Localization of Light in subradiant Dicke states: a mobility edge in the imaginary axis.

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03/09/2018. International Workshop Disordered Systems, Daejeon, South Korea

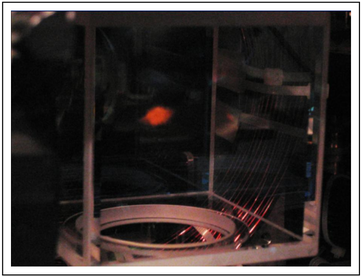
- Emergence: due to coopertative effects, new properties emerge which belong to the system as a whole and not to its parts.
- It refers to the appearance of (unexpected) features on a (bigger) scale that were not present on another (smaller) scale,
- Examples: Superconductivity, Superradiance, etc...Understanding Emergence is one of the main challenges of CMP. Robustness to noise and Functional role.
- Inrterplay of different cooperative effects: Imaginary Mobility Edge $\Rightarrow$ localization of Light in cold atomic systems.

Emergence of Fractal Crystal Structure

Localization of Light in Cold Atomic Clouds

Cold Atoms: $\lambda \ll L$

Robin Kaiser Lab



$$V_{nm} = -\frac{\cos(k_0 r_{nm})}{k_0 r_{nm}} - i\,\frac{\sin(k_0 r_{nm})}{k_0 r_{nm}},$$

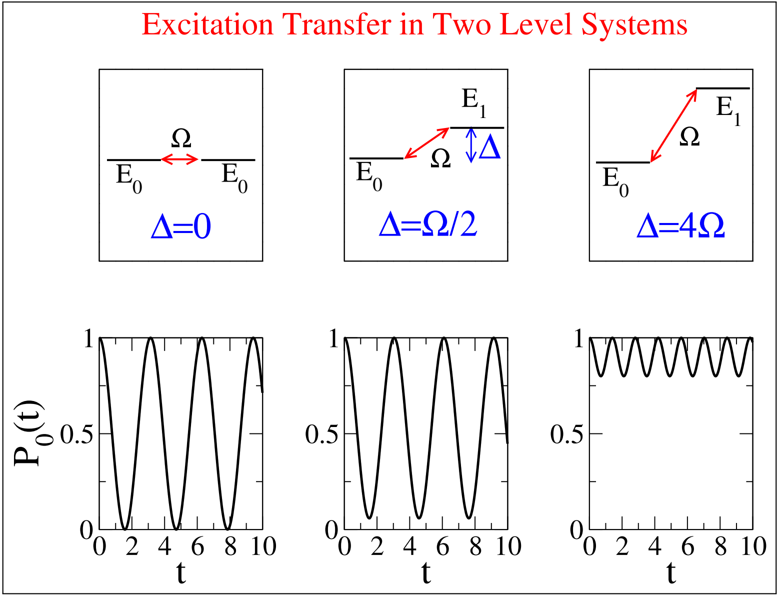
A Challenging Problem:

1. Disorder: positional and diagonal
2. Long Ramge Interactions
3. Open quantum systems
4. Cooperative effects: Super-subradiance
5. Qualifies as a complex systems!

ANDERSON LOCALIZATION

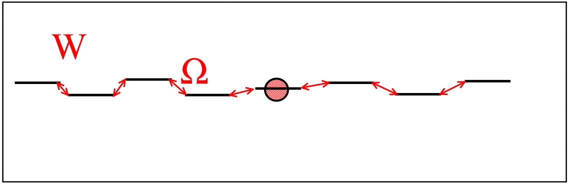
AND

ITS SIGNATURES

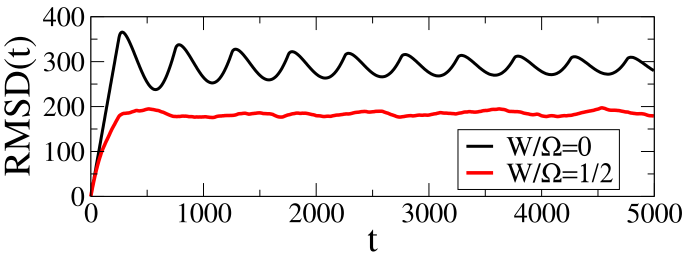

Excitation Transfer in Two Level Systems
Ω
E0
E0
Δ=0
E1
Δ
Ω
E0
Δ=Ω/2
E1
Ω
E0
Δ=4Ω
P0(t)
1
0.5
0
0 2 4 6 8 10
t

