Dynamical Glass

Carlo Danieli

Center for Theoretical Physics of Complex Systems
Institute for Basic Science
Daejeon, South Korea

Disordered Systems:
From Localization to Thermalization and Topology

People

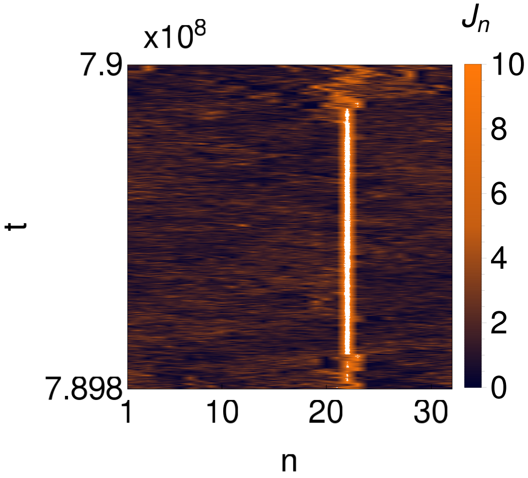
- Sergej Flach (PCS)
- David Campbell (Boston University)
- Mithun Thudiyangal (PCS)
- Yagmur Kati (PCS)
- Ihor Vakulchyk (PCS)
- Merab Malishava (PCS)

Some key words ...

Dynamical Glass

- Hamiltonian systems
- integrability (action/angle coordinates)
- weak non-integrable perturbations
- ergodicity
- ....

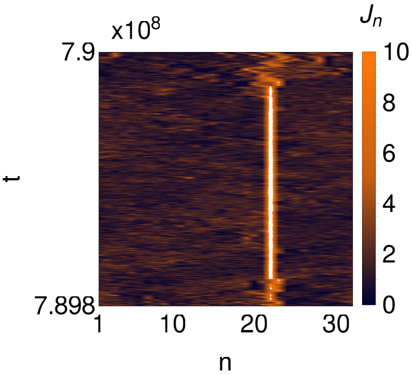
... a picture ...



... a picture (unfolded) ....

Klein-Gordon (KG) - periodic boundary condition

$$H = \sum_{n=1}^{N} \left[\frac{p_n^2}{2} + \frac{q_n^2}{2} + \frac{q_n^4}{4} + \frac{\varepsilon}{2}(q_{n+1} - q_n)^2 \right] = \sum_{n=1}^{N} J_n$$



CD, D.Campbell, S.Flach, PRE **95**, 060202(R) (2017)

... and then what?

A look back ...

The existence of these breather-like excitations in the equilibrium dynamics was known before ...

- M.Ivanchenko, O.Kanakov, V.Shalfeev, S.Flach, Physica D **198**, 120 (2004).
- S.Iubini, R.Franzosi, R.Livi, G.Oppo,A.Politi, New Journal of Physics **15**, 023032 (2013).
- C.Mulhern, S.Bialonski, and H.Kantz, PRE **91**, 012918 (2015).

What's new?

In these publications

- **CD**, D.Campbell, S.Flach, PRE **95**, 060202(R) (2017)
- T.Mithun, Y.Kati, **CD**, S.Flach, PRL **120**, 184101 (2018)

we introduced a method to detect these excursions out of equilibrium.

Question

How do they impact the ergodic dynamics of an Hamiltonian system (close to an integrable limit)?

Setting

Various ingredients

- Hamiltonian systems close to an integrable limit
  1. action/angle coordinates of the integrable part
  2. non-integrable perturbation
  3. network of actions
- measures to study the ergodic dynamics
  1. excursion out of equilibrium and their distributions
  2. finite time averages
  3. largest Lyapunov exponent

Setting

Weakly Non-Integrable Hamiltonian

$$H(J,\theta) = H_0(J) + \varepsilon P(J,\theta)$$

- J - action coordinates
- θ - angle coordinates
- N - numbers of degrees of freedom
- $X \cong \mathbb{R}^N \times \mathbb{R}^N$ - phase space
- the perturbation P defines a network of action

$$\dot{J}_n = -\varepsilon \frac{\partial P}{\partial \theta_n} \qquad \dot{\theta}_n = \frac{\partial H_0}{\partial J_n} + \varepsilon \frac{\partial P}{\partial J_n}$$

- Long range / Short range network of actions

Setting

Klein-Gordon (KG) - periodic boundary condition

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Low-energy limit $h = H/N \to 0$

