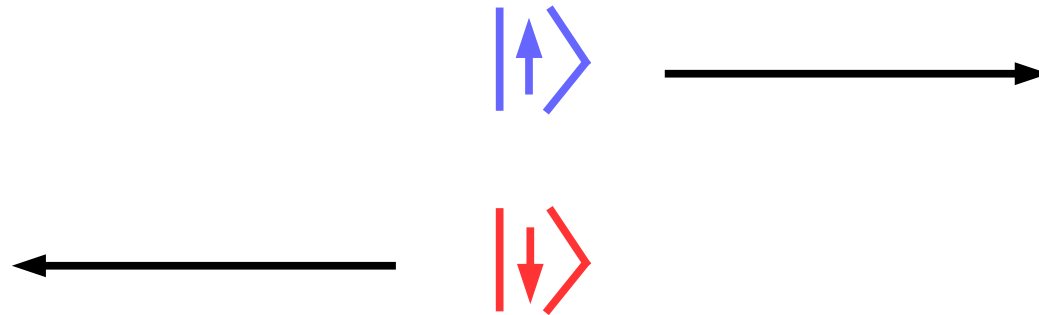


Disorder-robust helical coupled resonator waveguides

Daniel Leykam

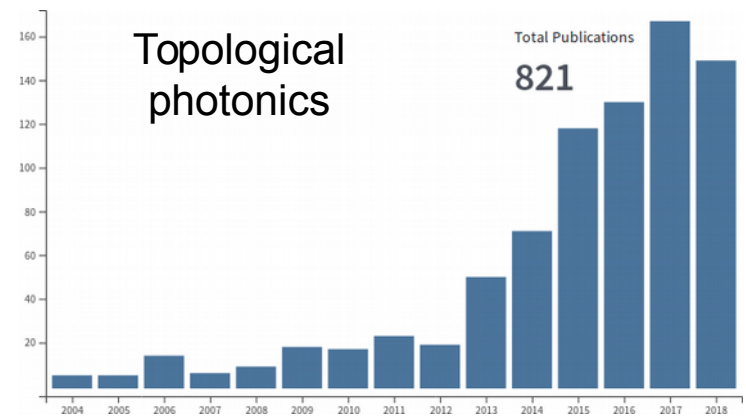
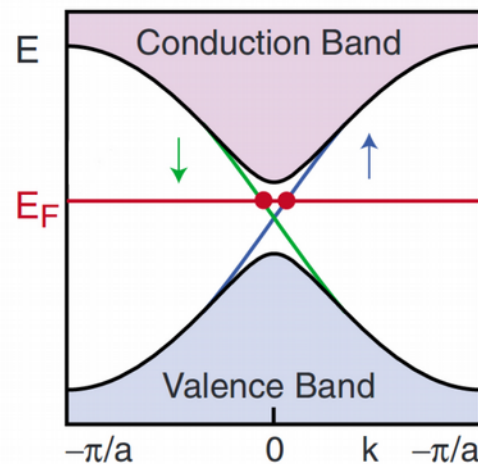
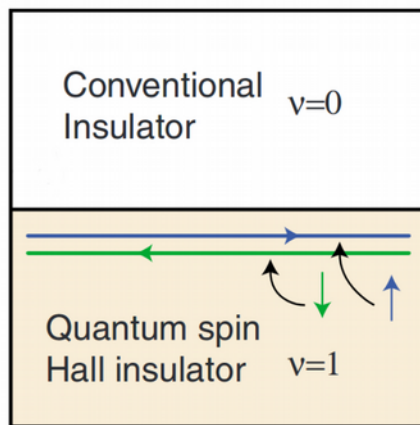
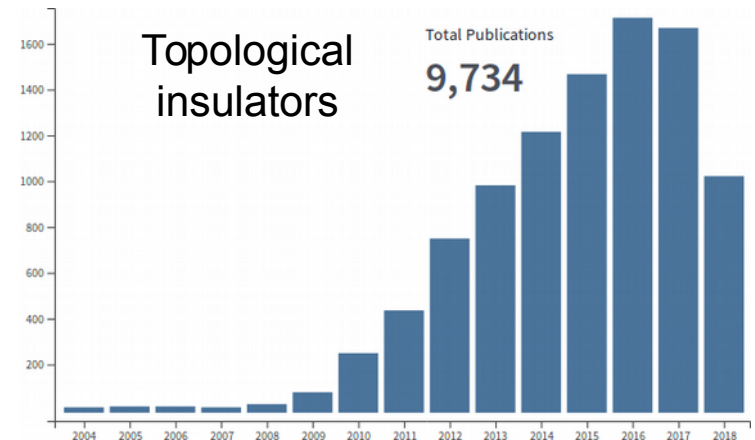
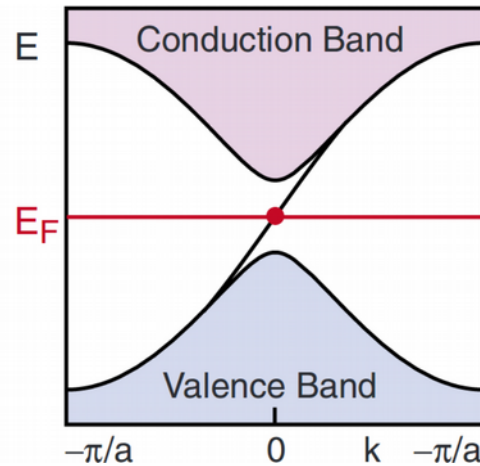
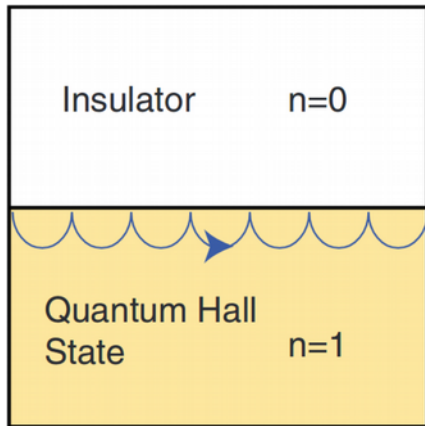


Center for Theoretical Physics of Complex Systems
Institute for Basic Science, Daejeon, Korea

Collaborators: Jungyun Han (IBS PCS), Clemens Gneiting (RIKEN)
Yidong Chong (Nanyang Technological University, Singapore)
Mohammad Hafezi & Sunil Mittal (Joint Quantum Institute, NIST/University of Maryland)

Topological protection against disorder

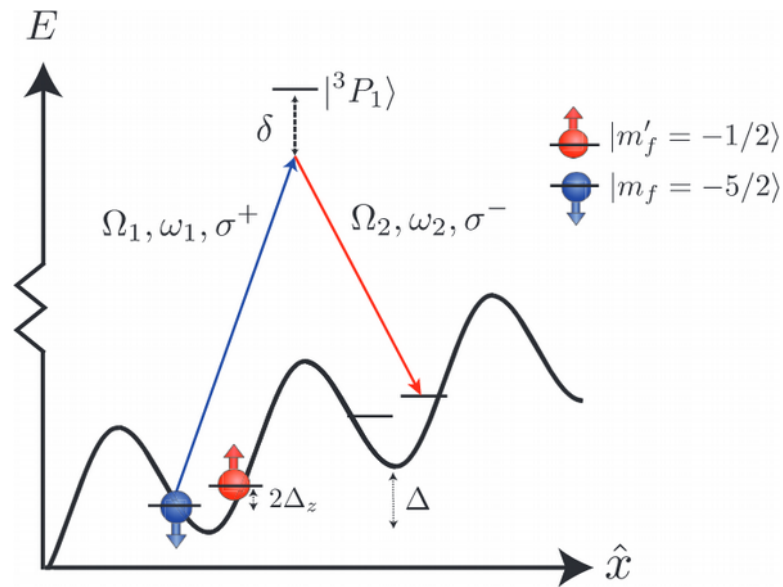
- Engineer 2D system with “topological” band structures
- One-way or spin-momentum locked edge modes
- Topological insulators, photonics, acoustics,



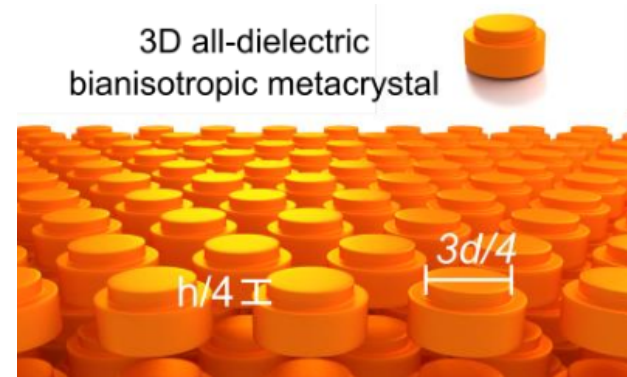
Kane & Mele, Phys. Rev. Lett. 95, 226801 (2005); Hasan & Kane, Rev. Mod. Phys. 82, 3045 (2010)

Motivation

- Are topological phases the “best” way to achieve disorder-robust transport?
- Drawbacks: system dimension, require exotic band structure engineering
- Can we use band structure engineering to directly achieve disorder robustness, without using topological phases?
- Benefits: reduced device size, can exploit fine-tuning



Budich et al., Phys. Rev. B 92, 245121 (2015)



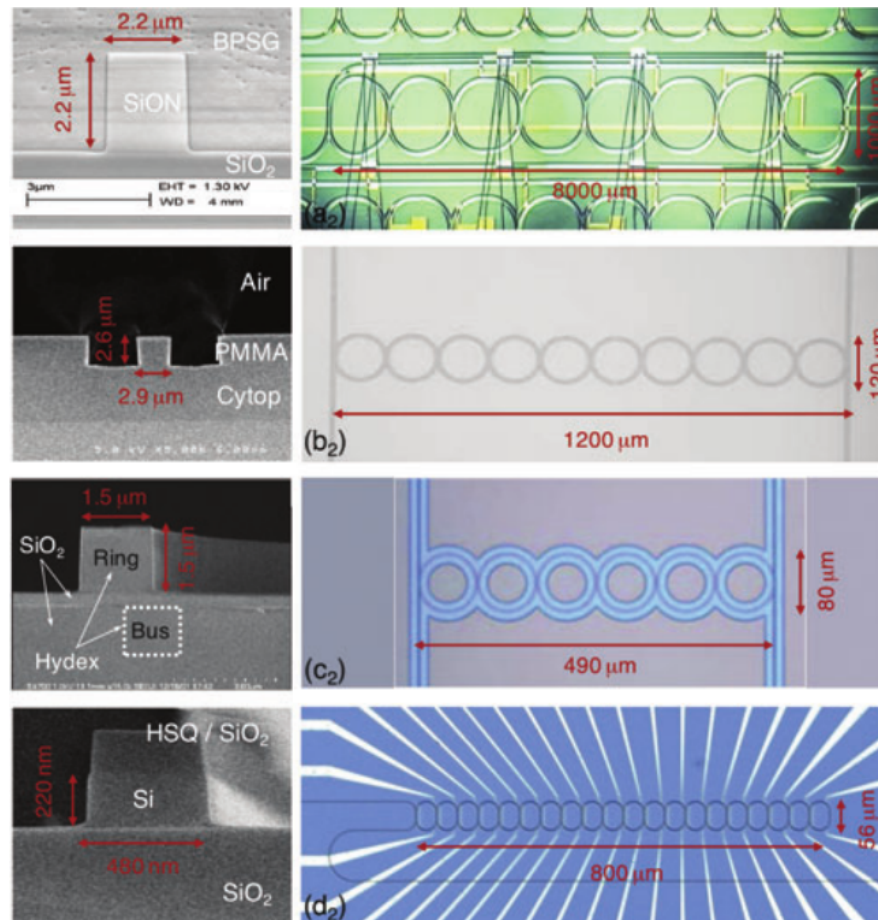
Slobozhanyuk et al, Nature Photonics 11, 130 (2017)

