String and Plaquette Valence-Bond-Solid States in the Frustrated J_1-J_2 Transverse Field Ising Model on the Square Lattice

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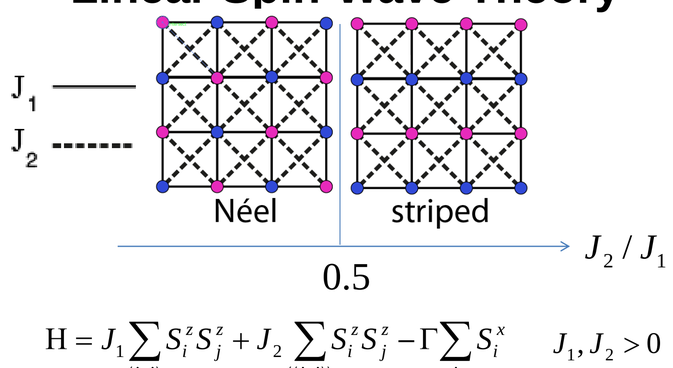
Topics

1. Frustrated Ising model with extensive degenerate ground states
2. Effect of quantum fluctuations to lift the ground state degeneracy
   i. Break down of LSWT with single-spin-flip excitations
   ii. Success of the cluster operator approach (COA) with multi-spin-flip excitations
3. Order by disorder : String and plaquette valence bond crystal phases
4. Mapping from checkerboard to diagonal square lattice.

Classical Background and Linear Spin-Wave Theory



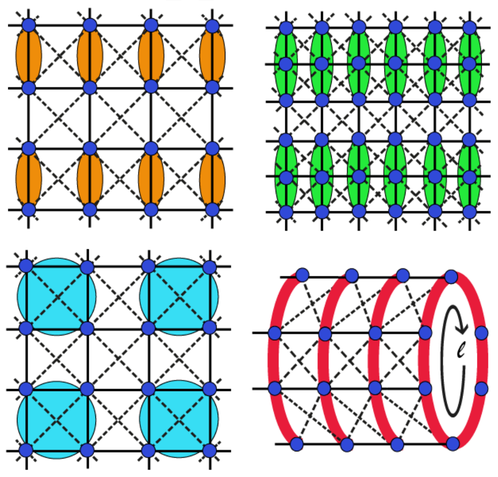
$$H = J_1 \sum_{\langle i,j \rangle} S_i^z S_j^z + J_2 \sum_{\langle\langle i,j \rangle\rangle} S_i^z S_j^z - \Gamma \sum_i S_i^x \qquad J_1, J_2 > 0$$

There is an exponential degeneracy of classical ground state at highly frustrated point J2/J1=0.5.

$$E_{cl}^{Néel} = E_{cl}^{striped} = -S^2 \cos^2\theta - \Gamma S \sin\theta$$

$$S^z = S - a^+ a, \quad S^x = \sqrt{\frac{S}{2}}(a^+ + a) \qquad E_{LSWT}^{Néel} = E_{LSWT}^{striped}$$

Cluster Operator Approach



$$H = H_0 + H_{int}, \quad H_0 = \sum_C H_C, \quad H_C = \sum_{\langle i,j \rangle} S_i^z S_j^z - \Gamma \sum_i S_i^x$$

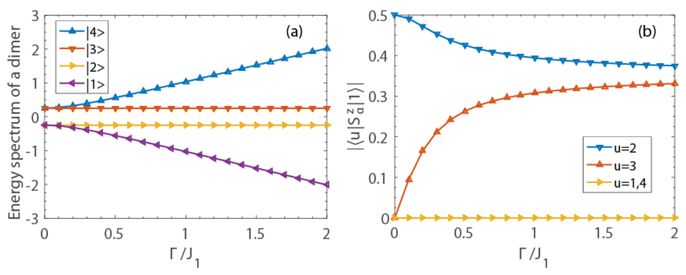


Fig. 1: (a) Energy levels of a single dimer versus transverse field (Γ). (b) Transition amplitudes versus magnetic field between the ground state (|1⟩) and four eigenstates (|u⟩, u =1,2,3,4) of a dimer.

