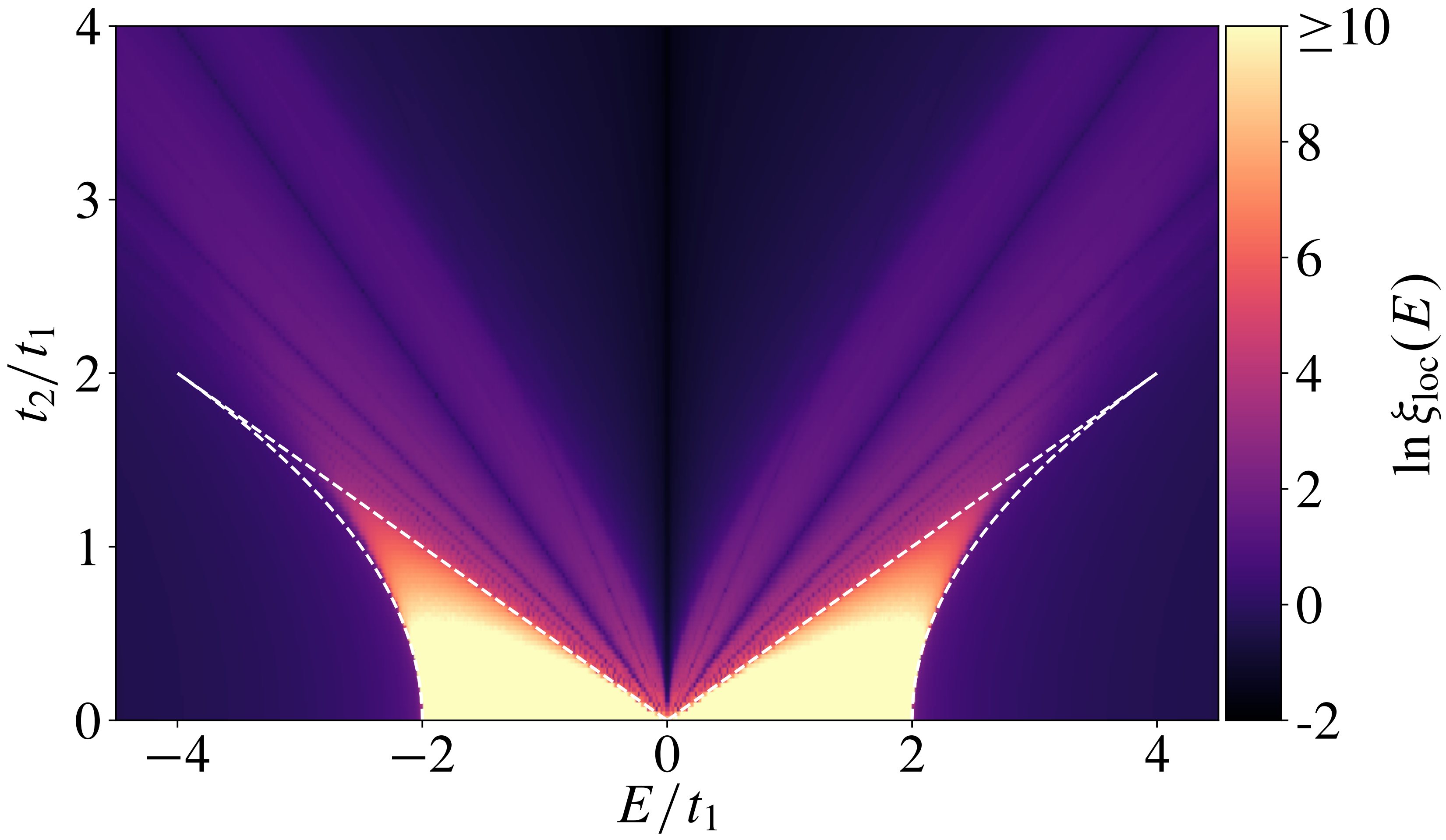
The random quantum comb:
From compact localized states to many-body scars

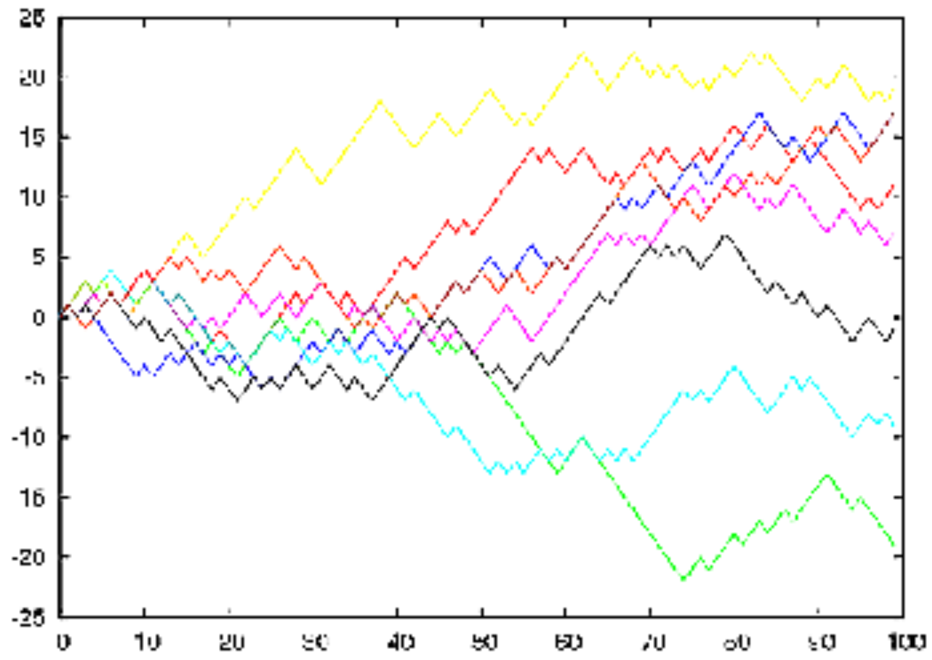


PRR 2, 043267 (2020)

