

# Quantum ratchet control — Harvesting on Landau-Zener transitions

L. MORALES-MOLINA<sup>1</sup>, S. FLACH<sup>2</sup> and J. B. GONG<sup>1,3</sup>

<sup>1</sup> *Department of Physics and Center for Computational Science and Engineering, National University of Singapore 117542, Republic of Singapore*

<sup>2</sup> *Max-Planck-Institut für Physik Komplexer Systeme - Nöthnitzer Str. 38, 01187 Dresden, Germany, EU*

<sup>3</sup> *NUS Graduate School for Integrative Sciences and Engineering - Singapore 117597, Republic of Singapore*

received 9 April 2008; accepted in final form 30 June 2008  
published online 19 August 2008

PACS 05.45.Mt – Quantum chaos; semiclassical methods  
PACS 05.60.-k – Transport processes  
PACS 37.10.Jk – Atoms in optical lattices

**Abstract** – We control the current of a single-particle quantum ratchet by designing ramping schemes for experimentally accessible control parameters. We harvest on Landau-Zener transitions between Floquet states. Adiabatic and diabatic ramping allow to control the resulting directed transport. We find strong changes of the current in the adiabatic regime. Simple loops in control parameter space with alternating adiabatic and diabatic ramping are proposed. We obtain also current reversal. Full desymmetrization of the quantum ratchet increases the critical ramping speed which separates the adiabatic from the diabatic regime.

Copyright © EPLA, 2008

The ratchet effect concerns the rectification of transport in the presence of zero mean forces. It was proposed to explain, *e.g.*, the molecular motility in biological systems, and has been generalized to various other areas in physics [1,2]. Applications of ratchets range from biological systems [3] to atomic physics [4]. Especially in the case of light-matter interaction and laser-induced symmetry breaking with applications to cold atoms [4], control parameters are experimentally well accessible.

A symmetry analysis of Hamiltonian and dissipative classical single-particle ratchets was performed in refs. [5,6]. A general sum rule to calculate the average velocity of classical ratchet transport was obtained [7]. Specific dynamical mechanisms were also proposed [8]. Experiments on ratchet transport have also been carried out using thermal cold rubidium and cesium atoms in optical lattices [4]. The time-dependent forces are applied to the atoms by phase modulating the laser beams which form the optical lattice [4]. The dissipationless case can be easily approached by using laser beams which generate far detuned standing waves. In the latter case quantum features become important. To address the quantum ratchet dynamics different models have been proposed mostly based on kicked systems [7,9,10].

In two recent studies of the quantum ratchet transport [11,12], it was found that a directed current appears

due to a desymmetrization of the Floquet states of the system. A main quantum feature of the rectification process is the resonant interaction between Floquet states close to avoided crossings, which leads to a strong enhancement of directed transport in a narrow interval of the control parameters. In a further extension to these studies, interaction between atoms has been considered as well [13]. In a different context, non-symmetric tunneling between bands in an asymmetric optical lattice has been reported [14]. The authors show that breaking the spatial symmetry induces changes of the band gap leading consequently to a modification of the tunneling rate between bands.

Recent studies have also shown that it is possible to control the states of the energy spectrum of electrons in semiconductor nanostructures by ramping the external electrical field [15]. The authors exploit the diabatic and adiabatic Landau-Zener transitions in avoided crossings.

We study the effect of Landau-Zener tunneling transitions between resonant Floquet states under non-symmetric ac forces for a quantum ratchet. These transitions are controlled by the ramping speed of a control parameter, which is related to the gap of the avoided crossing of Floquet states. We find that directed transport is significantly enhanced by adiabatically ramping parameters through avoided crossings between resonant Floquet

states, even when finally ramping the systems far away from the resonance. We obtain current reversal, and ways of increasing the critical ramping velocity which separates adiabatic from diabatic ramping. The results constitute an extension of the control approach in [15], from navigating between eigenstates to navigating between Floquet states.

