Compact localized states by local and latent symmetries

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> IBS Conference on Flatbands August 18, 2021





Acknowledgements

- The presented works have been done with Prof. Peter Schmelcher, Dr. Christian Morfonios, Maxim Pyzh (University of Hamburg).
- Briefly mentioned works have been done together with
 - Prof. Luca Dal Negro, Dr. Fabrizio Sgrignuoli (Boston University)
 - Prof. Fotios Diakonos (University of Athens), and Dr. Nikolas Palaiodimopoulos (Foundation for Research and Technology-Hellas, Crete)

Local symmetries

Latent symmetries 00000000000

Motivation and overarching question



Taken from S. Flach, D. Leykam, J. D. Bodyfelt, P. Matthies, and A. S. Desyatnikov, "Detangling flat bands into Fano lattices", EPL 105, 30001 (2014).

Which role do (local) symmetries play for compact localized states (CLSs) and flat bands?



In the past talks, many different systems were introduced: Magnons, interacting electrons, non-interacting electrons,

The concepts presented in this talk are system-independent, since we operate on the level of matrices (and not of the operator).



Preliminary remark

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The concepts presented in this talk are system-independent, since we operate on the level of matrices (and not of the operator).



F. Sgrignuoli, M. Röntgen, C. V. Morfonios, P. Schmelcher, and L. Dal Negro, "Compact localized states of open scattering media: a graph decomposition approach for an ab initio design", Opt. Lett. 44, 375 (2019) 5

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An easily understood example: The diamond chain



The global reflection P commutes with the Hamiltonian H of the lattice. Thus, the eigenstates of H can be chosen to have definite parity under P.

Latent symmetries

An easily understood example: The diamond chain



The global reflection P commutes with the Hamiltonian H of the lattice. Thus, the eigenstates of H can be chosen to have definite parity under P.

If w denotes one of the white sites, then each negative parity eigenstate $|\Phi\rangle$ fulfills

$$\langle w|\Phi\rangle = \langle w|P^2|\Phi\rangle = -\langle w|\Phi\rangle \Rightarrow \langle w|\Phi\rangle = 0.$$
(1)

Thus, the negative parity eigenstate vanishes on all white sites!

Local symmetries

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From global to local symmetries



Local symmetries

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From global to local symmetries

(There would a video on this slide; See the YouTube talk.)

The 1D pyrochlore lattice



Taken from S. Flach, D. Leykam, J. D. Bodyfelt, P. Matthies, and A. S. Desyatnikov, "Detangling flat bands into Fano lattices", EPL 105, 30001 (2014).

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Local symmetries

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Flat bands through local symmetries



Utilizing the equitable partition theorem $(EPT)^1$ from graph theory.

¹W. Barrett, A. Francis, and B. Webb, "Equitable decompositions of graphs with symmetries", Linear Algebra Its Appl. **513**, 409–434 (2017).

Local symmetries

Latent symmetries

Generalizations and further information

• M. Röntgen, C. V. Morfonios, and P. Schmelcher, "Compact localized states and flat bands from local symmetry partitioning", Phys. Rev. B **97**, 035161 (2018)

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Parity without symmetry? A simple example



Latent symmetry

u

There is no reflection symmetry for the right system. Yet, all eigenstates have definite parity on u and v.

Local symmetries

Making the latent symmetry visible



Latent symmetries can be made visible through the so-called *isospectral* reduction² (ISR)

$$\mathcal{R}_{S}(H, E) = H_{SS} + H_{S\overline{S}} \left(E I - H_{\overline{SS}} \right)^{-1} H_{\overline{SS}}$$

which is equivalent to an effective Hamiltonian obtained through subsystem partitioning.

Latent symmetry of two sites \Rightarrow Local parity!

²D. Smith and B. Webb, "Hidden symmetries in real and theoretical networks", Physica A 514, 855–867 (2019).

From latent symmetries to CLS

latent symmetry		Not pure permutation								
Latent Symmetry		10	1	0	0	0	0	0	0	0
u=1 6 7 3 4 5 8 v=2 9	Q =	1	0	0	0	0	0	0	0	0
		0	0	1	0	0	0	0	0	0
		0	0	0	1	0	0	0	0	0
		0	0	0	0	1	0	0	0	0
		0	0	0	0	0	0.5	-0.5	0.5	0.5
		0	0	0	0	0	-0.5	0.5	0.5	0.5
		0	0	0	0	0	0.5	0.5	0.5	-0.8
		$\left(0 \right)$	0	0	0	0	0.5	0.5	-0.5	0.5
Reflection symmetry	F	Pure	pern	nuta	tion					
1		$\sqrt{0}$	1	0	$0 \rangle$					
	P =	1 0 0 0								
		0	0	1	0					
		$\setminus 0$	0	0	1					

Due to the latent symmetry of u and v, there exists an orthogonal matrix Q with eigenvalues ± 1 which commutes with H. Q squares to one.

