

JOSEPHSON TRAVELING WAVE PARAMETRIC AMPLIFIERS AS NON-CLASSICAL LIGHT SOURCE FOR MICROWAVE QUANTUM ILLUMINATION

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28th October 2021

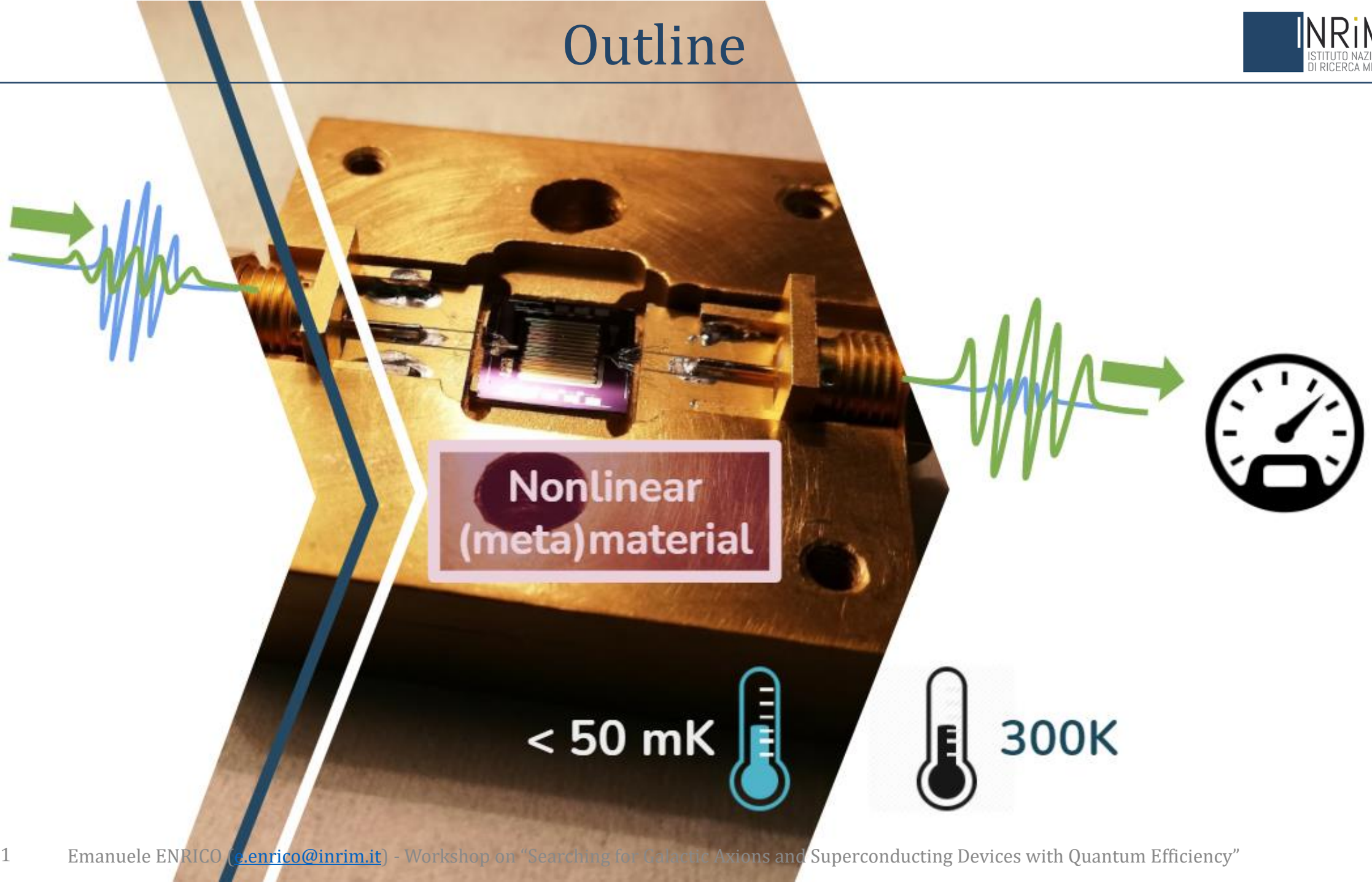
"Searching for Galactic Axions and Superconducting Devices with Quantum Efficiency"

EMPIR 10FUN17 ParaWave Project

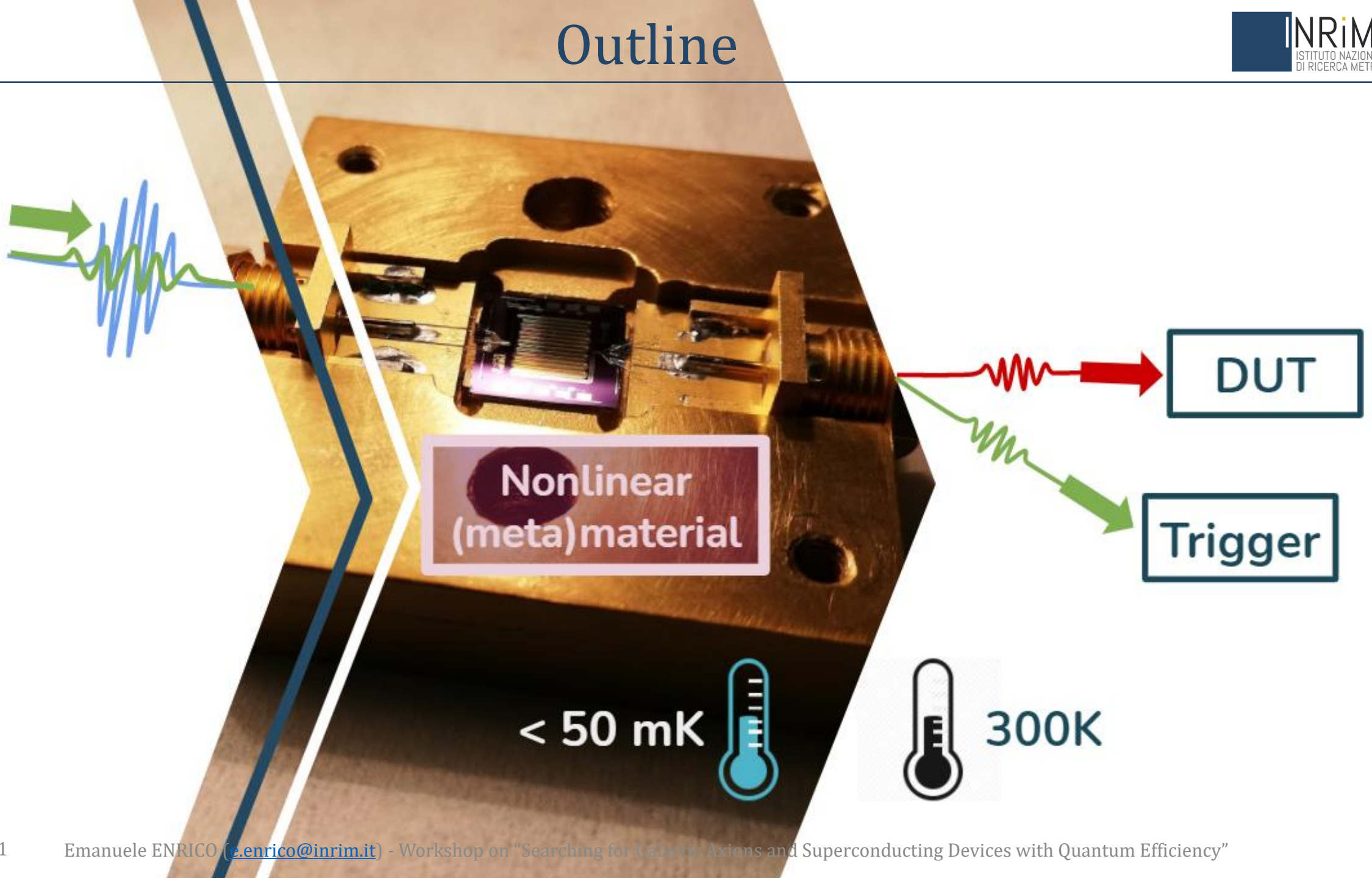
INRiM
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SUPERGALAX

