



Lifetime of flatband states Clemens Gneiting

in collaboration with

Z. Li and F. Nori

IBS conference *Flatbands: symmetries, disorder, interactions and thermalization*, 19. August 2021



(Brief) introduction to flatbands



Quantitative treatment: Disorder-dressed quantum evolution

Disorder-induced decay in the cross-stitch model



Flatbands



- increased sensitivity to interaction and disorder
- feature compact localized states
- rich interplay with topology, symmetries, geometry

this talk is about: interplay of flatbands and disorder

this talk is not about ..

- construction/identification of flatbands
- symmetry or topology
- interactions (we assume a single-particle picture)
- here: one-dimensional lattices (generalizations to higher dimensions conceivable)

(cf. many talks in this conference)

(In)stability of flatband states in the presence of disorder: generic decay mechanism

(In)stability of flatband states in the presence of disorder











 $|\psi_t(j)|^2$

(In)stability of flatband states in the presence of disorder



Intersecting dispersive band

 Dephasing-mediated momentum broadening couples state to the dispersive band



The backcoupling effectively diffuses the flatband state



Would like to have a dynamical description of what happens (for arbitrary initial states)

(In)stability of flatband states in the presence of disorder

e.g., S. Flach et al, Europhys. Lett. 105, 30001 (2014)



The cross-stitch lattice

Real-space Hamiltonian:

$$\hat{H} = -J\sum_{j\in\mathbb{Z}} \left\{ (|j\rangle\langle j+1| + |j\rangle\langle j-1|) \otimes (\mathbb{I}_2 + |\mathbf{a}\rangle\langle \mathbf{b}| + |\mathbf{b}\rangle\langle \mathbf{a}|) + t_{\mathbf{a}\mathbf{b}}|j\rangle\langle j| \otimes (|\mathbf{a}\rangle\langle \mathbf{b}| + |\mathbf{b}\rangle\langle \mathbf{a}|) \right\}$$



Momentum-space Hamiltonian:

$$\hat{H} = -J(4\cos[\hat{p}a/\hbar] + t_{\rm ab})|d\rangle\langle d| + t_{\rm ab}J|f\rangle\langle f$$

- $|d\rangle = (|a\rangle + |b\rangle)/\sqrt{2}$...dispersive band
- $|f
 angle = (|a
 angle |b
 angle)/\sqrt{2}$...flat band



- $t_{
 m ab}$ controls the position of the flatband
- one flatband and one (intersecting) dispersive band

Seek time-dependent evolution that includes disorder average

Quantitative treatment: disorder-dressed quantum evolution



The double-slit experiment







 Interference pattern delicate interplay of phases accumulated along different paths



The double-slit experiment with disorder





The double-slit experiment with disorder



Disorder potential distorts phase relations, interference pattern is lost

The disorder-averaged double-slit experiment



The disorder-averaged double-slit experiment



Disorder average recovers interference pattern, but with reduced visibility

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CG, F. Anger, A. Buchleitner, *PRA* 93, 032139 (2016)

The disorder-averaged double-slit experiment



This resembles the decoherence effect of an observing environment (obtaining which-way information)

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CG, F. Anger, A. Buchleitner, *PRA* 93, 032139 (2016) C. Kropf, CG, A. Buchleitner, *PRX* 6, 031023 (2016)

Quantum master equations

$$\partial_t \rho = -\frac{i}{\hbar} [\hat{H}, \rho] + \sum_{k=1}^N \gamma_k \mathcal{L}(\hat{L}_k, \rho)$$

 $\mathcal{L}(\hat{L},\rho)\equiv\hat{L}\rho\hat{L}^{\dagger}-\frac{1}{2}\hat{L}^{\dagger}\hat{L}\rho-\frac{1}{2}\rho\hat{L}^{\dagger}\hat{L}$...Lindblad form

usually used to describe open quantum systems (damping, decoherence)

- Lindblad form preserves Hermiticity, trace and positivity (if the rates are time-dependent, one has to be careful) of the state
- ullet Lindblad/jump operators \hat{L}_k describe incoherent part of dynamics
- for us: trace over environment is replaced by average over disorder realizations; while each realization evolves unitarily, the dynamics of the disorder-averaged state exhibits incoherent contributions (i.e., state is in general mixed)
- disorder effect in general induces non-Markovian evolution (time-dependent Lindblad operators, "negative rates"), which can give rise to coherence revivals