- Integrable part H_0 $\Rightarrow$ Harmonic oscillators
- non-integrable perturbation P $\Rightarrow$ quartic term
- network $\Rightarrow$ all normal modes to all (up to selections)

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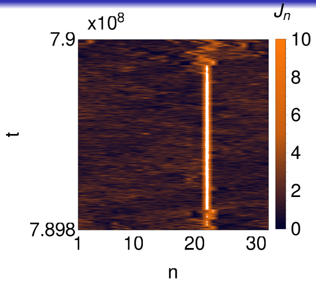
High-energy limit $h = H/N \to +\infty$

- Integrable part H_0 $\Rightarrow$ Non-linear decoupled oscillators
- non-integrable perturbation P $\Rightarrow$ harmonic coupling
- network $\Rightarrow$ nearest-neighbor

Measure 1: excursions out-of equilibrium



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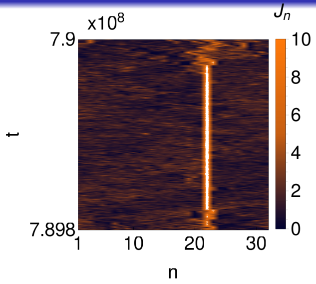

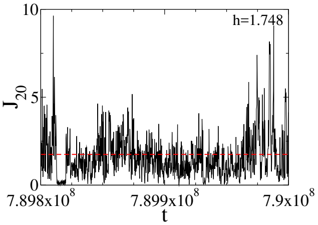

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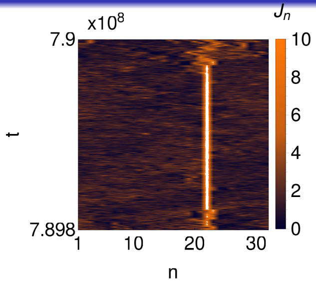

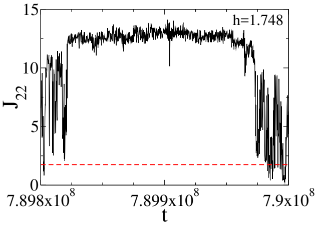

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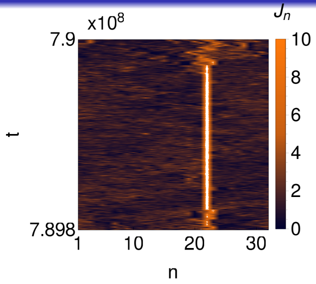

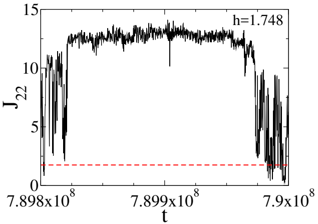

$$t \in \mathbb{R} \mid J_n(t) - h = 0$$

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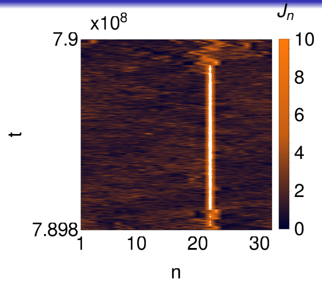

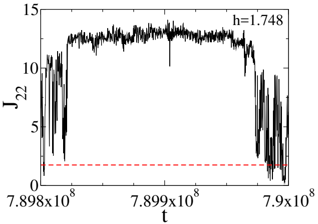

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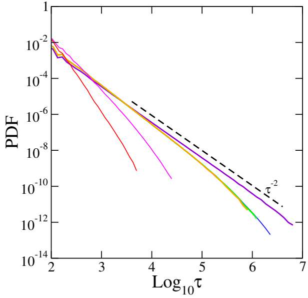

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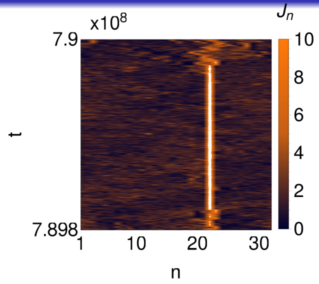

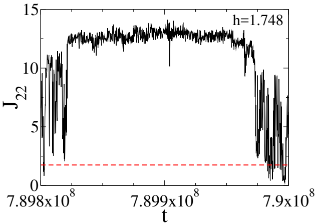

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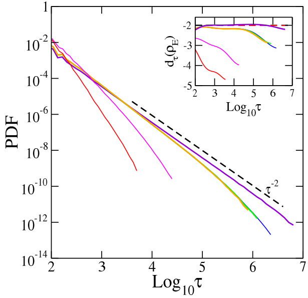

