Quantum chaos and time scales for thermalization in isolated many-body systems

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DYNSYSMATH
DYNamical systems and non equilibrium states of complex SYStems :
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Quantum chaos and thermalization in isolated systems of interacting particles

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Other old and new Reviews on the subject

- V. Zelevinsky, B.A. Brown, M. Horoi and N. Frazier,
  The nuclear shell model as a testing ground for many-body quantum chaos,
  Phys. Rep., **276** (1996) 85.
- V.K.B. Kota,
  Embedded random matrix ensembles for complexity and chaos in finite interacting particle systems,
  Phys. Rep. **347** (2001) 223.
- A. Polkovnikov, K. Sengupta, A. Silva, M. Vengalattore,
  Colloquium: Nonequilibrium dynamics of closed interacting quantum systems,
  Rev. Mod. Phys. **83** (2011) 863-883.
- R. Nandkishore, D.A. Huse,
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  Annu. Rev. Condens. Matter Phys. **6** (2015) 15.

Backstory

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- Interaction is necessary to describe equilibrium but is typically neglected, see for instance Bose-Einstein and Fermi-Dirac distributions for occupation numbers
- Canonical description at the end emerges, and the thermal bath shows itself only through $\beta = 1/k_B T$.

1. INTRODUCTION

In 1945 Bogolubov published a paper entitled "An Elementary Example of Relaxation to Statistical Equilibrium in a System Coupled to the Thermal Reservoir."[1] The model in question is specified by the Hamiltonian

$$H = H_S + H_\Sigma + H_{S\Sigma}$$
$$= \tfrac{1}{2}(p^2 + \omega^2 q^2) + \tfrac{1}{2} \sum_{n=1}^{N} (p_n^2 + \omega_n^2 q_n^2) + \varepsilon \sum_{n=1}^{N} \alpha_n q_n q \qquad (1)$$

It consists of a "probe" harmonic oscillator (H_S) weakly ($\varepsilon \to 0$) coupled ($H_{S\Sigma}$) to a "thermal reservoir" (H_Σ) made up of N also harmonic ("thermal") oscillators with some distribution in their frequencies ω_n and energies $E_n = \frac{1}{2}(p_n^2 + \omega_n^2 q_n^2)$.

Under the two assumptions (i) that the statistical properties of the thermal reservoir are described by the Gibbs canonical phase density

$$\rho_\Sigma = \exp\left(\frac{\psi - H_\Sigma}{T}\right) \qquad (2)$$

B.V. Chirikov, Transient Chaos in Quantum and Classical Mechanics, Foundations of Physics, **16**, 1, (1986)

and (ii) that the distributions of α_n and ω_n admit the limit

$$\sum_n \frac{\alpha_n^2}{\omega_n^2} \rightarrow \int d\omega \, J(\omega), \qquad N \rightarrow \infty \tag{3}$$

where $J(\omega)$ is some continuous spectral density of the perturbation, Bogolubov proved that the probe oscillator also approaches the Gibbs distribution as $t \rightarrow \infty$. In other words, in the limit $N \rightarrow \infty$, an infinitely small subsystem H_S of dynamical system (1) exhibits statistical relaxation. If initially, at $t = 0$, the energy of this subsystem $E_S(0) = 0$, the relaxation takes the especially simple form[(1)]:

$$\rho_S = \frac{\omega}{2\pi T_S(t)} e^{-E_S/T_S(t)}, \qquad T_S(t) = T(1 - e^{-\gamma t}) \tag{4}$$

where the relaxation rate

$$\gamma = \frac{\pi}{2} \varepsilon^2 J(\omega) \tag{5}$$

B.V. Chirikov, Transient Chaos in Quantum and Classical Mechanics, Foundations of Physics, **16**, 1, (1986)

The keystone

The keystone of the transient chaos mechanism, as Bogolubov is used to emphasize,[5] is the transition $N \to \infty$, which is called the thermodynamic limit in statistical physics. Some additional statistical hypotheses are also required for this mechanism to be efficient. In Bogolubov's example, they are specified by Eqs. (2) and (3). The first one, in particular, implies randomness and independence of the initial phases of thermal oscillators. It is thus clear that relaxation to the Gibbs distribution here is a particular case of the central limit theorem in probability theory (cf. Ref. 4).

B.V. Chirikov, Transient Chaos in Quantum and Classical Mechanics, Foundations of Physics, **16**, 1, (1986)

An old story

Recently, Bogolubov came back to his classical example in its quantum version.[5]

Presently, a remarkable peculiarity of Bogolubov's example is that the system (1) is completely (and trivially) integrable at any finite N. Such a system has a complete set of N commuting integrals of motion, and its trajectories fill up N-dimensional tori. Hence the motion is not even ergodic. Nevertheless, in the limit $N \to \infty$, the statistical relaxation does occur, which requires at least the mixing. Moreover, the relaxation is exponential in time, obeying (4), which is one of the strongest statistical properties according to modern ergodic theory. At present the possibility of such statistical properties in an infinite system which is completely integrable at any finite N has been rigorously proved indeed.[3] In Bogolubov's example it is explained by the fact that after the *earlier* transition to the limit $N \to \infty$ the perturbation spectrum of the probe oscillator becomes *continuous* [Eq. (3)], which is a necessary condition for the mixing. The importance of

B.V. Chirikov, Transient Chaos in Quantum and Classical Mechanics, Foundations of Physics, **16**, 1, (1986)

We do not need chaos to have statistical relaxation in an infinite system.

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7. N. N. Bogolubov and N. M. Krylov, "On Fokker–Planck's Equation Derived in Perturbation Theory by the Method Based on Spectral Properties of Perturbation Hamiltonian (Application to Classical and Quantum Mechanics)," in N. N. Bogolubov, *Selected Papers* (Naukova Dumka, Kiev, 1970), Vol. 2, p. 5 (Russian).

In classical finite systems a sufficiently chaotic interaction is necessary (instead of the thermodynamic limit) to provide statistical relaxation.

What about quantum systems? What is the relation between integrability and thermalization?

- Nuclear and atomic physics V. Zelevinsky, M. Horoi, and B.A. Brown, Phys. Lett. B **350**, (1995) 141; M. Horoi, V. Zelevinsky, and B.A. Brown, Phys. Rev. Lett. **74**, (1995) 5194; V. Zelevinsky, B.A. Brown, M. Horoi and N. Frazier, Phys. Rep. **276**, (1996) 85; V. Zelevinsky, Ann. Rev. Nucl. Part. Sci. **46**, (1996) 237., V. V. Flambaum, A. A. Gribakina, G. F. Gribakin, M. G. Kozlov, Phys. Rev. A 50 (1994) 267; V. V. Flambaum and F. M. Izrailev, Statistical Theory of Finite Fermi-Systems Based on the Structure of Chaotic Eigenstates, Phys. Rev. E **56**, 5144 (1997)
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On the Connection between Quantization of Nonintegrable Systems and Statistical Theory of Spectra (*).

G. Casati and F. Valz-Gris
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(ricevuto l'8 Aprile 1980)

The energy spectrum of a quantum system possessing an integrable classical limit can be conveniently approximated by means of semi-classical quantization rules (1-5).

