Flat Bands, Sharp Physics

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Angelakis Group https://www.quantumlah.org/research/group/dimitris

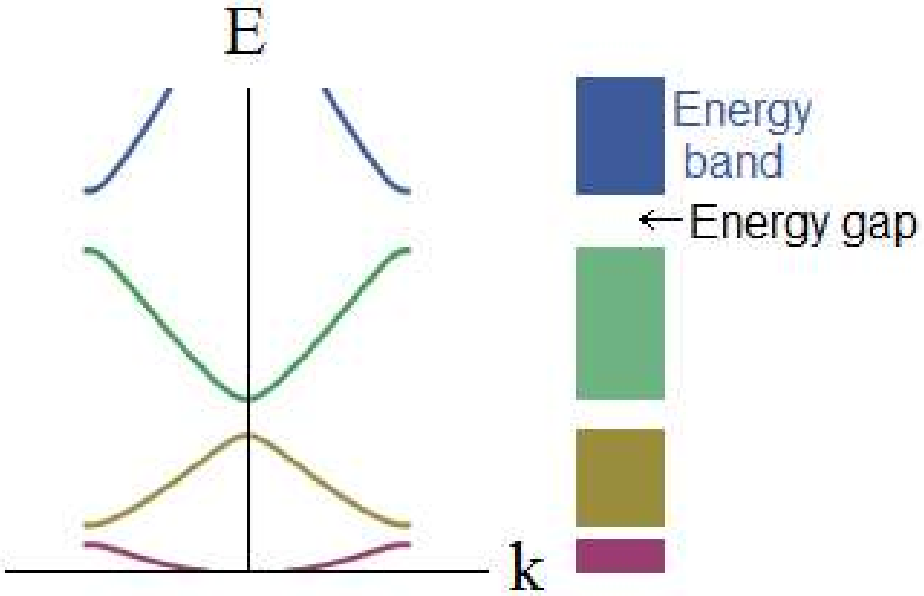
Outline

1. How and why I became interested in flat bands
2. Recent progress in creating flat bands for light, electrons, and atoms
3. Future directions: hyperbolic lattices, quantum circuits, Floquet systems

Wave propagation in lattices

$$i\partial_t \psi(\mathbf{r},t) + \nabla^2 \psi(\mathbf{r},t) + V(\mathbf{r})\psi(\mathbf{r},t) = 0,$$

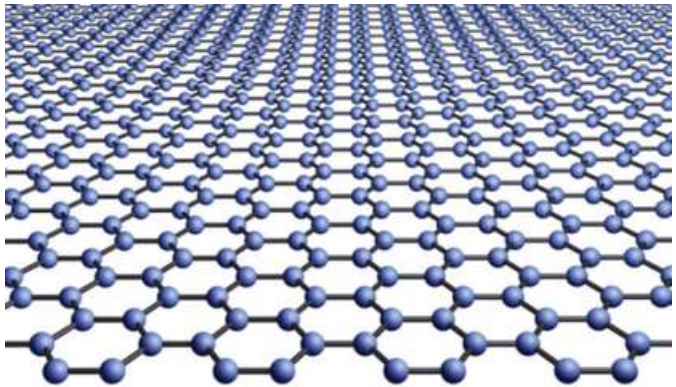
- Periodic potentials V(r): modes are Bloch waves $\psi_{\mathbf{k}} = \exp(i\mathbf{k}.\mathbf{r})u_{\mathbf{k}}(\mathbf{r})$
- Spectrum of energy eigenvalues *E* forms discrete set of bands
- Control wave propagation via band structure (e.g. group velocity $\boldsymbol{v}_g = \nabla_{\boldsymbol{k}} E$)



Dirac materials

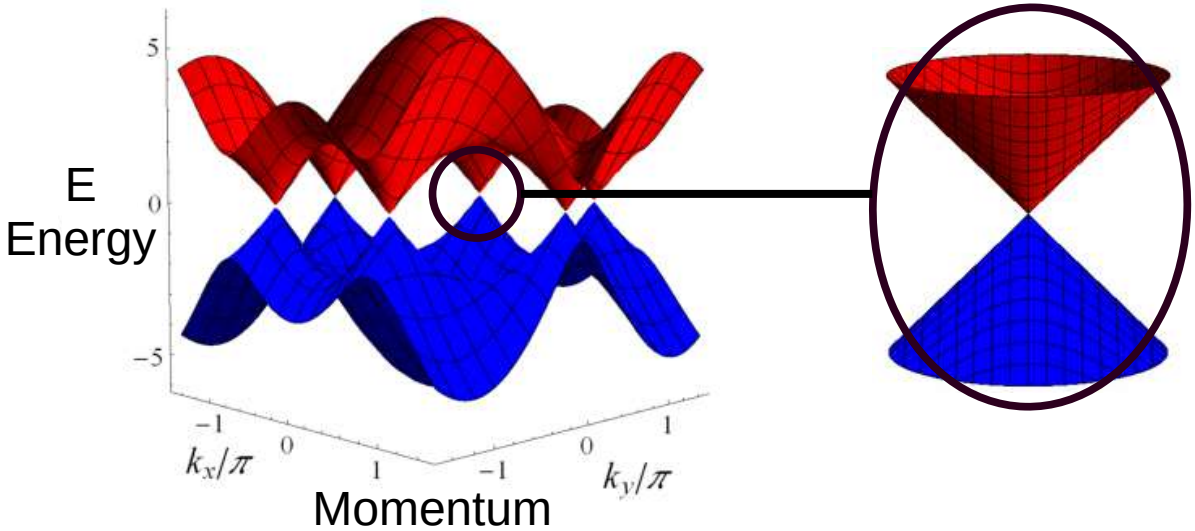
- Lattices hosting relativistic (massless) dispersion for non-relativistic (massive) particles
- Relativistic dispersion occurs near band crossing points
- Band crossings may be protected by symmetry or created by fine tuning

Graphene honeycomb lattice



Band structure

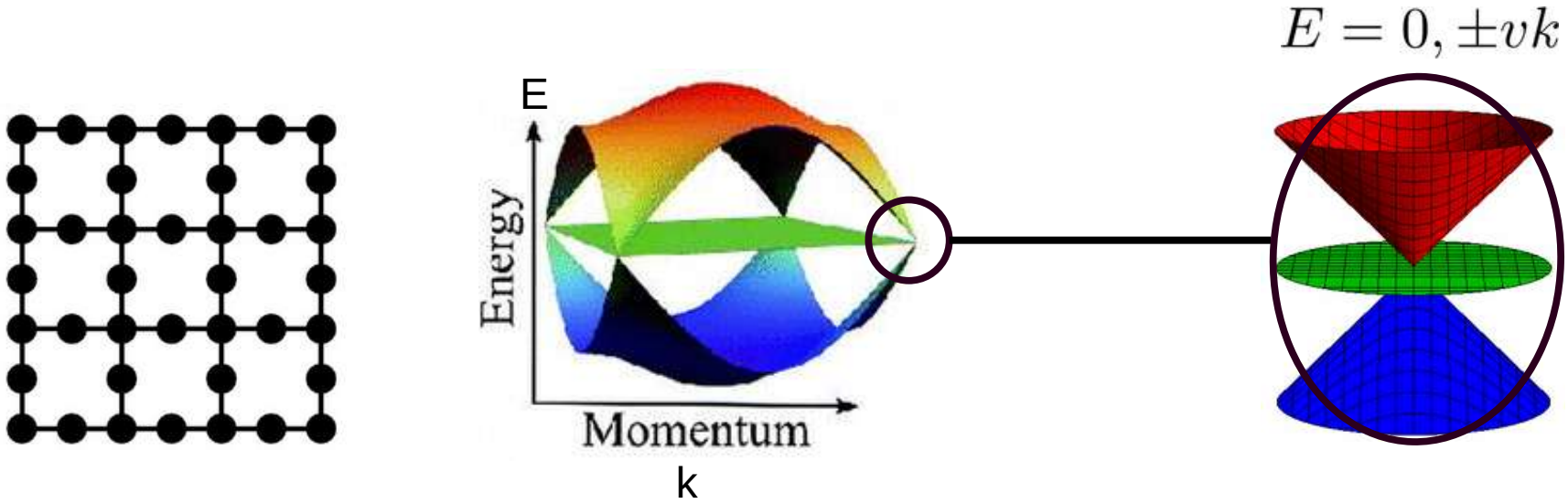
$E \propto k$



M. I. Katsnelson, K. S. Novoselov & A. K. Geim, Nature Phys. 2, 620 (2006); K. S. Novoselov, Rev. Mod Phys. 83, 837 (2011)

The face-centred square (Lieb) lattice

- Pair of conically-intersecting bands similar to graphene
- Additional perfectly flat band; infinite wave effective mass
- Wave energy in flat band is independent of momentum



E.H. Lieb, Phys. Rev. Lett. 62, 1201 (1989); R. Shen et al., Phys. Rev. B 81, 041410 (2010); V. Apaja et al., Phys. Rev. A 82, 041402 (2010); C. Weeks & M. Franz, Phys. Rev. B 82, 085310 (2010); N. Goldman et al., Phys. Rev. A 83, 063601 (2011); K. Asano & C. Hotta, Phys. Rev. B 83, 245125 (2011).

