IBS conference on

"Flatbands: symmetries, disorder, interactions and thermalization"

# Localization, phase and transitions in the three-dimensional extended Lieb lattices

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# **Background** - Anderson Localization

## The Nobel Prize in Physics 1977

Philip Warren Anderson
Sir Nevill Francis Mott
John Hasbrouck van Vleck

Fundamental theoretical investigations of the electronic structure of **magnetic** & **disordered systems**

### Absence of Diffusion in Certain Random Lattices

P. W. Anderson
*Bell Telephone Laboratories, Murray Hill, New Jersey*
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

P. W. Anderson

### Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions

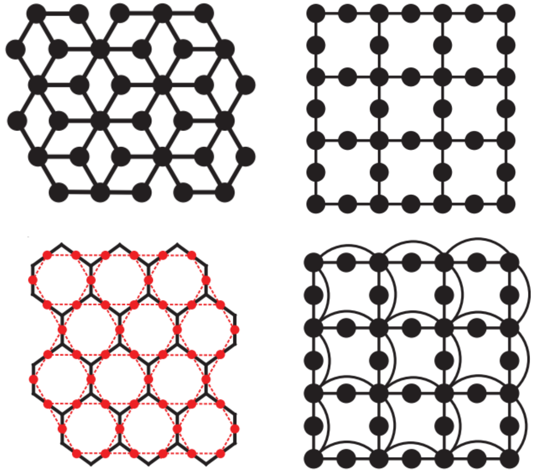
E. Abrahams, P.W. Anderson, D.C. Licciardello, T.V. Ramakrishnan

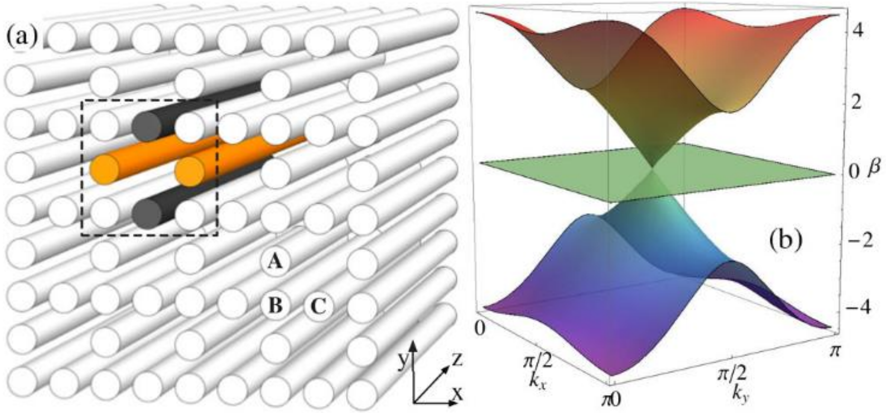
Physical Review Letters, vol. 42, p. 673 (1979)

# **Background** - Flat Band

**Flat bands generation**(TBM)
✓ Internal symmetries
✓ Fine-tuned coupling

**Theoretical prediction**
A. Dice lattice(1986)
B. Lieb lattice(1989)
C. Line graph -- Kagome Lattice
D. Tasaki's decorated square lattice
…






**Experimental**
- Wigner Crystals
- High-temperature superconductivity
- Photonic wave guide arrays
- Bose-Einstein condensates
- Ultra-cold atoms in optical lattices
- Electronic systems

…

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**Artificial flat band systems: from lattice models to experiments**

Daniel Leykam, Alexei Andreanov and Sergej Flach

Center for Theoretical Physics of Complex Systems, Institute for Basic Science (IBS), Daejeon, Republic of Korea