Outline

1. Coupled resonator optical waveguides (CROWs)
2. Topological protection in 2D resonator lattices
3. New approach: 1D CROWs with spin-momentum locking

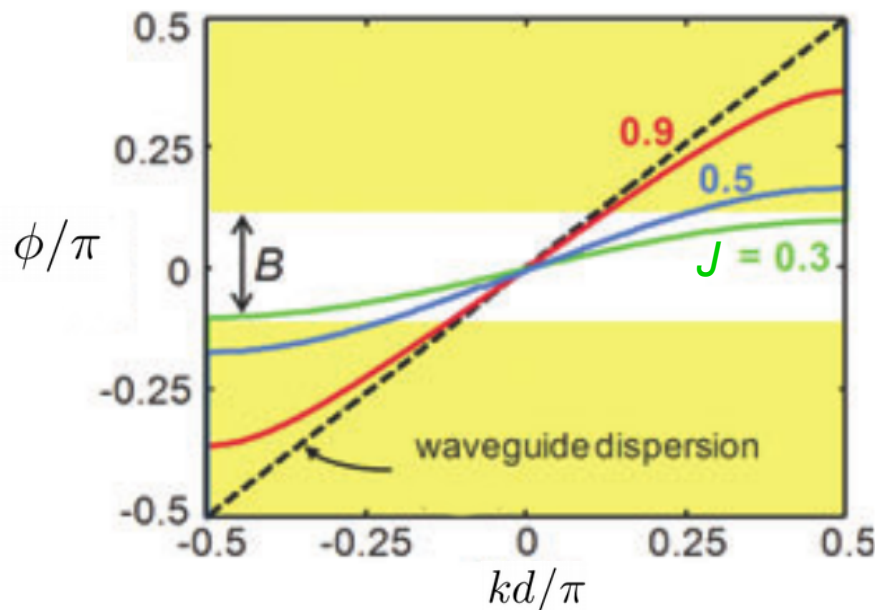
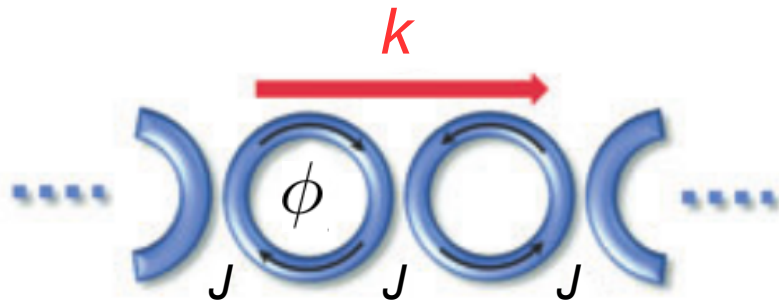
Coupled resonator optical waveguides (CROWs)

- Evanescently coupled high Q microresonators
- Applications: optical filtering & buffering, slow light, nonlinear signal processing



Coupled resonator optical waveguides (CROWs)

- Resonator modes: clockwise or anticlockwise (negligible intra-ring backscattering)
- Coupling between neighboring rings: scattering matrices, coupling angle $\sim J/\text{FSR}$
- Weak coupling limit $J \ll \text{FSR}$: tight binding Hamiltonian



Energy $\phi = \omega nd/c$

Free spectral range $\text{FSR} = c/2nd$

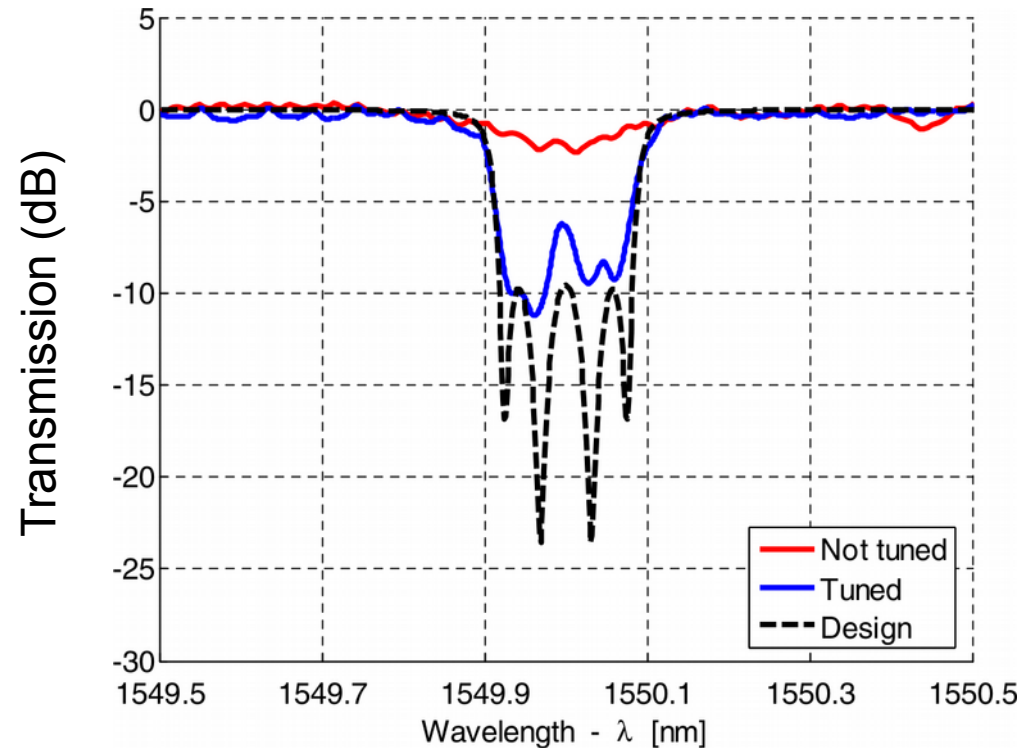
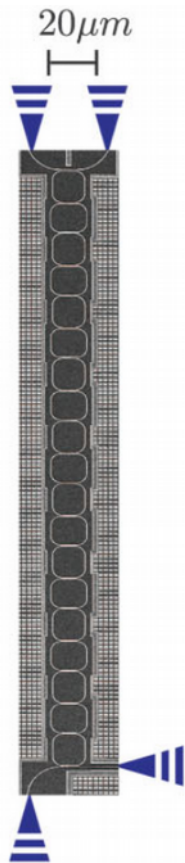
Bloch momentum $\cos(kd) = \frac{\text{FSR}}{J} \sin \phi$

Bandwidth $B = \frac{2\text{FSR}}{\pi} \sin^{-1}(J/\text{FSR})$

$$\hat{H}a_n = J(a_{n-1} + a_{n+1})$$

Limits to CROWs: absorption & disorder

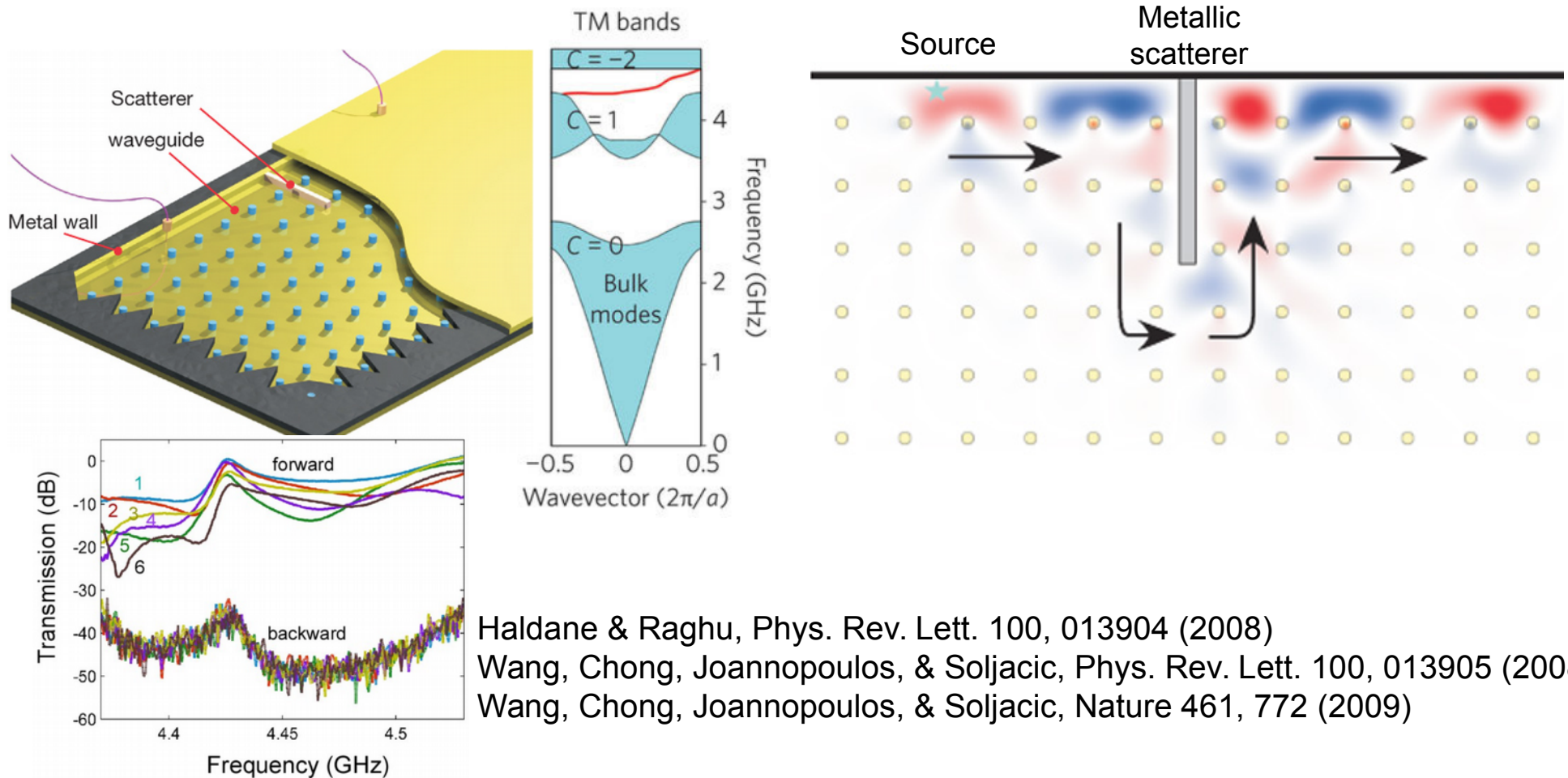
- Intrinsic absorption $\kappa \sim 2\text{GHz}$
- Coupling (off-diagonal) disorder $\Delta J \sim 1\text{GHz}$
- Resonance misalignment (on-site disorder) $\Delta\omega \sim \mathbf{30\text{GHz}}$
- Thermal tuning of $\Delta\omega$: additional power consumption, limited bandwidth