Giuseppe De Tomasi

IBS Conference
20/08/2021

Diffusion in classical system

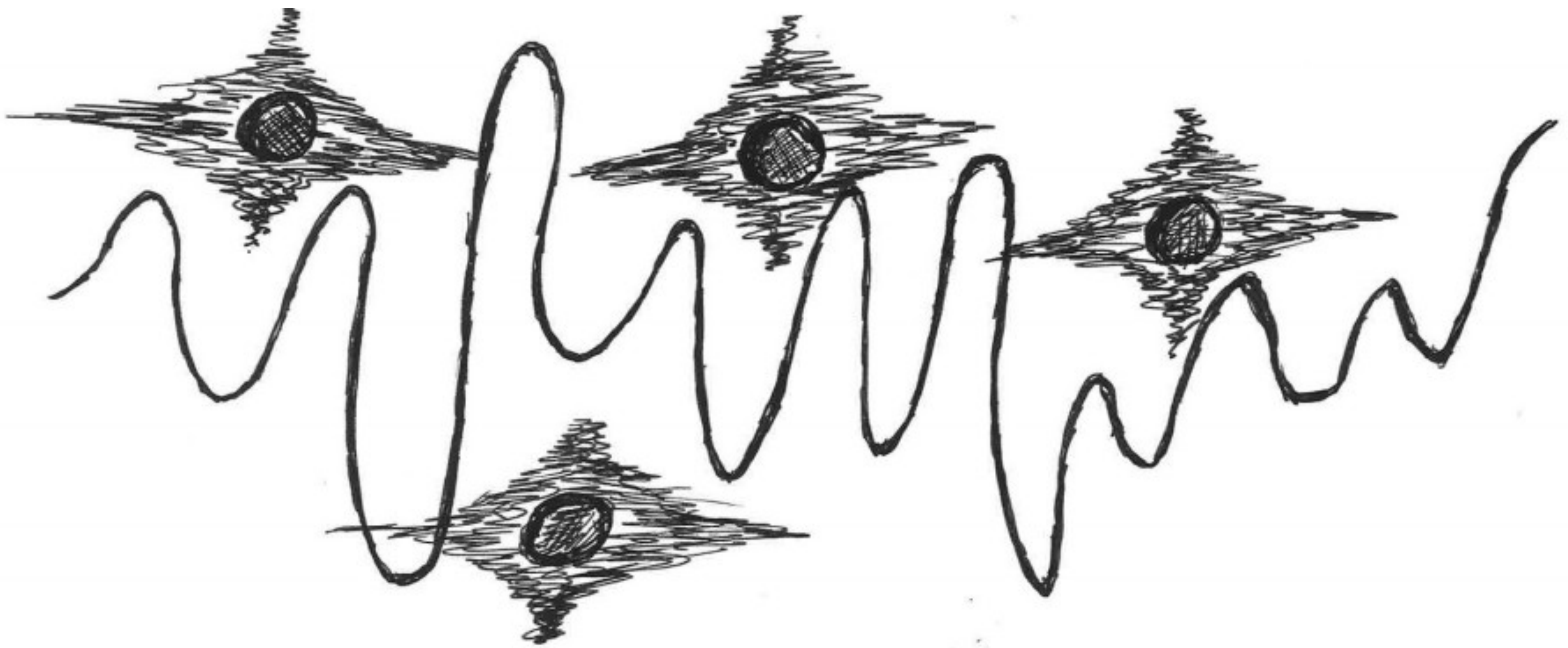


$$\Delta X^2(t) \sim Dt$$

Diffusion

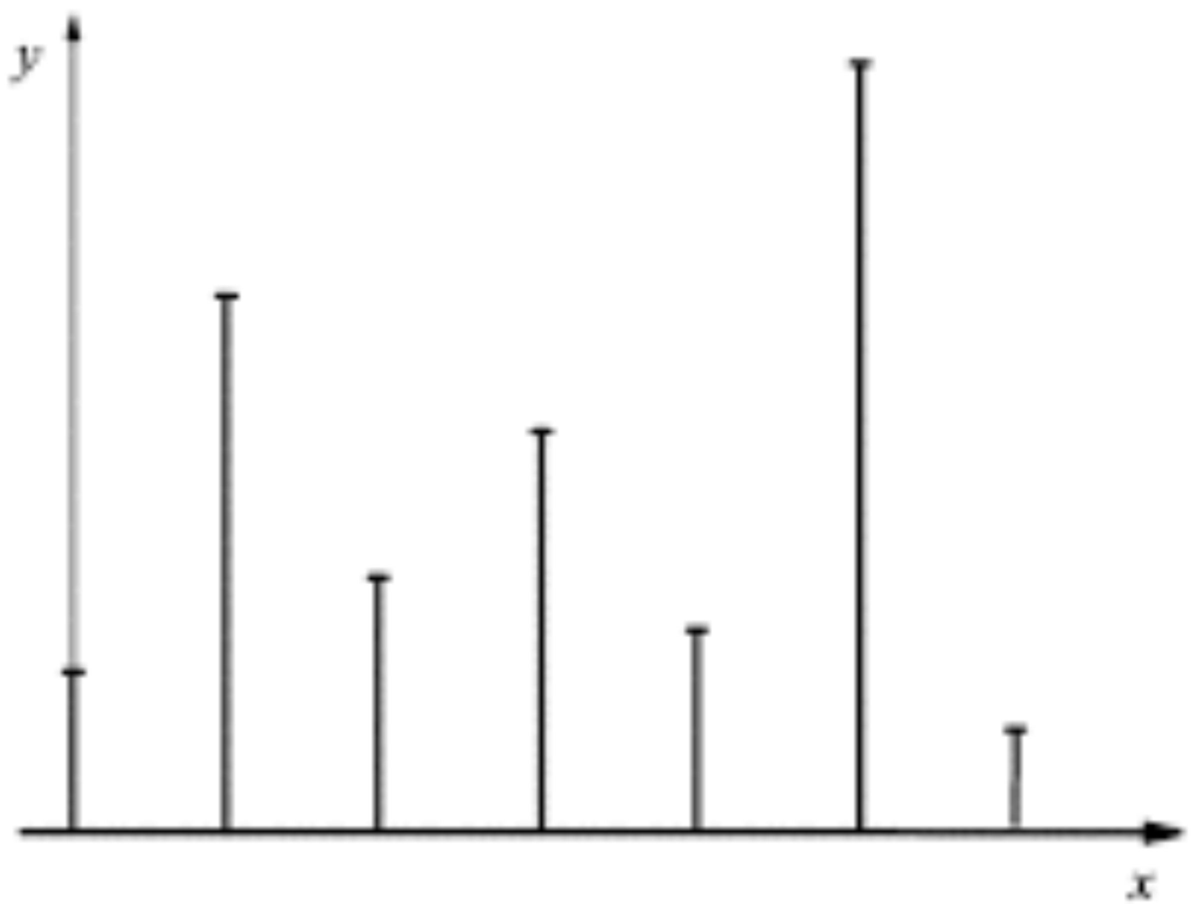
Anderson Localization

Noninteracting fermions subject to a random potential can be localized!



P. W. Anderson, Phys. Rev. 109, 1492

Subdiffusion in the Random Comb Model



$$\Delta X^2(t) \sim t^{\frac{1+\gamma}{2}}$$

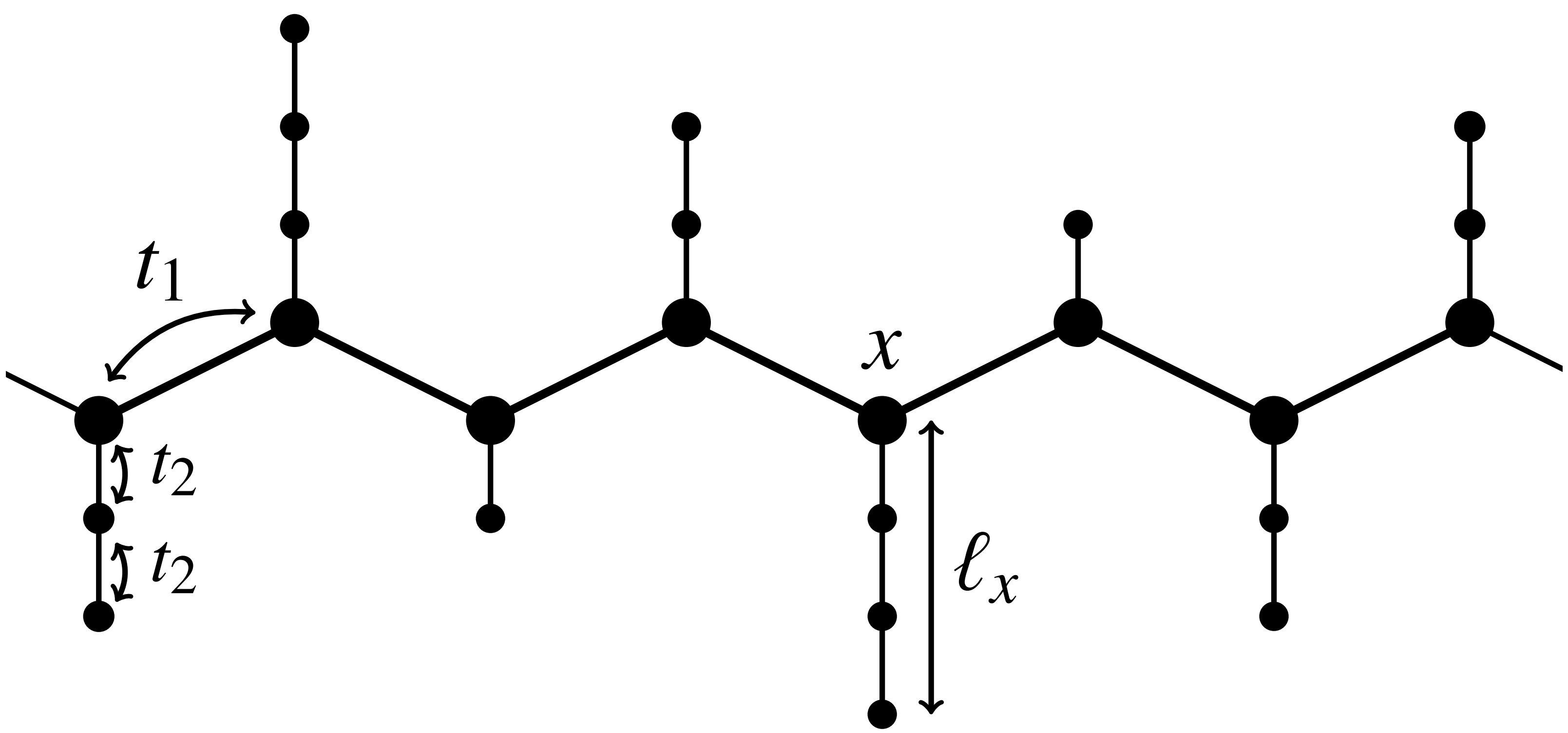
Subdiffusion

$$p(\ell) \sim \ell^{-(1+\gamma)}$$

Fat tails distribution

J.-P. Bouchaud and A. Georges, Physics Reports 195, 127 (1990)

The random quantum comb

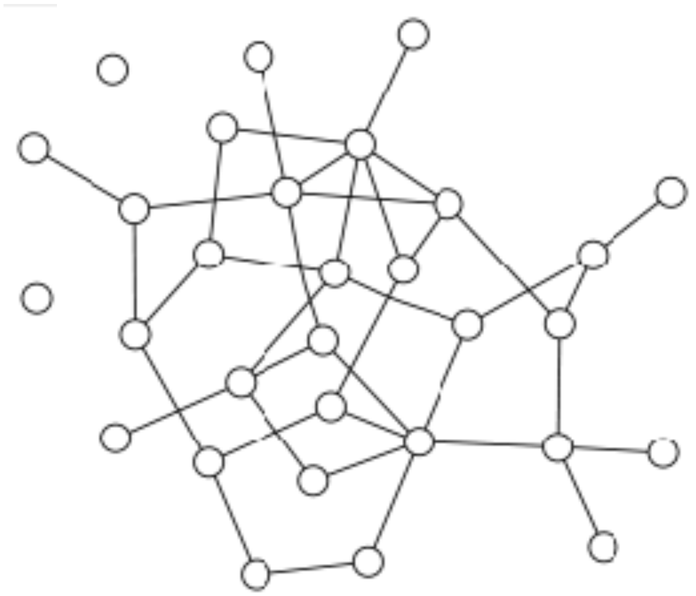


(tight-binding model)

O. Hart, GDT and C. Castelnovo, PRR 2, 043267 (2020)

The random quantum comb

Disorder due to the structure
or
configuration of the system



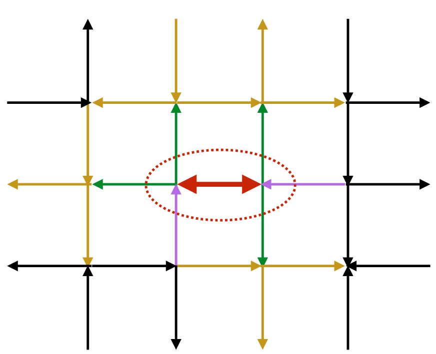
A. B. Harris, Phys. Rev. Lett. 49, 296 (1982)

