AC-Josephson effect as a source of entangled microwave photons

Lisa Arndt and Fabian Hassler

JARA Institute for Quantum Information, RWTH Aachen University

Outline

- Introduction
- Setup for Josephson parametric down conversion
- Method
- Results
- Conclusion

Parametric resonance

$$\ddot{\phi}(t) + \Gamma\dot{\phi}(t) + \left[\Omega_0^2 + A\cos(\Omega t)\right]\phi(t) = 0$$

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$\rightarrow$ phase-locking between drive and oscillations

$\rightarrow$ ansatz: $\phi(t) \approx C\cos(\Omega_0 t)$

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Parametric resonance

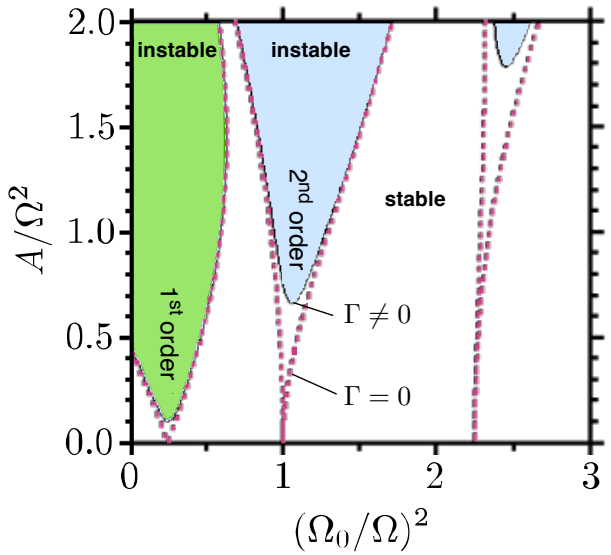
$$\ddot{\phi}(t) + \Gamma\dot{\phi}(t) + \left[\Omega_0^2 + A\cos(\Omega t)\right]\phi(t) = 0 \quad \rightarrow \quad \ddot{\phi}(t) + \Gamma\dot{\phi}(t) + \Omega_0^2\,\phi(t) = -\frac{AC}{2}\cos(\Omega_0 t)$$

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$$C = C_0 \exp\left[\left(\frac{A}{4\Omega_0} - \frac{\Gamma}{2}\right)t\right]$$



http://www.nld.ds.mpg.de

Parametric resonance

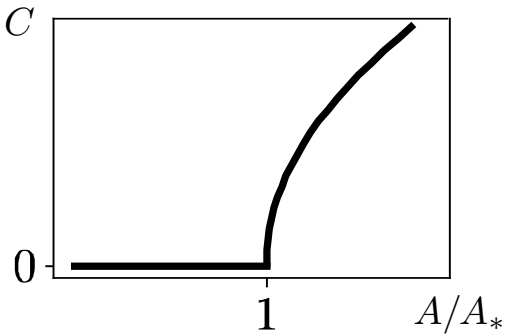
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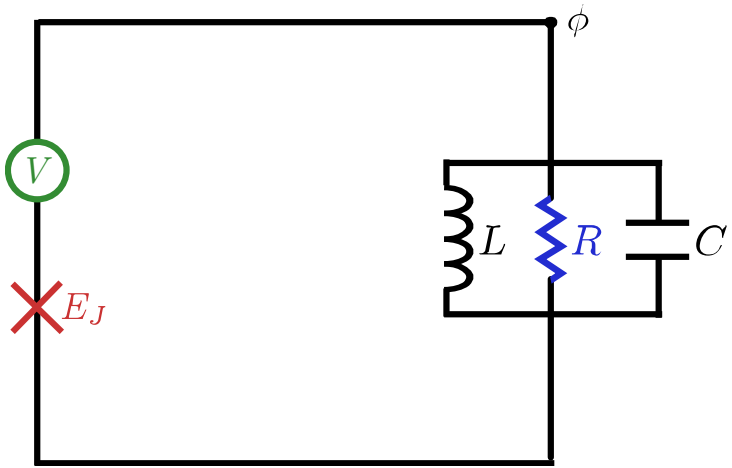
$\rightarrow$ instability threshold: $A_* = 2\Omega_0\Gamma$

$A > A_*$ self-sustained oscillations

$A < A_*$ no coherent response

Josephson circuit – degenerate

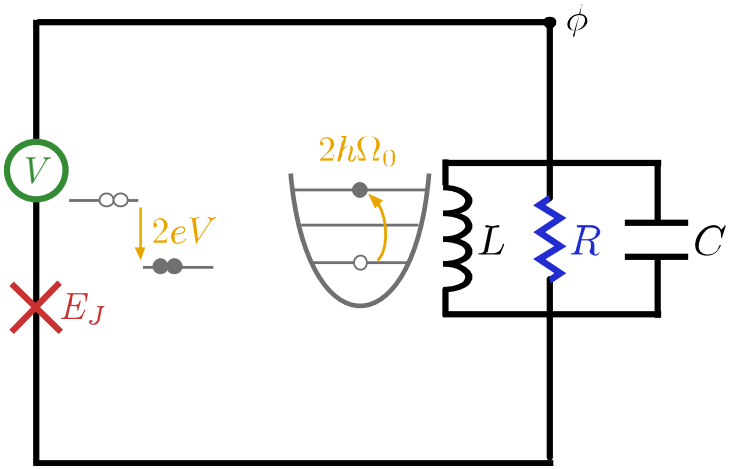
$$\ddot{\phi}(t) + \Gamma\dot{\phi}(t) + \left[\Omega_0^2 + A\cos(\Omega t)\right]\phi(t) = 0$$



$$\Omega_0 = \frac{1}{\sqrt{LC}} \qquad \Omega = \frac{2e}{\hbar}V \qquad \Gamma = \frac{1}{RC} \qquad A = \frac{E_J}{C}\left(\frac{2e}{\hbar}\right)^2$$