PHYSICAL REVIEW A
covering atomic, molecular, and optical physics and quantum information

PHYSICAL REVIEW A **86**, 031805(R) (2012)

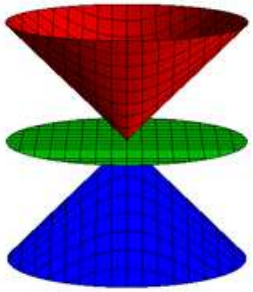
Pseudospin and nonlinear conical diffraction in Lieb lattices

Daniel Leykam,[1] Omri Bahat-Treidel,[1,2] and Anton S. Desyatnikov[1]

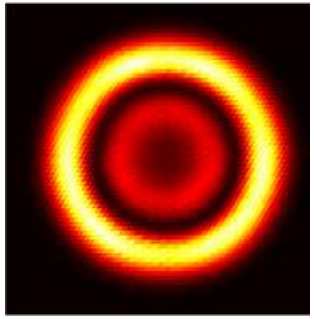
[1]*Nonlinear Physics Centre, Research School of Physics and Engineering, The Australian National University, Canberra ACT 0200, Australia*

[2]*School of Mathematics and Physics, The University of Queensland, Brisbane, Queensland 4072, Australia*

(Received 4 July 2012; published 24 September 2012)



We study linear and nonlinear wave dynamics in the Lieb lattice, in the vicinity of an intersection point between two conical bands and a flat band. We define a pseudospin operator and derive a nonlinear equation for spin-1 waves, analogous to the spin-1/2 nonlinear Dirac equation. We then study the dynamics of wave packets that are associated with different pseudospin states, and find that they are distinguished by their linear and nonlinear conical diffraction patterns.



DOI: 10.1103/PhysRevA.86.031805 PACS number(s): 42.25.Fx, 42.65.Hw

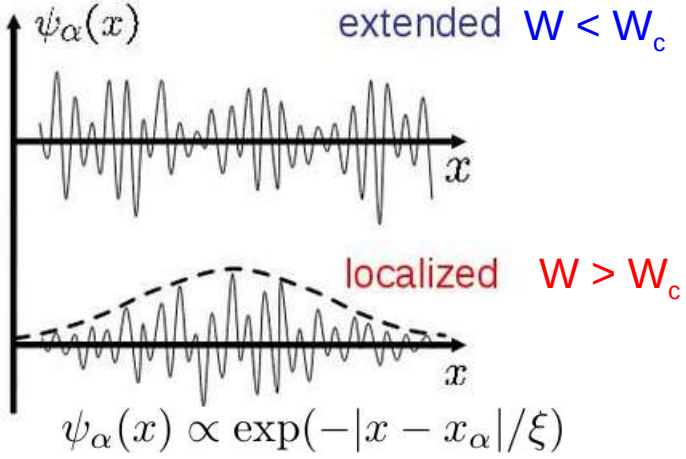
Referee: “Interesting, but experimental imperfections and disorder will make the predicted effect extremely challenging to observe...”

Anderson model of disorder

- Periodic tight binding lattice + random site-dependent perturbation
- 1D & 2D lattices: all modes localized if disorder is nonzero ($W_c = 0$)
- 3D lattices: transition from extended to localized modes at critical disorder strength W_c

$$E\psi_n = \epsilon_n \psi_n + \psi_{n-1} + \psi_{n+1}$$

$$\epsilon \in [-W/2, W/2]$$



http://lpmmc.grenoble.cnrs.fr/

P. W. Anderson, Phys. Rev. 109, 1492 (1958)

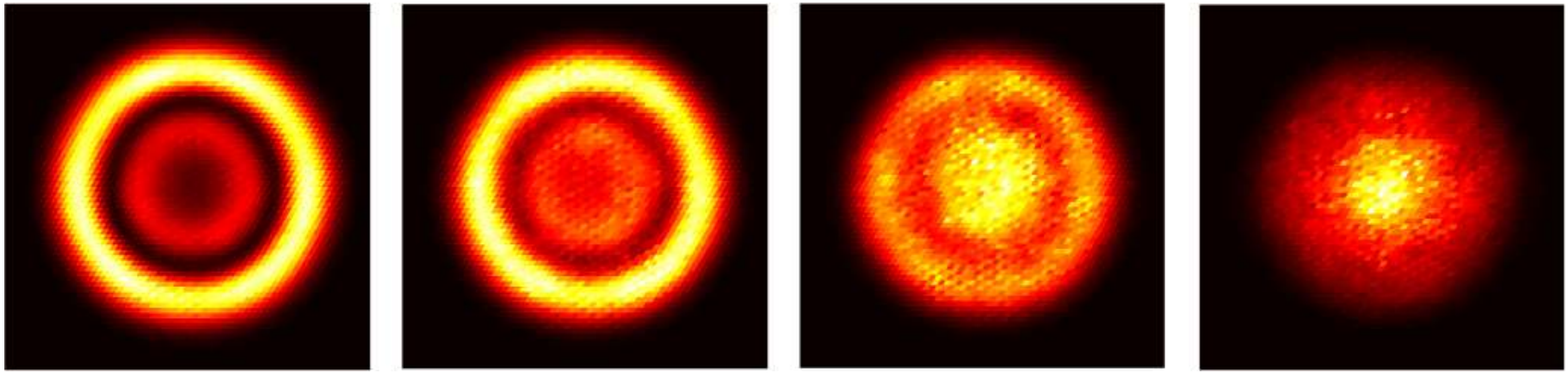
Wave propagation in disordered honeycomb and Lieb photonic lattices

Daniel Leykam[1], Omri Bahat-Treidel[2], and Anton S. Desyatnikov[1]
[1] *Nonlinear Physics Centre, Research School of Physics and Engineering, The Australian National University, Canberra ACT 0200, Australia*
[2] *School of Mathematics and Physics, The University of Queensland, Brisbane, Queensland 4072, Australia*

(Dated: February 26, 2013)

We study numerically the effect of disorder and nonlinearity on the conical diffraction of wavepackets in the honeycomb and Lieb lattices. Disorder-enhanced spreading occurs in the Lieb lattice but not the honeycomb lattice. The reason for this is their different density of states close to the band intersection point: infinite vs zero. Effect of nonlinearity....

Increasing disorder



Australian National University

Event Horizon

Research School of Physics and Engineering

Weekly newsletter | Volume 39 Number 6 | 15 - 21 February 2013

DEPARTMENTAL SEMINARS

Nonlinear Physics Centre

Nonlinear Waves in Complex Systems: essentials, problems, perspectives

Professor Sergej Flach

Institute for Advanced Study, Centre for Theoretical Physics and Chemistry, Massey University, New Zealand

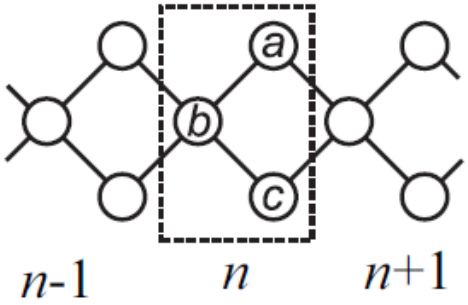
11:00am Thursday 28 February

Oliphant Building Seminar Room

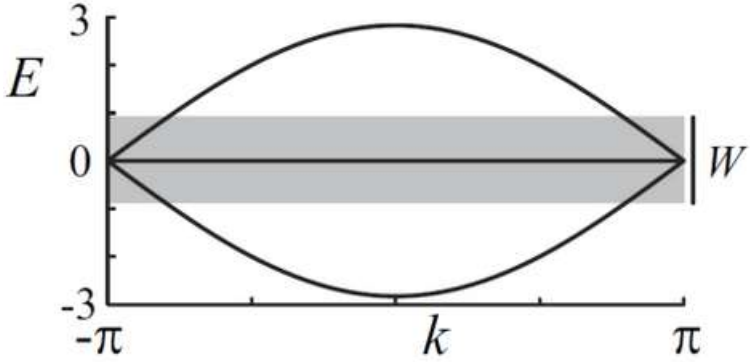
Anderson localization in the 1D diamond lattice

- Disorder with strength W gives flat band a finite width ~ W
- Close to flat band: strong localization, unconventional mode profiles
- Far from flat band: weak localization, conventional Anderson localization