Mittal et al., Phys. Rev. Lett. 113, 087403 (2014); Canciamilla et al., J. Opt. 12, 104008 (2010)

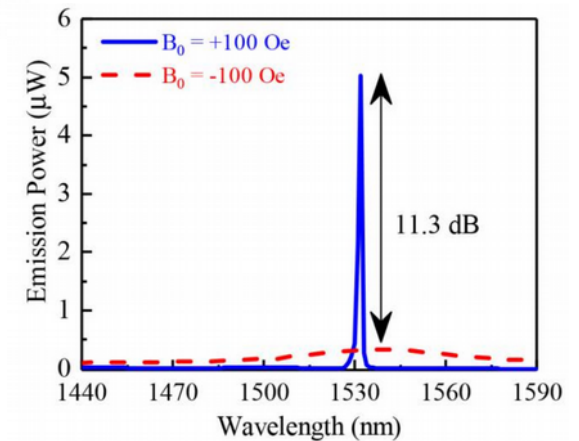
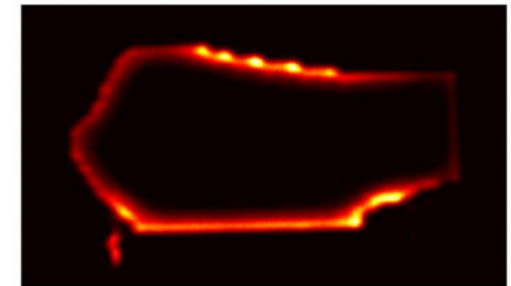
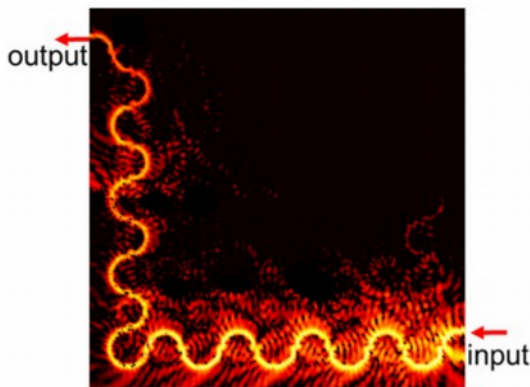
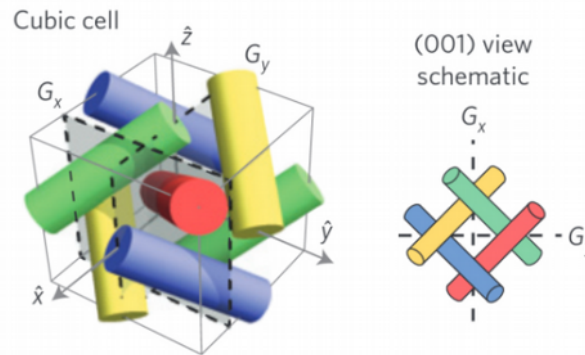
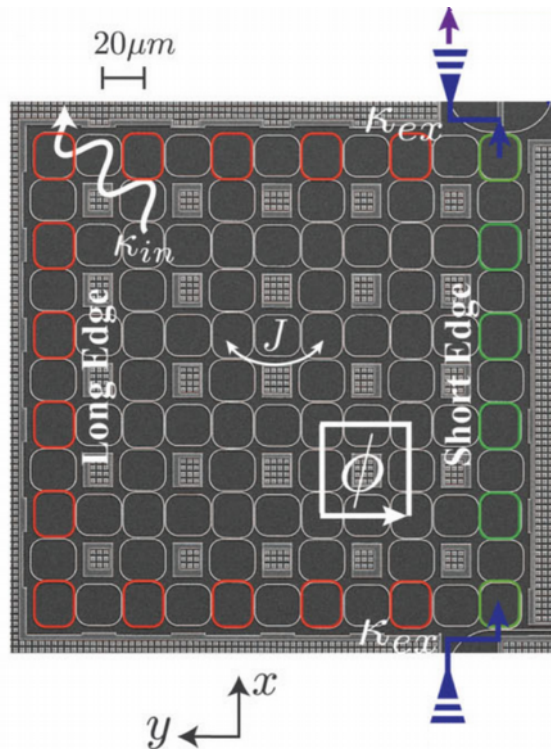
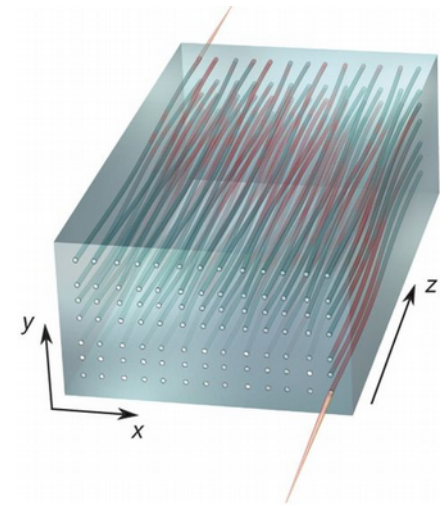
Topological photonics

- Photonic lattices: “insulators” for light, can have topological band structures
- Photonic crystals, waveguide arrays, resonator lattices, metamaterials
- Device applications: Disorder-robust waveguides, lasers, ...



Photonic topological insulators

- Now generalized to various non-magnetic designs
- Waveguide arrays, resonator lattices, metamaterials
- 1D, 2D, 3D, 4D (!)
- New physics, inaccessible in condensed matter?
- Device applications? Unidirectional waveguides, lasers, ...

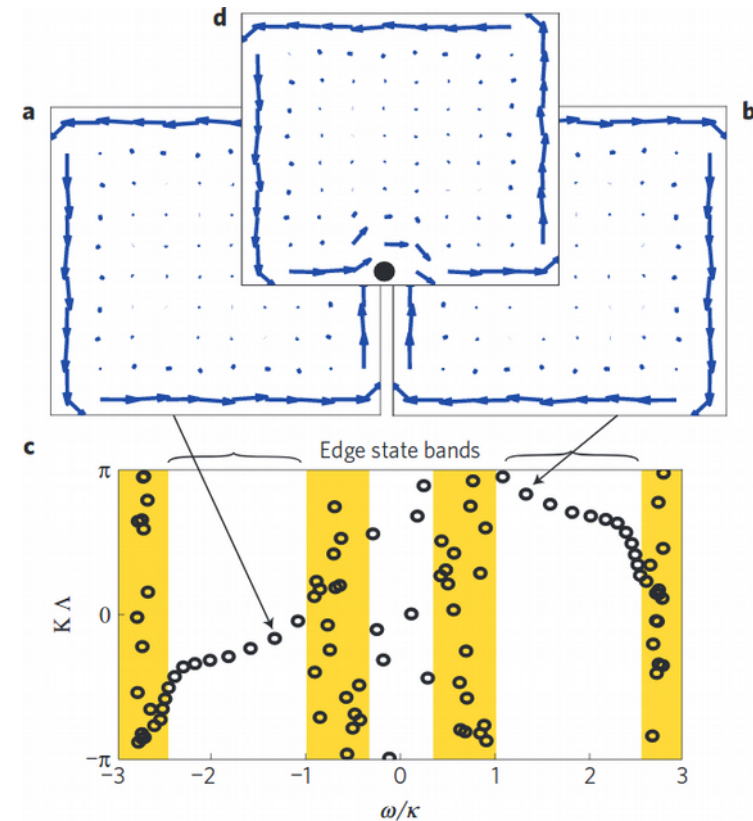
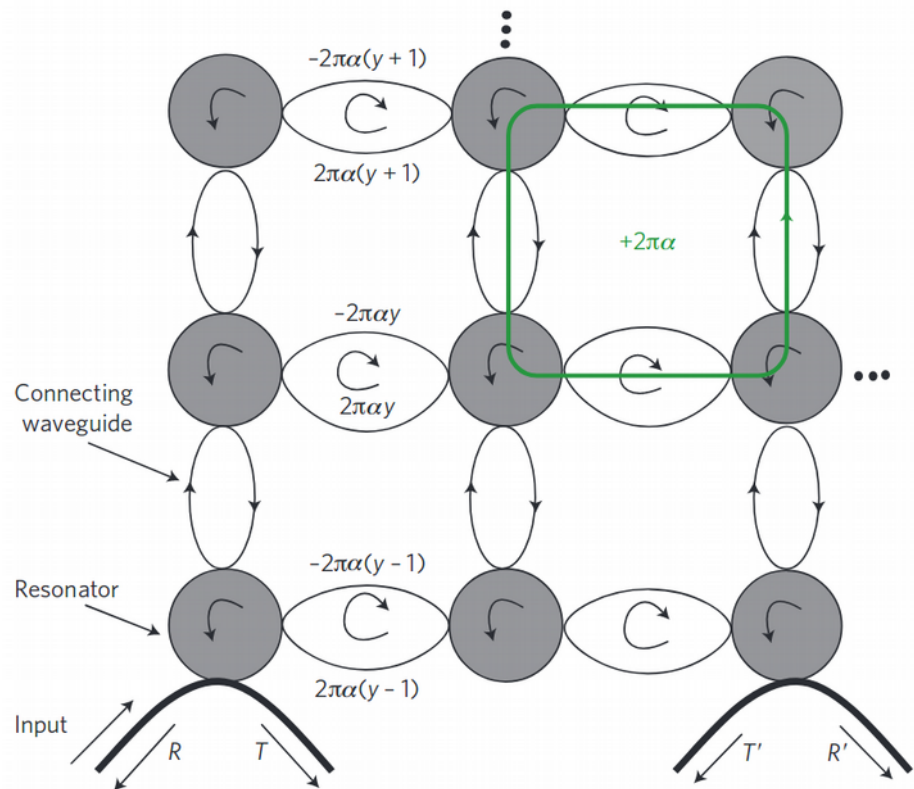


