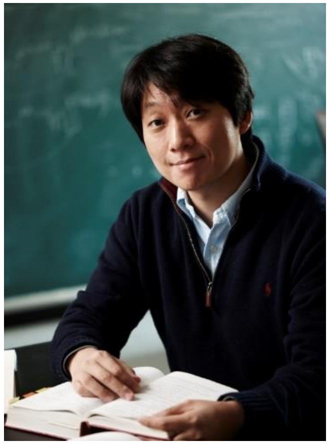
Many-Body Invariants of

Multipolar Higher-order Topological Insulators

Gil Young Cho
(POSTECH)

Byungmin Kang (KIAS)

Hyun Woong Kwon (KIAS)



Prof. Kwon Park (KIAS)

Ref. Byungmin Kang, Hyun Woong Kwon, Kwon Park, and **GYC**, in progress

Main Research Themes:

1. Developing Novel Theories

for Topological States/Strongly-Correlated States

Ex: new types of quantum field theory (geometric deg. of. freedom)

anomalies, topological field theory, and so on

Today, but no equation/no field theory

2. Designing Models for Topological States

Ex: Topological superconductors [Majorana fermions]

Anyons in fractional quantum Hall states, and/or heterostructures

Contents:

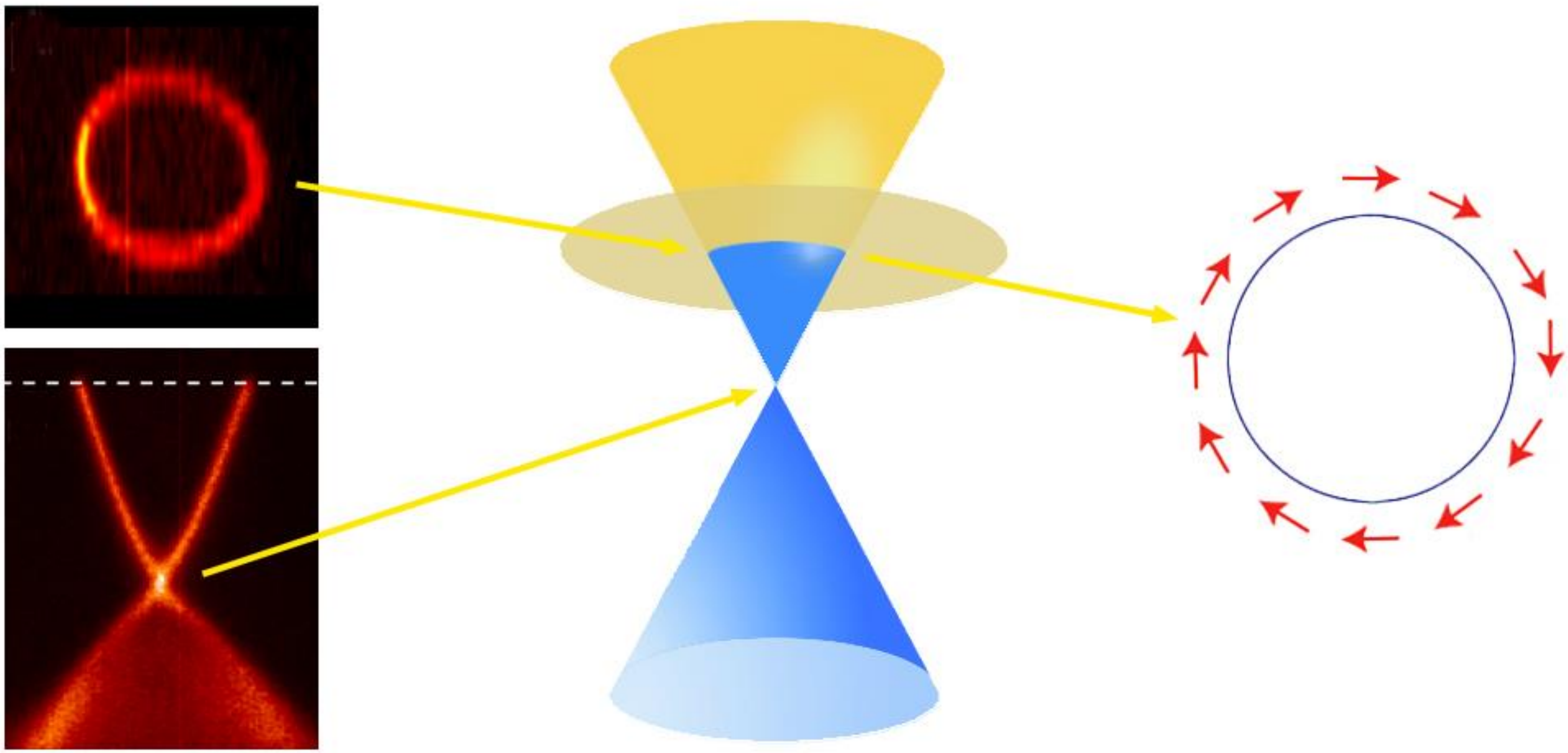
Ref. Byungmin Kang, Hyun Woong Kwon, Kwon Park, and **GYC**, in progress

1. Introductions

Topological Insulators

:**Topological properties** often manifested by **“Surface States”**

EX: 3D Topological Insulator



Surface: Symmetry-protected Dirac cone

New Generation of Topological Insulators:

"Higher-Order Topological Insulators"

RESEARCH

TOPOLOGICAL MATTER

Quantized electric multipole insulators

Wladimir A. Benalcazar,[1] B. Andrei Bernevig,[2] Taylor L. Hughes[1*]

[Science, 2017]

SCIENCE ADVANCES | RESEARCH ARTICLE

MATERIALS SCIENCE

Higher-order topological insulators

Frank Schindler,[1] Ashley M. Cook,[1] Maia G. Vergniory,[2,3*] Zhijun Wang,[4] Stuart S. P. Parkin,[5] B. Andrei Bernevig,[4,2,6†] Titus Neupert[1†]

[Science, 2018]

Reflection-Symmetric Second-Order Topological Insulators and Superconductors

Josias Langbehn, Yang Peng, Luka Trifunovic, Felix von Oppen, and Piet W. Brouwer
Phys. Rev. Lett. **119**, 246401 – Published 11 December 2017

[PRL, 2017]

Observation of a phononic quadrupole topological insulator

Marc Serra-Garcia, Valerio Peri, Roman Süsstrunk, Osama R. Bilal, Tom Larsen, Luis Guillermo Villanueva & Sebastian D. Huber

[Nature, 2018]