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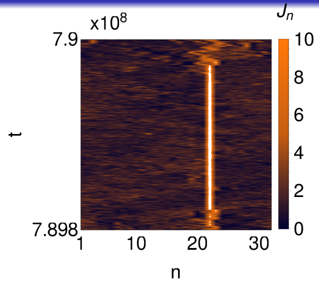

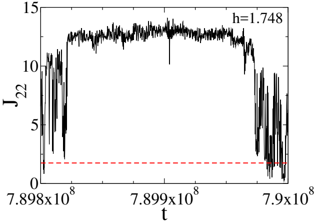

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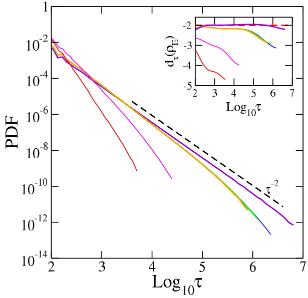

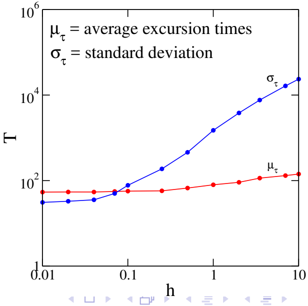

Measure 2: finite time averages

Question

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Ergodicity

Let $R(t) = (q(t), p(t))$. For any observable f defined over the phase space X:

$$\langle f \rangle_X = \langle f \rangle_\infty = \lim_{T\to\infty} \frac{1}{T}\int_0^T f(R(t))dt$$

time-average $\langle f \rangle_\infty$

ensemble-average $\langle f \rangle_X$

Measure 2: finite time averages

Finite time averages $T < +\infty$ of an observable f

$$\langle f \rangle_T = \frac{1}{T} \int_0^T f(R(t))dt \quad \Rightarrow \quad \rho(\langle f \rangle_T) \quad \text{distribution}$$

$$\text{Average of } \rho: \quad \Rightarrow \quad \mu_f(T)$$

$$\text{Variance of } \rho: \quad \Rightarrow \quad \sigma_f^2(T)$$

$$\text{Ergodicity} \quad \Rightarrow \quad \lim_{T \to \infty} \rho(\langle f \rangle_T) = \delta\left(\langle f \rangle_\infty - \langle f \rangle_X\right)$$

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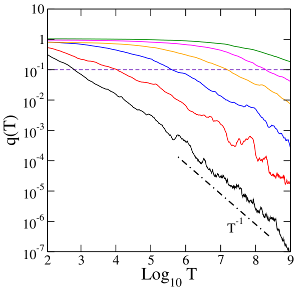
Fluctuation Index

$$q(T) = \frac{\sigma_f^2(T)}{\mu_f^2(T)}$$

Measure 1 + 2

Fluctuation Index



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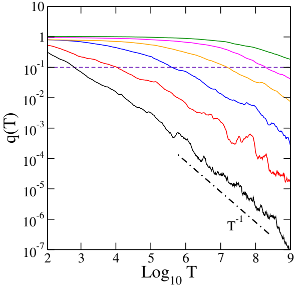

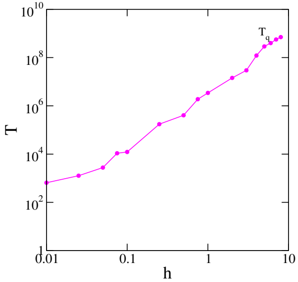

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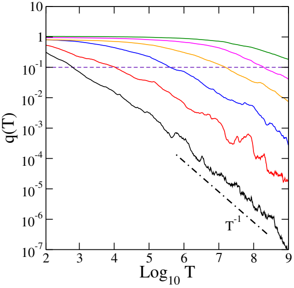