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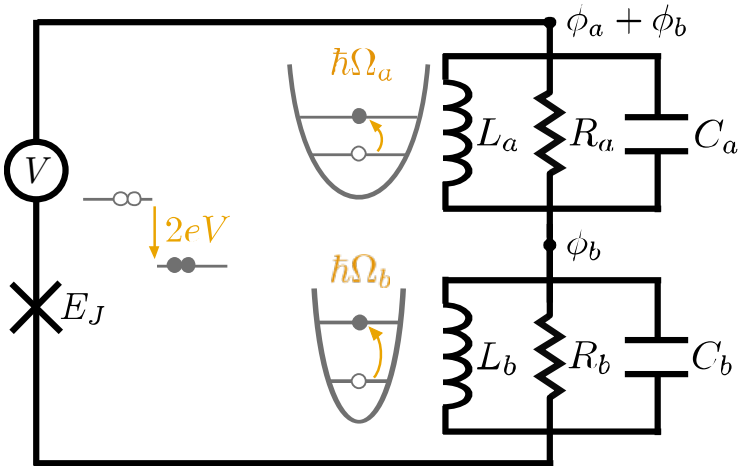


parametric resonance:

$$2eV = 2\hbar\Omega_0$$

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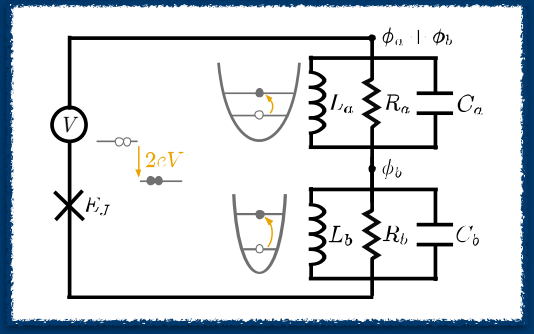
Josephson circuit – non degenerate



parametric resonance:

$$2eV = \hbar\Omega_a + \hbar\Omega_b$$

Setup for Josephson parametric down conversion



Experimental results

PRL **119,** 137001 (2017) PHYSICAL REVIEW LETTERS week ending 29 SEPTEMBER 2017

Emission of Nonclassical Radiation by Inelastic Cooper Pair Tunneling

M. Westig,[1] B. Kubala,[2] O. Parlavecchio,[1] Y. Mukharsky,[1] C. Altimiras,[1] P. Joyez,[1] D. Vion,[1] P. Roche,[1] D. Esteve,[1] M. Hofheinz,[1,*] M. Trif,[3] P. Simon,[3] J. Ankerhold,[2,†] and F. Portier[1,‡]

[1]*SPEC, CEA, CNRS, Université Paris-Saclay, CEA Saclay, 91191 Gif-sur-Yvette, France*
[2]*Institute for Complex Quantum Systems and IQST, University of Ulm, 89069 Ulm, Germany*
[3]*Laboratoire de Physique des Solides, Université Paris-Sud, 91405 Orsay, France*

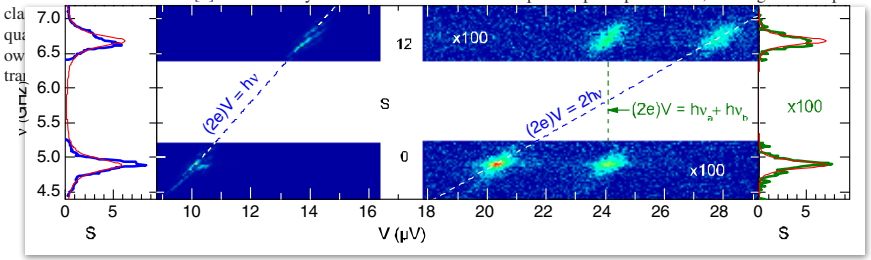
(Received 21 March 2017; published 26 September 2017)

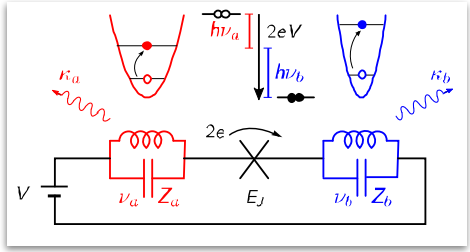
We show that a properly dc-biased Josephson junction in series with two microwave resonators of different frequencies emits photon pairs in the resonators. By measuring auto- and intercorrelations of the power leaking out of the resonators, we demonstrate two-mode amplitude squeezing below the classical limit. This nonclassical microwave light emission is found to be in quantitative agreement with our theoretical predictions, up to an emission rate of 2 billion photon pairs per second.