Disorder-dressed evolution equations ho_1 –

• Quantum map:
$$\overline{\rho}(t) = \int d\varepsilon \, p_{\varepsilon} e^{-\frac{i}{\hbar}\hat{H}_{\varepsilon}t} \rho_0 e^{\frac{i}{\hbar}\hat{H}_{\varepsilon}t}$$

Coupled disorder channels:

 $\rho_{\varepsilon} = \overline{\rho} + \Delta \rho_{\varepsilon} \qquad \hat{H}_{\varepsilon} = \overline{\overline{H}} + \hat{V}_{\varepsilon}$

Perturbative limit:

$$\begin{aligned} \partial_t \overline{\rho}(t) &= -\frac{\mathrm{i}}{\hbar} [\hat{H}_{\mathrm{eff}}(t), \overline{\rho}(t)] \\ &+ \sum_{\alpha \in \{\pm 1\}} \frac{2\alpha}{\hbar^2} \int \mathrm{d}\varepsilon \, p_\varepsilon \int_0^t \mathrm{d}t' \mathcal{L} \big(\hat{L}_{\varepsilon,t'}^{(\alpha)}, \overline{\rho}(t) \big). \end{aligned} \qquad \begin{aligned} H_{\mathrm{eff}}(t) &= \overline{H} - \frac{\mathrm{i}}{2\hbar} \int \mathrm{d}\varepsilon \, p_\varepsilon \int_0^t \mathrm{d}t' \, [\hat{V}_\varepsilon, \hat{\tilde{V}}_\varepsilon(t')] \\ \hat{L}_{\varepsilon,t}^{(\alpha)} &= \frac{1}{2} \big(\hat{V}_\varepsilon + \alpha \, \hat{\tilde{V}}_\varepsilon(t) \big), \\ \hat{\tilde{V}}_\varepsilon(t) &= \overline{U}(t) \hat{V}_\varepsilon \overline{U}(t)^{\dagger} \end{aligned}$$

 ρ_0



Applicable under weak disorder and up to finite times

 General: applicable to arbitrary systems and disorder characteristics

CG, F. Nori, *PRA* 96, 022135 (2017) CG, *PRB* 101, 214203 (2020)

 ρ_m

 $\Delta \rho_m$

 $\overline{\rho}$

Example: Topologically protected transport



$$\int \mathrm{d}\varepsilon \, p_{\varepsilon} \, V_{\varepsilon}(x) V_{\varepsilon}(x') \equiv C(x - x') = \int_{-\infty}^{\infty} \mathrm{d}q \, e^{\frac{i}{\hbar}q(x - x')} G(q)$$

Example: Topologically protected transport

• Master equation
$$\partial_t \overline{\rho}(t) = -\frac{i}{\hbar} [v\hat{p}, \overline{\rho}(t)] + \int_{-\infty}^{\infty} dq \, \frac{2tG(q)}{\hbar^2} \operatorname{sinc}\left[\frac{qvt}{\hbar}\right] \left\{ e^{\frac{i}{\hbar}q\hat{x}} \overline{\rho}(t) e^{-\frac{i}{\hbar}q\hat{x}} - \overline{\rho}(t) \right\}$$

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• Solution
$$\langle x|\overline{\rho}(t)|x'\rangle = \langle x - vt|\rho_0|x' - vt\rangle \exp\left[-\overline{F}_t(x - x')\right]$$

 $\overline{F}_t(x) = \frac{t^2}{\hbar^2} \int dq \, G(q) \operatorname{sinc}^2\left[\frac{qvt}{2\hbar}\right] \left\{1 - \cos\left[\frac{qx}{\hbar}\right]\right\}$



bounded momentum broadening:

$$\langle (\Delta \hat{p})^2 \rangle(t) = \langle (\Delta \hat{p})^2 \rangle_0 + \frac{4}{v^2} \int_{-\infty}^{\infty} dq \ G(q) \ \sin^2 \left[\frac{qvt}{2\hbar} \right]$$

CG, F. Nori, PRL 119, 176802 (2017)