# Models

## Anderson Model:

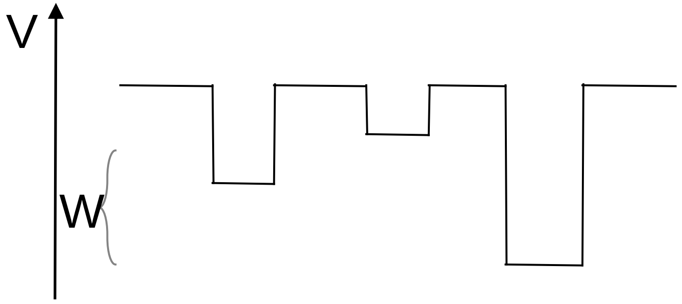
$$H = \sum_{\boldsymbol{i}} V_{\boldsymbol{i}} |\boldsymbol{i}\rangle\langle \boldsymbol{i}| - \sum_{\boldsymbol{i}\neq \boldsymbol{j}} t_{\boldsymbol{ij}} |\boldsymbol{i}\rangle\langle \boldsymbol{j}|$$

Diagonal disorder:

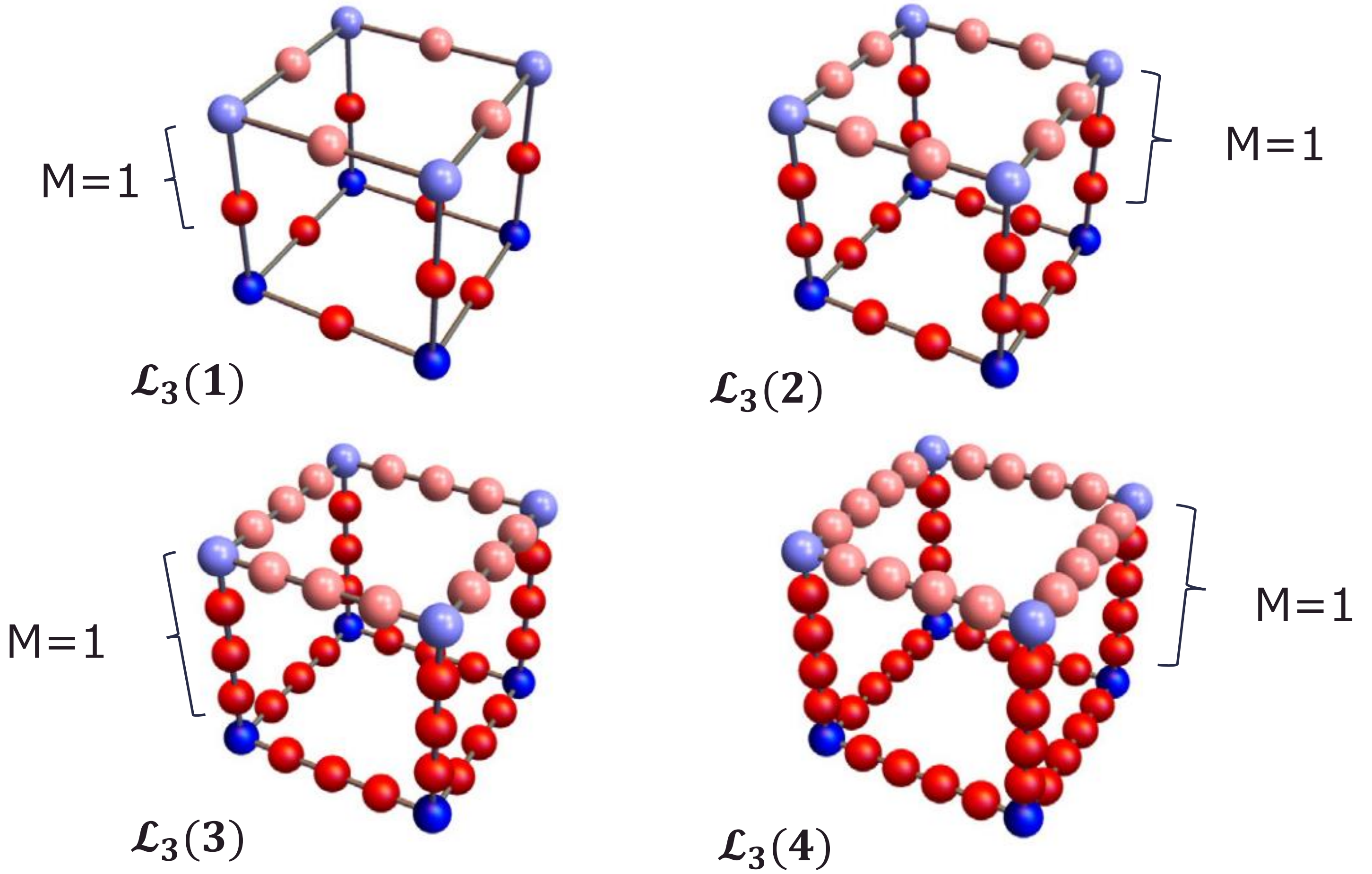
$$V_{\boldsymbol{i}} \in [-\frac{W}{2}, \frac{W}{2}]$$

