An Overview Of The SYK Model And Its Relatives

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Based on: many works, just few of them are mine

IBS conference on Flatbands, August 16 - 20 2021

ibs Institute for Basic Science

pcs Center for Theoretical Physics of Complex Systems

Motivation: few numbers

Slogan: SYK model has received **huge** attention in the last years

How huge? Which numbers are we talking about?

- Kitaev, Apr. 2015: two talks at Santa Barbara (no paper on them) ``**A simple model of quantum holography''**
- Maldacena and Stanford, Apr. 2016: paper on Kitaev's talk ``**Remarks on the Sachdev-Ye-Kitaev model''**, *Phys. Rev. D*

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Around 1200 citations by now!

OUTLINE

- **Quantum chaos** in many-body systems: early and late time scales
- **SYK model**: strongly chaotic but yet a solvable model
- Experimental setups: **flatbands** at work
- Adding structure to SYK: **generalizations**
  - **Mass-deformed** model $\longrightarrow$ Fock space localization
  - Finite distance model $\longrightarrow$ Solvable, genuine, MBL?
  - SYK on **graphs** $\longrightarrow$ sparse SYK, small-world SYK ...

Quantum chaos in many-body systems

- System with N degrees of freedom, **many-body** Hamiltonian, $\hat{\mathcal{H}}$
  $\Rightarrow \approx e^N$ energy levels, **mean level spacing:** $\Delta \approx e^{-N}$

Non-equilibrium dynamics: prepare a state $|\psi\rangle$ and let it evolve.

RMT universality at late times: [Bohigas, Giannoni, Schmit, 1984]

At **late times**, dynamics is described by random matrix theory (RMT).
Thouless time: $\tau_{\mathrm{Th}} \approx 1/\Delta \approx e^N$

$\Rightarrow$ **spectral correlations**: Finite N studies work well (ED)

Early-time chaos: [Larkin, Ovchinnikov, 1969; Kitaev '14; Maldacena, Shenker, Stanford, '15]

- Spreading of **local operators** $\Rightarrow$ **OTOCs**

$$(\hat{V},\, \hat{W}) \text{ local} \rightarrow \langle [\hat{V}(t), \hat{W}(0)]^2 \rangle = F(t)$$

- $F(t)$ has **exponential** behavior for chaotic systems:

$$\boxed{F(t) \propto e^{\lambda_L t} \text{ for } O(1) < t < O(\log N)}$$

Problem at finite N: small time window at finite N.

Almost impossible via ED. **Analytic models** required $\Rightarrow$ **SYK** enters

Few words on SYK [Kitaev, '15; Maldacena and Stanford, '16; ...]

0-dimensional model of N Majorana **fermions**

$$\hat{\gamma}^i, \ i = 1, \dots, N \quad \text{s.t.} \ \left\{\hat{\gamma}^i, \hat{\gamma}^j\right\} = \delta^{i,j}$$

All-to-all, q-body Hamiltonian, with **Gaussian disorder**

$$\hat{\mathcal{H}}^{(q)} = \sum J_{i_1 \dots i_q} \, \hat{\gamma}^{i_1} \cdots \hat{\gamma}^{i_q} \ , \ \left\langle J^2_{i_1 \dots i_q} \right\rangle \propto \frac{1}{N^{q-1}}$$

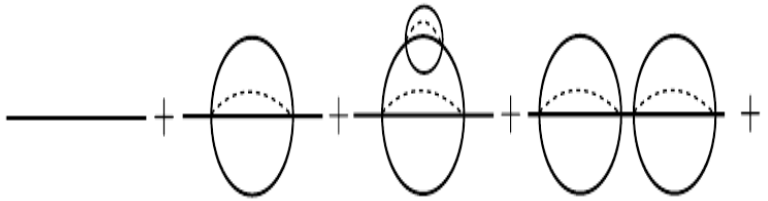
$\mathbf{q \geq 4}$: no hopping term $\Rightarrow$ **strongly interacting**, our focus now

$\mathbf{q = 2}$: **random mass** term, integrable dynamics

SYK solvability: $N \to \infty$ limit

- $T = 1/\beta$, averaged over J_{ijkl} ($q = 4$), **2-point function**

$$G(\tau) \equiv \frac{1}{N} \sum_i \overline{\langle \hat{\gamma}^i(\tau) \hat{\gamma}^i(0) \rangle}_\beta$$