Open Systems
and
Fractional Schrödinger equation

$$i\partial_t^\alpha \psi = \hat{H}\psi$$

V. Méndez et al. Chaos, Solitons & Fractals 53, 46 (2013), A. Iomin, A. Chaos, Solitons & Fractals 44, 348 (2011)

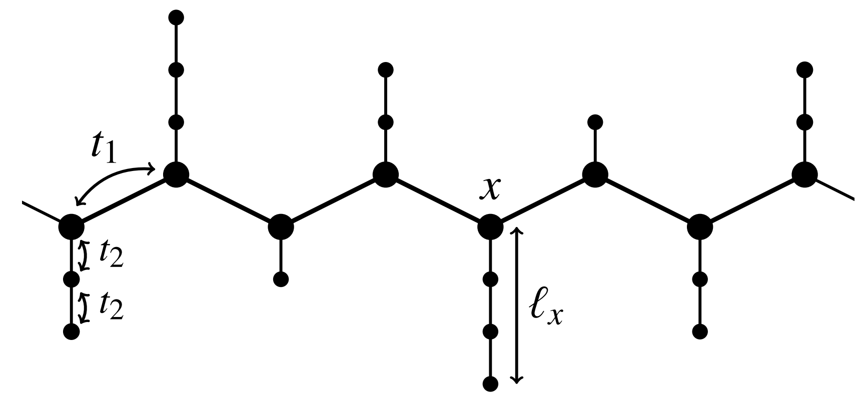
Dynamics of quasi-particles
in dimer, vertex and ice models



B. Tomasello et al. PRL 123, 067204 (2019),
O. Hart et al., PRB 101, 064428 (2020),
M. Stern, arXiv:1911.05742 (2019)

Localization properties

$$-t_1\psi_{x-1,0}-t_1\psi_{x+1,0}-t_2\psi_{x,1}=E\psi_{x,0}$$ **(backbone)**



$$\begin{cases} -t_2\psi_{x,0}-t_2\psi_{x,2}=E\psi_{x,1} \\ \vdots \\ -t_2\psi_{x,\ell_x-1}=E\psi_{x,\ell_x} \end{cases}$$ **(Offshoots)**

$$-t_1\psi_{x-1,0}-t_1\psi_{x+1,0}=[E+W_\epsilon(\ell_x)]\psi_{x,0}$$

$$W_\epsilon(\ell_x)=-\frac{t_2}{\epsilon+\sqrt{\epsilon^2-1}\left[1+2\left(\left(\frac{\epsilon+\sqrt{\epsilon^2-1}}{\epsilon-\sqrt{\epsilon^2-1}}\right)^{\ell_x}-1\right)^{-1}\right]};$$

$$\epsilon\ \equiv\ E/2t_2$$

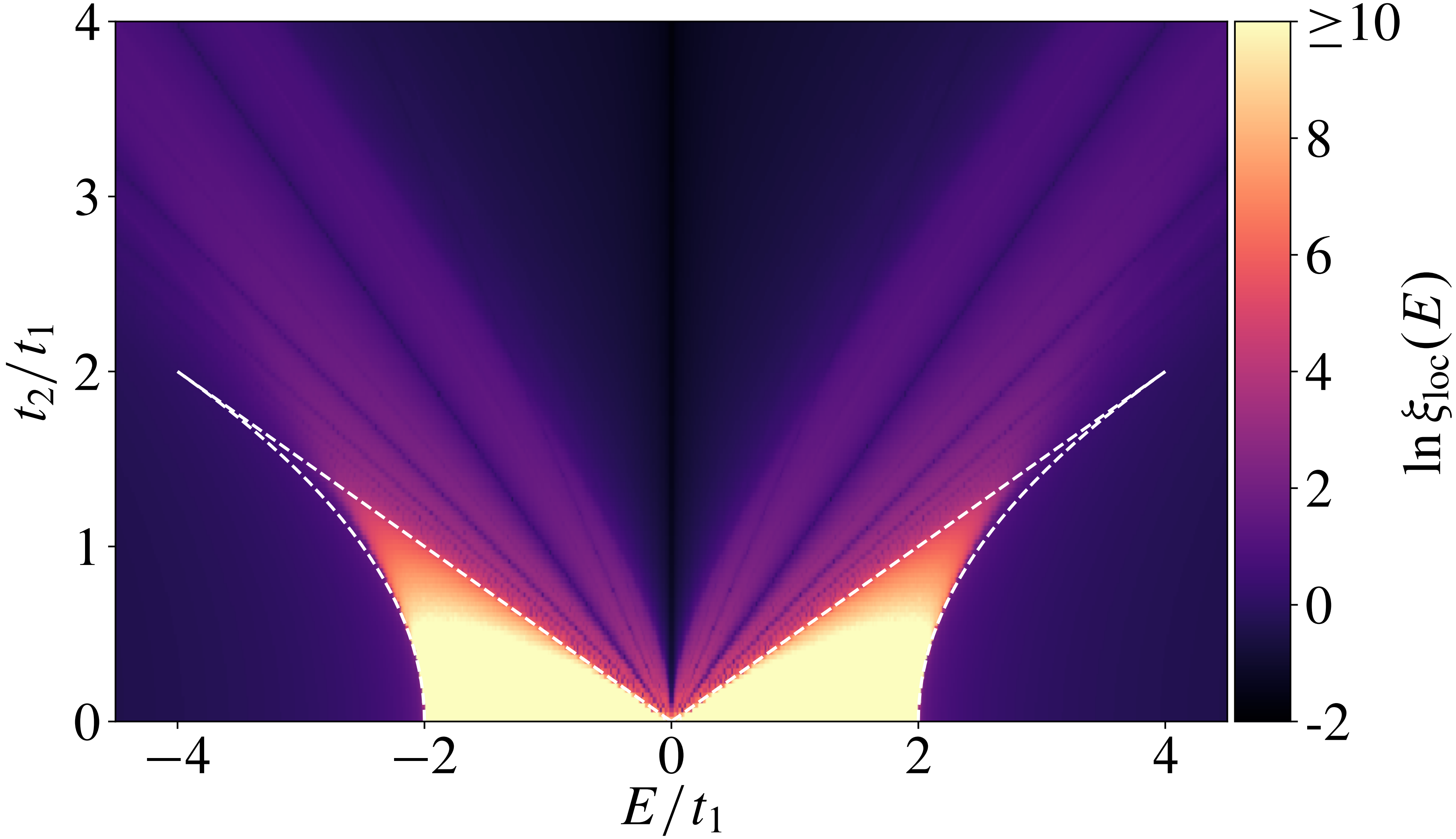
Effective Schrödinger equations on the backbone with "fractional" time-derivative.

Transfer matrix

$$\begin{pmatrix} \psi_{x+1,0} \\ \psi_{x,0} \end{pmatrix} = T_x(E)T_{x-1}(E)\cdots T_1(E)\begin{pmatrix} \psi_{1,0} \\ \psi_{0,0} \end{pmatrix}$$

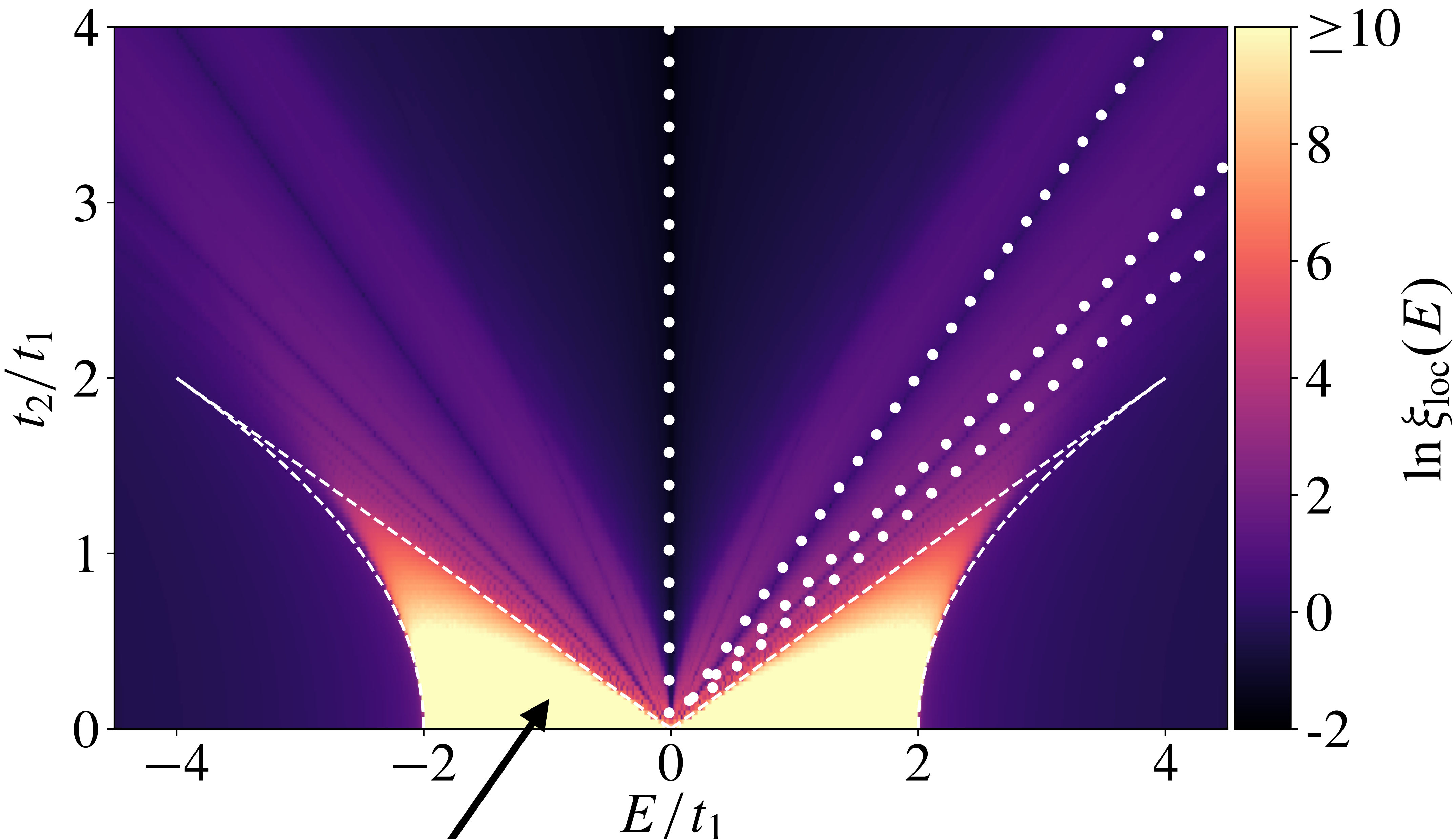
$$\psi_x \sim e^{-x/\xi_{\mathrm{loc}}}$$

