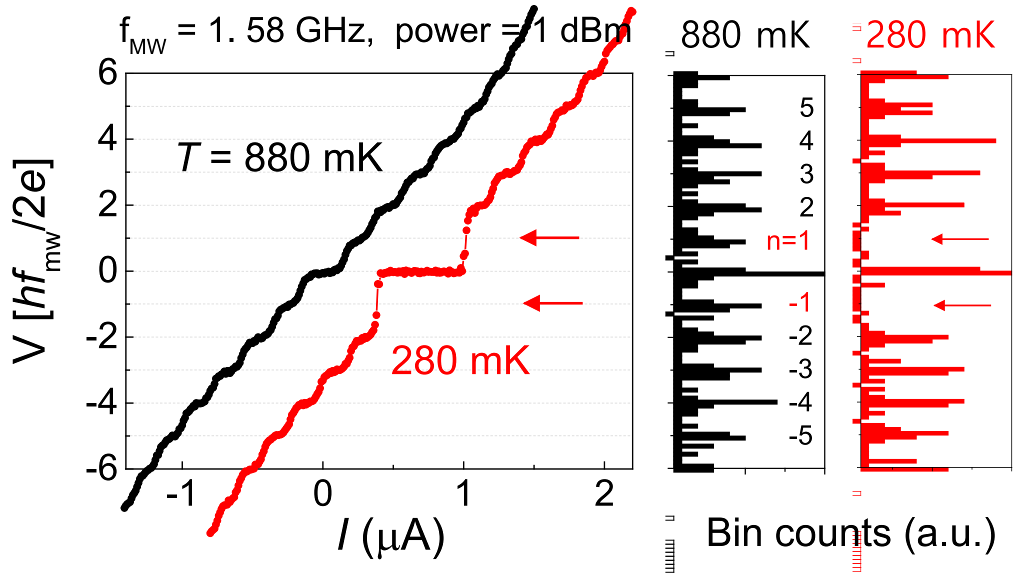

- 10 μm-long segments of nanoribbon exhibit h/e- and h/2e-periodic magnetoconductance oscillations.
- Critical current $I_c$ oscillates with the same periodicity of $\Phi$ = h/e and h/2e under $B_{axial}$ field.
- The first observation of the supercurrent ABO's in TI NR ~ Evidence of the supercurrent flow via the topological surface state of TI nanoribbon !

H.-S. Kim, YJD et al. (in preparation)

# First Shapiro Step Missing (FSSM)

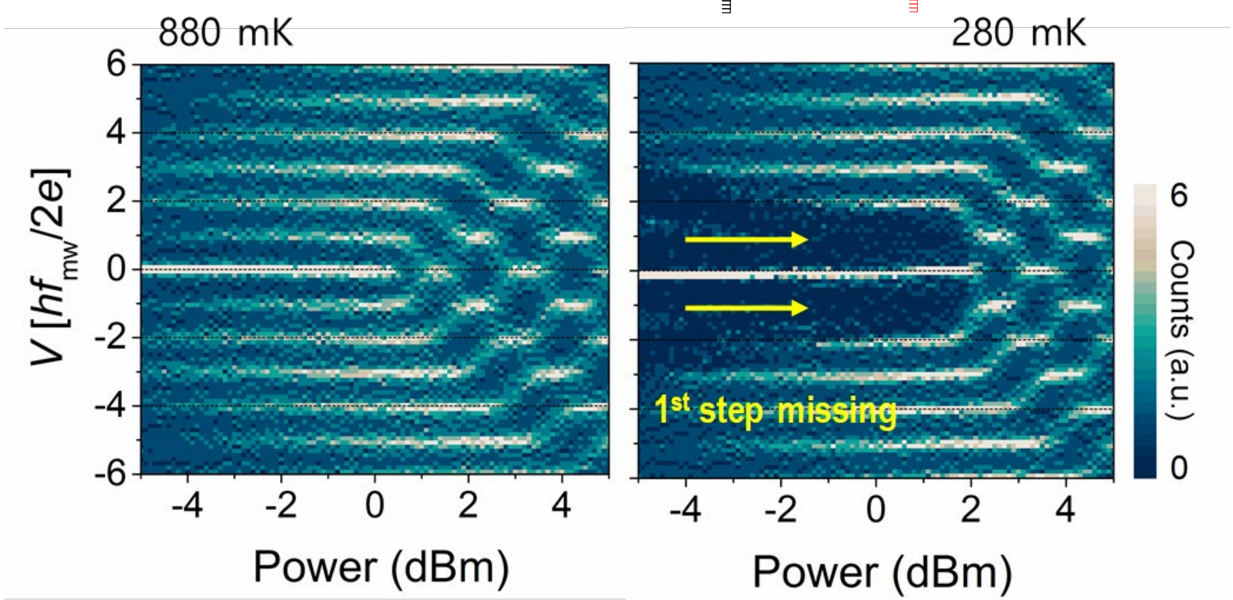


JJ-A




$$V_n = n\frac{hf_{MW}}{2e},$$

$$n = \text{integer}$$

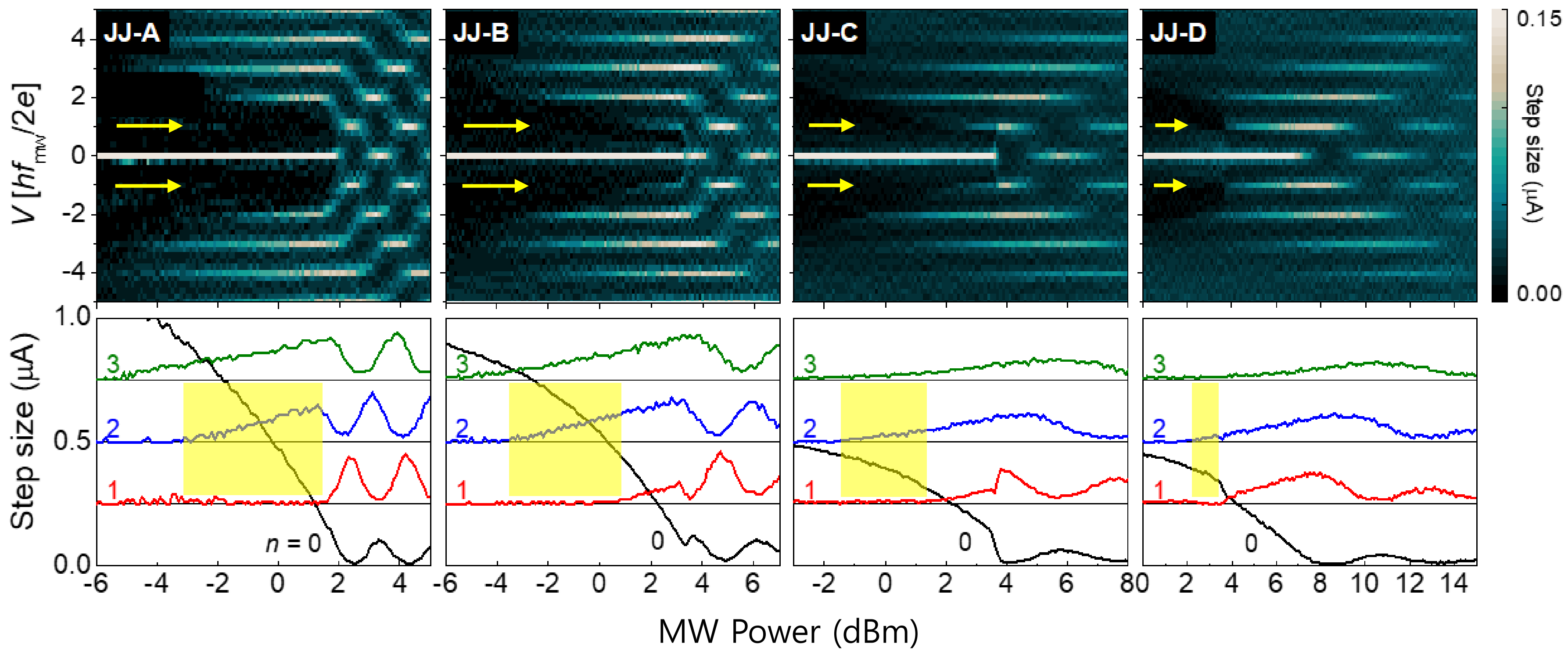



Shapiro steps with $n$ = ± 1 are missing at low temperatures and low-power MW

# Four JJ's on the same TI NR ~ $L_{ch}$ dependence

$T$ = 280 mK, $f_{MW}$ = 1.58 GHz,

H.-S. Kim, YJD et al. (in preparation)

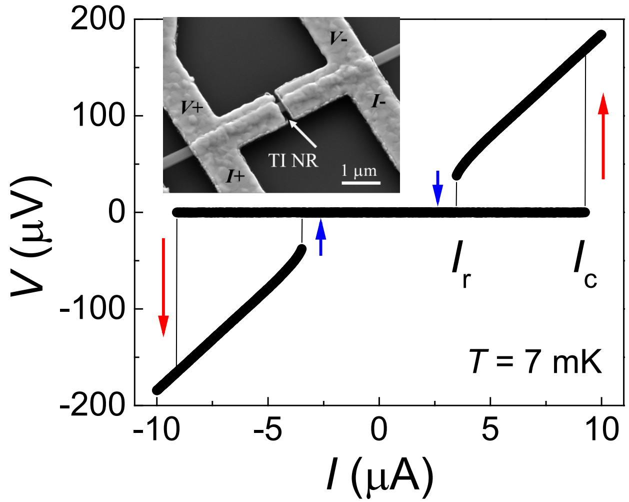



| | JJ-A | JJ-B | JJ-C | JJ-D |
|---|---|---|---|---|
| $L_{ch}$ (nm) | 240 | 280 | 380 | 430 |
| $I_c$ (μA) | 2.0 | 1.5 | 0.5 | 0.5 |
| $I_cR_n$ (μV) | 53 | 38 | 19 | 16 |