Hafezi et al., Nature Photon. 7, 1001 (2013); Gao et al., Nature Comm. 7, 11619 (2016); Lu et al, Nature Physics 12, 337 (2016); Bahari et al, Science eaao4551 (2017); Zilberberg et al, Nature 553, 59 (2018);
Lu, Joannopoulos, & Soljacic, Nature Photon. 8, 821 (2014); Ozawa et al., arXiv:1802.04173

Topological coupled resonator lattices

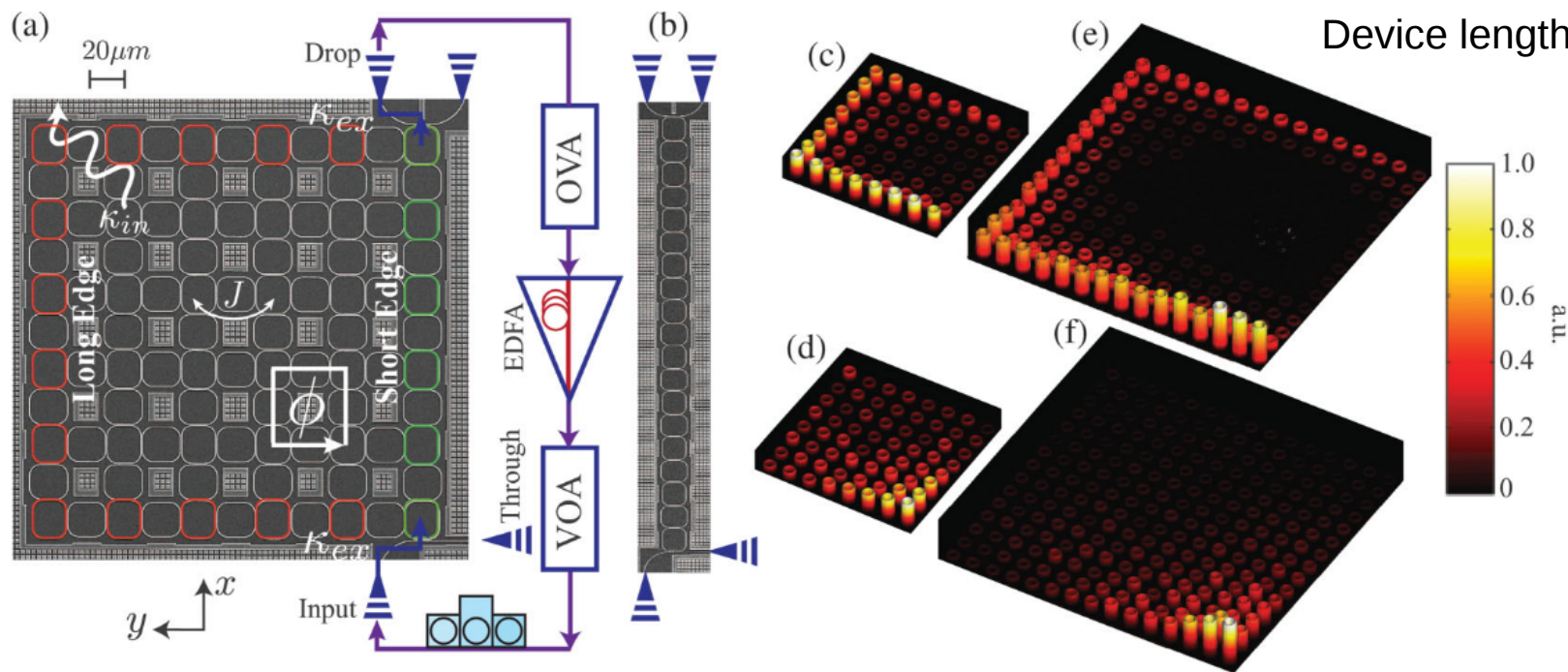
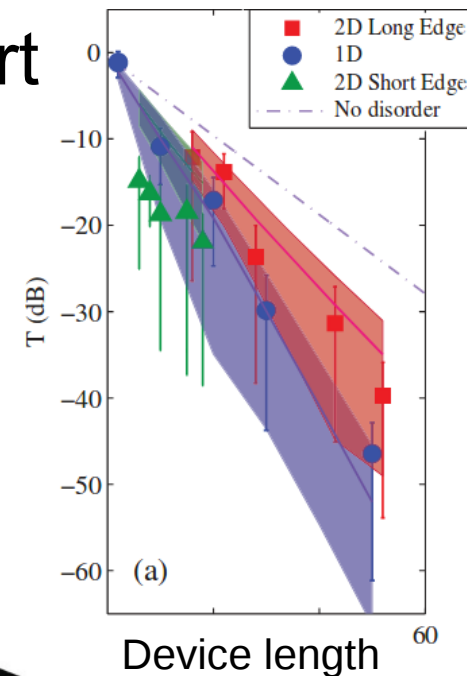
- Light localized to resonant “site” rings, weak coupling via off-resonant “link” rings
- Asymmetric link rings: relative hopping phase \sim vector potential
- Effective magnetic flux emulates quantum Hall effect

$$H_0 = -\kappa \left(\sum_{\sigma, x, y} \hat{a}_{\sigma x+1, y}^\dagger \hat{a}_{\sigma x, y} e^{-i2\pi\alpha y\sigma} + \hat{a}_{\sigma x, y}^\dagger \hat{a}_{\sigma x+1, y} e^{i2\pi\alpha y\sigma} + \hat{a}_{\sigma x, y+1}^\dagger \hat{a}_{\sigma x, y} + \hat{a}_{\sigma x, y}^\dagger \hat{a}_{\sigma x, y+1} \right)$$



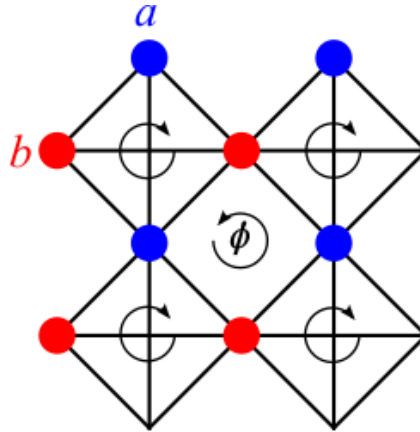
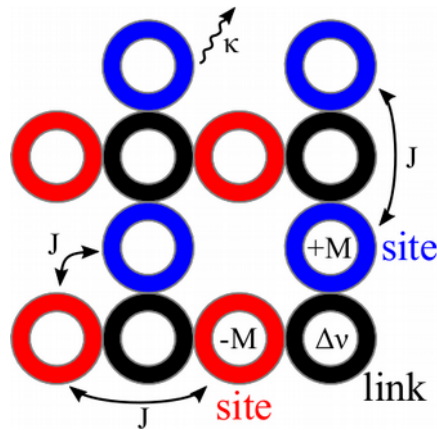
Topologically-protected transport

- Transmission: 1D vs topological 2D devices
- Both have similar losses due to absorption
- 2D edge states protected against disorder-induced localization



Topological phases from next-nearest neighbor coupling

- Off-resonant “link” rings induce NNN coupling
- Checkerboard lattice with effective gauge field
- Tunable Chern insulator, emulates the Haldane model

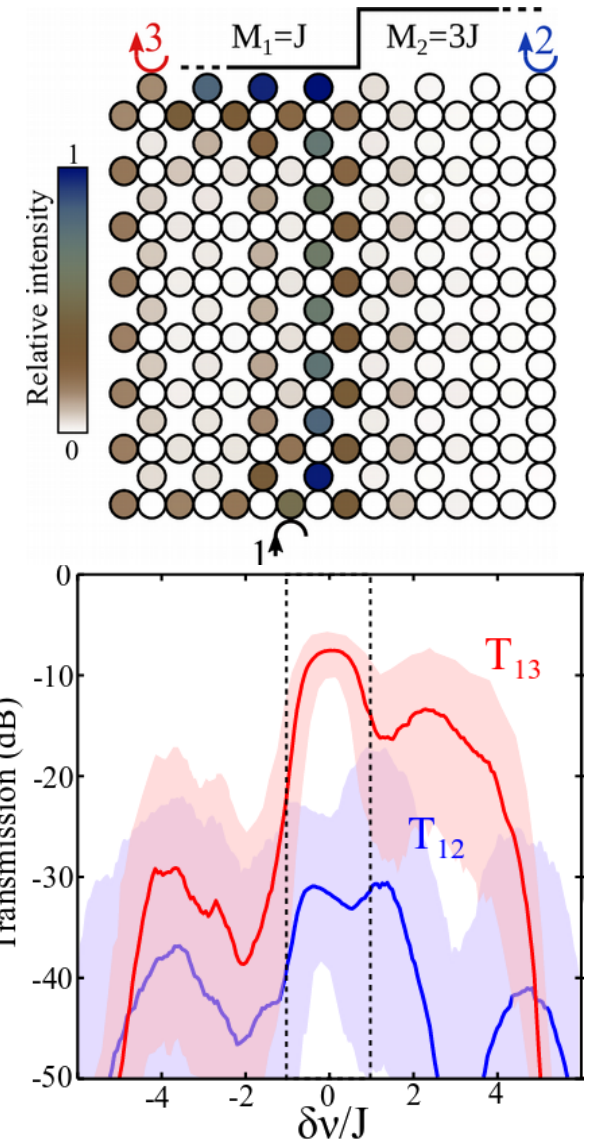


$$\hat{H} = \sum_{r,u} \left(\hat{H}_a + \hat{H}_b + \hat{H}_{ab} + \hat{H}_{ab}^\dagger \right)$$

$$\hat{H}_a = \hat{a}_{x,y}^\dagger \left[(2J \cot \frac{\phi}{2} + M) \hat{a}_{x,y} + J \csc \frac{\phi}{2} \sum_{\pm} \hat{a}_{x,y \pm 1} \right]$$