In recent years the problem of finding similar rules for nonintegrable systems has attracted considerable interest (6-9). This particular problem fits naturally into the more general question as to what properties of a quantum system, if any, should be assumed as a counterpart of classical ergodic properties (10-14).

It is now generally agreed that in the discrete-energy spectrum a «regular» and an «irregular» part should be distinguished. The former is suitably described by semi-classical quantization in the region of phase-space filled with invariant tori (15,16).

G.Casati, I.Guarneri and F. Valz-Gris, Lettere a Nuovo Cimento 1, 1980 (279); O. Bohigas, M.J. Giannoni, C. Schmit, Phys. Rev. Lett. 52 (1984)

- Integrable and chaotic systems are characterized, in general, by different spectral properties
- while the former display a Poisson nearest neighbor level distribution, the latter are characterized by a Wigner-Dyson distribution (level repulsion).

Is Wigner-Dyson distribution necessary for statistical relaxation?

PRL **108**, 094102 (2012) PHYSICAL REVIEW LETTERS week ending 2 MARCH 2012

Chaos and Statistical Relaxation in Quantum Systems of Interacting Particles

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[2]*Dipartimento di Matematica e Fisica and Interdisciplinary Laboratories for Advanced Materials Physics, Universitá Cattolica, via Musei 41, 25121 Brescia, and INFN, Sezione di Pavia, Italy*
[3]*Instituto de Física, Universidad Autónoma de Puebla, Apartado Postal J-48, Puebla, 72570, Mexico*
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(Received 20 October 2011; published 1 March 2012)

We study the transition to chaos and the emergence of statistical relaxation in isolated dynamical quantum systems of interacting particles. Our approach is based on the concept of delocalization of the eigenstates in the energy shell, controlled by the Gaussian form of the strength function. We show that, although the fluctuations of the energy levels in integrable and nonintegrable systems are different, the global properties of the eigenstates are quite similar, provided the interaction between particles exceeds some critical value. In this case, the statistical relaxation of the systems is comparable, irrespective of whether or not they are integrable. The numerical data for the quench dynamics manifest excellent agreement with analytical predictions of the theory developed for systems of two-body interactions with a completely random character.

DOI: 10.1103/PhysRevLett.108.094102 PACS numbers: 05.45.Mt, 02.30.Ik, 05.30.−d, 05.70.Ln

An annoying counter-example : Spin models

$$H_1 = H_0 + \mu V_1,$$
$$H_0 = \sum_{i=1}^{L-1} J\left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y\right),$$
$$V_1 = \sum_{i=1}^{L-1} J S_i^z S_{i+1}^z$$

Figure: Model 1 : Interacting fully-integrable

$$H_2 = H_1 + \lambda V_2,$$
$$V_2 = \sum_{i=1}^{L-2} J\left[\left(S_i^x S_{i+2}^x + S_i^y S_{i+2}^y\right) + \mu S_i^z S_{i+2}^z\right].$$

Figure: Model 2 : Interacting fully chaotic

Ref. L.F.Santos, F.Borgonovi, F.M.Izrailev, PRL **108**, 094102 (2012)

Level statistics for both models

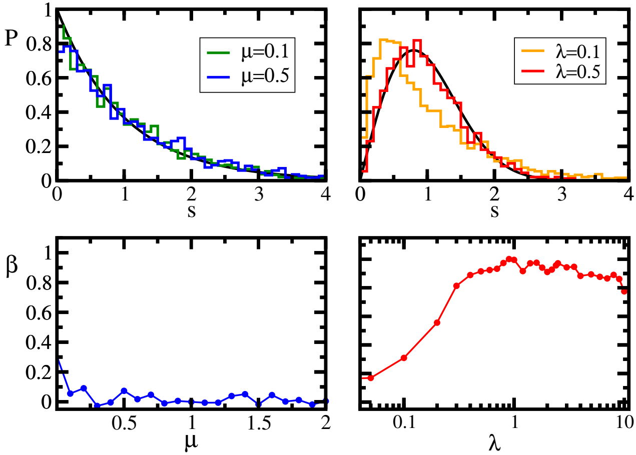


Figure: left column : model 1 (integrable), right column (chaotic). Top panels : nearest neighbor level spacing distribution. Bottom panels : Brody parameter. Ref. L.F.Santos, F.Borgonovi, F.M.Izrailev, PRL **108**, 094102 (2012)

Definition of Shannon entropy for a state $|\psi\rangle$ in the basis $|n\rangle$

$$S(\psi) = -\sum_n |\langle n|\psi\rangle|^2 \ln |\langle n|\psi\rangle|^2, \tag{1}$$

Note that

- for $|\psi\rangle = |n_0\rangle$ (localized in one site of the $|n\rangle$ basis) then $S(\psi) = 0$.
- for $|\psi\rangle = 1/\sqrt{N}$ (fully delocalized in the $|n\rangle$ basis) then $S(\psi) = \ln N$.

Statistical relaxation for observables : Shannon entropy

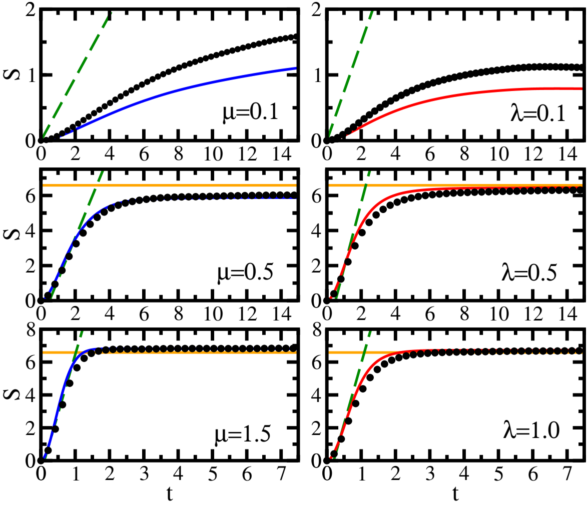


Figure: left column : model 1 (integrable), right column (chaotic). Ref. L.F.Santos, F.Borgonovi, F.M.Izrailev, PRL **108**, 094102 (2012)

Take home

In finite systems, statistical relaxation occurs for some observable even for integrable models. It is more important the presence of sufficiently extended and irregular eigenstates than the integrability/non-integrability of the model.