$$|u\rangle_I = b_{I,u}^T |0\rangle, \quad u = 1,\ldots,16 \qquad \bar{p}^2 = \langle b_{I,1}^T b_{I,1} \rangle \qquad N\bar{p}^2 + \sum_{I,u} b_{I,u}^T b_{I,u} = N$$

$$H_{eff} = N\mu + N\bar{p}^2(\varepsilon_1 - \mu) - \frac{1}{2}N\sum_u(\varepsilon_u - \mu) + \sum_{\vec{k}}\sum_{\nu=1}^{4}(\frac{1}{2} + \gamma_{\vec{k},\nu}^T \gamma_{\vec{k},\nu})\omega_{\vec{k},\nu}(\mu, \bar{p})$$

$$\frac{\partial\langle H_{eff}\rangle}{\partial\mu} = 0, \frac{\partial\langle H_{eff}\rangle}{\partial\bar{p}} = 0$$

Results

A. String Condensation

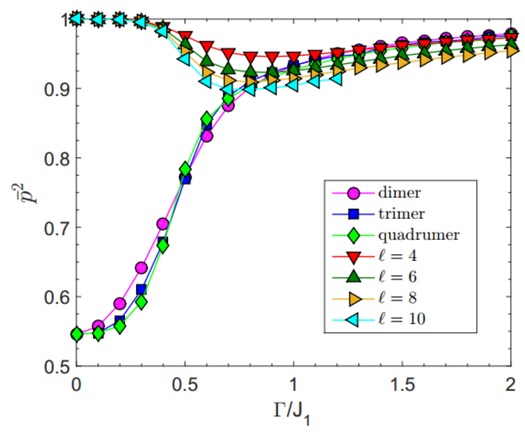


Fig. 2: The condensation probability for different cluster formation.

B. Nature of string-VBC phase

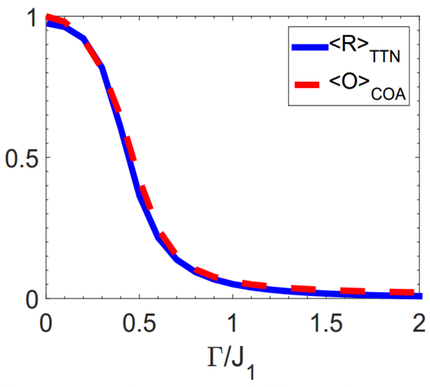


Fig. 3: The expectation value of (R) obtaind by tree tensor network (TTN) simulation on a 4*4 lattice. The expectation value of resonating operator (O) obtained from COA calculation on a l=8 cylinder are also shown.

Breaking of Lattice Rotational Symmetry

$$\hat{R} = \frac{1}{S^2}(S_A^z S_B^z - S_A^z S_C^z)$$

A resonating string valence bond phase

$$\hat{O} = |\varphi\rangle\langle\bar{\varphi}| + |\bar{\varphi}\rangle\langle\varphi| \qquad |\varphi\rangle = |\uparrow\downarrow\uparrow\downarrow\ldots\rangle, \; |\bar{\varphi}\rangle = |\downarrow\uparrow\downarrow\uparrow\ldots.\rangle$$

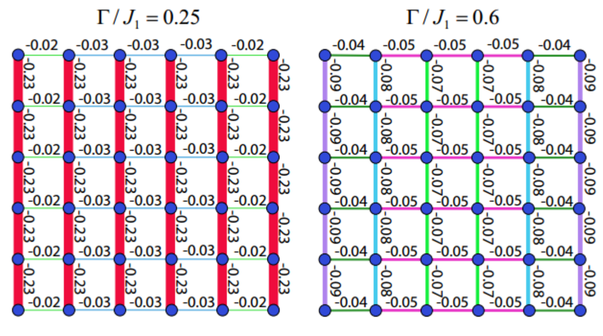


Fig. 4: Nearest-neighbor correlations , $C_{NN} = \langle S_i^z S_j^z \rangle$, obtained by TTN numerical simulation, for low and high field regimes.