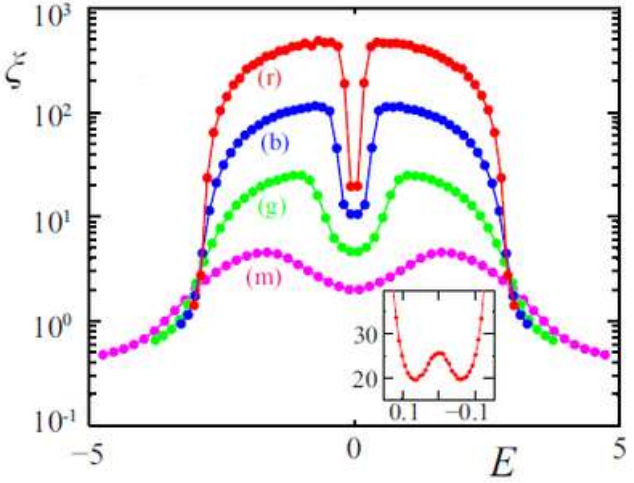
1D diamond lattice



Band structure



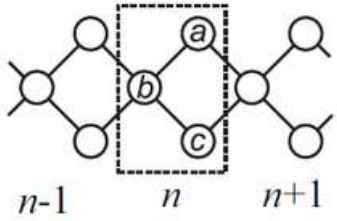
Localization length ξ



Disorder strength W = 0.5, 1, 2, 4

D. Leykam, S. Flach, O. Bahat-Treidel, and A. S. Desyatnikov, Phys. Rev. B, 88, 224203 (2013)

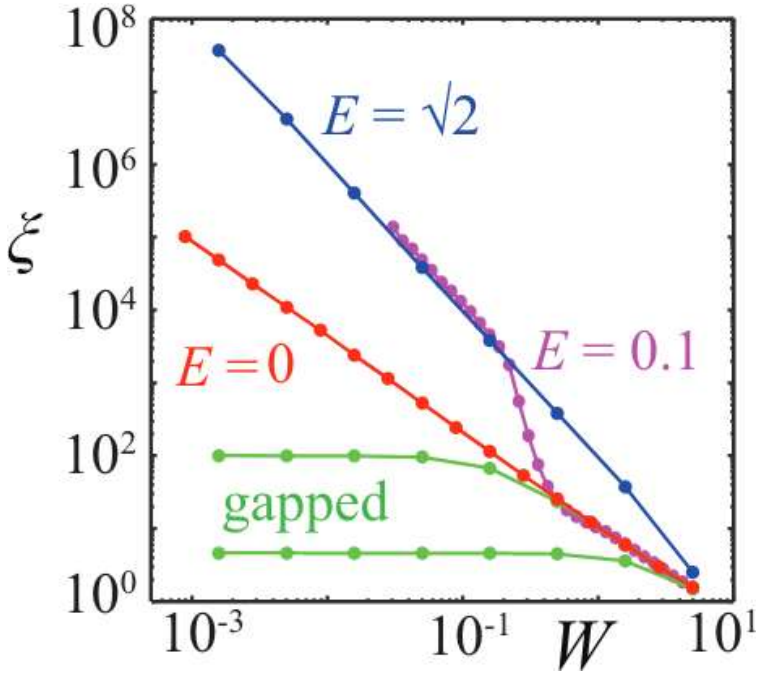
Localization length scaling



- Power law scaling with disorder strength: $\xi \sim 1/W^{\nu}$
- Exponent depends on mode energy: ν ≈ 2 (dispersive bands), ν ≈ 1.3 (flat band)
- Reduction to regular Anderson model does not explain...

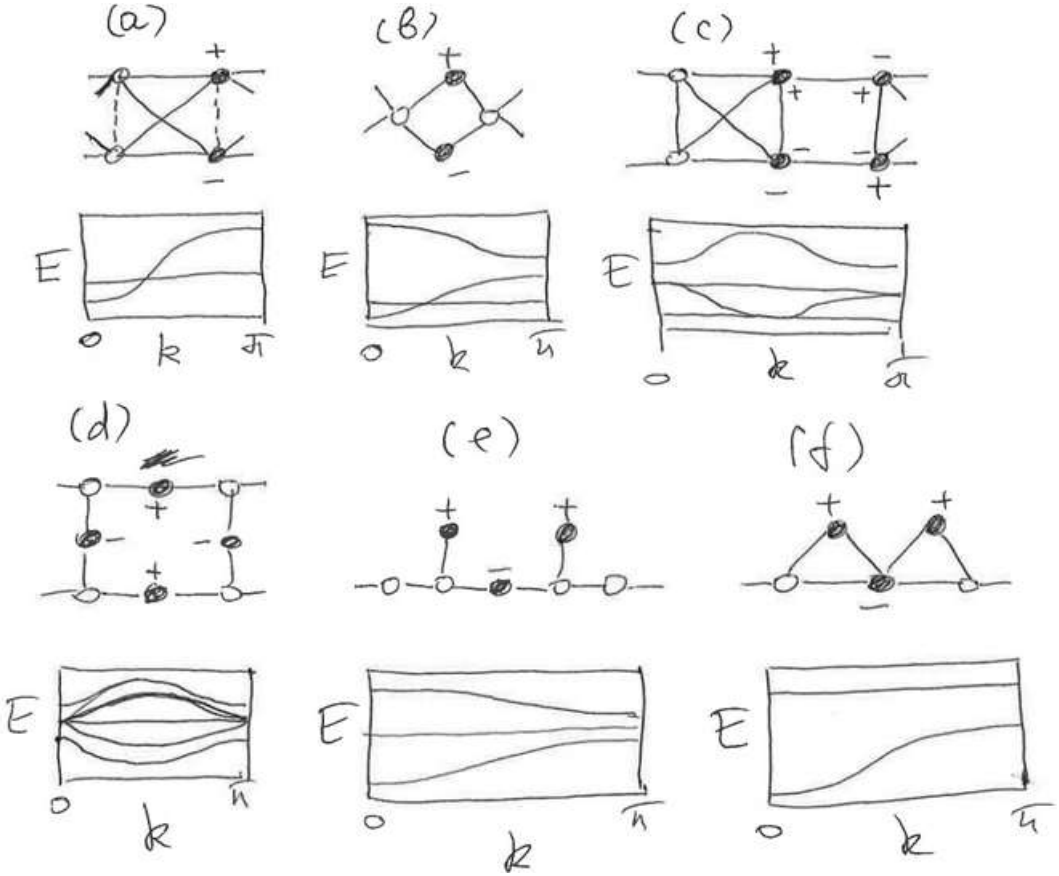
$$a_n = \frac{b_n + b_{n+1}}{E - \epsilon_{a,n}}, \qquad c_n = \frac{b_n + b_{n+1}}{E - \epsilon_{c,n}},$$

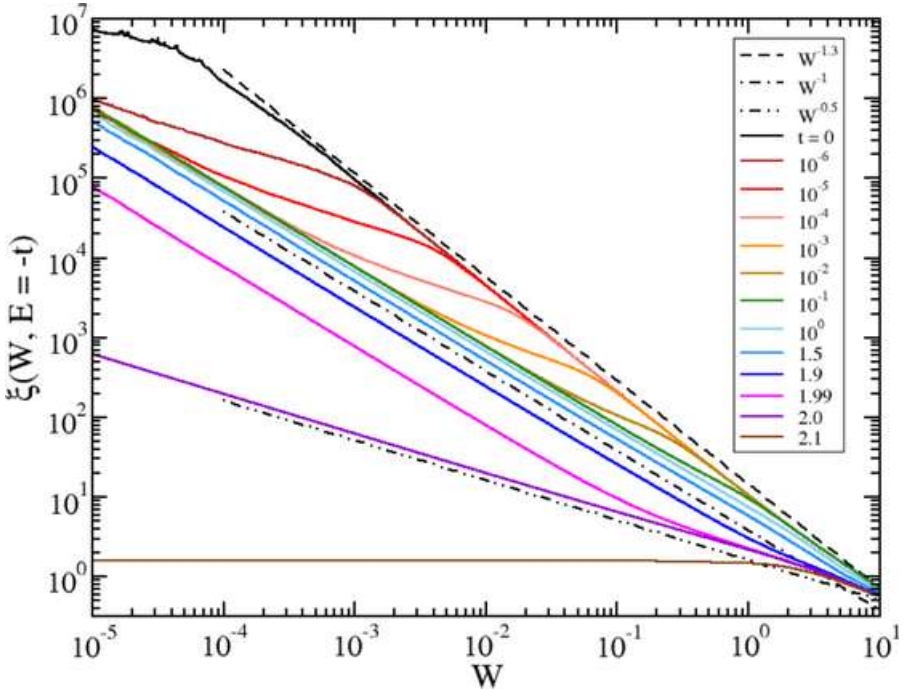
$$\varepsilon_n b_n = C_n b_{n+1} + C_{n-1} b_{n-1}$$
$$C_n = (\epsilon_{a,n} - E)^{-1} + (\epsilon_{c,n} - E)^{-1},$$
$$\varepsilon_n = \epsilon_{b,n} - E - C_n - C_{n-1},$$