Higher-Order Topology in Bismuth

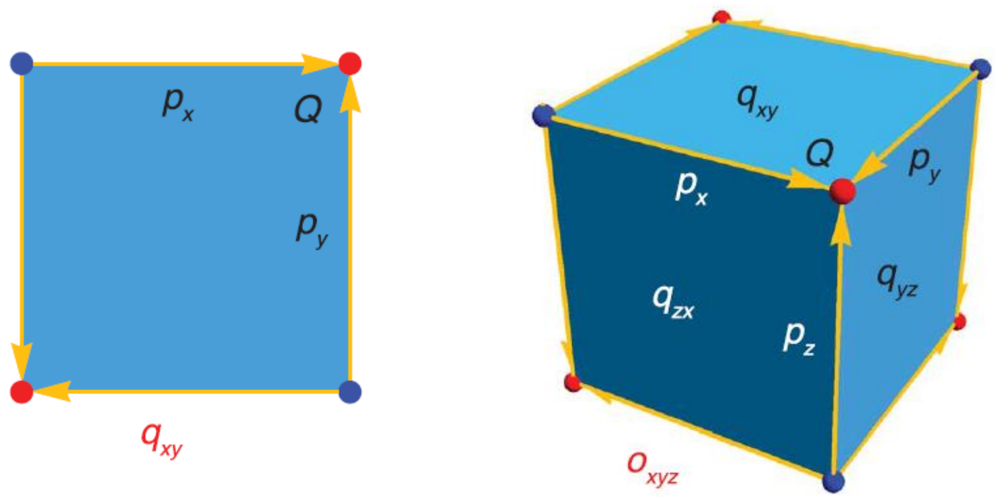
Frank Schindler,[1] Zhijun Wang,[2] Maia G. Vergniory,[3,4,5] Ashley M. Cook,[1] Anil Murani,[6] Shamashis Sengupta,[7] Alik Yu. Kasumov,[6,8] Richard Deblock,[6] Sangjun Jeon,[9] Ilya Drozdov,[10] Hélène Bouchiat,[6] Sophie Guéron,[6] Ali Yazdani,[9] B. Andrei Bernevig,[9] and Titus Neupert[1]

[Nat. Phys., 2018]

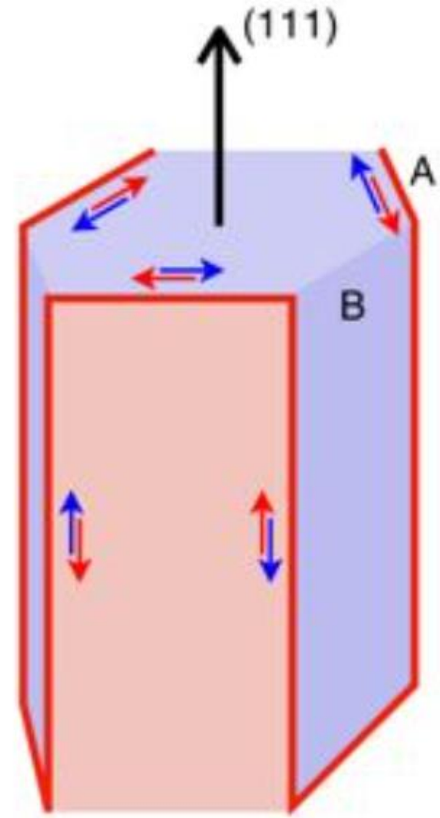
…and so on.

Higher-Order Topological Insulators (HOTI):

"Topology" = Non-trivial Edge of Edge states



Dangling states at "Corner" [Science 2017]

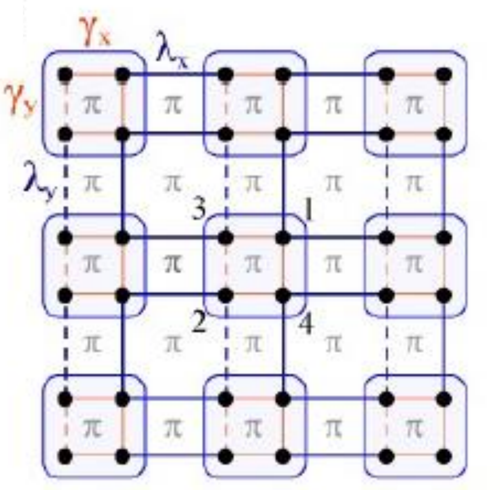


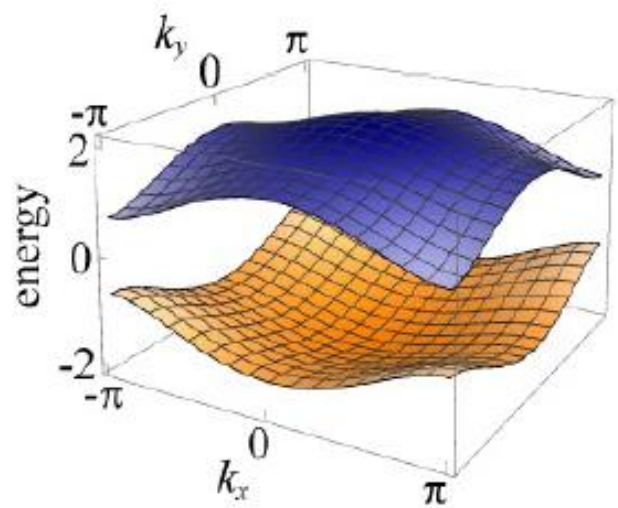
Bismuth [*Nat. Phys. 2018*]

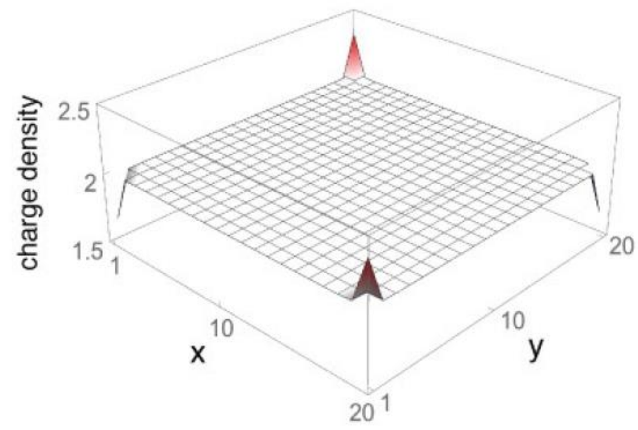
Many variants, but we consider today only prototypical HOTI

“Multipolar Charge Distribution” from Corner States

[Ref. Benalcazar-Bernevig-Hughes, Science, 2017]