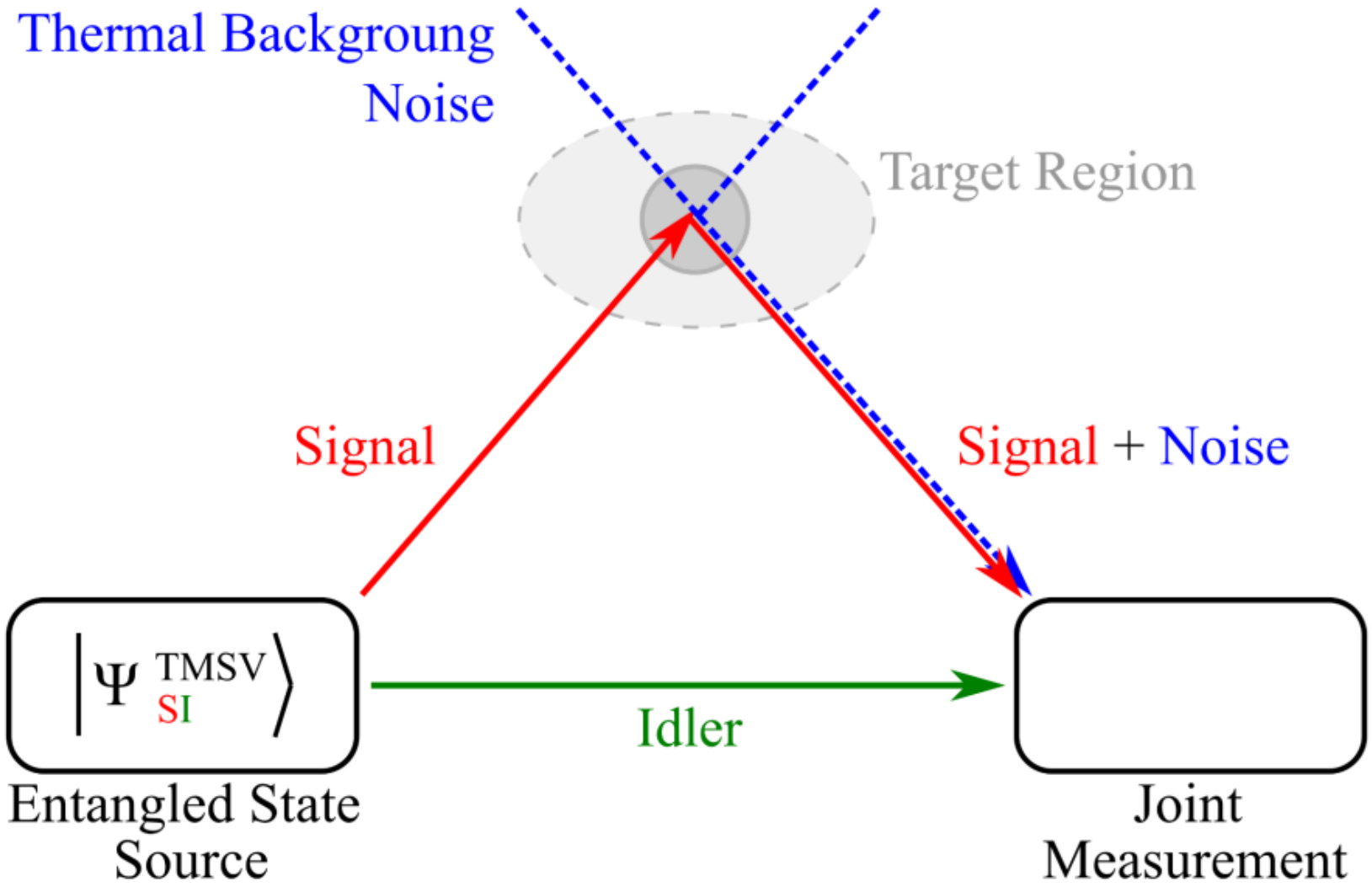
Outline



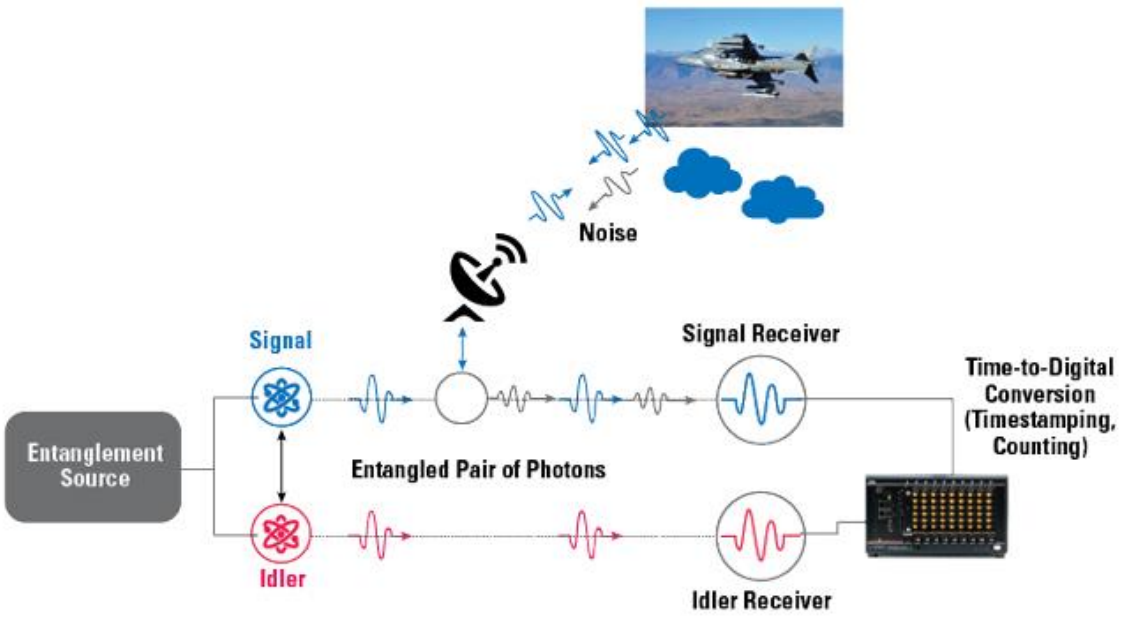
Outline



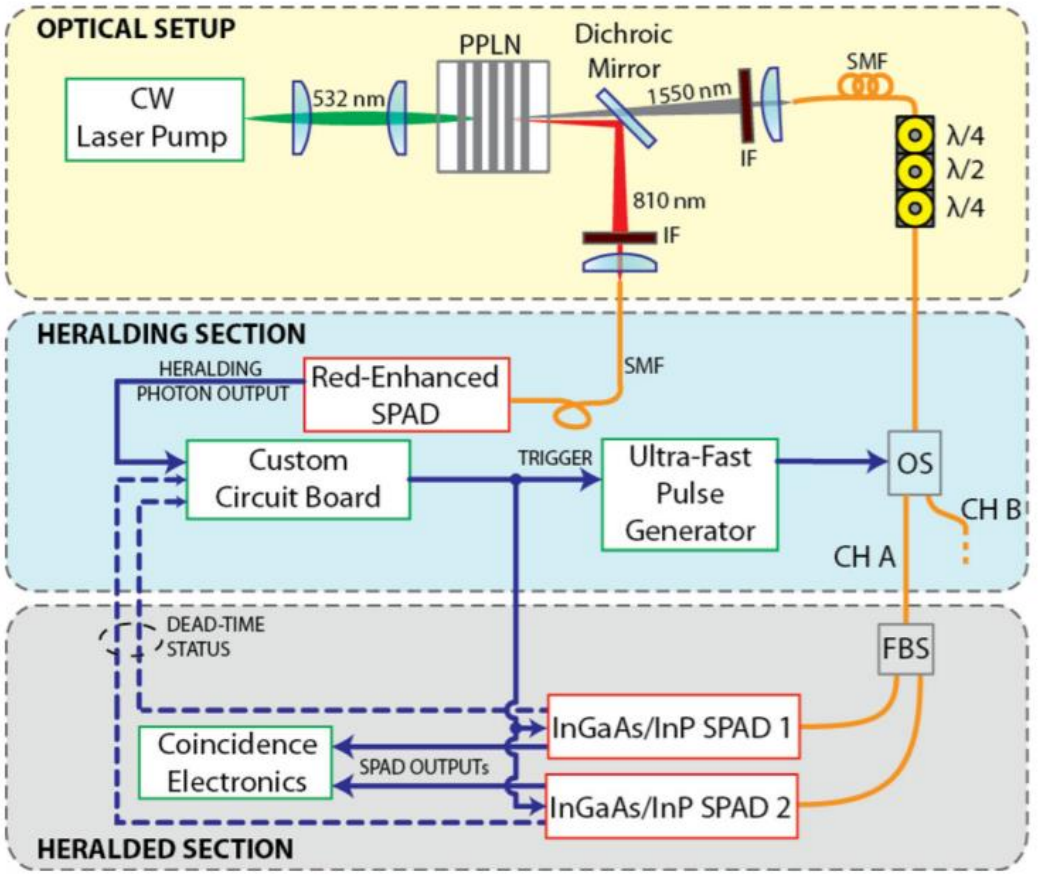
Quantum illumination protocol with TMSV



Quantum Illumination and Heralding Protocols

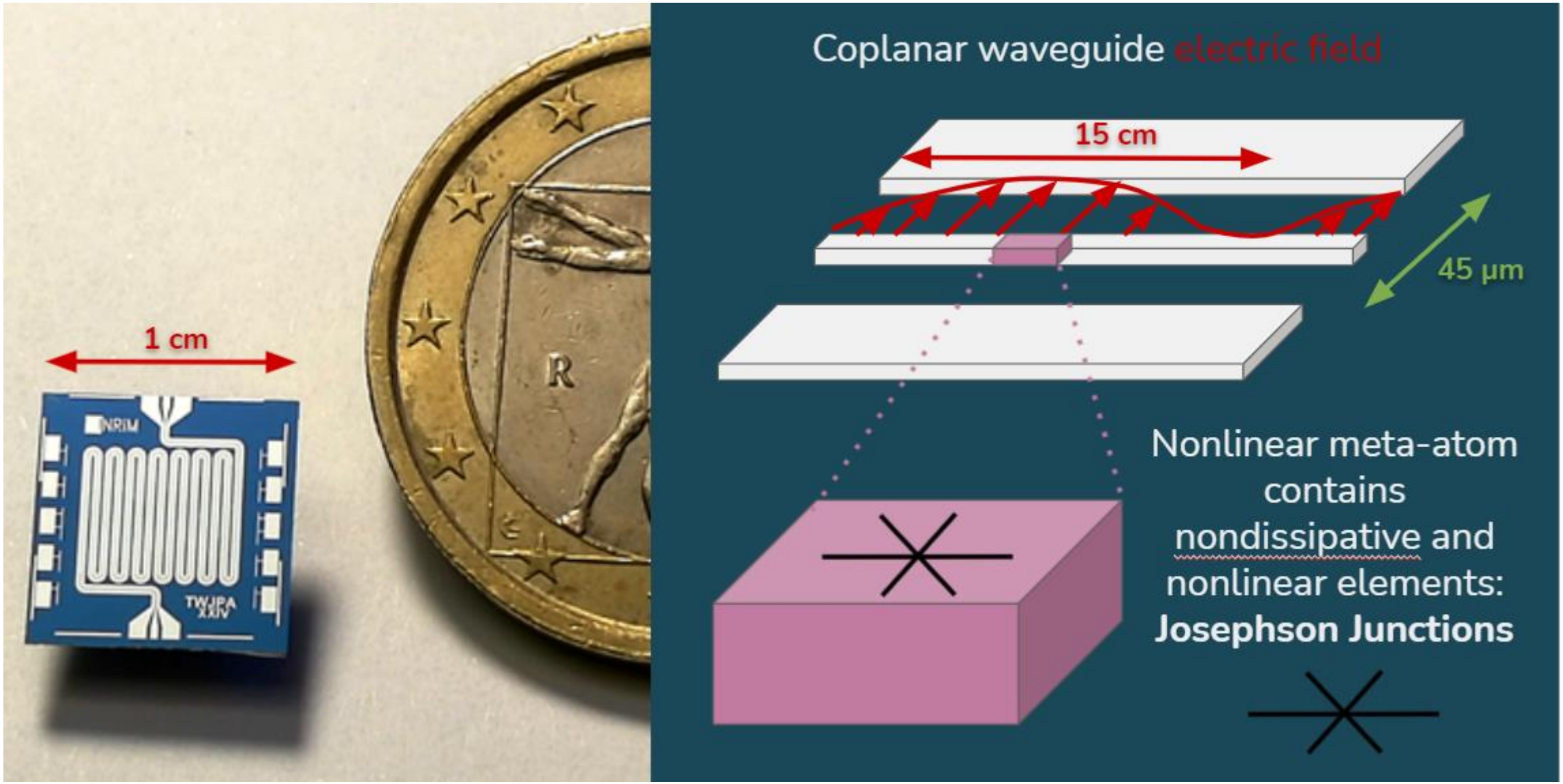


Suren Singh, *Instrument Applications in Quantum for the Aerospace and Defense Industry*, Microwave Journal, Keysight Technologies, Santa Rosa, Calif. (2020)

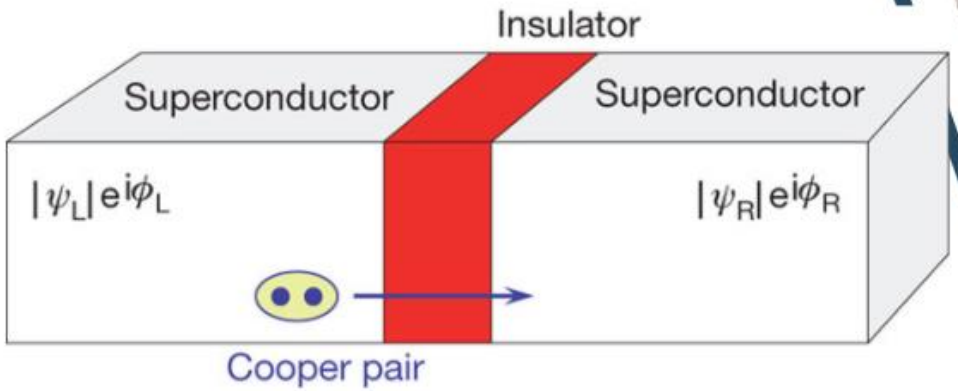


G. Brida et al., *An extremely low-noise* ***heralded single-photon source:*** *A breakthrough for quantum technologies* Appl. Phys. Lett. 101, 221112-1-11 (2012). doi:10.1063/1.4768288