Disorder strength

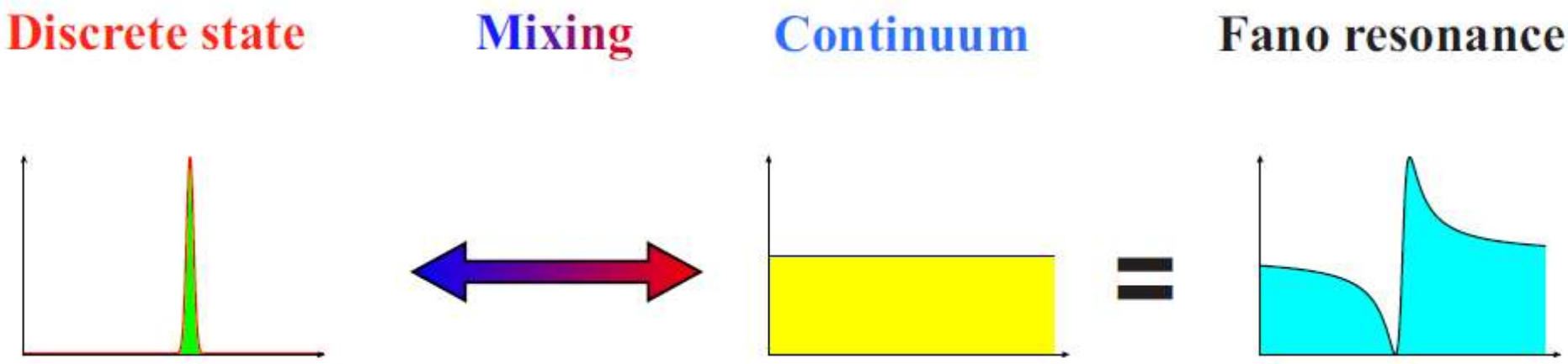
D. Leykam, S. Flach, O. Bahat-Treidel, and A. S. Desyatnikov, Phys. Rev. B, 88, 224203 (2013)


(a)
(b)
(c)
(d)
(e)
(f)
E
k
0
π


ξ(W, E = -t)
W
W^-1.3
W^-1
W^-0.5
t = 0
10^-6
10^-5
10^-4
10^-3
10^-2
10^-1
10^0
1.5
1.9
1.99
2.0
2.1

Fano resonance picture

- Flat band: localized modes embedded within a continuum of dispersive band modes
- Symmetry-breaking disorder induces weak coupling between flat and dispersive bands
- Fano resonance-like enhancement of effective disorder strength at flat band energy

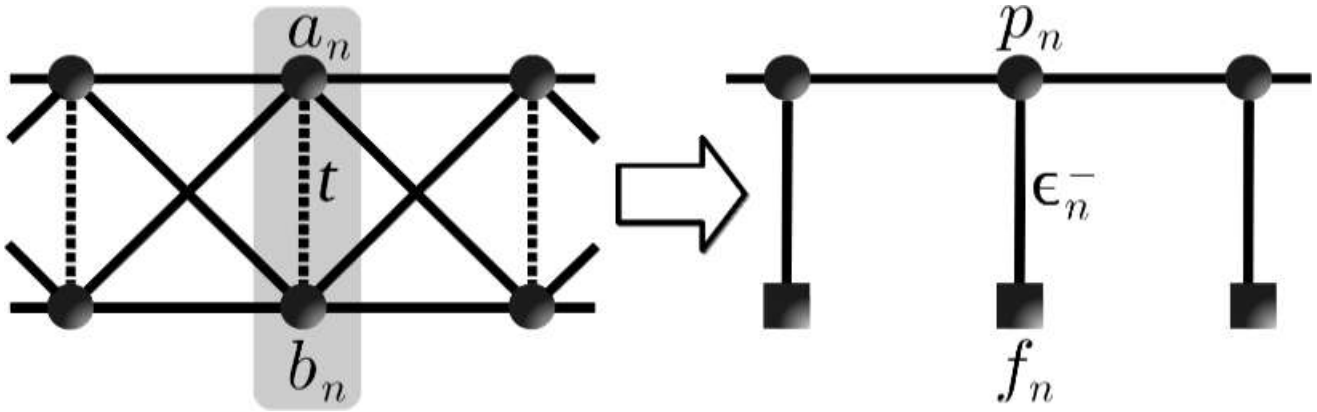


A. E. Miroshnichenko et al., Rev. Mod. Phys. 82, 2257 (2010)

Anomalous localization length scaling laws

- Eliminating flat band states from eigenvalue problem yields effective Anderson model
- Scaling of localization length with disorder strength depends on position of flat band

$$Ep_n = \left(V_n + \frac{(\epsilon_n^-)^2}{E - E_{FB} - \epsilon_n^+} \right) p_n + p_{n-1} + p_{n+1},$$



S. Flach et al., EPL 105, 30001 (2014); D.J. Thouless, J. Phys. C **5**, 77 (1972); K. Ishii, Prog. Theor. Phys. **53**, 77 (1973).

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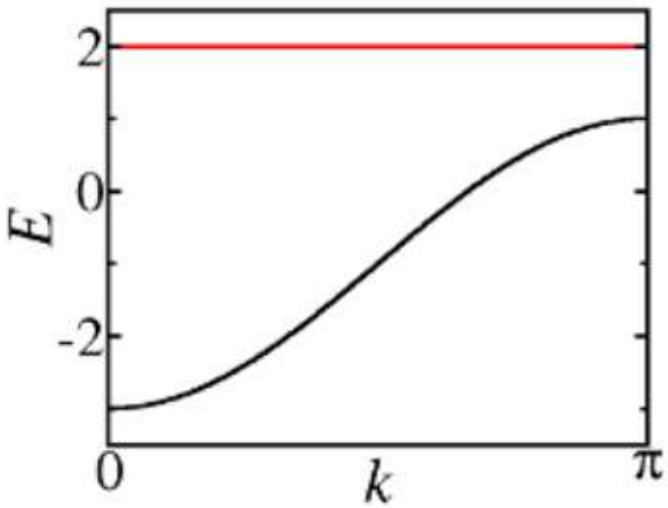

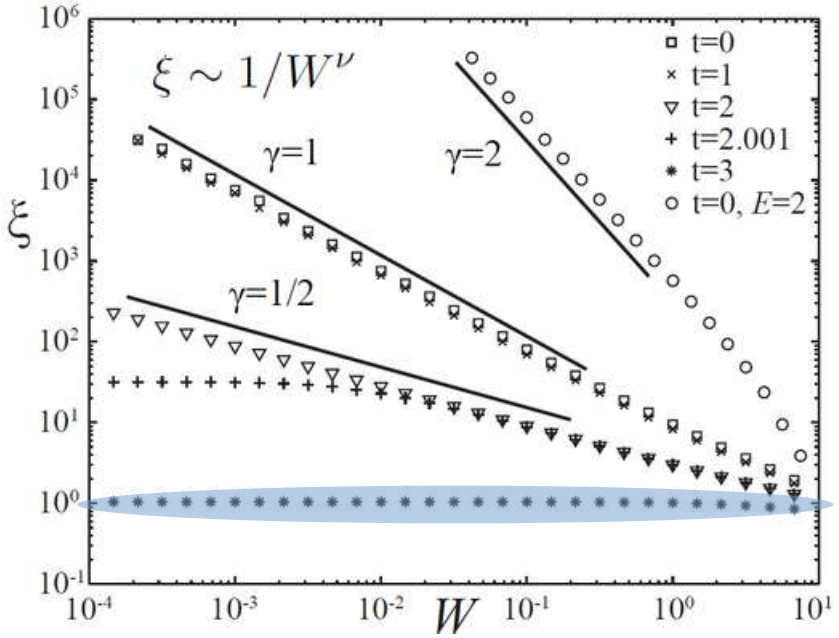

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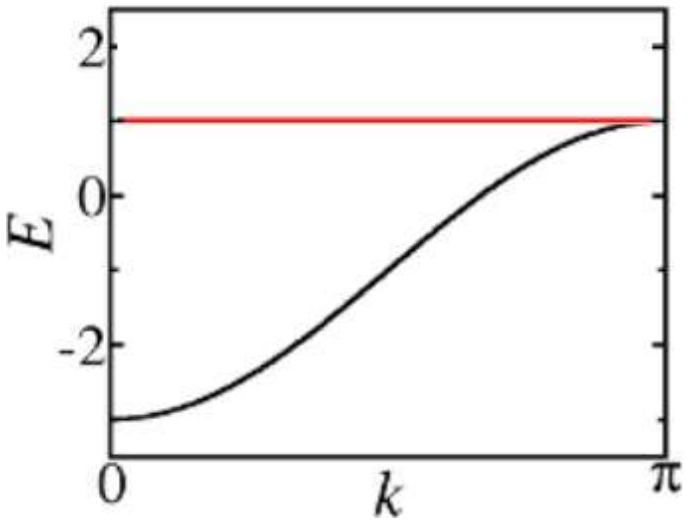