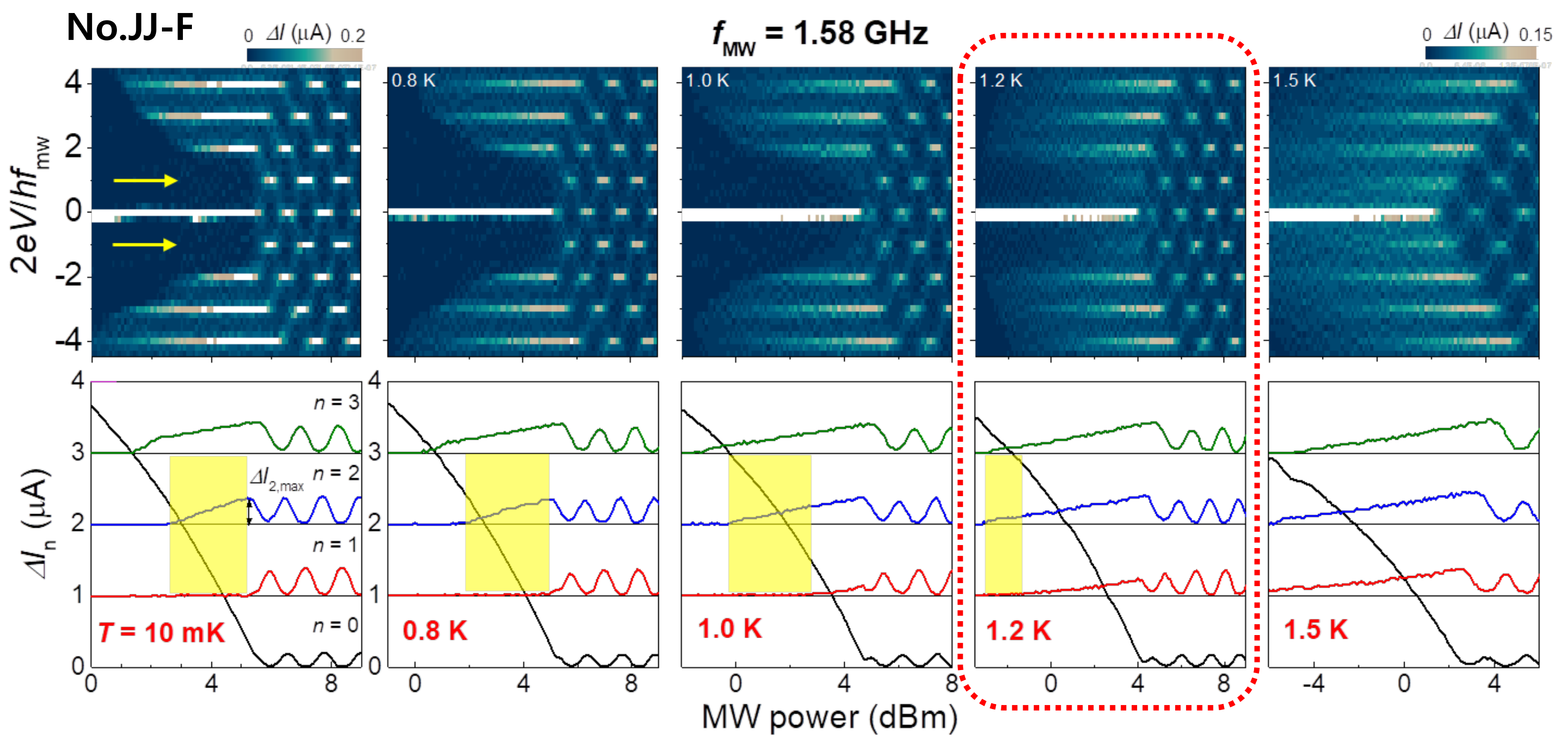
- All four JJ's exhibit complete absence of the first step, FSSM, indicated by yellow box
- FSSM is suppressed by increasing $L_{ch}$, but almost same transparency ~ LZT ruled out

# TI NR JJ's with $L_{ch}$ = 100 nm

N.-H. Kim, YJD et al. (in preparation)



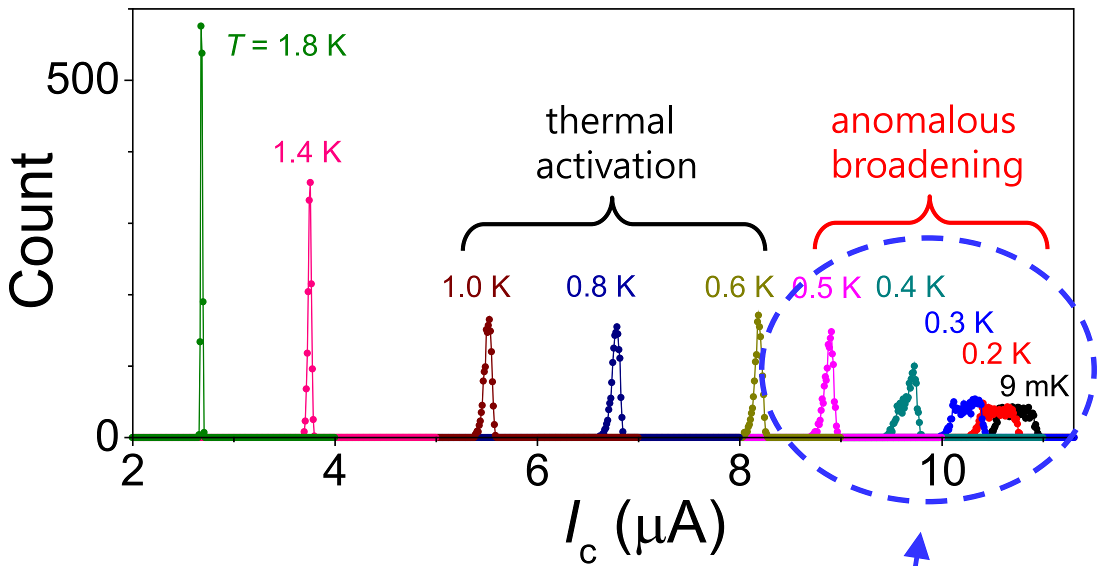

- Sb-$Bi_2Se_3$ TI NR, width = 281 nm, thickness = 88 nm
- Channel length, $L_{ch}$ = 100/125/129 nm for **D1/D2/D3**
- Critical current, $I_c$ = 9.1 µA
- $I_cR_N$ = 230 µV at $T$ = 7 mK
- $\Delta_0$ = 680 µeV ~ multiple Andreev reflection (MAR)



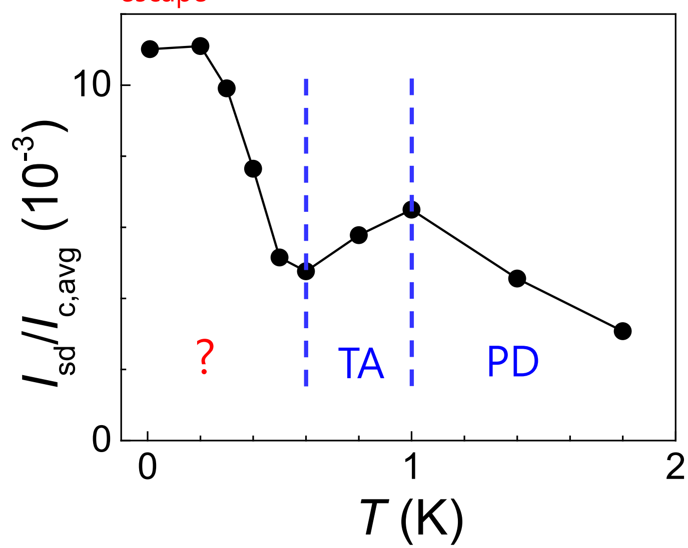

FSSM is observed up to T* = 1.2 K, the highest T

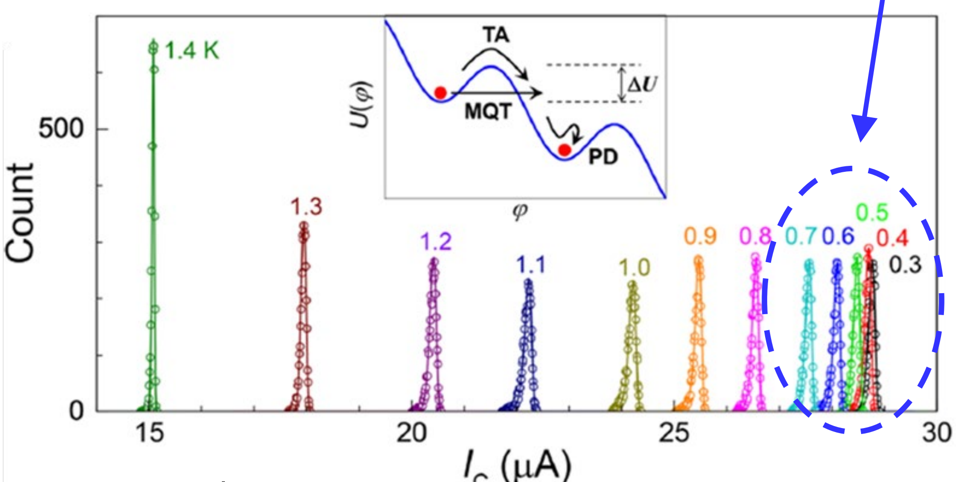
# Switching Current Distributions of TI NR JJ's

No.**D1** (1,450 switching events)




$T_{escape}$ ~ 5.0 K



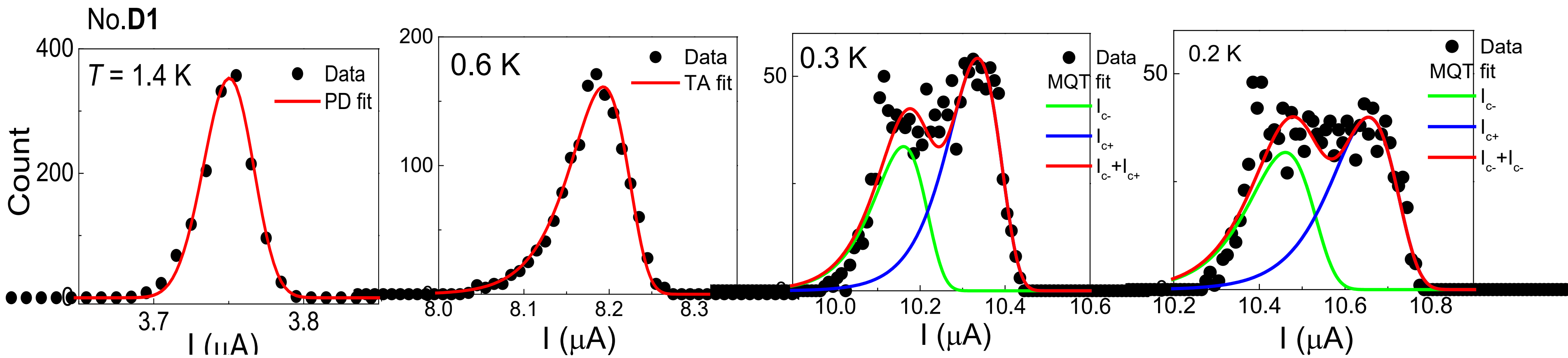



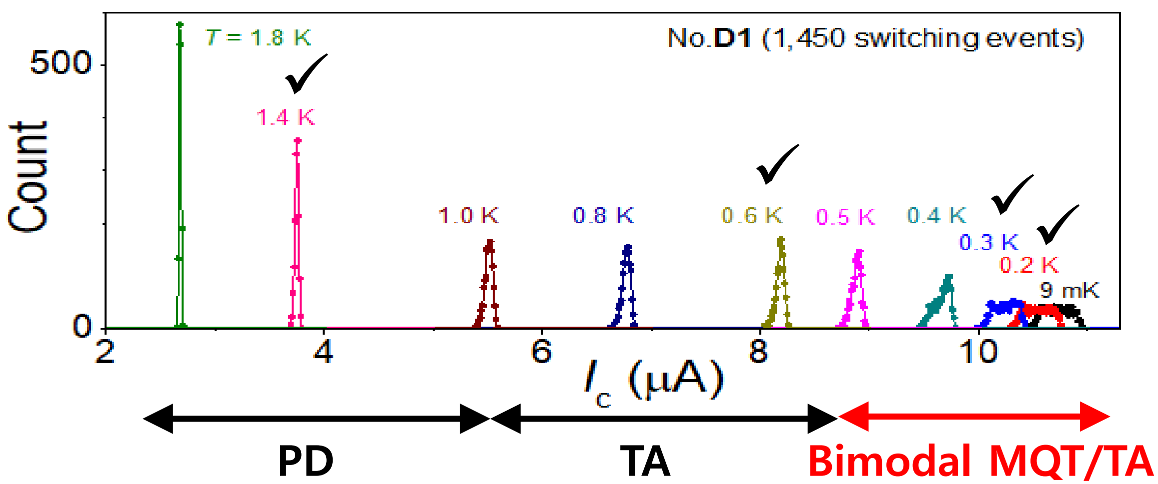

Conventional JJ's

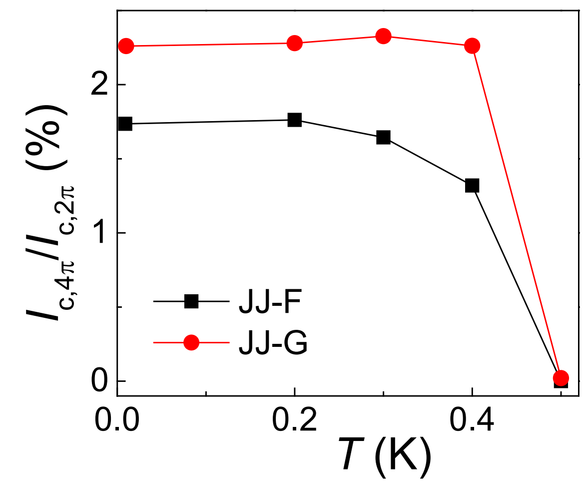
J. Kim, YJD et al., Nano Lett. (2017);
B.-K. Kim, YJD et al., ACS Nano (2017)