Nonlinear metamaterial in the microwave regime



Josephson Junction fabrication

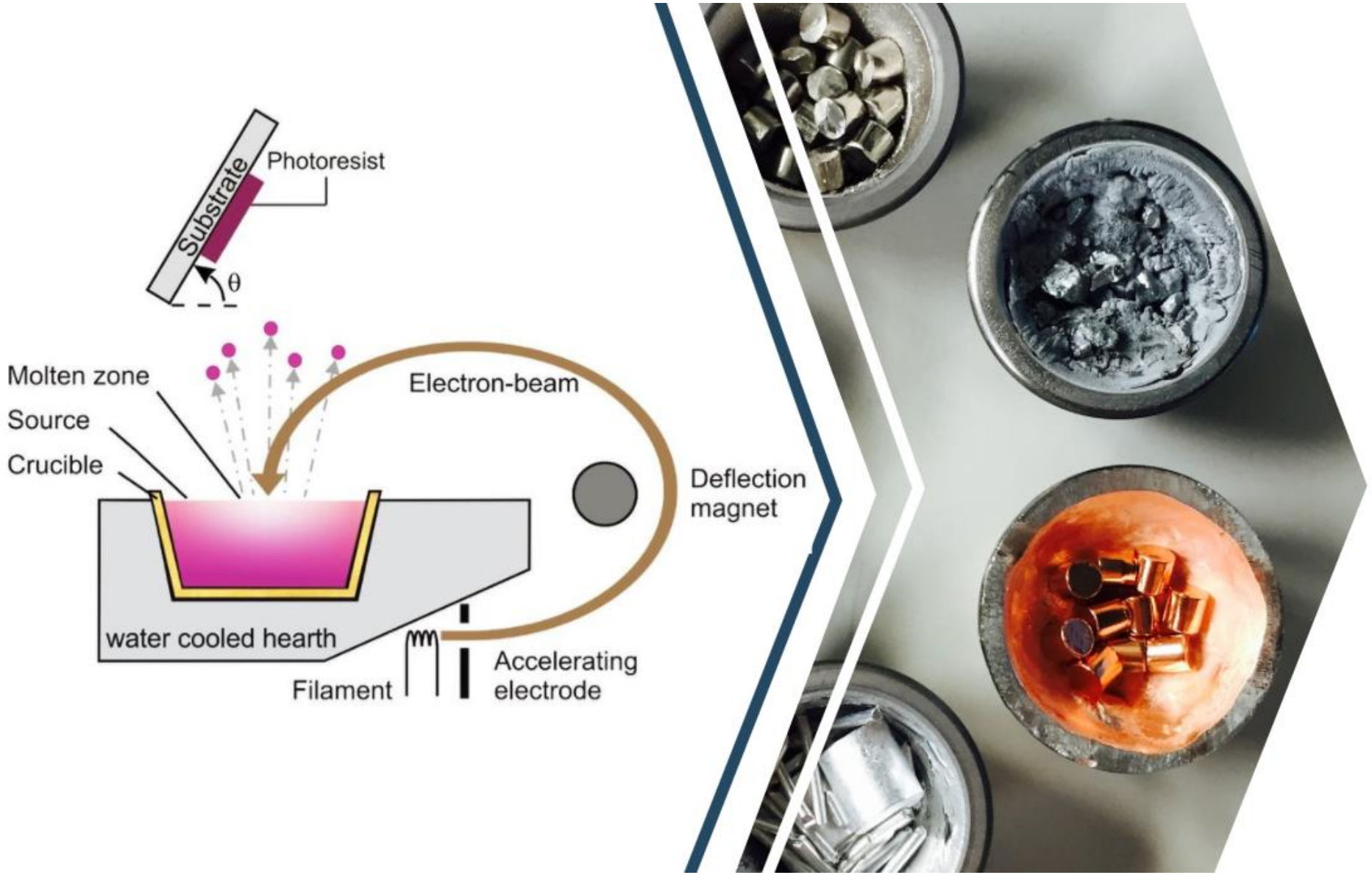


- **Niobium technology**
  Multi-step sputtering-based process
  (Nb/Al/AlOx/Nb Technique, '80)

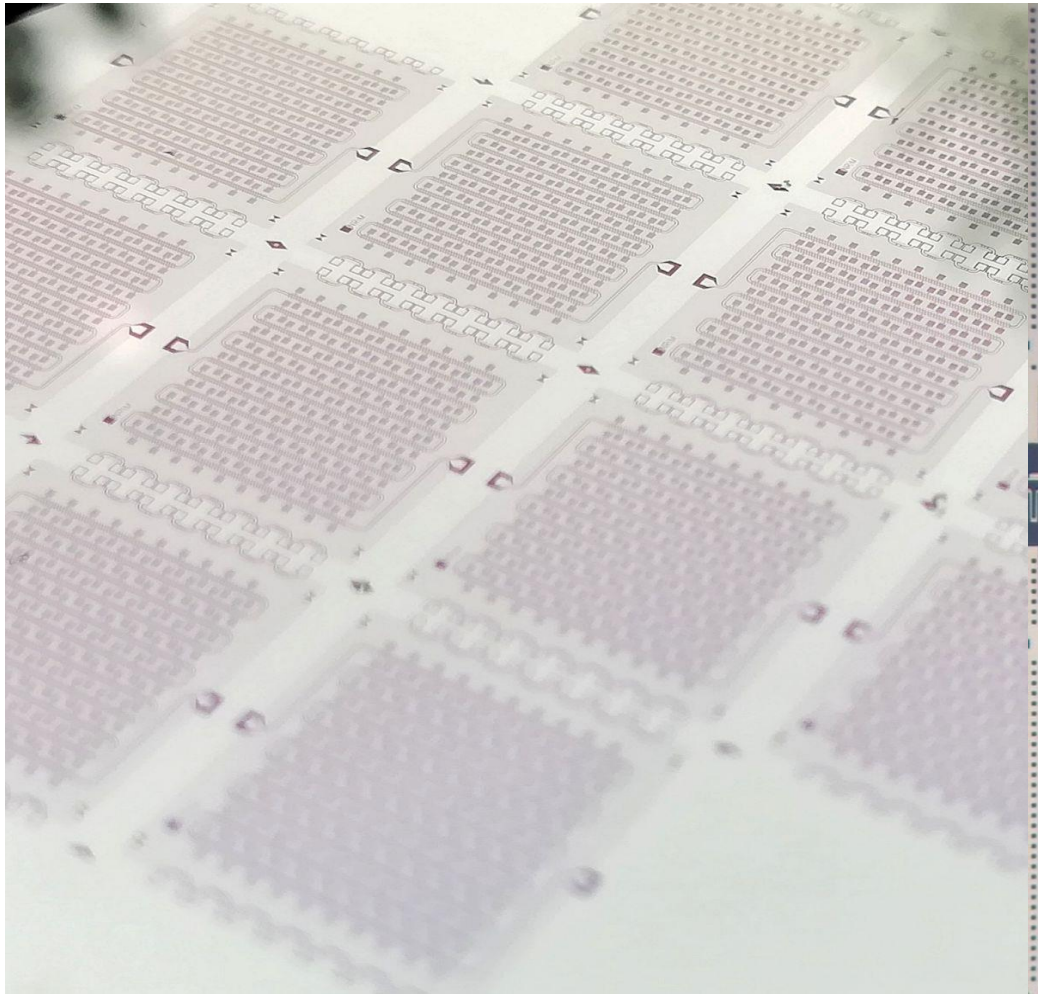
- **Aluminum technology**
  Single step e-beam evap-based process
  (Niemeyer-Dolan Technique, 1987)