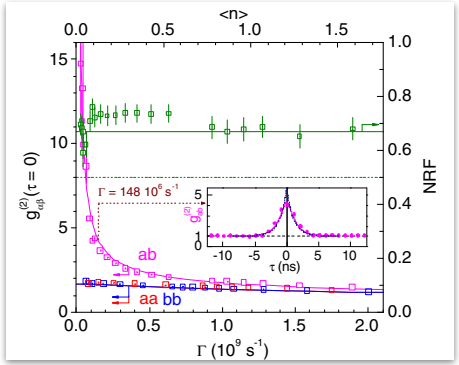
DOI: 10.1103/PhysRevLett.119.137001

Microwave radiation is usually produced by ac driving a conductor like a wire antenna. The radiated field is then a so-called coherent state [1] that closely resembles a cla… qua… ow… tra…

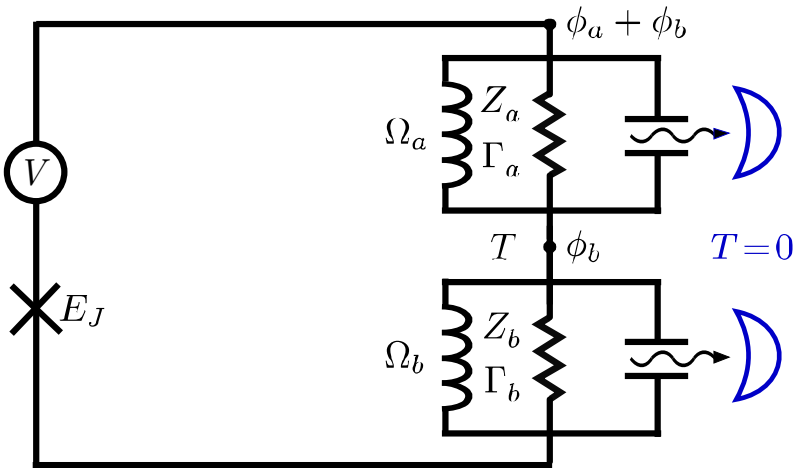
suppressed below the classical limit. This two-mode amplitude squeezing is observed for emission rates as high as 2×10^9 photon pairs per second, making our setup a





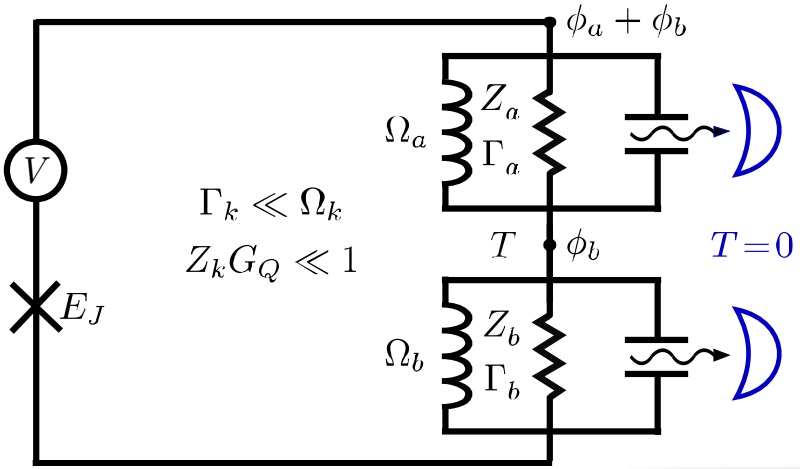


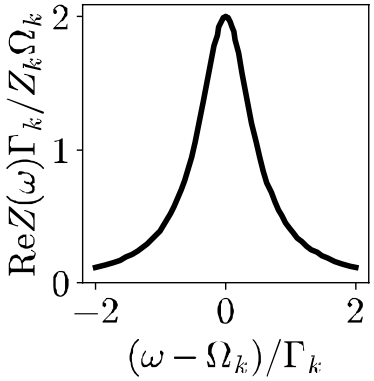
Model Setup



Astafiev et al. (2007), Leppäkangas et al. (2013), Armour et al. (2015), Wustmann et al. (2017), Westig et al. (2017), …

Model Setup





$k \in (a, b)$

parametric resonance:

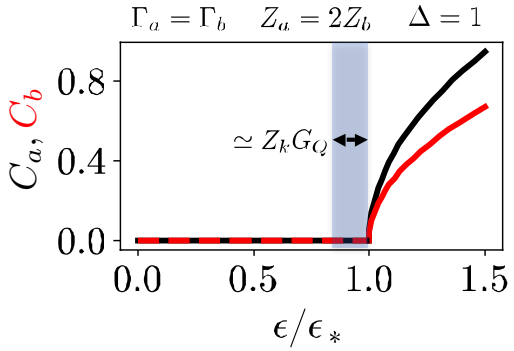
$$2eV = \hbar\Omega_a + \hbar\Omega_b + \hbar\Delta$$

