# Finetuning localization in interacting flatband networks

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MAX-PLANCK-GESELLSCHAFT

Flatbands: symmetries, disorder, interactions and thermalization
20/08/2021

mpipks

# People

- Alexei Andreanov (IBS-PCS)
- Sergej Flach (IBS-PCS)
- Ihor Vakulchyk (IBS-PCS)
  Yesterday's talk: Percolation Transitions in Interacting Many-Body Flatband Systems
- Mithun Thudiyangal (University of Massachusett)

# Unfolding the title

- **Title**:
  Finetuning localization in interacting flatband networks
- **Fine-tuning**:
  process in which parameters of a model are adjusted (very precisely) in order to obtain certain phenomena, or fit with certain observations
- **Our aim**:
  Tailor classes of interacting flatband networks (i.e. fine-tune flatband networks and/or interaction terms) in order to obtain certain localization phenomena

# Flatband Networks

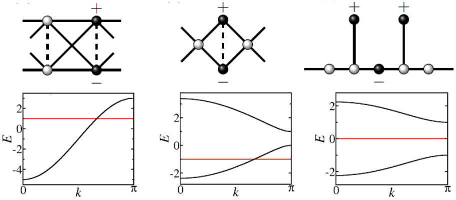
**Features**

1. translation invariant lattices which admit at least one *dispersionless* band $E_j(k) = Const$
2. flat band eigenstates are *spatially compact* - dubbed Compact Localized States (CLS)

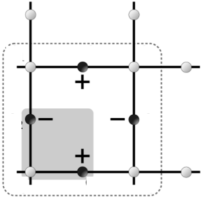
Eigenvalue problem with $\psi_n = (\psi_{n,1}, \ldots, \psi_{n,\nu})$ and $\nu$ number of sites for unit-cell

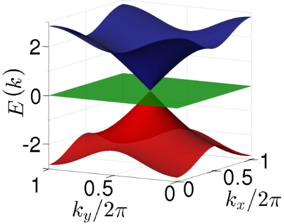
$$E\psi_n = -H_0\psi_n - H_1\psi_{n+1} - {H_1}^{\dagger}\psi_{n-1}$$

Examples of flatband lattices with nearest-neighbor hopping




EPL 105, 30001 (2014)



# Perturbing flatband networks

1D flatband lattice defined via the matrixes $H_0, H_1$ ($H_0$ is Hermitian)

$$E\psi_n = -H_0\psi_n - H_1\psi_{n+1} - {H_1}^\dagger\psi_{n-1}$$

Adding perturbations

$$E\psi_n = -H_0\psi_n - H_1\psi_{n+1} - {H_1}^\dagger\psi_{n-1} + P(\psi, \gamma, \mu....)$$

**Typical outcomes:**

1. compact localized states are lost
2. flat band disappears (sometimes with the whole band structure)

**Then**: check what's happening ....

# Finding (and generating) flatband networks

**Question:** how to get flatband networks?

$$E\psi_n = -H_0\psi_n - H_1\psi_{n+1} - H_1{}^{\dagger}\psi_{n-1}$$

**In general:** for generic $H_0, H_1$ all $\nu$ bands are dispersive, and all the eigenstates are extended

**Then:** the entrees of the matrixes $H_0, H_1$ have to be carefully selected (fine-tuned)

**(Some) Methods:**

- line-graph theory
  A. Mielke, J.Phys.A **24**,3311 (1991)
- local cell construction
  H. Tasaki, PRL **69**,1608 (1992)
- "origami rules" in decorated lattices
  R.G. Dias *et.al.*, Sci. Rep. **5**,16852 (2015)
- repetitions of mini arrays
  L. Morales-Inostroza *et.al.*, PRA **94**, 043831(2016)
- local symmetries
  M. Röntgen *et.al.*, PRB **97**, 035161 (2018)
- ... many more ...

**See:** Adv.Phys.X 3, 1473052 (2018)

# Generating flatband networks – PRB 95, 115135 (2017)

**Generator scheme (sketch)**

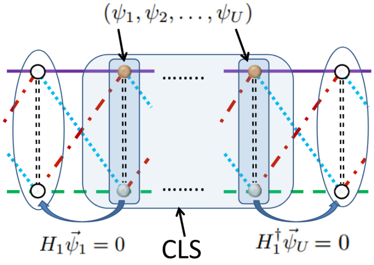
- eigenvalue problem for 1D networks

$$E\psi_n = -H_0\psi_n - H_1\psi_{n+1} - {H_1}^\dagger\psi_{n-1}$$

- parametrize a compact state (CLS) of size $U$

$$\begin{cases} \psi_n \neq 0 & 1 \leq n \leq U \\ \psi_n = 0 & n < 1 \ \&\& \ n > U \end{cases}$$