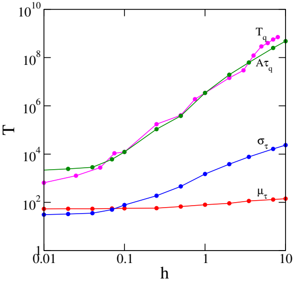

$$q(T) = \frac{\sigma_f^2(T)}{\mu_f^2(T)} \sim \begin{cases} q(0) & T \leq \tau_q \\ \frac{\tau_q}{T} & T \geq \tau_q \end{cases}$$

where $\tau_q = \frac{\sigma_\tau^2}{\mu_\tau}$

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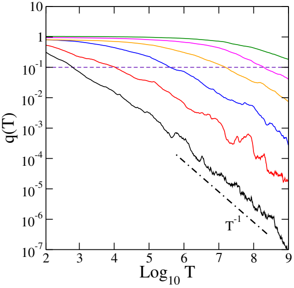

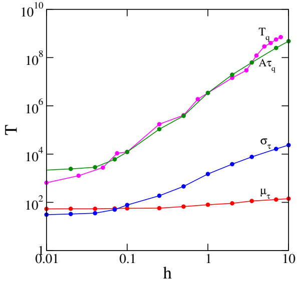

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Λ_{max} = largest Lyapunov exponent

Lyapunov Time $\quad T_\Lambda = \frac{1}{\Lambda_{\text{max}}}$

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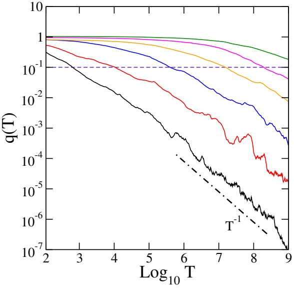

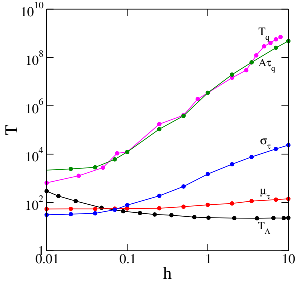

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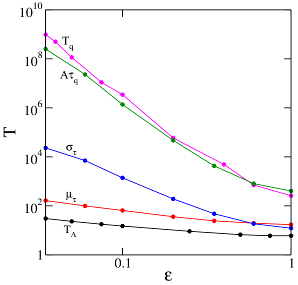

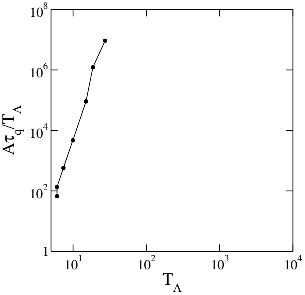

CD, Mithun T, Y Kati, D.Campbell, S.Flach, unpublished

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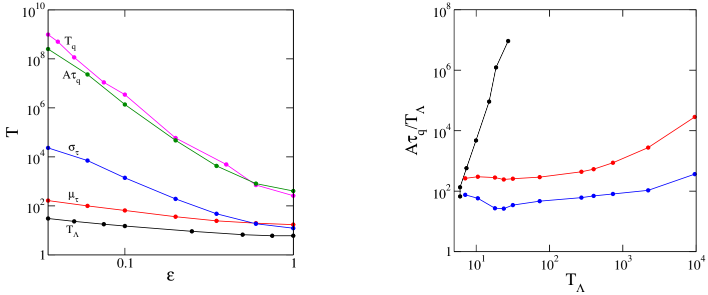

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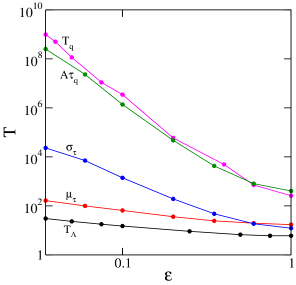
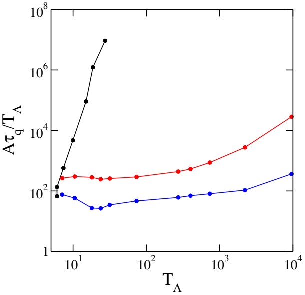


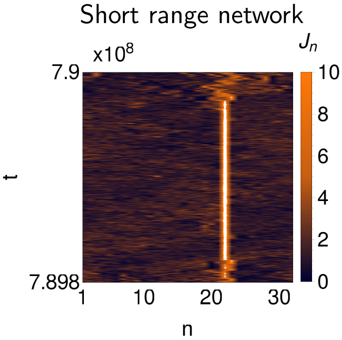


Open point

Why is the network of action playing such a role?