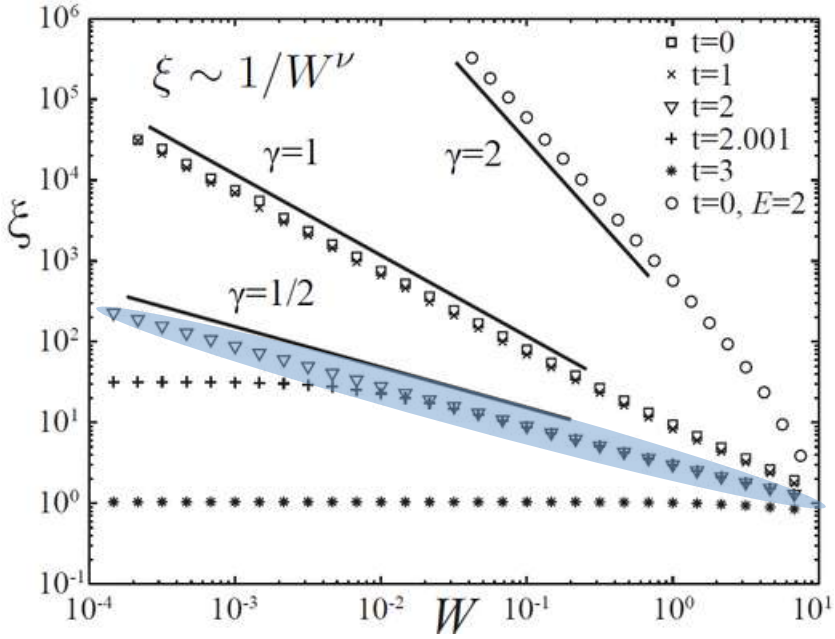

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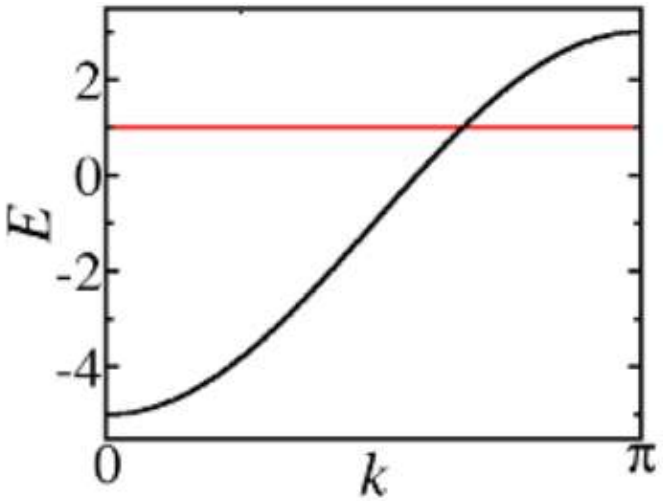
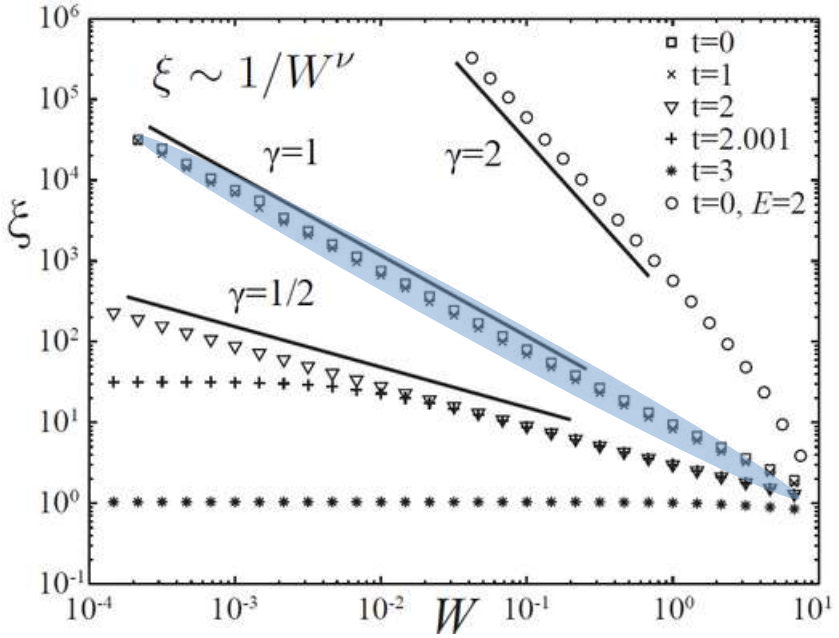




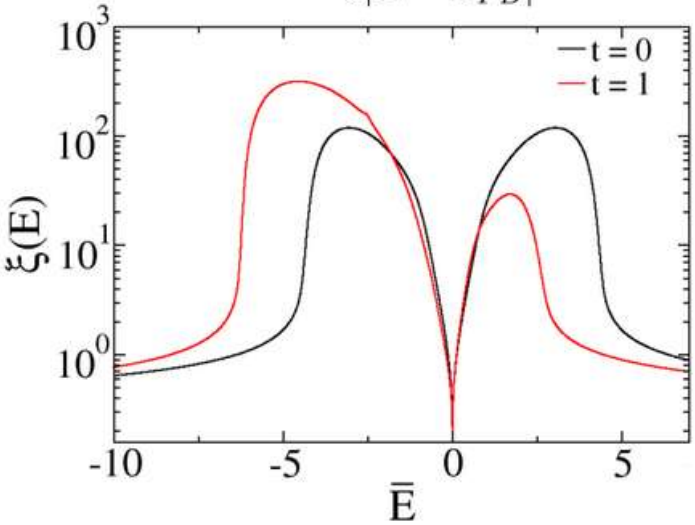
S. Flach et al., EPL 105, 30001 (2014); D.J. Thouless, J. Phys. C **5**, 77 (1972); K. Ishii, Prog. Theor. Phys. **53**, 77 (1973).

Correlated disorder: modified scaling laws

- Disorder correlations can fine-tune coupling between flat band and dispersive bands
- Exact solutions for singularities of localization length & density of states

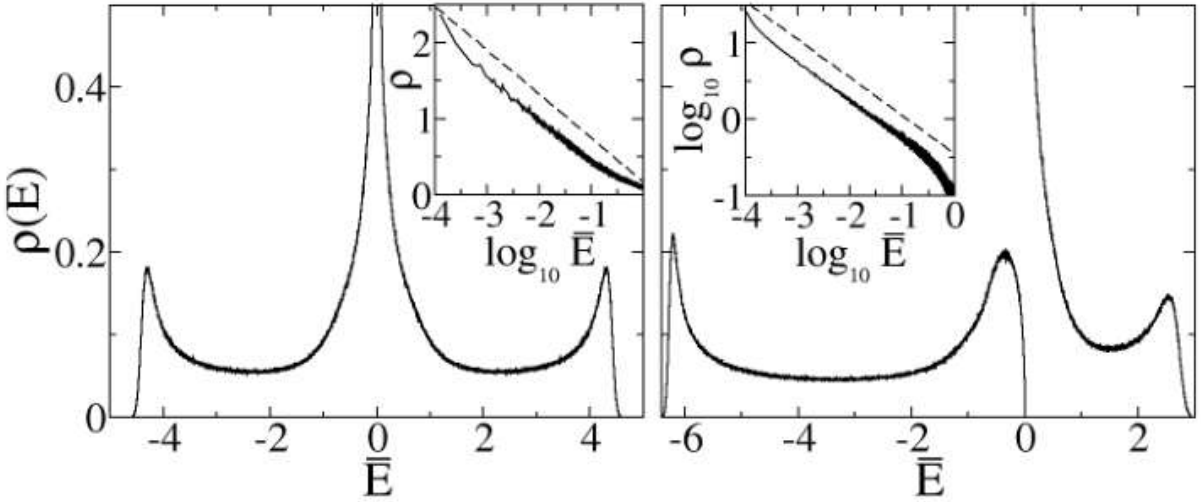
Localization length

$$\xi^{-1} = \ln \frac{W^2}{8|E - E_{FB}|} - 2.$$



Density of states

$$\rho(\bar{E}) = \frac{4}{W^2}\left(\ln\frac{W}{2} - \ln\frac{4|E - E_{FB}|}{W}\right) \qquad \rho(\bar{E}) = \frac{1}{W}\sqrt{\frac{2t}{\bar{E}}},$$

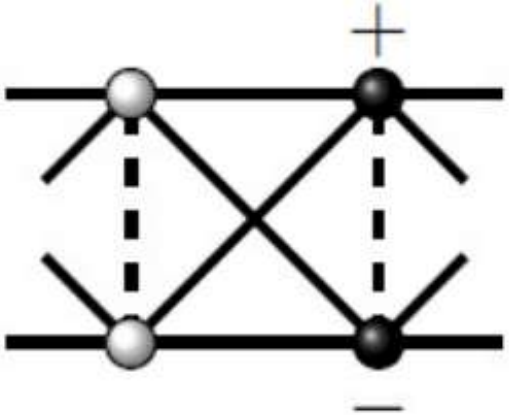


J. D. Bodyfelt, D. Leykam, C. Danieli, X. Yu, and S. Flach, Phys. Rev. Lett. 113, 236403 (2014).

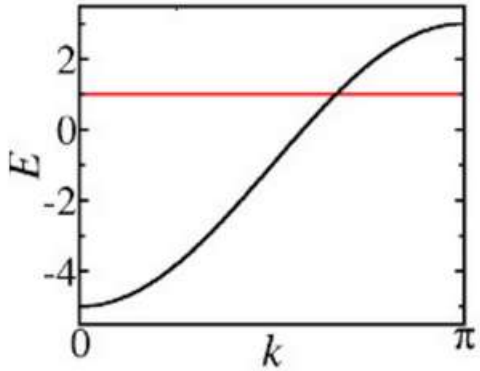
Summary: Properties of flat band lattices

- Large degeneracy of flat bands leads to sharp sensitivity to perturbations
- Response to perturbations depends on position of flat band with respect to other bands
- Basis transformations enable elegant exact solutions of non-perturbative phenomena!

