Non-symmetric diamond chains with magnetic and electric fields: flat bands, edge states and topology

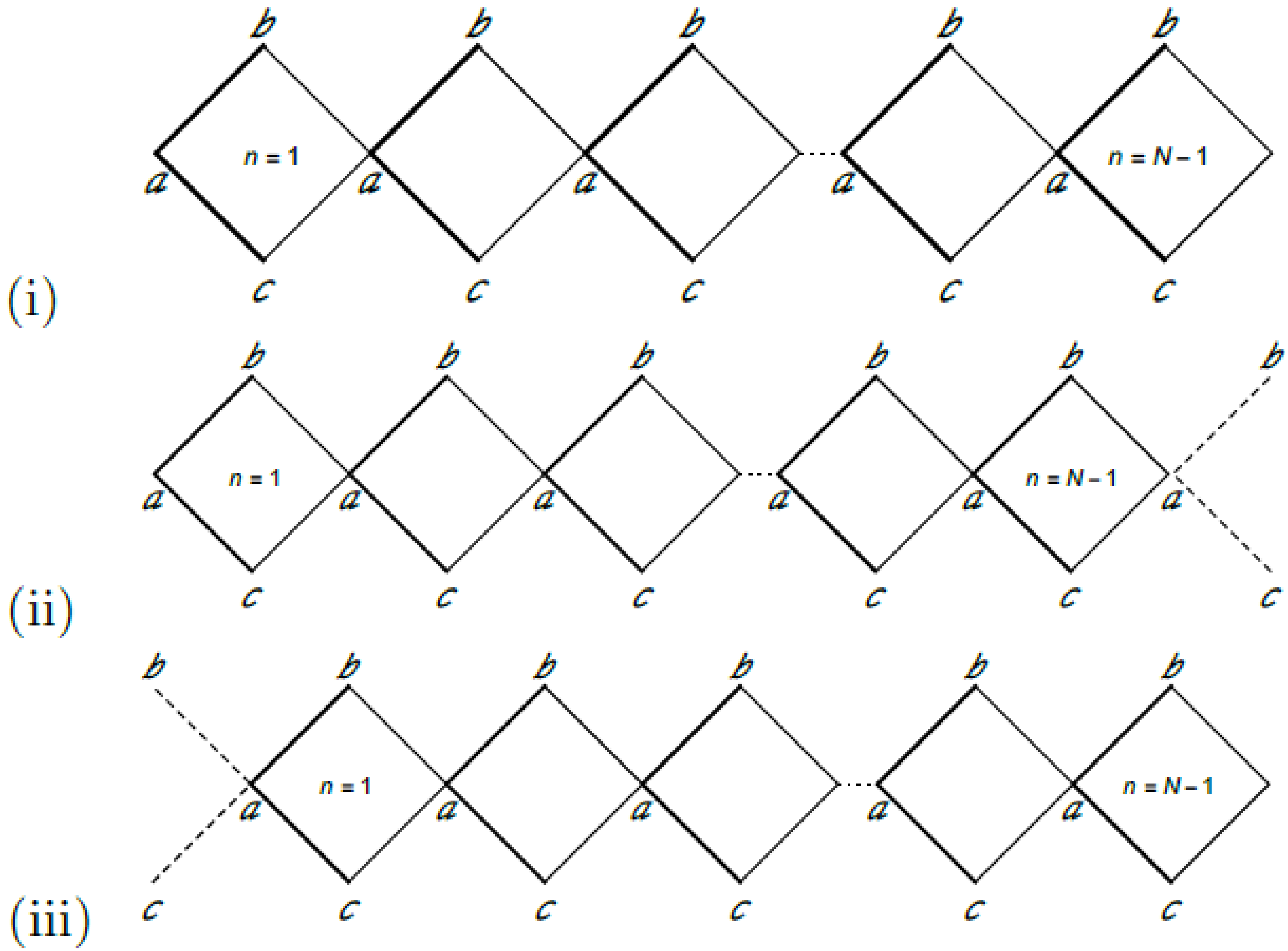
Amnon Aharony and Ora Entin-Wohlman

TEL AVIV UNIVERSITY אוניברסיטת תל-אביב

Also with Yasuhiro Tokura and Shingo Katsumoto (2008-2010) and Ovadya Bettoun

IBS conference on “Flatbands: symmetries, disorder, interactions and thermalization”

DIAMOND CHAINS



Disclosure: work in progress, no full literature review, appreciate advice

Outline

- History: Aharonov-Bohm cages, Spin-orbit cages, spin filtering
- Su-Schrieffer-Heeger (SSH) model – topology
- Tight-binding equations for diamond chains
- Limits: decoupled trimers and edge states
- Flat bands – Compact localized states (CLS's)
- Dispersive bands
- Edge states
- Scattering
- Outlook

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The Aharonov-Bohm (**AB**) effect

Magnetic field generates AB phase $\Delta\Phi = \frac{1}{\hbar}\int L dt = \frac{1}{\hbar}\int \left[\mathbf{p} + \frac{e}{c}\mathbf{A}\right] \cdot d\mathbf{s}$

$$\Rightarrow \quad \psi \to e^{i\frac{eA}{\hbar c}x}\psi$$

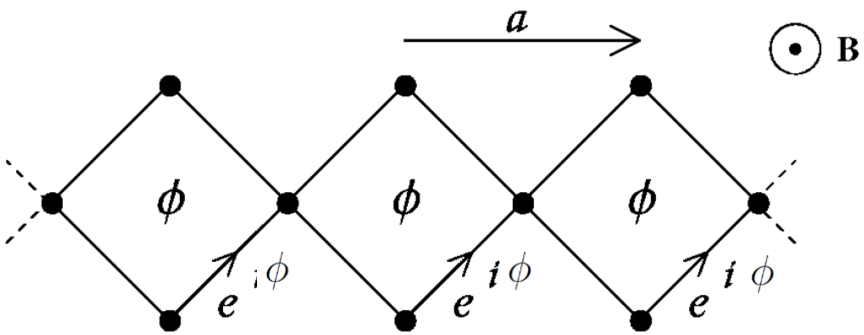
Periodic diamond chains – AB cages

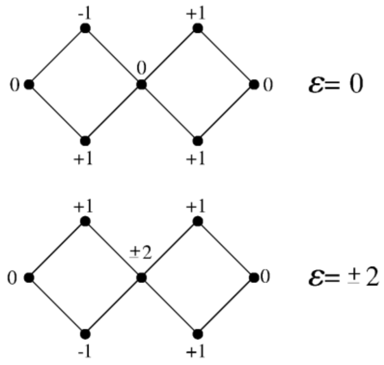
VOLUME 85, NUMBER 18 PHYSICAL REVIEW LETTERS 30 OCTOBER 2000

CLS

Interaction Induced Delocalization for Two Particles in a Periodic Potential

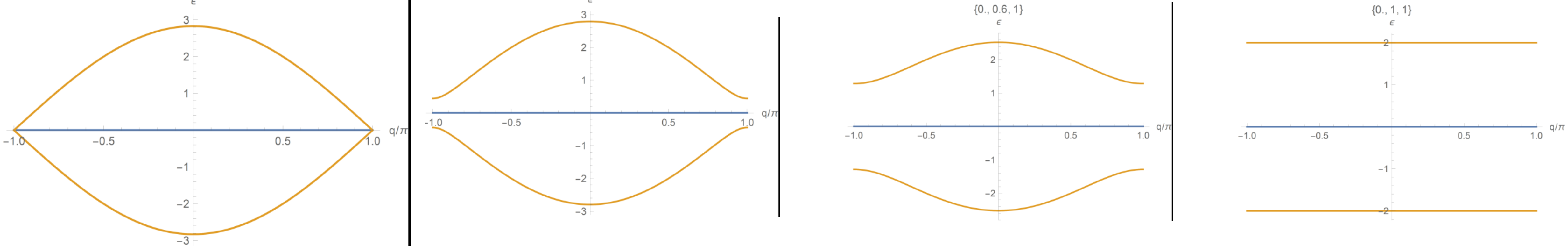
Julien Vidal,[1] Benoît Douçot,[2] Rémy Mosseri,[3] and Patrick Butaud[4]





Aharonov-Bohm cages – flat bands

ϵ = Fermi level



The Aharonov-Casher (**AC**) effect

$$\mathbf{E} = -\boldsymbol{\nabla} V = E\hat{\mathbf{n}}$$

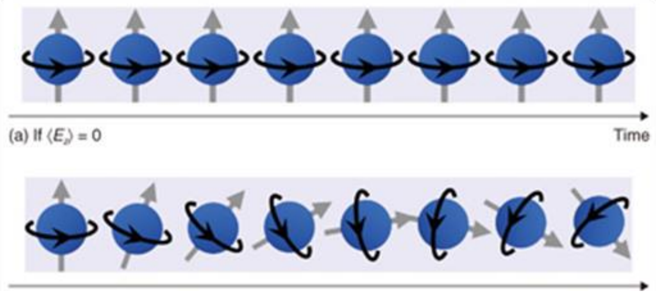
Rashba spin-orbit interaction in a plane

$$\mathcal{H}_R = \frac{\hbar k_{\rm so}}{m^*}\hat{\mathbf{n}} \cdot [\boldsymbol{\sigma} \times \mathbf{p}]$$

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m^*} + \mathcal{H}_R = \frac{\left(\mathbf{p} + \hbar k_{\rm so}[\hat{\mathbf{n}} \times \boldsymbol{\sigma}] \cdot \mathbf{p}\right)^2}{2m^*}$$

generates the AC phase,

$$e^{i\mathbf{k}\cdot\mathbf{R}}|\chi\rangle \quad \Rightarrow \quad e^{i[\mathbf{k}\cdot\mathbf{R}}e^{ik_{\rm so}[\hat{\mathbf{n}}\times\boldsymbol{\sigma}]\cdot\mathbf{R}}|\chi\rangle$$

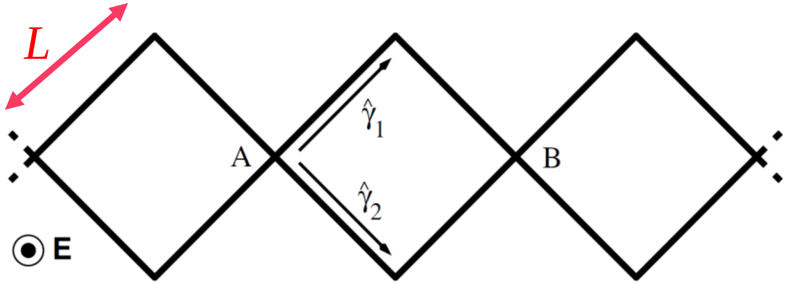


Periodic diamond chains – Aharonov-Casher AC cages

VOLUME 93, NUMBER 5 PHYSICAL REVIEW LETTERS week ending 30 JULY 2004

Rashba-Effect-Induced Localization in Quantum Networks

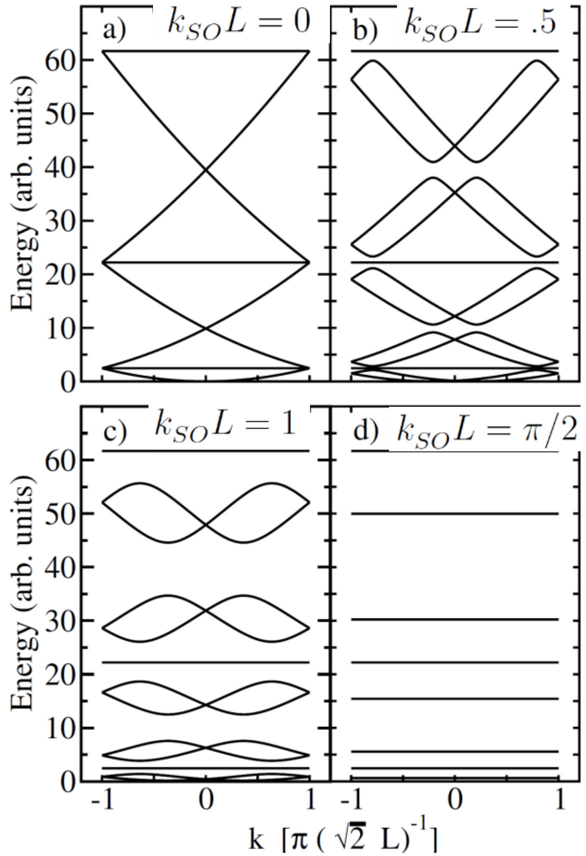
Dario Bercioux,[1] Michele Governale,[2] Vittorio Cataudella,[1] and Vincenzo Marigliano Ramaglia[1]