$$P\left(V_i\right) = \frac{1}{W} \left(V_i < \left|\frac{W}{2}\right|\right)$$

$$P\left(V_i\right) = 0 \quad \left(V_i > \left|\frac{W}{2}\right|\right)$$




## Lieb Model and its extensions $\mathcal{L}_3(\mathbf{n})$:



# **Methods** - Transfer matrix method & Finite Size Scaling

$$\mathbf{H} = \sum_i V_i |i\rangle\langle i| - \sum_{ij} t_{ij} |i\rangle\langle j|$$

$$\psi_{n+1} = (E - V_i)\psi_n - \psi_{n-1}$$

$$\begin{pmatrix} \psi_{n+1} \\ \psi_n \end{pmatrix} = \begin{pmatrix} E - V_n & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \psi_n \\ \psi_{n-1} \end{pmatrix} = \mathbf{T_n} \begin{pmatrix} \psi_n \\ \psi_{n-1} \end{pmatrix}$$

$$\begin{pmatrix} \psi_{L+1} \\ \psi_L \end{pmatrix} = \mathbf{T_L} \ldots \mathbf{T_2} \mathbf{T_1} \begin{pmatrix} \psi_1 \\ \psi_0 \end{pmatrix} = \mathbf{\Gamma_L} \begin{pmatrix} \psi_1 \\ \psi_0 \end{pmatrix}$$

Lyapunov exponent, $\gamma$ as the exponent of eigenvalues $e^{\pm\gamma}$ for matrix $\left(\Gamma_L {\Gamma_L}^{+}\right)^{1/L}$ as $L \to \infty$.

The localization length, as inverse of the minimum Lyapunov exponent - $\lambda = 1/\gamma_{min}$

- **Finite Size Scaling**[1]:

According to renormalization group equation

$$\Lambda = F(\chi_r M^{1/\nu}, \chi_i M^y)$$

where $\chi_r$ and $\chi_i$ are relevant and irrelevant scaling variables, respectively.