Quench dynamics for a completely random two-body interacting system of N bosons in M single particle energy levels with average distance $\langle d \rangle = 1$:

TBRI

$$H = H_0 + V = \sum \epsilon_s \, a_s^\dagger a_s + \sum V_{s_1 s_2 s_3 s_4} \, a_{s_1}^\dagger a_{s_2}^\dagger a_{s_3} a_{s_4} \,, \qquad (2)$$

and for a completely deterministic and chaotic (Wigner-Dyson statistics, chaotic eigenstates) spin system:

SPIN

$$\begin{aligned} H = H_0 + V \quad &= \quad \frac{J}{4} \sum_s \left(\sigma_s^x \sigma_{s+1}^x + \sigma_s^y \sigma_{s+1}^y + \Delta \sigma_s^z \sigma_{s+1}^z \right) + \qquad (3) \\ & \lambda \frac{J}{4} \sum_s \left(\sigma_s^x \sigma_{s+2}^x + \sigma_s^y \sigma_{s+2}^y + \Delta \sigma_s^z \sigma_{s+2}^z \right) \,, \qquad (4) \end{aligned}$$

The eigenstates $|\alpha\rangle = \sum_k C_k^{(\alpha)} |k\rangle$ of H can be written in terms of the basis states $|k\rangle$ of H_0, where

Unperturbed *vs* full Hamiltonian

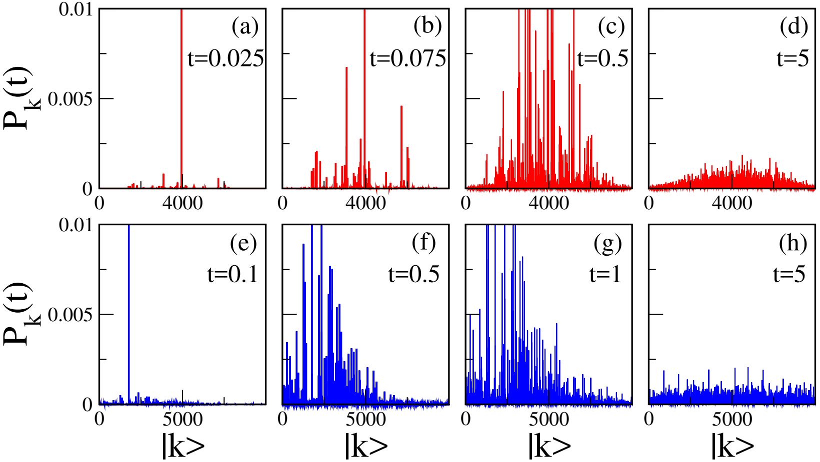
$$H|\alpha\rangle = E^{\alpha}|\alpha\rangle; \quad H_0|k\rangle = E_k^0|k\rangle. \tag{5}$$

An eigenstate $|\alpha\rangle$ of the total Hamiltonian is called chaotic when its number N_{pc} of principal components C_k^{α} is sufficiently large and C_k^{α} can be considered as random and non-correlated ones.

Initial state

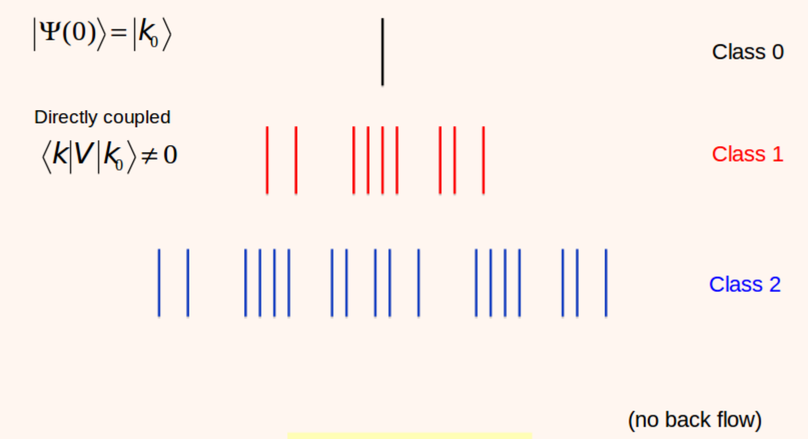
As initial state an eigenstate of H_0 is chosen and the dynamics observed under the evolution of the full Hamiltonian H.

Exploring the Many-body Hilbert space from an initial unperturbed state $|k_0\rangle$ of H_0 : top TBRI, bottom SPIN



$$P_k(t) = |\langle k|e^{-iHt}|k_0\rangle|^2 \quad (6)$$

Partitioning the many-body space: The Cascade Model



V.V. Flambaum and F.M. Izrailev, *Statistical Theory of Finite Fermi-Systems Based on the Structure of Chaotic Eigenstates*, Phys. Rev. E **56**, (1997) 5144

Definitions

W_k is the probability to be in the k-th class

The flow dynamics: a phenomenological approach

$$\frac{dW_0}{dt} = -\Gamma(W_0 - \overline{W_0^\infty}),$$
$$\frac{dW_1}{dt} = -\Gamma(W_1 - \overline{W_1^\infty}) + \Gamma(W_0 - \overline{W_0^\infty}), \tag{7}$$
$$\ldots$$

where $\overline{W_k^\infty} = \lim_{T\to\infty}(1/T)\int_0^T \, dt \, W_k(t)$

Only two classes are needed to describe the dynamics up to saturation, since the number of elements in the class 2, $\mathcal{N}_2$, is equal to the dimension of the Hilbert space $\mathcal{D}$ and $W_2 = 1 - W_0 - W_1$, due to conservation of probability. Backflow is neglected.

Analytical Estimates for two classes

Class Probabilities

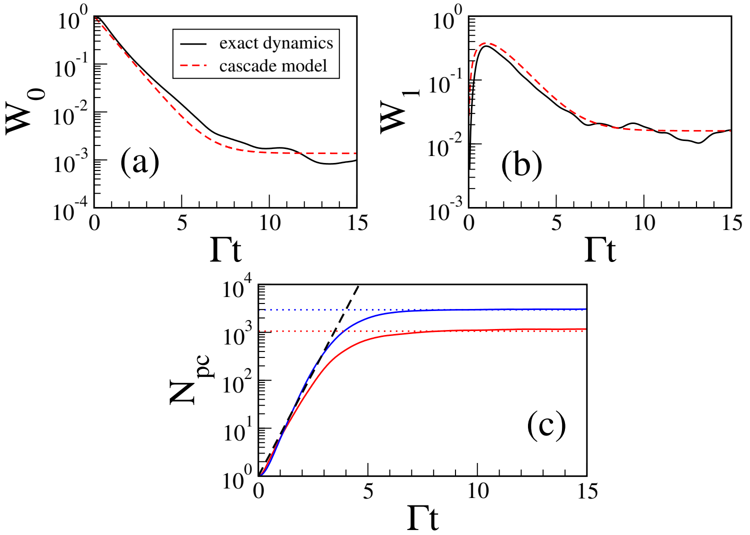
$$\begin{aligned} W_0(t) &= e^{-\Gamma t}(1-\overline{W_0^\infty}) + \overline{W_0^\infty}, \\ W_1(t) &= \Gamma t e^{-\Gamma t}(1-\overline{W_0^\infty}) + \overline{W_1^\infty}(1-e^{-\Gamma t}). \end{aligned} \tag{8}$$

With these expressions one can derive the time dependence for $N_{pc}(t)$,

Exponential growth of the number of principal components

$$N_{pc}(t) = \frac{1}{\sum_n |\langle n|\psi\rangle|^4} \simeq \frac{1}{\sum_n W_n^2/\mathcal{N}_n} \simeq \left[W_0^2 + W_1^2/\mathcal{N}_1\right]^{-1} \sim e^{2\Gamma t}, \tag{9}$$