$$T_x(E) = \begin{pmatrix} -[E+W_\epsilon(\ell_x)]/t_1 & -1 \\ 1 & 0 \end{pmatrix}$$



H. Furstenberg, Trans. Amer. Math. Soc. 108, 377 (1963),
I. Y. Gol'dshtein et al., Funct. Anal. Appl. 11, 1 (1977)

$$\boxed{-t_1\psi_{x-1,0} - t_1\psi_{x+1,0} = [E + W_\epsilon(\ell_x)]\psi_{x,0}}$$



$-2t_2 < E < 2t_2$

$W_\epsilon(\ell) \simeq -t_2 e^{-k}(1 - 2e^{-2\ell k})$

$\epsilon = \cosh k$ **(Weak Disorder)**

Compact Localized States

$$W_\epsilon(\ell_x) = -\frac{t_2}{\epsilon + \sqrt{\epsilon^2 - 1}\left[1 + 2\left(\left(\frac{\epsilon+\sqrt{\epsilon^2-1}}{\epsilon-\sqrt{\epsilon^2-1}}\right)^{\ell_x} - 1\right)^{-1}\right]};$$

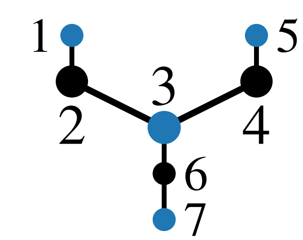
(proportional to the Green function on the offshoots)

$$\longrightarrow$$

Poles

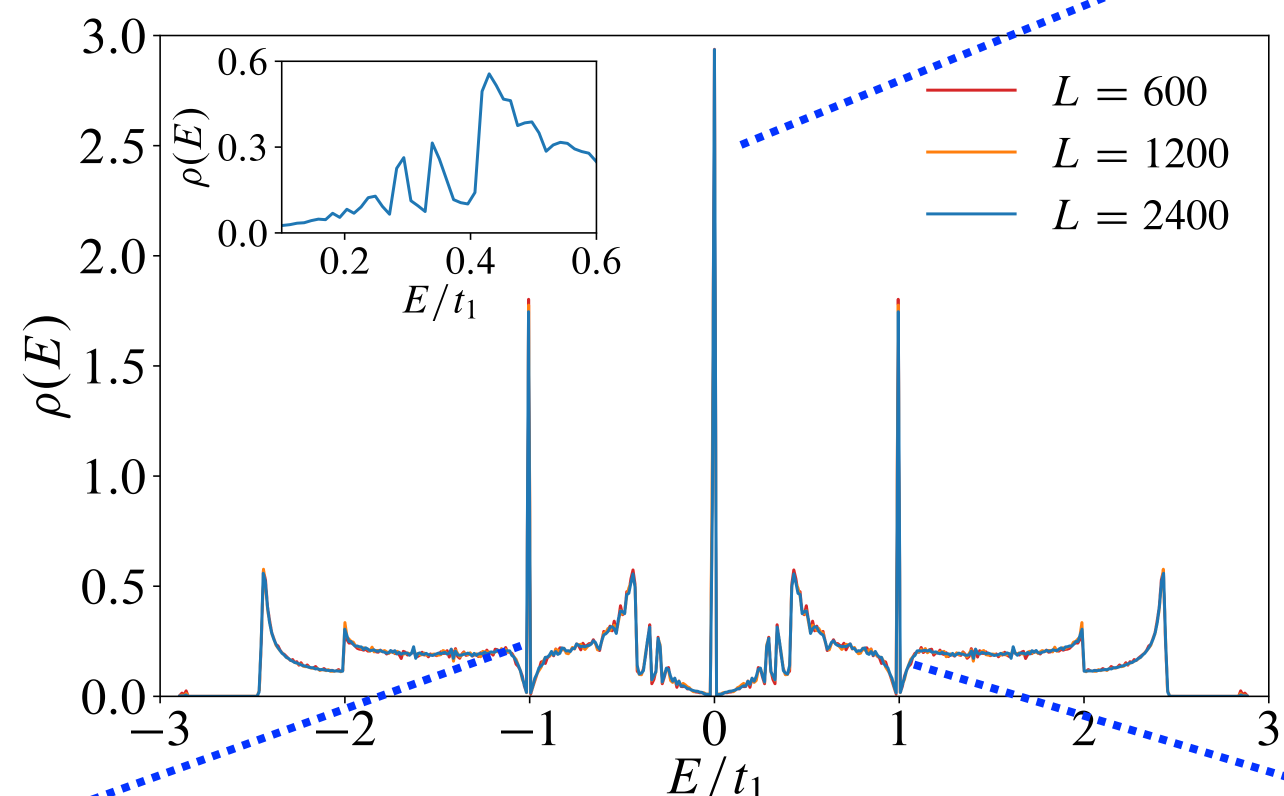
$$E_n^{\mathrm{cl}}(\ell) = -2t_2 \cos\left(\frac{n\pi}{\ell+1}\right)$$

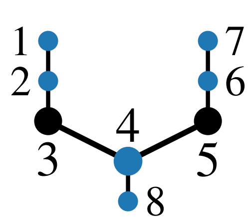
$$\epsilon \equiv E/2t_2$$

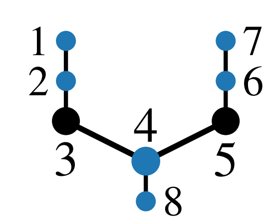


$$\left(\frac{t_1}{t_2}, 0, -1, 0, \frac{t_1}{t_2}, 0, 1\right)^T$$

Density of states



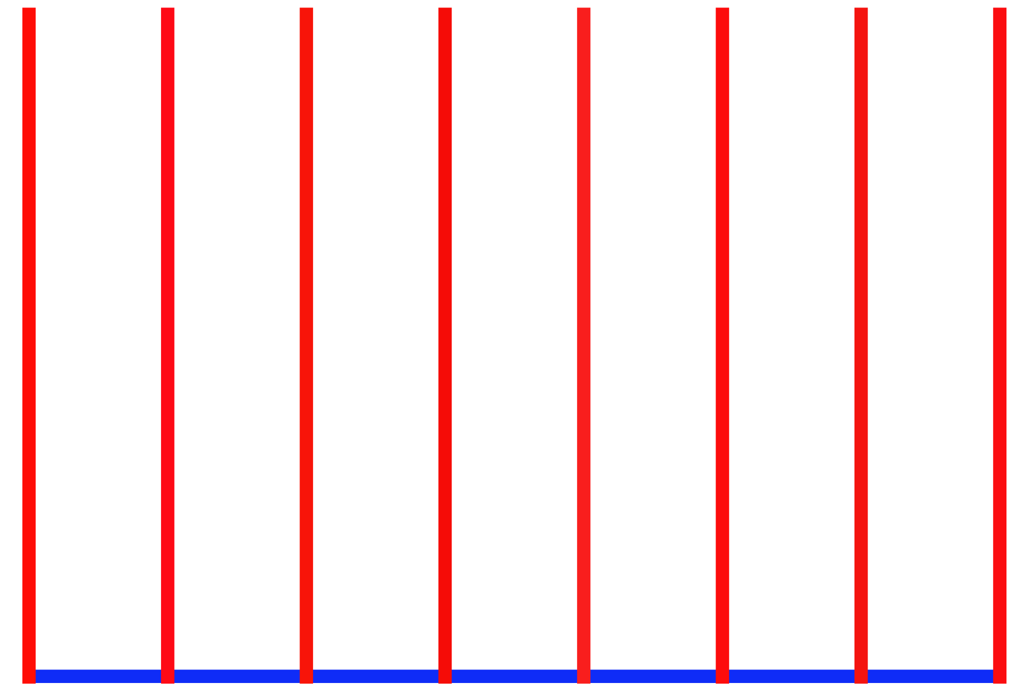




Fractional Schrödinger Equation

$$\hat{H}_0 = \sum_{x,k_y} E(k_y)\hat{a}^\dagger_{x,k_y}\hat{a}_{x,k_y} + \sum_{x,x'} h_{x,x'}\hat{c}^\dagger_{x,0}\hat{c}_{x',0}$$