- solve an "inverse problem" to compute $H_0, H_1$



# Generating flatband networks

**An example:** 1D, $\nu = 2$ bands, $U = 2$ size

- parametrized CLS for $0 \leq \varphi, \theta, \gamma, \delta \leq \pi$

$$\psi_1 = \begin{pmatrix} \cos\varphi \\ e^{i\gamma}\sin\varphi \end{pmatrix} \qquad \psi_2 = \begin{pmatrix} \cos\theta \\ e^{i\delta}\sin\theta \end{pmatrix}$$

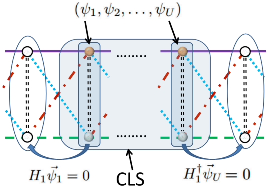
- blend in the eigenvalue problem

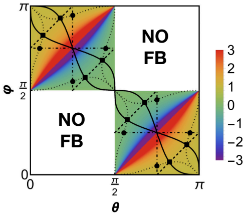
$$E\psi_n = -H_0\psi_n - H_1\psi_{n+1} - H_1{}^{\dagger}\psi_{n-1}$$

- resulting matrixes

$$H_0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \; H_1 = \alpha \begin{pmatrix} \cos\theta\cos\varphi & e^{i\gamma}\cos\theta\sin\varphi \\ e^{i\delta}\sin\theta\cos\varphi & e^{i(\delta-\gamma)}\sin\theta\sin\varphi \end{pmatrix}$$

- details in PRB 95, 115135 (2017)




**Upshot:** there exist parametric families of flatband lattices!

**For more**: PRB 99 125129 (2019); PRB 103, 165116 (2021); PRB 104, 035115 (2021)

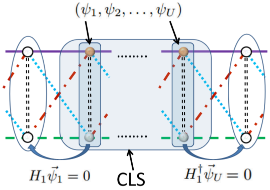
# Flatband networks

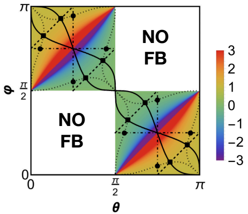
Generated 1D $\nu = 2$ flatband networks for $0 \leq \varphi, \theta, \gamma, \delta \leq \pi$

$$E\psi_n = -H_0\psi_n - H_1\psi_{n+1} - {H_1}^\dagger\psi_{n-1}\ , \qquad H_1 = \alpha\begin{pmatrix} \cos\theta\cos\varphi & e^{i\gamma}\cos\theta\sin\varphi \\ e^{i\delta}\sin\theta\cos\varphi & e^{i(\delta-\gamma)}\sin\theta\sin\varphi \end{pmatrix}$$

Hierarchy of fine-tuning

1. **in general** all $\nu$ bands are dispersive.
   All the single particle eigenstates are extended
2. **fine-tuning** of $H_0, H_1$ yields at least one *dispersionless* band $E_j(k) = Const$.
   Coexistence of extended eigenstates (dispersive bands) and compact eigenstates (flat band)
3. **additional fine-tuning** of $H_0, H_1$ yields (for example)
   (a) lattices with specific types of CLS
   (b) lattices with specific hopping profiles




W. Maimaiti *et.al.*, PRB 95, 115135 (2017)

# Impact of the interaction on flatband networks

We studied

- the impact of classical nonlinearity
- the impact of quantum two-body interaction

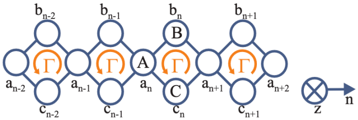
In particular, we focused on the case of all band flat networks

Let's begin with the nonlinear case

$$i\dot{\psi}_n = -H_0\psi_n - H_1\psi_{n+1} - H_1{}^{\dagger}\psi_{n-1} + \gamma\mathcal{F}(\psi_n)\psi_n$$

**Question:** what is the impact of nonlinearity in all-flat-band networks?

- nonlinear symmetry breaking of Aharonov-Bohm cages
  G. Gligoric *et.al.*, PRA A 99, 013826 (2019)
- nonlinear dynamics of Aharonov-Bohm cages
  M. Di Liberto *et.al.*, PRA 100, 043829 (2019).