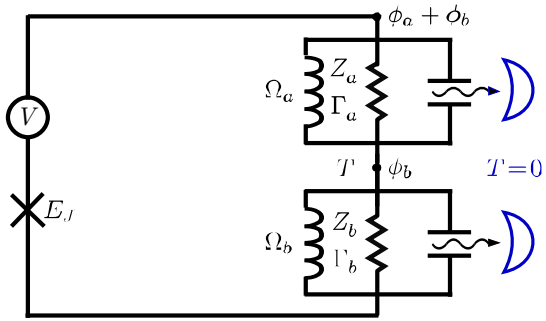
$$\Delta \ll \Omega_k$$

Astafiev et al. (2007), Leppäkangas et al. (2013), Armour et al. (2015), Wustmann et al. (2017), Westig et al. (2017), …

Close to threshold

ansatz: $\phi_k = C_k \cos(\Omega_k t + \xi_k)$

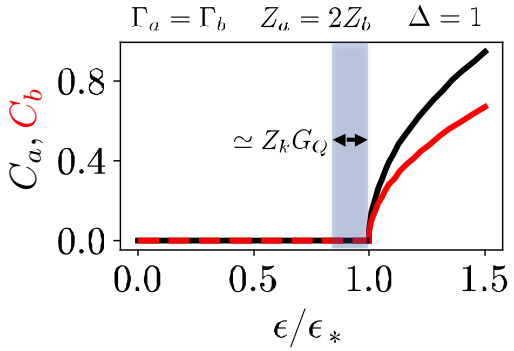




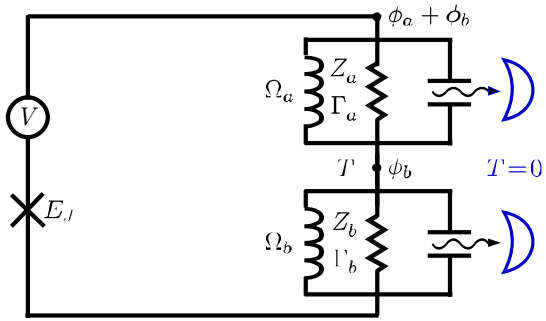
$$\epsilon = \frac{4\pi E_J G_Q \sqrt{Z_a Z_b}}{\sqrt{\Gamma_a \Gamma_b}} \qquad \Gamma = \frac{\Gamma_a + \Gamma_b}{2}$$

Close to threshold

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→ instability threshold: $\epsilon_* = \sqrt{1 + \Delta^2/\Gamma^2}$

→ phase fluctuations: $(\delta\phi_k)^2 \simeq Z_k G_Q \ll 1$

↳ Expand in ϕ_k

→ valid up to interval $|\epsilon_* - \epsilon| \simeq Z_k G_Q \epsilon_* \ll \epsilon_*$

Method: Keldysh path integral

$$\rho(t) = e^{-iHt}\rho(0)e^{iHt}$$

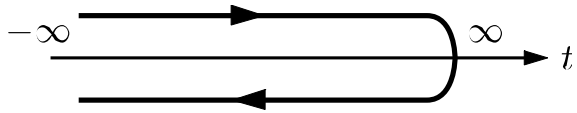
Kamenev, Many-body theory of non-equilibrium systems (2005), Paduraru *et al.* (2012)

Method: Keldysh path integral

$$\rho(t) = \underbrace{e^{-iHt}}_{\text{forward}} \rho(0) \underbrace{e^{iHt}}_{\text{backward}}$$

$$\mathcal{Z} = \int \mathcal{D}[\phi_k]\, e^{iS}$$

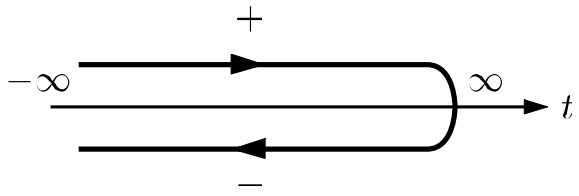
$$S = \int_{\mathcal{C}} dt\, \mathcal{L}[\phi_k(t), \dot{\phi}_k(t), t]$$



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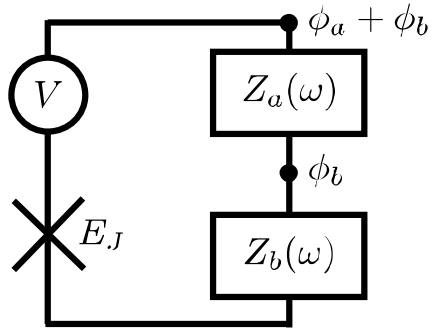
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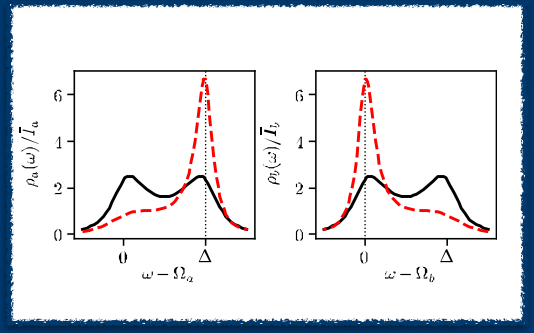


$$S = S_J + S_a + S_b$$

$$S_J = E_J \int dt \Big\{ \cos\big[2eVt + \phi_a^+(t) + \phi_b^+(t)\big] - \cos\big[2eVt + \phi_a^-(t) + \phi_b^-(t)\big] \Big\}$$

Kamenev, Many-body theory of non-equilibrium systems (2005), Padurariu *et al.* (2012)