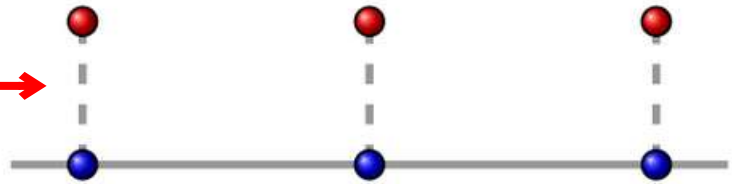
Lattice



Band structure



Basis transformation:
regular lattice + defect chain



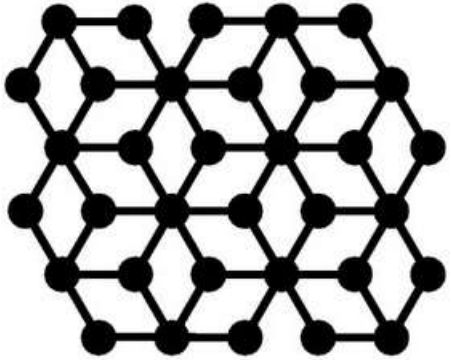
Outline

1. How and why I became interested in flat bands
2. Recent progress in creating flat bands for light, electrons, and atoms
3. Future directions: hyperbolic lattices, quantum circuits, Floquet systems

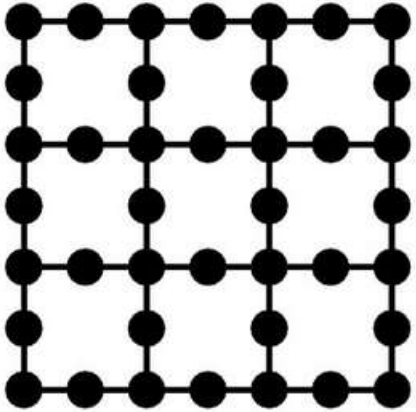
Early predictions of flat bands

- Bipartite lattices [Sutherland (1986), Lieb (1989)]
- Quasicrystals [Nori and Niu (1990)]
- Line graphs [Mielke (1991)]
- Decorated lattices [Tasaki (1992)]

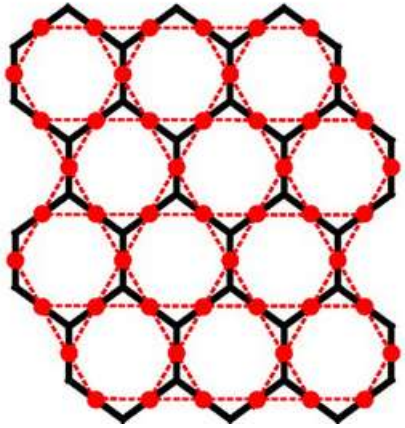
Dice lattice



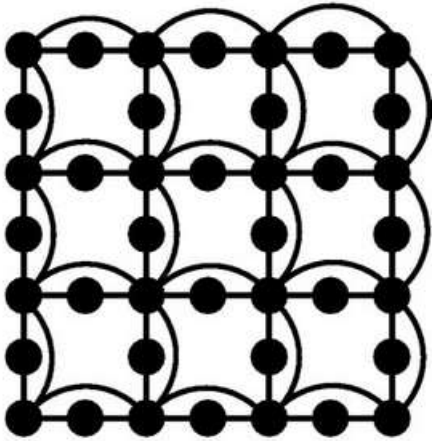
Lieb lattice



Line graph



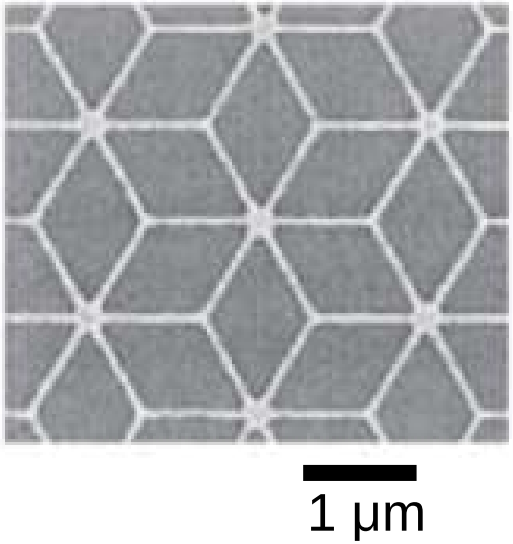
Decorated lattice

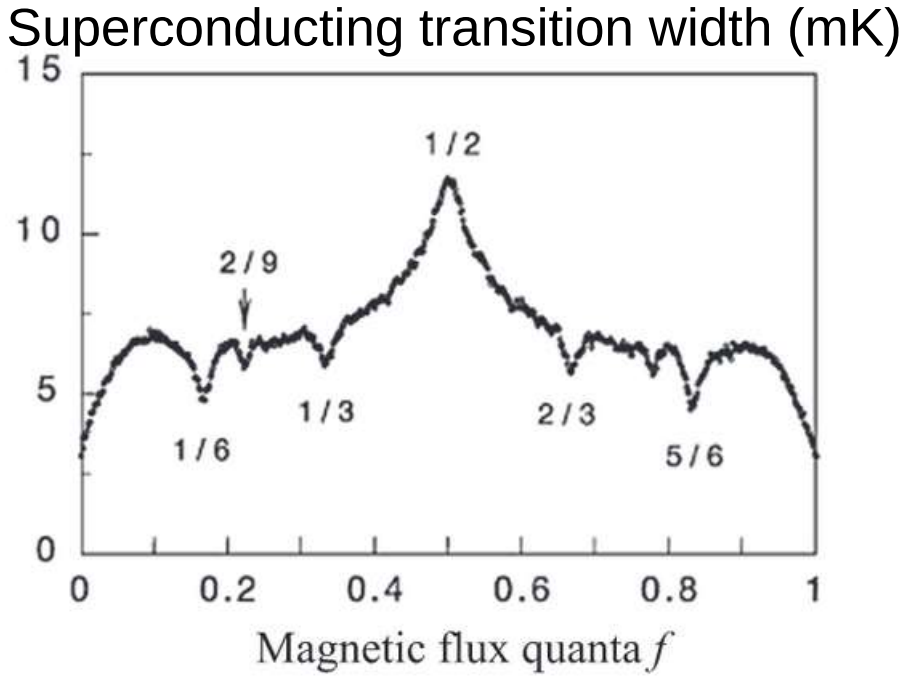


D. Leykam, A. Andreanov, and S. Flach, Adv. Phys.: X 3, 1473052 (2018)

First experiments using superconducting networks (1999)

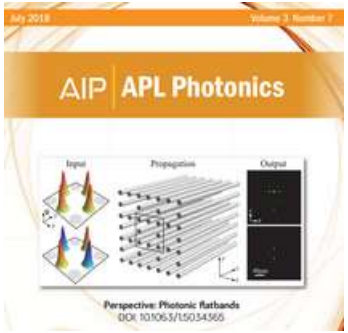
- Dice lattice + fine-tuned mT magnetic field: **all** bands become flat
- Superconducting wire lattice fabricated via electron beam lithography
- Flat spectrum: many incoherent superconducting domains, broader transition width





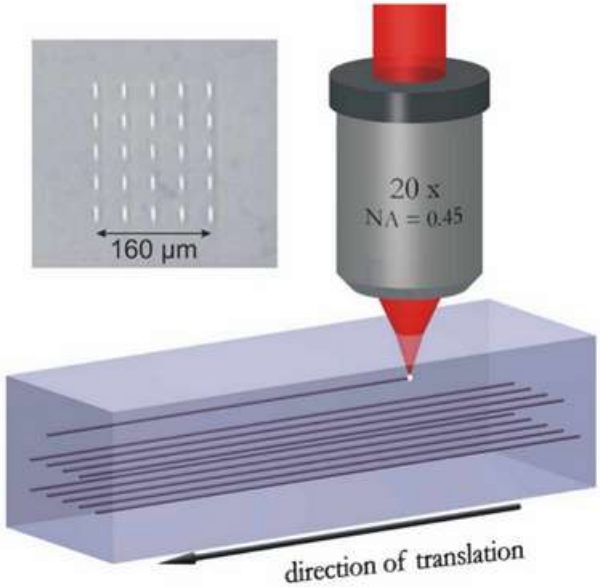
J. Vidal et al., Phys. Rev. Lett. 81, 5888 (1998); C. C. Abilio et al., Phys. Rev. Lett. 83, 5102 (1999).