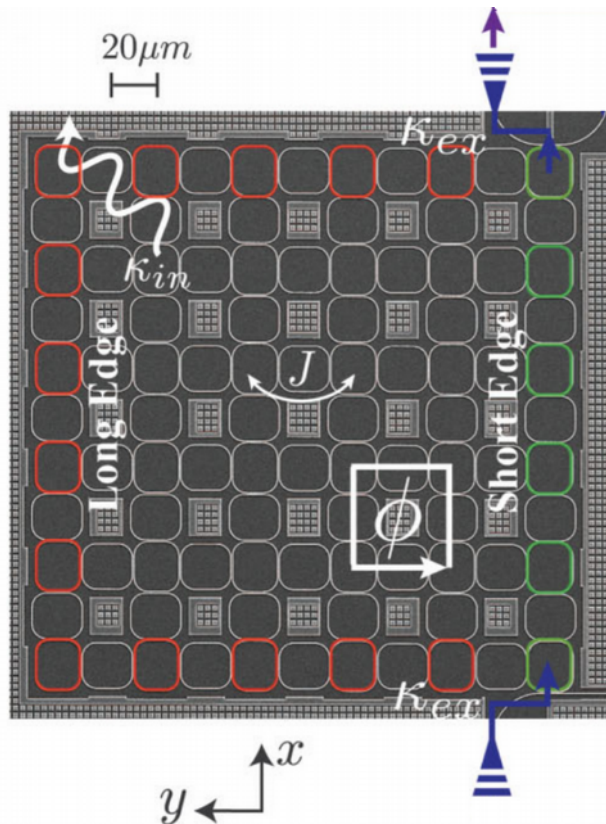
$$\hat{H}_b = \hat{b}_{x,y}^\dagger \left[(2J \cot \frac{\phi}{2} - M) \hat{b}_{x,y} + J \csc \frac{\phi}{2} \sum_{\pm} \hat{b}_{x \pm 1, y} \right]$$

$$\hat{H}_{ab} = J e^{i\phi/4} \csc \frac{\phi}{2} \left[\hat{a}_{x,y}^\dagger (\hat{b}_{x,y} + \hat{b}_{x+1,y+1}) + \hat{b}_{x,y}^\dagger (\hat{a}_{x-1,y} + \hat{a}_{x,y-1}) \right]$$

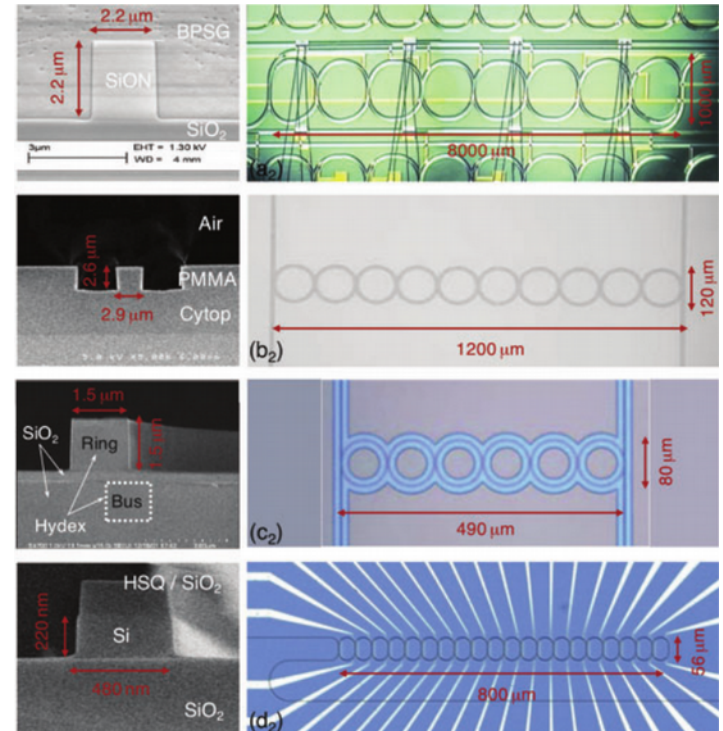


Limits to topological protection

- Topological protection requires a 2D bulk
- Overhead: delay line of length $\sim L$ requires $\sim c \cdot L$ lattice sites
- Increased device footprint compared to 1D CROWs
- Can we find a way to eliminate backscattering in 1D systems?



VS

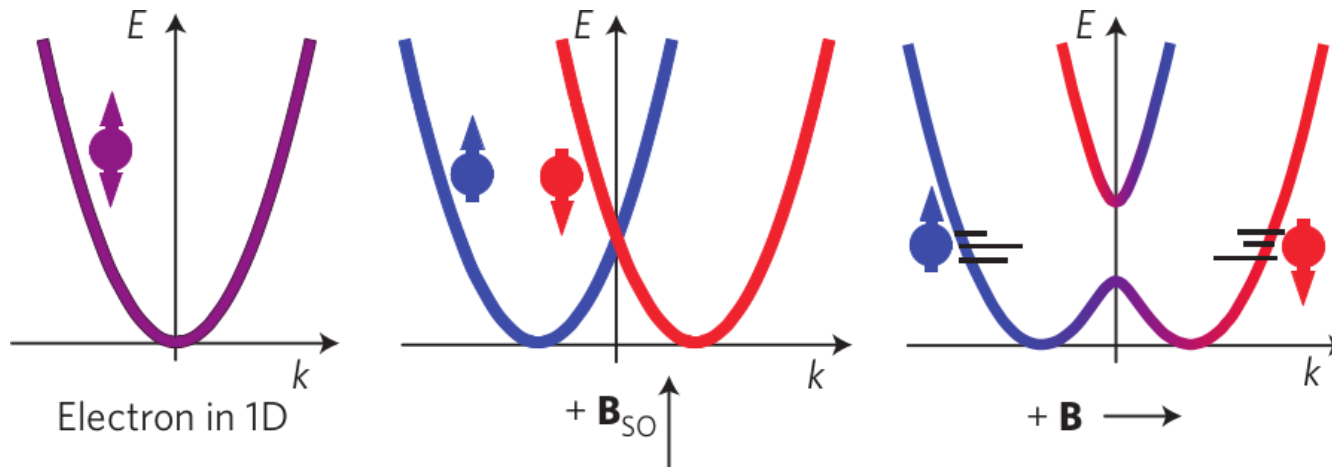
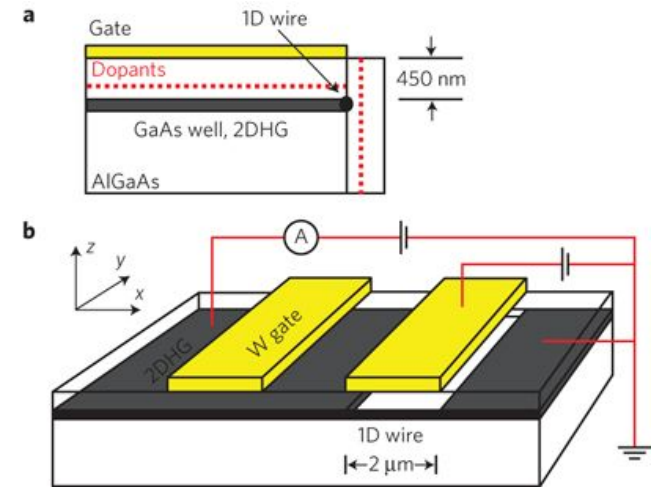


1D helical transport

- Spin-orbit coupling + magnetic field = “spin-orbit gap”
- Backscattering requires a spin flip
- Analogous to edge states of 2D topological insulators

$$H_{\text{tot}} = H_0 + H_{\text{SO}} + H_Z$$

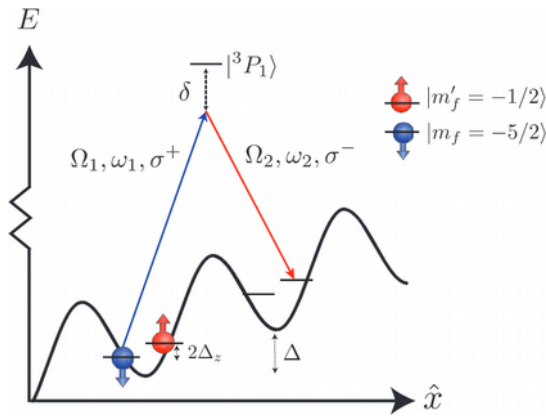
$$H_0 = \hbar^2 k^2 / 2m \quad H_{\text{SO}} = \beta \boldsymbol{\sigma} \cdot (\mathbf{k} \times \nabla V) \quad H_Z = g\mu_B \mathbf{B} \cdot \hat{\boldsymbol{\sigma}}$$



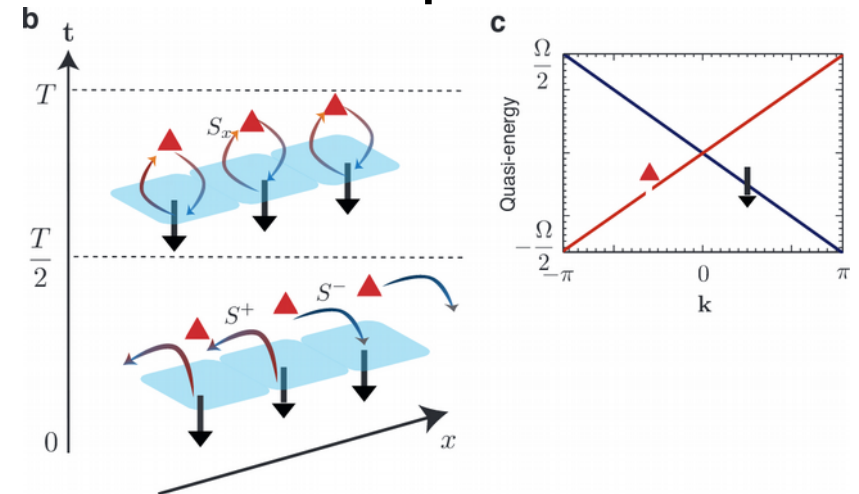
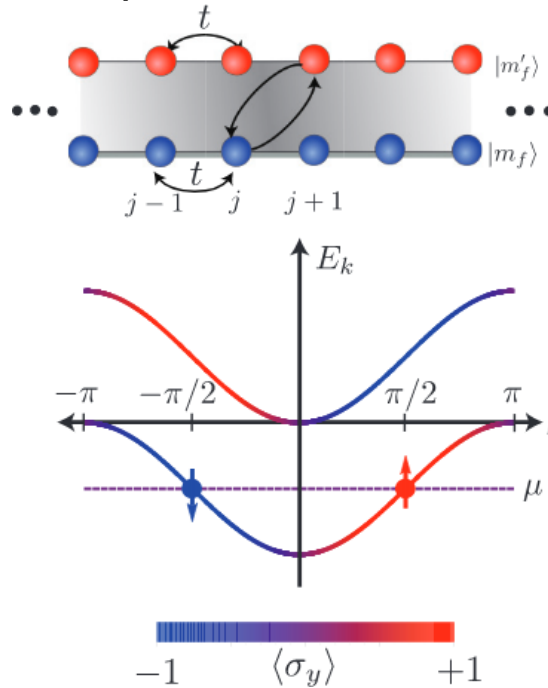
Quay et al., Nature Physics 6, 336 (2010)