Topological if $|\lambda_a| > |\gamma_a|$ **for** $a = x, y$

Quadrupolar Corner Charges

Using *semi-classical* arguments, it has been shown that:

[Ref. Benalcazar-Bernevig-Hughes, Science, 2017]

This state has “quantized quadrupolar moments $Q_{xy} = \frac{1}{2}$ mod 1”

Key claims: [Ref. Benalcazar-Bernevig-Hughes, Science, 2017]

1. Multipoles can be quantized.

1. With proper symmetries: mirrors $\{M_x, M_y\}$ and/or C_{4z}

2. When lower poles are vanishing:

E.g., Translation by $\vec{d}$: $\boldsymbol{Q_{xy}} = \sum \boldsymbol{xy\, q} \rightarrow \boldsymbol{Q_{xy}} + \boldsymbol{d_x P_y} + \boldsymbol{d_y P_x} + \boldsymbol{d_x d_y Q_{tot}}$

[Invariance (well-defined): P_x, P_y vanish (no polarization)]

2. Topologically Trivial/non-trivial Multipoles

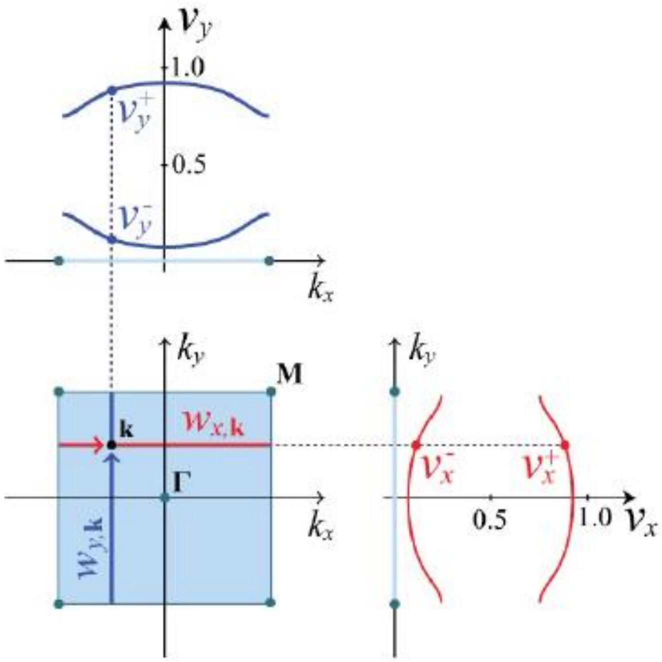
Ex: quadrupole in 2d with C_{4z} and no polarization

$\boldsymbol{Q_{xy} = \frac{1}{2}}$ **mod 1 for higher-order topological states**

$\boldsymbol{Q_{xy} = 0}$ **mod 1 for trivial states**

Topological Band Indices:

"Polarizations" of "Wilson Loop Operators"



$$\mathcal{W}_{\mathcal{C},\mathbf{k}} \equiv e^{iH_{\mathcal{W}_\mathcal{C}}(\mathbf{k})}.$$

Not clear enough for me…

Can we diagnose "Multipoles"

in Condensed Matter Systems?

[Generically "Quantum" + "Many Body"

I.E., Beyond *"free fermion + momentum space"* definitions]