Photonic flatbands (2013 - present)

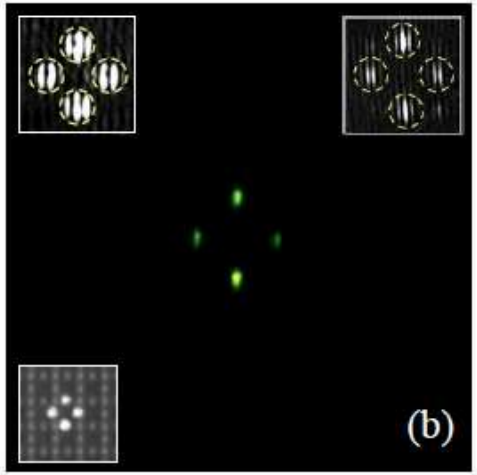


- Prepare initial states and observe their time (z) evolution

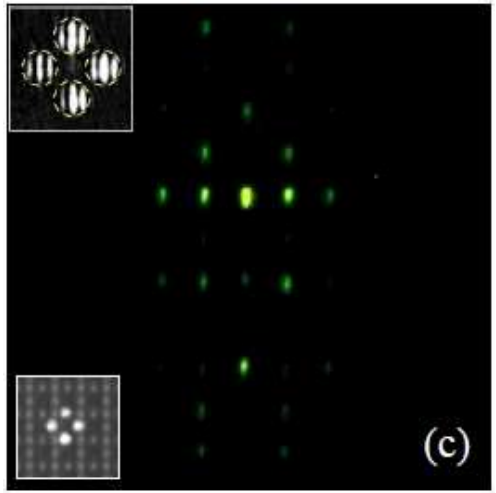
$$i\partial_z\psi = -\frac{1}{2k_0}\nabla^2\psi - \frac{k_0\Delta n(x,y,z)}{n_0}\psi,$$



Flat band output



Dispersive band output



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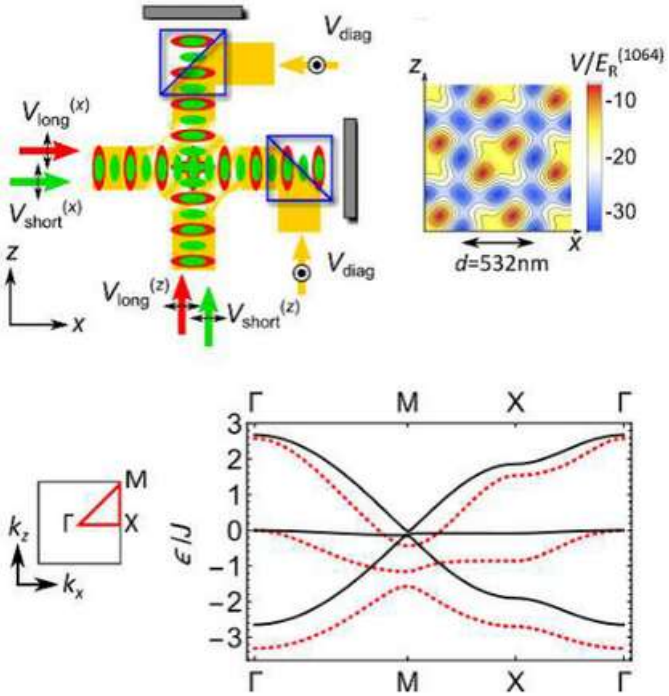
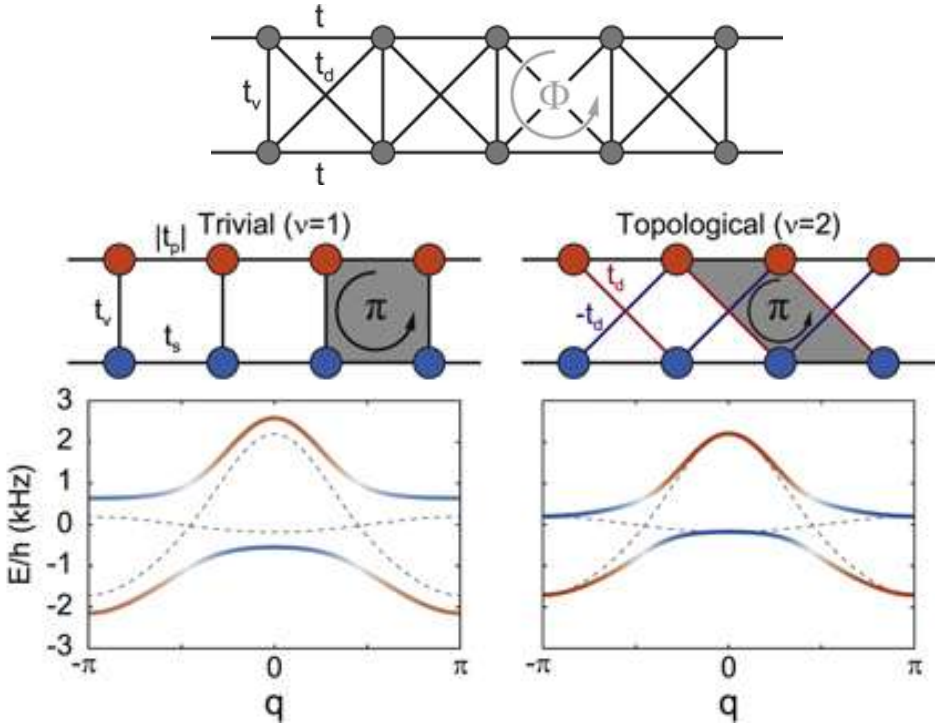
Optical lattices for cold atoms (2015 - present)

- Trade-off between lattice depth and ability to resolve dynamics
- Challenge: use multi-component atoms to implement more quasi-1D flat band models



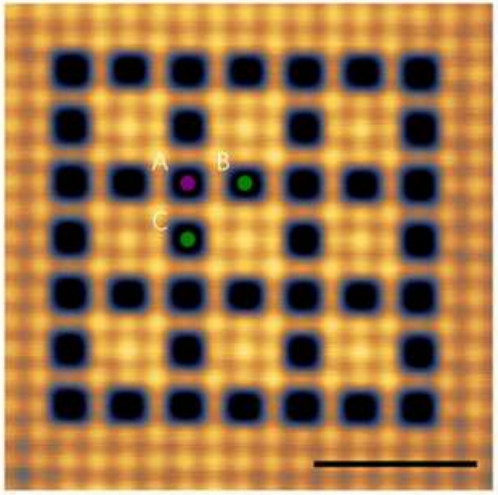


Taie et al., Sci. Adv. 1, e1500854 (2015); Ozawa et al., Phys. Rev. Lett. 118, 175301 (2017); Taie et al., Nature Commun. 11, 1 (2020); Leung et al., Phys. Rev. Lett 125, 133001 (2020); Kang et al., New J. Phys. 22, 013023 (2020);

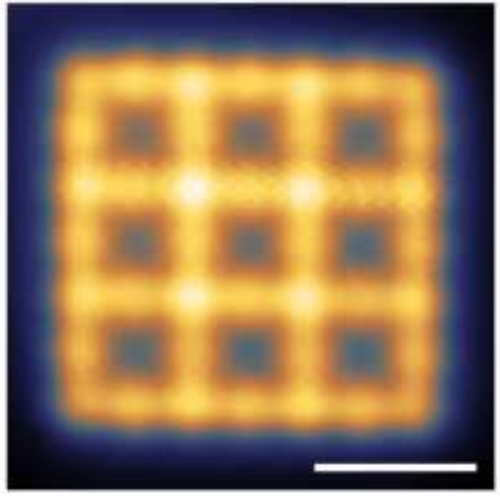
Engineered atomic lattices (2017 - present)

- Scanning tunnelling microscope used to add or remove individual atoms from monolayer
- Energy-resolved conductance mapping to reconstruct eigenstates
- 2020: realization of quasi-1D diamond, cross, and stub chains