Offshoots **Backbone**



$$\hat{H}_0 \left|\psi\right\rangle = i\partial_t \left|\psi\right\rangle$$

$$\updownarrow$$

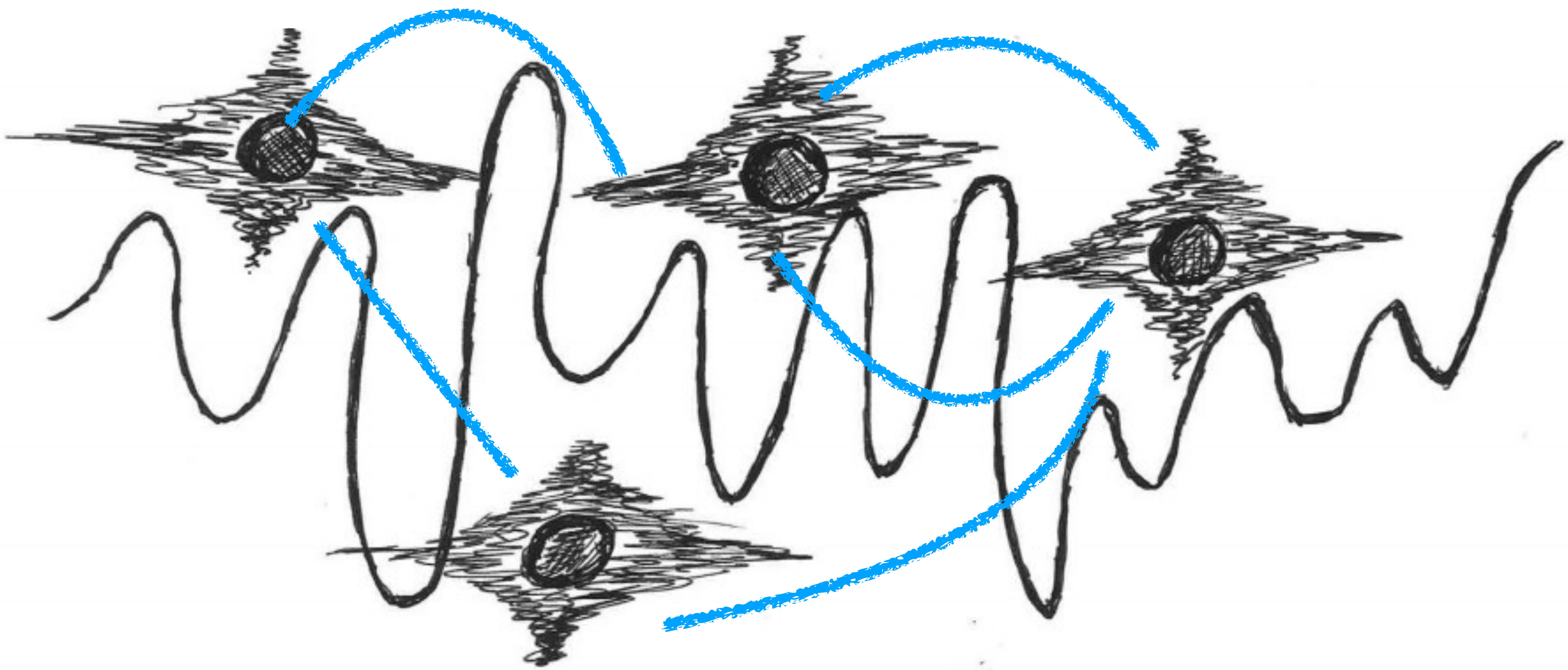
$$\frac{1}{N}\sum_{k_y} \frac{is[\sum_{x'} h_{x,x'}\Psi_{x',0}(s) + i\psi_{x,0}(0)]}{is - E(k_y)} = is\Psi_{x,0}(s)$$

$$\boxed{\begin{array}{l} i^\alpha \partial_t^\alpha \psi = \mathcal{H}\psi \\ \alpha = 1/2 \end{array}}$$

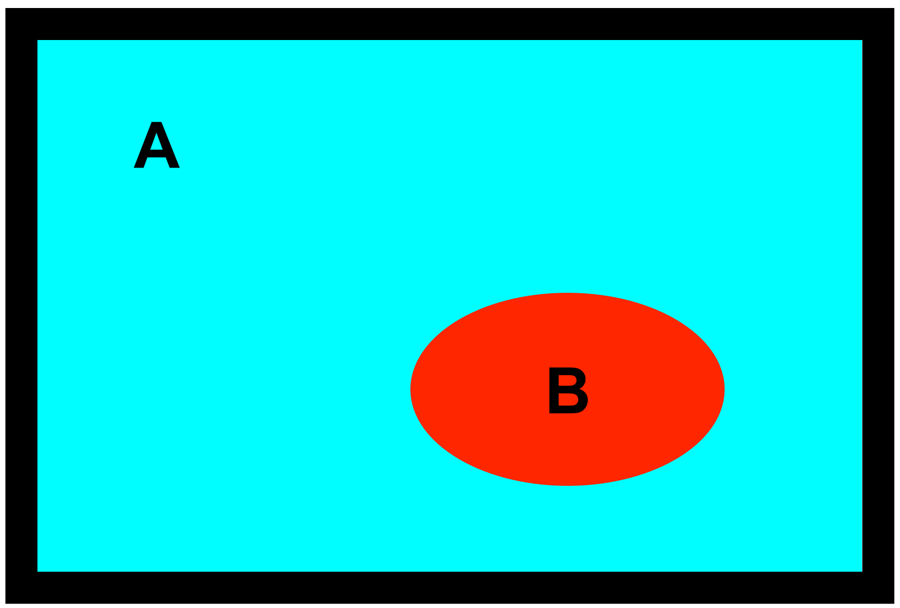
$$E(k_y) = k_y^2/2m$$

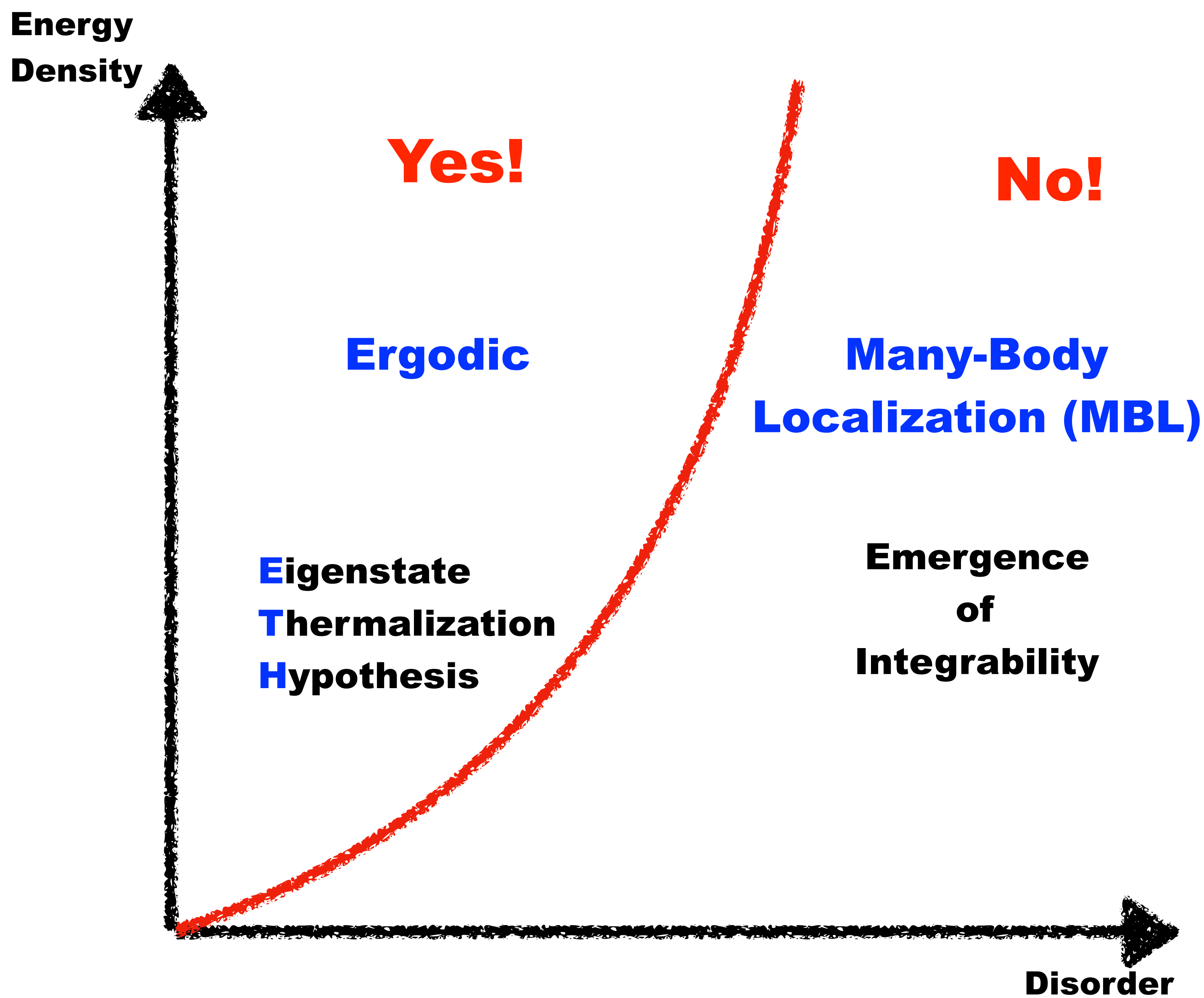
Continuum limit

What about interactions ?



Can generic isolated quantum many-body systems undergoing to reversible unitary time evolution thermalize?