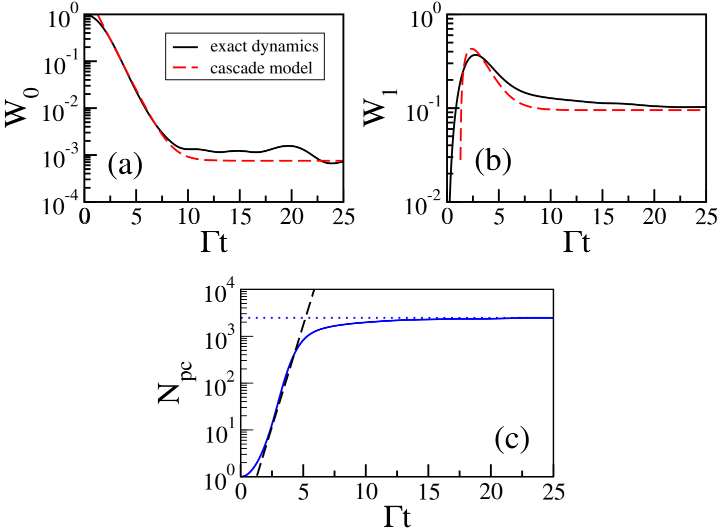
F.Borgonovi, F.M. Izrailev, L.F. Santos, *Exponentially fast dynamics in the Fock space of chaotic many-body systems*, arXiv:1802.08265



F.Borgonovi, F.M. Izrailev, L.F. Santos, *Exponentially fast dynamics in the Fock space of chaotic many-body systems*,

arXiv:1802.08265

Exponential growth, SPIN



F.Borgonovi, F.M. Izrailev, L.F. Santos, *Exponentially fast dynamics in the Fock space of chaotic many-body systems*,

arXiv:1802.08265

Time scales for relaxation

Our data clearly manifest the existence of two time scales. The first one, $t_\Gamma \simeq 1/\Gamma$, corresponds to the characteristics decay time of W_0, as shown in Eq. (8). The second, t_S, is the time scale for the saturation of the dynamics and can be estimated from $e^{2\Gamma t} \simeq N_{pc}^{\infty}$, which gives

$$t_S \simeq \ln(N_{pc}^{\infty})/2\Gamma. \tag{10}$$

Assuming a Gaussian shape for both the density of states and the LDOS, we show that the maximal value N_{pc}^{max} is

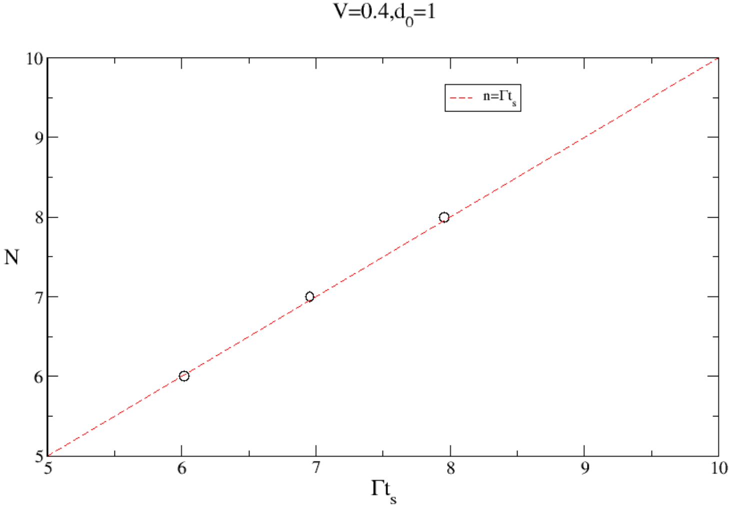
$$N_{pc}^{max} = \eta\sqrt{1-\eta^2}\mathcal{D} \tag{11}$$

where $\eta = \Gamma/\sigma\sqrt{2}$ and σ is the width of the density of states. For $M \sim 2N$ and for $M, N \gg 1$ one gets the estimate

Time scale for statistical relaxation

$$t_S \sim N/\Gamma = Nt_\Gamma. \tag{12}$$

Dependence on the number of particles, TBRI (Fermi particles)



J.C. Molina, F.Borgonovi, F.M.Izrailev, in preparation

In order to describe the process of thermalization we consider the dynamics of the single particle occupation number distribution,

$$n_s(t) \equiv \langle \hat{n}_s \rangle_t = \langle \psi(t) \,|\, \hat{n}_s \,| \psi(t) \,\rangle = \sum_k n_s^k |\langle k|\psi(t)\rangle|^2. \qquad (13)$$

and we studied the statistical properties after relaxation,

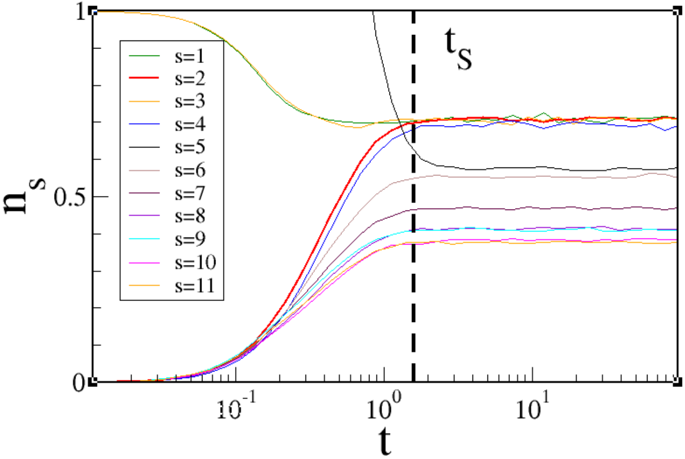
fluctuations of the classical observable $n_s(t)$ about equilibrium

$$\Delta^2 n_s = \int dn_s \, P(n_s) n_s^2 - \left[\int dn_s \, P(n_s) n_s \right]^2 \qquad (14)$$

and long-time averaged quantum fluctuations

$$\delta^2 n_s = \lim_{T \to \infty} \frac{1}{T} \int_0^T dt \left[\langle \hat{n}_s^2 \rangle_t - \langle \hat{n}_s \rangle_t^2 \right] \qquad (15)$$

Single-particle occupation number distribution



Fausto Borgonovi, Felix M. Izrailev, *Emergence of correlations in the process of thermalization of interacting bosons*, arXiv:1806.00435

