Wannier-Stark Flatbands

Arindam Mallick

In collaboration with:

N. Chang [Tsinghua University, China], W. Maimaiti [Rutgers University, USA], A. Andreanov, S. Flach.

Content :

- Bravais lattices
- Interaction
- $\bullet \, \mathsf{Anti-}\mathcal{PT} \mathsf{protection}$





Wannier-Stark flatbands on Bravais lattices

 $I Bravais lattice \Rightarrow single site per unit cell$

- **2** Band: dispersion relation $E(\vec{k})$
 - Dispersive band: width $\Delta E > 0$
 - Flatband: width $\Delta E = 0$

③ Single particle tight-binding Hamiltonian on a Bravais lattice \Rightarrow single band.

Square lattice:

Dispersive band \Rightarrow transport



4 Uniform DC field $\vec{\mathcal{E}}$ along commensurate direction

 \equiv Translational invariance in perpendicular directions $\vec{\mathcal{E}}^{\perp} \Rightarrow$ Wannier-Stark bands

(5) Hopping allowed along $\vec{\mathcal{E}}^{\perp}$



() No hopping along $\vec{\mathcal{E}}^{\perp}$



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Wannier-Stark flatbands on Bravais lattices

 $\textcircled{0} No hopping along perpendicular to the field <math>\Rightarrow$ all bands flat \Rightarrow no transport



2 Flatband eigenstates: noncompact, super-exponentially localized



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"Wannier-Stark flatbands in Bravais lattices", Phys. Rev. Research 3, 013174 (2021)

Two-interacting particles: Theory

With two-particle interactions:

 $\mathcal{H}_{\text{two}} = \mathcal{H}_1 \otimes \mathbbm{1}_2 + \mathbbm{1}_1 \otimes \mathcal{H}_2 + V \sum_{\vec{n}_1 = \vec{n}_2} \left| \vec{n}_1, \vec{n}_2 \right\rangle \left\langle \vec{n}_1, \vec{n}_2 \right|,$

 $H_{1,2}$ single-particle Hamiltonians with DC field.

2 Interaction V induces dispersion and pair motion perpendicularly to the field.

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Two-interacting particles: Numerics

() $V = 0.1, t = 1, \mathcal{F} = 9$

2 Initial state: Two particles around the center of a square lattice



Final state: Single particle probability distribution at final time = 4000

Ocenter of mass (c.m.) coordinates

- Along field: $X = \vec{\mathcal{E}} \cdot (\vec{n_1} + \vec{n_2})$ —no motion
- Perpendicular to field: $X = \vec{\mathcal{E}}^{\perp} \cdot (\vec{n}_1 + \vec{n}_2)$ —ballistic motion

Relative (rel.) coordinates

- Along field: $X = \vec{\mathcal{E}} \cdot (\vec{n_1} \vec{n_2})$ —no motion
- Perpendicular to field: $X = \vec{\mathcal{E}}^{\perp} \cdot (\vec{n_1} \vec{n_2})$ —no motion



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Anti- \mathcal{PT} symmetric flatbands

- Wannier-Stark flatband on non-Bravais lattices
- ② Non-Bravais lattices ⇒ more than one lattice site per unit cell
- **③** Point-reflection $\mathcal{P}: \vec{r} \mapsto -\vec{r}$ and complex conjugation $\mathcal{T}: |\psi\rangle \mapsto |\psi^*\rangle$



- Anti- \mathcal{PT} symmetry (anti-unitary): $(\mathcal{T} \cdot \mathcal{P}) \cdot \mathcal{H} \cdot (\mathcal{T} \cdot \mathcal{P})^{-1} = -\mathcal{H}$ (different from unitary e.g. chiral symmetry)
 - \Rightarrow pairwise eigenvalues $\{E(\vec{k}), -E(\vec{k})\}$
- **(3)** Odd number of sublattices \Rightarrow flatband: $E(\vec{k}) = 0$
- InitPT symmetry persists in DC field
 ⇒ Flatband exists within Wannier-Stark band structure

(5)

Anti- \mathcal{PT} symmetric flatbands

3D generalization of kagome lattice with anti- \mathcal{PT} symmetry—formed by vertically stacking 2D kagome planes "Anti- \mathcal{PT} Flatbands", arXiv:2108.01845 (2021)



Band structure $E(k_1, k_2, k_3 = \pi/7)$, no DC field



Wannier-Stark bands (DC field)



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- Uniform commensurate DC field induces Wannier-Stark flatbands on Bravais lattices
- Flatband eigenstates are super-exponentially localized
- Interaction induces motion of two-particle pair along the direction perpendicular to the field
- Anti- \mathcal{PT} symmetry + odd number of sublattices \Rightarrow flatband
- DC field: Anti- \mathcal{PT} symmetry protects Wannier-Stark flatbands

References:

- (1) "Wannier-Stark flatbands in Bravais lattices", Phys. Rev. Research 3, 013174 (2021).
- (2) "Anti-*PT* Flatbands", arXiv:2108.01845 (2021).

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