(Simpler) ***Definitions of Multipoles?***

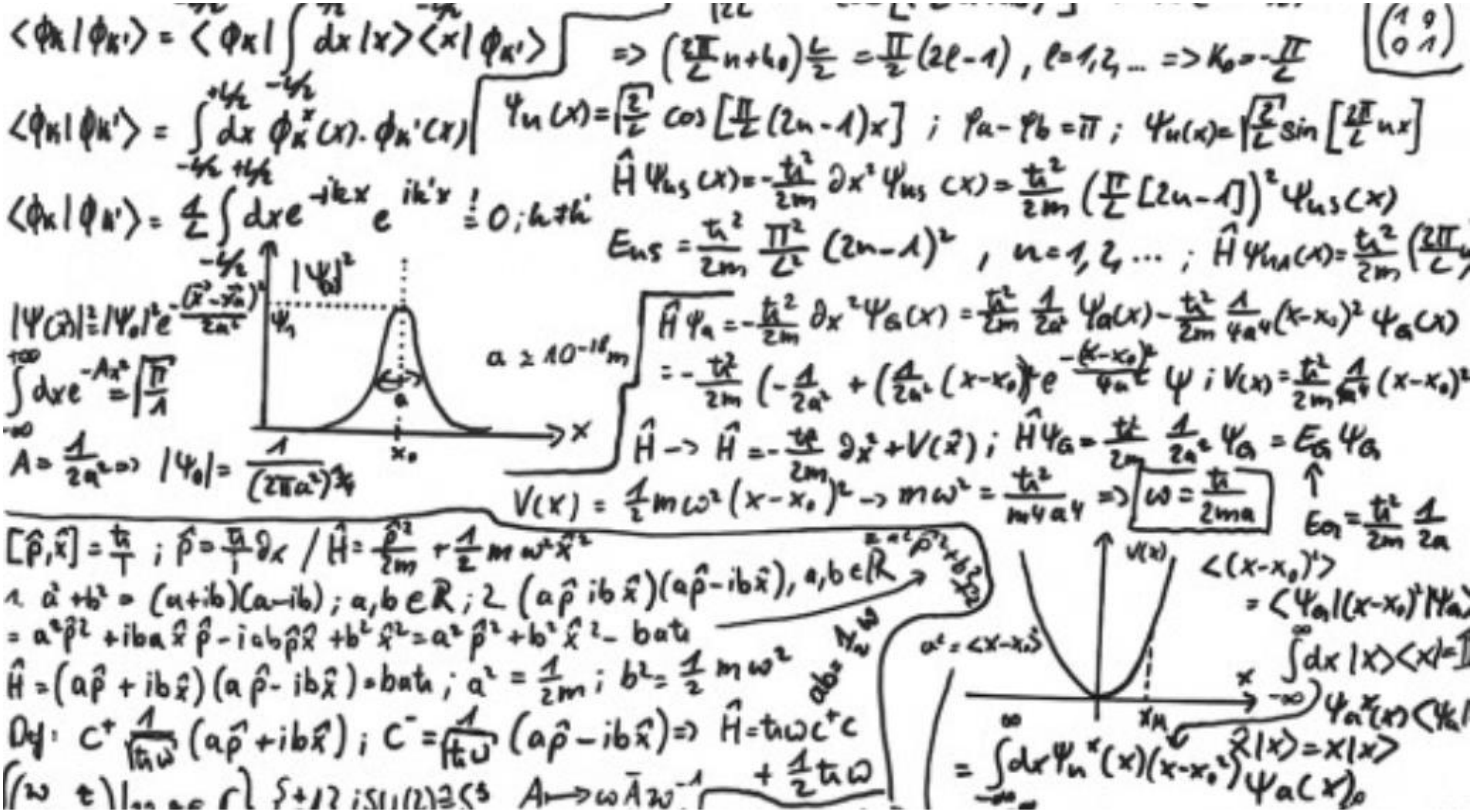
In a fully-quantum, interacting set-up

Ref. Byungmin Kang, Hyun Woong Kwon, Kwon Park, and **GYC**, in progress

2. Conjectures & Numerical Results

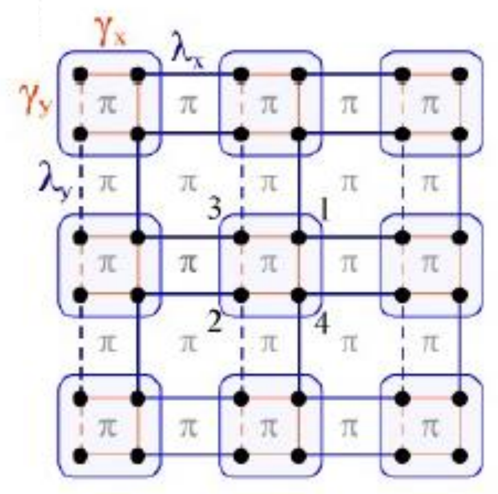
Ref. Byungmin Kang, Hyun Woong Kwon, Kwon Park, and **GYC**, in progress

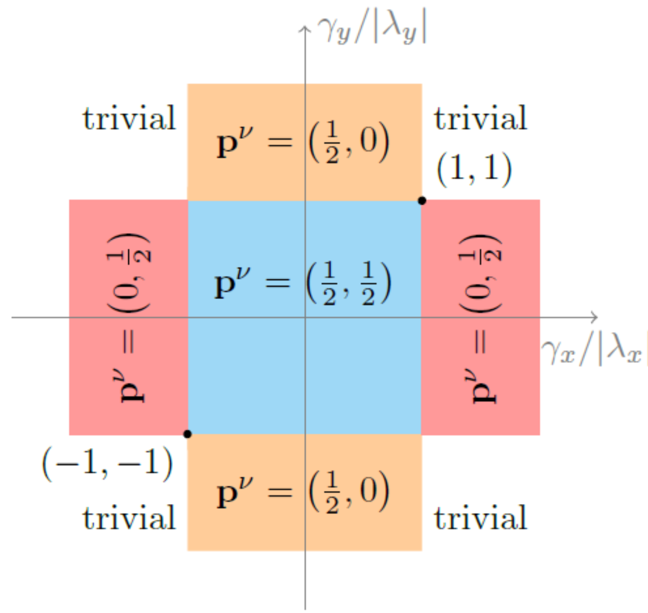
Instead of ***showing you lengthy arguments*** for this



…which you probably don’t care

"seeing" is "believing"



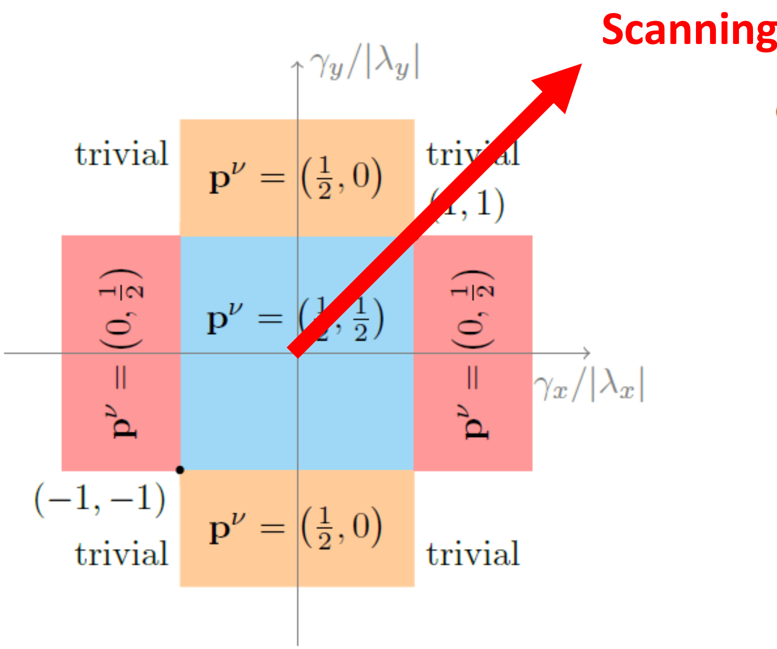


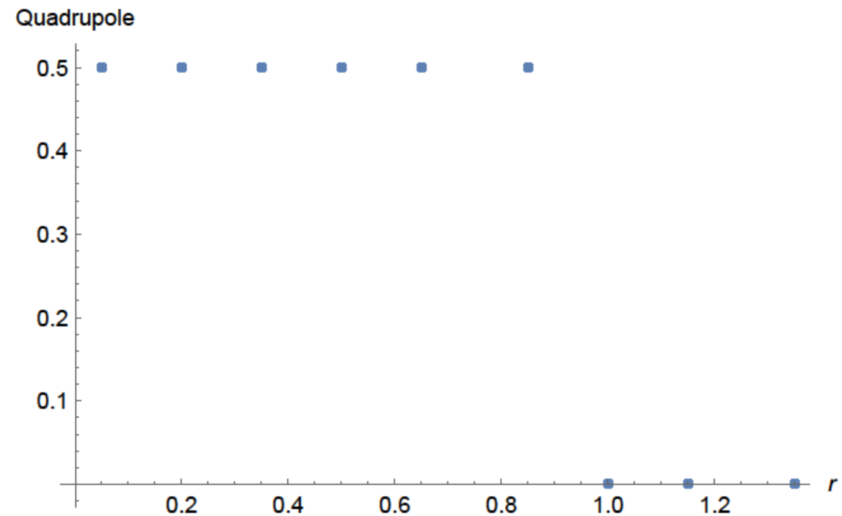
Blue Region: $Q_{xy} = \frac{1}{2}$ mod 1 (topological)

Others: $Q_{xy} = 0$ mod 1 (trivial)

Ref. Byungmin Kang, Hyun Woong Kwon, Kwon Park, and **GYC**, in progress

“seeing” is “believing”