- Typical SCD behaviors of phase diffusion (PD) and thermal activation (TA) processes are observed down to T = 0.6 K
- Significant broadening of SCD at low temperatures below T = 0.5 K, resulting $T_{escape}$ = 5.0 K
- Uni-modal SCD's cannot explain the anomalous broadening below T = 0.5 K

# Bimodal Switching Current Distributions

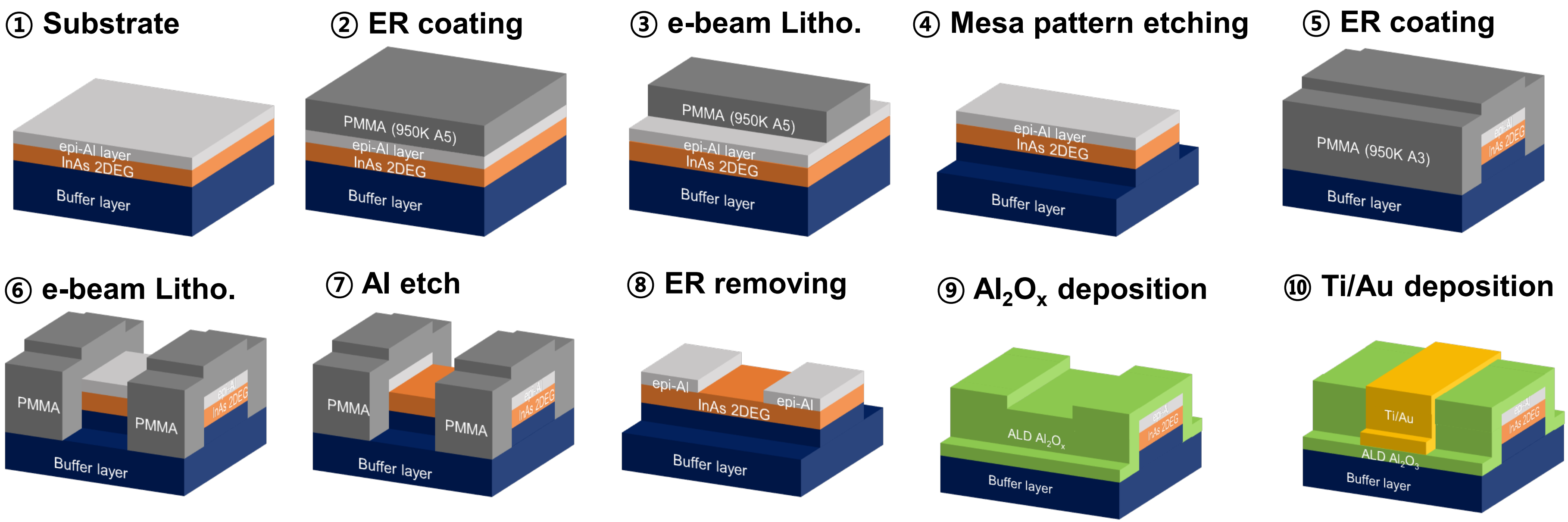


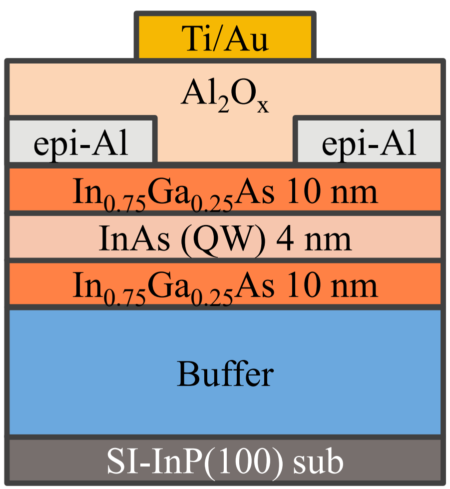



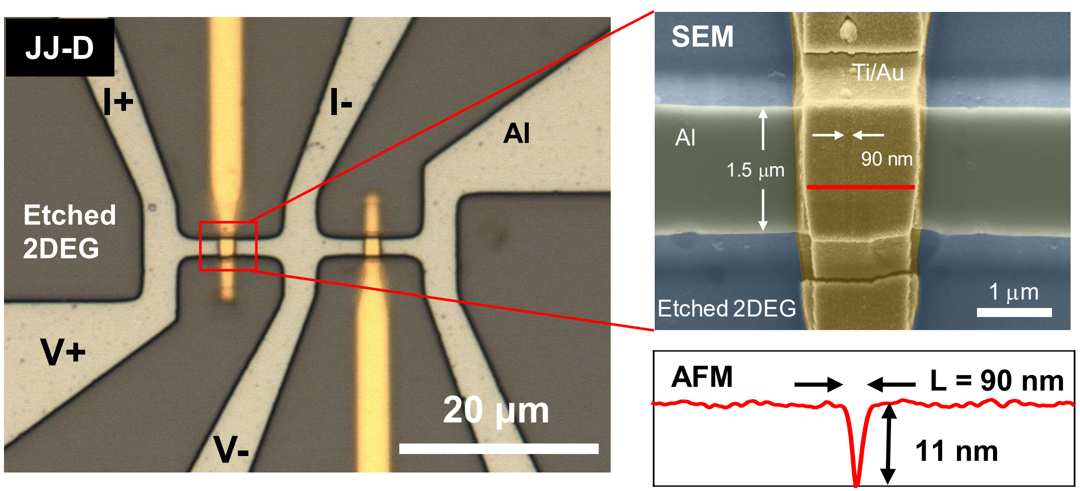




- SCD data fit well to PD and TA models down to T = 1.0 K and 0.6 K, respectively
- SCD data below T = 0.5 K fit well to a bimodal MQT model, indicating the existence of two distinct values of critical current, which are $I_{c\pm} \sim I_{c,2\pi} \pm I_{c,4\pi}/2$
- **Topological critical current is measured ($I_{c,4\pi}/I_{c,2\pi}$ = 2.3 %) for the first time**
- Note spin-fileter SFS JJ's ~ double-well potential but π-periodic supercurrent (fractional SS's)