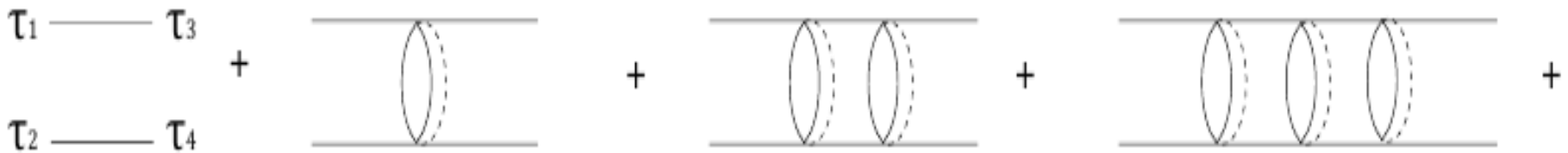
- Only **melonic diagrams** contribute: $G(\tau) =$ 

  $\Rightarrow$ Compact **SD equations**:

$$\boxed{G(\omega)^{-1} = -i\omega - \Sigma(\omega) \quad \Sigma(\tau) = J^2 G(\tau)^3}$$

Higher point functions: [Kitaev '15; Maldacena, Stanford, '16; Gross, Rosenhaus, '17]

- Melonic dominance extends to **all-point functions**:
  - OTOC is a **4-point function**:



$$\Rightarrow \boxed{\langle [\hat{V}(t), \hat{W}(0)]^2 \rangle \propto e^{\lambda_L t} \qquad \lambda_L = \frac{2\pi k_B T}{\hbar}}$$

  - λ_L saturates the **MSS bound** [Maldacena, Shenker, Stanford, '15]

    $\Rightarrow$ **Fastest scrambling**

Experimental proposals

- Key features and challenges:
  - 0-dimensional, **no momentum** $\Rightarrow$ **flatbands** at work!
  - Random couplings, all-to-all
- Main proposals:
  - Interface topological insulator/superconductor [Pikulin, Franz, '17]
  - Graphene flake in a magnetic field [Chan et al., '18]
  - Digital quantum simulation [García-Álvarez et al., '17]
  - Few more [Chew et al., '17; Danshita et al., '17]

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Optical lattice platform for the Sachdev-Ye-Kitaev model

Chenan Wei and Tigran A. Sedrakyan
Department of Physics, University of Massachusetts, Amherst, Massachusetts 01003, USA

- A more familiar setup: $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\text{imp}} + \hat{\mathcal{H}}_{\text{int}}$
  - $\hat{\mathcal{H}}_0$: tight-binding on **kagome** lattice with flux $\Rightarrow$ **flatband**
  - $\hat{\mathcal{H}}_{\text{imp}} = u \sum_{r_m} a^{\dagger}_{r_m} a_{r_m}$: **random** impurities on few sites
  - $\hat{\mathcal{H}}_{\text{int}}$: quartic **interaction** term

N particles on the flatband feel a low-temperature SYK-like physics

Adding structure to SYK: generalizations

- SYK is **highly chaotic** and **solvable**:
  a **boring model**: everything thermalize very fast!

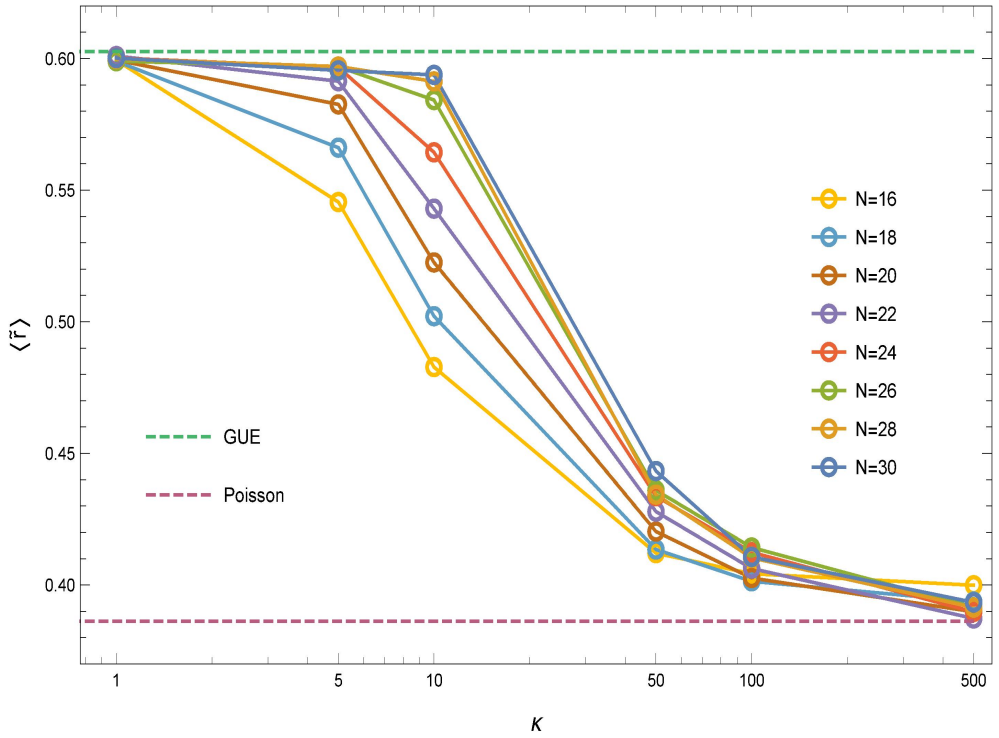
Can we add more structure and keep solvability? Less chaos?