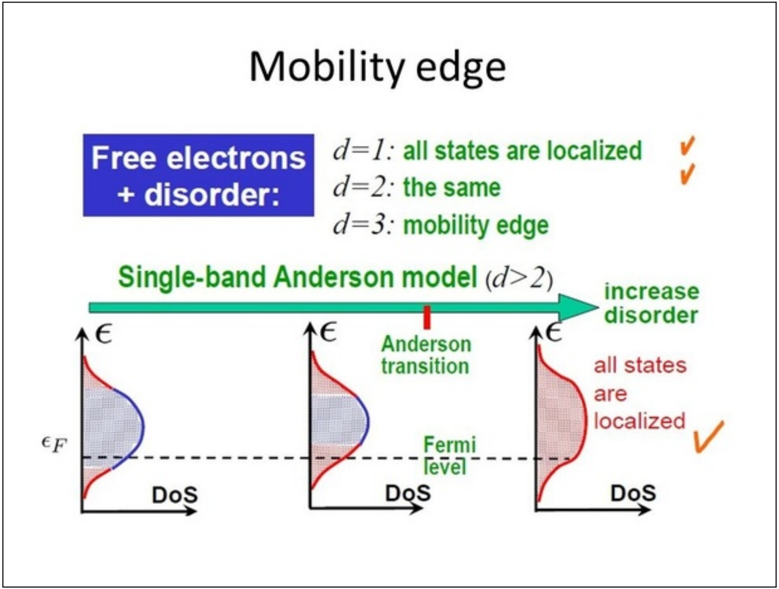
Anderson Localization as an emergent phenomenon

Linear Chain of N sites with small disorder



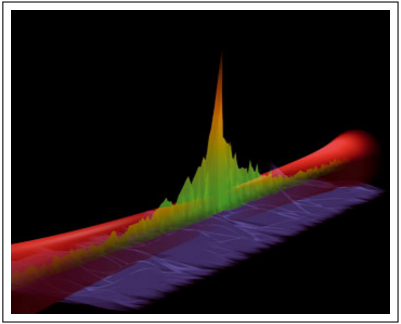
$H = \Sigma \; E_i \, | i \rangle \langle i | + \Omega \, \Sigma \; | i \rangle \langle i+1 | + h.c.$




Mobility edge
Free electrons + disorder:
d=1: all states are localized
d=2: the same
d=3: mobility edge
Single-band Anderson model (d>2)
increase disorder
ϵ
ϵF
DoS
Anderson transition
Fermi level
all states are localized

Characterization of Anderson localization

Localized wave functions



J. Billy et al., Nature, 2008. 453: 891-4.

$$|\psi| \sim e^{-|x-x_0|/\xi}$$

ξ : localization length.

Participation Ratio:

$$PR = \frac{1}{\sum_{i=1}^{N} |\psi(i)|^4}.$$

EXTENDED:

$$\langle i|\psi\rangle = \frac{1}{\sqrt{N}} \Rightarrow PR \propto N$$

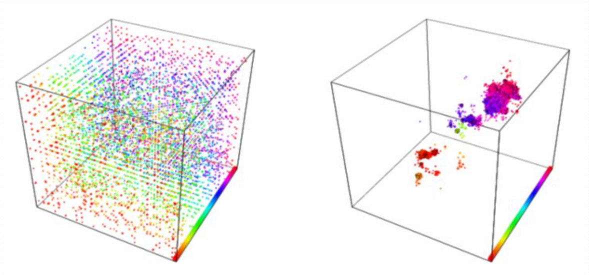
LOCALIZED:

$$\langle i|\psi\rangle = 1 \Rightarrow PR = const.$$

M.I.T.: DIVERGENCE OF PR AT THE MOBILITY EDGE

Localization in 3D

Eigenstates for weak and strong W



extended state
weak disorder, band center

localized state
strong disorder, band edge

(L=240) R.A.Römer

ANDERSON LOCALIZATION

AND

LONG RANGE INTERACTION

Localization and long range.