After Taylor-expand,

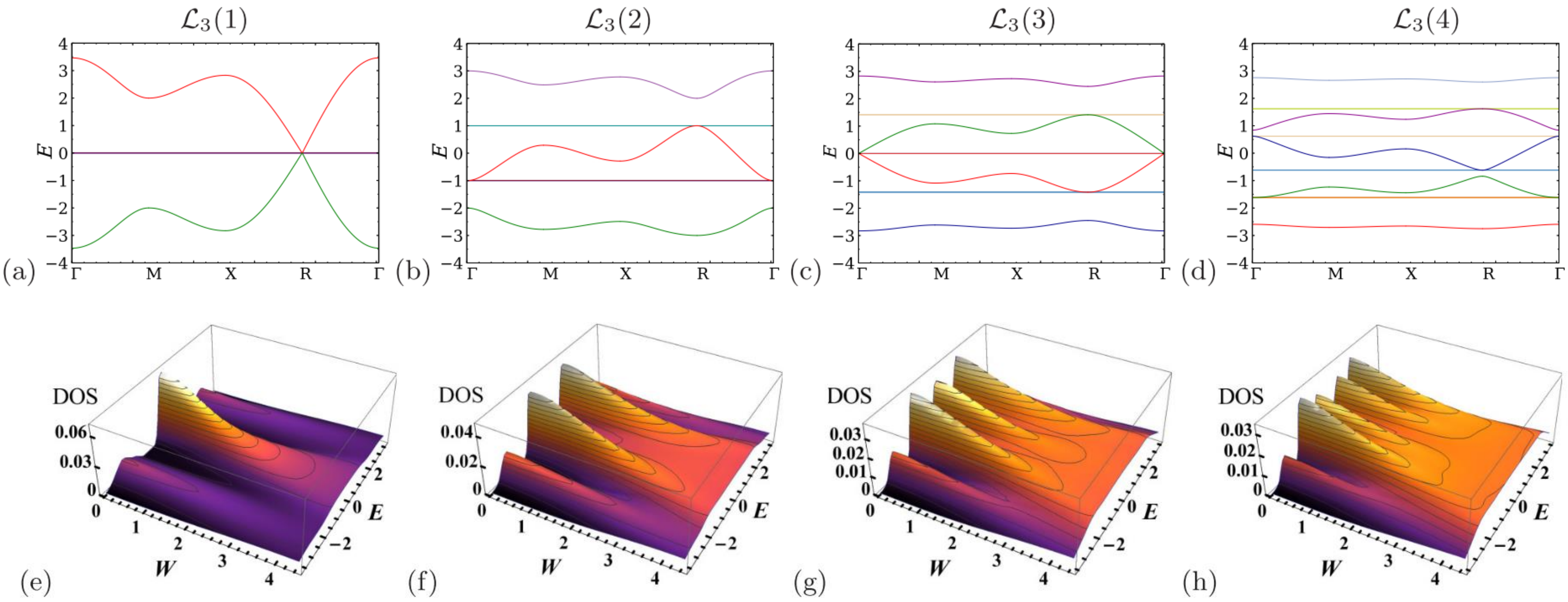
$$\Lambda = \sum_{n=0}^{n_i} \chi_i^n M^{ny} F_n(\chi_r M^{1/\nu}), \quad F_n = \sum_{k=0}^{n_r} a_{nk} \chi_r^k M^{k/\nu}.$$