- The answer is **positive**:
  many **generalizations** have been developed over the years
- Impossible to give a complete overview
  $\Rightarrow$ I will describe some of them guided by my personal taste

Mass deformed SYK [García-García et al., '17; DR et al. '18]

$$\hat{\mathcal{H}}(\kappa) = \hat{\mathcal{H}}^{(4)} + \kappa\,\hat{\mathcal{H}}^{(2)}, \qquad \kappa \equiv \text{mass parameter}$$

- Two terms with **opposite** behaviors
- Mass deformation controlled by κ
- **Chaos/integrability** transition with κ
- **ETH/MBL transition?**
  but no notion of space in the model,
  where is localization taking place?



Fock space localization [Altland et al., '20]

- SYK mapped to $N/2$ **spinless** fermions: $(\hat{\gamma}^{2i}, \hat{\gamma}^{2i-1}) \Rightarrow (\hat{c}^{i}, \hat{c}^{\dagger i})$
- In occupation number representation: $\hat{\mathcal{H}}^{(2)}$ **on-site** energy
- $\hat{\mathcal{H}}^{(4)}$ **hopping**. Distance given by **Hamming distance**, $h(|n\rangle, |m\rangle)$

$$\boxed{|n\rangle, |m\rangle \text{ connected if } h(|n\rangle, |m\rangle) \leq 4}$$

- **MBL scheme**: competition between hopping and disorder
- MBL in Fock space. Some **analytical results** at large N

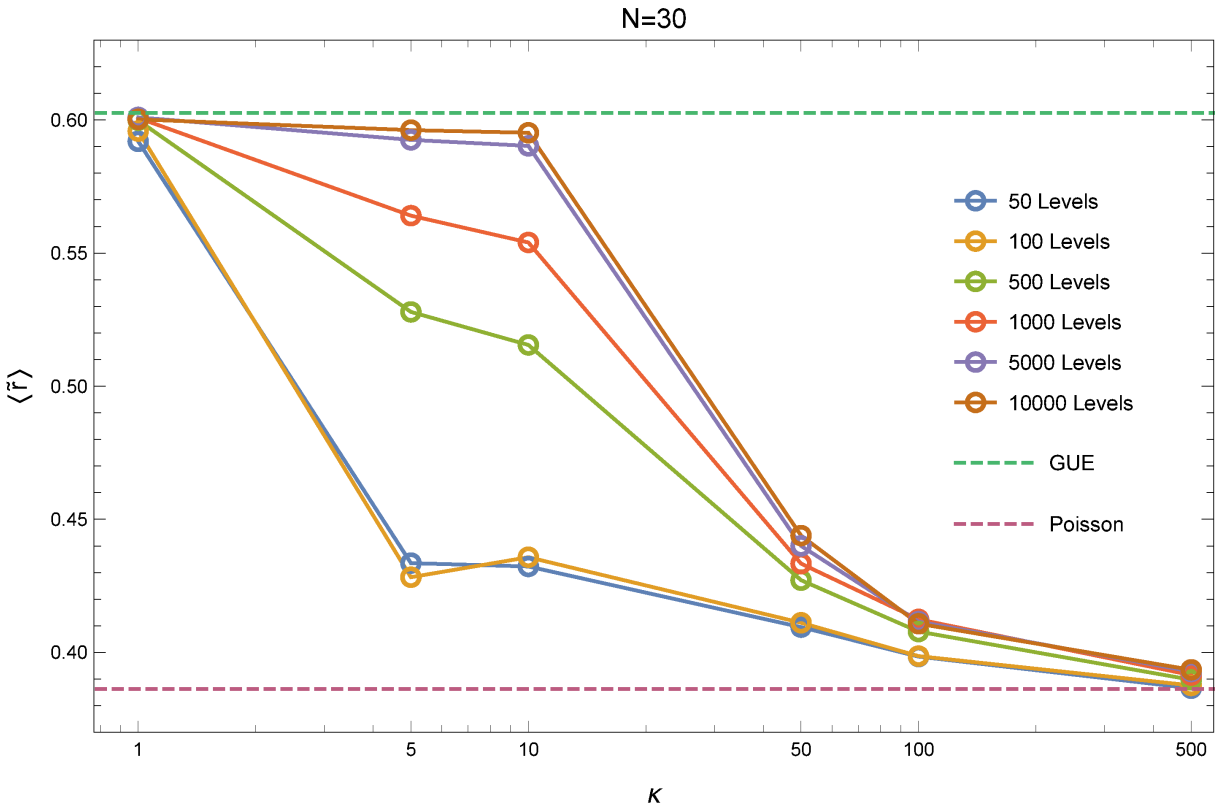
Mobility edge?