# Device fabrications of 2DEG JJ's

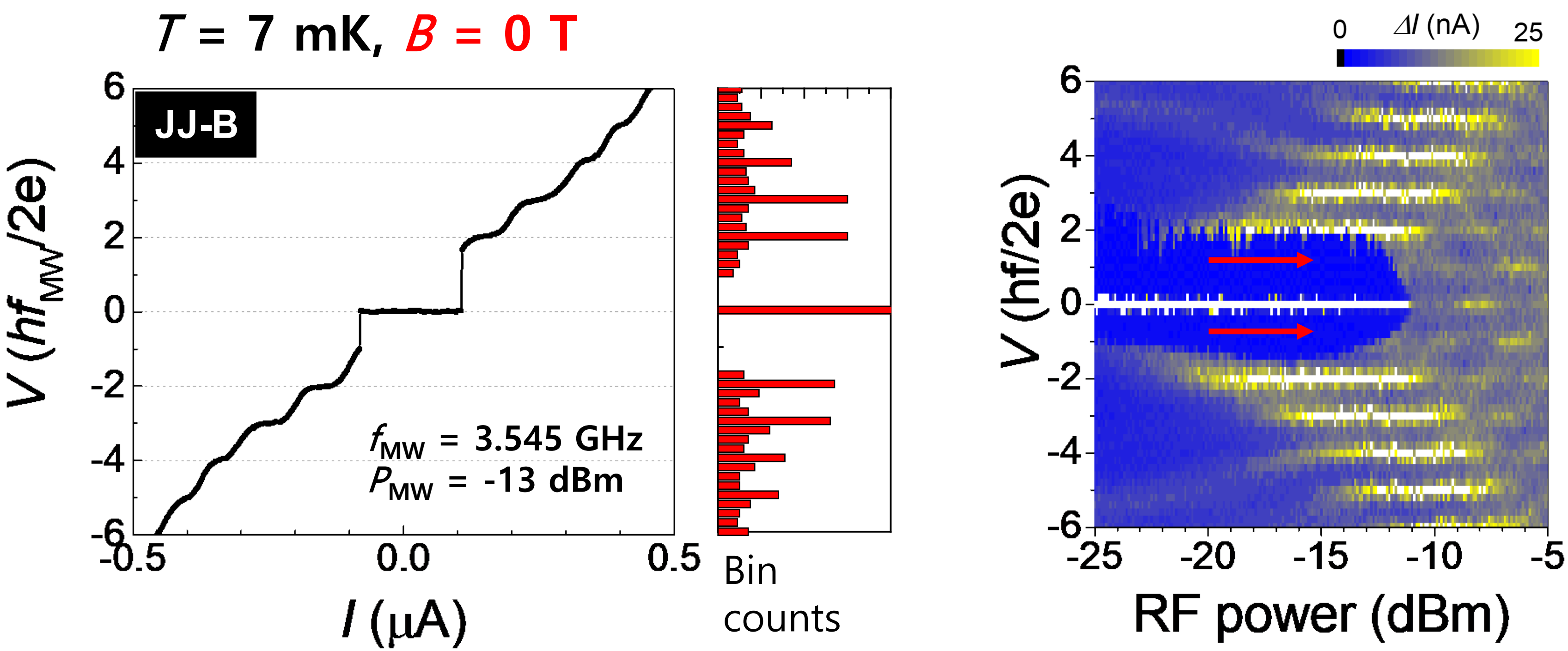



**2DEG growth by prof. Joon Sue Lee @ Univ. Tennessee**

# First Shapiro step missing



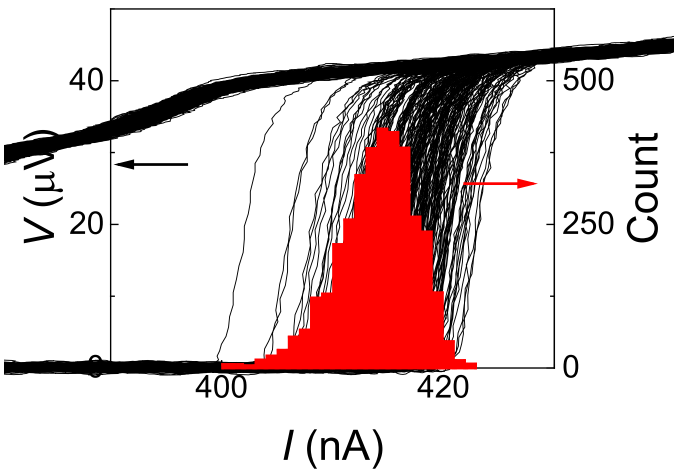

- First Shapiro step missing under zero B field

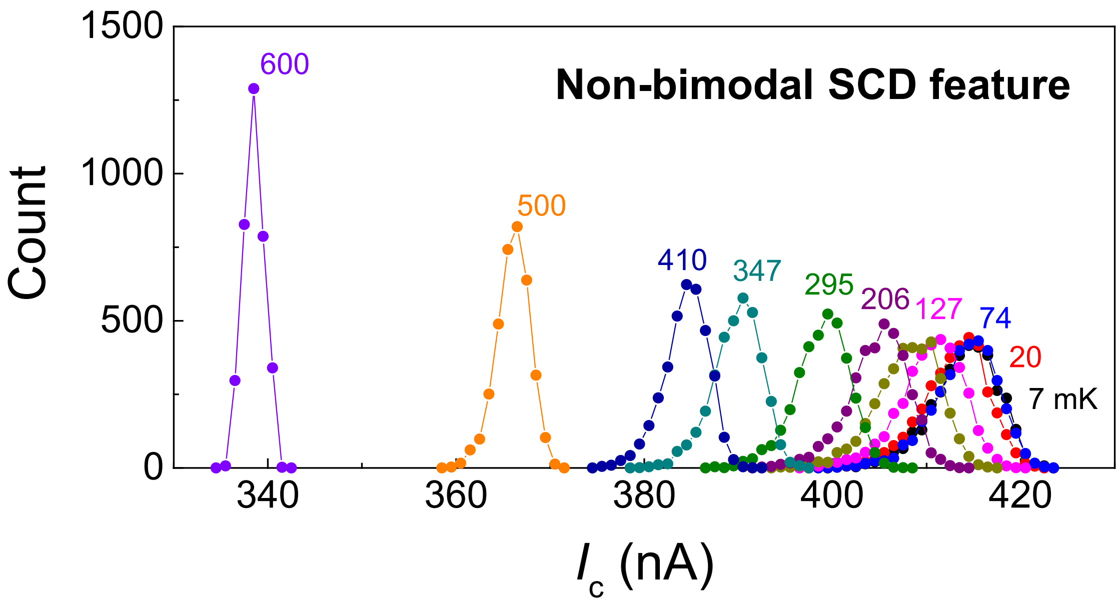
  ~ due to a topologically trivial origin.

# Switching current distribution (SCD)



I-V meas. 3600 times



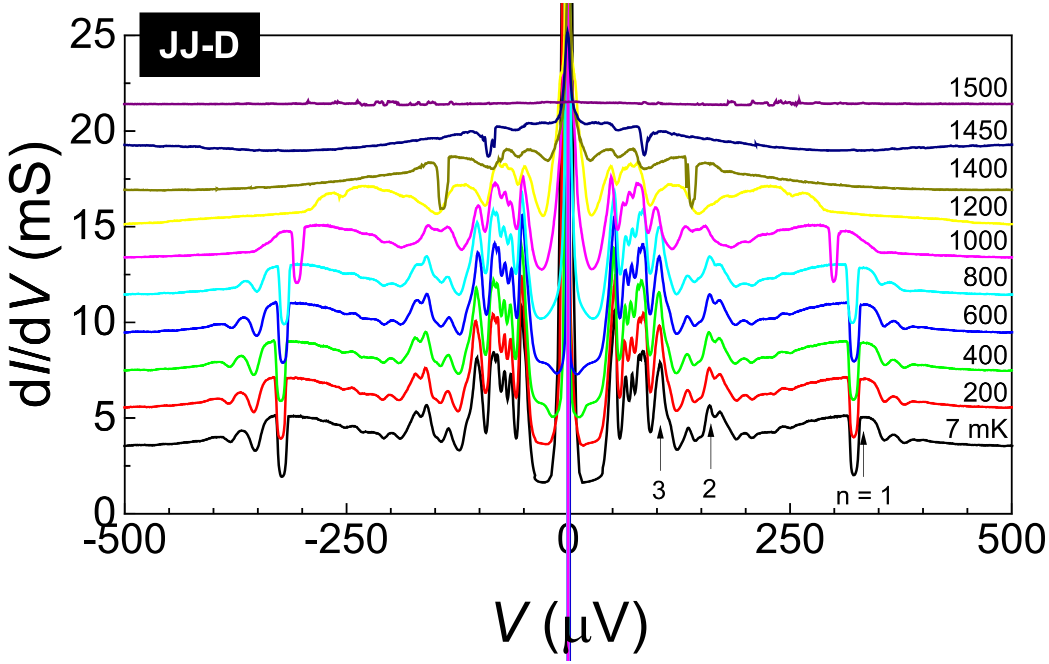



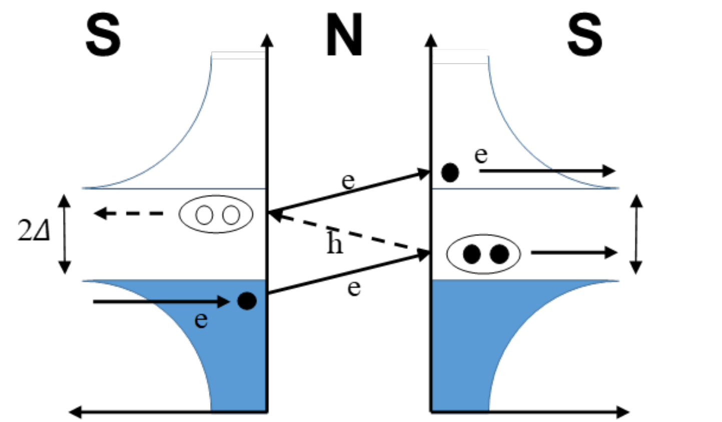

**No bimodal SCD's in 2DEG JJ's under zero B field**

- ✓ τ ~ 0.82
- ✓ First Shapiro step missing without B field
- ✓ Non-bimodal SCD feature

**➔ 4π-periodic Josephson effect by Landau-Zener transition**

# Multiple Andreev reflection (MAR)




**Conductance enhancement at** $eV_n = \frac{2\Delta}{n}$

| Device | $W$ (μm) | $L$ (nm) | $I_c$ (nA) | $R_n$ (Ω) | $I_cR_n$ (μV) | $\Delta_{BCS}$ (μeV) | τ |
|---|---|---|---|---|---|---|---|
| JJ-B | 1.5 | 83 | 418 | 308 | 129 | 162 | > 0.80 |
| JJ-D | 1.55 | 96 | 420 | 289 | 121 | 165 | > 0.82 |

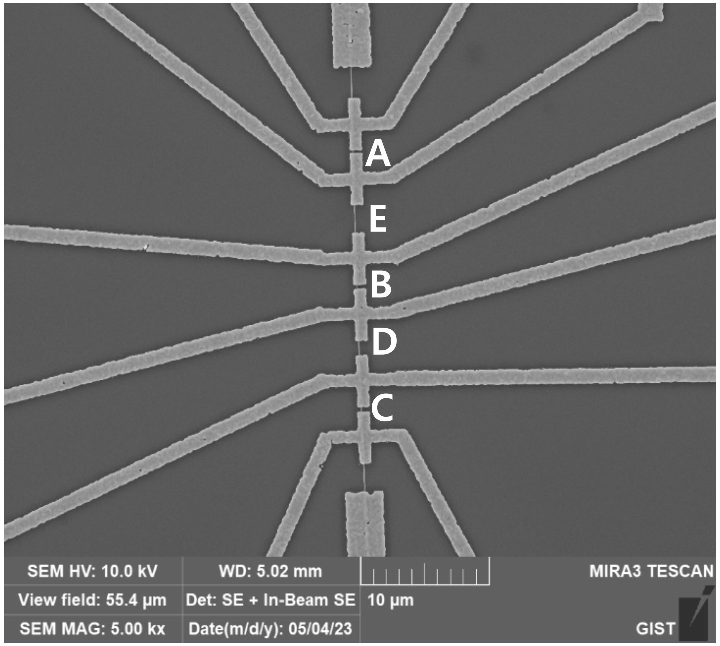
- Mean free path $l_e \sim 200$ nm
- Superconducting coherence length $\xi_0 \sim 740$ nm

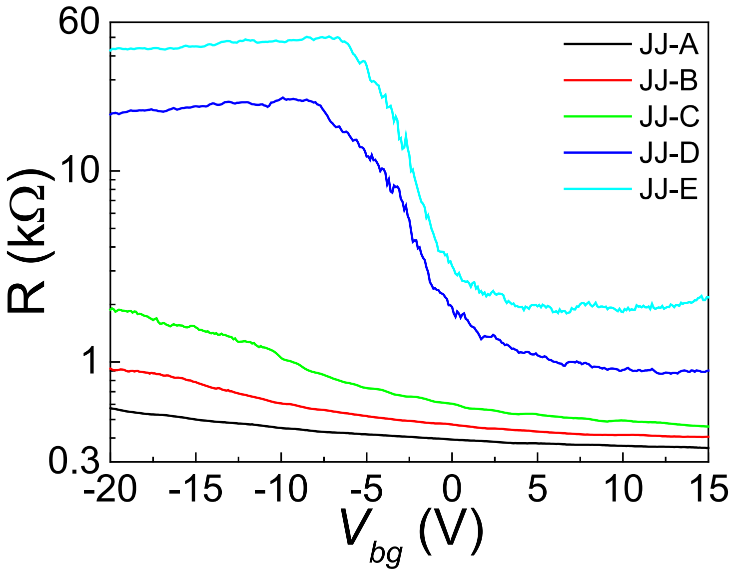
→ **Short and ballistic Josephson junction** $(L < l_e, \xi_0)$