Other mechanisms for 1D helical transport

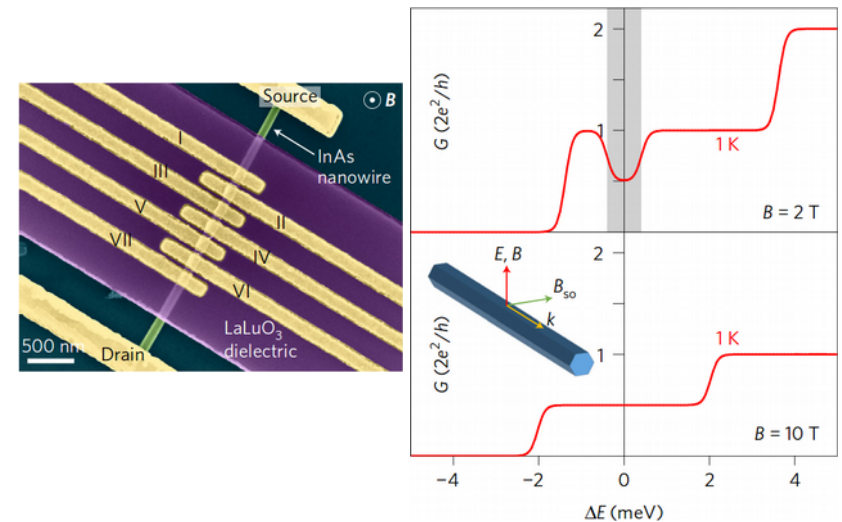
- Strong electron-electron interactions
- Cold atoms + Raman-assisted tunneling
- Periodic driving
- What about with coupled optical resonators?



Budich et al., Phys. Rev. B 92, 245121 (2015)



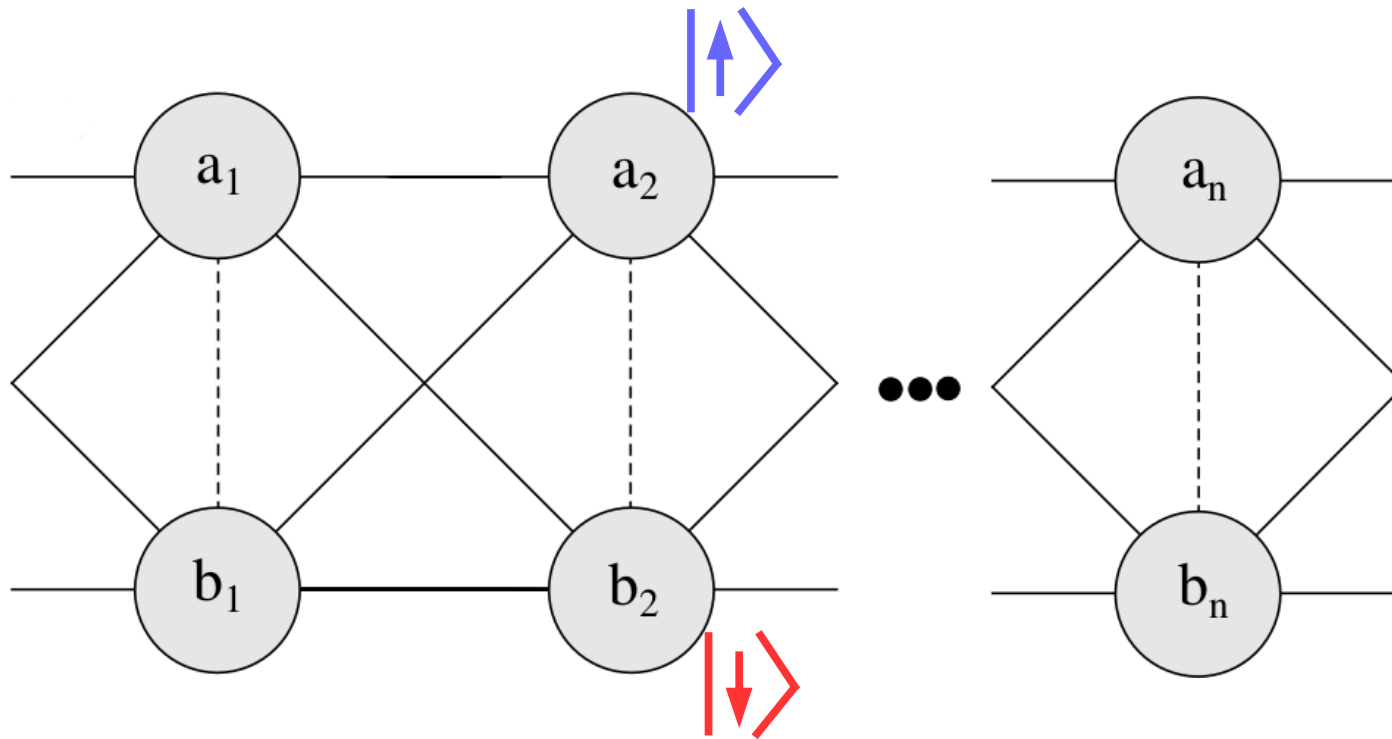
Budich et al, Phys. Rev. Lett. 118, 105302 (2017)



Heedt et al., Nature Physics 13, 563 (2017)

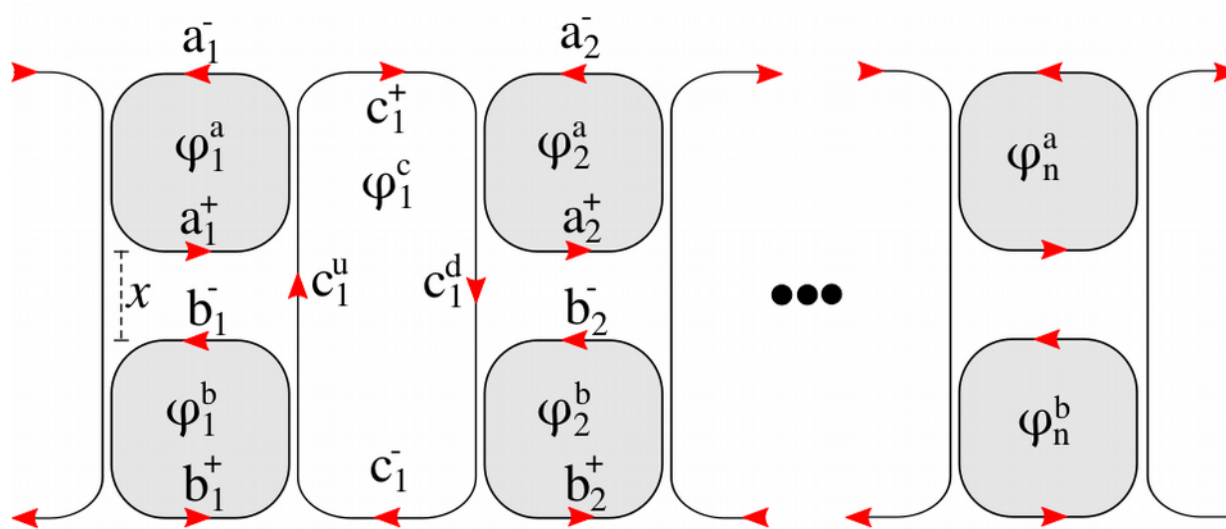
Pseudospin-orbit coupling in a two leg ladder

- Sublattice degree of freedom: pseudo-spin
- Generic Bloch wave Hamiltonian: $\hat{H}(k) = J\mathbf{d}(k) \cdot \hat{\boldsymbol{\sigma}}$
- d_0 : intra-leg, symmetric hopping $\sim H_0$ (effective mass)
- d_x : intra-leg, leg-dependent hopping $\sim H_{\text{SO}}$ (spin-orbit coupling)
- d_z : inter-leg hopping $\sim H_Z$ (Zeeman shift of magnetic field B_x)



Coupled resonator implementation

- Two sublattices formed by resonant site rings
- Coupling mediated by an anti-resonant link ring

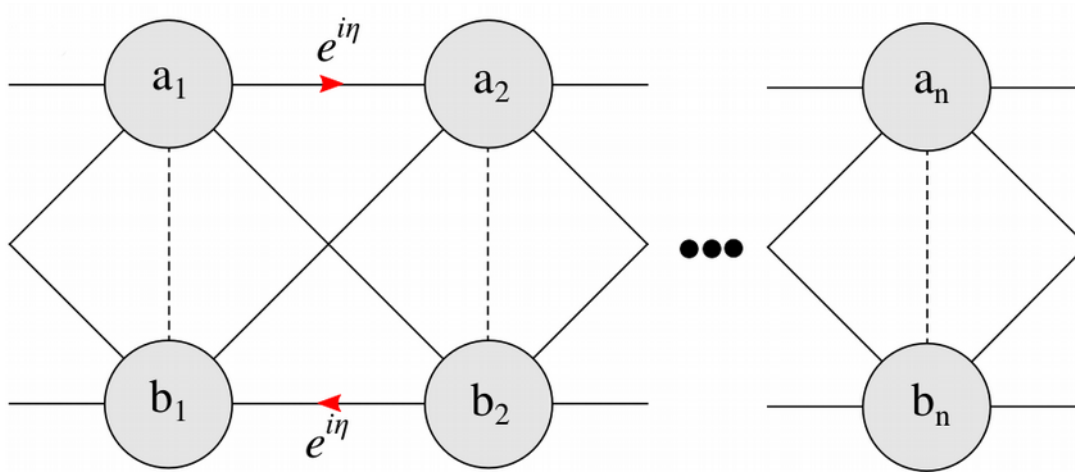


$$\begin{pmatrix} a_n^- e^{-i\varphi_n^a/2} \\ c_n^+ e^{-i(\varphi_n^c/2 - \eta)/2} \end{pmatrix} = \hat{S} \begin{pmatrix} a_n^+ e^{i\varphi_n^a/2} \\ c_n^u e^{i\eta/2} \end{pmatrix}, \quad \begin{pmatrix} a_{n+1}^+ e^{-i\varphi_{n+1}^a/2} \\ c_n^d e^{-i\eta/2} \end{pmatrix} = \hat{S} \begin{pmatrix} a_{n+1}^- e^{i\varphi_{n+1}^a/2} \\ c_n^+ e^{i(\varphi_n^c/2 - \eta)/2} \end{pmatrix},$$

$$\begin{pmatrix} b_n^- e^{-i\varphi_n^b/2} \\ c_n^u e^{-i\eta/2} \end{pmatrix} = \hat{S} \begin{pmatrix} b_n^+ e^{i\varphi_n^b/2} \\ c_n^- e^{i(\varphi_n^c/2 - \eta)/2} \end{pmatrix}, \quad \begin{pmatrix} b_{n+1}^+ e^{-i\varphi_{n+1}^b/2} \\ c_n^- e^{-i(\varphi_n^c/2 - \eta)/2} \end{pmatrix} = \hat{S} \begin{pmatrix} b_{n+1}^- e^{i\varphi_{n+1}^b/2} \\ c_n^d e^{i\eta/2} \end{pmatrix},$$