Example: Topologically protected transport

• Master equation
$$\partial_t \overline{\rho}(t) = -\frac{i}{\hbar} [v\hat{p}, \overline{\rho}(t)] + \int_{-\infty}^{\infty} dq \, \frac{2tG(q)}{\hbar^2} \operatorname{sinc}\left[\frac{qvt}{\hbar}\right] \left\{ e^{\frac{i}{\hbar}q\hat{x}} \overline{\rho}(t) e^{-\frac{i}{\hbar}q\hat{x}} - \overline{\rho}(t) \right\}$$

• Solution
$$\langle x|\overline{\rho}(t)|x'\rangle = \langle x - vt|\rho_0|x' - vt\rangle \exp\left[-\overline{F}_t(x - x')\right]$$

Disorder-induced dephasing measured by state purity $r = \text{Tr}[\overline{\rho}^2]$





Parabolic wave guide

 $\hat{H}_{\varepsilon} = \frac{\hat{p}^2}{2m} + \hat{V}_{\varepsilon}(\hat{x})$

 Backscattering and ongoing purity decay

CG, F. Nori, PRA 93, 032139 (2016)



Propagation along wave guide

Disorder-robust entanglement transport

 $\hat{H}_{\varepsilon} = v\hat{p}_1 + v\hat{p}_2 + V_{\varepsilon}(\hat{x}_1) + V_{\varepsilon}(\hat{x}_2)$

Identified disorder-prone and disorder-robust classes of entangled states

CG, D. Leykam, F. Nori, PRL 122, 066601 (2019)

Dirac particle

 $\hat{H}_{\varepsilon} = v\hat{p}\sigma_z + m_{\varepsilon}(\hat{x})v^2\sigma_x$





Two entangled edge particles



Propagation exposed to mass fluctuations

Disorder in the cross-stitch model

 $\hat{H} = -J(4\cos[\hat{p}a/\hbar] + t_{\rm ab})|d\rangle\langle d| + t_{\rm ab}J|f\rangle\langle f|$



• on-site disorder potential $\hat{H}_{\varepsilon} = \hat{H} + \hat{V}_{\varepsilon}$ with $\hat{V}_{\varepsilon} = \hat{V}^{a}_{\varepsilon}(\hat{x}) \otimes |a\rangle \langle a| + \hat{V}^{b}_{\varepsilon}(\hat{x}) \otimes |b\rangle \langle b|$

 $\textbf{matrix-valued correlation function} \quad C_{\sigma\sigma'}(x-x') = \int d\varepsilon \, p_{\varepsilon} \, V_{\varepsilon}^{\sigma}(x) V_{\varepsilon}^{\sigma'}(x') = \int_{-\infty}^{\infty} dq \, e^{\frac{i}{\hbar}q(x-x')} G_{\sigma\sigma'}(q)$

interband coupling vanishes with perfectly correlated sublattice potentials $\hat{V}_{\varepsilon} = \hat{V}_{\varepsilon}^{+}(\hat{x}) \otimes \mathbb{I}_{2} + \hat{V}_{\varepsilon}^{-}(\hat{x}) \otimes (|\mathbf{f}\rangle\langle \mathbf{d}| + |\mathbf{d}\rangle\langle \mathbf{f}|)$ with $\hat{V}_{\varepsilon}^{\pm}(\hat{x}) = \frac{1}{2}[\hat{V}_{\varepsilon}^{a}(\hat{x}) \pm \hat{V}_{\varepsilon}^{b}(\hat{x})]$ intraband correlations: $\tilde{G}_{0}(q) = \frac{1}{2}[G_{aa}(q) + G_{ab}(q)]$ interband correlations: $\tilde{G}_{1}(q) = \frac{1}{4}[G_{aa}(q) - G_{ab}(q)]$



Single-intersection approximation:

$$\hat{H} = -J(4\cos[\hat{p}a/\hbar] + t_{ab})|d\rangle\langle d| + t_{ab}J|f\rangle\langle f|$$
$$\approx v(\hat{p} - p_1) \otimes |d\rangle\langle d|$$