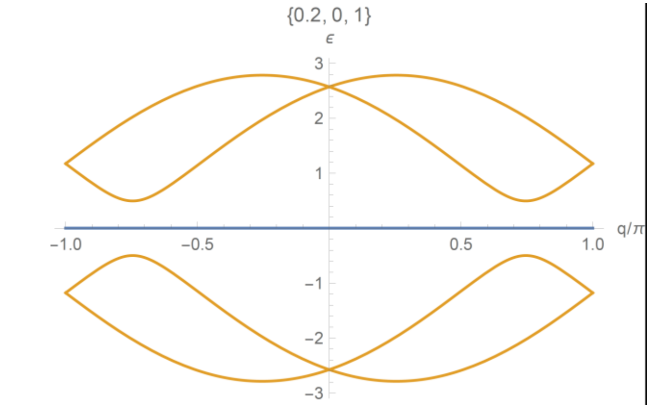


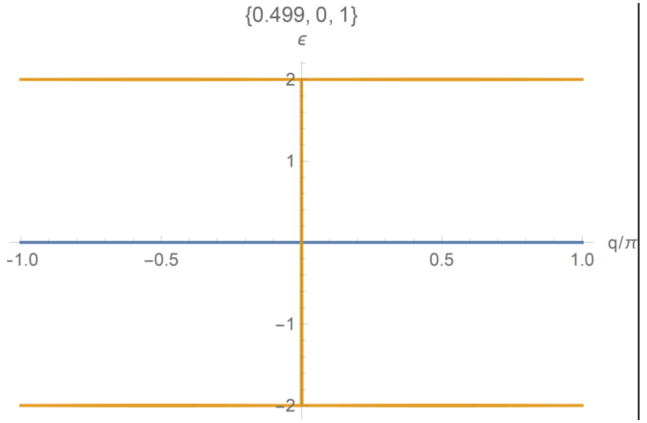
$$\mathcal{H} = \frac{p_\gamma^2}{2m} - \frac{\hbar k_{\rm SO}}{m} p_\gamma (\vec{\sigma} \times \hat{z}) \cdot \hat{\gamma},$$

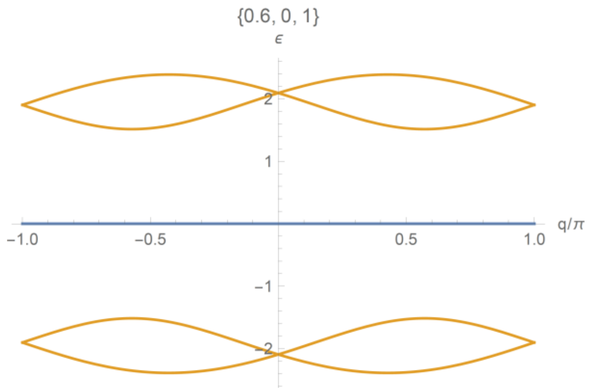
$$\alpha = k_{\rm so} L$$

Note: time-reversal symmetric









PHYSICAL REVIEW B **78**, 125328 (2008)

Spin filtering by a periodic spintronic device

Amnon Aharony,[1,*] Ora Entin-Wohlman,[1,*] Yasuhiro Tokura,[2] and Shingo Katsumoto[3]

Periodic diamond chains

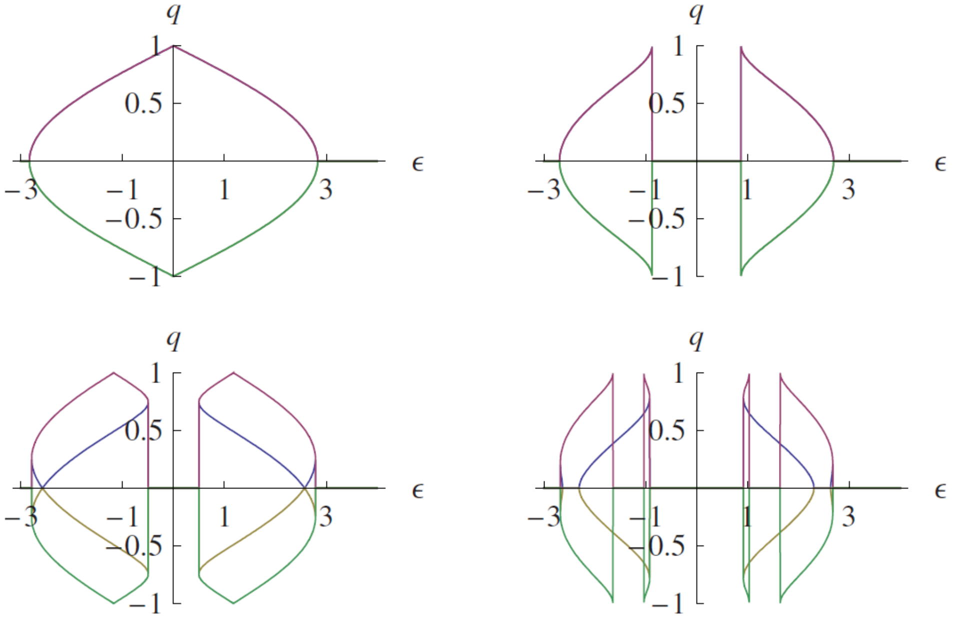
AB+AC cages

(ask about AA)

$\alpha = k_{so} L$

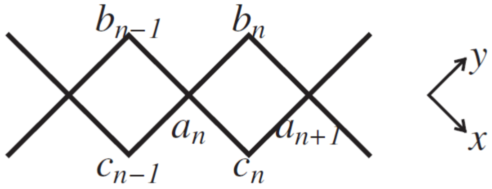
$\alpha = 0$

$\alpha = .2\pi$



ϕ=0.

ϕ=0.4π.



Each band has definite spin

Full ballistic spin filtering

Need both AB and SOI for filtering!

Contents lists available at ScienceDirect

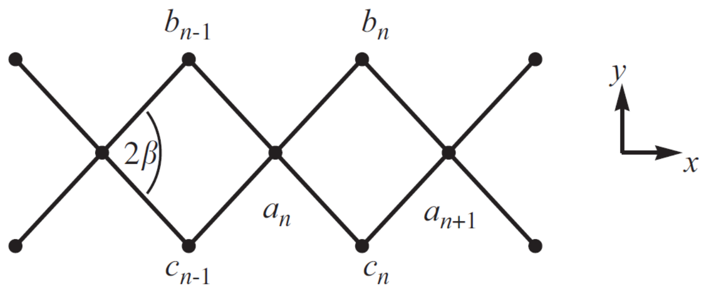
Physica E

journal homepage: www.elsevier.com/locate/physe

2010

Spin filtering due to quantum interference in periodic mesoscopic networks

Amnon Aharony[a,*,1], Ora Entin-Wohlman[a,1], Yasuhiro Tokura[b], Shingo Katsumoto[c]



$$\epsilon_a = \epsilon_c = 0, \quad \epsilon_b = .5$$

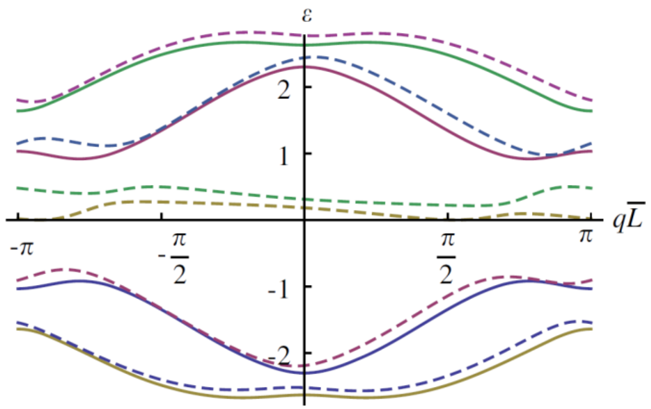


Fig. 2. Spectra for $\beta = \pi/4$, $\phi = .4\pi$, $\alpha = .2\pi$. Four full lines: $\varepsilon_b = \varepsilon_c = 0$. Six dashed lines: $\varepsilon_b = .5$, $\varepsilon_c = 0$. All energies are in units of J.

For asymmetric diamonds- flat band splits and becomes dispersive

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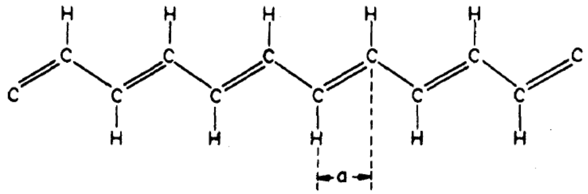
Volume 42, Number 25 PHYSICAL REVIEW LETTERS 18 June 1979

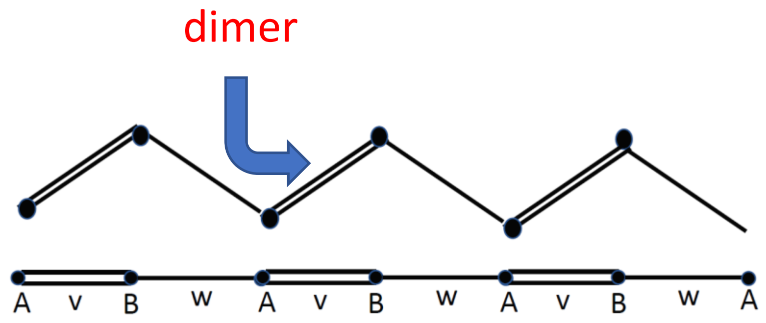
Solitons in Polyacetylene

W. P. Su, J. R. Schrieffer, and A. J. Heeger

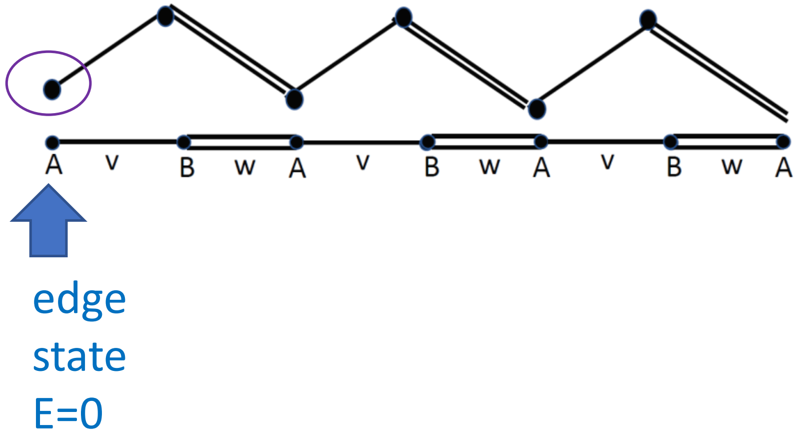


Su-Schrieffer-Heeger (SSH) model

Trivial for v>w – no edge states



Topological for v<w – edge states



$$H = v\sum_n (A_n^\dagger B_n + B_n^\dagger A_n) + w\sum (A_{n+1}^\dagger B_n + B_n^\dagger A_{n+1}).$$

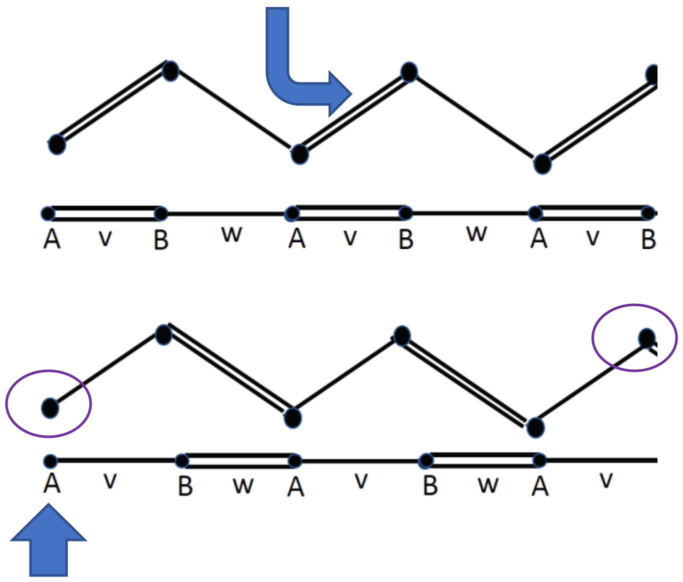
Topology: edge states are **robust**

SSH model

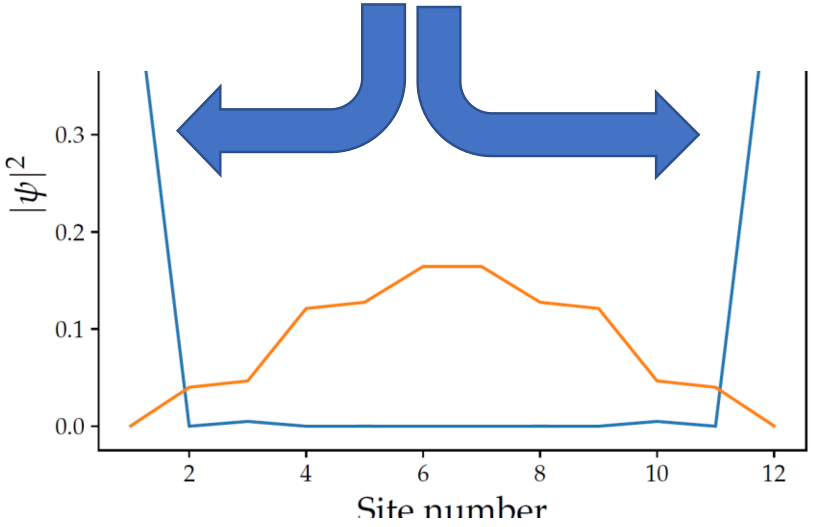
$$H = v\sum_n (A_n^\dagger B_n + B_n^\dagger A_n) + w\sum (A_{n+1}^\dagger B_n + B_n^\dagger A_{n+1}).$$