# $Cd_3As_2$ Dirac-semimetal JJ's

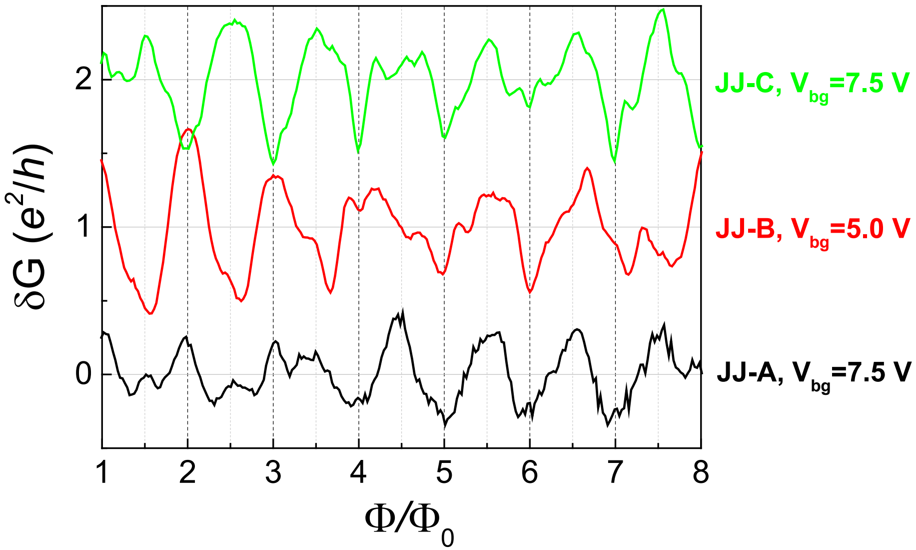


## Growth by prof. Dong Yu at UC Davis



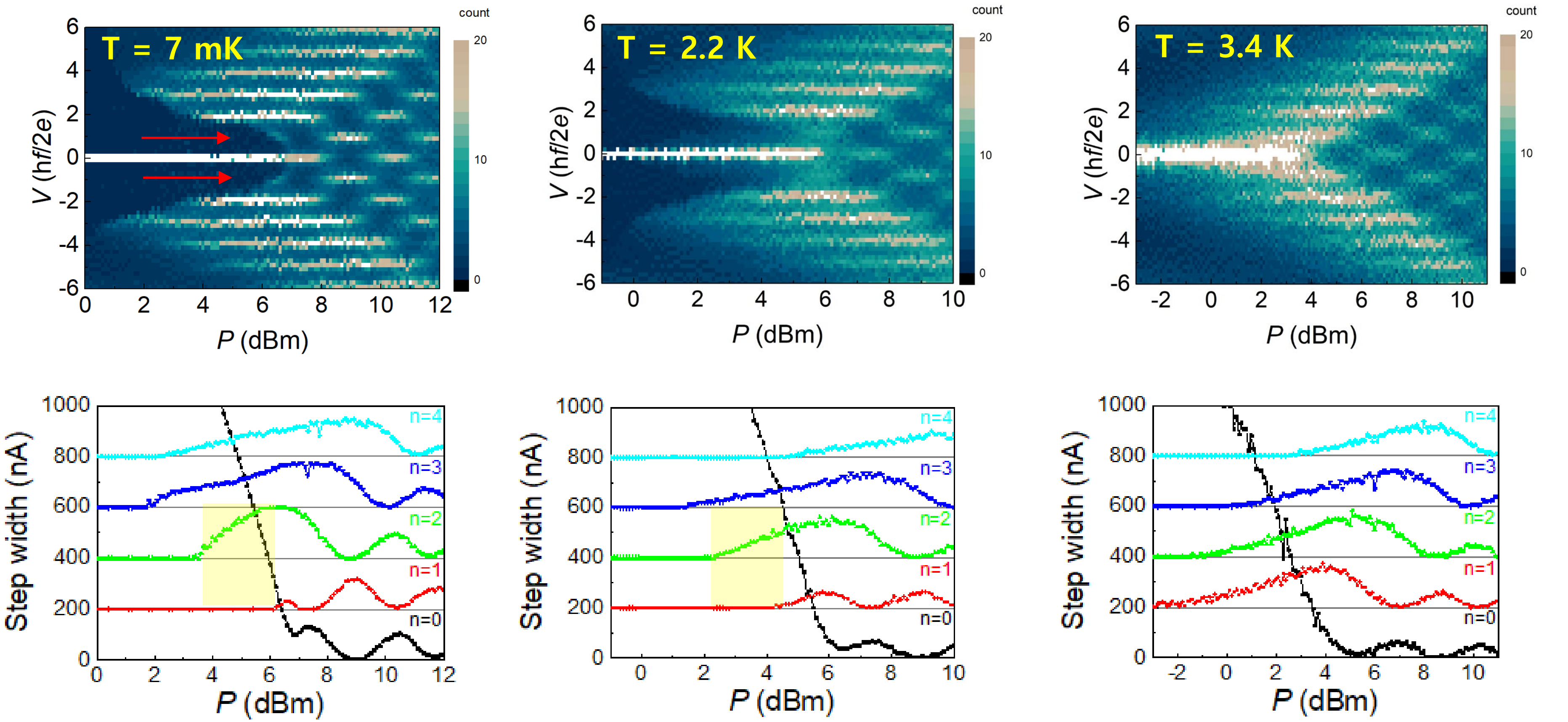




| | JJ-A | JJ-B | JJ-C | JJ-D | JJ-E |
|---|---|---|---|---|---|
| $L$ (nm) | 150 | 264 | 345 | 1050 | 2070 |
| $W$ (nm) | 91 | 92 | 93 | 92 | 92 |
| $t$ (nm) | 86 | 87 | 89 | 88 | 86 |




- PbIn superconducting electrodes
- Aharonov-Bohm oscillations with the axial B-field

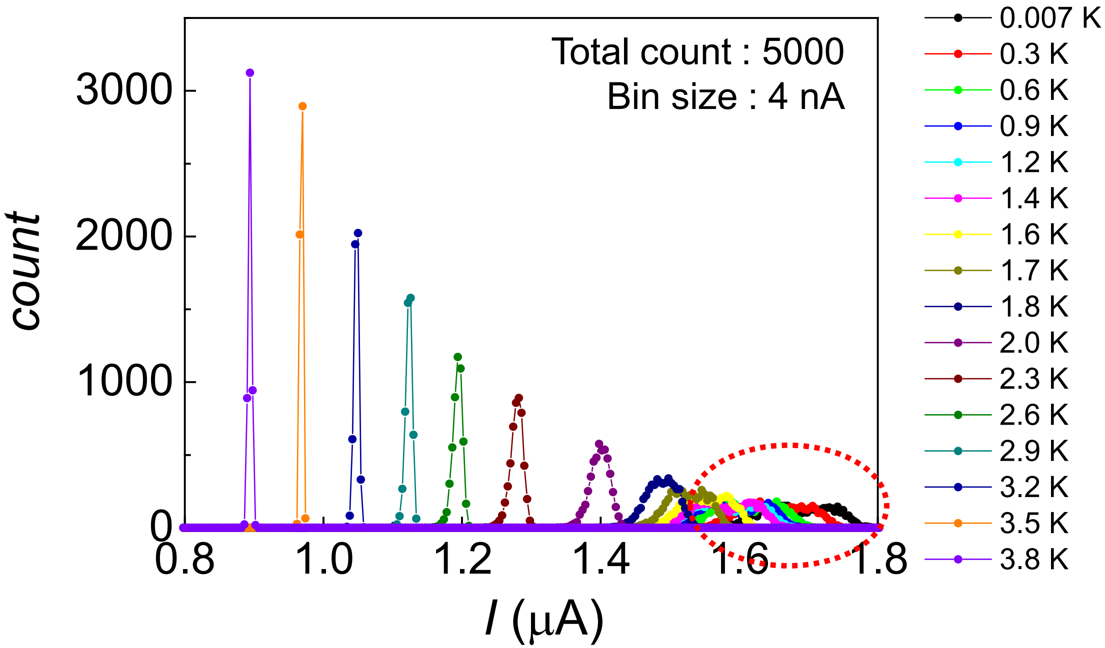
# Temperature dependence of Shapiro steps JJ-A, $f_{rf}$ = 2.52 GHz

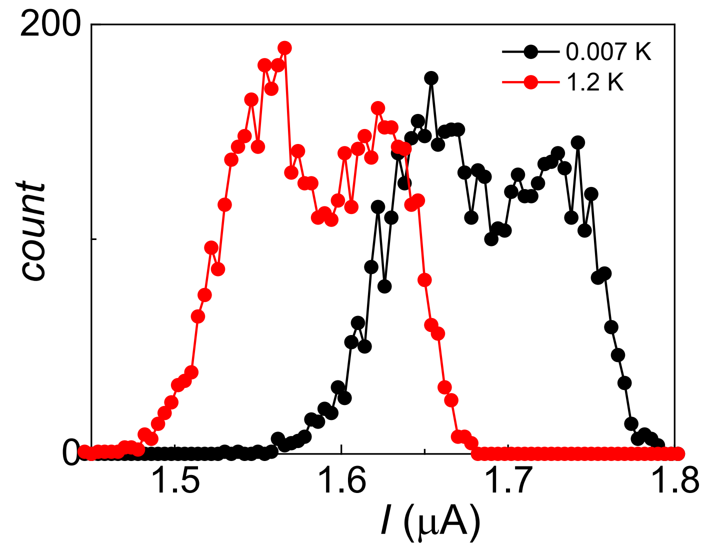



- FSSM behavior observed up to ***T* = 3.0 K** ~ the highest temperature so far

# Switching current distribution ~ Temperature dependence

✓ **JJ-A, T dependence**



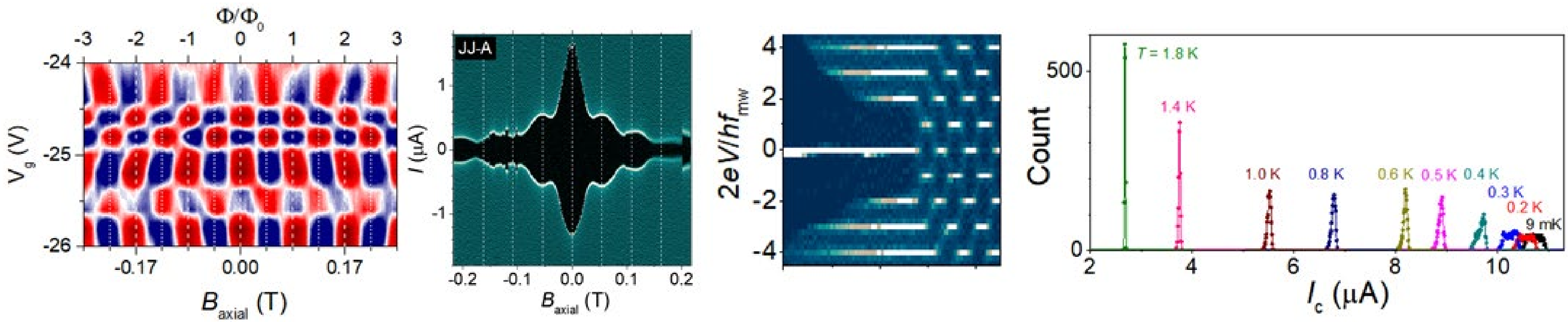




- **Bimodal SCD's are observed up to $T$ ~ 2.0 K**
- **Topological critical current is estimated to be $I_{c,4\pi}/I_{c,2\pi}$ ~ 5.9 %**

# Summary

* yjdoh@gist.ac.kr






- TI NR JJ's exhibited the 1st Shapiro step missing (FSSM) behavior, the so-called fractional Josephson effect, due to the mixed supercurrents.
- We observed bimodal switching current distributions (SCD's) at low temperatures due to the mixed supercurrents. The critical current of 4π-periodic topological supercurrent was estimated to be ~ 250 nA from the bimodal SCD's for the 1st time in our best knowledge.
- Fractional Josephson effect due to Majorana bound states can be confirmed by complementary measurements of Shapiro steps and the SCD's

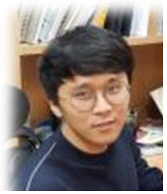
# Group members & Collaborators



**Dr. Hong-Seok Kim** (now at ETRI)
TI NR JJ

**Dr. Tae-Ha Hwang** (now at ETRI)
TI NP JJ

**Dr. Nam-Hee Kim** (now at KRISS)
TI NR JJ



**Mr. Rak-Hee Kim**
DS NW JJ

**Mr. Yeoungmin Jang**
2DEG JJ

**Mr. Hyeongseop Kim**
~ theory (KAIST)

**Prof. H.-S. Sim** (KAIST) ~ theory

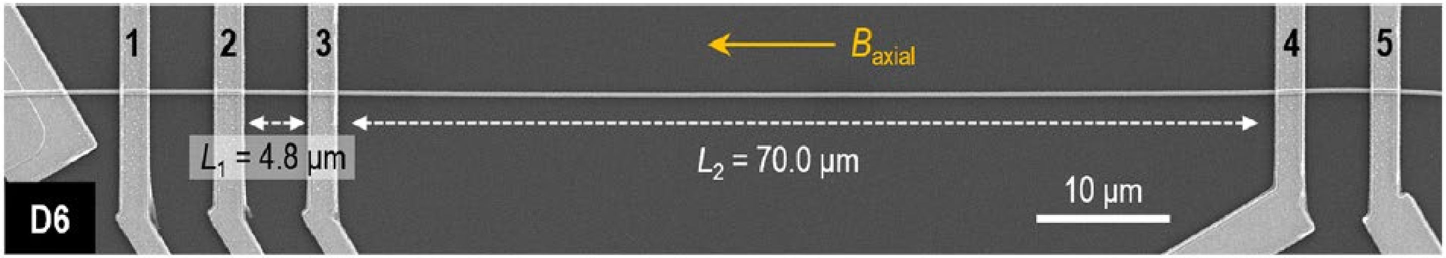
**Dr. Sang-Jun Choi** (now at Wurzburg Univ.) ~ theory