By considering the nonlinearities,

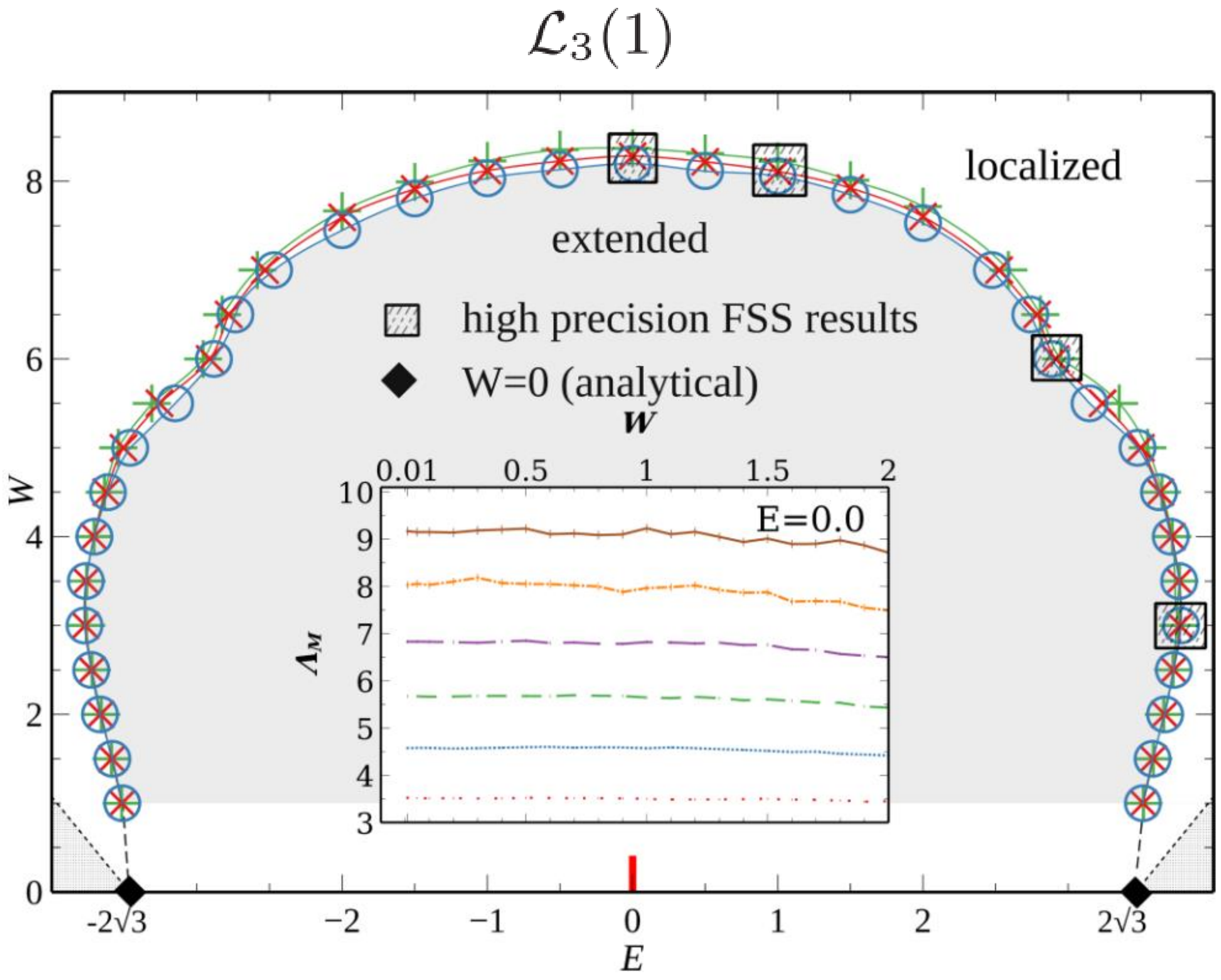
$$\chi_r(\omega) = \sum_{m=1}^{m_r} b_m \omega^m, \quad \chi_i(\omega) = \sum_{m=0}^{m_i} c_m \omega^m.$$