$$\gamma = v/w$$

dimer

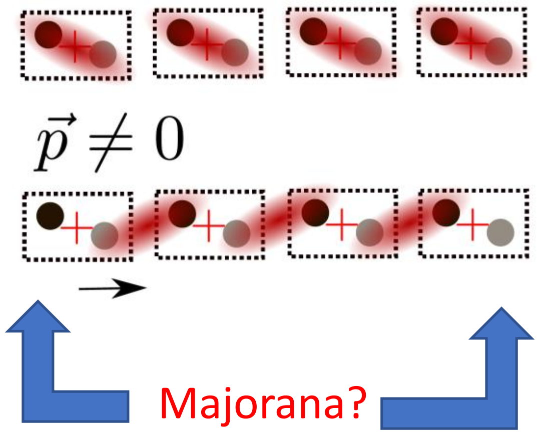


Edge state, E=0



$\vec{p} = 0$

$\vec{p} \neq 0$



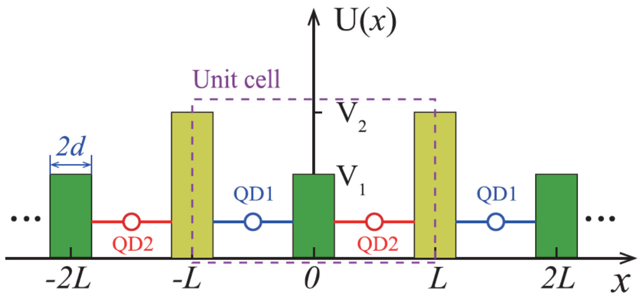
Majorana?

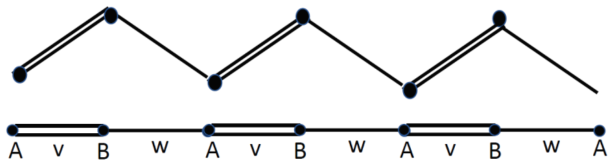
PHYSICAL REVIEW B **104**, 085302 (2021)

published 6 August 2021

Topological states and interplay between spin-orbit and Zeeman interactions in a spinful Su-Schrieffer-Heeger nanowire

Zhi-Hai Liu,[1,*] O. Entin-Wohlman,[2] A. Aharony,[2] J. Q. You,[3] and H. Q. Xu[1,4,†]

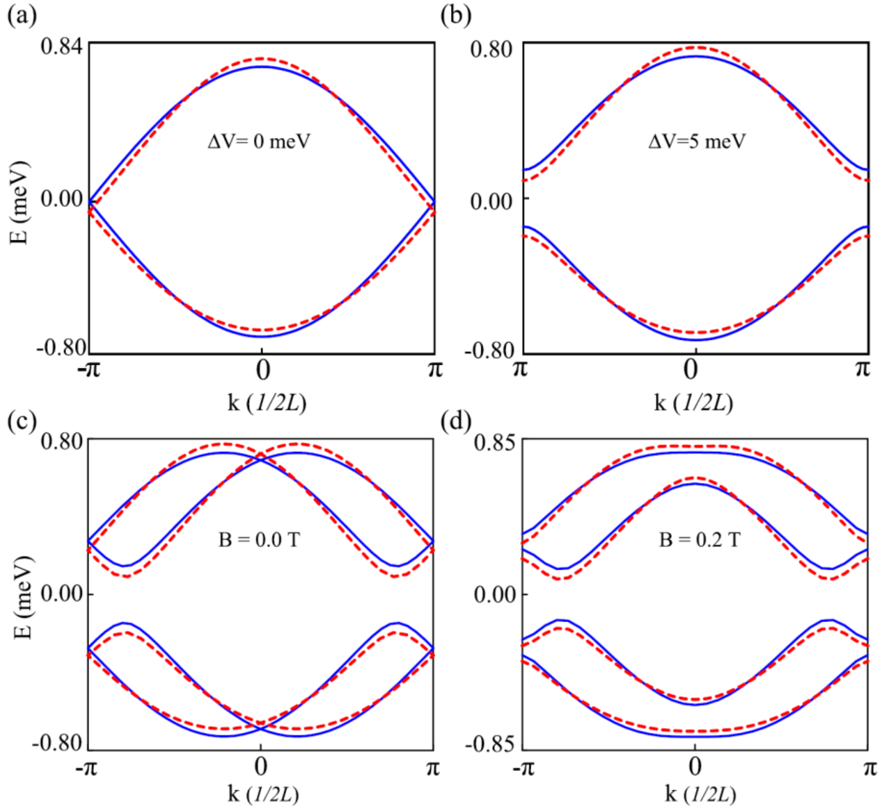




$$H_T = \sum_n (a_n^\dagger t_{\rm in} b_n + a_{n+1}^\dagger t_{\rm ex} b_n + \text{H.c.}) + \frac{\Delta_z}{2}\sum_n (a_n^\dagger \sigma_z a_n + b_n^\dagger \sigma_z b_n),$$

$$H = \frac{p^2}{2m_e} - V_c + U(x) + \alpha p \sigma_y + \frac{\Delta_z}{2}\sigma_\varsigma$$

Zeeman



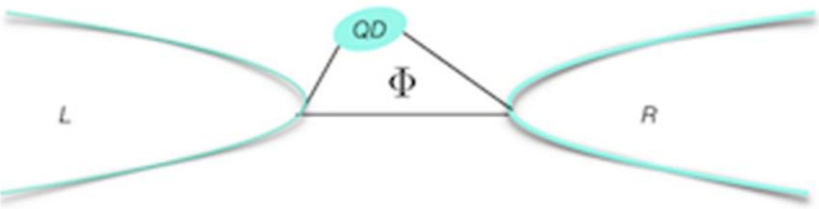
Continuum vs tight binding

PHYSICAL REVIEW RESEARCH **1**, 033112 (2019)

Editors' Suggestion

Spin geometric phases in hopping magnetoconductance

O. Entin-Wohlman* and A. Aharony



Tunneling matrix for a straight segment with a magnetic field

$$U = -\pi m^* a e^{-as}\left(\cos(k_2 s) + \frac{\sin(k_2 s)}{k_2}[ik_{so}\hat{\mathbf{e}}\cdot\boldsymbol{\sigma} + m^* B a \sigma_z]\right).$$

Non-unitary!

AA+AB+AC

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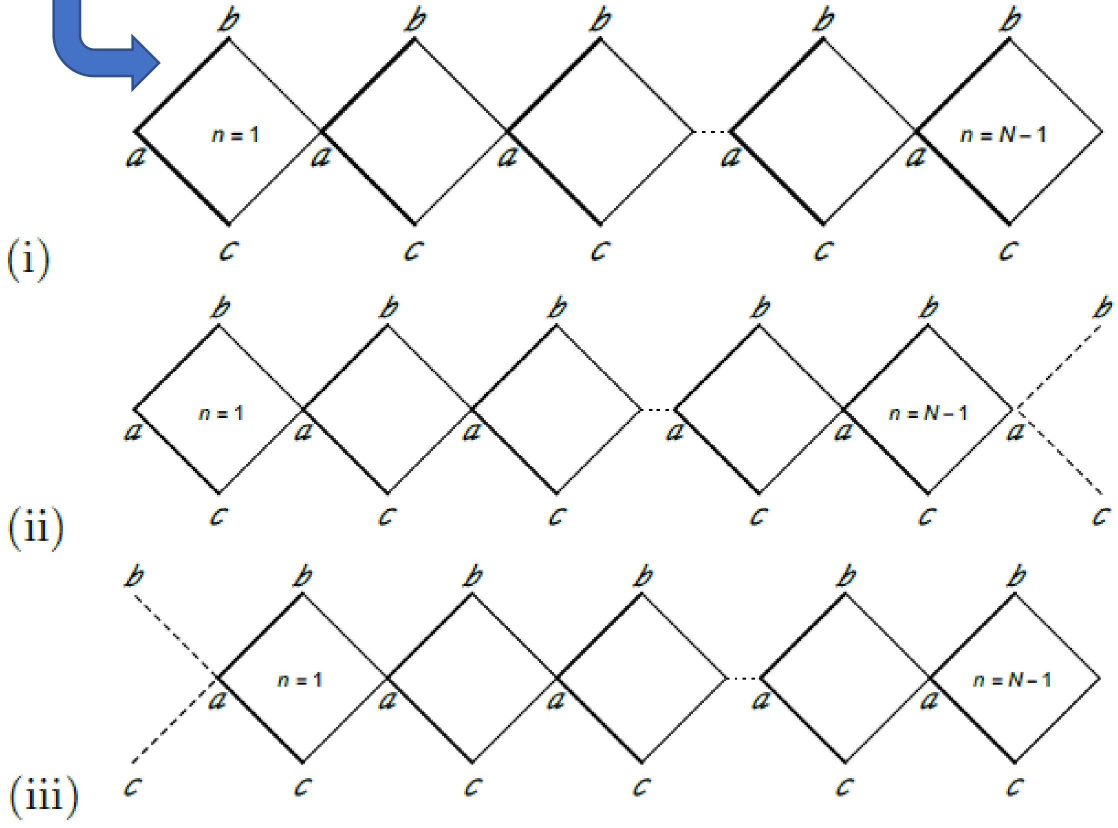
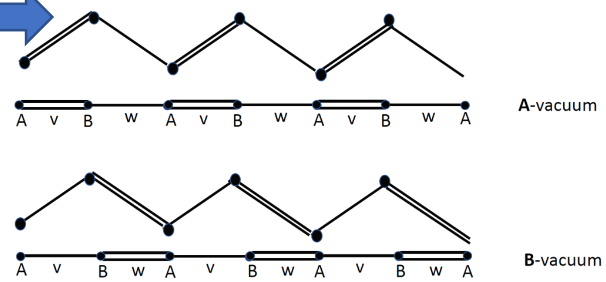
OUR DIAMOND CHAINS

trimer



dimer



We solve with AB, AC, SSH, Zeeman

AC

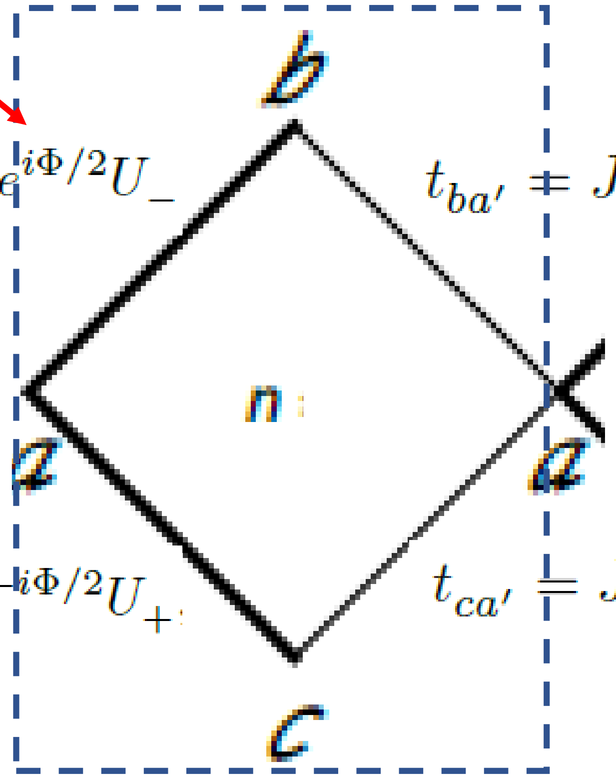
$$U_\pm = e^{i\alpha\sigma_\pm} \equiv \cos(\alpha) + i\sin(\alpha)\sigma_\pm \qquad \sigma_\pm = (\sigma_y \pm \sigma_x)/\sqrt{2},$$