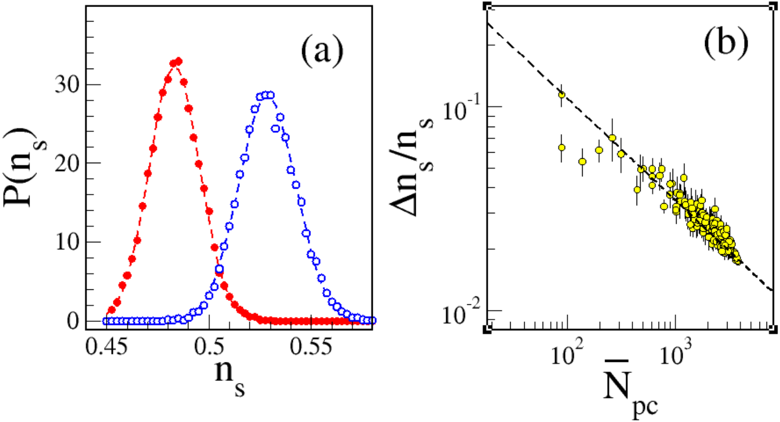
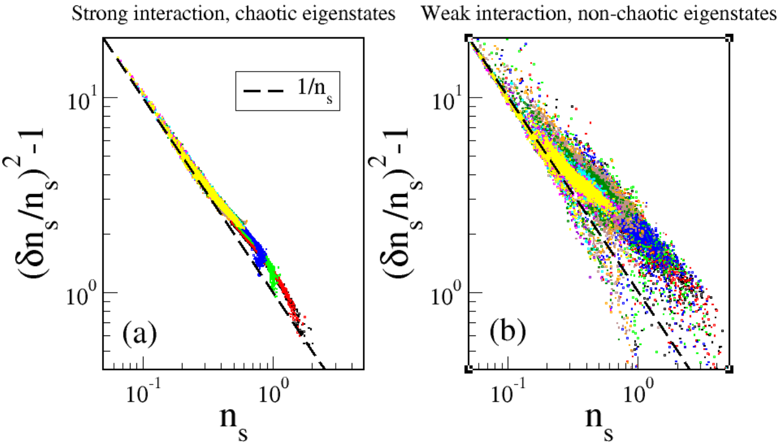
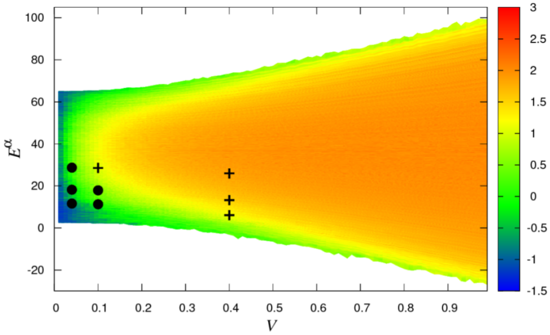


Figure: a) Gaussian fluctuations about the long-time average. b) Relative classical fluctuations decay as $1/\sqrt{\overline{N}_{pc}}$. $\overline{N}_{pc}$ in finite systems plays the role of the number of particles in statistical mechanics. Ref. : Fausto Borgonovi, Felix M. Izrailev, *Emergence of correlations in the process of thermalization of interacting bosons*, arXiv:1806.00435



Theoretical prediction for bosons in the canonical ensemble

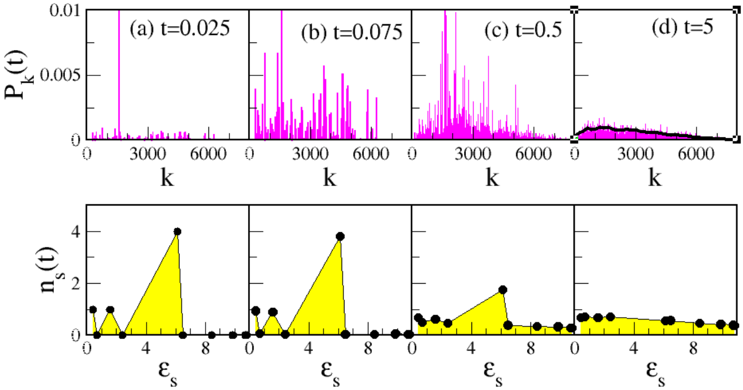
$$\delta^2 n_s / n_s^2 = 1 + 1/n_s \quad (16)$$



Even if the saturation time t_S, depends on the particularly chosen observable, we can consider as thermalized, in the sense given by statistical mechanics, any finite system, for sufficiently strong chaos, and for $t > N/\Gamma$.

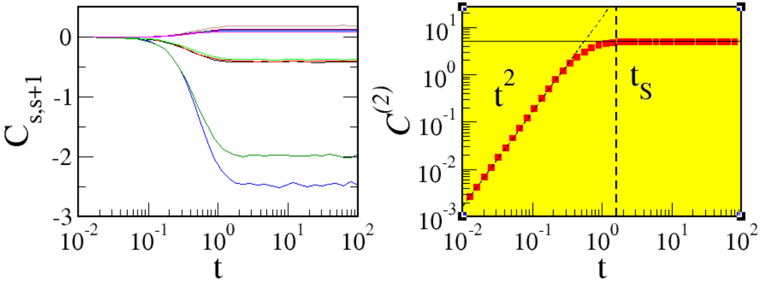
Fausto Borgonovi and Felix M. Izrailev *Localized Thermal States*, AIP Conference Proceedings 1912, 020003 (2017)

Creation of correlations between levels : emergence of BE distribution



Local two-levels correlation function

$$C_{s,s+1}(t) = \langle k_0 | [\hat{n}_s(t) - \hat{n}_s][\hat{n}_{s+1}(t) - \hat{n}_{s+1}] | k_0 \rangle. \tag{17}$$



Global two-levels correlation function

$$\mathcal{C}^{(2)}(t) = |\sum_{s=1}^{M-1} C_{s,s+1}(t)| \tag{18}$$

1. the perturbative behavior (t^2) lasts up to the saturation time t_S
2. absence of exponential growth
3. non-zero asymptotical global correlation

take home

Our results show how the information initially encoded in a local unperturbed state, spreads over the whole system and transforms onto global correlations specified by the BE distribution of occupation numbers.

Fausto Borgonovi, Felix M. Izrailev, *Emergence of correlations in the process of thermalization of interacting bosons*,

arXiv:1806.00435

OTOC on arxiv last three months

1. arXiv:1808.04383 [pdf, other] quant-ph
Semiclassical theory of out-of-time-order correlators for low-dimensional classically chaotic systems
Authors: Rodolfo A. Jalabert, Ignacio García-Mata, Diego A. Wisniacki
Abstract: The out-of-time-order correlator (**OTOC**), recently analyzed in several physical contexts, is studied for low-dimensional chaotic systems through semiclassical expansions and numerical simulations. The semiclassical expansion for the **OTOC** yields a leading-order contribution in $\hbar^2$ that is exponentially increasing w... ▽ More
Submitted 13 August, 2018; **originally announced** August 2018.
Comments: 24 pages (one column), 7 figures

2. arXiv:1808.04375 [pdf, other] quant-ph
Sensitivity of quantum information to environment perturbations measured with the out-of-time-order correlation function
Authors: Mohamad Niknam, Lea F. Santos, David G. Cory
Abstract: ...between the system and the environment. Quantum information shared between the two is sensitive to environment perturbations. The out-of-time-order correlation function (**OTOC**) is used to measure this sensitivity. By analyzing the decay of the... ▽ More
Submitted 13 August, 2018; **originally announced** August 2018.
Comments: 9 pages, 10 figures

3. arXiv:1807.11085 [pdf, other] quant-ph
Detection of out-of-time-order correlators and information scrambling in cold atoms: Ladder-XY model
Authors: Ceren B. Dağ, L. -M. Duan
Abstract: Out-of-time-order correlators (**OTOC**), recently being the center of discussion on quantum chaos, are a tool to understand the information scrambling in isolated many-body quantum systems. Quantum chaotic models are expected to scramble the information fastest and hence show exponential decay of... ▽ More
Submitted 29 July, 2018; **originally announced** July 2018.
Comments: 5 pages, 5 figures, Supplemental material (8 pages)