Consider a dilute gas of atoms. We can then, in good approximation, neglect the interaction between particles. The quantum dynamics for a particle moving in a periodic potential under the influence of an ac force is described by the dimensionless Schrödinger equation [11,12]

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi, \quad (1)$$

where  $H$  is the dimensionless one-particle Hamiltonian

$$H = \frac{1}{2} \hat{p}^2 + v_0 \cos(x) - xE(t) \quad (2)$$

with  $\hat{p} = -i\hbar \frac{\partial}{\partial x}$ , and  $v_0 = 1$  being the potential depth.  $\hbar$  is a dimensionless effective Planck constant. The dimensionless ac field satisfies  $E(t+T) = E(t)$  with period  $T$ . As in [11,12], we consider  $E(t) = E_1 \cos[\omega(t-t_0)] + E_2 \cos[2\omega(t-t_0) + \theta]$  with  $t_0$  as initial time. By applying a convenient gauge transformation [11,12,16], we get

$$H = \frac{1}{2} [\hat{p} - A(t)]^2 + \cos(x), \quad (3)$$

where  $A(t) = -E_1 \sin[\omega(t-t_0)]/\omega - E_2 \sin[2\omega(t-t_0) + \theta]/2\omega$  is the vector potential [11,12].

The Hamiltonian in eq. (2) is a periodic function of time. Hence the solutions of eq. (1),  $|\psi(t+t_0)\rangle = U(t_0, t+t_0)|\psi(t_0)\rangle$ , can be characterized by the eigenfunctions of  $U(t_0, T+t_0)$ , which satisfy the relation:

$$|\psi_\beta(t)\rangle = e^{-i\frac{\epsilon_\beta}{T}t} |\phi_\beta(t)\rangle, \quad |\phi_\beta(t+T)\rangle = |\phi_\beta(t)\rangle. \quad (4)$$

The quasienergies  $\epsilon_\beta$  ( $-\pi < \epsilon_\beta < \pi$ ) and the Floquet eigenstates can be obtained as solutions of the eigenvalue problem of the Floquet operator  $U(0, T)|\psi_\beta(0)\rangle = e^{-i\epsilon_\beta} |\psi_\beta(0)\rangle$ .

Due to Bloch's theorem, discrete translational invariance of eq. (3) implies that Floquet states are characterized by a quasimomentum  $\hbar\kappa$  with  $|\psi_\beta(x+L)\rangle = e^{i2\pi\kappa} |\psi_\beta(x)\rangle$ . We choose  $\kappa = 0$  (a treatment excluding studies of evolution of wavepackets) because our interest is in how a nonzero flux of atoms emerge from an initial state with almost zero momentum. The  $\kappa = 0$  initial state can be produced by, for instance, the ground state of an elongated (shallow) trap potential of a condensate. This consideration allows us to use periodic boundary conditions for eq. (1), with spatial period  $L = 2\pi$ . Hence the wave function can be expanded in the plane-wave eigenbasis of the momentum operator  $\hat{p}$ ,  $|n\rangle = \frac{1}{\sqrt{2\pi}} e^{inx}$  [12]. The Floquet operator is obtained by solving eqs. (1)–(3). Computational details are given in [12].

Breaking the time reversal and shift symmetries of the Floquet operator [17] generates a nonzero average

momentum of the Floquet states, and results in a nonzero directed transport [11,12].

The asymptotic current of the system for  $t \gg T$  is obtained through the expression  $J(t_0) = \sum_\beta \langle p \rangle_\beta |C_\beta(t_0)|^2$  [12], where  $\langle p \rangle_\beta$  are the Floquet state momenta and  $C_\beta(t_0)$  are the expansion coefficients of the initial wave function in the basis of Floquet states. The current is a function of the relative phase  $\theta$  and the initial time  $t_0$ , *i.e.*  $J(t_0, \theta)$ . After averaging over the initial time the current becomes a function of  $\theta$  only and fulfills the relation  $J(\theta) = -J(\theta + \pi) = -J(-\theta)$  [11,12].