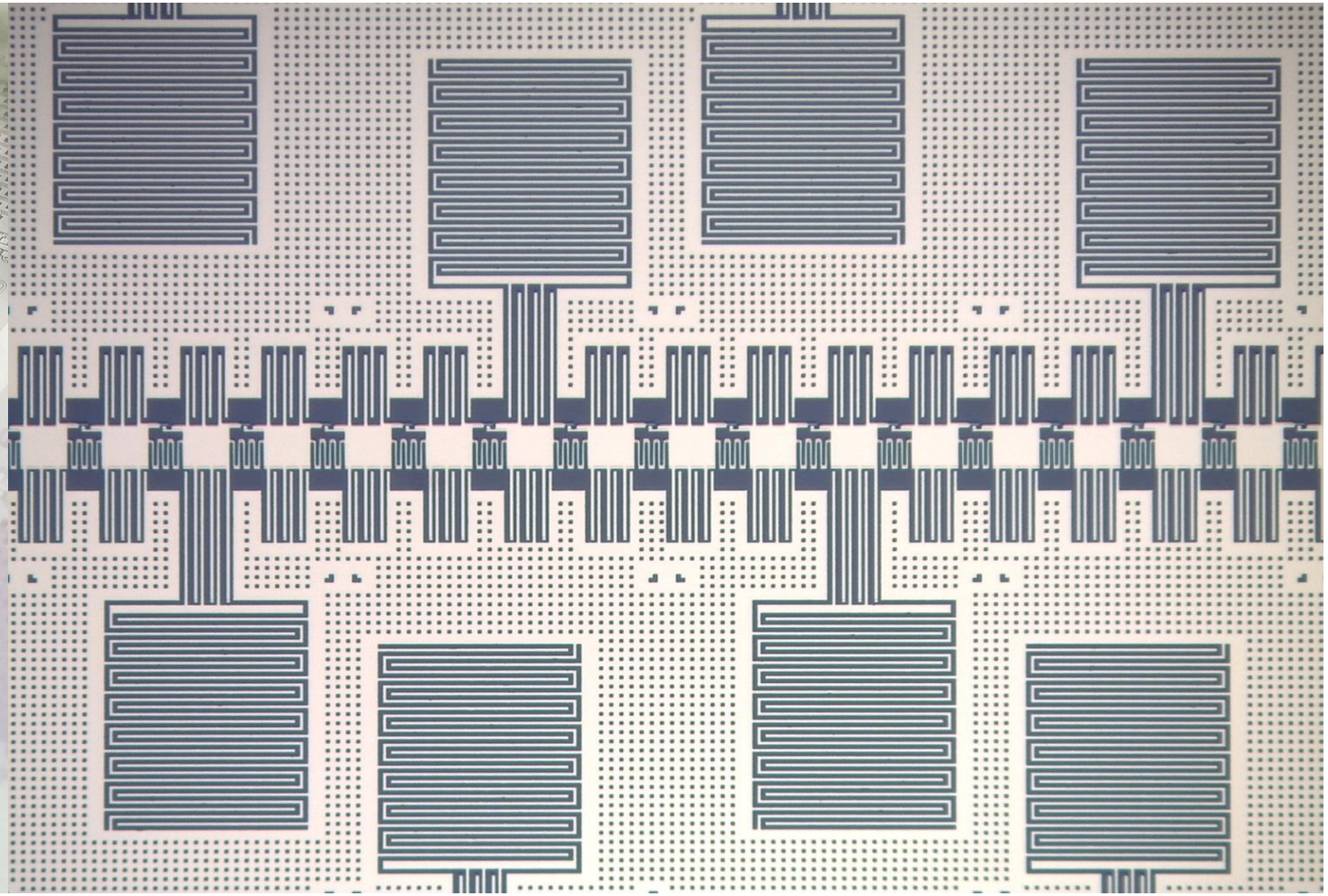


Josephson Junction fabrication

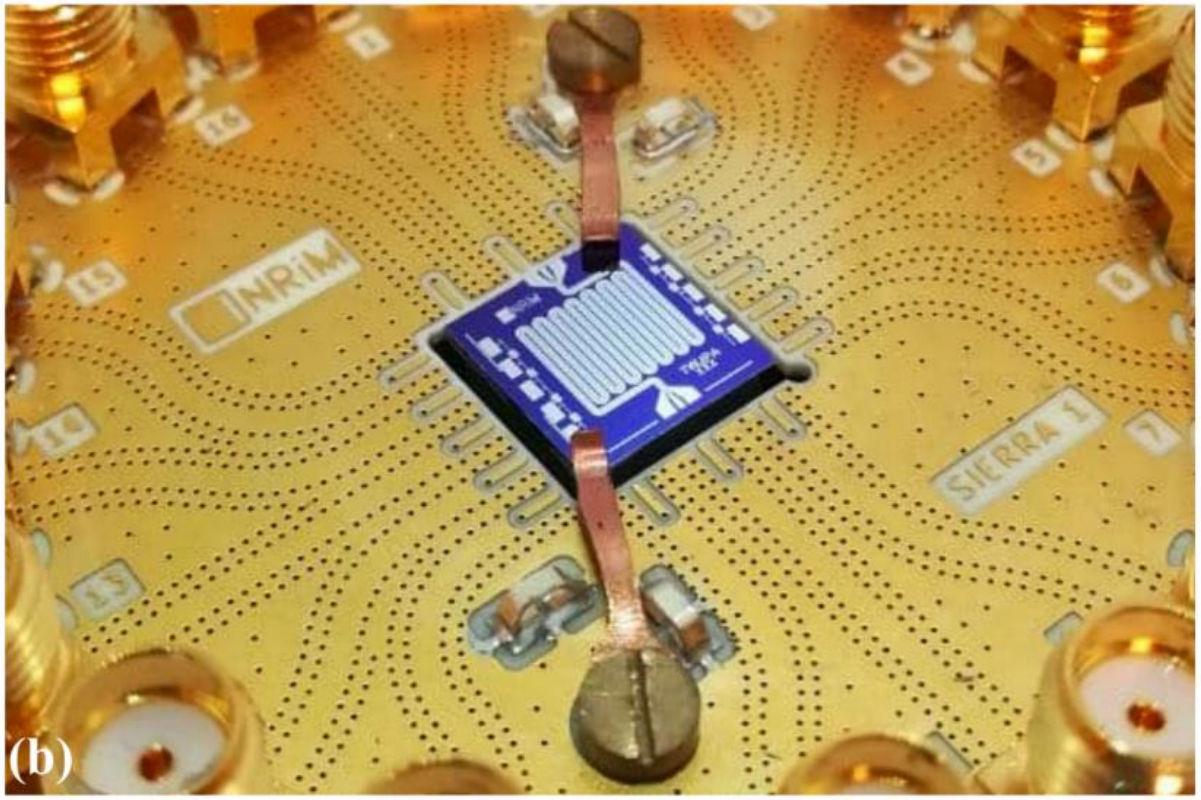


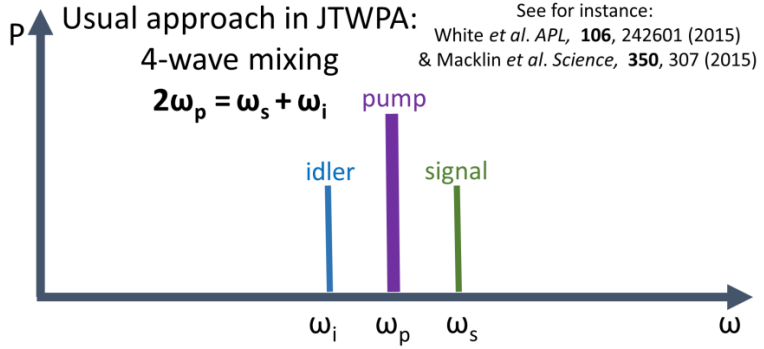
Josephson Travelling Wave Parametric Amplifiers



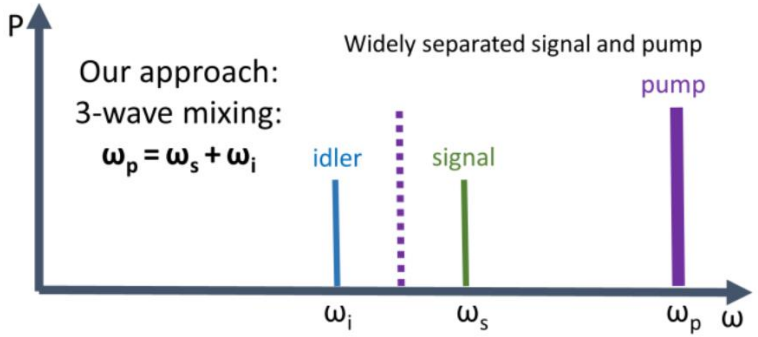


JTWPA - Physics





4-wave mixing



3-wave mixing

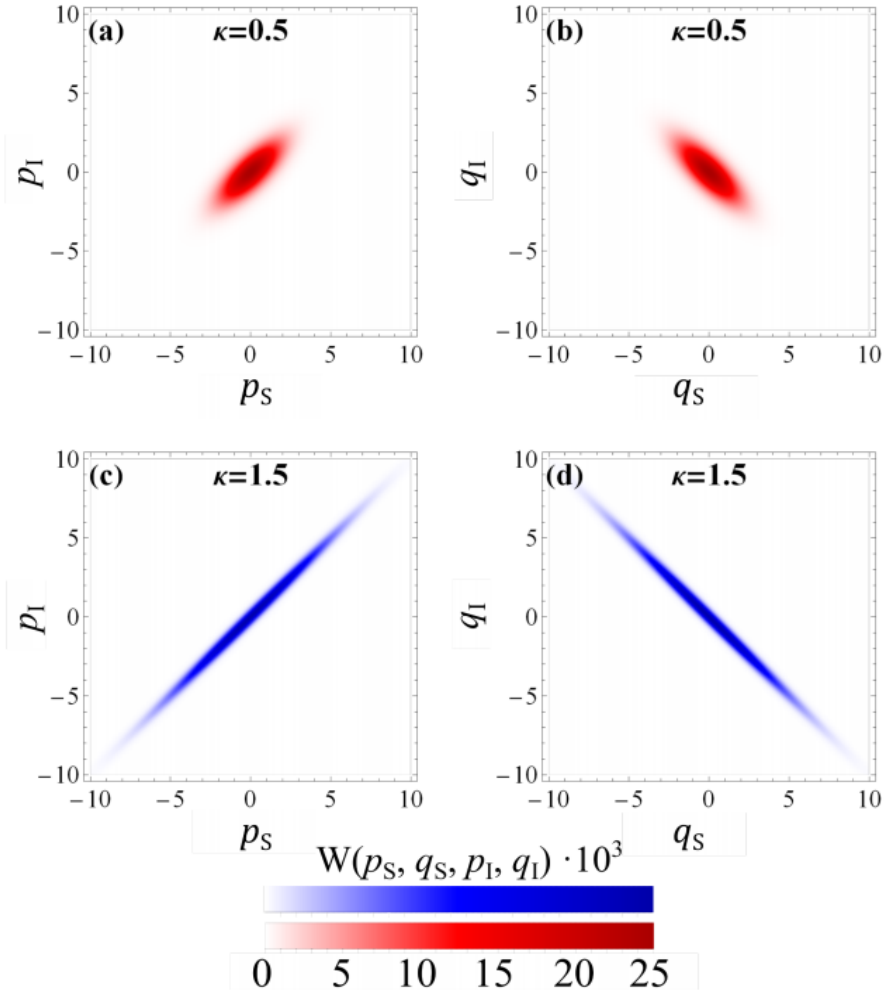
$$|\Psi_{SI}^{TMSV}> = \sum_{n=0}^{\infty} \frac{1}{cosh\,(\kappa)} (i\,tanh\,(\kappa))^n |n>_S |n>_I$$

A. Greco, L. Fasolo, A. Meda, L. Callegaro, and E. Enrico, *A quantum model for rf-SQUIDs based metamaterials enabling 3WM and 4WM Travelling Wave Parametric Amplification*. Phys. Rev. A (in press)
Preprint at arXiv:2009.01002 [cond-mat.supr-con].

JTWPA - Squeezing

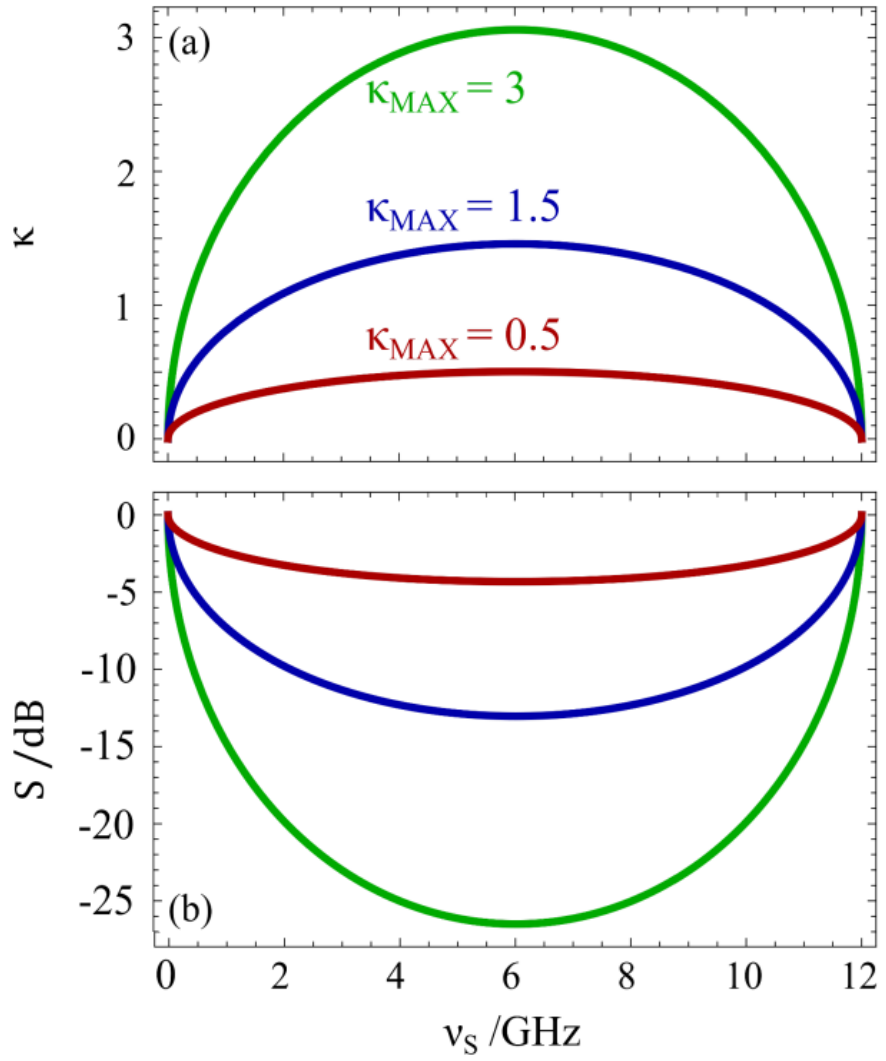
Quadratures of the *j*-th tone

$$q_j = a_j + a^{\dagger}_j, \qquad p_j = i\,(a^{\dagger}_j - a_j)$$



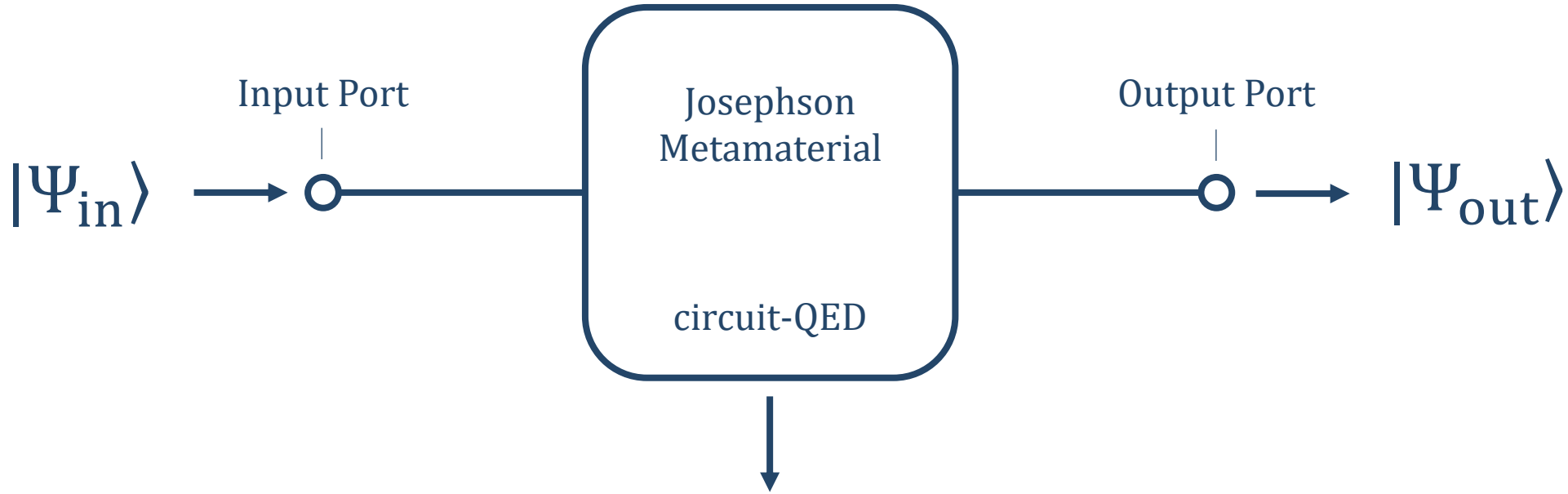
L. Fasolo, A. Greco, E. Enrico, F. Illuminati, R. Lo Franco, D. Vitali, P. Livreri, "Josephson Travelling Wave Parametric Amplifiers as Non-Classical Light Source for Microwave Quantum Illumination". Measurement:Sensors, 18, 100349 (2021).