CD, Mithun T, Y Kati, D.Campbell, S.Flach, unpublished

… pictures …



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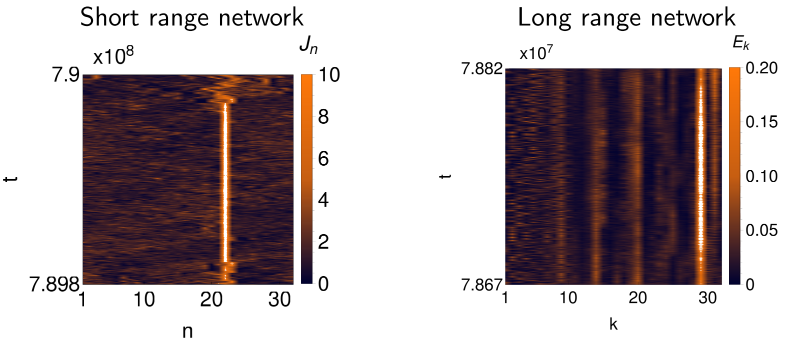
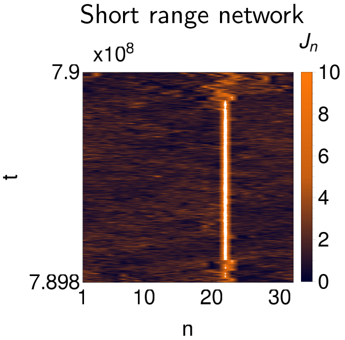
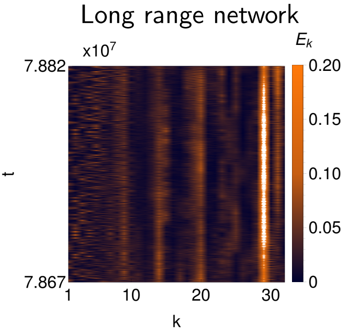


Short range network
x10^8
J_n
7.9
7.898
t
1
10
20
30
n
10
8
6
4
2
0
Long range network
x10^7
E_k
7.882
7.867
t
1
10
20
30
k
0.20
0.15
0.10
0.05
0

… pictures …





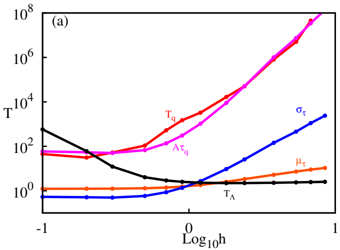
Open point

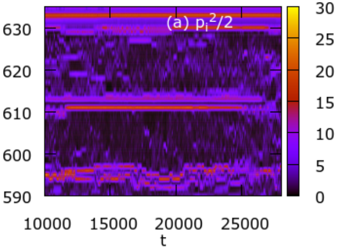
Resonances?

Rotors

Chains of couple rotors, high energy limit $h = H/N \to +\infty$

$$H(p,q) = \sum_{i=1}^{N} \left[\frac{p_i^2}{2} + E_J \left(1 - \cos(q_{i+1} - q_i)\right) \right]$$





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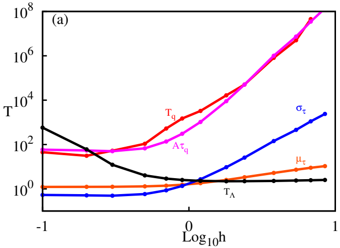

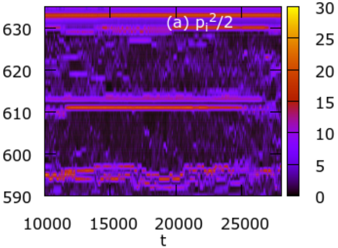

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Non-ergodic/bad-metal phase in chains of Josephson Junctions

M. Pino, L.B. Ioffe, B.L. Altshuler, PNAS 113, 536 (2016).

Open point: going quantum

Chains of couple rotors, high energy limit $h = H/N \to +\infty$

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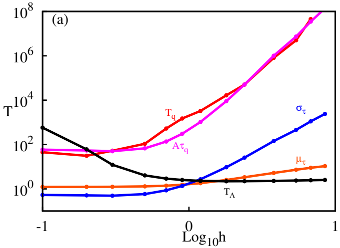

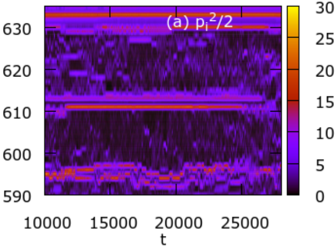

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Work in progress

Thank you