Free energy of broadened Landau levels of a two-dimensional electron gas

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ABSTRACT

- A broadened Landau level is considered.
- An analytic expression is derived for the free energy using a Gaussian density of states (DOS).

INTRODUCTION

- Two-dimensional electron gas (2DEG) is a system where the motion of electrons is confined in two directions. [1]
- The density of states (DOS) for a broadened 2DEG under a perpendicular magnetic field which is given as

$$D(E,B) = \frac{2eB}{h}\sum_n \left(\frac{1}{2\pi\Gamma^2}\right)^{1/2} \exp\left[-\frac{(E-E_n)^2}{2\Gamma^2}\right] \quad (1)$$

- Here, $E_n = \left(n+\frac{1}{2}\right)\hbar\omega_c$ is the nth Landau level, $\omega_c = \frac{eB}{m^*}$ is the cyclotron frequency for a given effective mass m^*, and Γ is the broadening parameter.

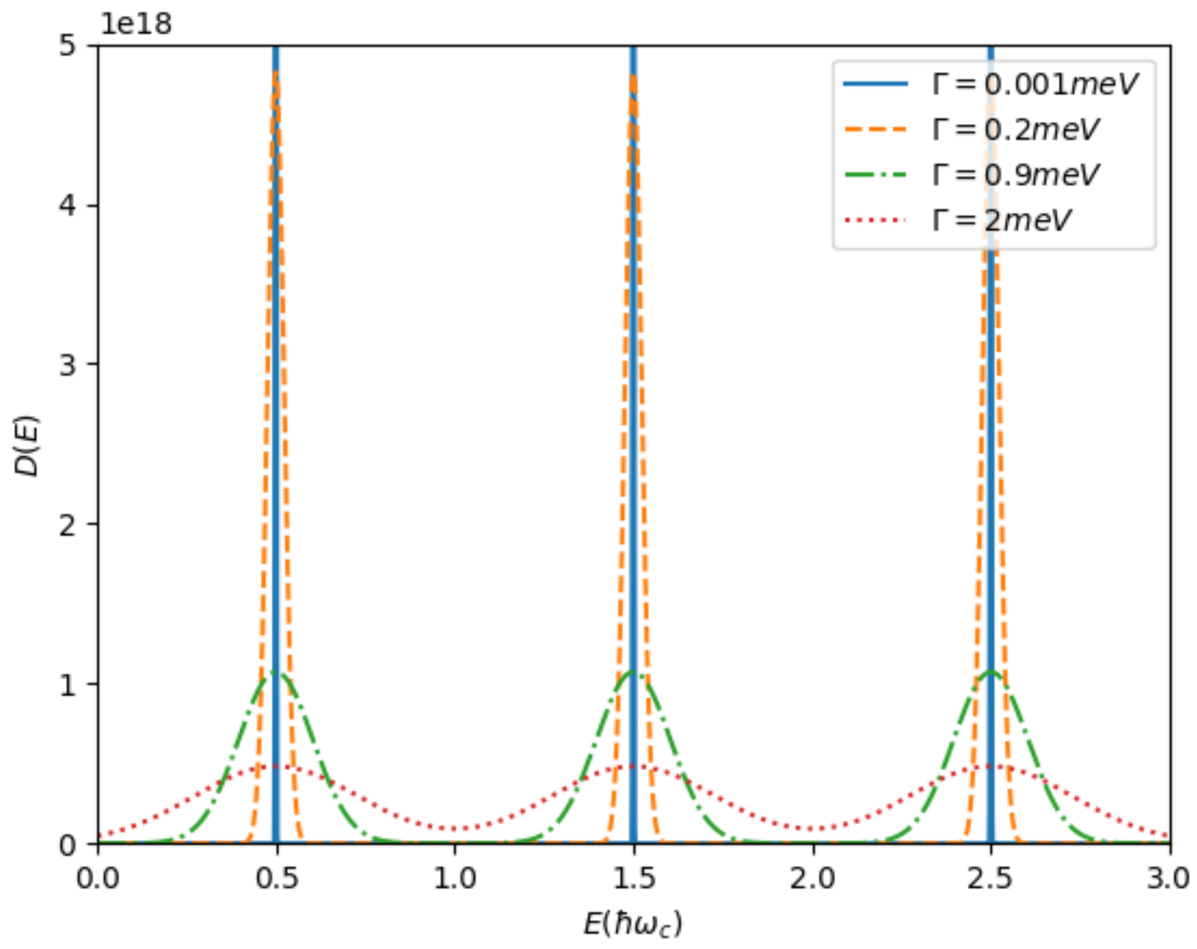

Fig. 1. . The plot for different values of Γ which is observed that the broadening parameter is the width of the peaks centered at E_n.