Does the mass deformed SYK model exhibit a mobility edge?

- critical mass, κ_c, looks **energy-dependent**

  [DR et al., '18]

- Further numerical and analytical results necessary



Adding space into SYK [García-García and Tezuka, '18]

- Several possibilities $\Rightarrow$ several models
- Most straightforward: **sharp cutoff** distance, D

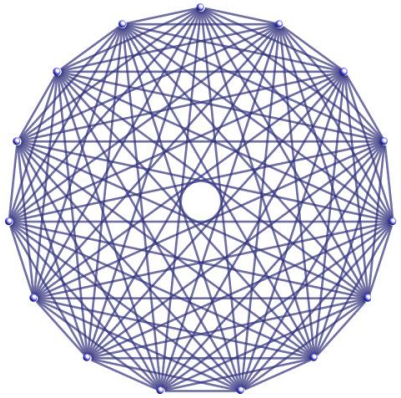
$$J_{i_1 \ldots i_q} \to \tilde{J}_{i_1 \ldots i_q}(D) = \begin{cases} 0 & i_q - i_1 \geq D \\ J_{i_1 \ldots i_q} & i_q - i_1 < D \end{cases}$$

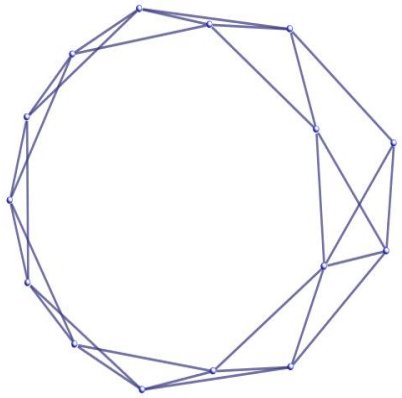
- Chaos/integrable transition by varying D

Is this transition an ETH/MBL transition? [WIP with Dillip Nandy]

SYK on graphs [Susskind, Swingle, Xu '20; García-García, Verbaarschot, DR, Jia, '20]

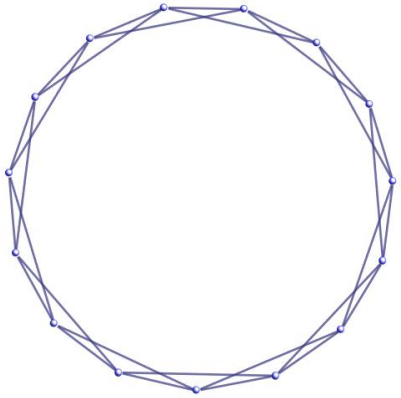
- All-to-all $\Rightarrow$ **complete hypergraphs**
  $\forall\ \hat{\gamma}_i \approx N^{q-1}$ couplings, **demanding!**

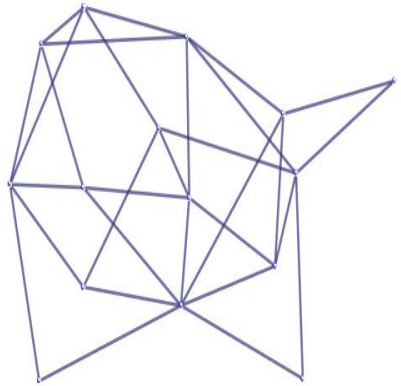




Reduce connectivity keeping physics?

- **Sparse SYK**:
  Random, **regular**, hypergraph
  $\forall\ \hat{\gamma}_i \to kq$ hyperedges only
  - Quantum chaos for $k \approx O(1)$
  - Regularity simplifies the **solvability**





New questions come out: [WIP with several collaborators]

- Hypergraph induced **ETH/MBL** transition?
- Which **geometric features** are crucial for the SYK physics?
  Hint: mean diameter of the hypergraph is the **most prominent**
  What about **other features**?
- Other **sparse graph** generating techniques (e.g. Watts-Strogats)
  $\Rightarrow$**Small-world** SYK models
  Still solvable?

Conclusions & Outlook

- SYK model: **strongly interacting** but solvable model.
  - Many **generalizations**: exotic physics laboratory
- Flatbands are the realm of **strong interactions**:
  - They should be the perfect arena to realize SYK models
    $\Rightarrow$ a single proposal for the **standard** SYK model exists.

Can we use flatband setups to engineer SYK-like models?