For a given value of the control parameter  $\theta$ , the initial state  $|n=0\rangle$  will most strongly overlap with *e.g.* a certain Floquet state, yielding a nonzero asymptotic current. Variation of  $\theta$  will lead to avoided crossings between quasienergies, *i.e.* to a resonant interaction between pairs of Floquet states. Avoided crossings are confined to a small interval in  $\theta$ . Inside the avoided crossing region the Floquet states hybridize. If the second (new) Floquet state has a large mean momentum, the asymptotic current will resonantly increase. But upon leaving the avoided crossing region, the Floquet states cease to interact and to hybridize, and the asymptotic current returns to its nonresonant (lower) value. We will therefore ramp the control parameter  $\theta$  in time. Depending on the ramping velocity we expect to either re-observe the results for the case without ramping (diabatic Landau-Zener regime), or to slowly populate the second state (adiabatic regime). In the second case, after leaving the avoided crossing region, the quantum state will reside mainly in the second Floquet state and keep the large asymptotic current.

Hereafter the analysis is done for  $\hbar = 0.2$ . Figure 1 displays an avoided crossing of Floquet states that leads to an enhancement of the current. The resonance takes place between a state  $A$  located in the chaotic layer of the corresponding classical model, and a transporting state  $C$  (see [11,12] for details). We first consider an initial quantum state which exactly equals the Floquet state  $A$  in fig. 1. Then we ramp  $\theta$  linearly in time, *viz*

$$\theta(t) = \Lambda(\theta_1, \theta_2, \alpha, t) \equiv \begin{cases} \theta_1, & t \leq t_{ini}, \\ \theta_1 + \alpha(t - t_{ini}), & t_{ini} < t \leq t_{ini} + (\theta_2 - \theta_1)/\alpha, \\ \theta_2, & t > t_{ini} + (\theta_2 - \theta_1)/\alpha, \end{cases} \quad (5)$$

where  $\alpha$  is the ramping rate and  $t_{ini}$  is the time at which the ramping process starts.  $\theta_1$  and  $\theta_2$  are the initial and final values.

In the adiabatic regime  $\alpha \rightarrow 0$ , state  $A$  transforms into the transporting state  $C$ . The ramping time being finite, we estimate the deviation from the strict adiabatic behavior. We define

$$\psi_f = a(t)\psi_C + b(t)\psi_B \quad (6)$$

as the final state in the basis of states  $C$  and  $B$ . Next, we define [18]

$$\Lambda_{C,B} = \frac{1}{T} \int_0^T dt |\langle \psi_f | \psi_{C,B} \rangle|^2, \quad (7)$$