4. arXiv:1807.10292 [pdf, other] cond-mat.stat-mech
Quantum and Classical Lyapunov Exponents in Atom-Field Interaction Systems
Authors: Jorge Chávez-Carlos, B. López-del-Carpio, Miguel A. Bastarrachea-Magnani, Pavel Stránský, Sergio Lerma-Hernández, Lea F. Santos, Jorge G. Hirsch
Abstract: The exponential growth of the out-of-time-ordered correlator (**OTOC**) has been proposed as a quantum signature of classical chaos. The growth rate is expected to coincide with the classical Lyapunov exponent. This quantum-classical correspondence has been corroborated for the kicked rotor and the stadium billiard, which are one-body chaotic systems. The conjec... ▽ More
Submitted 26 July, 2018; **originally announced** July 2018.
Comments: 5 pages, 4 figures

5. arXiv:1807.09799 [pdf, other] cond-mat.dis-nn
Non-linear sigma model approach to many-body quantum chaos: regularized and unregularized out-of-time-ordered correlators
Authors: Yunxiang Liao, Victor Galitski
Abstract: The out-of-time-ordered correlators (**OTOCs**) have been proposed and widely used recently as a tool to define and describe many-body quantum chaos. Here, we develop the Keldysh non-linear sigma model technique to calculate these correlators in interacting disordered metals. In particular, we focus on the regularized and unregularized... ▽ More

OTOC on arxiv last three months

6. arXiv:1807.09731 [pdf, ps, other] hep-th

Probing Out-of-Time-Order Correlators

Authors: Soumyadeep Chaudhuri, R. Loganayagam

Abstract: We present a method to probe the Out-of-Time-Order Correlators (**OTOCs**) of a general system by coupling it to a harmonic oscillator probe. When the system's degrees of freedom are traced out, the... ▽ More

Submitted 25 July, 2018; **originally announced** July 2018.

Comments: 5 pages+references

7. arXiv:1807.09087 [pdf, other] quant-ph

Probing scrambling using statistical correlations between randomized measurements

Authors: Benoît Vermersch, Andreas Elben, Lukas M. Sieberer, Norman Y. Yao, Peter Zoller

Abstract: ...between measurements, performed after evolving a quantum system from random initial states. We show that the resulting statistical correlations are directly related to **OTOCs** and can be used to probe scrambling in many-body systems. Our protocol, which does not require reversing time evolution or auxiliary degrees of freedom, can be realized in state-of-the-a... ▽ More

Submitted 24 July, 2018; **originally announced** July 2018.

8. arXiv:1807.08826 [pdf, other] cond-mat.str-el

Out-of-time-ordered correlators in short-range and long-range hard-core boson models and Luttinger liquid model

Authors: Cheng-Ju Lin, Olexei I. Motrunich

Abstract: We study out-of-time-ordered correlators (**OTOC**) in hard-core boson models with short-range and long-range hopping and compare the results to the **OTOC** in the Luttinger liquid model. For the density operator, a related `commutator function' starts at zero and decays back to zero after the passage of the wavefront in... ▽ More

Submitted 23 July, 2018; **originally announced** July 2018.

Comments: 10 pages of main text + 5 pages of appendices; 6 figures

9. arXiv:1806.10405 [pdf, ps, other] cond-mat.stat-mech

Effective dimension, level statistics, and integrability of Sachdev-Ye-Kitaev-like models

Authors: Eiki Iyoda, Hosho Katsura, Takahiro Sagawa

Abstract: ...attracts attention in the context of information scrambling, which represents delocalization of quantum information and is quantified by the out-of-time-ordered correlators (**OTOC**). The SYK model contains N fermions with disordered and four-body interactions. Here, we introduce a variant of the SYK model, which we refer to as the Wishart SYK model. We inves... ▽ More

Submitted 26 July, 2018; **v1** submitted 27 June, 2018; **originally announced** June 2018.

Comments: 12 pages, 6 figures

10. arXiv:1806.09637 [pdf, other] quant-ph

Out-of-time-ordered-correlator quasiprobabilities robustly witness scrambling

Authors: José Raúl González Alonso, Nicole Yunger Halpern, Justin Dressel

Abstract: Out-of-time-ordered correlators (**OTOCs**) have received considerable recent attention as qualitative witnesses of information scrambling in many-body quantum systems. Theoretical discussions of... ▽ More

Submitted 25 June, 2018; **originally announced** June 2018.

OTOC on arxiv last three months

11. arXiv:1806.04686 [pdf, other] cond-mat.stat-mech

Entanglement production and information scrambling in a noisy spin system

Authors: Michael Knap

Abstract: ...show that entanglement growth and its fluctuations are described by the Kardar-Parisi-Zhang equation. Moreover, we find that the wavefront in the out-of-time ordered correlator (**OTOC**), which is a measure for the operator growth, propagates linearly with the butterfly velocity and broadens diffusively, with a diffusion constant that is larger than the one of... ▽ More

Submitted 18 June, 2018; **v1** submitted 12 June, 2018; **originally announced** June 2018.

Comments: 6 pages, 3 figures + supplemental material

12. arXiv:1806.04147 [pdf, other] quant-ph

Reconciling two notions of quantum operator disagreement: Entropic uncertainty relations and information scrambling, united through quasiprobabilities

Authors: Nicole Yunger Halpern, Anthony Bartolotta, Jason Pollack

Abstract: ...quantum information theory, entropic uncertainty relations constrain measurement outcomes. (ii) In condensed matter and high-energy physics, the out-of-time-ordered correlator (**OTOC**) signals scrambling, the spread of information through many-body entanglement. We unite these measures, deriving entropic uncertainty relations for scrambling. The entropies are... ▽ More

Submitted 11 June, 2018; **originally announced** June 2018.

Comments: 14 pages (4 figures) + appendices

Report number: CALT-TH-2018-021

13. arXiv:1806.02807 [pdf, other] quant-ph

Verified Quantum Information Scrambling

Authors: Kevin A. Landsman, Caroline Figgatt, Thomas Schuster, Norbert M. Linke, Beni Yoshida, Norman Y. Yao, Christopher Monroe

Abstract: ...owing to the exponential complexity of ergodic many-body entangled states. One way to characterize quantum scrambling is to measure an out-of-time-ordered correlation function (**OTOC**); however, since scrambling leads to their decay, **OTOCs** do not generally discriminate between quantum scrambling and ordinary decoherence... ▽ More

Submitted 18 June, 2018; **v1** submitted 7 June, 2018; **originally announced** June 2018.