AB

$t_{ab} = \gamma J e^{i\Phi/2} U_-$ $\qquad t_{ba'} = JU_+$

SSH

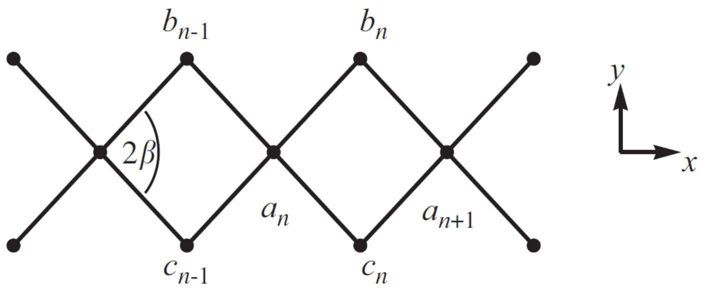
$t_{ac} = \gamma J e^{-i\Phi/2} U_+$ $\qquad t_{ca'} = JU_-$



Unit cell –

3 spinors,

6 components



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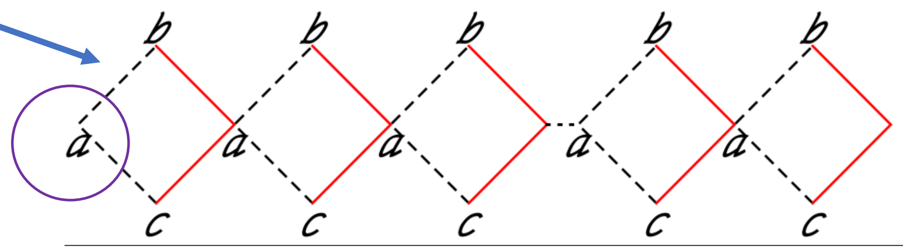
Limits:

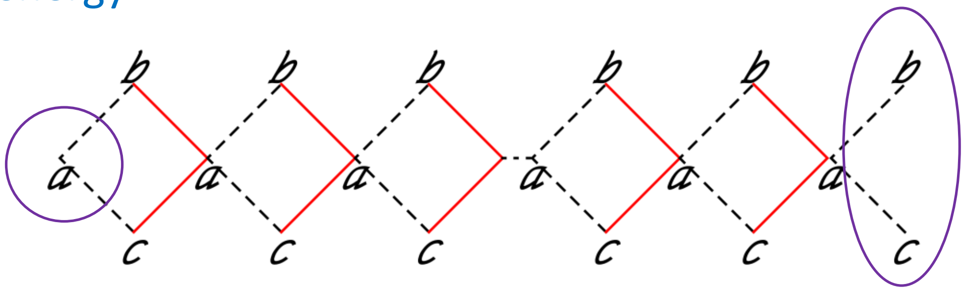
$$\gamma = 0$$

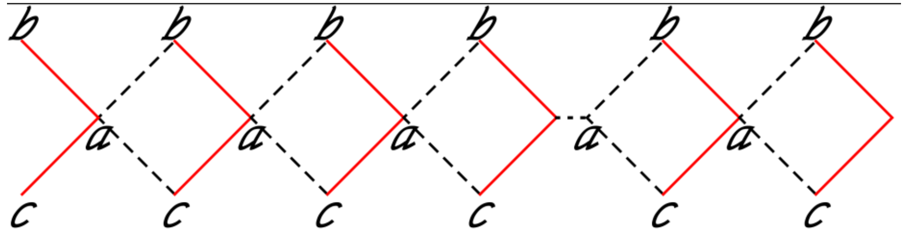
$\epsilon_b = \epsilon_c = 0, \quad B = 0$



Decoupled trimers

Edge states
with 0 energy



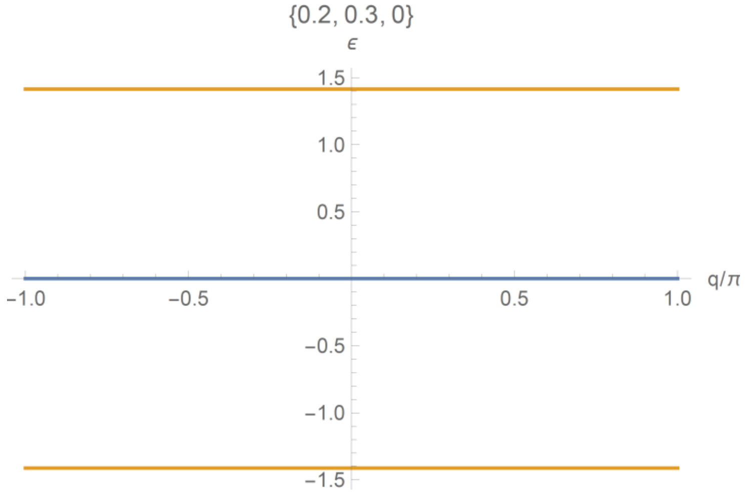


Each trimer has 3 eigen energies:

$\epsilon = 0$: $\boxed{\Psi_a(n) = 0, \quad \Psi_b(n) = \chi \quad \underline{\Psi_c(n) = -U_- U_+^\dagger \chi}}$

$\epsilon = \pm\sqrt{2}$: $\boxed{\Psi_a(n) = \pm\sqrt{2}\chi, \quad \underline{\Psi_b(n-1) = U_+\chi, \quad \Psi_c(n-1) = U_-\chi}}$

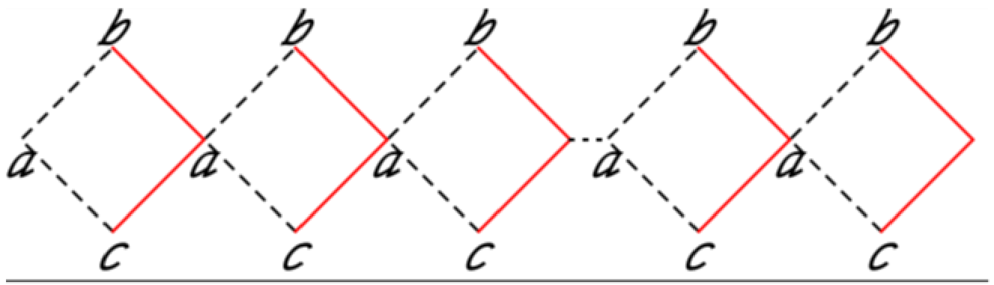
Flat bands



This also happens for any choice of site energies and Zeeman field!

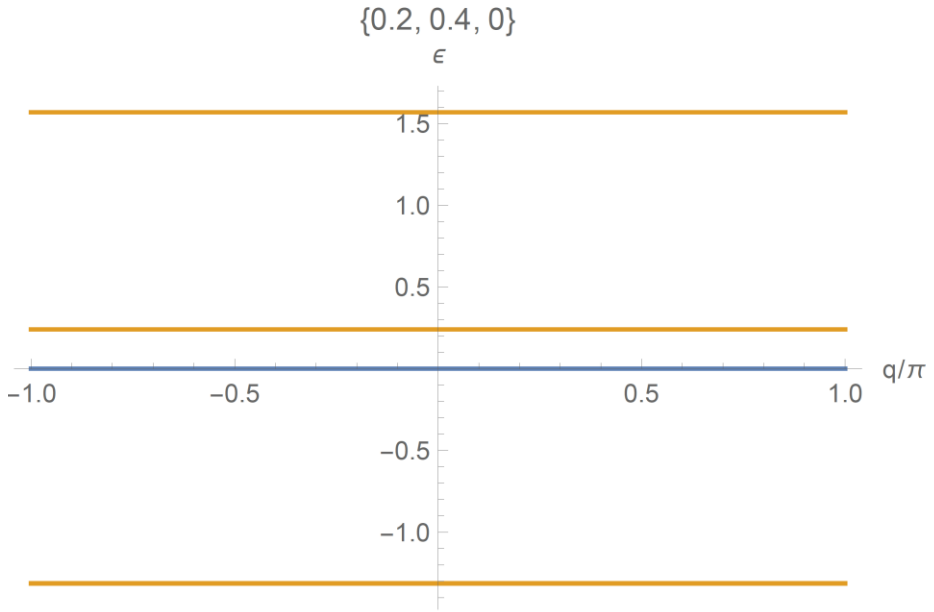
This also happens for any choice of site energies!

$$\gamma = 0$$



Decoupled trimers with

$$\epsilon_b = .5 \qquad \epsilon_c = 0$$

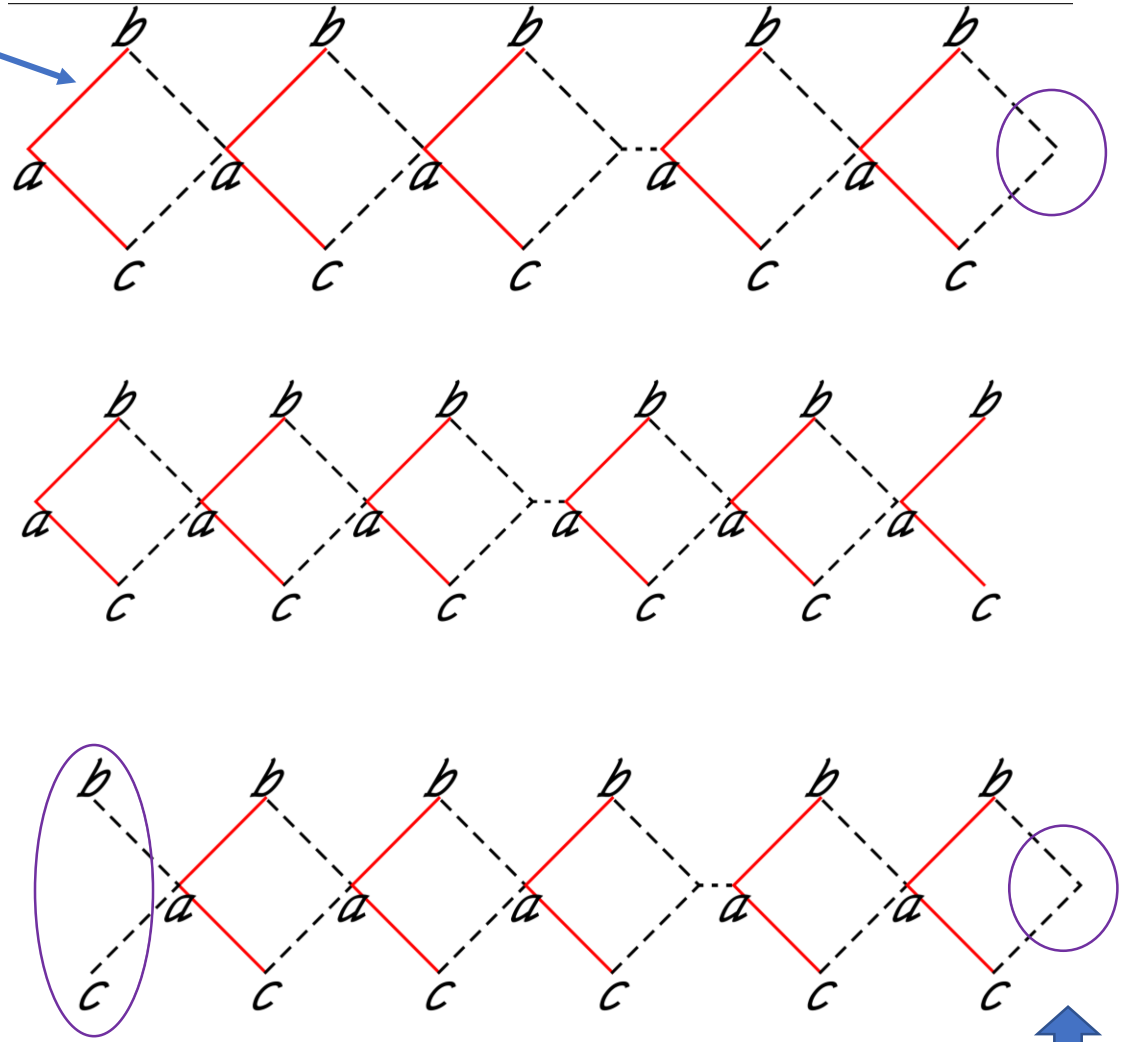


$\epsilon_b = \epsilon_c = 0, \quad B = 0$

Limits:

$\gamma \rightarrow \infty$

decoupled trimers

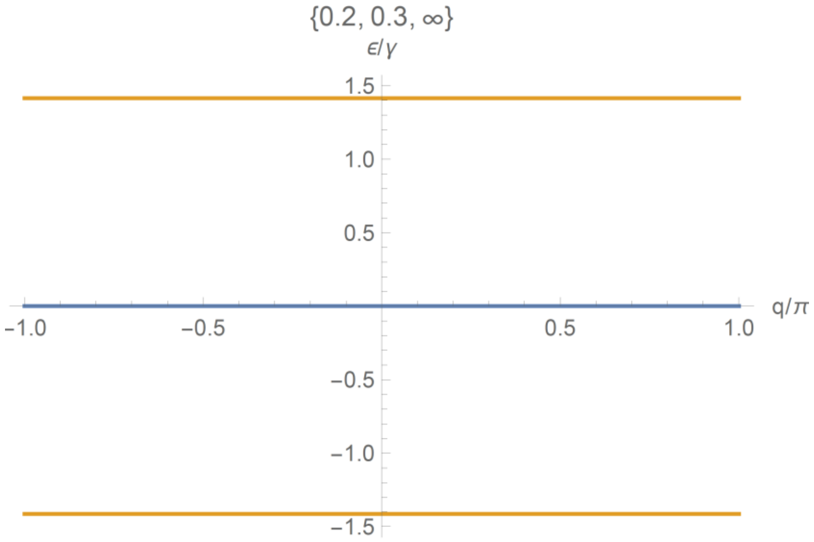


Each trimer has 3 eigen energies

$\epsilon = 0 :$ $\boxed{\Psi_a(n) = 0, \quad \Psi_b(n) = \chi \quad \Psi_c(n) = -U_+^\dagger U_- \chi}$

$\epsilon = \pm\sqrt{2}\gamma :$ $\boxed{\Psi_a(n) = \pm\sqrt{2}\chi, \quad \Psi_b(n) = U_-^\dagger \chi, \quad \Psi_c(n) = U_+ \chi}$

Flat bands



Edge states with 0 energy

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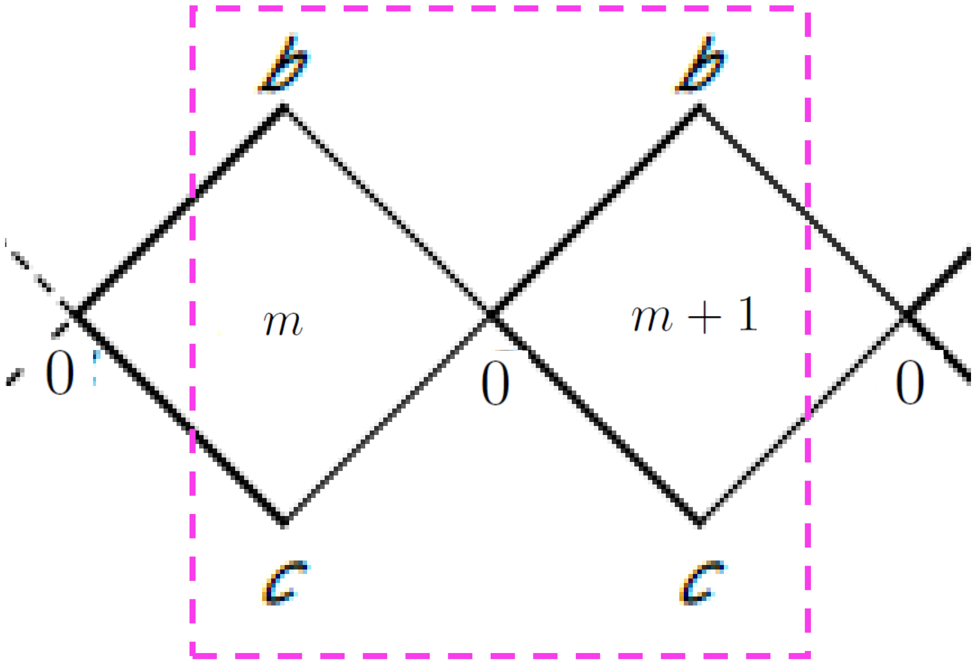
Flat bands in real space – CLS's, $\epsilon = 0.$

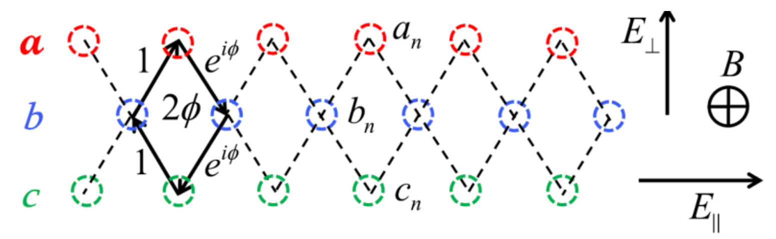
$\epsilon_b = \epsilon_c = 0, \quad B = 0$



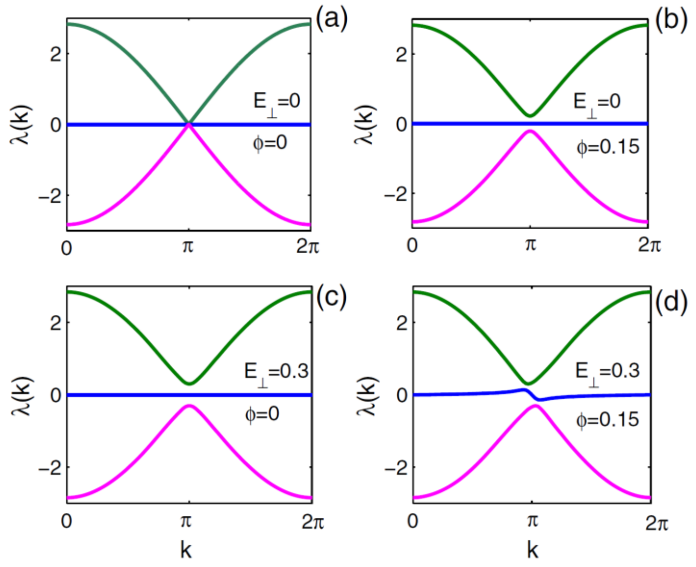
Minimal CLS requires 2 unit cells, U=2

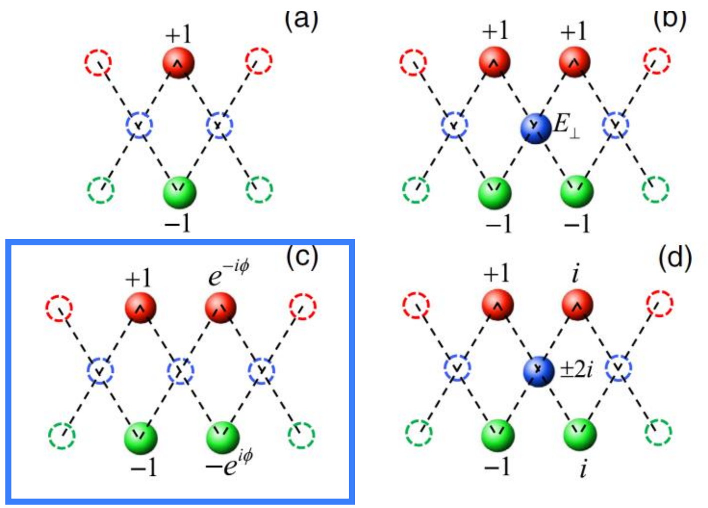
Landau-Zener Bloch Oscillations with Perturbed Flat Bands

Ramaz Khomeriki[1,2] and Sergej Flach[2,3]



$$
\begin{aligned}
\hat{\mathcal{H}} &= \hat{\mathcal{H}}_h + \hat{\mathcal{H}}_\perp + \hat{\mathcal{H}}_\parallel, \\
\hat{\mathcal{H}}_h &= -\sum (\hat{b}_n^+ \hat{a}_n + \hat{c}_n^+ \hat{b}_n) e^{i\phi} - \hat{b}_{n-1}^+ \hat{c}_n - \hat{a}_n^+ \hat{b}_{n-1} + \text{c.c.}, \\
\hat{\mathcal{H}}_\perp &= \sum E_\perp (\hat{a}_n^+ \hat{a}_n - c_n^+ \hat{c}_n), \\
\hat{\mathcal{H}}_\parallel &= \sum E_\parallel n (\hat{a}_n^+ \hat{a}_n + \hat{b}_n^+ \hat{b}_n + \hat{c}_n^+ \hat{c}_n) + \frac{E_\parallel}{2} \hat{b}_n^+ \hat{b}_n, \qquad (1)
\end{aligned}
$$





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Dispersive bands in momentum space – Bloch states

Bloch functions: each unit cell has 3 sites, each site has a spinor --- **six bands**

$$\widetilde{\mathbf{\Psi}}(q) = \mathcal{N}\begin{bmatrix} \widetilde{\Psi}_a(q) \\ \widetilde{\Psi}_b(q) \\ \widetilde{\Psi}_c(q) \end{bmatrix}, \qquad \mathcal{H}(q) = \begin{bmatrix} \epsilon_a & \rho_b^\dagger & \rho_c^\dagger \\ \rho_b & \epsilon_b & 0 \\ \rho_c & 0 & \epsilon_c \end{bmatrix}$$

$$\mathcal{H}(q)\widetilde{\mathbf{\Psi}}(q) = \epsilon\widetilde{\mathbf{\Psi}}(q)$$

$$\rho_b(q) = \gamma e^{-i\Phi/2}U_-^\dagger + e^{iq}U_+, \ \rho_c(q) = \gamma e^{i\Phi/2}U_+^\dagger + e^{iq}U_-.$$

Chiral symmetry

$\epsilon_b = \epsilon_c = 0, \quad B = 0$ $\Rightarrow$ $\mathcal{H}(q) = \begin{bmatrix} 0 & \rho_b^\dagger & \rho_c^\dagger \\ \rho_b & 0 & 0 \\ \rho_c & 0 & 0 \end{bmatrix}$ $\Rightarrow$ $\epsilon = 0$ or $\epsilon^2 \widetilde{\Psi}_a = [\rho_b^\dagger \rho_b + \rho_c^\dagger \rho_c] \widetilde{\Psi}_a$

$\Downarrow$

Diagonalize

$$V\mathcal{H}(q)V^\dagger = \mathcal{H}(q)_{diag} = \begin{bmatrix} \boldsymbol{\epsilon} & 0 & 0 \\ 0 & -\boldsymbol{\epsilon} & 0 \\ 0 & 0 & 0 \end{bmatrix} \Leftrightarrow \text{FB}$$

$$\boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_1 & 0 \\ 0 & \epsilon_2 \end{bmatrix}$$

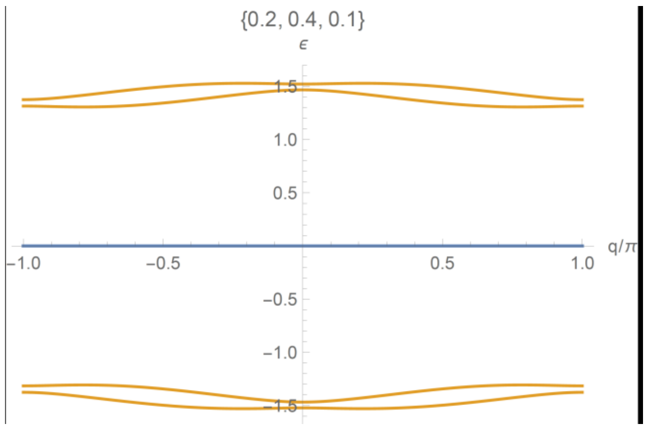
$$\Gamma \mathcal{H}_{diag} \Gamma^\dagger = -\mathcal{H}_{diag}$$

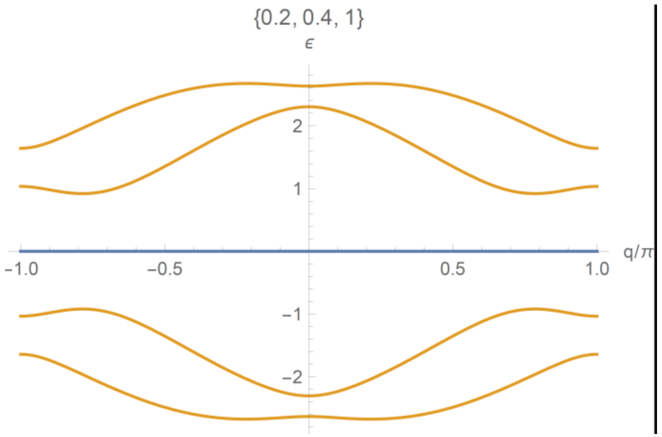
$$\Gamma = \begin{bmatrix} 0 & \mathbf{1}_2 & 0 \\ \mathbf{1}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i\mathbf{1}_2 & 0 \\ i\mathbf{1}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