JTWPA - Squeezing vs. Bandwidth



L. Fasolo, A. Greco, E. Enrico, F. Illuminati, R. Lo Franco, D. Vitali, P. Livreri, "Josephson Travelling Wave Parametric Amplifiers as Non-Classical Light Source for Microwave Quantum Illumination". Measurement:Sensors, 18, 100349 (2021).

Outline



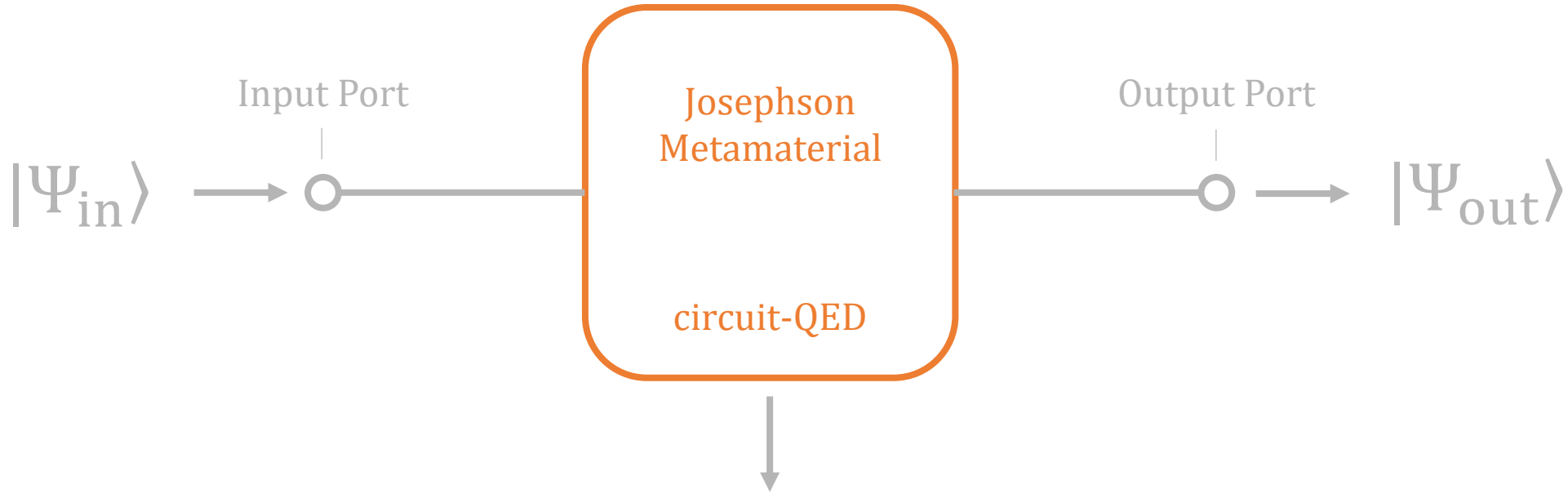
Classic observables:

- Gain
- Noise Figure
- Noise Temperature

Quantum behavior:

- Occupancy probability distribution

Outline



Classic observables:

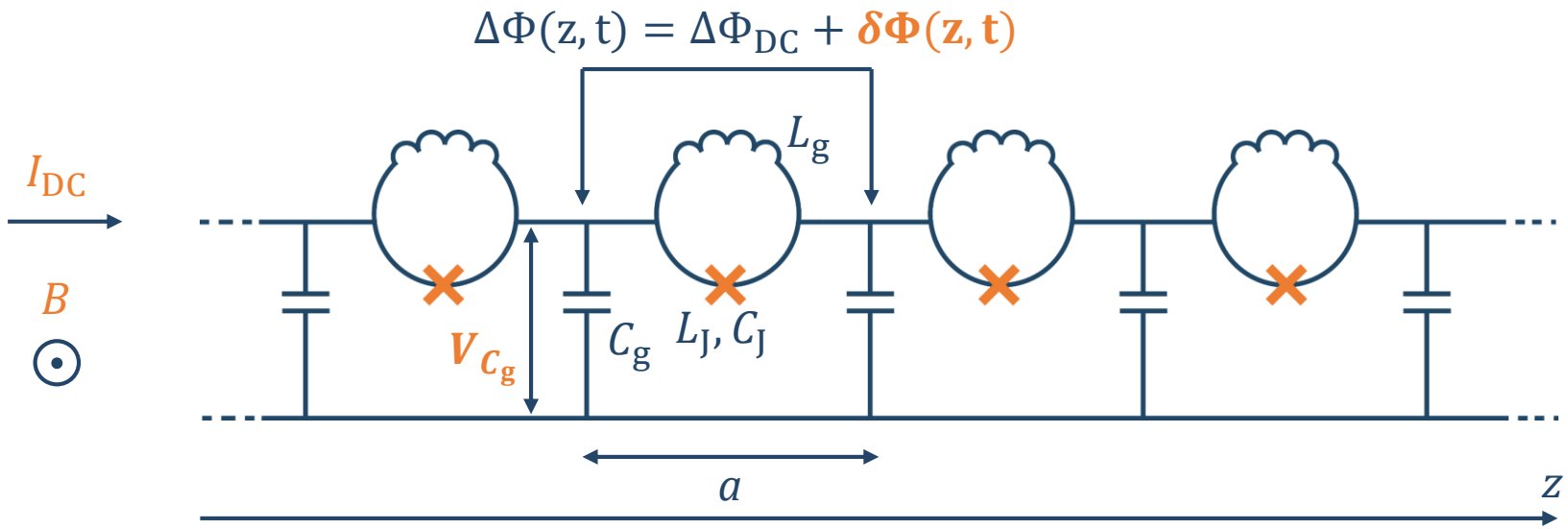
- Gain
- Noise Figure
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Quantum behavior:

- Occupancy probability distribution

First Quantization Hamiltonian

We adapt and further extend the work presented in [1] to an **rf-SQUID based Josephson Traveling Wave Parametric Amplifier** [2]



$\Delta\Phi_{DC}$ is the **constant** flux difference due to external bias

$\delta\Phi(z,t)$ is the **time-dependent** flux difference induced by the travelling waves

The **first quantization Hamiltonian** can be written as the sum of the electromagnetic energy stored in each component of the transmission line

$$H = \frac{1}{a}\int_0^{aN}\left[\frac{1}{2\boldsymbol{L_g}}\boldsymbol{\Delta\Phi(z,t)^2} + \varphi_0\boldsymbol{I_c}\left(1-\cos\left(\frac{\boldsymbol{\Delta\Phi(z,t)}}{\varphi_0}\right)\right) + \frac{\boldsymbol{C_J}}{2}\left(\frac{\partial\boldsymbol{\Delta\Phi(z,t)}}{\partial t}\right)^2 + \frac{\boldsymbol{C_g}}{2}\boldsymbol{V_{C_g}^2(z,t)}\right]dz$$

[1] T. H. A. van der Reep, "Mesoscopic Hamiltonian for Josephson traveling-wave parametric amplifier", Phys. Rev. A **99**, 063838 (2019)
[2] A. B. Zorin, "Josephson Traveling-Wave Parametric Amplifier with Three-Wave Mixing", Phys. Rev. App. **6**, 034006 (2016)

Second Quantization Hamiltonian

$$H(\delta\Phi, V_{C_g}) \xrightarrow[V_{C_g} = V_{C_g}(\hat{a}_n, \hat{a}_n^\dagger)]{\delta\Phi = \delta\Phi(\hat{a}_n, \hat{a}_n^\dagger)} \hat{H}(\hat{a}_n, \hat{a}_n^\dagger)$$

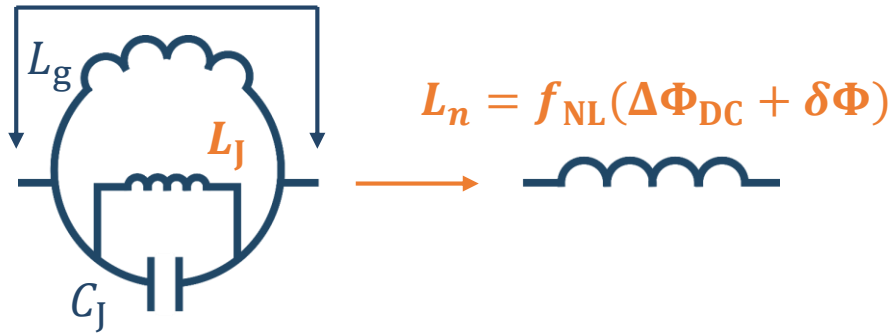
First Quantization — Second Quantization

Assuming that sinusoidal waves are propagating through the line, the voltage drop across Cg can be expressed as a **decomposition of n modes** (standard approach in c-QED [3]):

$$\hat{V}_{C_g}(z,t) = \sum_n \sqrt{\frac{\hbar\omega_n}{2C_g N}} \left(\hat{a}_n e^{i(k_n z - \omega_n t)} + \text{H. c.}\right)$$

While, to determine an expression for the flux difference one can pass through the definition of the current flowing through the line exploiting the **Telegrapher's Equation** ($\partial V_n/\partial z = -(L_n/a)\,\partial I_n/\partial t$):

$$\Delta\Phi = \Delta\Phi_{DC} + \delta\Phi$$



$$L_n = f_{NL}(\Delta\Phi_{DC} + \delta\Phi)$$

$$\hat{I}(z,t) = \sum_n \text{sgn}(n) \sqrt{\frac{\hbar\omega_n}{2L_n N}} \left(\hat{a}_n e^{i(k_n z - \omega_n t)} + \text{H. c.}\right)$$

[3] U. Vool and M. Devoret, "Introduction to quantum electromagnetic circuits", *International Journal of Circuit Theory and Applications* **45**, 897 (2017).

Second Quantization Hamiltonian - Linearization

Eventually, having a definition of the transmission line inductance ad of the flowing current, the **magnetic flux variation** induced by the traveling waves can be derived:

$$\delta\hat{\Phi}(z,t) = \sum_n L_n\, I_n(z,t) = \sum_n f_{\mathrm{NL}}(\Delta\Phi_{DC} + \delta\hat{\Phi}) \cdot I_n\big(f_{\mathrm{NL}}(\Delta\Phi_{DC} + \delta\hat{\Phi})\big)$$

$\delta\Phi(z,t)$ is defined by a **non-linear implicit relation**. To find an **explicit** expression for the $\delta\hat{\Phi}(z,t)$:

- First order approximation solution imposing $\boldsymbol{\delta\Phi = \delta\Phi^{(0)}}$, where $\boldsymbol{\delta\Phi^{(0)}}$ is the variation in phase due to a **single multimodal interaction** in the medium;

- Imposing a **Taylor expansion of f_{NL}** with respect $\delta\Phi^{(0)}$

$$\delta\hat{\Phi} = \sum_{n=0}^{\infty}\left[q_{0,n} + q_{1,n}\,\delta\Phi^{(0)} + q_{2,n}\left(\delta\Phi^{(0)}\right)^2 + q_{3,n}\left(\delta\Phi^{(0)}\right)^3 + O\left(\delta\Phi^{(0)}\right)^4\right]$$

3 modes interactions — 4 modes interactions

Parametric Down-Conversion

The Hamiltonian describes all the **energy preserving interactions** between 3 or 4 traveling waves (i.e., the **parametric down conversion**, the **sum frequency generation**, the **high order harmonics generation**, etc..)