Comments: 11 pages

14. arXiv:1806.00781 [pdf, other] quant-ph

Application of quantum scrambling in Rydberg atom on IBM quantum computer

Authors: Daattavya Aggarwal, Shivam Raj, Bikash K. Behera, Prasanta K. Panigrahi

Abstract: Quantum scrambling measured by out-of-time-ordered correlator (**OTOC**) has an important role in understanding the physics of black holes and evaluating quantum chaos. It is known that Rydberg atom has been a general interest due to its extremely favourable properties for building a quantum simulator. Fast and efficient quantum simulators can be developed by st... ▽ More

Submitted 6 July, 2018; **v1** submitted 3 June, 2018; **originally announced** June 2018.

Comments: 39 pages, 7 figures

15. arXiv:1806.00435 [pdf, other] cond-mat.stat-mech

Emergence of correlations in the process of thermalization of interacting bosons

Authors: Fausto Borgonovi, Felix M. Izrailev

Abstract: ...of the number of principal components of the wave function, recently discovered and explained in Ref.\cite{BIS18}. We also demonstrate that the out-

OTOC on arxiv last three months

16. arXiv:1806.00113 [pdf, ps, other] quant-ph

Tripartite mutual information, entanglement, and scrambling in permutation symmetric systems with an application to quantum chaos

Authors: Akshay Seshadri, Vaibhav Madhok, Arul Lakshminarayan

Abstract: ...for large subsystems across the Ehrenfest or log-time and saturate at the random state values if there is global chaos. During this time the out-of-time order correlators (**OTOC**) evolve exponentially implying scrambling in phase space. We discuss how positive TMI may coexist with such scrambling. ▽ More

Submitted 31 May, 2018; **originally announced** June 2018.

17. arXiv:1805.12299 [pdf, other] cond-mat.str-el

Universal properties of many-body quantum chaos at Gross-Neveu criticality

Authors: Shao-Kai Jian, Hong Yao

Abstract: Quantum chaos in many-body systems may be characterized by the Lyapunov exponent defined as the exponential growth rate of out-of-time-order correlators (**OTOC**). So far Lyaponov exponents around various quantum critical points (QCP) remain largely unexplored. Here, we investigate the Lyapunov exponent around QCPs of the Gross-Neveu (GN) model with N flavors... ▽ More

Submitted 30 May, 2018; **originally announced** May 2018.

Comments: 7 pages, 7 figures + appendix

18. arXiv:1805.06377 [pdf, other] cond-mat.stat-mech

Many-Body Quantum Interference and the Saturation of Out-of-Time-Order Correlators

Authors: Josef Rammensee, Juan-Diego Urbina, Klaus Richter

Abstract: Out-of-time-order correlators (**OTOCs**) have been proposed as sensitive probes for chaos in interacting quantum systems. They exhibit a characteristic classical exponential growth, but saturate beyond the so-called scrambling or Ehrenfest time τ_E in the quantum correlated regime. Here we present a path-integral approach for the entire time evolution o... ▽ More

Submitted 16 May, 2018; **originally announced** May 2018.

Comments: 6 + 10 pages, 2 figures

19. arXiv:1805.05376 [pdf, other] cond-mat.str-el

Locality, Quantum Fluctuations, and Scrambling

Authors: Shenglong Xu, Brian Swingle

Abstract: ...in complexity of initially simple Heisenberg operators. Operator growth is a manifestation of information scrambling and can be diagnosed by out-of-time-order correlators (**OTOCs**). However, the behavior of... ▽ More

Submitted 14 May, 2018; **originally announced** May 2018.

Comments: 12+8 pages, 8 figures

20. arXiv:1805.00667 [pdf, other] quant-ph — doi 10.1103/PhysRevA.98.012132

Strengthening weak measurement of qubit out-of-time-order correlators

Authors: Justin Dressel, José Raúl González Alonso, Mordecai Waegell, Nicole Yunger Halpern

Abstract: ...can be determined exactly from the average of a measurement sequence. As a relevant example, we provide quantum circuits for measuring multiqubit out-of-time-order correlators (**OTOCs**) using optimized control-Z or ZX -90 two-qubit gates common in superconducting transmon implementations. ▽ More

Submitted 2 May, 2018; **originally announced** May 2018.

Comments: 11 pages, 6 figures

In 2014, Kitaev proposed to quantify chaos in interacting quantum many-body systems in terms of the following out-of-time-ordered (four- point) correlation function (OTOC):

$$C(x,t) = -\langle [w_x(t), v_0(0)]^2 \rangle_\beta = -Tr\{e^{-\beta H}[w_x(t), v_0(0)]^2\}$$

where w_x , v_0 are local observables in the Heisenberg picture. The concept is based on a work by Larkin and Ovchinnikov where OTOC was connected to the instability of semi-classical trajectories of electrons scattered by impurities in a superconductor. According to that, extended quantum systems were defined as chaotic if there exists a pair of local observables, w and v, such that the OTOC grows exponentially at early times:

$$C(x,t) \propto e^{\lambda_L (t-|x|/v_B)}.$$

where λ_L is called *Lyapunov exponent* and v_B butterfly velocity.

A. Kitaev, *Hidden correlations in the Hawking radiation and thermal noise*, (2014), talk given at Fundamental Physics Prize Symposium; A. I. Larkin and Y. N. Ovchinnikov, Soviet Journal of Experimental and Theoretical Physics **28**, 1200 (1969), J. Maldacena, S. H. Shenker, and D. Stanford, JHEP **08**, 106 (2016).

Physically, it describes how much the perturbation introduced by $v_0(0)$ changes the value of the $w_x(t)$. At large times $C(x,t)$ goes to a zero (if $\langle w_x \rangle = \langle v_0 \rangle = 0$), because the state created by the consecutive action of the operators $w_x(t)v(0)$ is incoherent with the state obtained when these operators act in a different order. The anomalous time order in the correlator implies the evolution backward in time, so it is not **measureable** by direct physical experiments on one copy of the system in the absence of a time machine such as implemented in NMR experiments. One can view the decrease of the OTOC with time as the consequence of the dephasing between two initially almost identical Worlds evolving with the same Hamiltonian. In this respect it is different from the problems of fidelity and Loschmidt echo that study evolution forward and backwards with slightly different Hamiltonians.

Igor L. Aleiner, Lara Faoro and Lev B. Ioffe, *Microscopic model of quantum butterfly effect: out-of-time-order correlators and traveling combustion*, Annals of Physics **375**, (2016) 378-406.

Measure of OTOC via spin-echo techniques

- M. Garttner, J. G. Bohnet, A. Safavi-Naini, M. L. Wall, J. J. Bollinger, and A. M. Rey, Nat. Phys. **13**, 781 (2017),
- J. Li, R. Fan, H. Wang, B. Ye, B. Zeng, H. Zhai, X. Peng, and J. Du, Phys. Rev. X **7**, 031011 (2017).
- K. X. Wei, C. Ramanathan, and P. Cappellaro, Phys. Rev. Lett. **120**, 070501 (2018).
- M. Niknam, L. F. Santos, and D. G. Cory, ArXiv:1808.04375.