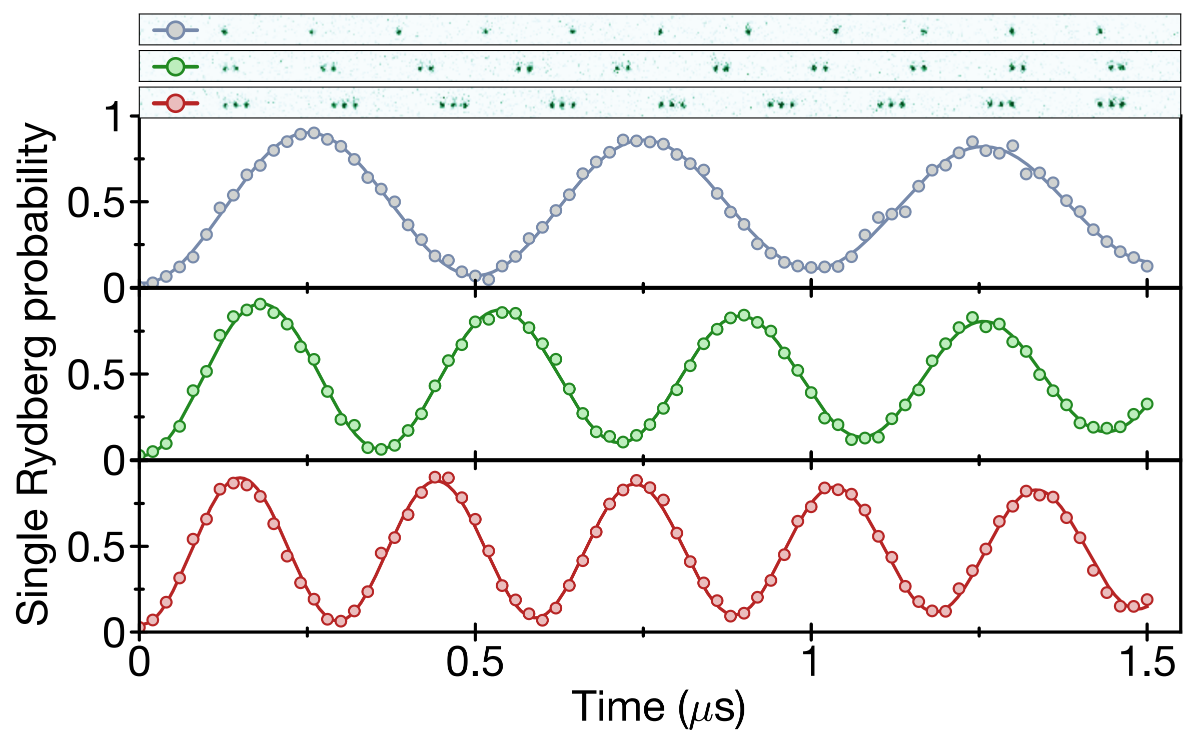


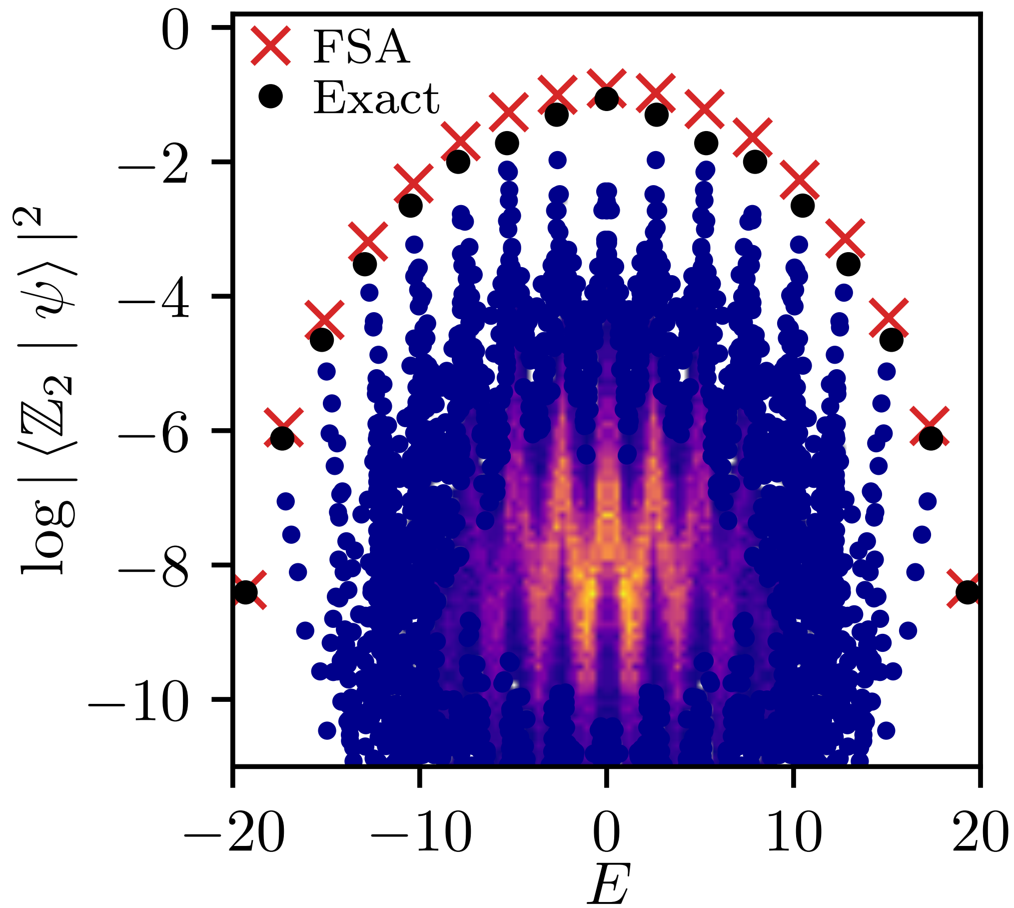
R. Nandkishore, D. A. Huse, Annu. Rev. Vol. 6: 15-38 (2015),

D. Basko et al., Annals of Physics 321, 1126 (2006)

+ many more

Many-Body Scars





H. Bernien et al., Nature 551, 579-584 (2017)

C. Turner et al., Nature Physics 14, 745–749 (2018)

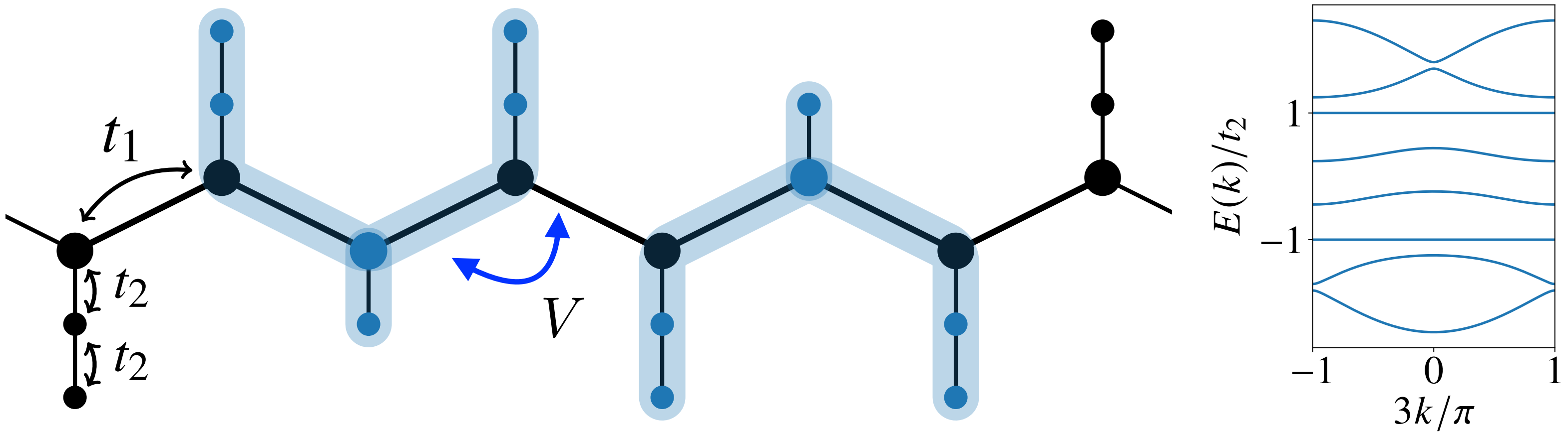
- **Violates the ETH**
- **Embedded in a sea of thermal states**