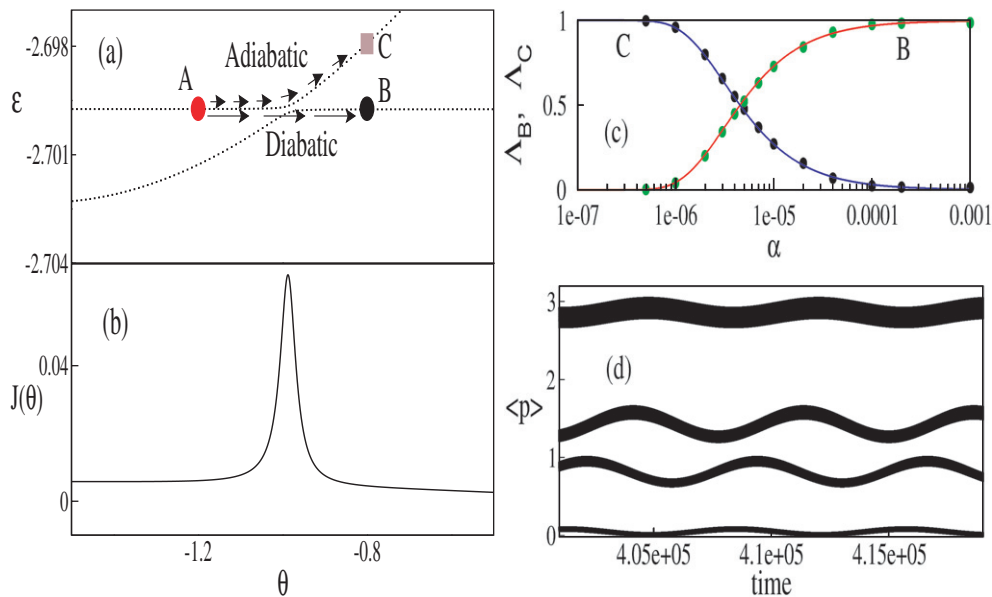


Fig. 1: (a) Avoided crossings between two Floquet states which lead to an enhancement of the current (no ramping). The arrows indicate the direction of the ramping with initial state  $A$ . (b) Peak of the current due to the resonant states in the above avoided crossing. (c) Transition probability  $\Lambda$  vs.  $\alpha$  (logarithmic scale) in the basis of states  $B$  and  $C$  (see text for details). Circles: numerical computation of eq. (7). The solid lines are the fitting of eq. (8) to the circles:  $P$  (red on-line line),  $1 - P$  (blue on-line line). (d) Momentum evolution of the final state. The ramp is performed from  $\theta = -1.2$  to  $\theta = -0.8$ . Momenta from top to bottom correspond to  $\alpha = 10^{-6}$ ,  $5 \times 10^{-6}$ ,  $10^{-5}$ ,  $10^{-3}$ , respectively.  $t_{ini} = 0$ . The parameters are  $E_1 = 3.26$ ,  $E_2 = 1.2$  and  $\omega = 3$ .

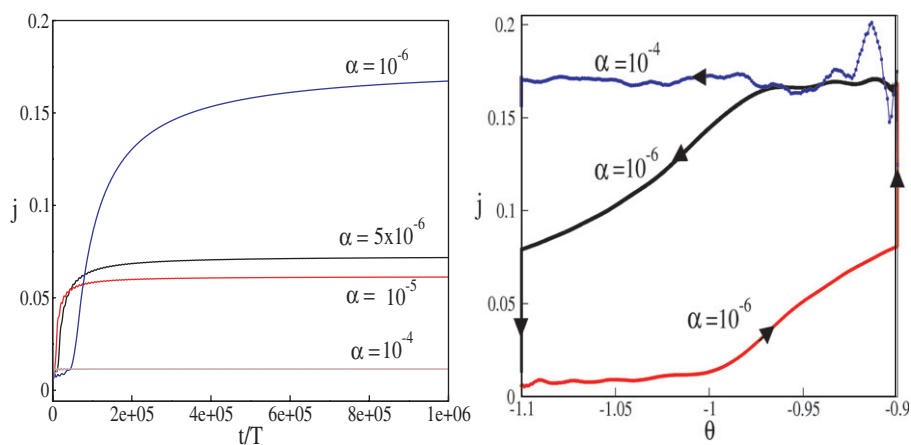


Fig. 2: Left panel: current of the state  $|0\rangle$  as a function of time in units of the period  $T$ . Ramping is performed from  $\theta = -1.1$  till  $\theta = -0.9$ .  $t_{ini} = 1000$ . Right panel: current  $j$  vs.  $\theta$  as we ramp forward and backward with different rates. We ramp forward from  $\theta = -1.1$  till  $\theta = -0.9$  with  $\alpha = 10^{-6}$ . Then we ramp from  $\theta = -0.9$  to  $\theta = -1.1$  with  $\alpha = 10^{-4}$ ,  $10^{-6}$ . The arrows indicate the tendency of the current value as time evolves. In both panels  $t_0 = 0$ . Parameters are the same as in fig. 1.

as the transition probability of the initial state  $A$  into the states  $B$  and  $C$ , after ramping  $\theta$  (fig. 1).