- Levitov, PRL **64**, 547 1990: "IT IS KNOWN THAT IN SYSTEMS WITH DIMENSION d WITH $r^{-\alpha}$ INTERACTION, LOCALIZATION CAN EXIST ONLY IF $\alpha > d$. FOR $\alpha \leq d$ A DIVERGING NUMBER OF RESONANCES DESTROYS LOCALIZED STATES".

- ANDERSON (1958): More distant sites are not important because the probability of finding one with the right energy increases much more slowly with distance than the interaction decreases

Number of Resonances:

$$N_{res} = \frac{V_k}{W} N_k \propto R^{d-\alpha} \rightarrow \infty \quad \text{for} \quad \alpha < d$$

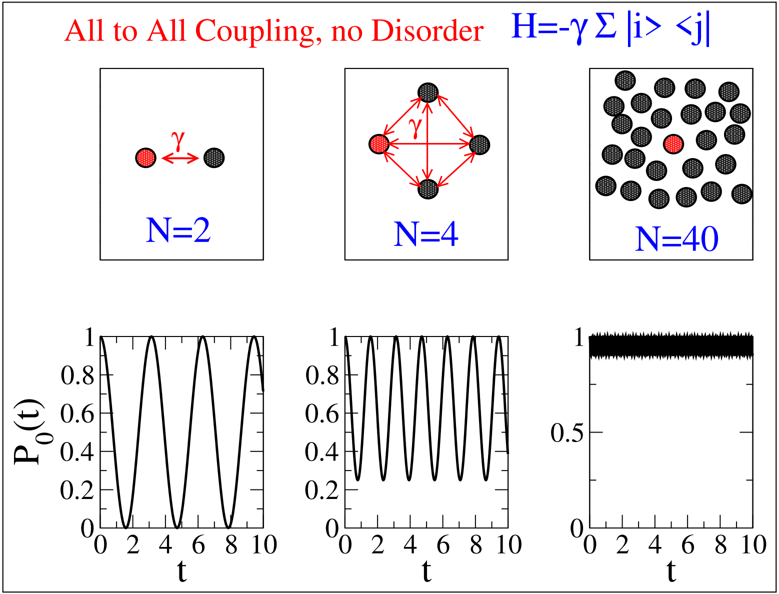
RANDOM VS NON RANDOM INTERACTIONS

- **Absence of Localization of Vibrational Modes Due to Dipole-Dipole Interaction**, L. S. Levitov, Europhys. Lett. 9, 83 (1989); Phys. Rev. Lett. 64, 547 (1990);
- **Anderson transitions**, F. Evers and A. D. Mirlin, Rev. Mod. Phys. 80, 1355 (2008).
- **Transition from localized to extended eigenstates in the ensemble of power-law random banded matrices**, A. D. Mirlin, Yan V. Fyodorov, F.-M. Dittes, J. Q., and T. H. Seligman Phys. Rev. E 54, 3221 (1996).
- Kastner, New J. Phys. 17 063021 (2015), PRX 3, 031015 (2013). Suppression of information spreading in long range systems (Lieb-Robinson Bounds).
- **Anderson localization on a simplex**, A Ossipov, Journal of Physics A: Mathematical and Theoretical, Volume 46, (2013) $H = \sum E_i^0 |i\rangle\langle i| - \gamma \sum |i\rangle\langle i|$

PR and all its moments independent of N.

How do we explain such contradiction?

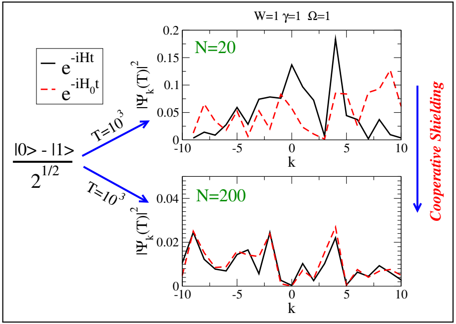
Infinite Range interactions



Cooperative Shielding

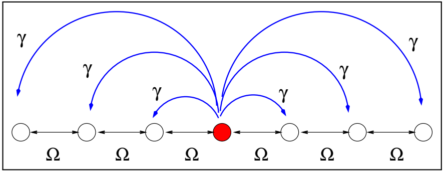
- Dynamical Evolution can be determined by an effective short range Hamiltonian, even in presence of long range.
- Given a system $H = H_0 + V$, we can eliminate V from the the dynamics up to a time scale diverging with the system size.
- $H = H_0 + V$, H_0: ANDERSON MODEL, V: LONG RANGE

Cooperative Shielding



Long Range: $H = H_0 + V$

V does not affect the evolution (*shielding*) up to a time scale that grows with N (*cooperativity*).