$$H = \hbar\chi_0 + \sum_n \hbar\chi_1^{(n)}\left(\hat{a}_n^\dagger \hat{a}_n + \frac{1}{2}\right) + \sum_{n,l,m} \hbar\,\chi_3^{(n,l,m)}\{\hat{a} + \hat{a}^\dagger\}_{n,l,m} \cdot \delta_{\Delta_{n,l,m},0} + \sum_{n,l,m,s} \hbar\,\chi_4^{(n,l,m,s)}\{\hat{a} + \hat{a}^\dagger\}_{n,l,m,s} \cdot \delta_{\Delta_{n,l,m,s},0}$$

Energy from **external bias**

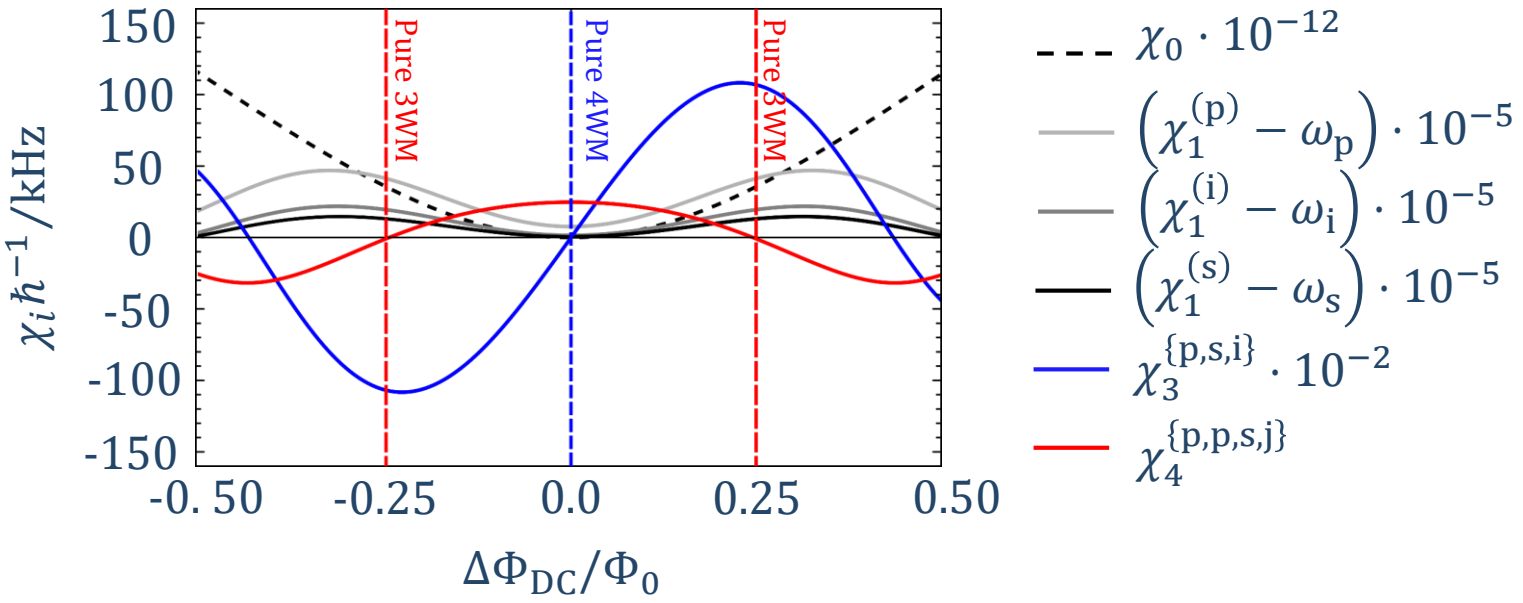
Energy of the **propagating waves**

All 3-wave mixing energy-preserving processes

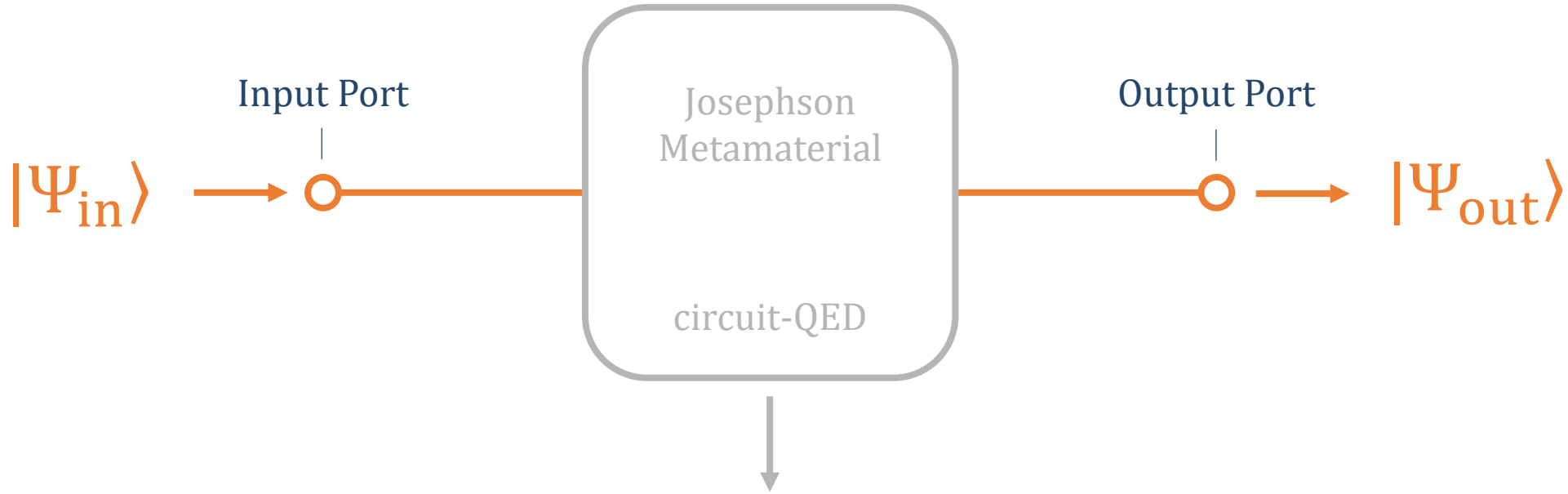
All 4-wave mixing energy-preserving processes

For **parametric down-conversion**

- **3WM** $n, l, m = \{\mathrm{p}, \mathrm{s}, \mathrm{i}\}$ with $\omega_\mathrm{p} = \omega_\mathrm{s} + \omega_\mathrm{i}$
- **4WM** $n, l, m, s = \{\mathrm{p}, \mathrm{s}, \mathrm{j}\}$ with $2\omega_\mathrm{p} = \omega_\mathrm{s} + \omega_\mathrm{j}$



Outline



Classic observables:

- Gain
- Noise Figure
- Noise Temperature

Quantum behavior:

- Probability distribution

Coupled Mode Equations from Heisenberg Equation

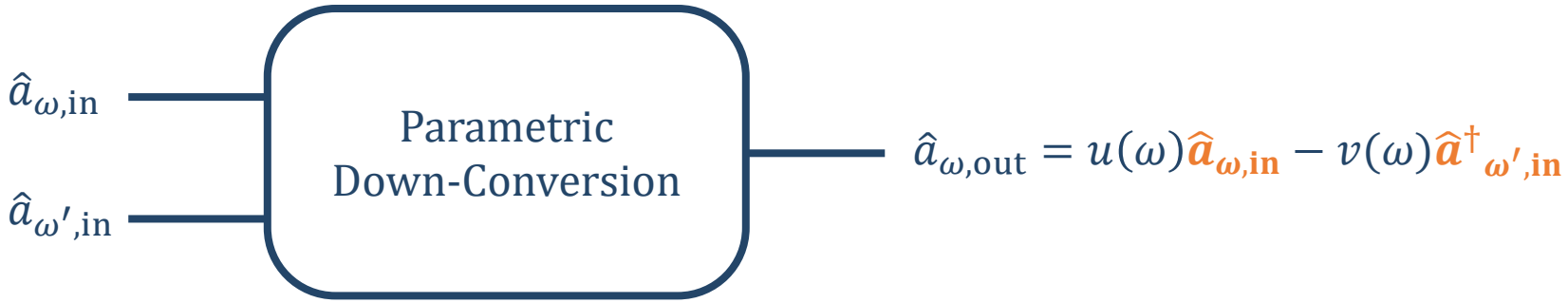
Selecting a proper bias condition, the amplifier can work as a pure 3-Wave Mixer or 4-Wave Mixer (H_{3WM}, H_{4WM}).
In this condition the **evolution of propagating modes** can be derived solving the **Heisenberg equation**:

$$\frac{d\hat{a}_n}{dt} = \frac{i}{\hbar}\left[H_{3WM(4WM)}, \hat{a}_n\right] + \frac{\partial \hat{a}_n}{\partial t} \qquad \text{where } n = \{p, s, i, j\}$$

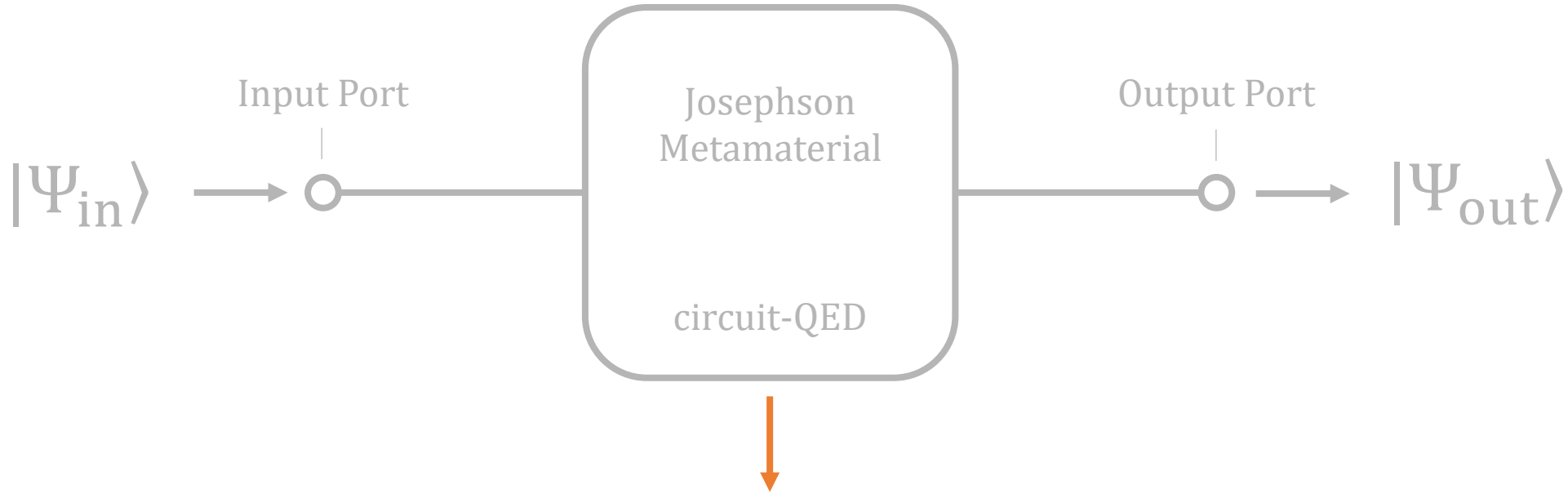
Under the **undepleted and classical pump** approximation, the output field at the **signal frequency ω** is:

$$\hat{a}_\omega(t) = \left[\left(\cosh(gt) + \frac{i\Psi}{2g}\sinh(gt)\right)\hat{a}_{\omega,in} - \left(\frac{i\Upsilon}{g}\sinh(gt)\right)\hat{a}^\dagger_{\omega',in}\right] e^{-i\left(\frac{\Psi}{2}\right)t}$$

where ***g*** is the ***complex gain factor***, **Ψ** is the ***density phase mismatch*** and **ϒ** is the ***interaction parameter***.