Single-intersection approximation:

$$\hat{H} = -J(4\cos[\hat{p}a/\hbar] + t_{ab})|d\rangle\langle d| + t_{ab}J|f\rangle\langle f|$$
$$\approx v(\hat{p} - p_1) \otimes |d\rangle\langle d|$$



$$\begin{aligned} & \bullet \text{ Coupled evolution equations:} \\ \partial_t \overline{\rho}_{\rm f} = \frac{2t}{\hbar^2} \int_{-\infty}^{\infty} dq \, \tilde{G}_0(q) \{ e^{iq\hat{x}/\hbar} \overline{\rho}_{\rm f} e^{-iq\hat{x}/\hbar} - \overline{\rho}_{\rm f} \} \\ & \overline{\rho}_{\rm d} = \langle {\rm f} | \overline{\rho} | {\rm f} \rangle \\ & \overline{\rho}_{\rm d} = \langle {\rm d} | \overline{\rho} | {\rm d} \rangle \\ & - \frac{2}{\hbar^2} \int_{-\infty}^{\infty} dq \, \tilde{G}_1(q) \int_0^t dt' \{ e^{ivt'q/\hbar} e^{-ivt'(\hat{p}-p_1)/\hbar} \overline{\rho}_{\rm f} \\ & - e^{iq\hat{x}/\hbar} \overline{\rho}_{\rm d} e^{-ivt'(\hat{p}-p_1)/\hbar} e^{-iq\hat{x}/\hbar} + h.c. \} \\ \partial_t \overline{\rho}_{\rm d} = -\frac{i}{\hbar} [v\hat{p}, \overline{\rho}_{\rm d}] + \frac{2t}{\hbar^2} \int_{-\infty}^{\infty} dq \, \tilde{G}_0(q) {\rm sinc} [\frac{vtq}{\hbar}] \{ e^{iq\hat{x}/\hbar} \overline{\rho}_{\rm d} e^{-iq\hat{x}/\hbar} - \overline{\rho}_{\rm d} \} \\ & - \frac{2}{\hbar^2} \int_{-\infty}^{\infty} dq \, \tilde{G}_1(q) \int_0^t dt' \{ e^{ivt'(\hat{p}-p_1)/\hbar} \overline{\rho}_{\rm d} \\ & - \frac{2}{\hbar^2} \int_{-\infty}^{\infty} dq \, \tilde{G}_1(q) \int_0^t dt' \{ e^{ivt'(\hat{p}-p_1)/\hbar} \overline{\rho}_{\rm d} \\ & 25 - e^{iq\hat{x}/\hbar} \overline{\rho}_{\rm f} e^{-iq\hat{x}/\hbar} e^{ivt'(\hat{p}-p_1)/\hbar} + h.c. \} \end{aligned}$$



Single-intersection approximation:

$$\hat{H} = -J(4\cos[\hat{p}a/\hbar] + t_{ab})|d\rangle\langle d| + t_{ab}J|f\rangle\langle f|$$
$$\approx v(\hat{p} - p_1) \otimes |d\rangle\langle d|$$



$$\begin{split} \partial_t \overline{\rho_{\rm f}} &\approx \frac{2t}{\hbar^2} \int_{-\infty}^{\infty} dq \, \tilde{G}_0(q) \big\{ e^{iq\hat{x}/\hbar} \overline{\rho_{\rm f}} e^{-iq\hat{x}/\hbar} - \overline{\rho_{\rm f}} \big\} \\ &- \frac{2}{\hbar^2} \int_{-\infty}^{\infty} dq \, \tilde{G}_1(q) \int_0^t dt' \Big\{ e^{ivt'q/\hbar} e^{-ivt'(\hat{p}-p_1)/\hbar} \overline{\rho_{\rm f}} \\ &- e^{iq\hat{x}/\hbar} \overline{\rho_{\rm d}} e^{-ivt'(\hat{p}-p_1)/\hbar} e^{-iq\hat{x}/\hbar} + h.c. \Big\} \end{split}$$

 $\overline{\rho}_{\rm d}\approx 0$

 initial state resides in the flatband, backcoupling from the dispersive band negligible





$$\hat{H} = v(\hat{p} - p_1) \otimes |\mathbf{d}\rangle \langle \mathbf{d}|$$



Decay into the dispersive band: perfectly anticorrelated sublattice potentials, $\tilde{G}_0(q) = 0$

$$\partial_t \overline{\rho}_{\rm f} \approx \frac{2t}{\hbar^2} \int_{-\infty}^{\infty} dq \, \tilde{G}_0(q) \left\{ e^{iq\hat{x}/\hbar} \overline{\rho}_{\rm f} e^{-iq\hat{x}/\hbar} - \overline{\rho}_{\rm f} \right\} \\ - \frac{2}{\hbar^2} \int_{-\infty}^{\infty} dq \, \tilde{G}_1(q) \int_0^t dt' \left\{ e^{ivt'q/\hbar} e^{-ivt'(\hat{p}-p_1)/\hbar} \overline{\rho}_{\rm f} + h.c. \right\}$$