[1] K. Slevin and T. Ohtsuki, Phys. Rev. Lett. 82, 382 (1999).

# Results – Dispersion Relation & DOS

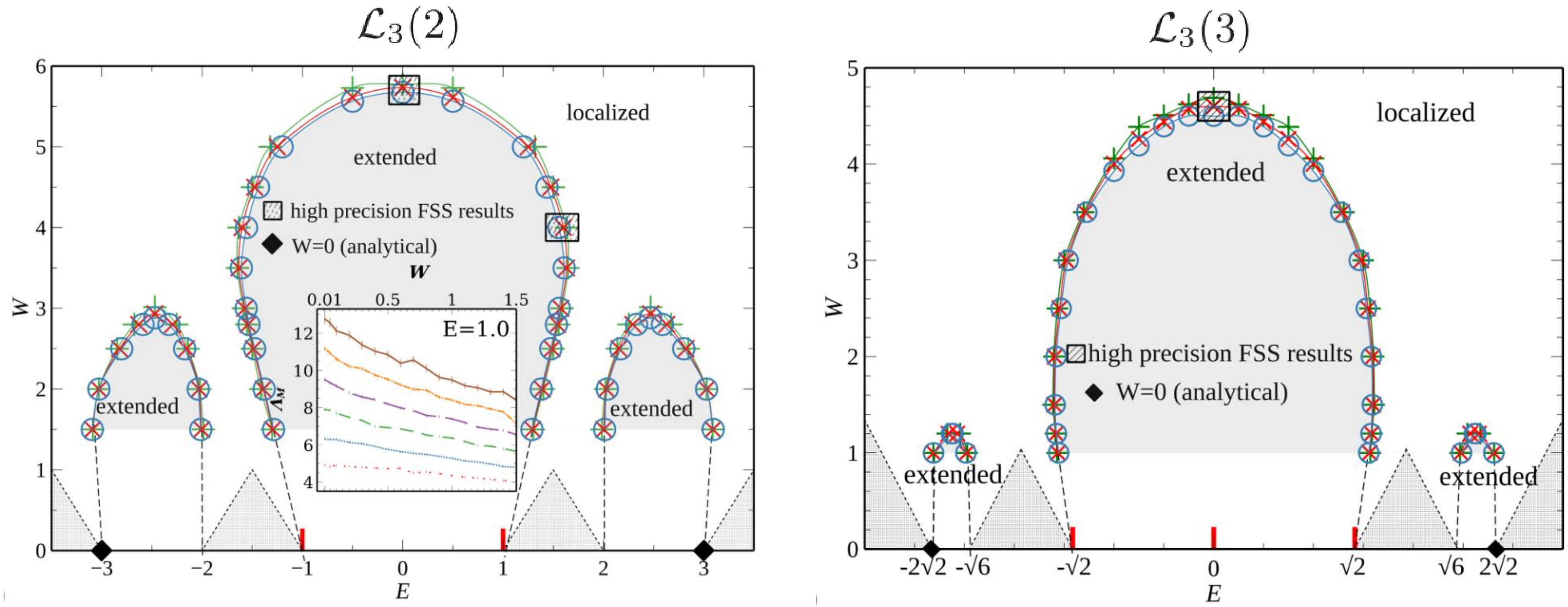


# Results – *E-W* Phase diagram




- The three solid and colored lines(determined by width *M* equal to 4, 6 and 8) represent the approximate location of the phase boundary.
- Inside of the phase boundary is extended while outside is localized.
- Inset: Strip width *M* vary from 4, 6, 8, 10, 12 from down to up.

# Results – *E-W* Phase diagram




➤ The three solid and colored lines(determined by width *M* equal to 4, 6 and 8) represent the approximate location of the phase boundary.

➤ Inset: Strip width *M* vary from 4, 6, 8, 10, 12 from down to up.