References

Shielding in Many Body Systems

PRL **116,** 250402 (2016) PHYSICAL REVIEW LETTERS week ending 24 JUNE 2016

Cooperative Shielding in Many-Body Systems with Long-Range Interaction

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In recent experiments with ion traps, long-range interactions were associated with the exceptionally fast propagation of perturbation, while in some theoretical works they have also been related with the suppression of propagation. Here, we show that such apparently contradictory behavior is caused by a general property of long-range interacting systems, which we name cooperative shielding. It refers to shielded subspaces that emerge as the system size increases and inside of which the evolution is unaffected by long-range interactions for a long time. As a result, the dynamics strongly depends on the initial state: if it belongs to a shielded subspace, the spreading of perturbation satisfies the Lieb-Robinson bound and may even be suppressed, while for initial states with components in various subspaces, the propagation may be quasi-instantaneous. We establish an analogy between the shielding effect and the onset of quantum Zeno subspaces. The derived effective Zeno Hamiltonian successfully describes the short-ranged dynamics inside the subspaces up to a time scale that increases with system size. Cooperative shielding can be tested in current experiments with trapped ions.

DOI: 10.1103/PhysRevLett.116.250402

Introduction.—A better understanding of the nonequilibrium dynamics of many-body quantum systems is central to a wide range of fields, from atomic, molecular, and condensed matter physics to quantum information and cosmology. New insights into the subject have been obtained thanks to the remarkable level of controllability and isolation of experiments with optical lattices [1–7] and trapped ions [8,9]. Recently there has been a surge of interest in the dynamics of systems with long-range interactions, triggered by experiments with ion traps [8,9], where the range of interactions in one-dimensional (1D) spin models can be tuned with great accuracy. Other realistic systems that contain long-range interaction include cold atomic clouds [10], natural light-harvesting complexes [11–13], helium Rydberg atoms [14], and cold Rydberg gases [15]. Long-range interacting systems display features that are not often observed in other systems, such as broken ergodicity [16–19] and long-lasting out-of-equilibrium regimes [20].

According to the usual definition [21], in d dimension, an interaction decaying as $1/r^{\alpha}$ (where r is the distance between two bodies), is short range when $\alpha > d$ and is long range when $\alpha \le d$. A major topic of investigation has been whether the propagation of excitations in systems with long-range interaction remains confined or not to an effective light cone [22–30], as defined by the Lieb-Robinson bound [31] and its generalizations ([30] and references therein). In the aforementioned experiments with trapped ions, it was observed that for short-range interaction, the propagation of perturbation is characterized by a constant maximal velocity, being bounded to an effective light cone. As α decreases, the propagation velocity increases and eventually diverges. For long-range interaction, $\alpha < 1$, the light-cone picture is no longer valid and the dynamics becomes nonlocal. However, examples of constraint dynamics in long-range interacting systems have also been reported, including logarithmic growth of entanglement [23], light-cone features [30], self-trapping [32], and slow decays at critical points [33].

Here, we show that these contradictory results are due to a general effect present in long-range interacting systems, which we name cooperative shielding. It corresponds to the onset of approximate superselection rules that cause a strong dependence of the dynamics on the initial state. Inside a superselection subspace, long-range interactions do not affect the system evolution (shielding) up to a time scale that grows with system size (cooperativity). The dynamics can then be described by an effective short-ranged Hamiltonian that either leads to a propagation within the Lieb-Robinson light cone or to localization. In contrast, for an initial state with components over several subspaces, the propagation of excitations is affected by long-range interactions and can be unbounded.