For adiabatic ramping, we assume that variations in  $\theta$  are negligible during one period of the force, which allows us to separate the time scales of ac driving and ramping [18]. Then, we can use the standard Landau-Zener approach [19] and derive the transition probability, from which we estimate the ramping rate needed for an adiabatic process. The transition probability for Floquet

states is given by [18,20],

$$P = \exp \left[ -\frac{\pi \delta \epsilon \delta \theta}{2 \hbar \alpha} \right], \quad (8)$$

where  $\delta \theta$  is the  $\theta$ -interval in which the quasienergy difference increases by a factor of  $\sqrt{2}$  [18].  $\delta \epsilon$  denotes the quasienergy gap. Thus, we find  $\delta \epsilon \approx 2 \times 10^{-4}$ ,  $\delta \theta \sim 0.01$  for the avoided crossing in fig. 1. This leads to the rough

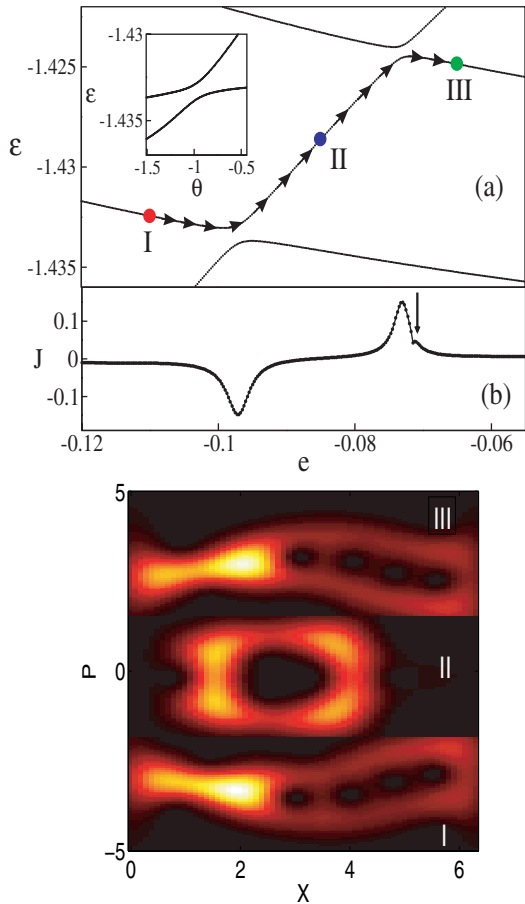


Fig. 3: Upper panel: (a) quasienergy  $\epsilon$  vs. amplitude  $e$  of the second harmonic in the potential  $v(x)$ . The arrows indicate the ramping process starting in I, then passing through the intermediate state II, finally ending up in III. Inset: quasienergy vs.  $\theta$  at the resonance point  $e = -0.097$ . (b) Current  $J$  vs.  $e$ . The arrow indicates a narrow peak which appears due to an avoided crossing not shown in the graphic. Lower panel: Husimi functions of the states labeled by Roman numbers in upper panel (a). The intensity scales, in descending order, from yellow to black (colour on-line). We use the Husimi function defined in [12]. The parameters are  $\theta = -0.99$  and  $\theta_p = -\pi/2$ . The other parameters are the same as in fig. 1.

estimate  $\frac{\delta\epsilon\delta\theta}{2\hbar} = 5 \times 10^{-6}$ . To be more accurate we fit the transition probabilities in fig. 1 using eq. (8) and obtain  $\frac{\delta\epsilon\delta\theta}{2\hbar} \approx 10^{-6}$ . Therefore, for  $\alpha \leq 10^{-6}$ , the adiabatic regime is realized. Indeed, from fig. 1c, we see that, as the ramping rate decreases, the transition probability to the state  $C$  increases consequently. A crossover takes place at  $\alpha \approx 5 \times 10^{-6}$  as the system reaches one half of the transition probability.