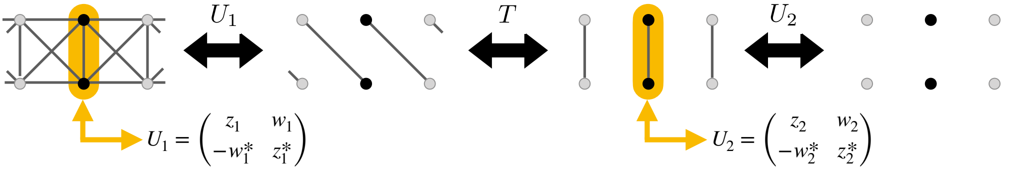


# All bands flat networks

**Detangling**: in one-dimension, every all band flat lattice can be rotated in decoupled local units via unit-cells redefinitions (local unitary transformations)

**Example**: $\nu = 2$ all band flat lattice

$$i\dot{\psi}_n = -H_0\psi_n - H_1\psi_{n+1} - H_1^\dagger\psi_{n-1}, \qquad \textit{e.g.}\ \text{Creutz} \quad H_0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad H_1 = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$




**Then**:

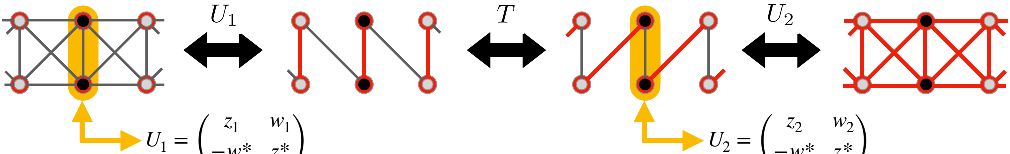
- **Caging**: compact excitations remain localized in compact sub-regions of the network
- **Generator**: obtained by inverting the detangling

CD, A. Andreanov, T. Mithun, S. Flach, PRB **104**, 085131 (2021)

# Nonlinear caging?

**In general**: nonlinearity induces coupling between the detangled components, it destroys caging and induce transport

**Example**: $\nu = 2$ flatband lattice $E_{FB} = \pm 2$ with Kerr nonlinearity




$U_1 = \begin{pmatrix} z_1 & w_1 \\ -w_1^* & z_1^* \end{pmatrix}$ $\qquad$ $U_2 = \begin{pmatrix} z_2 & w_2 \\ -w_2^* & z_2^* \end{pmatrix}$

**Nonlinearity**

$$\mathcal{H}_1 = \sum_{n,i} |\psi_{n,i}|^4 \qquad \Rightarrow \qquad \mathcal{H}_1 = \sum_{na,mb;kc,ld} V_{na,mb;kc,ld} \phi^*_{n,a} \phi^*_{m,b} \phi_{k,c} \phi_{l,d}.$$

**Unless** – fine tuning $H_0, H_1$ – i.e. setting $|z_1|^2 = |w_1|^2$, for any $z_2, w_2$ – avoids transport

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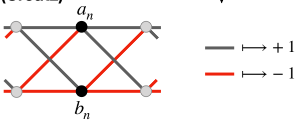
# Fine-tuned nonlinear caging

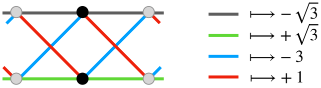
**1D** $\nu = 2$ **examples** – both with $z_2 = \frac{1}{\sqrt{2}} = w_2$

Fine-tuning condition – $|z_1|^2 = |w_1|^2$

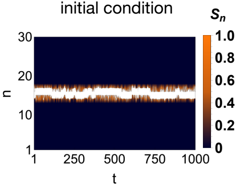
**Fine-tuned** — $z_1 = w_1 = \frac{1}{\sqrt{2}}$
**(Creutz)**

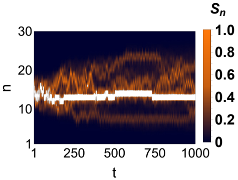



**Non fine-tuned** — $z_1 = \frac{\sqrt{3}}{2}, \quad w_1 = \frac{1}{2}$



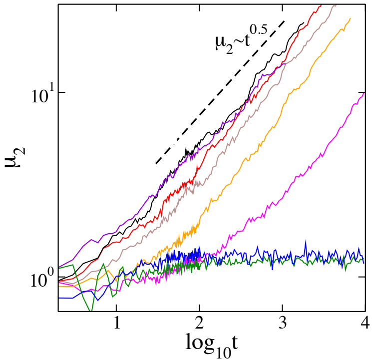

Time-evolution of a compact initial condition




$S_n = |a_n|^2 + |b_n|^2$

Time-evolution of the second moment

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# From classical to quantum

**Then**: from nonlinear dynamics, we moved to study the dynamics of interacting particles

- extended states of two interacting particles in all-band-flat rhombic lattice
  J. Vidal *et.al.*, PRL 85, 3906 (2000)
- number parity operators in certain interacting all-band-flat lattices
  M.Tovmasyan *et.al.*, PRB 98, 134513 (2018)