$$\overline{\rho}_{\rm f}(p) = \overline{\rho}_{\rm f,0}(p) e^{-\overline{\Gamma}_t(p-p_1)} \quad \text{with} \quad \overline{\Gamma}_t(p) = \frac{2}{\hbar^2} \int_{-\infty}^{\infty} dq \, \tilde{G}_1(q) t^2 {\rm sinc}^2 \left[\frac{vt(q-p)}{2\hbar}\right]$$

momentum-dependent decay into dispersive band (for finite correlation length); suppressed if flatband state is detuned from the intersection

$$|v|t \gg \ell \rightarrow \overline{\Gamma}_t(p) = \frac{4\pi t}{\hbar |v|} \tilde{G}_1(p)$$





$$\hat{H} = v(\hat{p} - p_1) \otimes |\mathbf{d}\rangle \langle \mathbf{d}|$$





Disorder-induced dephasing:

perfectly correlated sublattice potentials, $\tilde{G}_1(q) = 0$

$$\begin{split} \partial_t \overline{\rho}_{\mathbf{f}} &= \frac{2t}{\hbar^2} \int_{-\infty}^{\infty} dq \, \tilde{G}_0(q) \big\{ e^{iq\hat{x}/\hbar} \overline{\rho}_{\mathbf{f}} e^{-iq\hat{x}/\hbar} - \overline{\rho}_{\mathbf{f}} \big\} \\ &- \frac{2}{\hbar^2} \int_{-\infty}^{\infty} dq \, \tilde{G}_1(q) \int_0^t dt' \big\{ e^{ivt'q/\hbar} e^{-ivt'(\hat{p}-p_1)/\hbar} \overline{\rho}_{\mathbf{f}} \\ &- e^{iq\hat{x}/\hbar} \overline{\rho}_{\mathbf{d}} e^{-ivt'(\hat{p}-p_1)/\hbar} e^{-iq\hat{x}/\hbar} + h.c. \big\} \end{split}$$



disorder-induced dephasing

$$\langle x|\overline{\rho}_{\rm f}(t)|x'\rangle = \langle x|\overline{\rho}_{{\rm f},0}|x'\rangle e^{-\overline{F}_t(x-x')} \quad \text{with} \quad \overline{F}_t(x) = \frac{t^2}{\hbar^2} \int_{-\infty}^{\infty} dq \,\tilde{G}_0(q) \Big\{1 - \cos\big[\frac{qx}{\hbar}\big]\Big\}$$

 $\langle (\Delta \hat{p})^2 \rangle(t) = \langle (\Delta \hat{p})^2 \rangle_0 + \frac{t^2}{\hbar^2} \int_{-\infty}^{\infty} dq \, q^2 \tilde{G}_0(q)$ dephasing-induced momentum broadening



Disorder-induced lifetime of flatband states, detuning from the intersection controls "lifetime" (delayed decay into dispersive band)

 $\tau \lesssim \frac{(p_0 - p_1)\ell}{\sqrt{C_0(1 + \delta)}}$
 $\tau \lesssim \frac{(p_0 - p_1)\ell}{\sqrt{C_0(1 + \delta)}}$
 $\tau \lesssim \frac{(p_0 - p_1)\ell}{\sqrt{C_0(1 + \delta)}}$

Compared with numerically exact ensemble average

CG, Z. Li, F. Nori, PRB 98, 134203 (2018)



 Backcoupling from dispersive band into flatband diffuses the flatband state symmetrically or directionally ("disorder *de*localizes")

Conclusions



Identified mechanism for the destabilization and diffusion of flatband states, where disorder "delocalizes" flatband states, mediated through coupling into dispersive bands

- Dynamical description is based on the general formalism of disorder-dressed quantum evolution, where disorder is treated as incoherent modification of the dynamics (disorder acts as an "environment", while the system remains unchanged)
- Analysis based on one-dimensional lattices and flatband intersecting with dispersive band (cross-stitch lattice), but other generic cases (higher dimensions, touching parabolic band etc.) are also conceivable
- Interplay between topology and disorder?

Thank you very much for your attention!

Collaborators



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nonclassicality measure for open system dynamics

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