The final state is a superposition of Floquet states  $B$  and  $C$  (see (6)), and therefore displays interference effects [18]. A signature of that interference are the oscillations experienced by the momentum of the final state fig. 1d. These oscillations take place around a nonzero value, which increases as the system approaches to the adiabatic behavior. Using eqs. (6) and (4), it is

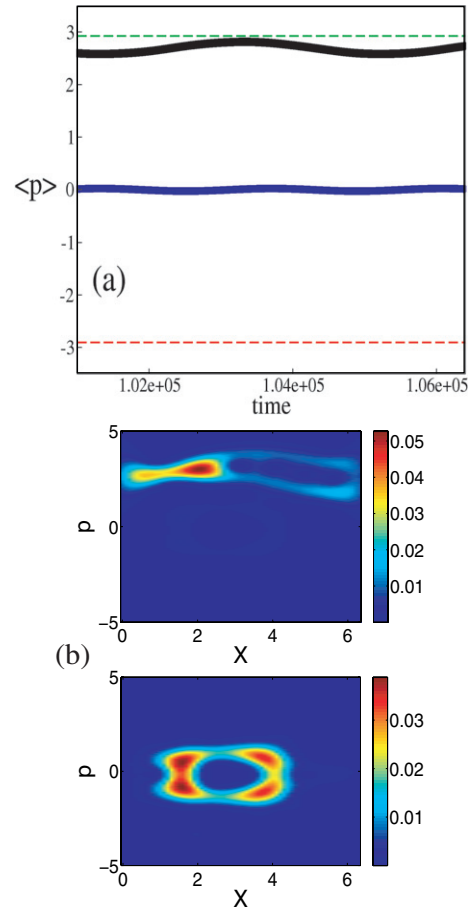


Fig. 4: (a) Momenta of the intermediate and final states while ramping  $e$  as a function of time depicted by thick curves. The thickness of the curves is a result of periodic oscillations of the momenta with period  $T$ , which are not resolved by the eye in this time scale. Mean momentum for the states I, II and III in fig. 3 are depicted by dashed lines. The dashed line for the state II appears superimposed to the momentum of the intermediate state. The ramping rate is  $\alpha_2 = 2.5 \times 10^{-7}$ . (b) Husimi functions for intermediate and final states at  $t = 50998 T$ . The other parameters are the same as in fig. 3.

straightforward to find

$$\begin{aligned} \bar{p} = & |a|^2 \bar{p}_C + |b|^2 \bar{p}_B + a^* b \exp\left(-i \frac{\epsilon_B - \epsilon_C}{T} t\right) \langle \phi_C | \hat{p} | \phi_B \rangle \\ & + a b^* \exp\left(-i \frac{\epsilon_C - \epsilon_B}{T} t\right) \langle \phi_B | \hat{p} | \phi_C \rangle. \end{aligned} \quad (9)$$

The last two terms in eq. (9) generate oscillations, which are slow compared to the period of the driving force. The period of the slow oscillations in fig. 1d is in agreement with the estimation  $\frac{1}{|\epsilon_C - \epsilon_B|} \sim \tau \approx 10^4$ . In the extreme adiabatic limit  $b = 0$ , and consequently the last two terms in eq. (9) disappear.

Let us now address the effect of adiabatic ramping on the generation of directed transport. To this end, we ramp  $\theta$  within the interval  $\theta \in (-1.1, -0.9)$ , where no other avoided crossings appear. We consider the initial state  $|0\rangle$  with zero momentum. It overlaps strongly with the

state  $A$  depicted by a circle in fig. 1a. By adiabatically ramping through the resonant region, the quantum state will strongly overlap with the transporting state  $C$ , thus leading to an increase of the current.