Results


6
4
2
0
0
Δ
$\omega - \Omega_a$
$\rho_a(\omega)/\bar{I}_a$
6
4
2
0
0
Δ
$\omega - \Omega_b$
$\rho_b(\omega)/\bar{I}_b$

Frequency spectrum

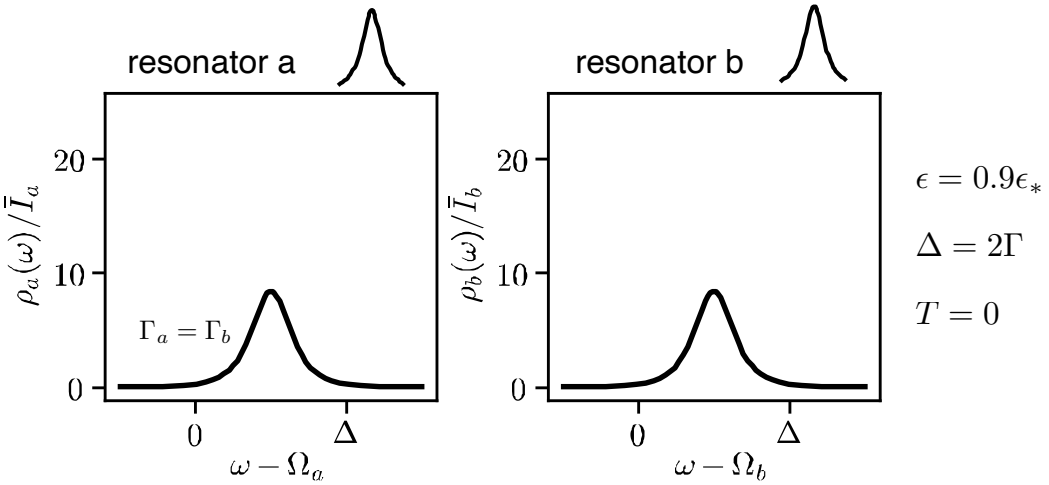
photon current $\bar{I}_k \overset{T=0}{=} \dfrac{\Gamma_a \Gamma_b f_k \epsilon^2}{2\Gamma(\epsilon_*^2 - \epsilon^2)} = \displaystyle\int \frac{d\omega}{2\pi} \rho_k(\omega)$

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$\epsilon \approx \epsilon_*$:

detuning is split between both resonators



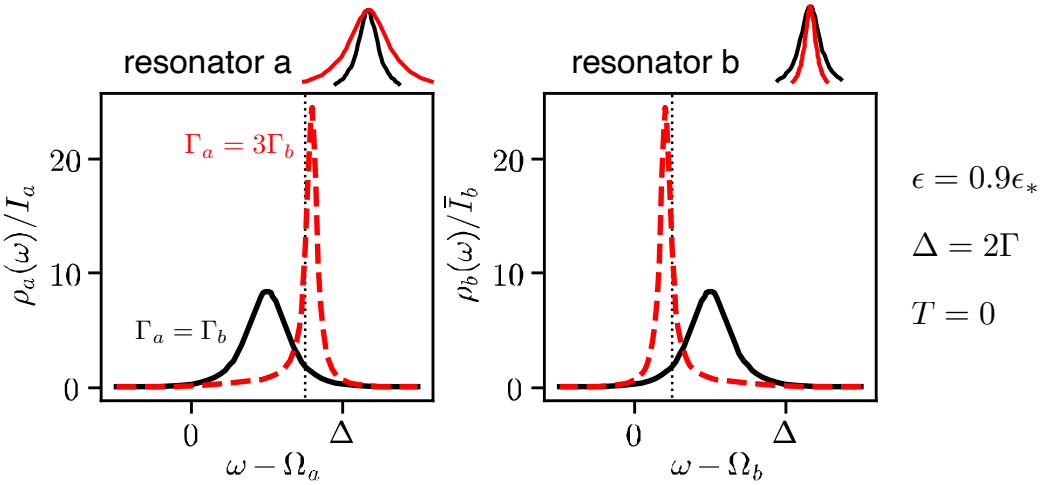
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$\epsilon \approx \epsilon_*$:

detuning is split between both resonators according to linewidth

$$\Omega_k + \frac{\Gamma_k}{\Gamma_a + \Gamma_b}\Delta$$

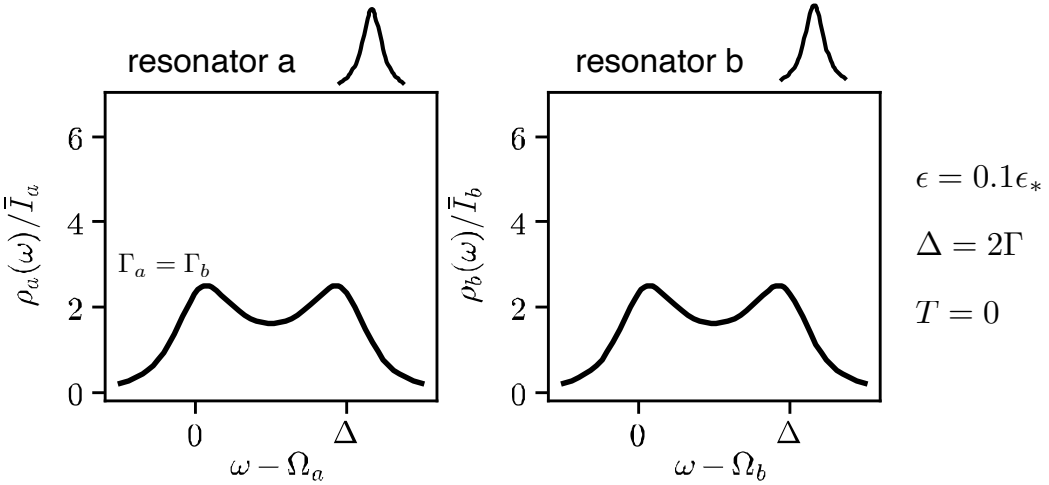


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$\epsilon \ll \epsilon_*$:

one resonator takes total detuning

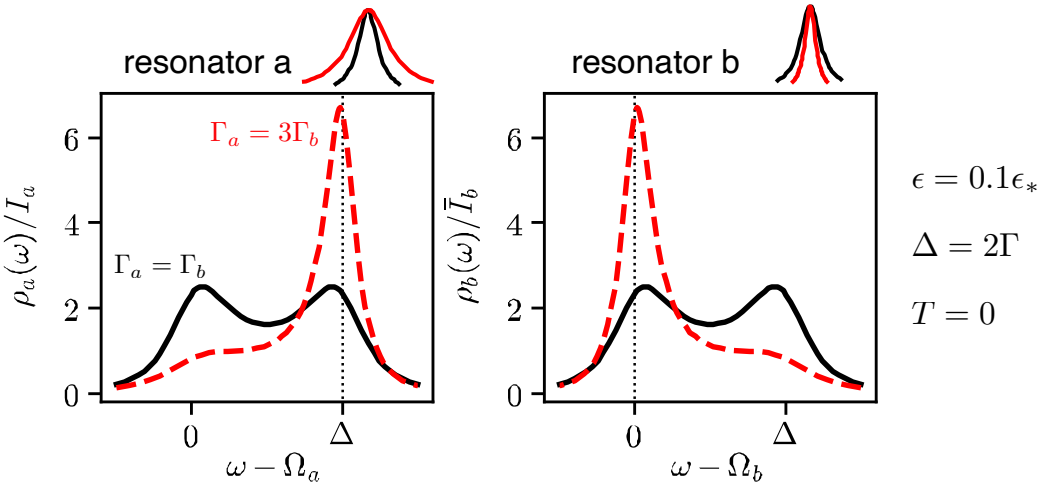


Frequency spectrum

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$\epsilon \ll \epsilon_*$:

resonator with larger linewidth takes total detuning



Second-order coherences

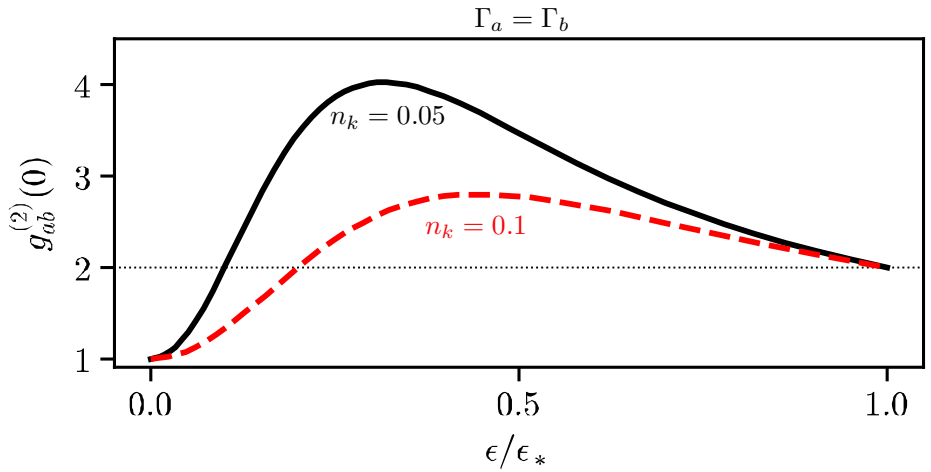
second-order coherences:

$$g_{kl}^{(2)}(\tau) = \frac{\langle : I_l(\tau) I_k(0) : \rangle}{\langle I_k \rangle \langle I_l \rangle}, \quad k, l \in \{a, b\}$$

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→ Cauchy-Schwarz inequality:

$$\sqrt{g^{(2)}_{aa}(0) g^{(2)}_{bb}(0)} \geq g^{(2)}_{ab}(0)$$

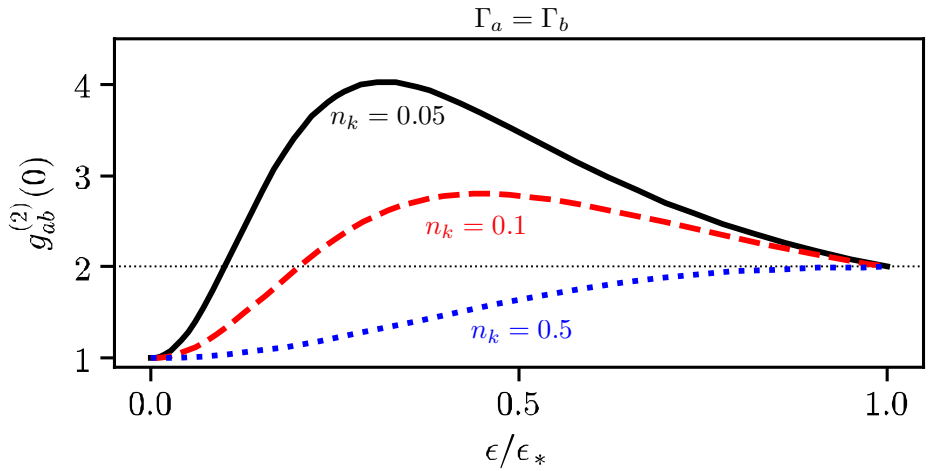
(valid for classical radiation)

$$g^{(2)}_{aa}(0) = g^{(2)}_{bb}(0) = 2$$

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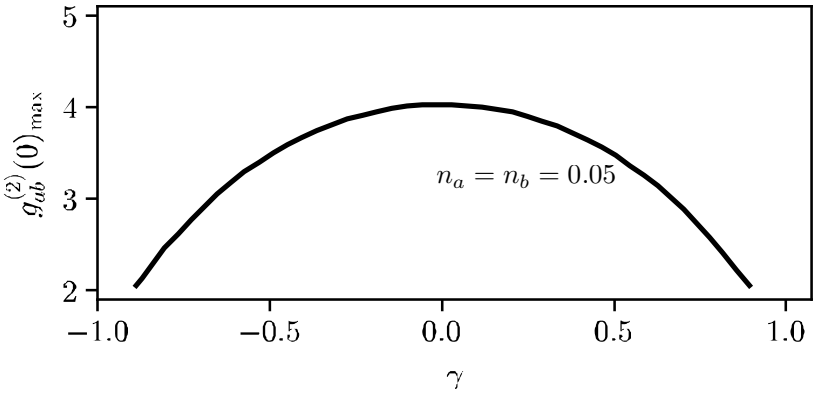
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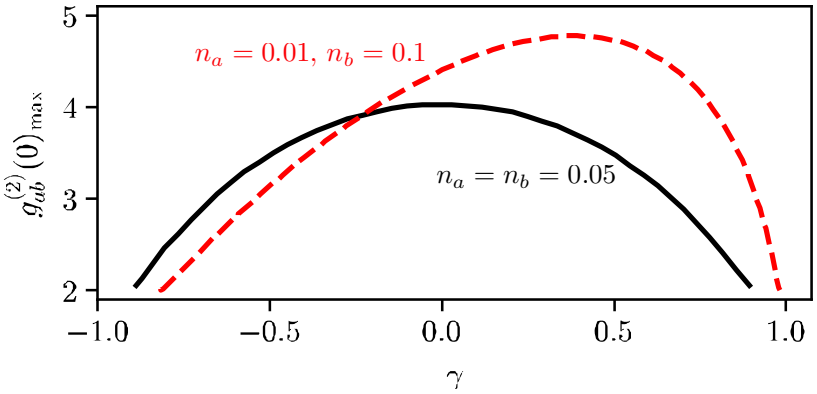
→ Never violated for $n_a\frac{\Gamma_a}{\Gamma_b} + n_b\frac{\Gamma_b}{\Gamma_a} \geq 1$

Asymmetry in linewidth



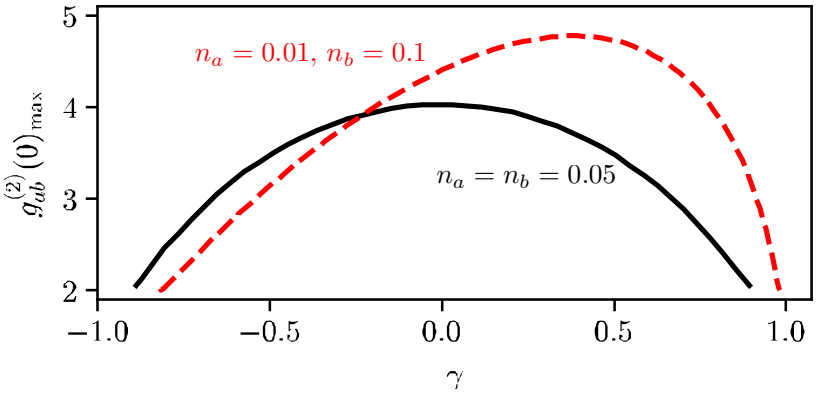
$$\gamma = \frac{\Gamma_a - \Gamma_b}{\Gamma_a + \Gamma_b}$$

Asymmetry in linewidth



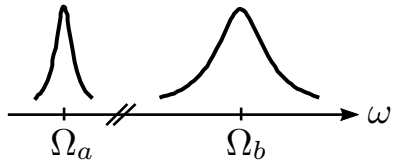
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Asymmetry in linewidth