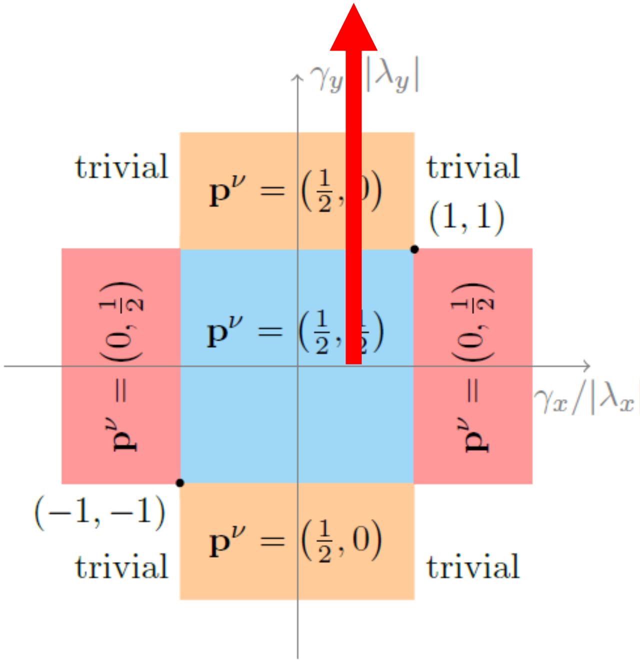
$$Q_{xy} = \frac{1}{2\pi} \text{Im} \log \langle GS | U_2 | GS \rangle$$

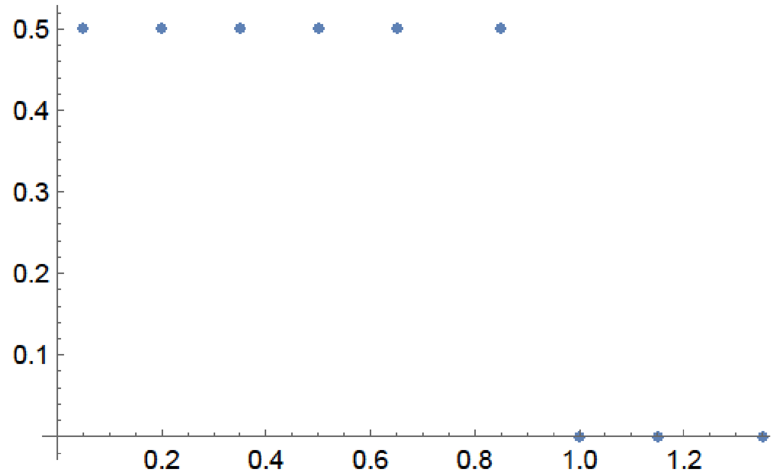
Reproduces (i) quantization, (ii) phase transition, (iii) topological/non-topological dichotomy

Ref. Byungmin Kang, Hyun Woong Kwon, Kwon Park, and **GYC**, in progress

“seeing” is “believing”

Scanning **[No gap closing in energy spectrum]**



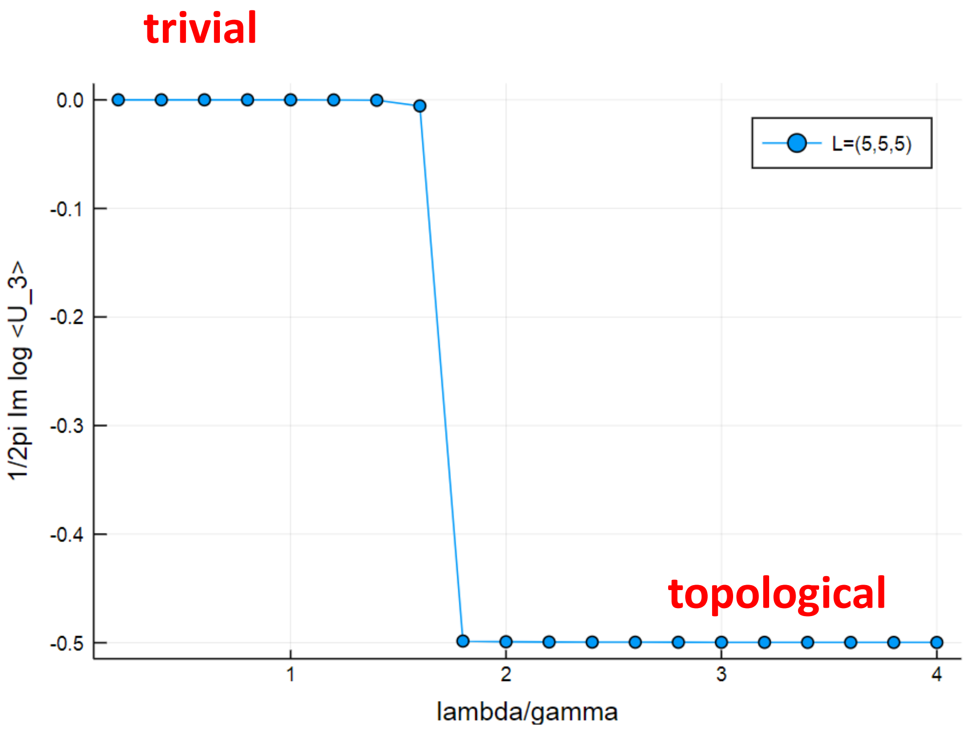


$$Q_{xy} = \frac{1}{2\pi} \text{Im} \log \langle GS|\, U_2 \,|GS\rangle$$

Reproduces (i) quantization, (ii) phase transition, (iii) topological/non-topological dichotomy

Ref. Byungmin Kang, Hyun Woong Kwon, Kwon Park, and **GYC**, in progress

“seeing” is “believing”