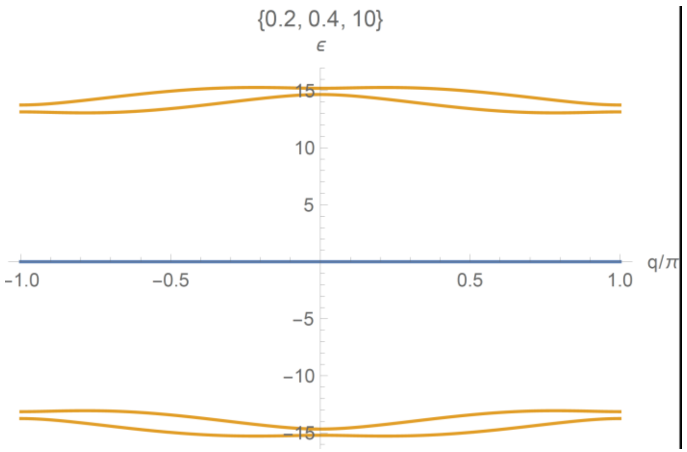
$$\Rightarrow \quad V^\dagger \Gamma V \underline{\mathcal{H}(q)} V^\dagger \Gamma V \underline{\;=\; -}\mathcal{H}(q)$$

Also particle-hole

$\epsilon_b = \epsilon_c = 0, \quad B = 0$ $\qquad$ $\epsilon =$ Fermi level $\qquad$ $\alpha/\pi, \phi/\pi, \gamma$

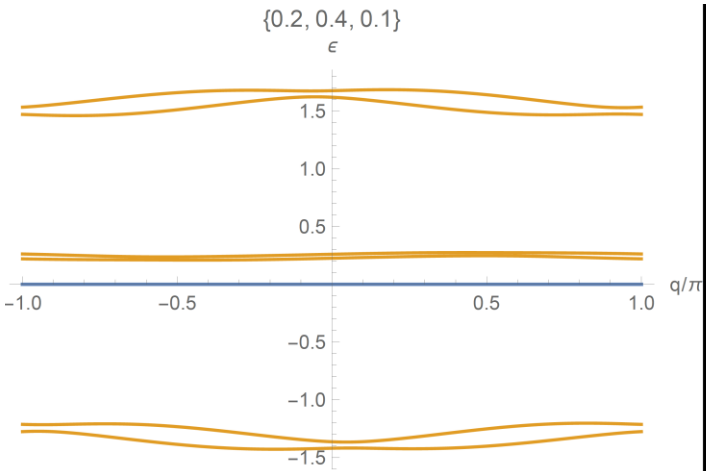




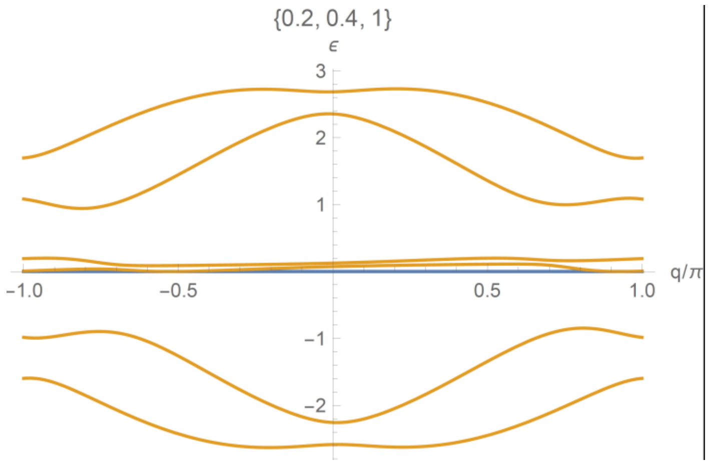


$\epsilon_b = \epsilon_c = 0$

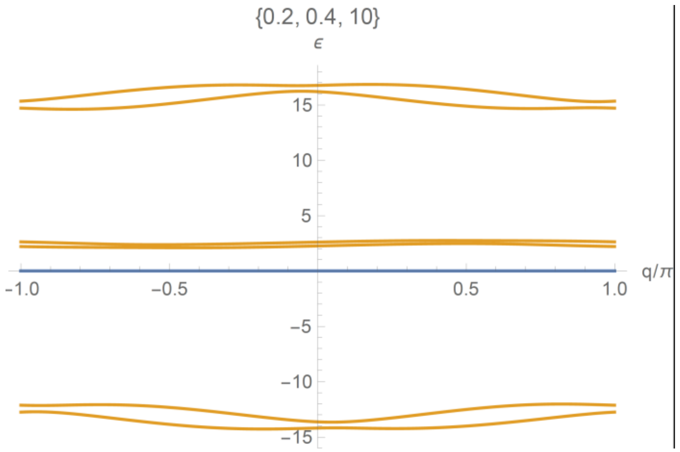
Role of site energy at *b*



$\gamma = .1, \quad \epsilon_b = .5$

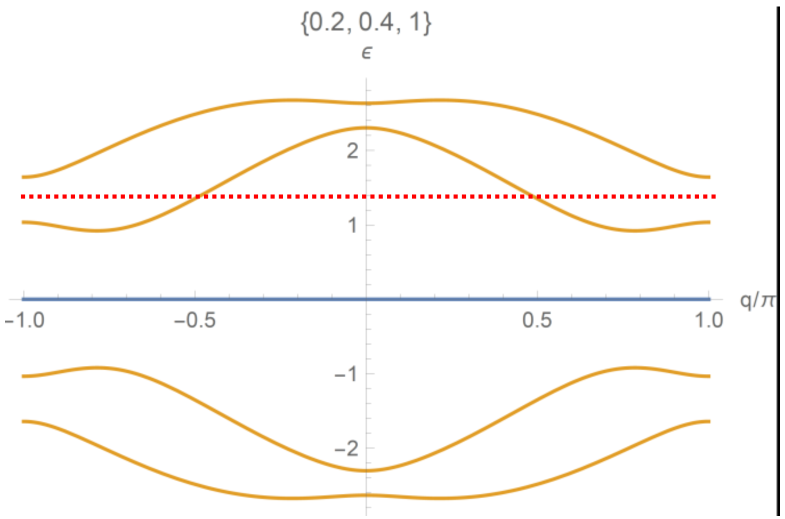


$\gamma = 1, \quad \epsilon_b = .2$



$\gamma = 10, \quad \epsilon_b = 5$

ϵ = Fermi level



PHYSICAL REVIEW B **78**, 125328 (2008)

Spin filtering by a periodic spintronic device

Amnon Aharony,[1,*] Ora Entin-Wohlman,[1,*] Yasuhiro Tokura,[2] and Shingo Katsumoto[3]

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Spin filtering due to quantum interference in periodic mesoscopic networks

Amnon Aharony[a,*,1], Ora Entin-Wohlman[a,1], Yasuhiro Tokura[b], Shingo Katsumoto[c]

Spin filtering:

Each band has a well defined spin,
In a direction which depends on
AB and AC phases.

For the Fermi energy on the otted line
has "running" solutions with a
definite spin polarization.
The other spin has only evanescent
Solutions, absent in the infinite
periodic case.

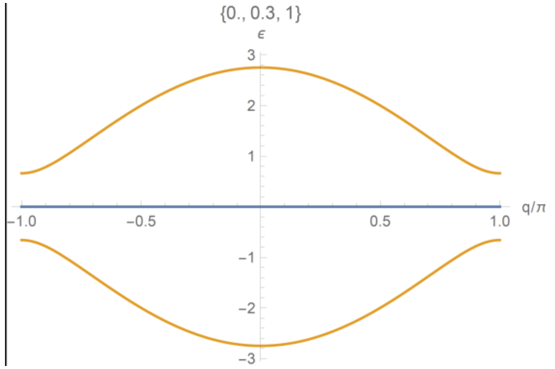
→ Ballistic full spin filtering.

Effects of Zeeman field

Ovadya Bettoun

$$\epsilon_b = \epsilon_c = 0$$

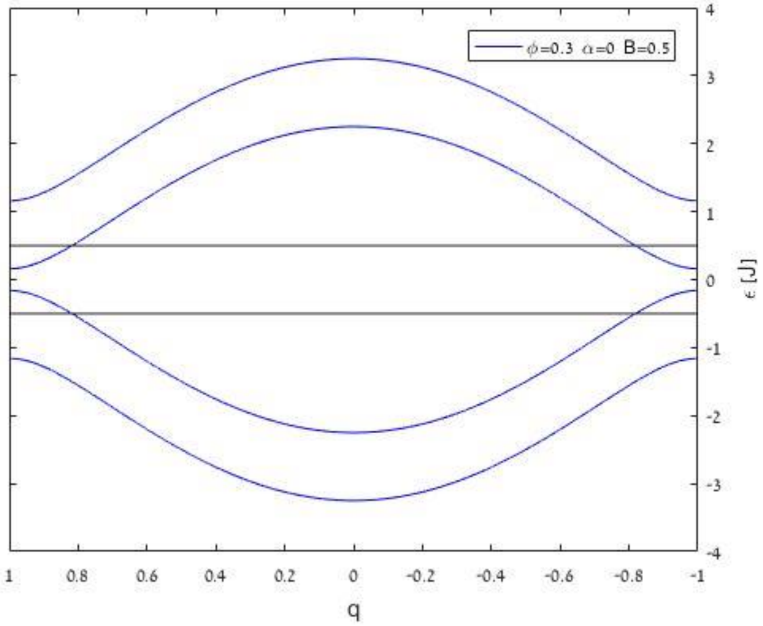
B=0



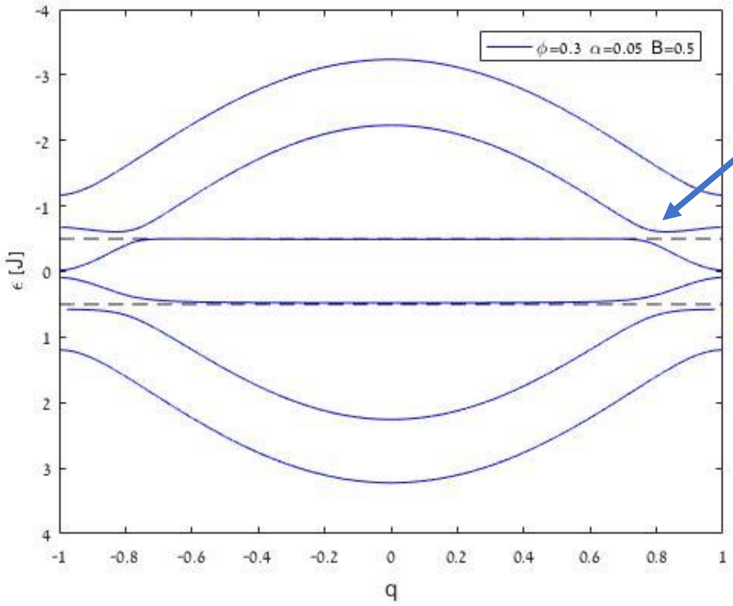
B=0.5

$$\phi = .3\pi, \quad \gamma = 1$$

$\alpha = 0$



$\alpha = .05\pi$



Level crossing due to SOI

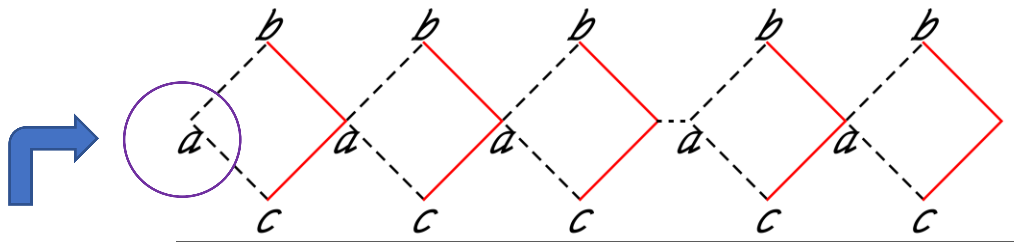
Outline

- History: Aharonov-Bohm cages, Spin-orbit cages, spin filtering
- Su-Schrieffer-Heeger (SSH) model – topology
- Tight-binding equations for diamond chains
- Limits: decoupled trimers and edge states
- Flat bands – Compact localized states (CLS's)
- Dispersive bands
- Edge states
- Scattering
- Outlook

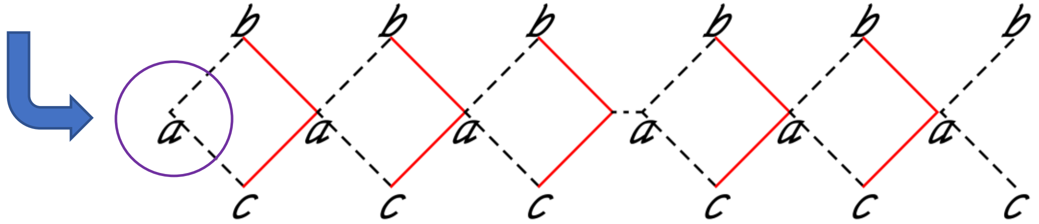
Edge states --- Example 1: semi-infinite chain with SSH and AB, no SOI