We compute the time-dependent current as  $j = \frac{1}{(t-t_0)} \times \int_{t_0}^t \bar{p} dt$  with  $\bar{p} = \langle \psi | \hat{p} | \psi \rangle$  which becomes the asymptotic current  $J$  in the limit  $t \rightarrow \infty$ . Figure 2 displays the current evolution as we ramp with different rates. We find that the final current increases by an order of magnitude when the ramping velocity is lowered from the diabatic into the adiabatic regime. If we ramp again, but backwards, with the same adiabatic rate, we will more or less return to the original state and momentum. However, if we ramp fast (diabatically) backwards, we will perform a Landau-Zener transition, and arrive at the initial value  $\theta = -1.1$ , but now with a much larger current than the one we started with! Figure 2 shows the corresponding numerical results, as we ramp forward and backward with different ramping velocities. We ramp forward and backward between  $\theta = -1.1$  and  $\theta = -0.9$ . When ramping forward we use  $\alpha = 10^{-6}$  which corresponds to an adiabatic regime. During the backward ramping we use two values of  $\alpha$ . Whereas in an adiabatic backward ramp the current becomes small again, for the diabatic ramp the current remains around its maximum value (fig. 2).

The gap of an avoided crossing determines the maximum adiabatic ramping rate. In order to increase that rate, the avoided crossing gap has to be increased. At the same time, for a Landau-Zener description to be applicable, the gap value should remain considerably smaller than the average level spacing. The gap value is determined by the strength of interaction between Floquet states. The interaction strength in turn is the larger, the more asymmetric the Floquet operator is. Motivated by this observation as well as the work of ref. [14], we further consider a bichromatic optical lattice potential to enhance the gap value. In particular, we consider the potential  $v(x) = \cos(x) + e \cos(2x + \theta_p)$  as in ref. [14]. We compute the quasienergy spectrum at  $\theta = -0.99$  and ramp  $e$ . Figure 3 shows two avoided crossings of different states which lead to peaks with opposite current signs. The corresponding Husimi functions (see [11,12] for details) reveal that such peaks appear due to resonances between a state in the chaotic layer and transporting states that carry opposite mean momenta.

We use that structure and ramp adiabatically through this parameter window. As a result the initial state I with nonzero momentum transforms into a state with opposite momentum, and thus we reverse the current ( $\alpha_2$  denotes the ramping rate for  $e$ ).

We first ramp  $e$  from  $-0.11$  to  $-0.085$  with state I as initial input. After that, we compute the momentum of the obtained intermediate state as a function of time. Then we ramp  $e$  from  $-0.085$  to  $-0.065$ , with the intermediate state as initial input. Then, we compute the evolution of the momentum of the final state.

Figure 4 shows the momenta of the intermediate and final states as a function of time. Both show good agreement with the momenta of the states II and III of fig. 3, respectively. The oscillations indicate interference due to the coupling between the states. Husimi functions for states in adiabatic ramping show a good correspondence to the Floquet states II and III in fig. 3. We therefore manage to reverse and increase the current value significantly.

If ramping  $\theta$  instead within the interval  $(-1.5, -0.5)$  (see inset of fig. 3a), we find that, for an adiabatic ramping, the transition rate should be below  $\alpha = 4 \times 10^{-5}$ . This value is much larger than the one for the symmetric case  $e = 0$ . Certainly, further increase of the gap values can be obtained by tuning other system parameters as well, thus further reducing the time needed for the adiabatic ramping.

To summarize, we ramped control parameters through avoided crossings of Floquet states with different average current characteristics. When diabatically ramping, the crossing is barely visible due to Landau-Zener transitions. For adiabatic ramping, the final quantum state will strongly change its transport properties. One can therefore switch the average current from small to large values or vice versa. By first ramping adiabatically, and then ramping back diabatically, we can therefore reach a completely different quantum state at one and the same value of the control parameter. Harvesting on several consecutive crossings, it is possible to achieve current reversals. In addition, by considering a bichromatic optical lattice potential, we show it is possible to increase the ramping speed while maintaining adiabaticity.