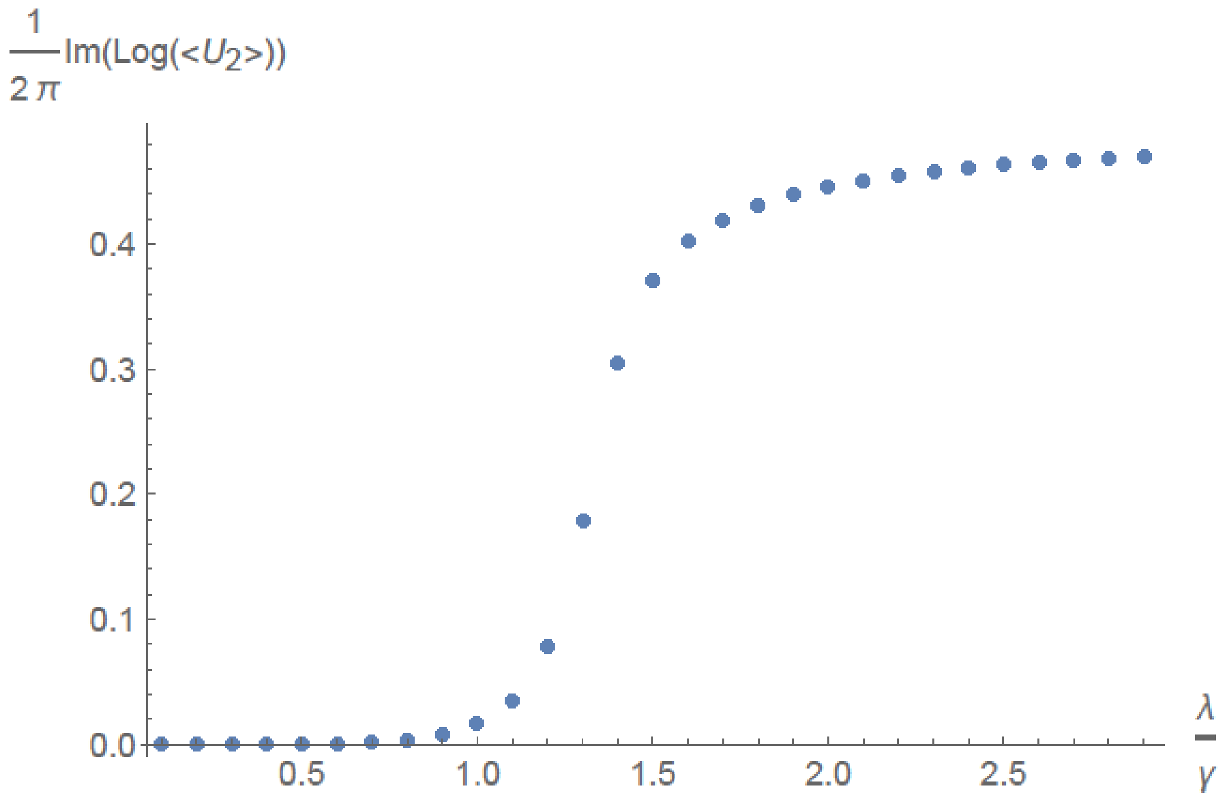


$$O_{xyz} = \frac{1}{2\pi} \mathrm{Im} \log \langle GS | U_3 | GS \rangle$$

Reproduces: (i) quantization, (ii) phase transition (with finite size effect) (iii) topological/non-topological dichotomy

Ref. Byungmin Kang, Hyun Woong Kwon, Kwon Park, and **GYC**, in progress

Note: with (weak) symmetry breaking terms



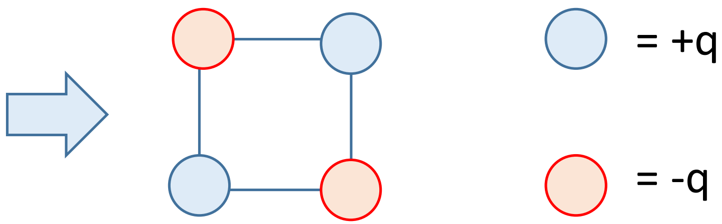
Smooth phase boundary + non-quantization (as expected).

Physically, what can we learn from this?

If $Q_{xy} = q$ is **the quadrupolar density**, then...

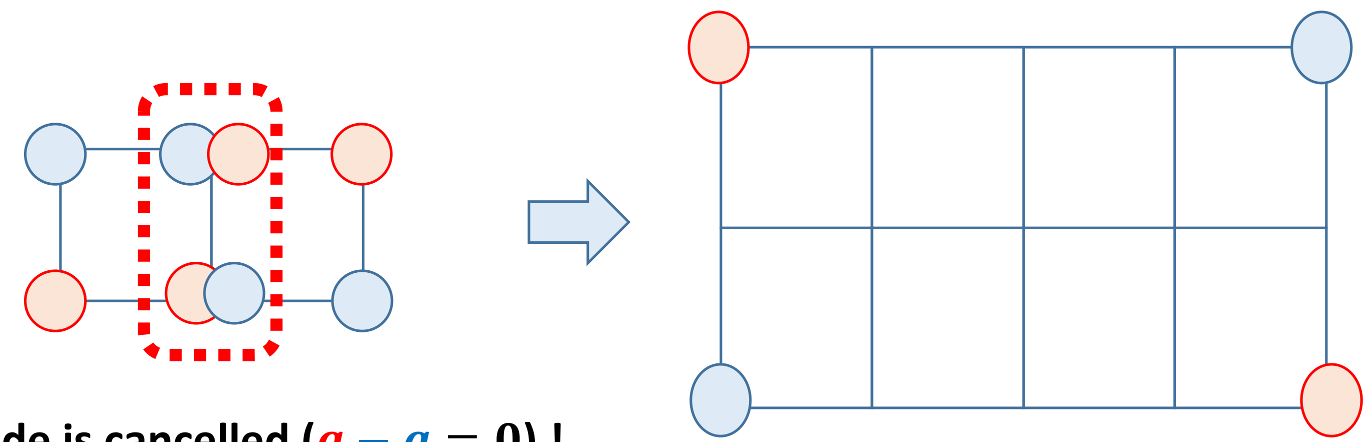
q is **the charge localized at the boundary**

$$Q_{xy} = q$$



When the quadrupolar density is uniformly stacked,

Ex:

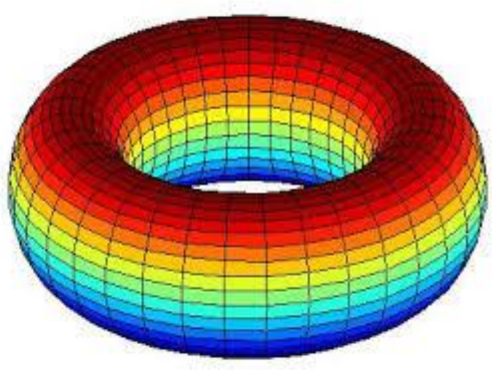


Inside is cancelled ($q - q = 0$) !