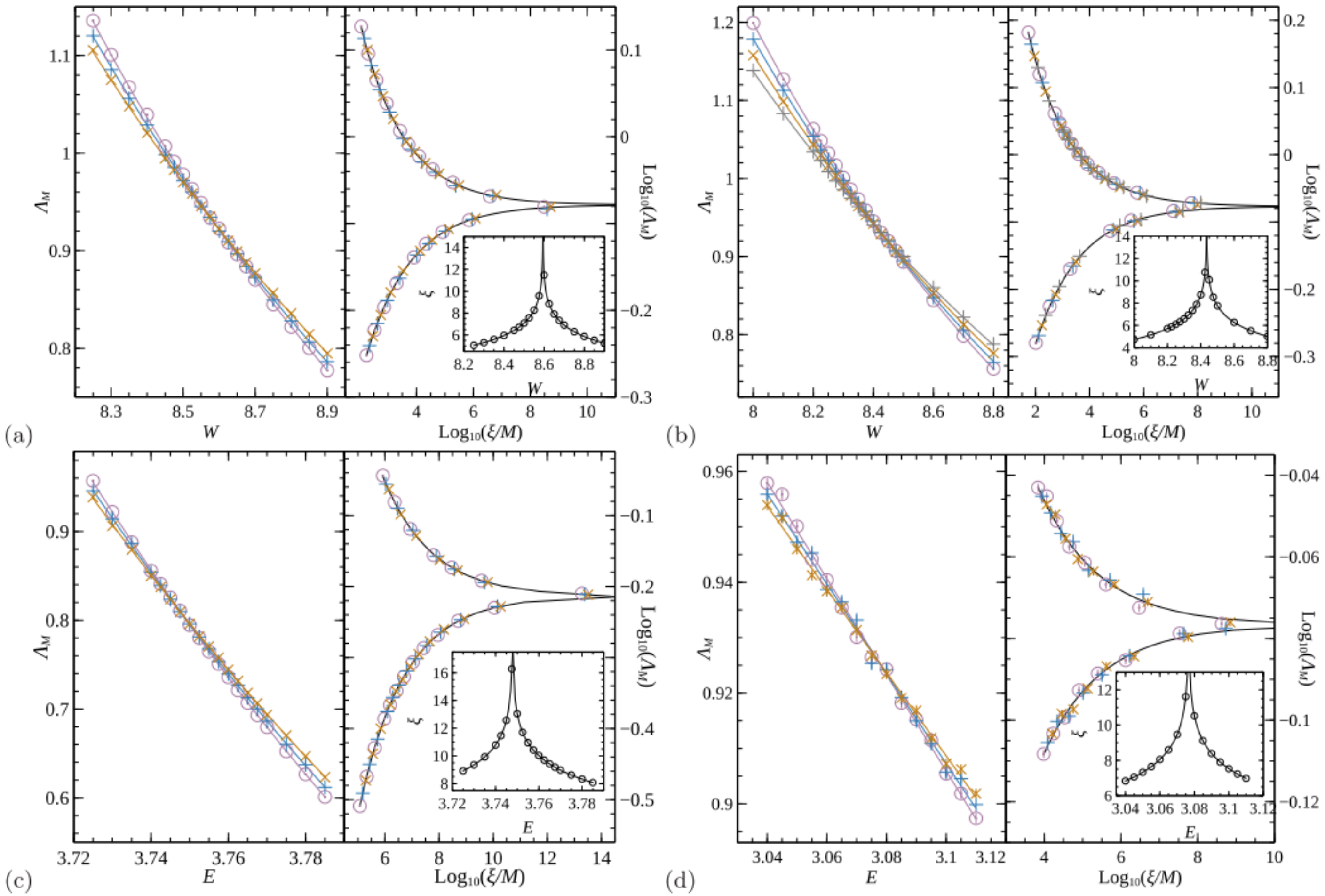
# Results – Finite Size Scaling

| | $n_r$ | $m_r$ | $W_c$ |
|---|---|---|---|
| | 3 | 1 | 8.594 |
| $\mathcal{L}_3(\mathbf{1})$ | 2 | 2 | 8.598 |
| | 3 | 2 | 8.595 |
| Average: | | | **8.596(4)** |
| | 2 | 2 | 5.964 |
| $\mathcal{L}_3(\mathbf{2})$ | 2 | 3 | 5.965 |
| | 3 | 2 | 5.963 |
| Average: | | | **5.964(3)** |
| | 2 | 1 | 4.79 |
| $\mathcal{L}_3(\mathbf{3})$ | 1 | 2 | 4.791 |
| | 2 | 2 | 4.791 |
| Average: | | | **4.790(2)** |

- For models $\mathcal{L}_3(n)$, the critical disorder $W_c$ decrease as *n* increase.
- All FSS results produce stable and robust.