To explain how shielding can arise in a very trivial case, let us consider the total Hamiltonian $H = H_0 + V$, describing a many-body quantum system, where H_0 has one-body terms and possible short-range interactions, and V corresponds to some additional interactions. If $[H_0, V] = 0$ and V is highly degenerate in one of its eigensubspaces $\mathcal{V}$, so

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Shielding in single excitation

PHYSICAL REVIEW B **94**, 144206 (2016)

Shielding and localization in the presence of long-range hopping

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We investigate a paradigmatic model for quantum transport with both nearest-neighbor and infinite-range hopping coupling (independent of the position). Due to long-range homogeneous hopping, a gap between the ground state and the excited states can be induced, which is mathematically equivalent to the superconducting gap. In the gapped regime, the dynamics within the excited-state subspace is shielded from long-range hopping, namely it occurs as if long-range hopping would be absent. This is a cooperative phenomenon since shielding is effective over a time scale that diverges with the system size. We named this effect cooperative shielding. We also discuss the consequences of our findings on Anderson localization. Long-range hopping is usually thought to destroy localization due to the fact that it induces an infinite number of resonances. Contrary to this common lore we show that the excited states display strong localized features when shielding is effective even in the regime of strong long-range coupling. A brief discussion on the extension of our results to generic power-law decaying long-range hopping is also given. Our preliminary results confirm that the effects found for the infinite-range case are generic.

DOI: 10.1103/PhysRevB.94.144206

I. INTRODUCTION

In recent years, technological advancement has allowed us to engineer several systems in which the role of quantum coherence is essential to understanding their dynamics. In view of these considerations, searching for novel coherent effects is fundamental to exploit quantum properties in technological devices such as quantum wires, quantum computers, and quantum sensors. Of particular interest is the topic of transport of energy or charge in the quantum coherent regime, due to its relevance in many technological applications, such as in light-harvesting systems [1], molecular wires [2], and in other mesoscopic systems [3].

Recently, great attention has been devoted to quantum transport in models with long-range interactions due to their relevance in many condensed matter physical systems. Indeed long-range interactions between the constituents of a system do not arise only from microscopic interactions, but in many condensed matter systems they can be induced by the coupling with environmental modes having a wavelength larger than the system size. This mediated long-range interaction arises in several systems: in ion traps [4] due to the coupling of the trapped ions with large wavelength phonon modes, in cold atoms [5], and in natural light harvesting systems [1], due to the coupling with the electromagnetic field (EMF) when the wavelength of the photon is much larger than the system size. Long-range interacting systems display particular features that are not often observed in other systems, such as broken ergodicity [6] and long-lasting out-of-equilibrium regimes [7]. Out-of-equilibrium dynamics of such models have been widely analyzed both experimentally [4] and theoretically [8–10], showing nontrivial cooperative effects and strong dependence of the dynamical evolution on the initial state. Together with a very fast spreading of information [4] and the destruction of localization [11], also the opposite behavior has been reported in case of long-range interacting system: the suppression of information spreading [8,9,12] and strong signatures of localization [13–15].

In a recent publication [10] by some of the authors of the present paper, a common feature of long-range interacting systems was found, named cooperative shielding. This effect has been discussed in a many-body spin system in Ref. [10], where it was shown that shielding is able explain many contradictory dynamical and transport features in systems with long-range interactions, as the ones mentioned above. Indeed, contrary to the common lore, which claims that propagation of perturbation is very fast in long-range interacting systems, it was found that even in the regime of very large long-range interaction strength there are subspaces where the evolution is determined by an emergent short-ranged Hamiltonian.

Here we analyze the cooperative shielding effect in a different model: a single excitation model of transport with long-range hopping. We also discuss the consequences of such effect on transport and localization. Specifically, here we focus on models with an infinite interaction range, which are representative of the whole class of long-range interacting systems [16–18]. Despite its apparent simplicity, infinite-range hopping can be realized experimentally in ion traps [4] where linear spin chains have been recently emulated with a spin-spin interaction decaying with the distance as $1/r^{\alpha}$ with $0 \leqslant \alpha \leqslant 3$. The case $\alpha = 0$ corresponds to an infinite interaction range, which is discussed here. Moreover, it is routinely used to model superconductivity in ultrasmall metallic grains [19] and nonequilibrium phenomena around a phase transition in strongly correlated materials [20].