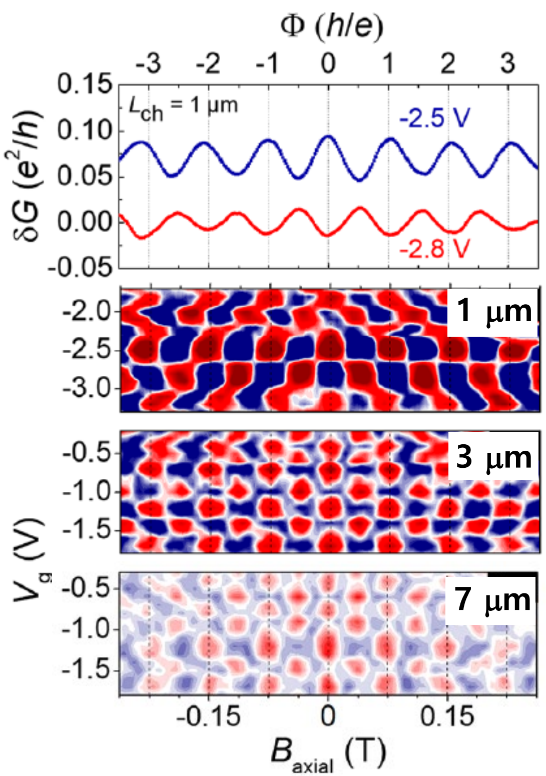
**Prof. Dong Yu** (UC Davis) ~ Growth of TI NR & DSM

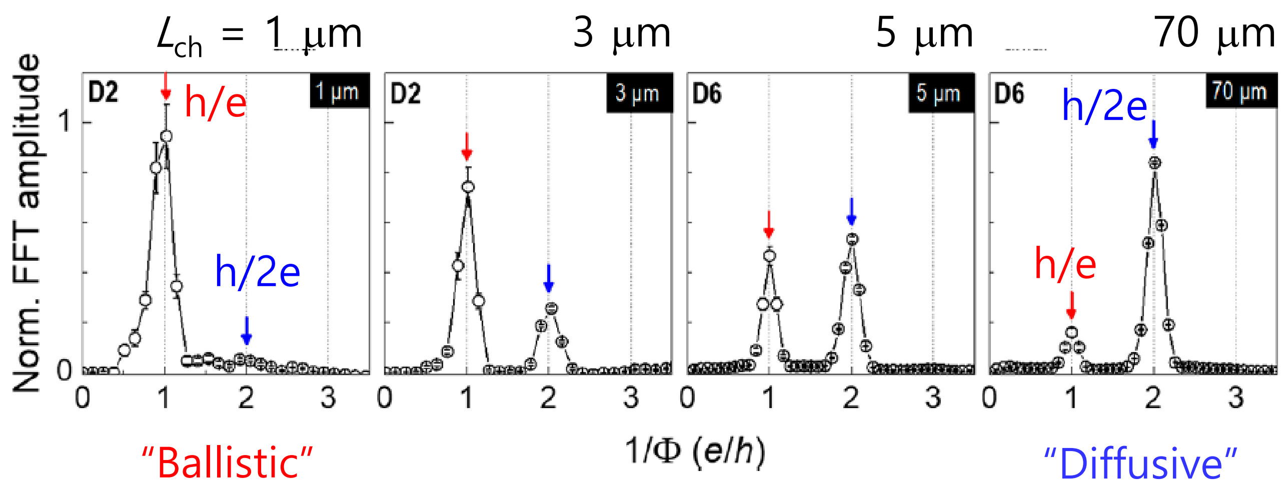
**Prof. Joon Sue Lee** (Univ. Tennessee) ~ 2DEG growth

# Supplementary

# h/e and h/2e oscillations in TI nanoribbon (NR)



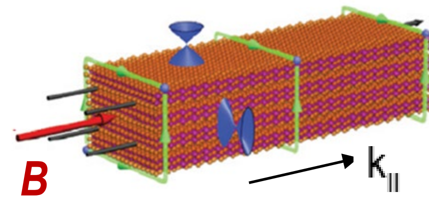




- Quantum AB & AAS oscillations in TI NR with $L_{ch}$ = 1 ~ 70 µm
- h/2e periodic Altshuler-Aronov-Spivak (AAS) oscillations
- Ballistic/diffusive quantum transport is selective with $L_{ch}$

H.-S. Kim*, T.-H. Hwang*, YJD et al., ACS Nano (2020)

# Aharonov-Bohm oscillations in TI nanoribbon



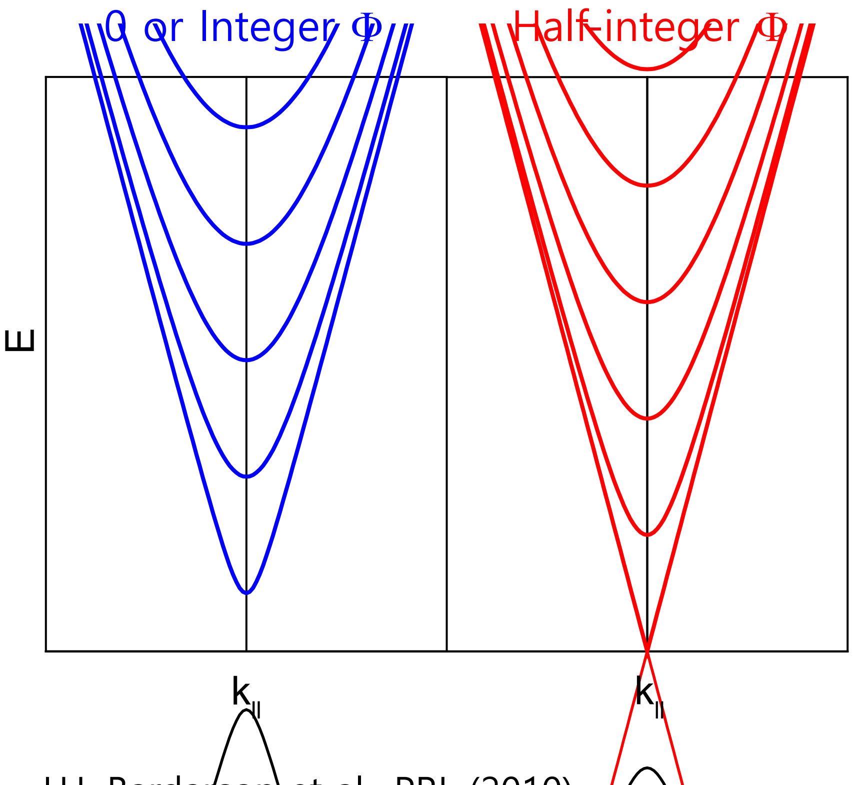

- Dispersion relation for the 1-dimensional subbands in TI NR

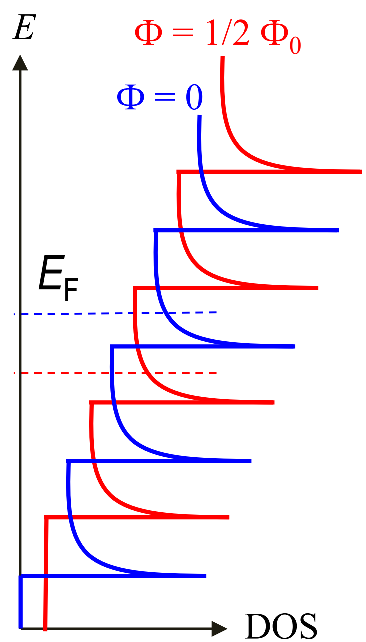
π Berry's phase

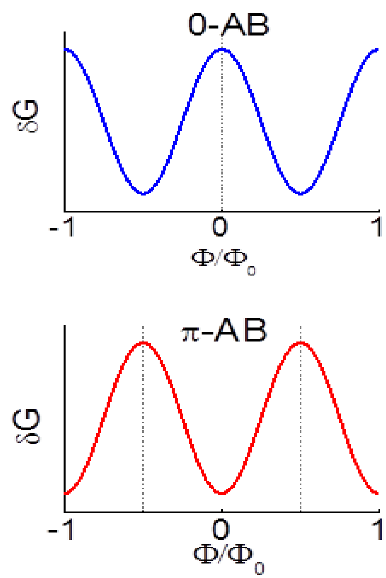
$$E(n,k,\Phi) = \pm\hbar v_{\mathrm{F}}\left[k_{\parallel}^{2} + \frac{4\pi^{2}}{L_{\mathrm{p}}^{2}}(n + 1/2 - \Phi/\Phi_{0})^{2}\right]^{1/2}$$

A-B phase

$\Phi_0 = h/e$ = magnetic flux quantum
$v_F$ = Fermi velocity
$L_p$ = nanoribbon perimeter length
$n$ = 0, 1, 2 ... (integer)



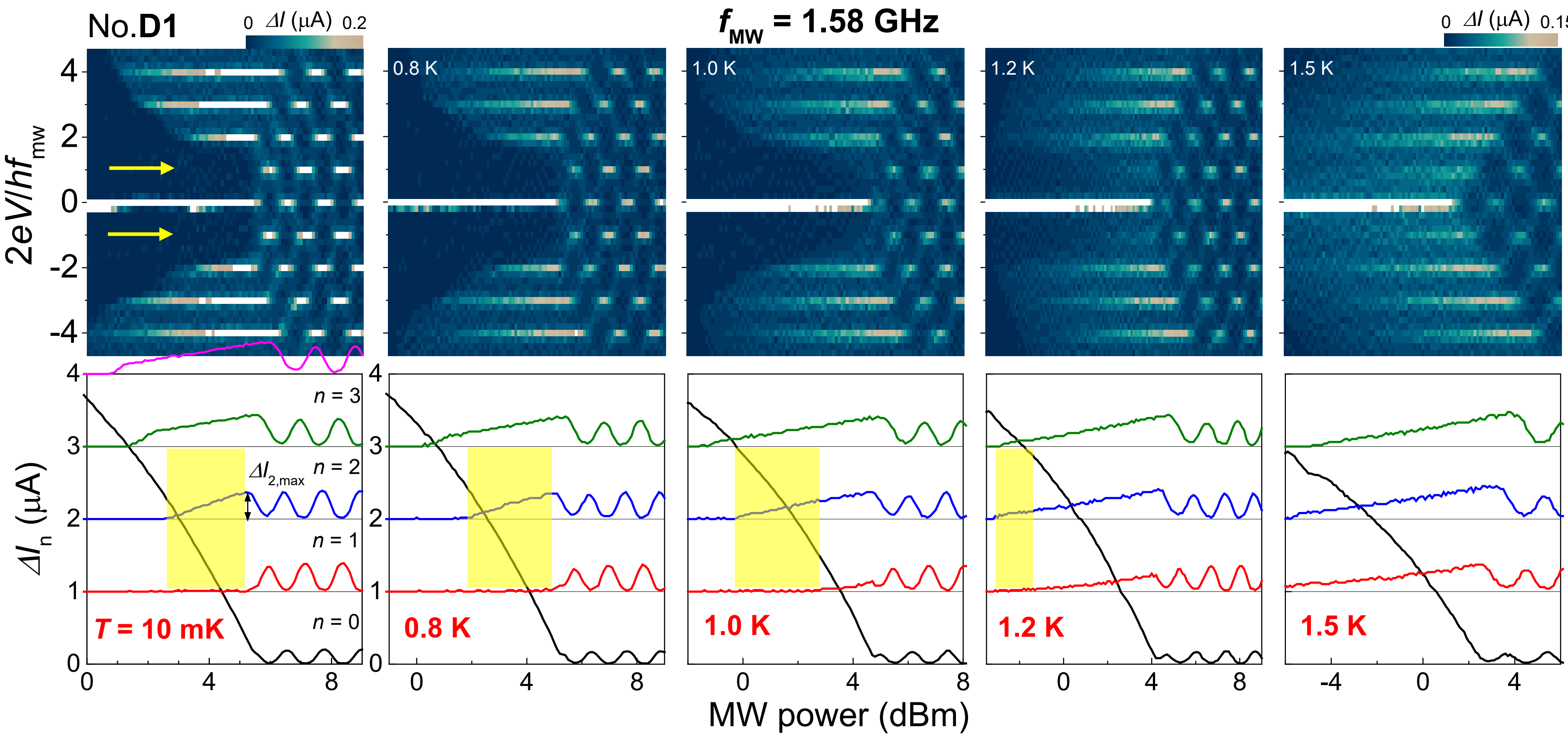




J.H. Bardarson et al., PRL (2010)

ABO phase alternates with the Fermi level or $V_g$.