\*\*\*

One of the authors (JBG) is supported by the start-up funding (WBS grant No. R-144-050-193-101 and No. R-144-050-193-133) and the NUS “YIA” funding (WBS grant No. R-144-000-195-123), both from the National University of Singapore.

## REFERENCES

- [1] REIMANN P., *Phys. Rep.*, **361** (2002) 57.
- [2] ASTUMIAN R. D. and HANGGI P., *Phys. Today*, **55**, No. 11 (2002) 33.
- [3] JÜLICHER F., AJDARI A. and PROST J., *Rev. Mod. Phys.*, **69** (1997) 1269.
- [4] SCHIAVONI M., SANCHEZ-PALENCIA L., RENZONI F. and GRYNBERG G., *Phys. Rev. Lett.*, **90** (2003) 094101; JONES P. H., GOONASEKERA M. and RENZONI F., *Phys. Rev. Lett.*, **93** (2004) 073904; GOMMERS R., BERGAMINI S. and RENZONI F., *Phys. Rev. Lett.*, **95** (2005) 073003; GOMMERS R., DENISOV S. and RENZONI F., *Phys. Rev. Lett.*, **96** (2006) 240604.
- [5] FLACH S., YEVTUSHENKO O. and ZOLOTARYUK Y., *Phys. Rev. Lett.*, **84** (2000) 2358.



- [6] YEVTUSHENKO O., FLACH S., ZOLOTARYUK Y. and OVCHINNIKOV A. A., *Europhys. Lett.*, **54** (2001) 141.
- [7] SCHANZ H., OTTO M. F., KETZMERICK R. and DITTRICH T., *Phys. Rev. Lett.*, **87** (2001) 070601; SCHANZ H., DITTRICH T. and KETZMERICK R., *Phys. Rev. E*, **71** (2005) 026228.
- [8] DENISOV S. *et al.*, *Phys. Rev. E*, **66** (2002) 041104.
- [9] MONTEIRO T. S., DANDO P. A., HUTCHINGS N. A. C. and ISHERWOOD M. R., *Phys. Rev. Lett.*, **89** (2002) 194102.
- [10] GONG J. and BRUMER P., *Phys. Rev. E*, **70** (2004) 016202; *Annu. Rev. Phys. Chem.*, **56** (2005) 1; *Phys. Rev. Lett.*, **97** (2006) 240602.
- [11] DENISOV S., MORALES-MOLINA L. and FLACH S., *Europhys. Lett.*, **79** (2007) 10007.
- [12] DENISOV S., MORALES-MOLINA L., FLACH S. and HÄNGGI P., *Phys. Rev. A*, **75** (2007) 063424.
- [13] MORALES-MOLINA L. and FLACH S., *New. J. Phys.*, **10** (2008) 013008.
- [14] SALGER T., GECKERLER C., KLING S. and WEITZ M., *Phys. Rev. Lett.*, **99** (2007) 190405.
- [15] MURGIDA G. E., WISNIACKI D. A. and TAMBORENA P. I., *Phys. Rev. Lett.*, **99** (2007) 036806.
- [16] LATKA M., GRIGOLINI P. and WEST B. J., *Phys. Rev. A*, **50** (1994) 1071; HENSELER M., DITTRICH T. and RICHTER K., *Phys. Rev. E*, **64** (2001) 046218.
- [17] GRAHAM R. and KEYMER J., *Phys. Rev. A*, **44** (1991) 6281.
- [18] BREUER H. P. and HOLTHAUS M., *Z. Phys. D*, **11** (1989) 1.
- [19] LANDAU L. D., *Phys. Z. Sowjetunion*, **2** (1932) 46; ZENER C., *Proc. R. Soc. London, Ser. A*, **137** (1932) 696.
- [20] BREUER H. and HOLTHAUS M., *Phys. Lett. A*, **140** (1989) 507.