II. MODEL AND ENERGY GAP

We discuss the shielding effect by means of a paradigmatic model for quantum transport, e.g., a one-dimensional (1D)

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We found effective Short Range Hamiltonian for $\alpha = 0$ and signatures of Shielding for $\alpha \neq 0$

Literature

Duality Between short and long range

PHYSICAL REVIEW LETTERS **120**, 110602 (2018)

Duality in Power-Law Localization in Disordered One-Dimensional Systems

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The transport of excitations between pinned particles in many physical systems may be mapped to single-particle models with power-law hopping, $1/r^a$. For randomly spaced particles, these models present an effective peculiar disorder that leads to surprising localization properties. We show that in one-dimensional systems almost all eigenstates (except for a few states close to the ground state) are power-law localized for any value of $a > 0$. Moreover, we show that our model is an example of a new universality class of models with power-law hopping, characterized by a duality between systems with long-range hops ($a < 1$) and short-range hops ($a > 1$), in which the wave function amplitude falls off algebraically with the same power γ from the localization center.

"Power-law localization emerges because long-range ($1/r^\alpha$) hops are not fully shielded, but rather become effectively short range."
Wave functions have power law tail with

$$\gamma(\alpha) = \gamma(2 - \alpha)$$

Shielding in 3D and role of Anisotropy

The effect of anisotropy of long-range hopping on localization in three-dimensional lattices

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(Dated: May 29, 2018)

It has become widely accepted that particles with long-range hopping do not undergo Anderson localization. However, several recent studies demonstrated localization of particles with long-range hopping. In particular, it was recently shown that the effect of long-range hopping in 1D lattices can be mitigated by cooperative shielding, which makes the system behave effectively as one with short-range hopping. Here, we show that cooperative shielding, demonstrated previously for 1D lattices, extends to 3D lattices with *isotropic* long-range $r^{-\alpha}$ hopping, but not to 3D lattices with dipolar-like *anisotropic* long-range hopping. We demonstrate the presence of localization in 3D lattices with uniform ($\alpha = 0$) *isotropic* long-range hopping and the absence of localization with uniform *anisotropic* long-range hopping by using the scaling behaviour of eigenstate participation ratios. We use the scaling behaviour of participation ratios and energy level statistics to show that the existence of delocalized, non-ergodic extended, or localized states in the presence of disorder depends on both the exponents α and the isotropy of the long-range hopping amplitudes.

Long range dipole like interactions.
Localization and Shielding in 3D.
Anisotropy may break localization and shielding.

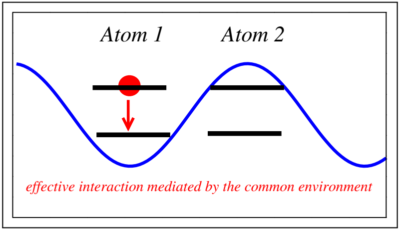
SUPER and SUB-RADIANCE

AND

INTERPLAY WITH DISORDER

Super and Sub-Radiance

Dicke, PR **93**, 99 (1954).



One atom:

$$P(t) \propto e^{-\gamma t/\hbar}$$

with $\gamma/\hbar = \frac{2\pi}{\hbar}|A|^2\rho$ from FGR:
Two atoms: If I start with one atom

$$P_{1,2} \to 1/4$$

Single Excitation Superradiance: The Super of Superradiance Marlan O. Scully et al., Science, **325**, 1510 (2009). Single Atom:

$$e^{-\gamma t/\hbar}$$

$$|k\rangle = |0\rangle_1 |0\rangle_2 .... |1\rangle_k .... |0\rangle_N$$

Cooperative Emission of N entangled atoms:

$$|Superradiant\rangle = \frac{1}{\sqrt{N}} \sum_{k=1,N} |k\rangle,$$

$$e^{-\Gamma_{SR} t/\hbar}, \quad \Gamma_{SR} = N\gamma$$

Subradiant, $\Gamma_{sub} = 0$

The effective Non-Hermitian Hamiltonian:

$$H = \sum_{i=1}^{N} e_0 |i\rangle\langle i| + \sum_{i\neq j} \Delta_{ij} |i\rangle\langle j| - \frac{i}{2} \sum_{i,j=1}^{N} Q_{ij} |i\rangle\langle j|. \qquad (1)$$