# Temperature Dependence of FSSM

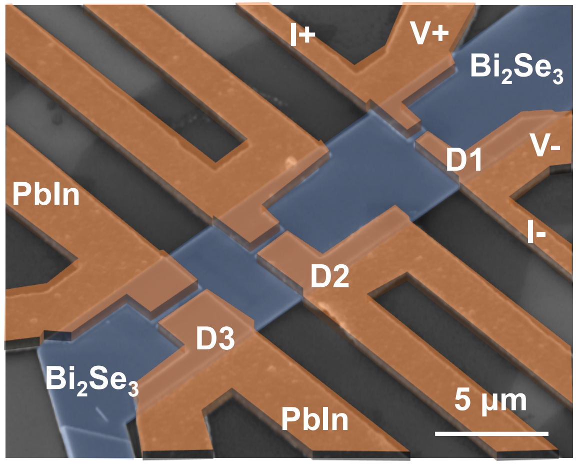
$f_{mw}$ = 1.58 GHz

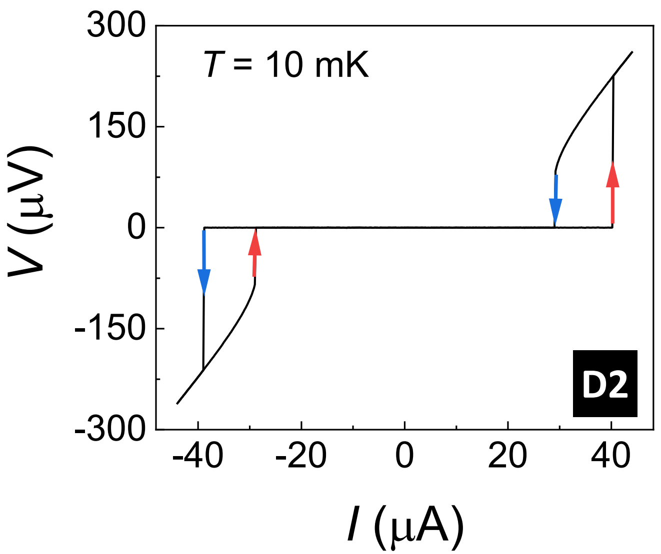



- JJ dev **D1** with $L_{ch}$ = 100 nm, irradiated with $f_{MW}$ = 1.58 GHz
- FSSM is observed up to T* = 1.2 K, the highest record of FSSM temperature so far
- The 1st lobe of the n=1 step is completely absent up to T = 0.8 K

N.-H. Kim, YJD et al. (in preparation)

# Topological insulator (TI) based Josephson junction



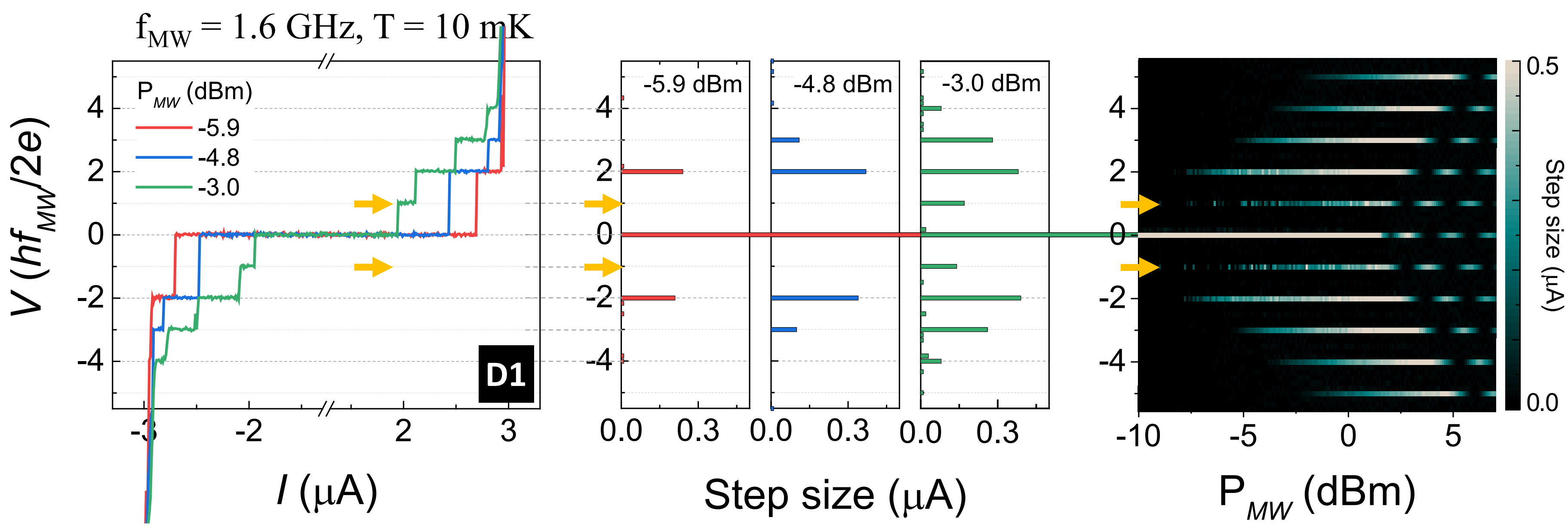




| Device | $W$ (μm) | $L$ (μm) | $I_C$ (μA) | $R_N$ (Ω) | $T_C$ (K) |
|---|---|---|---|---|---|
| **D1** | 1.0 | 210 | 4.3 | 28 | 4.0 |
| **D2** | 2.0 | 210 | 37.8 | 14 | 6.0 |
| **D3** | 3.0 | 210 | 151 | 9.9 | 6.4 |

- TI-based Josephson junction
  - Sb-doped $Bi_2Se_3$ nanoplate + PbIn
- Clear supercurrent branch below critical temperature

Session D5 (PM 14:00)
*Similar experiment with*
*$Bi_2Se_3$ nanoribbon*
*by Dr. Kim Namhee*

# Fractional Josephson effect in TI-based Josephson junction

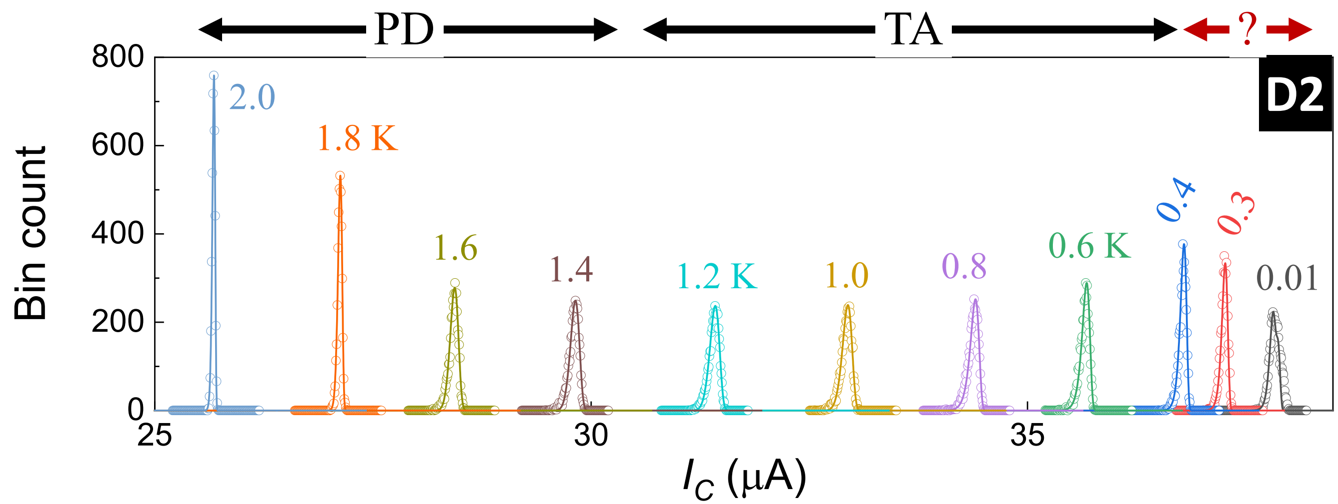


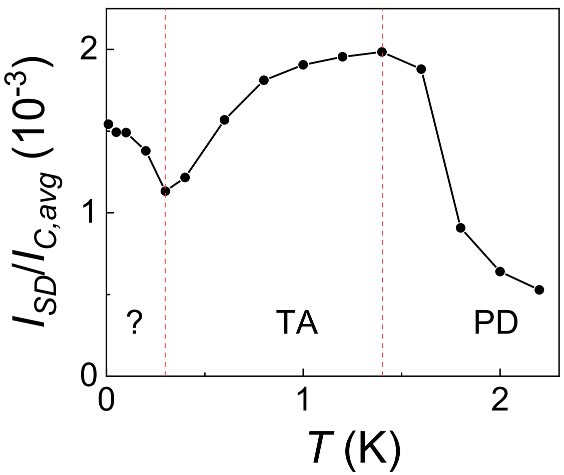

- Shapiro steps with n = ± 1 are missing
- The 1st steps are not fully suppressed