Only the boundary is left

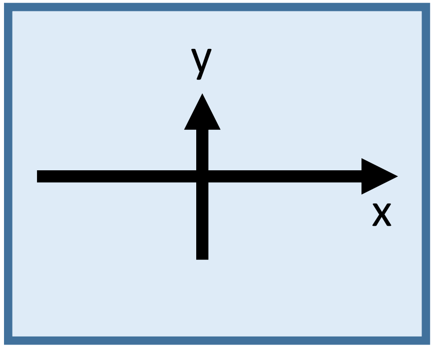
So we compare the following two quantities:

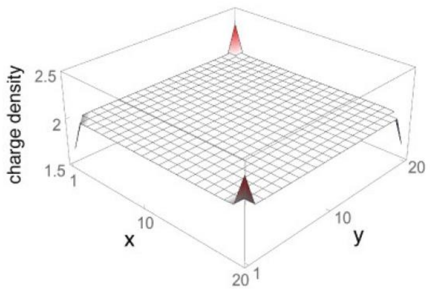
1. Periodic BC on Torus



Many-body Invariants

2. Open BC



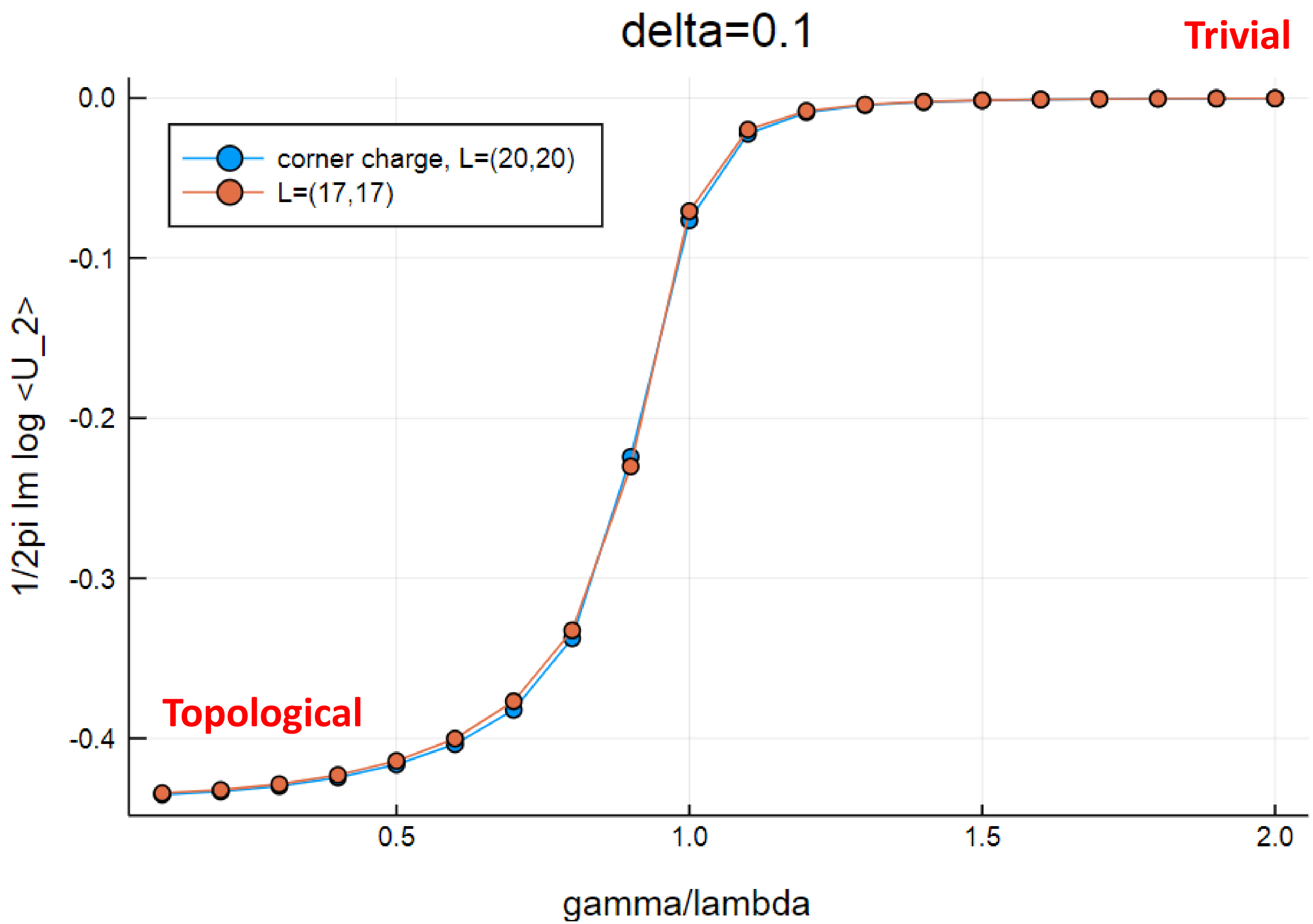


Single-particle Observable

$$q = \int d^2x \, \langle GS| \, \hat{\rho}(x) - \bar{\rho} \, |GS\rangle$$

Do I find $Q_{xy} = q$?