$$\Delta_{nm} = \frac{3\gamma}{4} \left[\left(-\frac{\cos(k_0 r_{nm})}{(k_0 r_{nm})} + \frac{\sin(k_0 r_{nm})}{(k_0 r_{nm})^2} + \frac{\cos(k_0 r_{nm})}{(k_0 r_{nm})^3} \right) \hat{\mu}_n \cdot \hat{\mu}_m + \right.$$

$$\left. - \left(-\frac{\cos(k_0 r_{nm})}{(k_0 r_{nm})} + 3\frac{\sin(k_0 r_{nm})}{(k_0 r_{nm})^2} + 3\frac{\cos(k_0 r_{nm})}{(k_0 r_{nm})^3} \right) (\hat{\mu}_n \cdot \hat{r}_{nm}) (\hat{\mu}_m \cdot \hat{r}_{nm}) \right],$$

$$Q_{nm} = \frac{3\gamma}{2} \left[\left(\frac{\sin(k_0 r_{nm})}{(k_0 r_{nm})} + \frac{\cos(k_0 r_{nm})}{(k_0 r_{nm})^2} - \frac{\sin(k_0 r_{nm})}{(k_0 r_{nm})^3} \right) \hat{\mu}_n \cdot \hat{\mu}_m + \right.$$

$$\left. - \left(\frac{\sin(k_0 r_{nm})}{(k_0 r_{nm})} + 3\frac{\cos(k_0 r_{nm})}{(k_0 r_{nm})^2} - 3\frac{\sin(k_0 r_{nm})}{(k_0 r_{nm})^3} \right) (\hat{\mu}_n \cdot \hat{r}_{nm}) (\hat{\mu}_m \cdot \hat{r}_{nm}) \right],$$

VECTORIAL AND SCALAR MODEL

Complex Eigenvalues and Eigenmodes

Dicke Example:

$$H_{eff} = \begin{pmatrix} E_0 - i\gamma/2 & \Omega - i\gamma/2 \\ \Omega - i\gamma/2 & E_0 - i\gamma/2 \end{pmatrix}$$

Complex Eigenvalues:
$\mathcal{E}_k = E_k^0 - i\Gamma_k/2$

Triplet: $|+\rangle = (|1\rangle + |2\rangle)/\sqrt{2}$, with $\Gamma_+ = 2\gamma$,

Singlet: $|-\rangle = (|1\rangle - |2\rangle)/\sqrt{2}$, with $\Gamma_- = 0$,

$$|\psi(t)\rangle = c_+ e^{-i\mathcal{E}_+ t/\hbar}|+\rangle + c_- e^{-i\mathcal{E}_- t/\hbar}|-\rangle$$

$|\mathcal{E}\rangle$: eigenstate of the non-Hermitian Hamiltonian,

$$P(k) = \frac{|\langle k|\mathcal{E}\rangle|^2}{\sum_k |\langle k|\mathcal{E}\rangle|^2} \qquad (2)$$

Conditional probability to find the excitation on site k given that the excitation is in the system and not in the photon field.

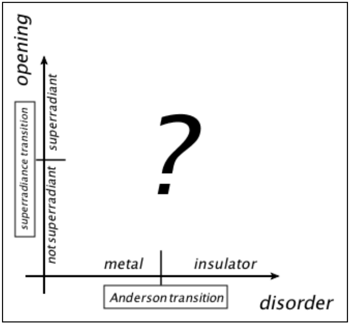
$$|\psi(t)\rangle = e^{-iH_{eff} t/\hbar}|\psi_0\rangle,$$

$|\psi(t)\rangle$: projection on the single excitation manifold of the molecular aggregate full wave function (including also the photon field degrees of freedom) at time t.

Interplay of Superradiance and Disorder: Shielding in Subradiant states

- Cooperativity can affect the response of the system to disorder in a drastic way: while superradiant states show robustness to disorder, in the subradiant subspace, long range interaction is effectively shielded and signature of localization can emerge
- *Subradiant hybrid states in the open 3D Anderson-Dicke model*, A. Biella, F. Borgonovi, R. Kaiser, G.L. Celardo, EuroPhys. Lett. **103** 57009, (2013).