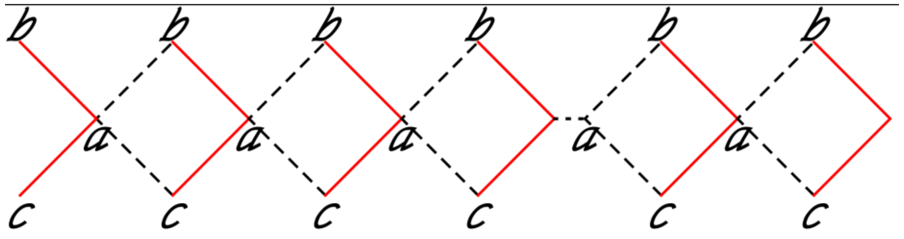
Cases(i) and (ii), left boundary condition:

$\epsilon_b = \epsilon_c = 0, \quad B = 0$



Edge states





$$\Psi_a(n) = Ce^{iqn} \qquad q = \pi + i\kappa$$

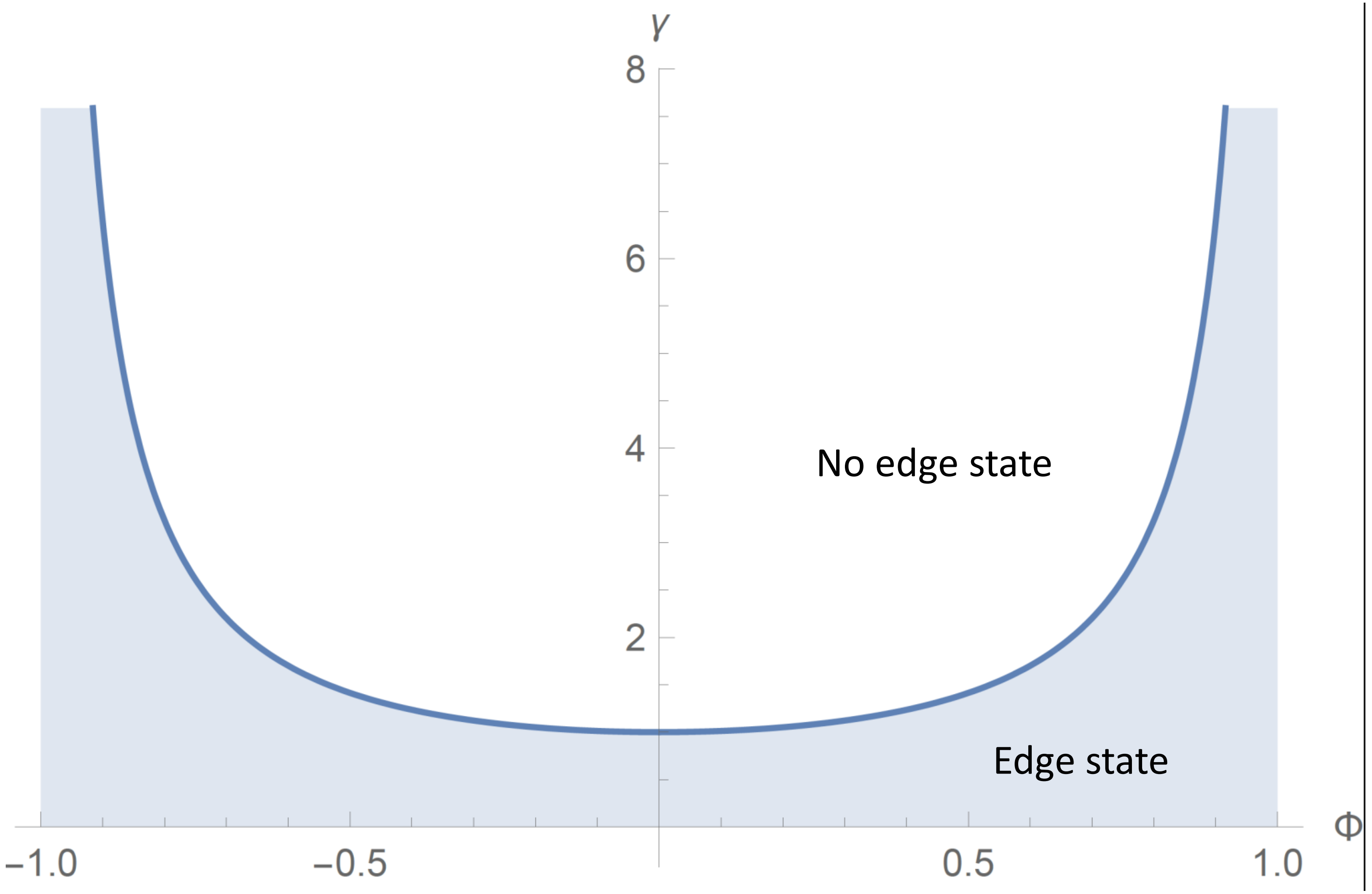
$$\Psi_a(n) = C(-1)^n e^{-\kappa n}$$

$$e^{-\kappa} = |\gamma \cos(\Phi/2)| \leq 1$$

Edge state exists for $\boxed{|\gamma| < 1/|\cos(\Phi/2)|}$

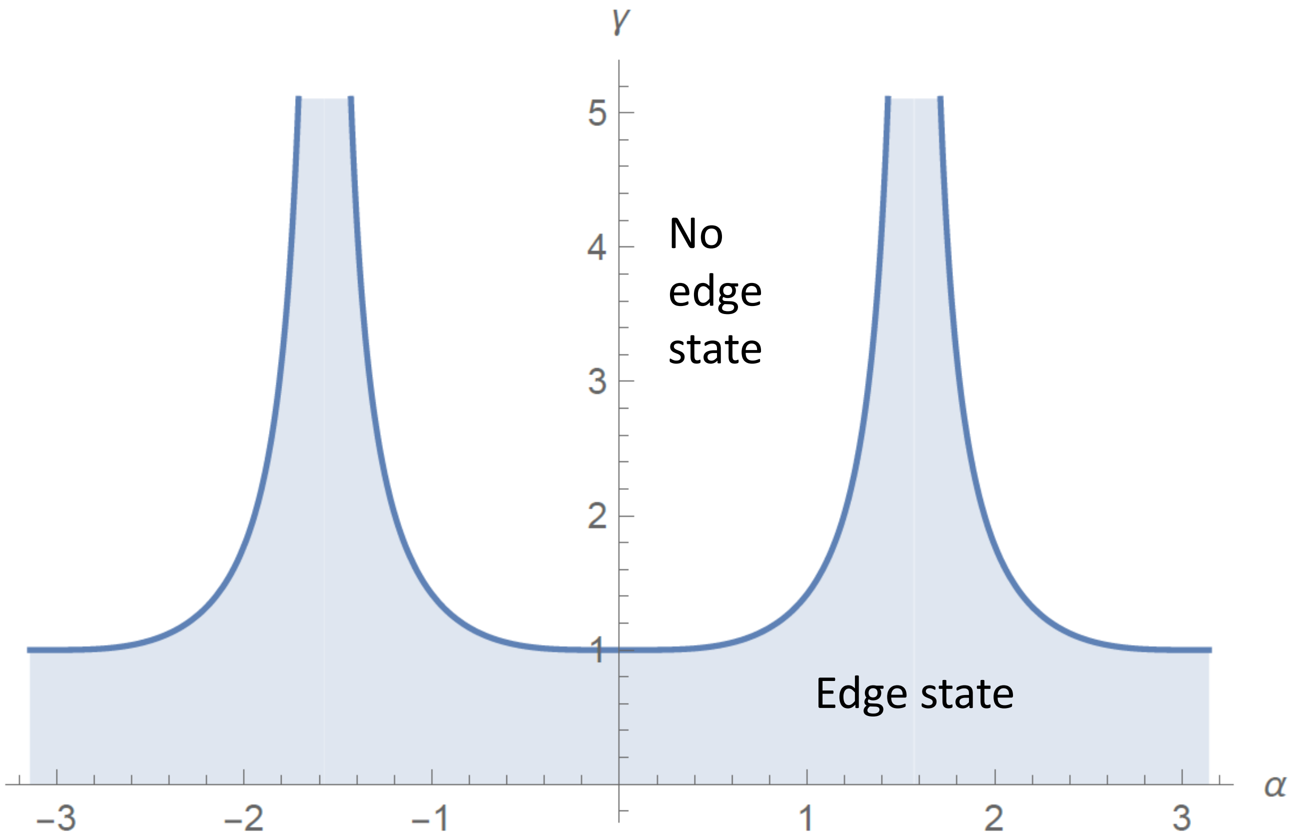
$$\epsilon_{edge}^2 = 2\gamma^2 \sin^2(\Phi/2)$$

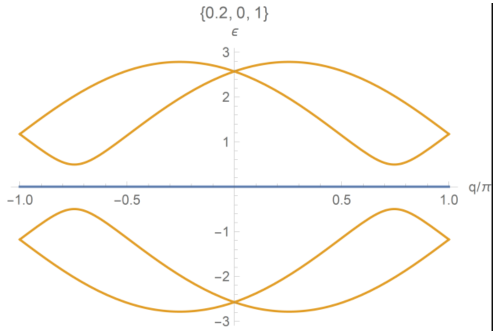
AB caging generates broader range for edge states!

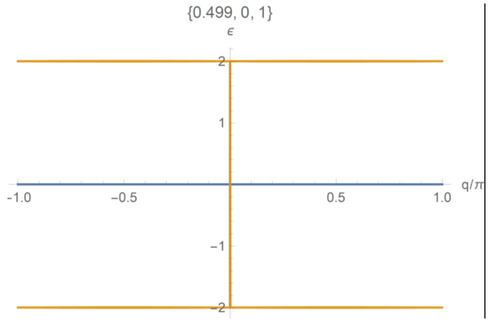


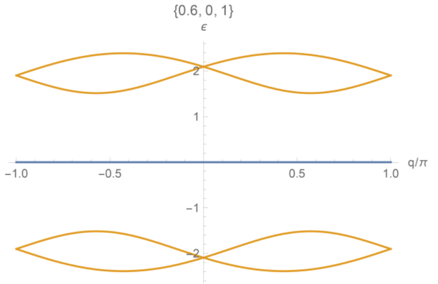
Edge states --- Example 2: semi-infinite chain with SSH and **AC**, no AB

AB caging generates broader region with edge states









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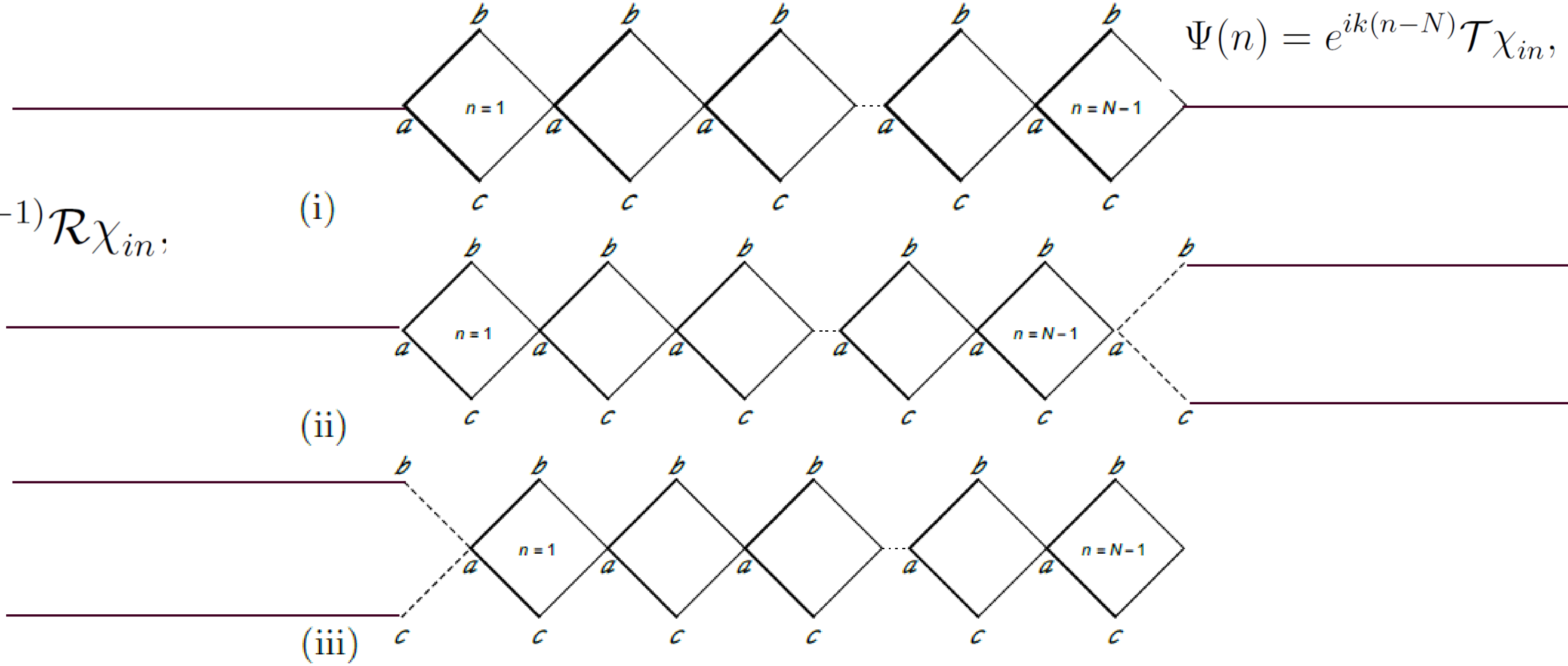
Scattering