Weak coupling limit: tight binding model

- Effective SOI strength η tunable via link ring parameters

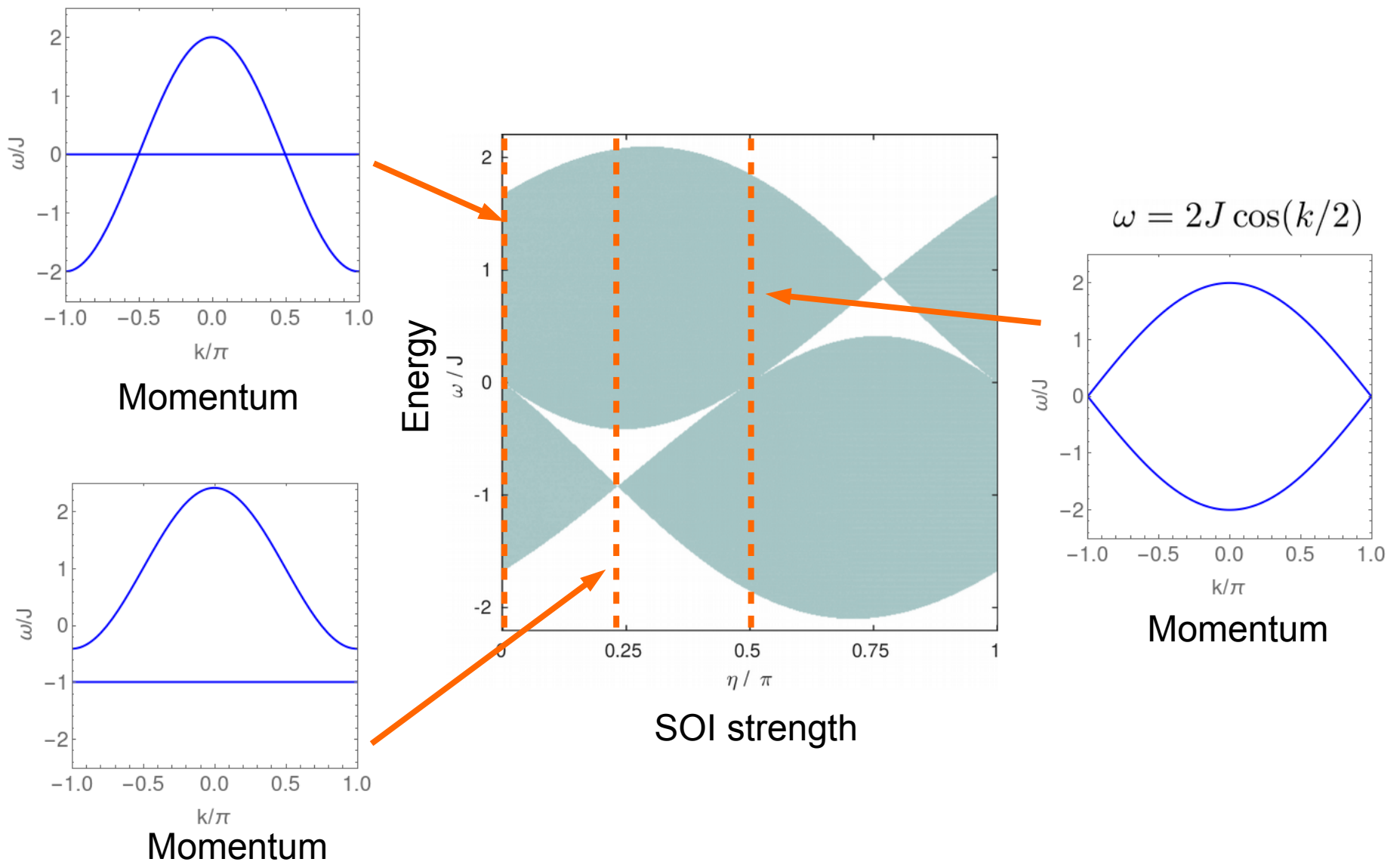


$$\omega a_n = \epsilon_n^a a_n + J \sin \eta b_n + \frac{J}{2} (e^{-i\eta} a_{n-1} + e^{i\eta} a_{n+1} + b_{n-1} + b_{n+1})$$

$$\omega b_n = \epsilon_n^b b_n + J \sin \eta a_n + \frac{J}{2} (e^{i\eta} b_{n-1} + e^{-i\eta} b_{n+1} + a_{n-1} + a_{n+1})$$

Band structure

- Effective SOI strength η tunes between flat band, gapped, & gapless dispersions



Helical Bloch waves

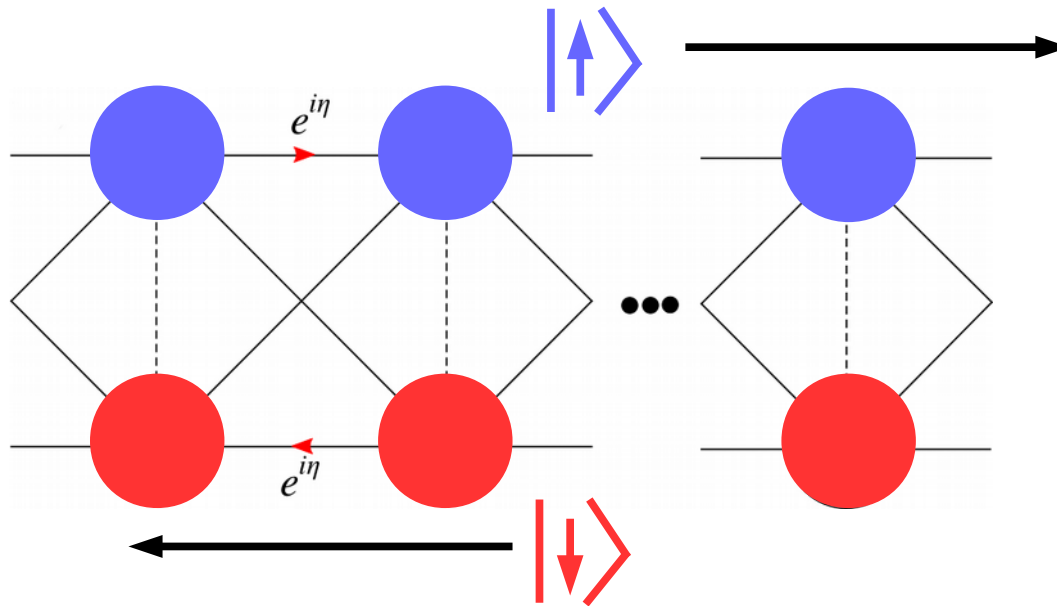
$$\hat{H}(k) = J\mathbf{d}(k) \cdot \hat{\boldsymbol{\sigma}}$$

$$d_0 = (\epsilon^a + \epsilon^b)/(2J) + \cos \eta \cos k,$$

$$d_x = \sin \eta + \cos k,$$

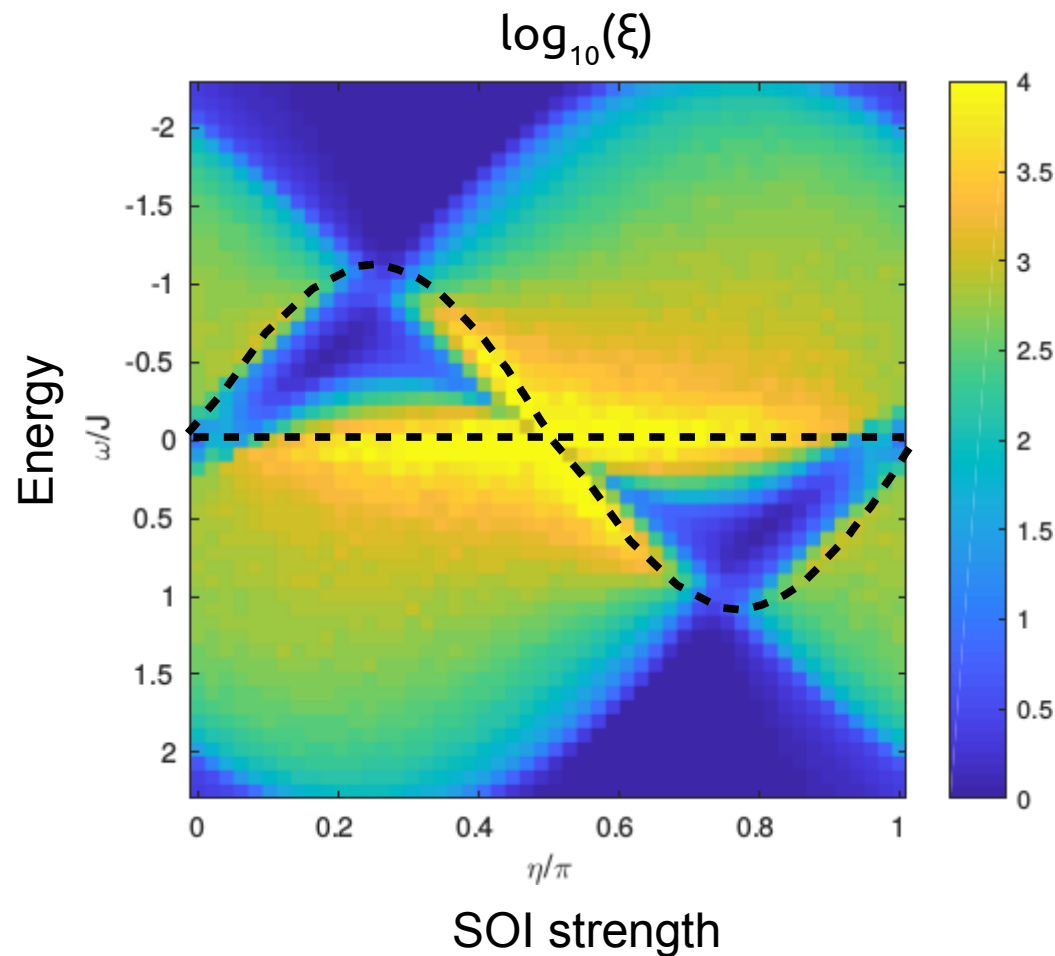
$$d_z = (\epsilon^a - \epsilon^b)/(2J) - \sin \eta \sin k.$$