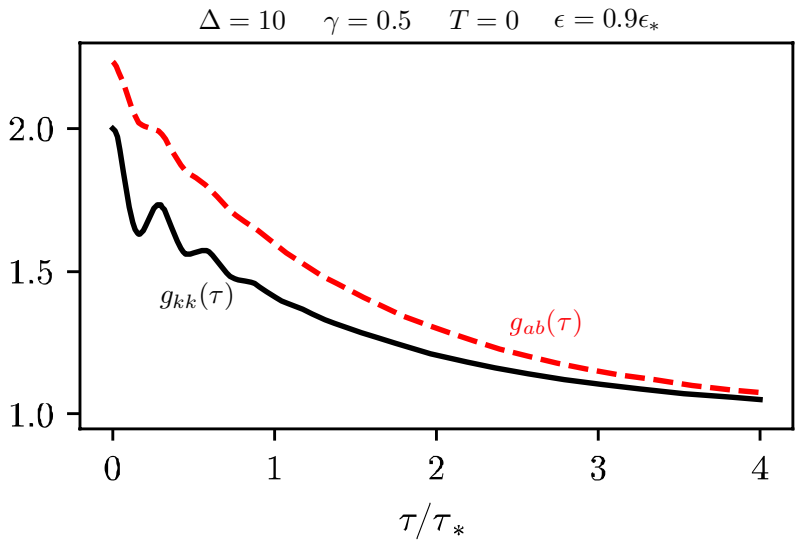


$$\gamma = \frac{\Gamma_a - \Gamma_b}{\Gamma_a + \Gamma_b}$$

To increase cross-coherence the resonator with larger resonance frequency (smaller occupation) should have the larger linewidth

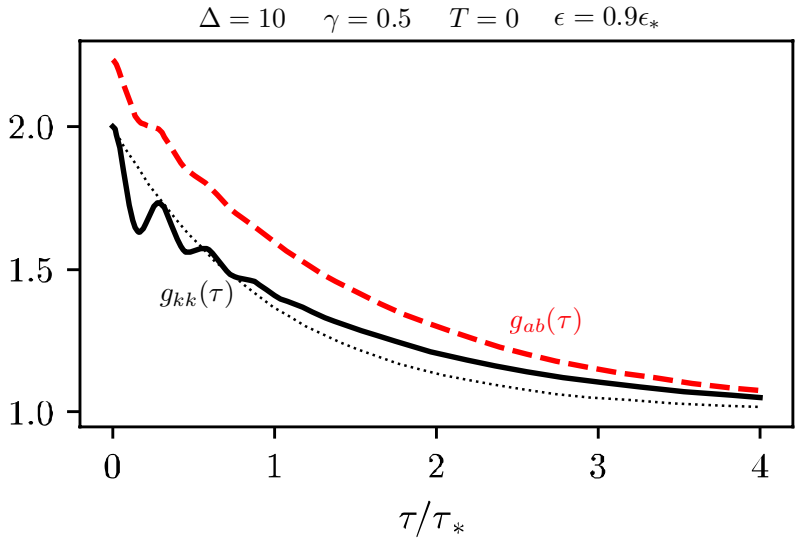


Divergent time-scale at threshold



more info: Arndt and FH, PRA **103**, 023506 (2021)

Divergent time-scale at threshold



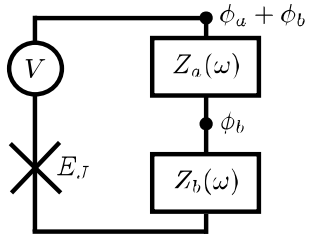
$\rightarrow$ close to threshold: $g_{kl}(\tau) = 1 + e^{-|\tau|/\tau_*}$

$$\tau_* = \frac{1+\gamma^2\Delta^2}{\Gamma\epsilon_*(\epsilon_*-\epsilon)(1-\gamma^2)}$$

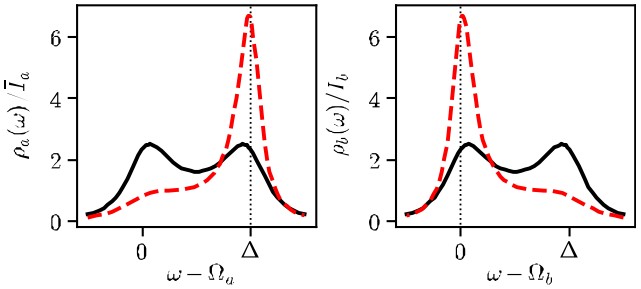
more info: Arndt and FH, PRA **103**, 023506 (2021)

Conclusion

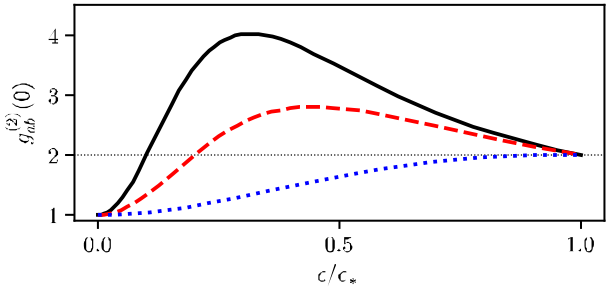
Keldysh path integral formalism to derive full counting statistics of non-degenerate oscillator



Asymmetry determines how **detuning** is split between both resonators



Cauchy-Schwarz inequality violated at small temperatures and strong drive



More information: Arndt & FH, PRB **100**, 014505 (2019); PRA **103**, 023506 (2021)