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Flatbands as an arena for superconducting and topological properties

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<u>Plan of the talk</u>

	Superconductivity (from repulsion)		Topological			
Equilibrium	✓ Flat-band SC		 Flat-band topological states 			
	🗸 Non-F	ermi liquid				
Non-equil	✓ Non-equil induced SC ?		 Floquet topological insulator 			
✓ Flat-band SC → topological SC ?						



2-band vs 1-band flat-band SC



 * Attractive model ← Suhl-Kondo mechanism for dispersive bands, but here we are talking about repulsion (spin-fl mediated pairing)
 * Flat band → highly-entangled interactions?

- * Flat band + dispersive band
 - \rightarrow higher Tc when flat band is incipient
 - (ie, close to, but away from, E_F)





<u>Topological when exactly 1/3 filled (ED result)</u>

t'/t=0

-π

(Kobayashi et al, PRB 2016)



Positioning flat bands in 2D for favouring SC

Usually, a flat band does not intersect the dispersive one



Can we have







(See also Hwang, Rhim & Yang, arXiv:2106.13057)

<u>Flat bands a la Lieb, MielKe, Tasaki are not just flat,</u> <u>but anomalous (no Wannier states exist) → very entangled</u>



Wannier spread for flat bands

- ✓ For topologically-trivial flat bands, see (Marzari & Vanderbilt: Maximally localized Wannier functions, PRB 1997; RMP 2012).
- ✓ For topological flat bands, see (Watanabe et al: Fragile topology, PNAS 2015; Nat Commun 2017).

In general, topological systems have no (spatially-localised) Wannier states i.e., no adiabatic route to the atomic limit.
 (Po, Vishwanath & Watanabe, Nat Commun 2017)
 * Historically, QHE systems have no Wannier as long known.

 ✓ "Quantum geomery" of flatband wavefunctions → Landau levels. (Rhim, Kim & Yan, Nature 2020; arXiv:2012.15132)

<u>Topological flat band \rightarrow can favour SC ?</u>



<u>A numerical result beyond mean-field</u>

Attractive Hubbard model with DMRG + ED (Mondaini et al, PRB 2018)

Creutz lattice





For Attractive Lieb lattice, see Julk, ..., Torma, PRL 2016: Huhtinen & Torma, PRB 2021.

 \rightarrow Repulsive Hubbard model ? --- an open question

<u>2-band vs 1-band flat-band SC</u>



<u>SC for repulsive U with FLEX+DMFT</u>

(Sayyad et al, PRB 2020)



Spin susceptibility χ_s



(Sayyad et al, PRB 2020)





Usually, SC from el-el repulsion works much better in 2D than in 3D



Flat-band SC has totally different dim-dep



<u>Plan of the talk</u>

	Superconductivity	Topological
Equilibrium	✓ Flat-band SC	✓ Flat-band topological states
	Non-Fermi liquid	
Non-equil		 Floquet topological insulator



Fermi liquids: Im $\Sigma(\omega) \sim \omega^2$ (real axis) \rightarrow Im $\Sigma(i\omega) \sim i\omega$ (Matsubara axis) (Werner et al , PRL 2008: PRB 2016 on a cuprate model)

<u>Plan of the talk</u>

	Superconductivity	Topological
Equilibrium	✓ Flat-band SC	Flat-band topological states
	Non-Fermi liquid	No-go theorem (finite-range hopping → no topological flat bands, Chen et al, JPA 2014), so we have to put in spin-orbit, etc
Non-equil		 Floquet topological insulator

Metal-organic framework (MOF) systems

(Yamada et al, PRB (R) 2016, a collaboration with MIT chemistry)



	Superconductivity	Topological				
Equilibrium	✓ Flat-band SC	✓ Flat-band topological states				
	✓ Non-Fermi liquid					
		A totally different way to make the system topological				
Non-equil		 Floquet topological insulator 				
		i = 0				
	ordinary bands (e.g. g	raphene) → Flat bands				

Floquet topological insulator

(Oka & Aoki, PRB 2009)