From latent symmetries to CLS



Due to the latent symmetry of u and v, all eigenstates have definite parity on them. For the present system, Q acts as the identity on each white site w, so that the negative parity eigenstate $|\Phi\rangle$ fulfills

$$\langle w|\Phi\rangle = \langle w|Q^2|\Phi\rangle = -\langle w|\Phi\rangle \Rightarrow \langle w|\Phi\rangle = 0.$$
 (2)

Thus, $|\Phi\rangle$ is compactly localized!

But why does Q act as the identity on the white sites???

Latent symmetries

Latent symmetries and singlets



When u, v are latently symmetric, then the eigenstates have definite parity on these two sites, and the following are equivalent

• The matrix Q acts as the identity on w.

•
$$(H^k)_{u,w} = (H^k)_{v,w} \forall k$$

In our nomenclature, the white sites are "singlets".

Local symmetries

Walks: A convenient interpretation of matrix powers

For³ a Hamiltonian with matrix elements 0 and 1: The matrix element $(H^k)_{i,i}$ gives the number of different walks of length k from i to j.



Taken from C. V. Morfonios, M. Pyzh, M. Röntgen, and P. Schmelcher, "Cospectrality preserving graph modifications and eigenvector properties via walk equivalence of vertices", Linear Algebra and its Applications 624, 53–86 (2021).

Thus, a singlet w has the same "distance" from the latently symmetric sites u, v, as is expressed by its defining equation

$$\left(H^{k}\right)_{u,w} = \left(H^{k}\right)_{v,w} \ \forall \ k \,. \tag{3}$$

³Note: This has been known in spectral graph theory for a long time!

Local symmetries 0000000 Latent symmetries

More on latent symmetries

Two sites u, v are latently symmetric if and only if⁴

$$\left(H^{k}\right)_{u,u} = \left(H^{k}\right)_{v,v} \forall k.$$
(4)

Equivalently, the eigenvalue spectra of $H \setminus u$ and $H \setminus v$ can be shown to be equal. The sites u and v are said to be cospectral.



 $^4M.$ Kempton, J. Sinkovic, D. Smith, and B. Webb, "Characterizing cospectral vertices via isospectral reduction", Linear Algebra Its Appl. 594, 226–248 (2020).

More on latent symmetries

- Each reflection symmetry is a latent symmetry, but the reverse is not true. ⇒ Latent symmetry is a broader concept!
- When a Hamiltonian features non-abelian latent symmetries, then it necessarily features degenerate eigenvalues⁴.

⁴M. Röntgen, M. Pyzh, C. V. Morfonios, N. E. Palaiodimopoulos, F. K. Diakonos, and P. Schmelcher, "Latent Symmetry Induced Degeneracies", Phys. Rev. Lett. **126**, 180601 (2021).₂₄

Ingredients for the design of CLSs through latent symmetries

Connecting subnetworks to singlets creates new singlets, and does not break reflection symmetry/latent symmetry.



Ingredients for the design of CLSs through latent symmetries

The simplest possible scheme:

- Pick a network with latent symmetries (there are plenty!).
- If necessary, equip this network with singlets (the procedure for this step is based, again, on an analysis of the matrix powers of H).
- Use the network as a unit cell; connect different unit cells via singlets.



Latent symmetries

Ingredients for the design of CLSs through latent symmetries



C. V. Morfonios, M. Röntgen, M. Pyzh, and P. Schmelcher, "Flat bands by latent symmetry", Phys. Rev. B 104, 035105 (2021)

Latent symmetries

Ingredients for the design of CLSs through latent symmetries



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Further information

- Applications to flat bands: C. V. Morfonios, M. Röntgen, M. Pyzh, and P. Schmelcher, "Flat bands by latent symmetry", Phys. Rev. B 104, 035105 (2021)
- Underlying mathematical theory: C. V. Morfonios, M. Pyzh, M. Röntgen, and P. Schmelcher, "Cospectrality preserving graph modifications and eigenvector properties via walk equivalence of vertices", Linear Algebra and its Applications 624, 53–86 (2021)

Local symmetries

Latent symmetries

Outlook

Can we also link other CLSs/flat bands to (latent) symmetries?
 U(n > 1)?



Taken from S. Flach, D. Leykam, J. D. Bodyfelt, P. Matthies, and A. S. Desyatnikov, "Detangling flat bands into Fano lattices", EPL 105, 30001 (2014).

• Generalization to other latent permutation symmetries.

Local symmetries

Latent symmetries

Thank you for your attention!

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