Outline



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Quantum behavior:

- Occupancy probability distribution

Expected value of signal photons at the output port

$$\langle \hat{n}_{\omega,out} \rangle = \langle \hat{a}^{\dagger}{}_{\omega,\,\mathrm{out}}\, \hat{a}_{\omega,\mathrm{out}} \rangle =$$
$$= |u(\omega)|^2 \langle \hat{n}_{\omega,\,\mathrm{in}} \rangle + |v(\omega)|^2 \langle \hat{n}_{\omega',\mathrm{in}} \rangle + i \left(u^*(\omega) v(\omega) \left\langle \hat{a}^{\dagger}_{\omega,\mathrm{in}}\, \hat{a}^{\dagger}_{\omega',\mathrm{in}} \right\rangle - u(\omega) v^*(\omega) \langle \hat{a}_{\omega',\mathrm{in}}\, \hat{a}_{\omega,\mathrm{in}} \rangle \right) + |v(\omega)|^2$$

Amplification of the input field at ω frequency

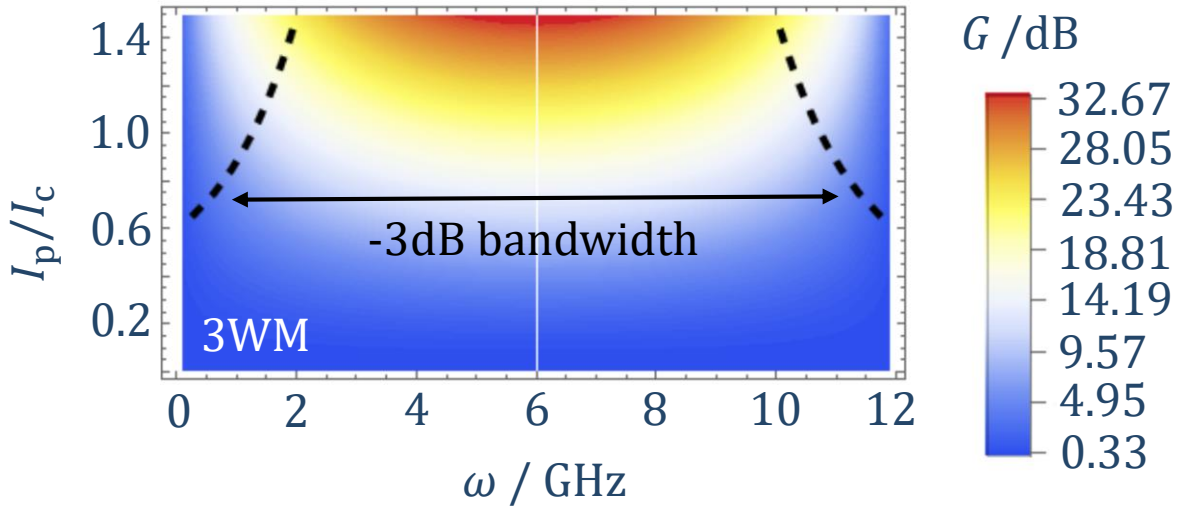
Contribution to the amplification of the **input field at ω** frequency due to the presence of an input signal at **ω' frequency**

Contribution from the **spontaneous annihilation** of a pump photon

Contribution from the **spontaneous creation** of a pump photon

Added noise photons ($\langle \mathrm{vac} | \hat{n}_{\omega,\mathrm{out}} | \mathrm{vac} \rangle$)

$$G(\omega) \equiv \frac{\langle \hat{n}_{\omega,\mathrm{out}} \rangle}{\langle \hat{n}_{\omega,\mathrm{in}} \rangle} = |u(\omega)|^2$$



Noise Figure

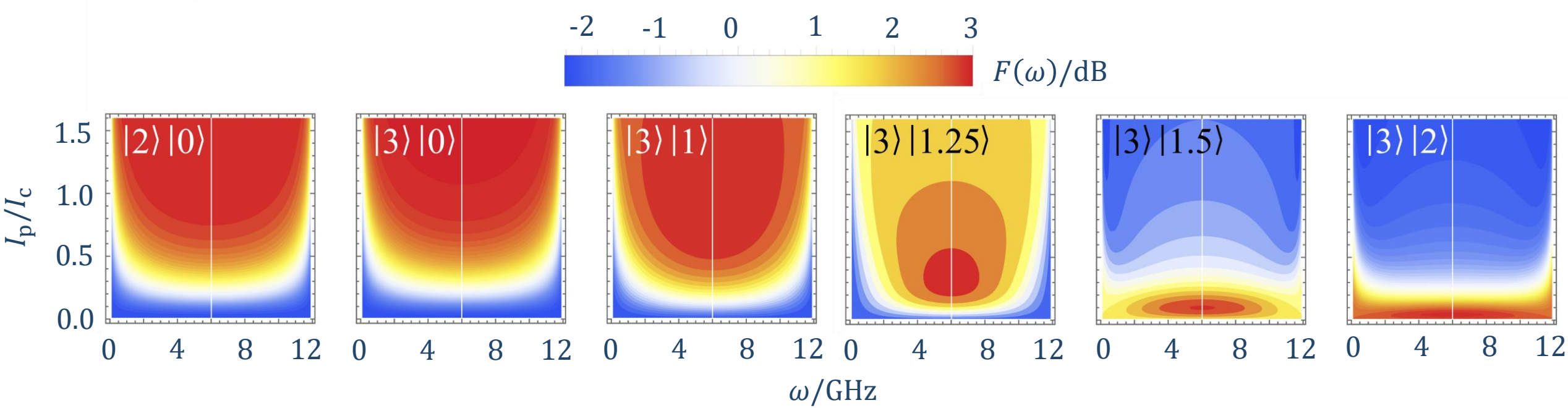
$$F(\omega) \equiv \frac{\mathrm{SNR_{in}}(\omega)}{\mathrm{SNR_{out}}(\omega)}$$

$$\mathrm{SNR_{in}}(\omega) \equiv \frac{\langle \hat{n}_{\omega,\mathrm{in}} \rangle^2}{\langle (\Delta \hat{n}_{\omega,in})^2 \rangle}$$

$$\mathrm{SNR_{out}}(\omega) \equiv \frac{\langle \hat{n}_{\omega,\mathrm{out}} \rangle^2 - \langle \hat{n}^{\mathrm{vac}}_{\omega,\mathrm{out}} \rangle^2}{\langle (\Delta \hat{n}_{\omega,\mathrm{out}})^2 \rangle} \quad [4]$$

↳ Variance of the input/output photon number

Supposing at the input a **Bimodal Coherent state**: $|\Psi_C\rangle = D(\alpha_\omega)|0_\omega\rangle D(\alpha_{\omega'})|0_{\omega'}\rangle = |\alpha_\omega\rangle|\alpha_{\omega'}\rangle$



An **idler** tone at the input port **changes the Noise figure** $F(\omega)$ of the amplifier

[4] Z. Shi *et al.*, "Quantum noise properties of non-ideal optical amplifiers and attenuators", *J. Opt.* **13** (2011)

Noise Temperature

The **effective temperature $T_{\mathrm{eff}}(\omega)$** of the amplifier is the temperature that a **Bose-Einstein distribution** should have to equal the output ω mode occupancy generated by a vacuum input state [5]:

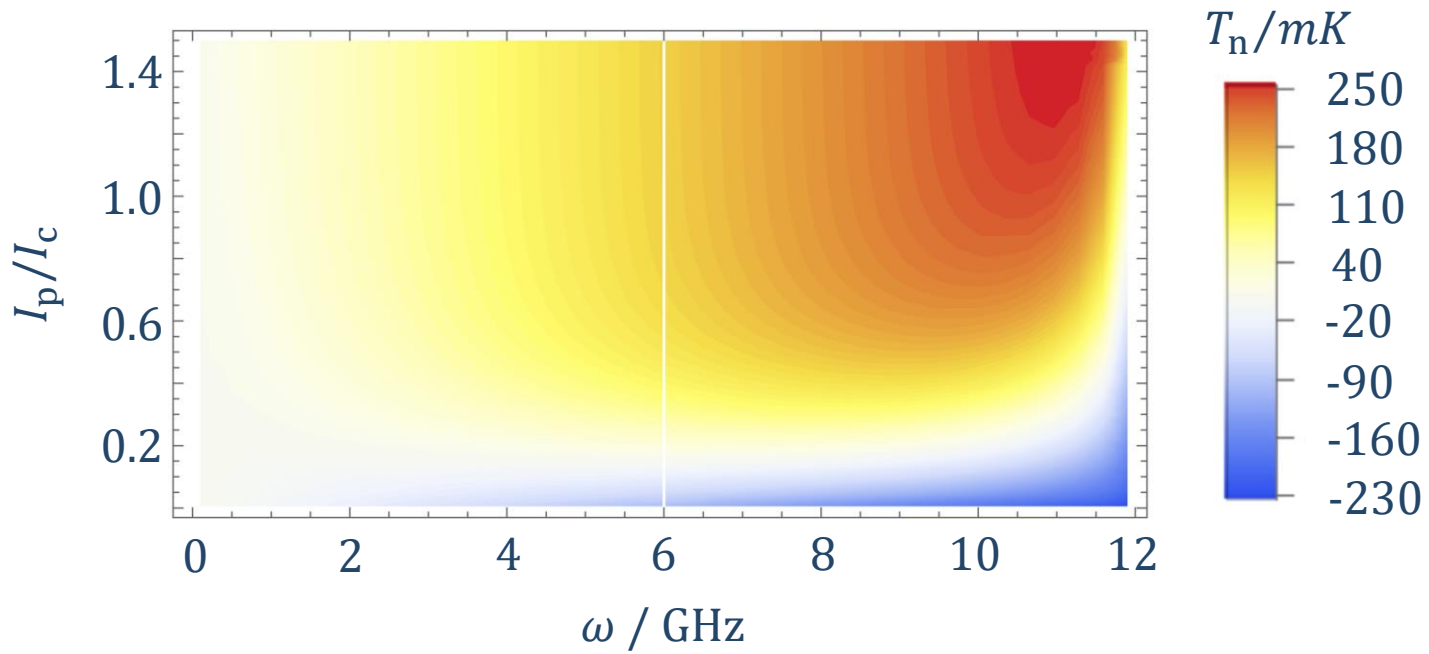
$$\frac{1}{e^{\hbar\omega/k_{\mathrm{B}}T_{\mathrm{eff}}(\omega)}-1} = |v(\omega)|^2$$

The **noise temperature $T_{\mathrm{n}}(\omega)$** is the effective temperature normalized on the gain minus the contribution given by the fluctuation of the input vacuum state:

$$T_{\mathrm{n}}(\omega) = \frac{T_{\mathrm{eff}}(\omega)}{G(\omega)} - \frac{1}{2}\frac{\hbar\omega}{k_{\mathrm{B}}}$$