  but, the suppressions are prevalent at low-power microwave regime < ~ 5dBm

  - Superposition of 2π- and 4π-periodic supercurrents

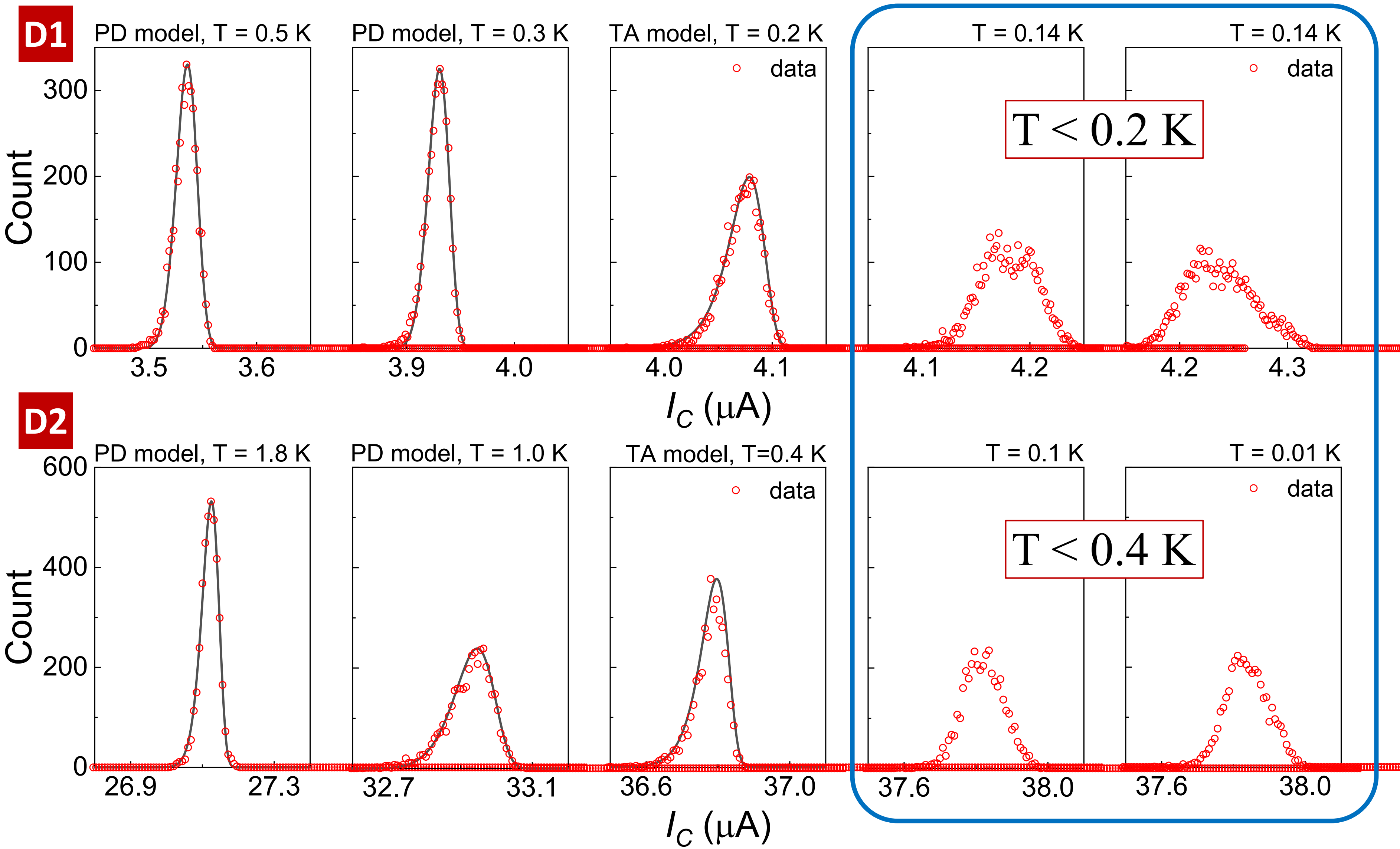
# Switching current distributions in TI-based Josephson junction




- Conventional TA and PD behavior
- Broadened switching current distributions < 0.4 K

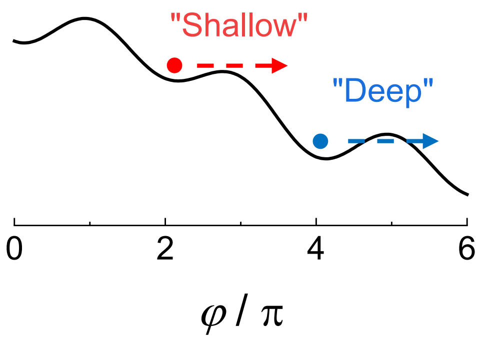
# Broadened switching current distributions

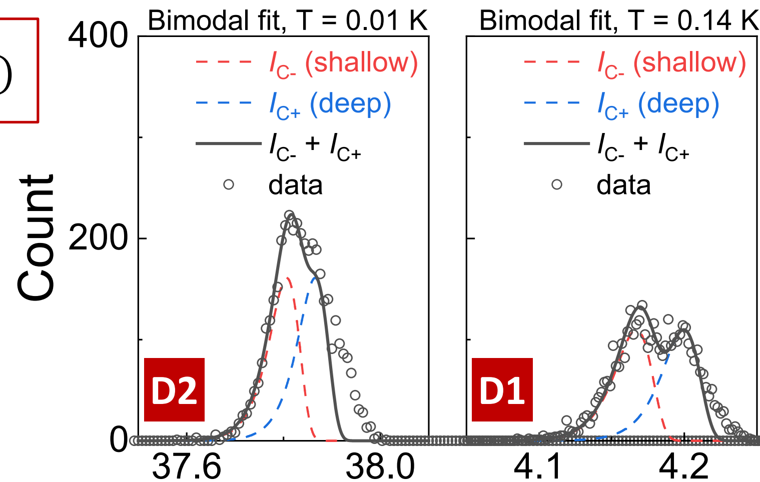


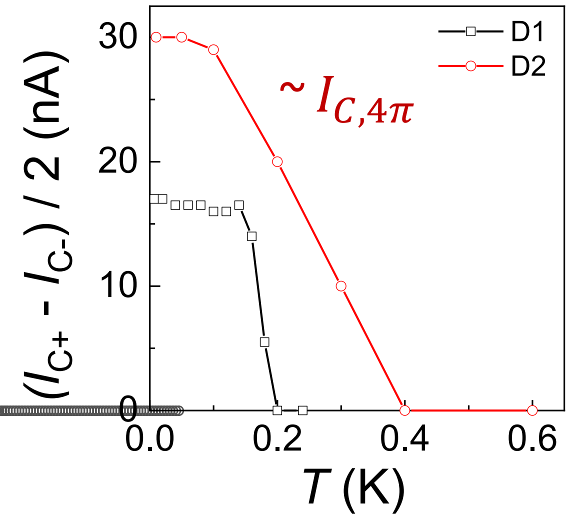

- Distributions in TA and PD regimes follow the each model calculation
- At low *T*, Broadened switching current distributions in different devices

# Bimodal switching current distribution

$$I = I_{C,2\pi}\sin\varphi + \alpha I_{C,4\pi}\sin\left(\frac{\varphi}{2}\right)$$

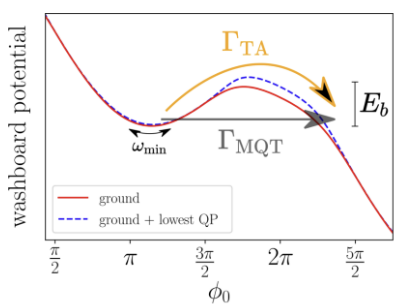


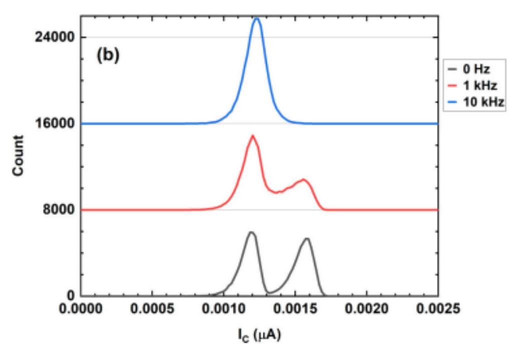




- Mixture of 2π- and 4π-periodic supercurrents
- Two different potential wells, *"shallow"* and *"deep"*
- Bimodal distribution with two critical currents, $I_{C-}$ and $I_{C+}$
- $I_{C,4\pi} \sim (I_{C+} - I_{C-})$ / 2

$$\frac{I_{C,4\pi}}{I_{C,total}} \sim \begin{matrix} 0.4\ \%\ (D1) \\ 0.1\ \%\ (D2) \end{matrix}$$

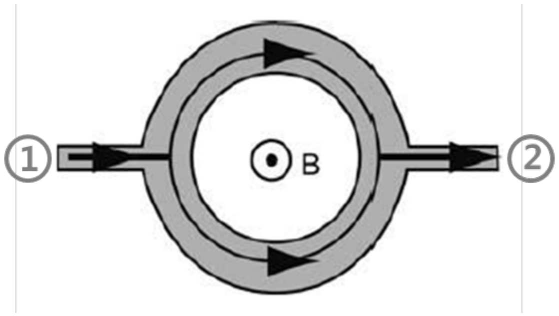



*"electron parity-dependent bimodal switching current distribution, reminiscent of the fractional Josephson effect"*

*- N. Abbound, D. Van Harlingen et al., Phys. Rev. B. 105, 214521 (2022)*

# Aharonov-Bohm (AB) oscillations



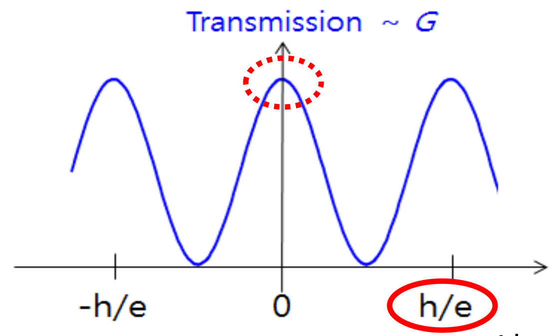

Phase difference between two partial waves along the ring,

$$\delta\phi_{\mathrm{AB}} = \frac{e}{\hbar}\int_{1,up}^{2}\vec{A}\cdot d\vec{r} - \frac{e}{\hbar}\int_{1,low}^{2}\vec{A}\cdot d\vec{r} = \frac{e}{\hbar}\oint\vec{A}\cdot d\vec{r} = \frac{e}{\hbar}\int\nabla\times\vec{A}\cdot\vec{s} = 2\pi\frac{\Phi}{\Phi_0}$$

$\Phi$ = magnetic flux, $\Phi_0 \equiv \frac{h}{e} = 4.1\times10^{-15}$ Wb ~ flux quantum

Total transmission probability maximized at

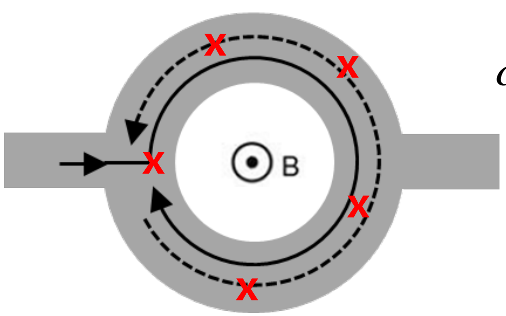
$$\Phi = n\frac{h}{e} \quad (n : \text{integer})$$




Aharonov & Bohm (1959)

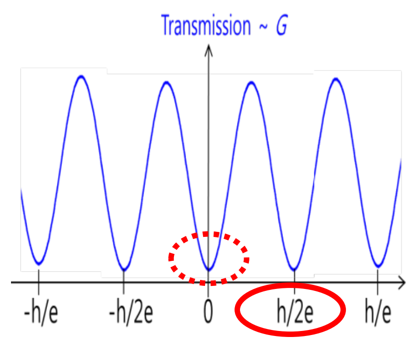
## AAS oscillations

For a diffusive system with time-reversal symmetry preserved,




$$\delta\phi_{\mathrm{AAS}} = \phi_{clockwise} - \phi_{counter-clockwise}$$

$$= 4\pi\frac{\Phi}{\Phi_0} = 2\pi\frac{\Phi}{\left(\frac{\Phi_0}{2}\right)}$$




Altshuler, Aronov & Spivak (1981)