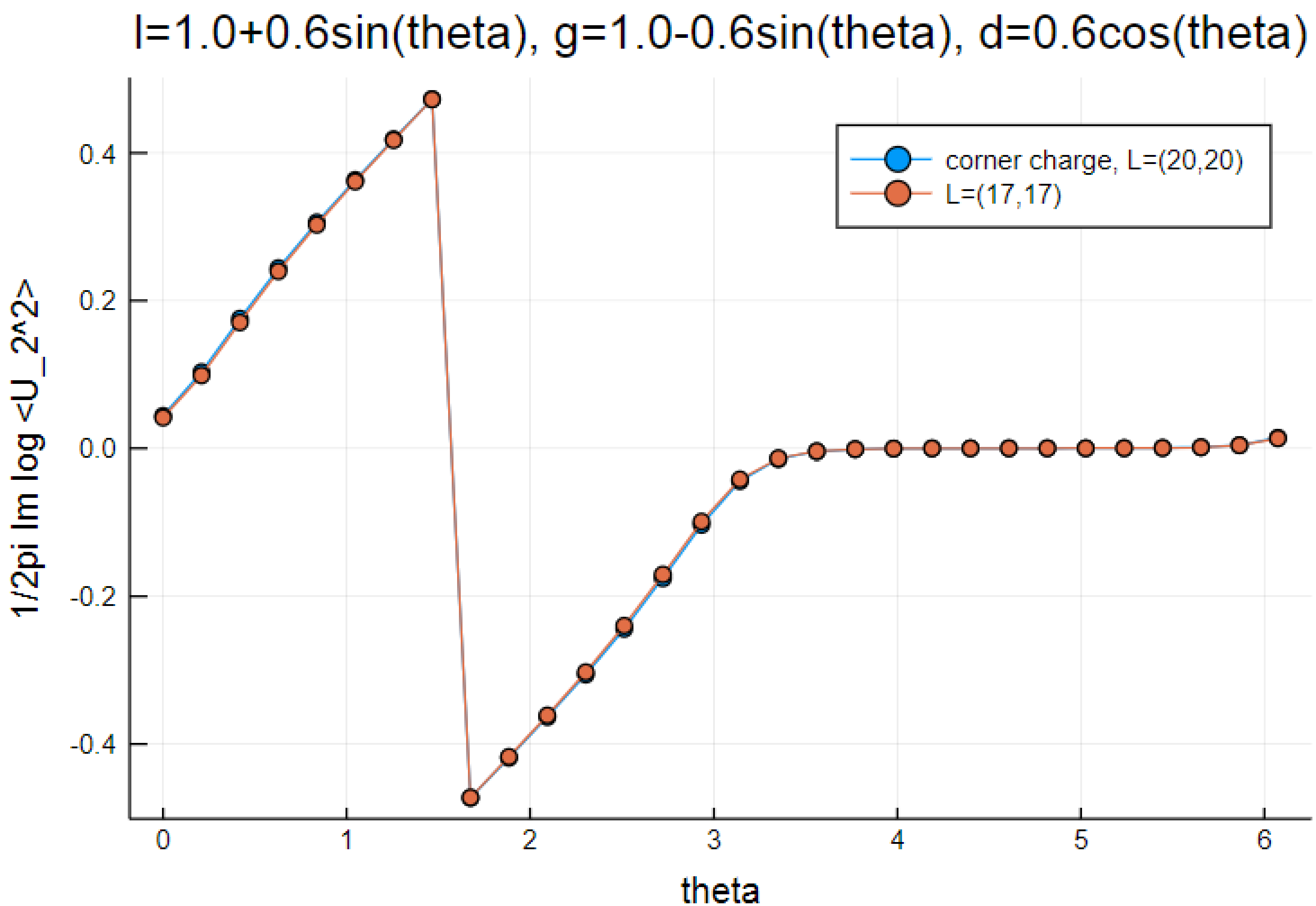
From bulk invariant to boundary charge



Many-body invariant exactly matches localized boundary charge!

Ref. Byungmin Kang, Hyun Woong Kwon, Kwon Park, and **GYC**, in progress

Even for more complicated processes, e.g., Thouless pumping

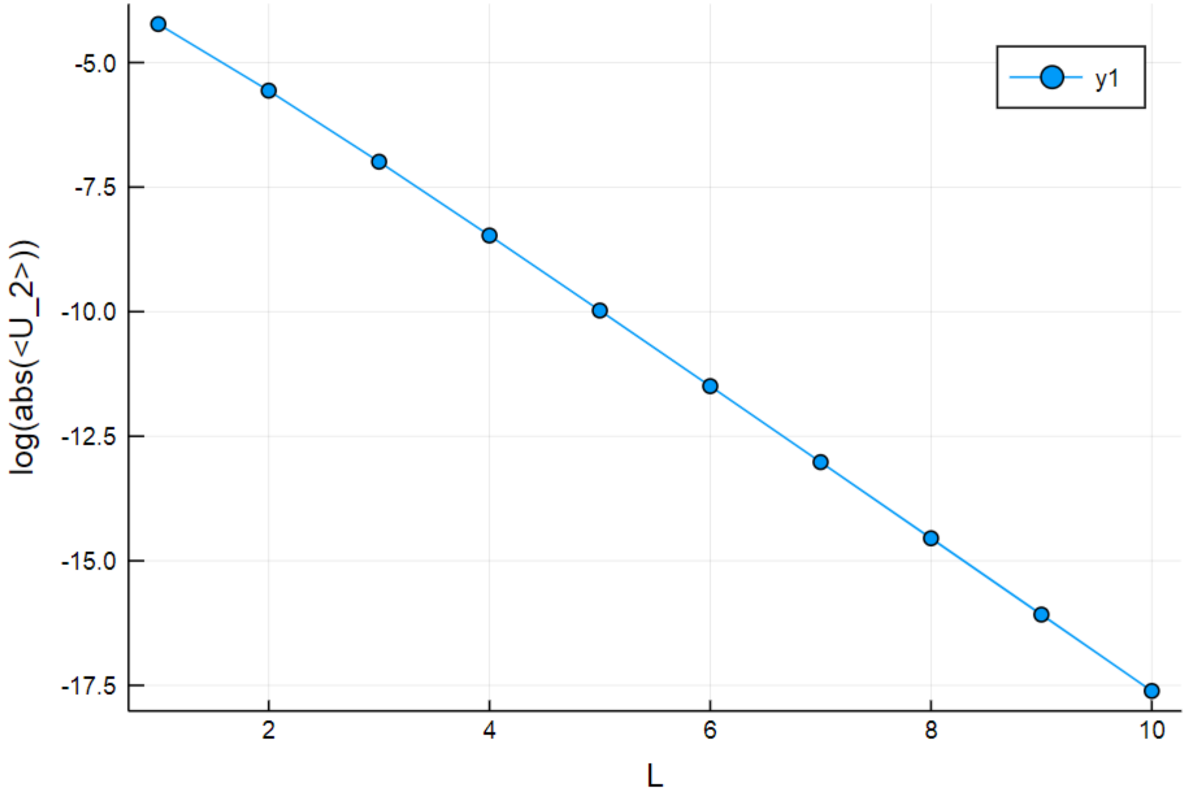


Q_{xy} tracks well of the charge over the full parameter space !

Ref. Byungmin Kang, Hyun Woong Kwon, Kwon Park, and **GYC**, in progress

New Order parameter for Metals & Insulators

At the critical point: $|\langle U_2 \rangle| \to 0$ as $L_x \to \infty$



2d critical: $|\langle U_2 \rangle| \sim c_1 \exp\left(-aL_x\right) \to 0$

[2d insulator: $|\langle U_2 \rangle| \sim const.$**]**

This is analogous with the original Resta's conjecture.

In Summary:

$\langle U_2 \rangle = |\langle U_2 \rangle| \ \mathrm{Exp}\left(2\pi i Q_{xy}\right)$ **is an order parameter**

(1) Phase = (Detail-free) Topology = Multipoles

(2) Amplitude = Spectral Property, i.e. Metal/Insulator

Ref. Byungmin Kang, Hyun Woong Kwon, Kwon Park, and **GYC**, in progress

3. Conclusions

Conclusions:

1. **Conjectured (definition of) many-body invariants for multipoles**

2. **Numerically confirmed the invariants**

3. **Novel order parameter for "metal" & "insulator"**

Ref. Byungmin Kang, Hyun Woong Kwon, Kwon Park, and **GYC**, in progress

4. **Several future directions**

Thank you for your attention !