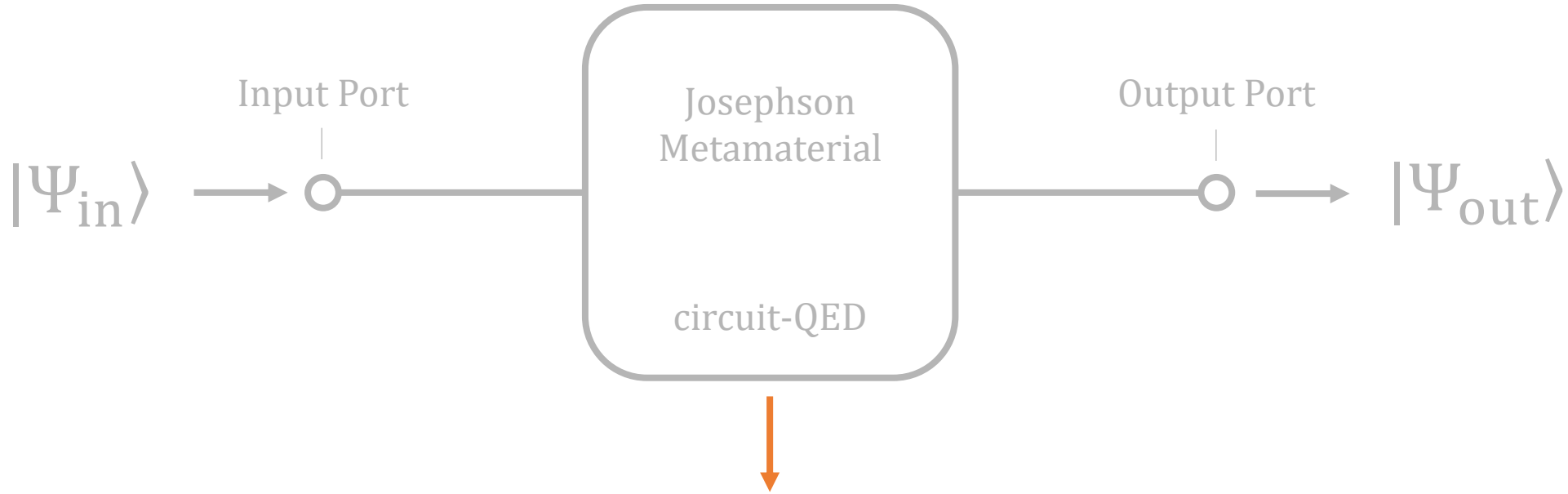
For high gain $T_{\mathrm{n}}(\omega)$ approaches the **Standard Quantum Limit**:

$$T_{\mathrm{n,SQL}} = \frac{1}{2}\frac{\hbar\omega}{k_{\mathrm{B}}}$$



[5] A. A. Clerk *et al.*, "Introduction to quantum noise, measurement, and amplification", *Rev. Mod. Phys.* **82,** 1155 (2010)

Outline



Classic observables:

- Gain
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- Noise Temperature

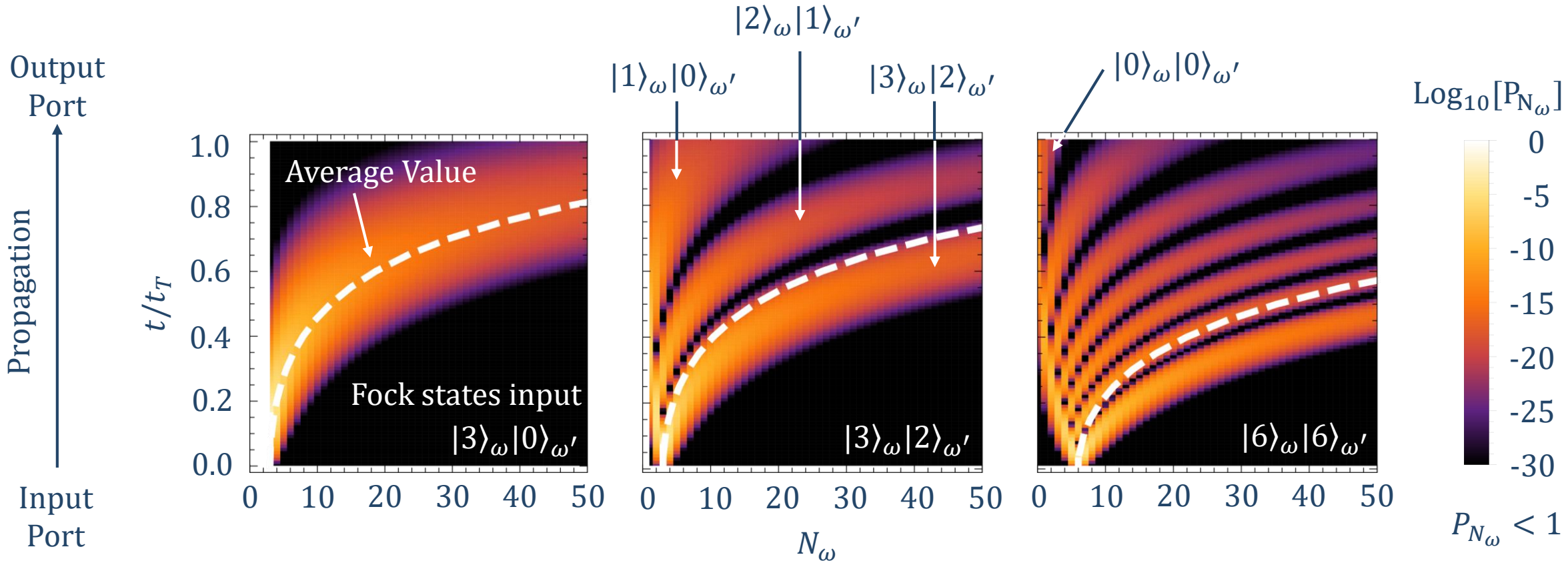
Quantum behavior:

- Occupancy probability distribution

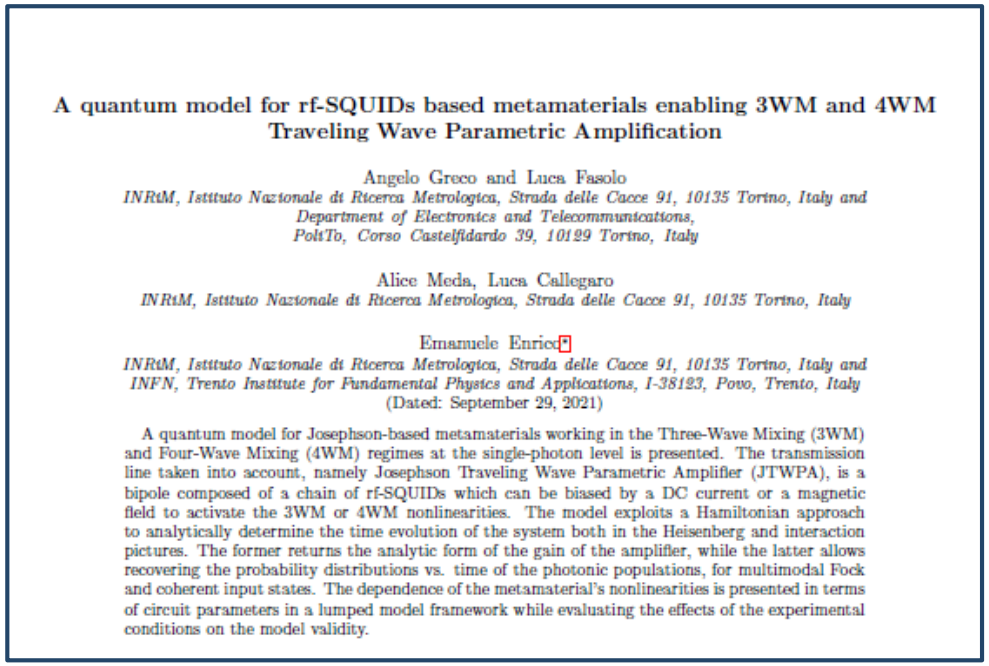
Time-evolution of the occupancy prob. distrib.

The average photon number at frequency ω at a certain time t can be written in terms of the **probability P_{N_ω}** to find N_ω photons at a given time

$$\langle n_\omega(t)\rangle = \sum_{N_\omega} P_{N_\omega}(t) \cdot N_\omega$$



For more details…



A quantum model for rf-SQUIDs based metamaterials enabling 3WM and 4WM Traveling Wave Parametric Amplification

Angelo Greco and Luca Fasolo
INRiM, Istituto Nazionale di Ricerca Metrologica, Strada delle Cacce 91, 10135 Torino, Italy and Department of Electronics and Telecommunications, PoliTo, Corso Castelfidardo 39, 10129 Torino, Italy

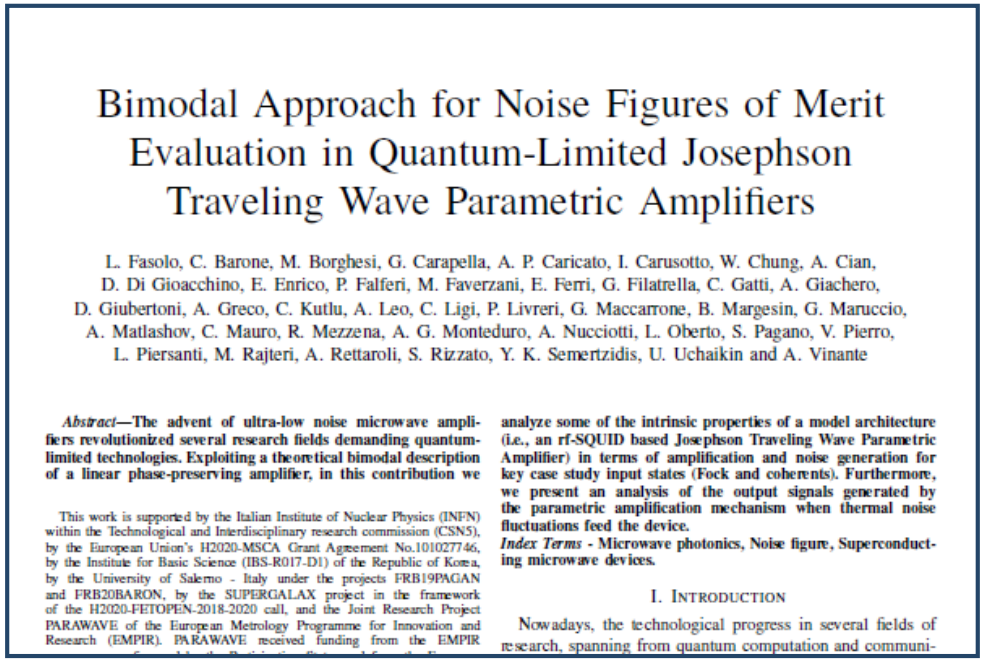
Alice Meda, Luca Callegaro
INRiM, Istituto Nazionale di Ricerca Metrologica, Strada delle Cacce 91, 10135 Torino, Italy

Emanuele Enrico*
INRiM, Istituto Nazionale di Ricerca Metrologica, Strada delle Cacce 91, 10135 Torino, Italy and INFN, Trento Institute for Fundamental Physics and Applications, I-38123, Povo, Trento, Italy
(Dated: September 29, 2021)

A quantum model for Josephson-based metamaterials working in the Three-Wave Mixing (3WM) and Four-Wave Mixing (4WM) regimes at the single-photon level is presented. The transmission line taken into account, namely Josephson Traveling Wave Parametric Amplifier (JTWPA), is a bipole composed of a chain of rf-SQUIDs which can be biased by a DC current or a magnetic field to activate the 3WM or 4WM nonlinearities. The model exploits a Hamiltonian approach to analytically determine the time evolution of the system both in the Heisenberg and interaction pictures. The former returns the analytic form of the gain of the amplifier, while the latter allows recovering the probability distributions vs. time of the photonic populations, for multimodal Fock and coherent input states. The dependence of the metamaterial's nonlinearities is presented in terms of circuit parameters in a lumped model framework while evaluating the effects of the experimental conditions on the model validity.

arXiv:2009.01002v5

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Bimodal Approach for Noise Figures of Merit Evaluation in Quantum-Limited Josephson Traveling Wave Parametric Amplifiers

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***Abstract*—The advent of ultra-low noise microwave amplifiers revolutionized several research fields demanding quantum-limited technologies. Exploiting a theoretical bimodal description of a linear phase-preserving amplifier, in this contribution we analyze some of the intrinsic properties of a model architecture (i.e., an rf-SQUID based Josephson Traveling Wave Parametric Amplifier) in terms of amplification and noise generation for key case study input states (Fock and coherents). Furthermore, we present an analysis of the output signals generated by the parametric amplification mechanism when thermal noise fluctuations feed the device.**

***Index Terms* - Microwave photonics, Noise figure, Superconducting microwave devices.**

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I. Introduction

Nowadays, the technological progress in several fields of research, spanning from quantum computation and communi-

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Conclusions

- JTWPA as an Heralded source for MQI
- Model for quantum (vacuum) states interaction with JTWPA
- TMSV generated by tuning experimental parameters
- MQI protocols and Heralded Microwave sources potentially enhanced by JTWPA



Thanks for your attention

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DARTWARS
Detector Array Readout with Traveling Wave AmplifieRS

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