Many-Body Scars

$$\hat{H} = \hat{H}_0 + V\hat{H}_{\rm int}$$

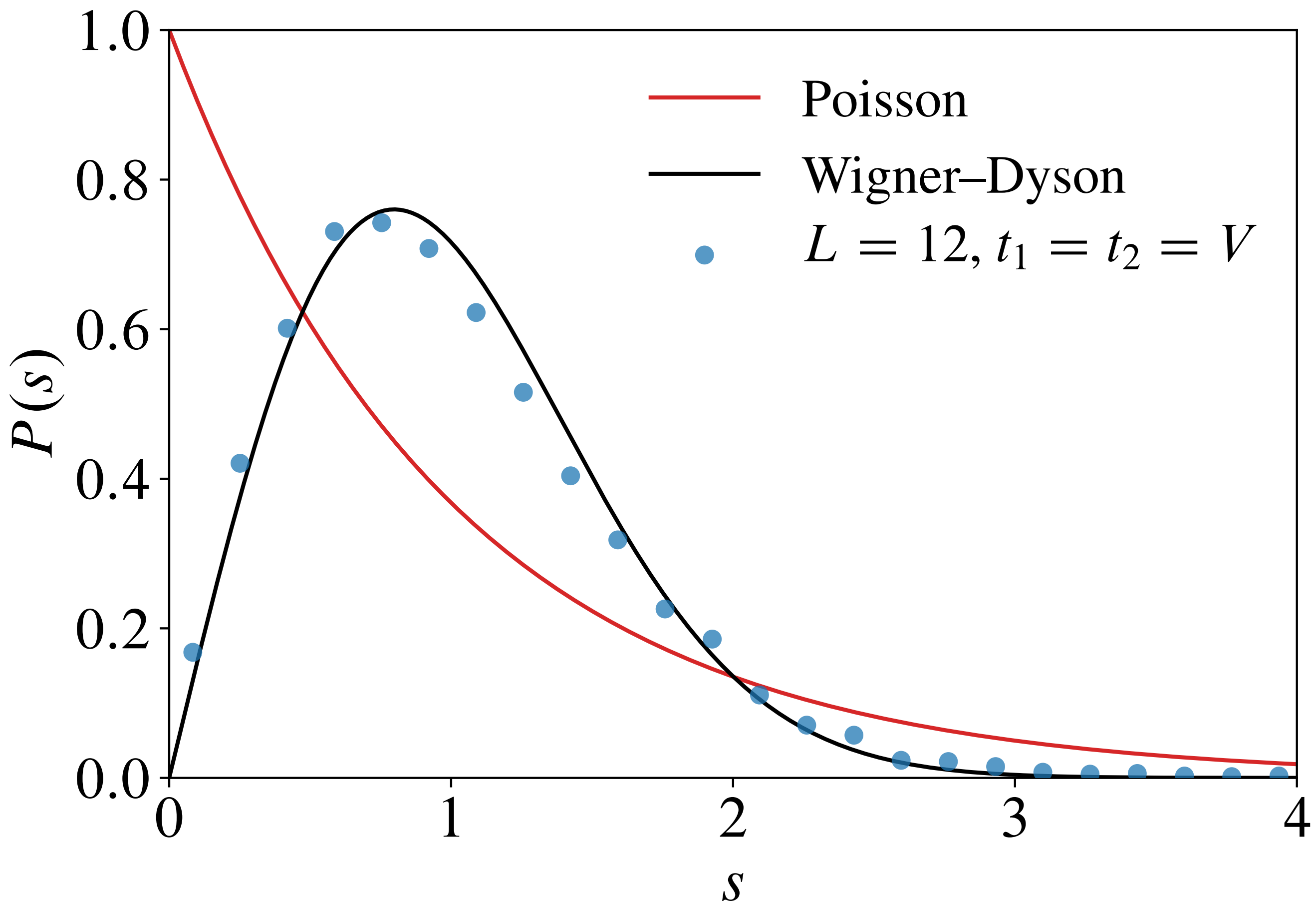
$$\hat{H}_{\rm int} = \sum_x \hat{n}_{x,0}\hat{n}_{x+1,0}$$

(Interactions only on the Backbone)



Flat Bands

Level Spacing Statistics

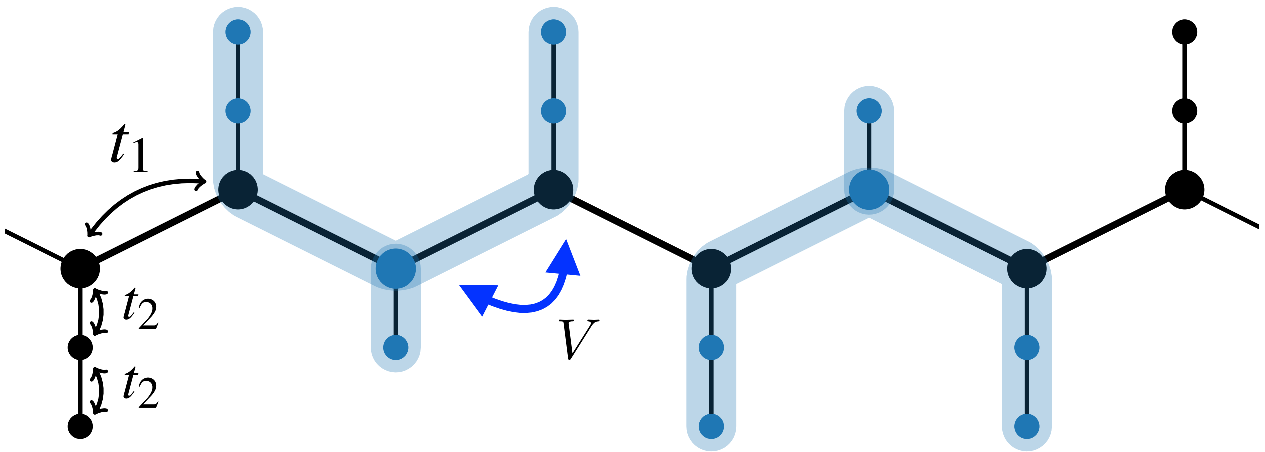


Non-Integrable

$$\hat{H} = \hat{H}_0 + V\hat{H}_{\text{int}}$$

$$\hat{H}_{\text{int}} = \sum_x \hat{n}_{x,0}\hat{n}_{x+1,0}$$

(Interactions only on the Backbone)



$$|\psi_{\text{cl}}\rangle = \prod_{n,s} \hat{\eta}_s^\dagger(E_n^{\text{cl}})|0\rangle$$

(Slater Determinant with CLS)

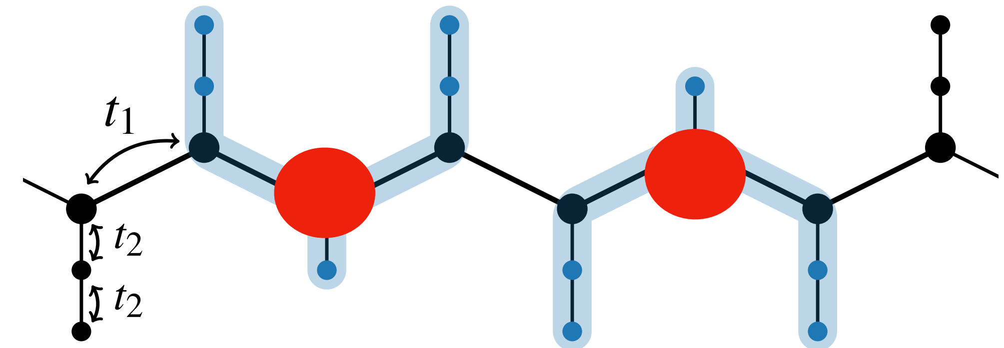
$$\hat{H}_{\text{int}}\,|\psi_{\text{cl}}\rangle = 0$$

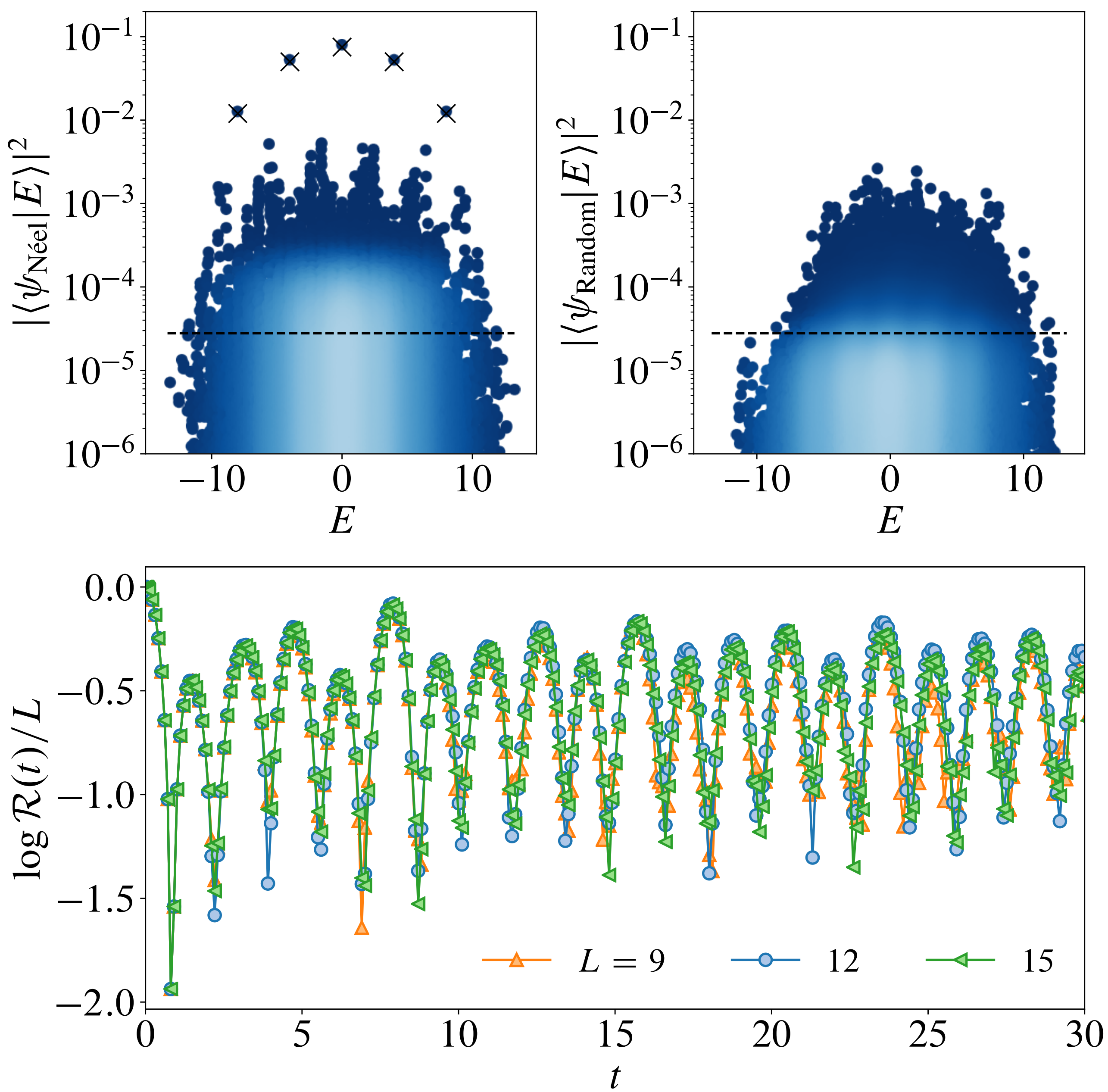
$\longrightarrow$

Exact Eigenstates

- $\binom{\frac{2L}{3}}{\frac{L}{3}} \sim 2^{2L/3}/\sqrt{\pi L/3}$
  $N = L/3$
- **Low Entanglement (Area law)**