**Case:** Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1$

- 1D all band flat lattice – $\hat{C}_n = (\hat{c}_{n,1}, \ldots, \hat{c}_{n,\nu})$, $\hat{C}_n^\dagger = (\hat{c}_{n,1}^\dagger, \ldots, \hat{c}_{n,\nu}^\dagger)$

$$\mathcal{H}_0 = \sum_n \left[ \frac{1}{2} \hat{C}_n^{\dagger\,T} H_0 \hat{C}_n + \hat{C}_n^{\dagger\,T} H_1 \hat{C}_{n+1} + \text{h.c.} \right]$$

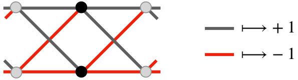
- Hubbard interaction – $\mathcal{H}_1 = \sum_{n,i} \hat{c}_{n,i}^\dagger \hat{c}_{n,i}^\dagger \hat{c}_{n,i} \hat{c}_{n,i}$

# … beginning with 2 interacting particles

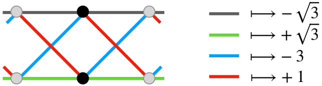
**1D $\nu = 2$ examples** – both with $z_2 = \frac{1}{\sqrt{2}} = w_2$

Fine-tuning condition – $|z_1|^2 = |w_1|^2$

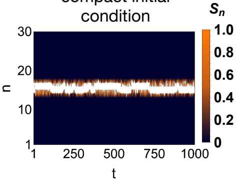
**Fine-tuned** — $z_1 = w_1 = \frac{1}{\sqrt{2}}$
**(Creutz)**

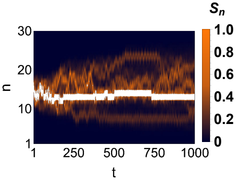



**Non fine-tuned** — $z_1 = \frac{\sqrt{3}}{2}, \quad w_1 = \frac{1}{2}$



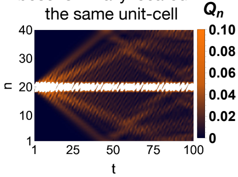

**Classical Case**
Time-evolution of a compact initial condition

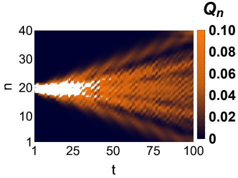



$S_n = |a_n|^2 + |b_n|^2$

**Quantum Case**
Time-evolution of 2 bosons initially located in the same unit-cell




$Q_n =$ PDF of the particle density

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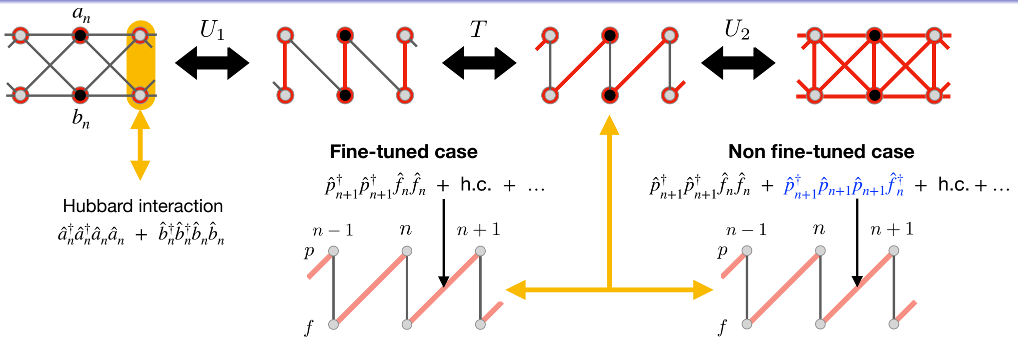
# Detangling the interacting problem




**Consequences** (for fine-tuned ABF lattices):

- transport of paired bosons (no exact quantum caging!)
- number parity operators – oddness/evenness nr. of bosons per unit-cell is preserved
  M.Tovmasyan *et.al.*, Phys. Rev. B 98, 134513 (2018)
- single bosons in neighboring cell form renormalized many-body CLS (quantum scars?)
  O. Hart *et.al.*, PRR **2**, 043267 (2020)
  Y. Kuno *et.al.*, PRB **102**, 241115(R) (2020)

CD, A. Andreanov, T. Mithun, S. Flach, PRB **104**, 085132 (2021)

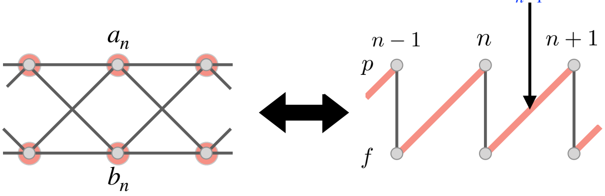
# What about quantum caging?