FREE ENERGY

- The free energy has the form,

$$F = \mu N - \frac{1}{\beta}\int_0^\infty D(E,B)\ln\{1+\exp[\beta(\mu-E)\}\,dE \quad (2)$$

where $\beta = \frac{1}{k_B T}$.

- Substitute $D(E)$ of eq. (2) by eq. (1).The compressed form of the free energy,

$$F = \mu N - \frac{2eB}{\beta h}\sum_n I_n \quad (3)$$

where

$$I_n = \int_0^\infty \left(\frac{1}{2\pi\Gamma^2}\right)^{1/2} \exp\left[-\frac{(E-E_n)^2}{2\Gamma^2}\right]\ln\{1+\exp[\beta(\mu-E)]\}\,dE \quad (4)$$

- The integral I_n can be evaluated by integration by parts (IBP) which leads to

$$I_n = uv_n\Big|_0^\infty + \frac{\beta}{2}\int_0^\infty 2v_n f(E)\,dE \quad (5)$$

where $u = \ln\{1+\exp[\beta(\mu-E)]\}$, $v_n = \frac{1}{2}\mathrm{erf}\left(\sqrt{\left(\frac{1}{2\Gamma^2}\right)}(E-E_n)\right)$ and $f(E)$ is the Fermi-Dirac distribution function.

- To evaluate the remaining integral, the Fermi-Dirac distribution function can be expanded by geometric series [2]

$$f(E) = \begin{cases} \sum_{k=0}^{\infty}(-1)^k \exp[\beta k(E-\mu)] & E<\mu \\ \sum_{k=0}^{\infty}(-1)^k \exp[\beta(k+1)(\mu-E)] & E>\mu \end{cases} \quad (6)$$

where the remaining integral in eq. (5)

$$J_n = \frac{\beta}{2}\int_0^\mu 2v_n\,dE + \frac{\beta}{2}\sum_{k=1}^{\infty}(-1)^k(\mathcal{I}_{nk} - \mathcal{J}_{nk}) \quad (7)$$

where

$$\mathcal{I}_{nk} = \int_0^\mu 2v_n \exp[\beta k(E-\mu)]\,dE \quad (8)$$

and

$$\mathcal{J}_{nk} = \int_\mu^\infty 2v_n \exp[\beta k(\mu-E)]\,dE \quad (9)$$

- The first term of eq. (7) can be evaluated by using integral no. 5.41 of Ref. [3]. Eqs. (8) and (9) can be evaluated by IBP simultaneously, and using the integral no. 1, 2.33 of Ref. [3].
- Upon evaluating all of the integration limits for eq. (8) and eq. (9),

$$\mathcal{I}_{nk} - \mathcal{J}_{nk} = \frac{e^{-\beta k\mu}}{\beta k}\left\{\mathrm{erf}\left(\sqrt{\frac{1}{2\Gamma^2}}E_n\right) - \exp\left(\frac{(\Gamma\beta k)^2}{2}\right)\times \left[\mathrm{erf}\left(\sqrt{\frac{1}{2\Gamma^2}}(\mu-E_n-\Gamma^2\beta k)\right) + \mathrm{erf}\left(\sqrt{\frac{1}{2\Gamma^2}}(E_n+\Gamma^2\beta k)\right)\right]\right\} + \frac{1}{\beta k}\exp\left(\frac{(\Gamma\beta k)^2}{2} + \beta k(\mu-E_n)\right)\left[\mathrm{erf}\left(\sqrt{\frac{1}{2\Gamma^2}}(\mu-E_n+\Gamma^2\beta k)\right) + 1\right] \quad (10)$$

- Evaluating all integration limits, the final form of the free energy

$$F = \mu N - \frac{eB}{h}\sum_{n,k=1}^{\infty}\left\{\left[\frac{1}{\beta}\ln\left(1+e^{\beta\mu}\right)\right]\mathrm{erf}\left(\sqrt{\frac{1}{2\Gamma^2}}E_n\right) + (\mu-E_n)\,\mathrm{erf}\left(\sqrt{\frac{1}{2\Gamma^2}}(\mu-E_n)\right) + \sqrt{\frac{2\Gamma^2}{\pi}}\left(\exp\left(-\frac{(\mu-E_n)^2}{2\Gamma^2}\right) - \exp\left(-\frac{E_n^2}{2\Gamma^2}\right)\right) + (-1)^k(\mathcal{I}_{nk} - \mathcal{J}_{nk})\right\} \quad (11)$$

CONCLUSIONS

- An analytic expression is derived for the free energy of broadened Landau levels of a 2DEG.
- This result can be use to obtain the magnetization and heat capacity closed forms having a Gaussian DOS and compare it with the numerical simulation [4].
- The form can examine the limiting case by deriving the specific heat and comparing the form obtained from Ref. [5].

References

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