Light-induced anomalous Hall effect

<u>Quantum anomalous Hall effect (QHE in B=0)</u>



Flat-band Floquet topological insulators



(Mikami, ..., Aoki, PRB 2016)

<u>Non-equilibrium phase diagram:</u>

(Mikami et al, PRB 2016)



Another flat band: Kagome + CPL

(Mikami et al, PRB 2016)







<u>Flat-band (eg Kagome) Floquet topological \rightarrow Chern # behaves wildly</u>



(Mikami et al, PRB 2016)

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In general, topological systems have no (spatially-localised) Wannier states i.e., no adiabatic route to the atomic limit.

- (Po, Vishwanath & Watanabe, nat commun 2017)
- * Historically, QHE systems have no Wannier as long known.

✓ Floquet topological insulators have Wannier states? (Nakagawa, ..., Oka, PRB 2020) Floquet-Bloch states → Fourier tr → "Topological obstruction" (spatial- and temporal-localised Wannier = absent)

i.e., Driven system —(adiabatic deformation)—

→ undriven insulator

<u>"Optical imprinting:</u> <u>Spatially-periodic CPL illuminated on 2D materials</u> <u>> in situ control of Floquet topological insulators</u>

(Kim, ..., Aoki & Hafezi, PRR 2020)



Topological phase diag



square \rightarrow hexagonal (loss of reflection symm) \rightarrow complex hopping as in Haldane's model

(Kim, ···, Aoki & Hafezi, PRR 2020)



General experimental feasibility of Floquet topological states (Can we have intense enough laser for sizeable topological gaps?)



Summary Flat bands do indeed harbour unique opportunities					
	Superconductivity	Topological			
Equilibrium	✓ Flat-band SC	✓ Flat-band topological states			
	🗸 Non-Fermi liquid 🔶	non-Fermi liquid SC ?			
Long-range & nonlocal interactions enhanced in flat bands ?					
Non-equil	✓ Non-equil induced SC ?	 Floquet topological insulator 			
Open questions Image: Comparison of the second					

		T-rever	sal Particl	le-hole Chiral	Spa <i>d</i> = 1	tial dime d =2	nsion d = 3
Standard (Wigner-Dyson)	A (unitary)	—Θ ² — Χ	— <u>Н</u> 2— х	х х		Z	IQHE
	AI (orthogonal)	+1	х	Х		Kane-I	Melē, QSHE
	All (symplectic)	-1	x	x		Z_2	Z_2 Z_2TI
Chiral	AIII (chiral unitary)	x	x	1	Z		Z
	BDI (chiral orthogonal)	+1	+1	1	(Z)	SSH <u>,</u> Cre	eut <u>z</u>
	CII (chiral symplectic)	-1	-1	1	Z		Z ₂
BdG (SC)	D (p-wave SC)	х	+1	х	Z ₂	Z	p+ip SC, v=5,-2 FQHE topological SC
	C (d-wave SC)	Х	-1	Х		(Z)	l+id_SC
	DIII (p-wave TRS SC)	- 1	+1	1	Z ₂	Z ₂	(Z) ³ He-B
	CI (d-wave TRS SC)	+1	- 1	1			Z

(Altland-Zirnbauer 1997; Schnyder-Ryu-Furusaki-Ludwig 2008; Hasan-Kane, RMP 2010)

 $V_{\text{group}} = 0$ at a point: van Hove sing. \rightarrow topological SC (d+id, etc) (Liu et al, PRL 2018)

(Sayyad et al, PRB 2020)

<u>Superconductivity in bilayer graphene</u>

(Cao…, Jarillo-Herrero, nature 2018)

doi:10.1038/nature26160

Unconventional superconductivity in magic-angle graphene superlattices

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ARTICLE

Nonzero Chern #s in twisted bilayer graphene

(Zhang et al, PRB 2019)

<u>Non-Fermi liquid SC in Hatsugai-Kohmoto model</u>

Keita Kobayashi, Masahiko Okumura, Masahiko Machida Japan Atomic Energy Agency

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