$$\Psi(n) = e^{ik(n-1)}\chi_{in} + e^{-ik(n-1)}\mathcal{R}\chi_{in};$$

$$\Psi(n) = e^{ik(n-N)}\mathcal{T}\chi_{in};$$

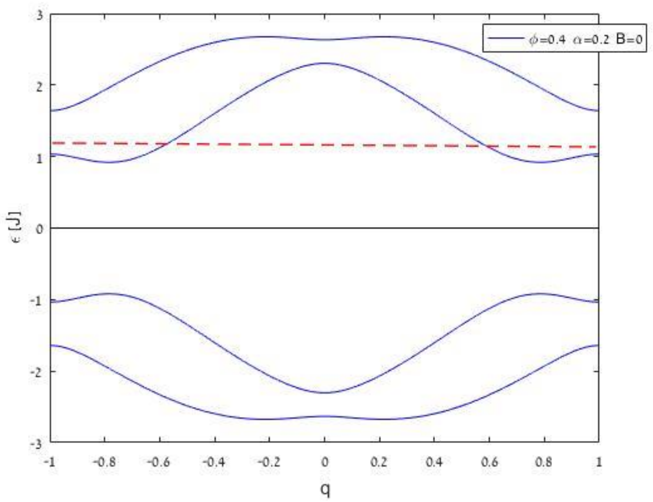


$$\Psi(n) = \sum_q A_q e^{iqn}\chi_\mu(q)$$

Transmission and reflection 2x2 matrices

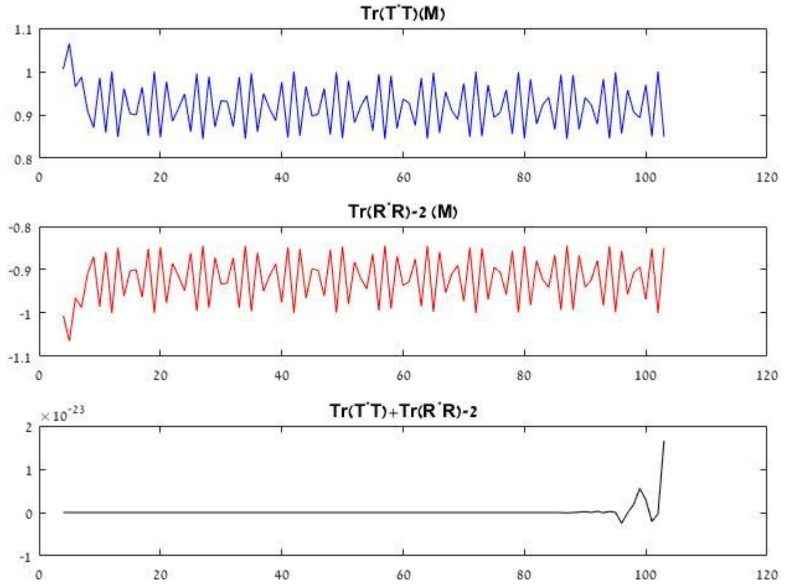
Ovadya Bettoun

$\gamma = 1$ $\epsilon_b = \epsilon_c = 0$ B=0

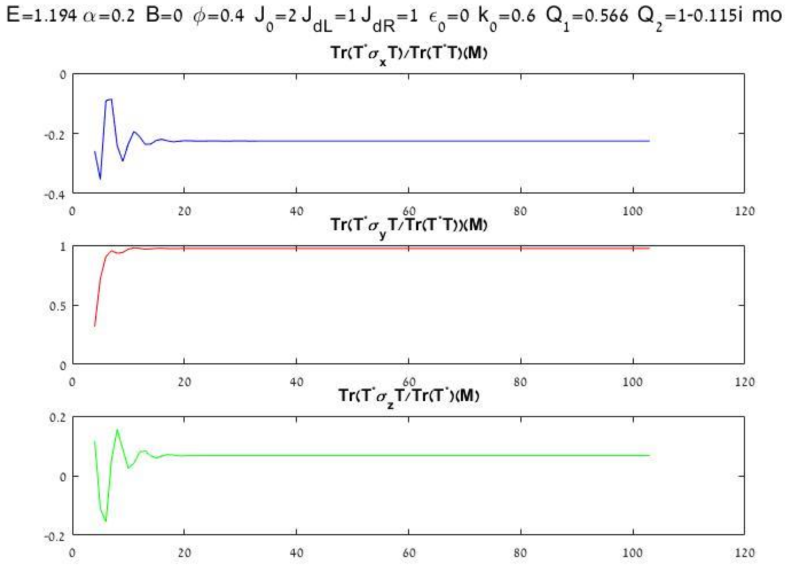


Long chain also spin filter

$$\text{Transmission probability} = \mathrm{Tr}[T^\dagger T]$$



$$\langle \mathbf{S} \rangle = \frac{\mathrm{Tr}[T^\dagger \boldsymbol{\sigma} T]}{\mathrm{Tr}[T^\dagger T]}$$



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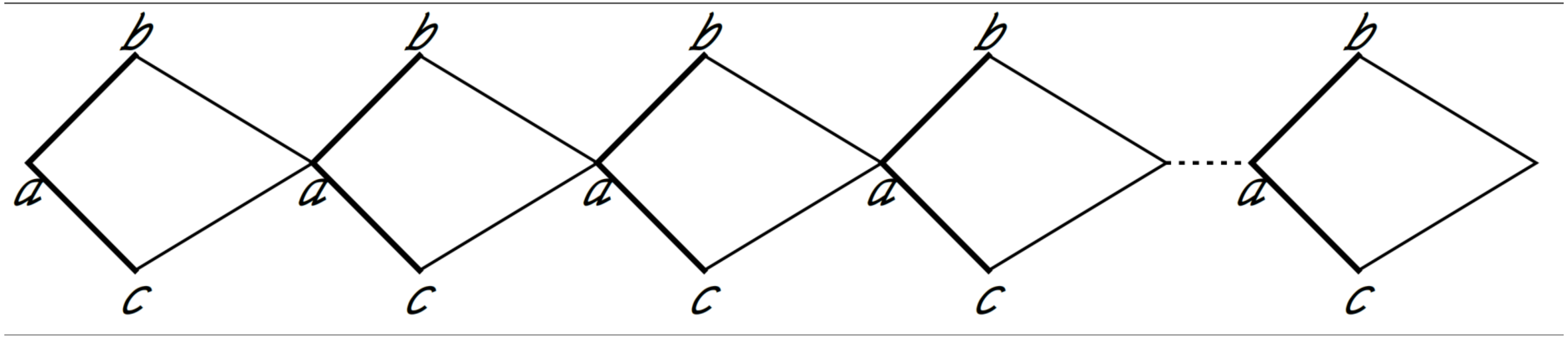
Outlook



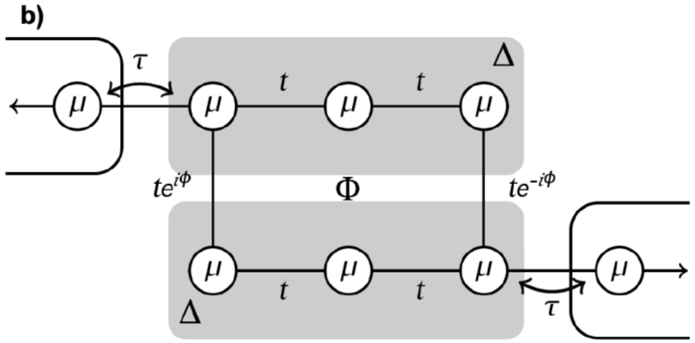
Symmetry – chirality - topology

More scattering, spin filtering

…

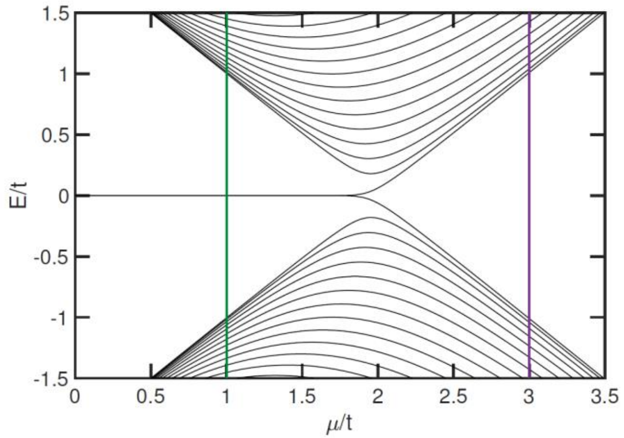
Aharonov-Bohm interference as a probe of Majorana fermions

T. C. Bartolo,[1,*] J. S. Smith,[1] B. Muralidharan,[2] C. Müller,[3,4] T. M. Stace,[4] and J. H. Cole[1,†]



$$\hat{H}_{\mathrm{NW}} = \sum_{j=1}^{\mathrm{N}} \left[-t\left(c_j^\dagger c_{j+1}\right) - \mu\left(c_j^\dagger c_j - \frac{1}{2}\right) + \mathrm{h.c.} \right]$$

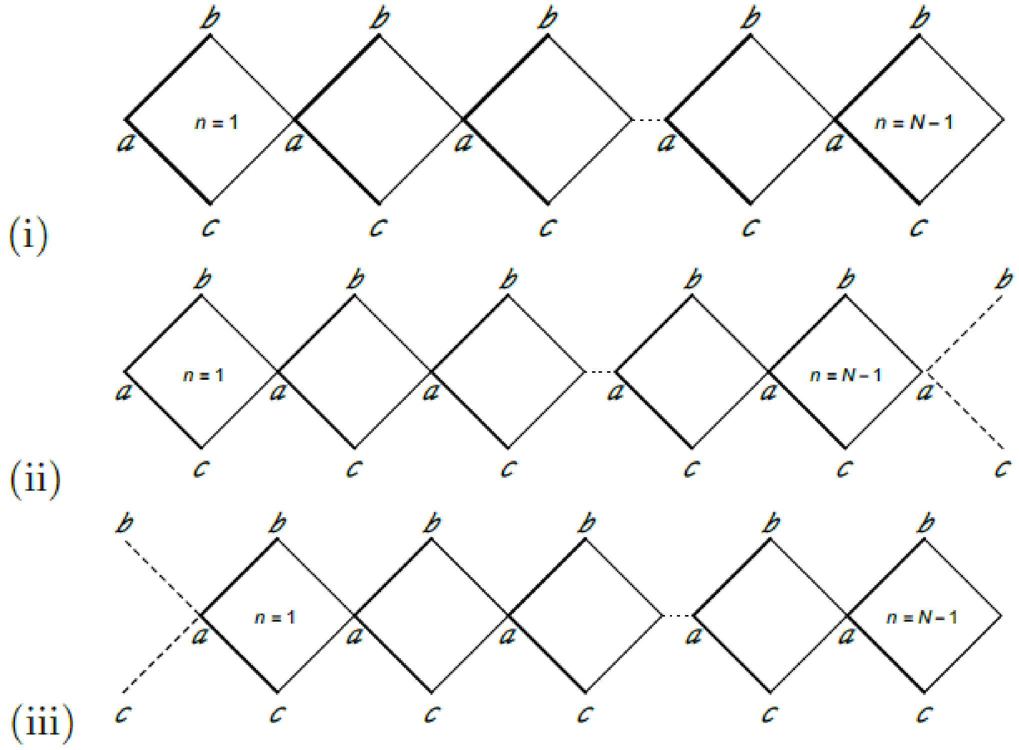
$$\hat{H}_{\mathrm{KC}} = \hat{H}_{\mathrm{NW}} + \sum_{j=1}^{\mathrm{N}} \left[\Delta e^{i\theta} c_j c_{j+1} + \Delta e^{-i\theta} c_{j+1}^\dagger c_j^\dagger \right]$$



감사 해요

Thank you

תודה רבה



Possible room for postdocs