# Results – Finite Size Scaling

$\mathcal{L}_3(1)$ $E = 0$ $E = 1$

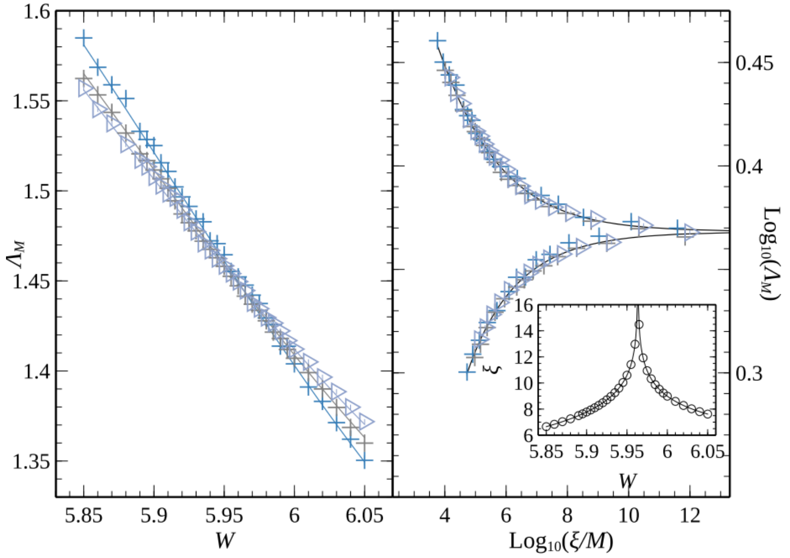



$W = 3$ $W = 6$

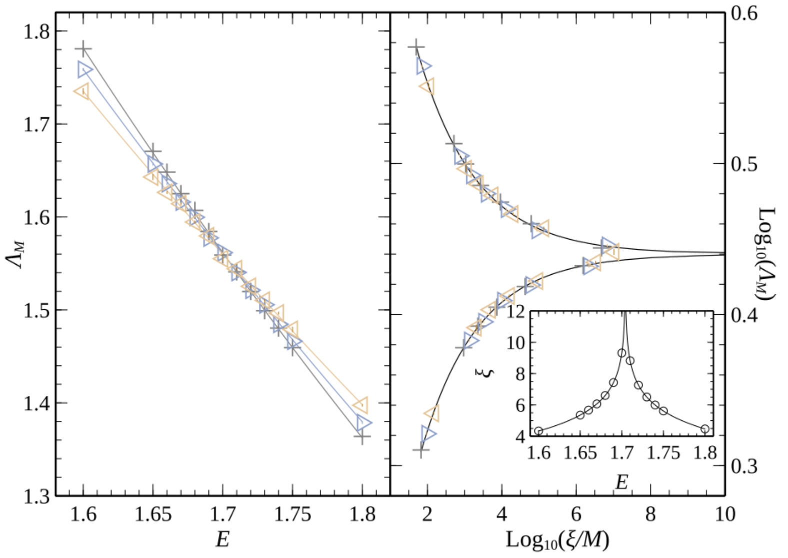
- The left half in each panel denotes a plot of $\Lambda_M$ versus disorder W or energy E.
- The right half shows the scaling function F (solid line).
- Each inset gives the scaling parameter $\xi$ as a function of disorder strength W, or energy E.

# Results – Finite Size Scaling

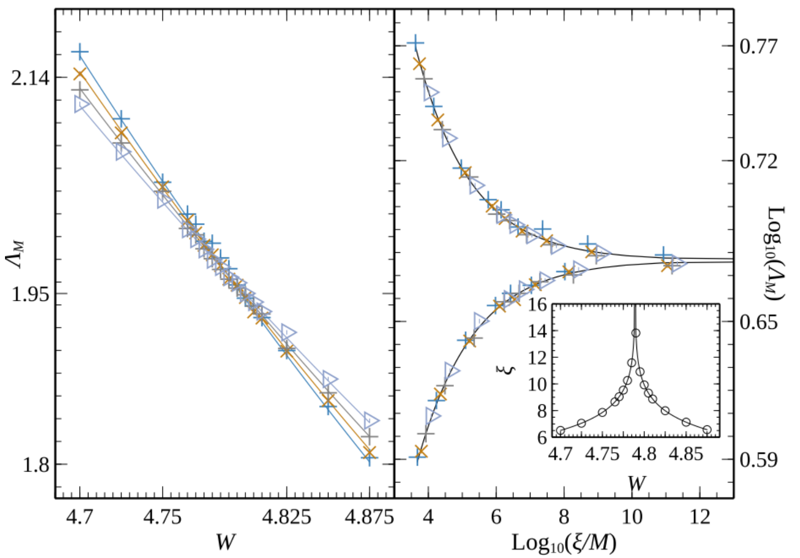
$\mathcal{L}_3(2)$ $E = 0$




$\mathcal{L}_3(2)$ $E = 1$




$\mathcal{L}_3(3)$ $E = 0$



# Conclusion

- For extended Lieb lattice $\mathcal{L}_3(n)$, their critical disorder $W_c$ at energy $E$=0 is from 8.596(4) for $\mathcal{L}_3(1)$, to 5.964(3) for $\mathcal{L}_3(2)$, and to 4.790(2) for $\mathcal{L}_3(3)$.
- Our research is instructive to study the MIT of 3D to 1D case as $n \to \infty$.
- In dispersion relations, the overall band width decreases as n increases, and the number of flat bands increases and the extremal energy of these bands extends as well towards $|E|$=2.