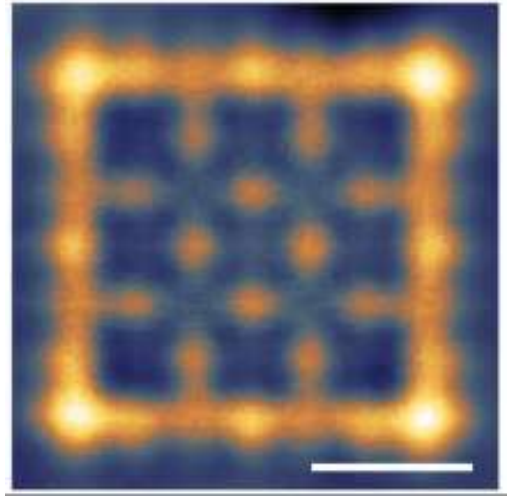
Lieb lattice



Dispersive band



Flat band



2 nm

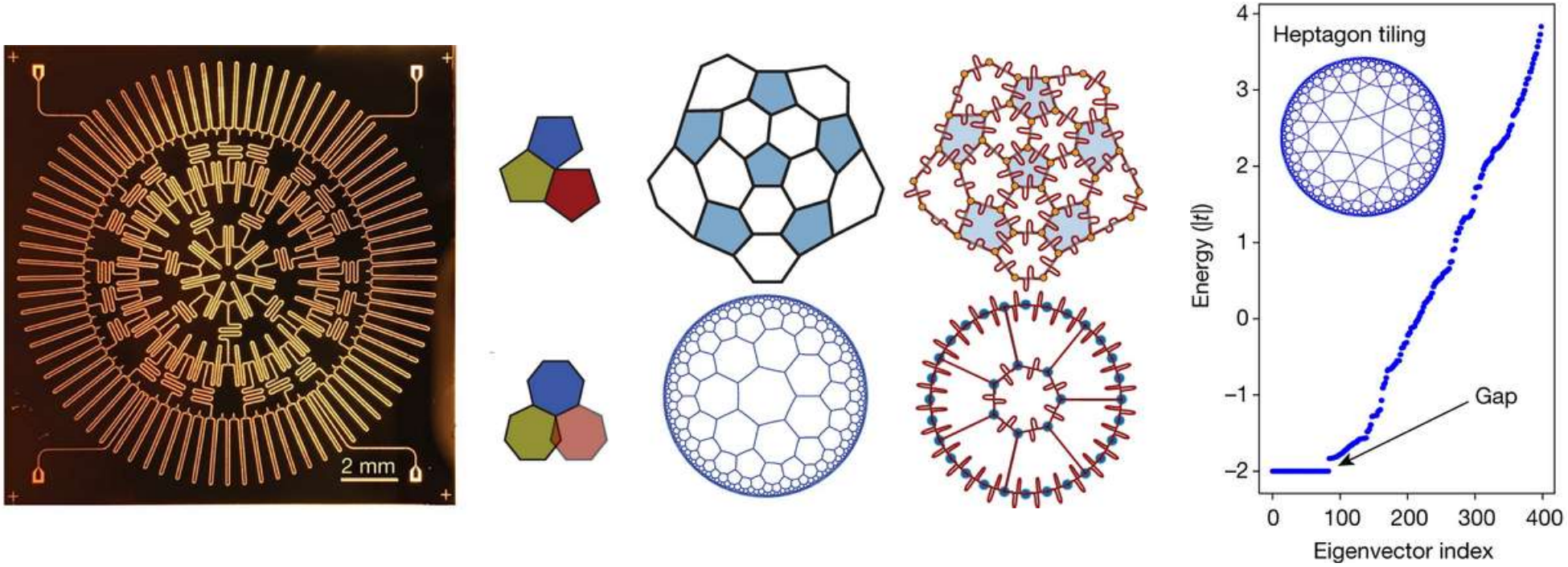
Drost et al, Nature Phys. 13, 668 (2017); Slot et al, Nature Phys. 13, 672 (2017);
Huda et al., Phys. Rev. Res. 2, 043426 (2020);

Outline

1. How and why I became interested in flat bands
2. Recent progress in creating flat bands for light, electrons, and atoms
3. Future directions: hyperbolic lattices, quantum circuits, Floquet systems

Flat bands in hyperbolic lattices

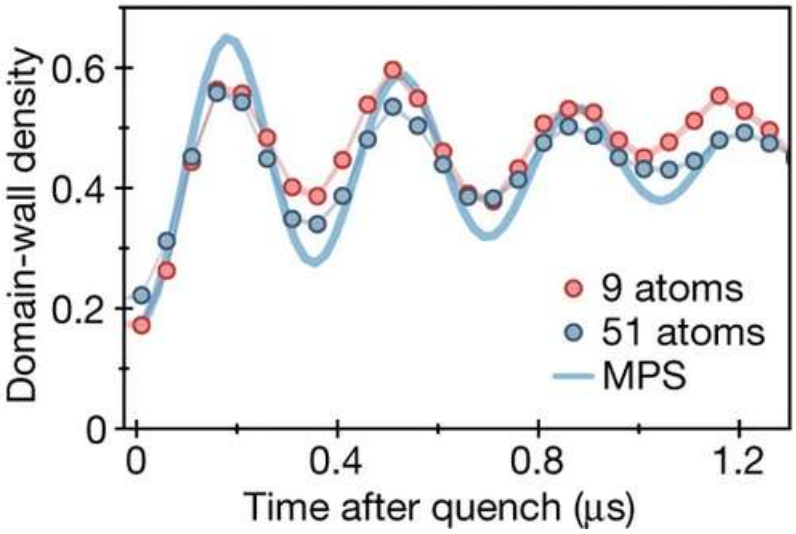
- Near arbitrary inter-site connectivity of superconducting circuits: lattices on hyperbolic space
- Heptagonal kagome-like lattice: gapped flat band

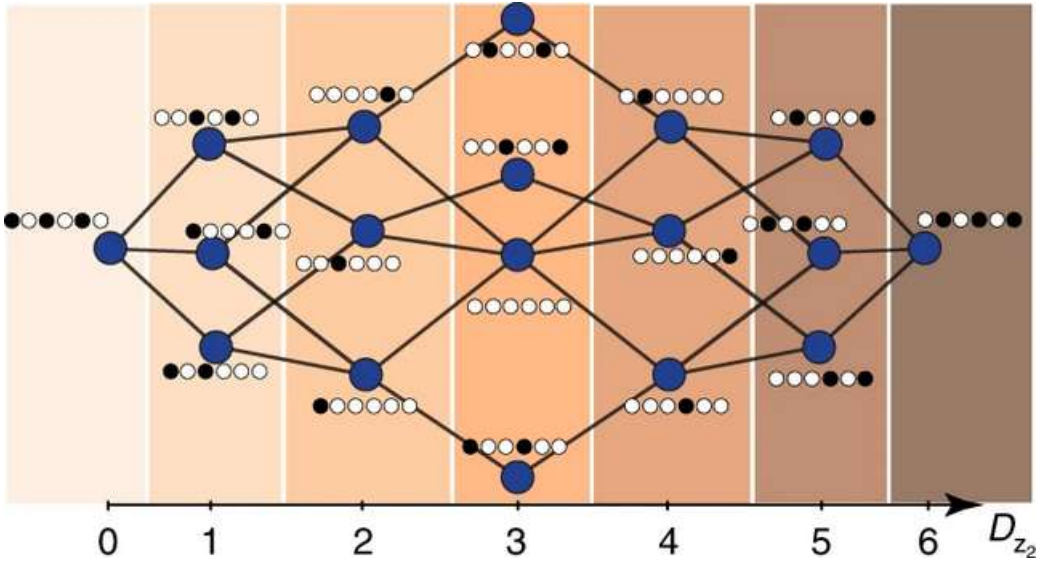


Kollár et al., Nature 571, 45 (2019); Kollár et al., Commun. Math. Phys. 376, 1909 (2020); Bienias et al., arXiv:2105.06490

Flat bands in quantum circuits

- Strong & tunable quantum interactions: many-body localization, quantum scars
- Quantum scarring: dynamics constrained to finite subset of Fock space





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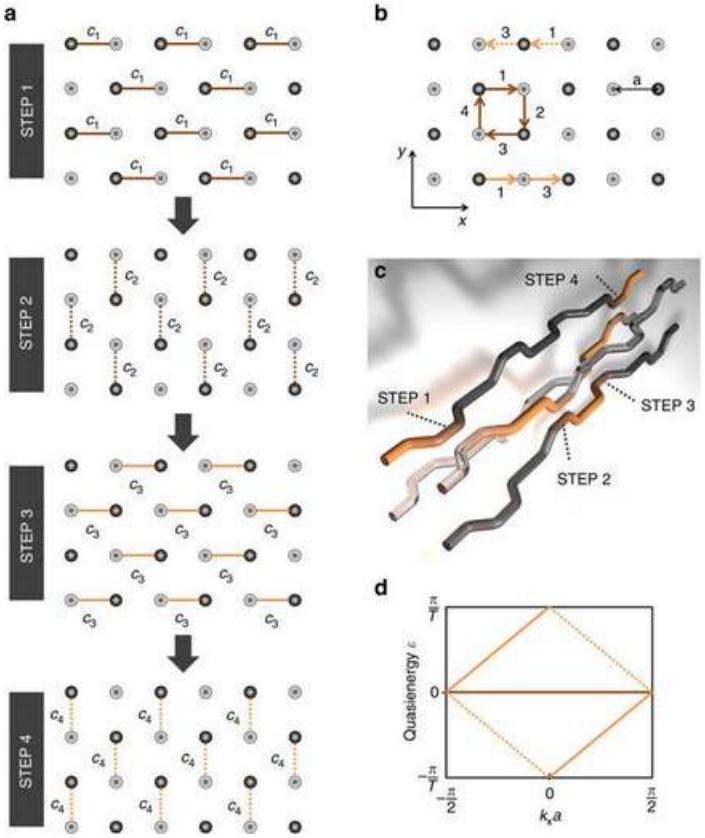
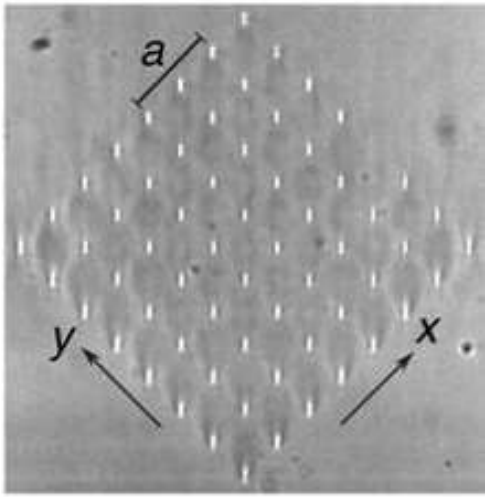
Floquet flat bands

- Quantum gates ~ unitary evolution ~ Floquet band structures
- Use flat band Floquet systems to test performance of quantum processors





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Summary

1. Flat bands: playground for analytically-tractable non-perturbative phenomena

2. Once-abstract flat band models now routinely realized in the lab

3. Quantum processors & circuits: a new frontier for flat band physics

D. Leykam, A. Andreanov, and S. Flach, Adv. Phys.: X 3, 1473052 (2018)
D. Leykam and A. Flach, APL: Photonics 3, 070901 (2018)

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