References

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C. Relation between square and checkerboard lattices and the presence of Plaquette valence-bond-solid state

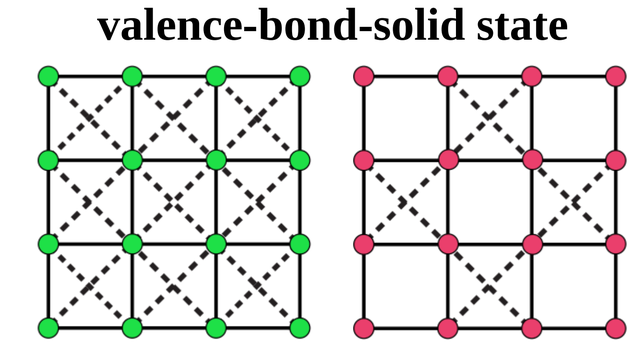
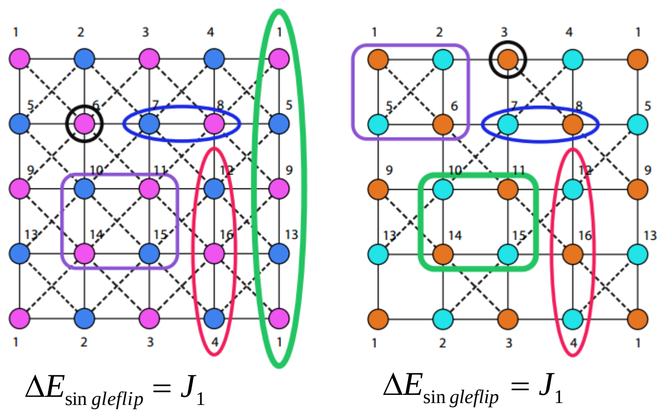


Fig. 5: J_1 - J_2 model on the square and checkerboard lattices. The solid and dashed lines are $J1$ and $J2$ bonds,respectively.



$\Delta E_{single flip} = J_1$ $\quad$ $\Delta E_{single flip} = J_1$
$\Delta E_{dimer flip} = J_1$ $\quad$ $\Delta E_{dimer flip} = J_1$
$\Delta E_{trimer flip} = J_1$ $\quad$ $\Delta E_{trimer flip} = J_1$
$\Delta E_{crossedplaquette flip} = J_1$ $\quad$ $\Delta E_{crossedplaquette flip} = 2J_1$
$\Delta E_{string flip} = 0$ $\quad$ $\Delta E_{emptyplaquette flip} = 0$

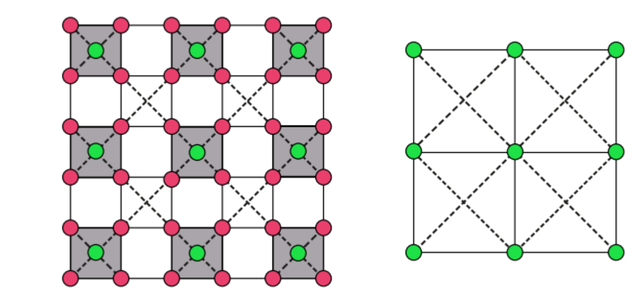


Fig. 6: Hatched crossed plaquettes of the checkerboard lattice form the quasi spins and construct a square lattice.

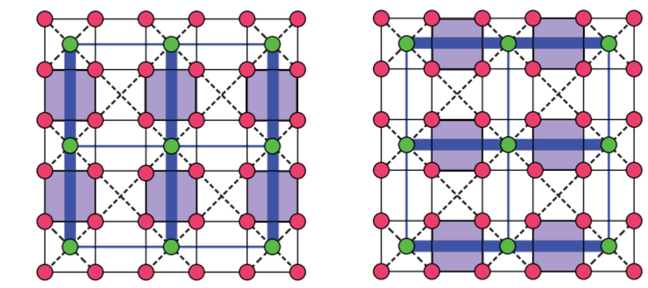
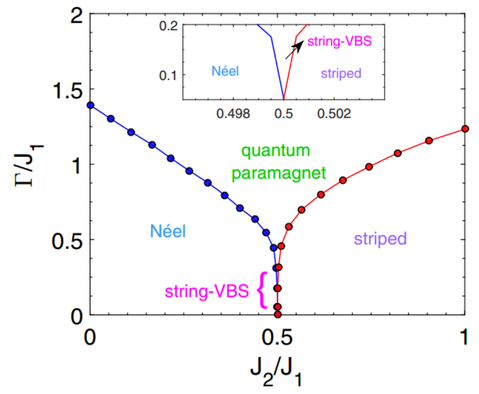


Fig. 7: Plaquette-VBS phase of the checkerboard lattice with broken translational symmetry with twofold degeneracy, which is mapped to the string-VBS phase of the square lattice with broken rotational symmetry and twofold degeneracy.

Square lattice phase diagram



Checkerboard lattice phase diagram