Dynamical Probes





$$|\psi_{\text{Néel}}\rangle = \prod_{x=1}^{L/3} \hat{c}^{\dagger}_{3x-2,0}|0\rangle$$

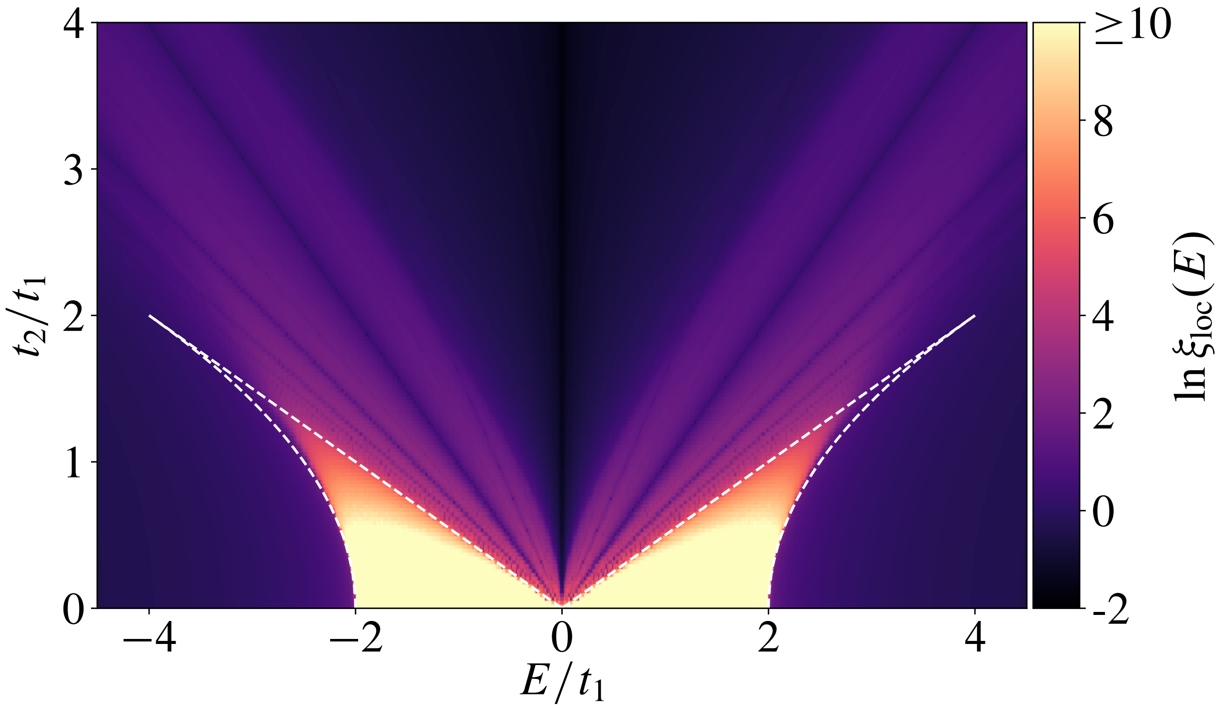
$$\mathcal{R}(t) = |\langle\psi_{\text{Néel}}|e^{-i\hat{H}t}|\psi_{\text{Néel}}\rangle|^2$$

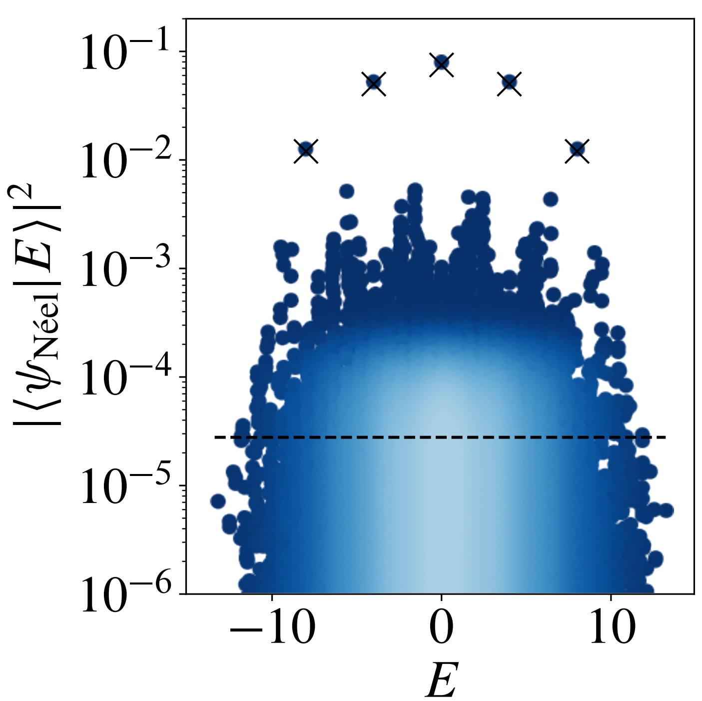
$$\boxed{\frac{\log\mathcal{R}(t)}{L} \simeq \frac{1}{3}\log\cos^2\left(\frac{t}{\tau_2}\right) - \frac{2}{3}\log\left[\frac{2t_1^2+t_2^2}{t_2^2}\right]}$$

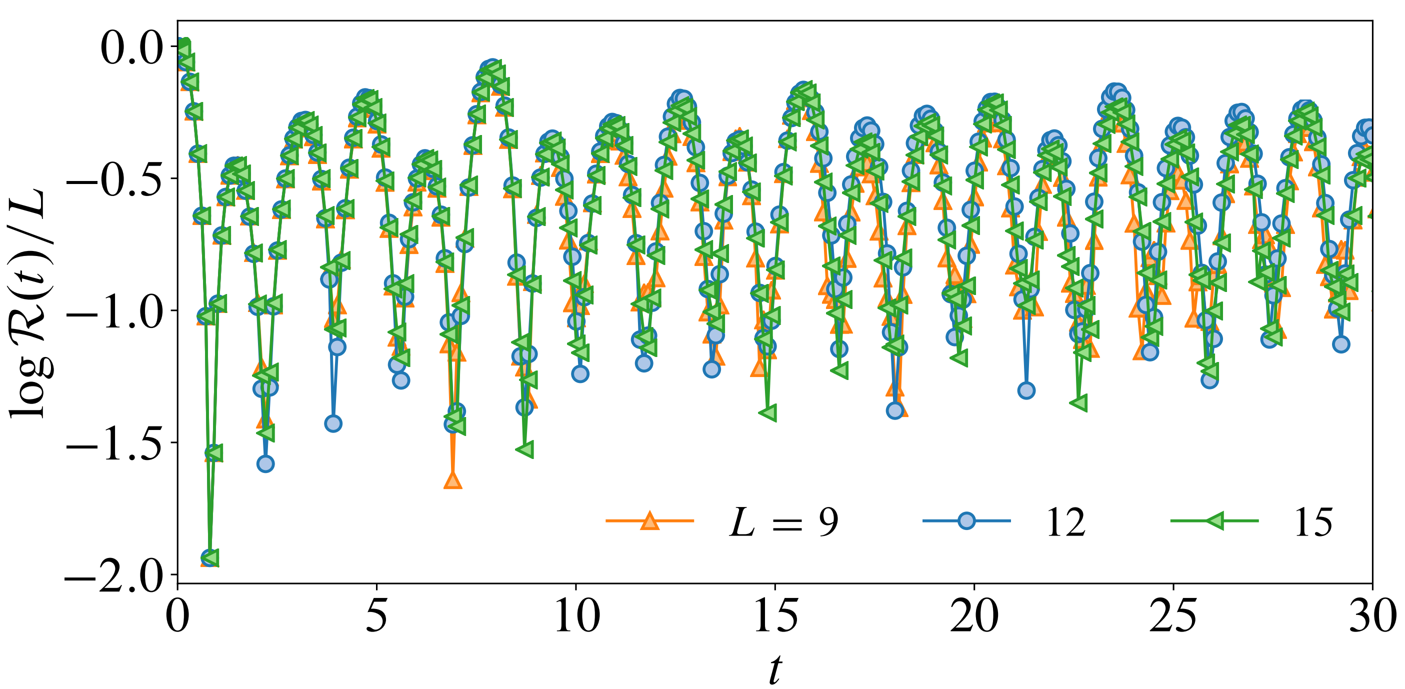
Oliver Hart
Cambridge

Claudio Castelnovo
Cambridge

Conclusion







O. Hart, GDT and C. Castelnovo, PRR 2, 043267 (2020)