Other proposed experimental setting for OTOC measuring

- Martin Garttner, Philipp Hauke, Ana Maria Rey, Phys. Rev. Lett. **120**, 040402 (2018)
- K. A. Landsman, C. Figgatt, T. Schuster, N. M. Linke, B. Yoshida, N. Y. Yao, and C. Monroe, *Verified Quantum Information Scrambling*, arXiv:1806.02807.
- S. V. Syzranov, A. V. Gorshkov, and V. Galitski, *Out-of-time-order correlators in finite open systems*, arXiv:1704.08442.
- Y.-L. Zhang, Y. Huang, and X. Chen, *Information scrambling in chaotic systems with dissipation*, arXiv:1802.04492.
- B. Swingle and N. Yunger Halpern, *Resilience of scrambling measurements*, Physical Review A 97, 062113 (2018)

For local Hamiltonian (so that distant parts of a system are not interacting directly with each other) and observables acting far from each other in real space, the correlator should decreases after the significant delay needed for the perturbation to spread over the distance separating the observables. When correlators of this type decayed for any separation between the operators in the real space the coherence is completely lost. The decay of OTOC at long times for all subsystems (i.e. for all separations) for all operators implies complete quantum information scrambling.

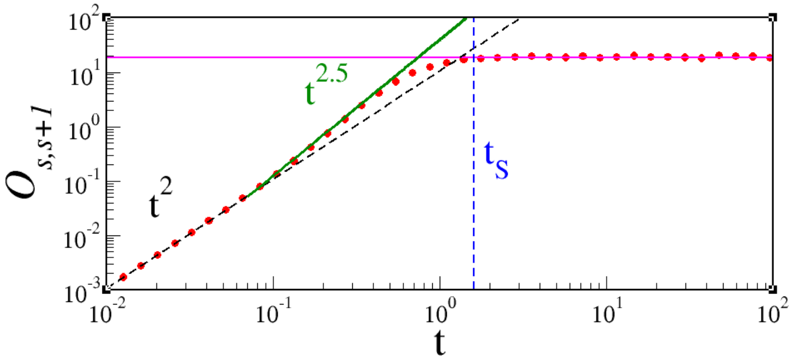
P. Hosur, X.-L. Qi, D. A. Roberts, B. Yoshida, Chaos in quantum channels, JHEP **02** (2016) 004

For local interactions $C(x,t)$ is bounded by the Lieb-Robinson theorem (LRT):

$$C(x,t) \leq 4||v||\ ||w||\ e^{-\mu\ max\{0,\ |x|-v_{LR}t\}}$$

and for $t > t^* = |x|/v_{LR}$ the OTOC is even more suppressed. t^* is the time in which $C(x,t)$ enters the causal cone. Before t^* , $C(x,t)$ is almost zero, while after t^* , it is bounded and saturates at a plateau. The dynamics can only be non-trivial near the edge of the causal-cone (or for $t \sim t^*$), where $C(x,t)$ can vary greatly.

I.Kukuljan, S.Grozdanov, and T.Prosen, *Weak Quantum Chaos* arxiv:1701.09147



Global two-levels correlation function

$$\mathcal{O}_{s,s+1}(t) = \langle k_0 | [\hat{n}_s(t), \hat{n}_{s+1}(0)] |^2 | k_0 \rangle. \tag{19}$$

Asymptotic regimes for OTOC

Perturbative regime

$$\mathcal{O}_{s,s+1}(t) \simeq t^2 \sum_{k\neq k_0} H^2_{k,k_0} \left(n_s^k - n_s^{k_0}\right)^2 \left(n_{s+1}^k - n_{s+1}^{k_0}\right)^2 . \qquad (20)$$

Long-time average

$$\overline{\mathcal{O}_{s,s+1}} = \sum_k \left(n_{s+1}^k - n_{s+1}^{k_0}\right)^2 \left\{ \left[\sum_\alpha C_k^\alpha C_{k_0}^\alpha \mathcal{N}_s^{\alpha,\alpha}\right]^2 + \sum_{\alpha\neq\beta} |C_k^\alpha|^2 |C_{k_0}^\beta|^2 \left(\mathcal{N}_s^{\alpha,\beta}\right)^2 \right\} \qquad (21)$$

with $\mathcal{N}_s^{\alpha,\beta} = \sum_k C_k^\alpha C_k^\beta n_s^k$,
$C_k^{(\alpha)} = \langle k|\alpha\rangle$,
$H_0 \,|k\rangle = E_k^0 \,|k\rangle$, $H\,|\alpha\rangle = E_\alpha \,|\alpha\rangle$.

1. a non-perturbative power-law behavior ($t^{2.5}$) emerges in the time-region characterized by the exponential growth of the number of principal components in the many-body Hilbert space.
2. absence of exponential growth

take home

OTOC have been found to increase exponentially for chaotic quantum many-body systems. Our preliminary results are at odd with these findings. Further investigations are needed.

Creation or loss of information?

Although the dynamics is completely reversible due to the unitarity of the evolution operator, it is practically impossible to extract the information about the initial state, by measuring the correlations between the components of the wave function. Indeed the full information about the initial state can be extracted only if there is an additional complete knowledge of the random operator V . Thus one can indeed speak of the loss of information due to scrambling. The process of this loss is accompanied by the emergence of global (thermodynamical) correlations.

Discussion of the results

1. Wave packets spread exponentially fast in the unperturbed basis before reaching saturation, when all states of the energy shell get populated.
2. the time scale for saturation $t_S \sim N/\Gamma$ is much larger than the characteristic decay time of the survival probability $t_\Gamma \sim 1/\Gamma$
3. the dynamical process is well described up to t_S by a phenomenological cascade model that allowed us to estimate the rate and the time scale of the relaxation, as well as the saturation value of the number of principal components in the wave packet
4. a single key parameter Γ – the width of LDOS – reproduces well the system dynamics at very different time scales.

Discussions/ansatz

1. In the context of quantum chaos, the LDOS has a well defined classical limit. In fact, the classical LDOS is nothing but the projection of the unperturbed Hamiltonian onto the total one and can be obtained by solving classical equations of motion.
2. The maximal value of the width of the LDOS is given by the width of the energy shell. In the classical description, the energy shell corresponds to the phase space volume obtained by the projection of the phase-space surface $H_0 = E_0$ onto the surface defined by the total Hamiltonian H.
3. our results for the exponential growth of N_{pc} can be treated in terms of the phase space occupied by the wave packet, $\mathcal{V}_E(t) \sim N_{pc}(t)/\rho(E)$, where $\rho(E)$ is the total density of states:
$$\mathcal{V}_E(t) = \mathcal{V}_E(0)e^{2\Gamma t} \sim \mathcal{V}_E(0)e^{h_{KS}t}. \tag{22}$$
4. 2Γ can be associated with the Kolmogorov-Sinai entropy, h_{KS}, which gives the exponential growth of phase-space volumes for classically chaotic Many-Body Systems.

- Statistical relaxation in finite systems, driven by interaction, depends on the initial state and not only on integrability/non integrability criteria. More precisely the initial state should be composed of many chaotic eigenstates.
- In quantum MBS thermalization occurs in a time $t_S \sim N/\Gamma \gg 1/\Gamma$.
- A unique parameter Γ, the width of LDOS is necessary to describe the short and long time dynamics.
- In the semiclassical limit this growth corresponds to the growth of phase space volumes with the rate $h_{KS} = 2\Gamma$

Thank you for your attention