**Upshot:**

ABF lattices + Hubbard interaction $\Rightarrow$ no strict caging

J. Vidal *et.al.*, PRL 85, 3906 (2000)
M.Tovmasyan *et.al.*, PRB 98, 134513 (2018)

$$\hat{p}_n\hat{p}_n\hat{f}^\dagger_n\hat{f}^\dagger_n + {\color{blue}\hat{p}^\dagger_n\hat{p}_n\hat{p}_n\hat{f}^\dagger_{n-1}} + \ldots + \text{h.c.}$$




**Question(s):**

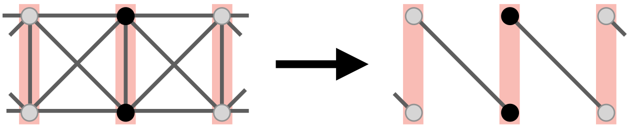
– can caging of interacting particles be enforced?

– in other words ... can we neglect particle/charge transport in interacting ABF networks by using the fine-tuning?

# Many-Body Flatband Localization

**(An) Answer:** there exists density-density interactions $\mathcal{H}_1$ invariant under unitary transformations

**Example**: $\nu = 2$ all band flat lattice



Interaction Hamiltonian $\hat{\mathcal{H}}_{\text{int}}$

$$\mathcal{H}_1 = \sum_{\kappa}\left[\hat{a}^\dagger_\kappa \hat{a}^\dagger_\kappa \hat{a}_\kappa \hat{a}_\kappa + \hat{b}^\dagger_\kappa \hat{b}^\dagger_\kappa \hat{b}_\kappa \hat{b}_\kappa + 2\hat{a}^\dagger_\kappa \hat{a}_\kappa \hat{b}^\dagger_\kappa \hat{b}_\kappa\right]$$
$$= \sum_{\kappa}\left[\hat{n}_{a,\kappa} + \hat{n}_{b,\kappa} - 1\right]\left[\hat{n}_{a,\kappa} + \hat{n}_{b,\kappa}\right]$$

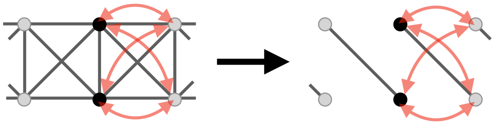
Unitary rotation - $|z_1|^2 + |w_1|^2 = 1$

$$U_1 : \begin{cases} \hat{c}_\kappa = z_1 \hat{a}_\kappa + w_1 \hat{b}_\kappa \\ \hat{d}_\kappa = -w_1^* \hat{a}_\kappa + z_1^* \hat{b}_\kappa \end{cases}$$

CD, A. Andreanov, S. Flach, PRB 102, 041116(R) (2020)

# Many-Body Flatband Localization

**Another example** (for spinless fermions)
Y. Kuno *et.al.*, NJP 22, 013032 (2020)



Invariant interaction

$$\mathcal{H}_1 = \sum_{\kappa} \left[\hat{n}_{a,\kappa} + \hat{n}_{b,\kappa}\right]\left[\hat{n}_{a,\kappa+1} + \hat{n}_{b,\kappa+1}\right]$$

**Properties:**

- any number of bands
- any dimension
- it holds for different many-body statistics (*e.g.* bosons, fermions)
- support the breaking of translation invariance (*e.g.* correlated disorder)
- for any interaction strength
- posses a extensive number of local integrals of motion
- but ... what about heat transport?
  Ihor's talk: Percolation Transitions in Interacting Many-Body Flatband Systems
  arXiv:2106.01664 (2021)

CD, A. Andreanov, S. Flach, PRB 102, 041116(R) (2020)

# Conclusions

**Take home messages:**

- flatband lattices exist in parametric families
  PRB 95, 115135 (2017) ... and many more ...
- this allows hierarchies of fine-tuning
- fine tuning flatband networks allows to tame perturbations (interaction terms)
- fine tuned all-band-flat lattices
  - with Kerr nonlinearity allows exact caging
    PRB **104**, 085131 (2021)
  - with Hubbard interaction yield interaction dependent many-body CLS
    PRB **104**, 085132 (2021)
- fine-tuned interaction in all-band-flat lattices yields many-body flatband localization
  PRB **102**, 041116(R) (2020)
  arXiv:2106.01664 (2021)

**Other works:**

- flatband-induced disorder-free localization
  arXiv:2104.11055 (2021)
- compact breather generator
  arXiv:2104.11458 (2021)