Spin-momentum locking when $d_x = 0 \Rightarrow \omega=0, -J \sin(2\eta)$



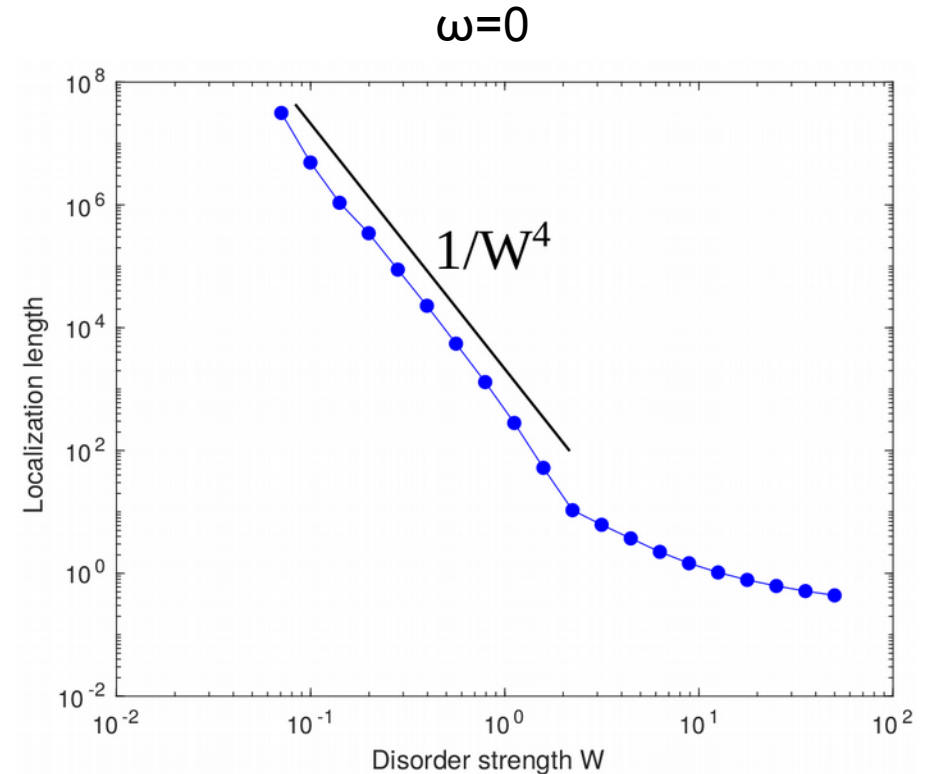
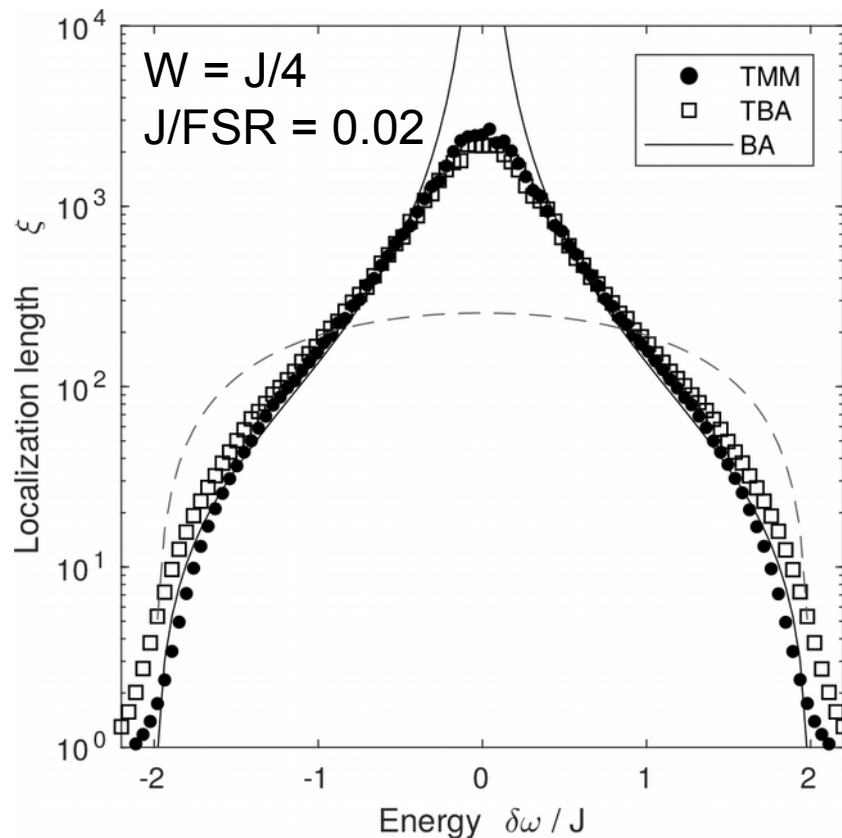
Disorder & Anderson localization

- Fabrication disorder: random resonator detunings $\sim W$
- Compute localization length ξ using transfer matrix method
- Large enhancement of ξ at spin-momentum locked energies



Anderson localization length: $\eta = \pi/2$

- Born approximation: divergence of ξ at $\omega=0$: $\xi = 48J^2(4J^2 - \omega^2)/(W^2\omega^2)$
- Anomalous scaling near band centre: $\xi \sim 1 / W^4$
- Conventional tight binding model: $\xi = 4(4J^2 - \omega^2)/W^2$
- Order of magnitude enhancement of ξ for typical device parameters



Locally-correlated disorder

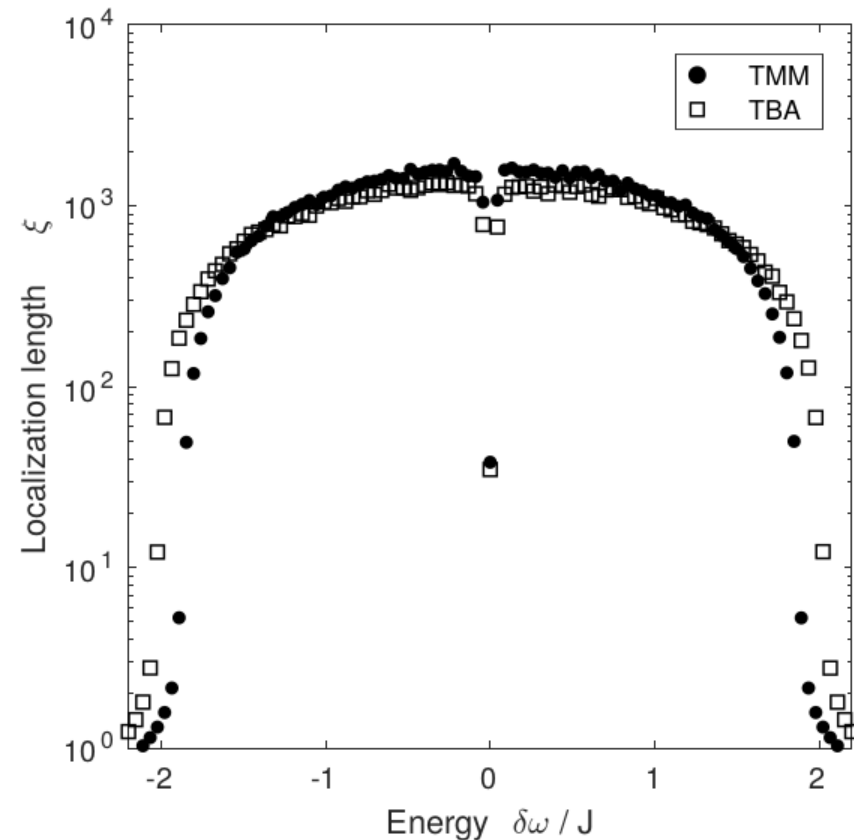
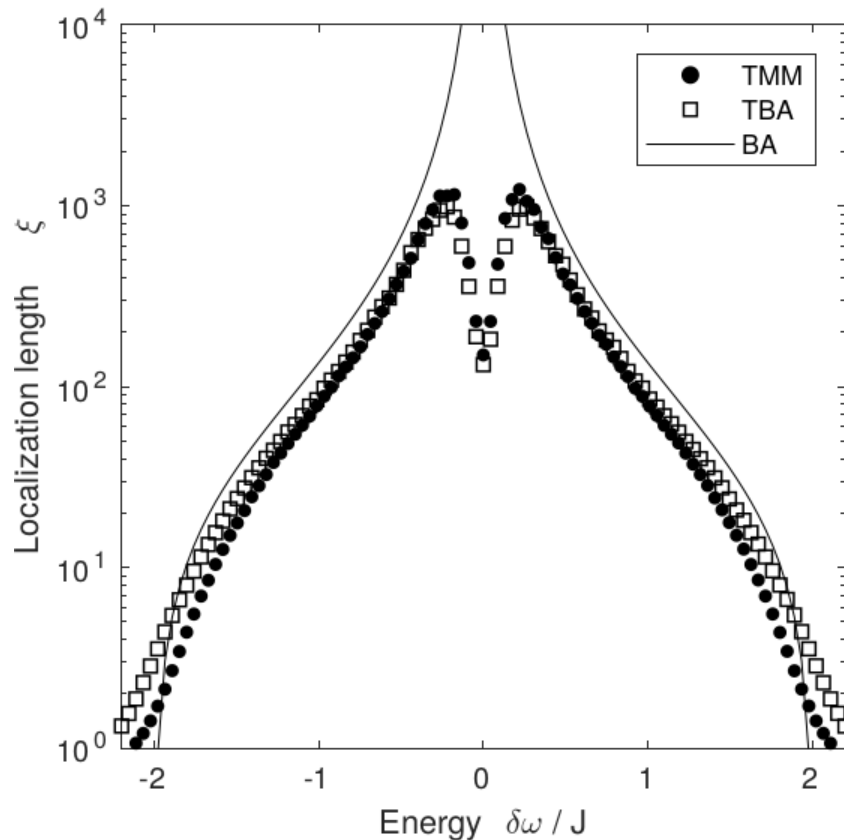
- Disorder can be split into inversion-symmetric and -antisymmetric parts
- Local correlations *enhance* localization at $\omega=0$

Symmetric disorder $\epsilon_n^a = \epsilon_n^b$

Antisymmetric disorder $\epsilon_n^a = -\epsilon_n^b$

BA: $\xi = 24J^2(4J^2 - \omega^2)/(W^2\omega^2)$

BA: $\xi \rightarrow \infty$



Summary

- Coupled resonator lattices can exhibit strong next-nearest neighbour coupling
- Novel way to implement topological phases & synthetic spin-orbit coupling
- 1D helical Bloch waves robust to on-site disorder
- Band structure engineering: 2D topological phases not required for robust transport

D. Leykam, S. Mittal, M. Hafezi, & Y. D. Chong, PRL 121, 023901 (2018); arXiv:1802.02253
J. Han, C. Gneiting, & D. Leykam, in preparation

<http://pcs.ibs.re.kr/>



Funding: IBS Young Science Fellowship (IBS-R024-Y1)