  Interplay of superradiance and disorder in the Anderson Model G.L. Celardo, A. Biella, L. Kaplan and F. Borgonovi, Fortschritte der Physik **61**, 250 (2013).

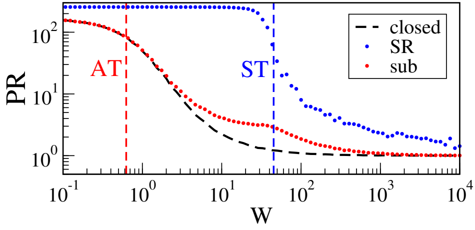


Open Anderson model:

$$H_{eff} = H_{AM} + Q$$

$$H_{AM} = \sum E_i^0 |i\rangle\langle i| + \Omega \sum |i\rangle\langle i+1|$$

$$Q = -i\gamma/2 \sum |i\rangle\langle j|$$



Fortschr. Phys. **61**, 250 (2013); EPL **103**, 57009 (2013); PRB **90**, 075113 (2014); PRB **90**, 085142 (2014); PRB **91**, 094301 (2015).

Hybrid subradiant states

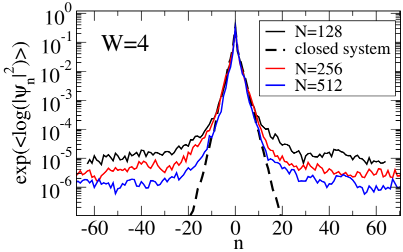


Figure: The averaged probability distribution of all eigenstates of the non-Hermitian Hamiltonian that are strongly peaked in the middle of the chain is shown. verage the logarithm of In all cases we fix $\Omega = 1$, $\gamma = 0.1$.

Common lore: no localization with long range..connection with CS

LIGHT LOCALIZATION

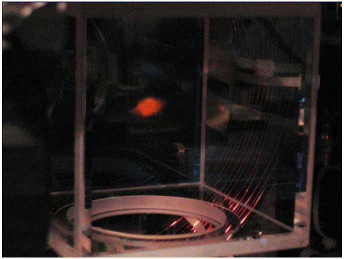
IN

COLD ATOMIC CLOUDS

Localization of Light in Cold Atomic Clouds

Cold Atoms: $\lambda \ll L$

Robin Kaiser Lab

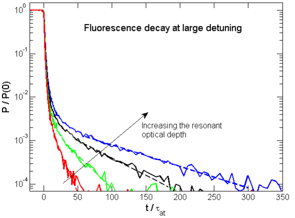


$$V_{nm} = -\frac{\cos(k_0 r_{nm})}{k_0 r_{nm}} - i\,\frac{\sin(k_0 r_{nm})}{k_0 r_{nm}},$$

Optical thickness:

$$b_0 = L/l = \rho \frac{4\pi}{k_0^2} \left(\frac{N}{\rho}\right)^{1/3} \approx \frac{N}{M}$$

Super and Subradiance: optical thickness



Décroissance de la fluorescence en fonction du temps. Le laser excitateur est coupé à t = 0 et l'axe des temps est en unité de la durée de vie d'un atome unique (26 ns). Les différentes courbes correspondent à différentes épaisseurs optiques à résonance et le désaccord du laser est de cinq fois la largeur naturelle de la transition.

$$\Gamma_{SR} \propto b_0$$
$$\Gamma_{Sub} \propto 1/b_0$$
$$\Delta_E \propto b_0$$

Comparison with Anderson Model

ANDERSON MODEL	COLD ATOMS
1) Sites in an ordered lattice	1) Atoms randomlly distributed in a 3D Volume
2) Nearest-Neighbor Interactions	2) Long range 1/r interactions
3) Closed Systems	3) Open System
	4) Cooperativity: Sub and Super-radiance

T. Kottos and A. Mendez-Bermudez

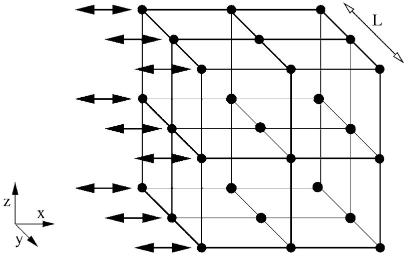
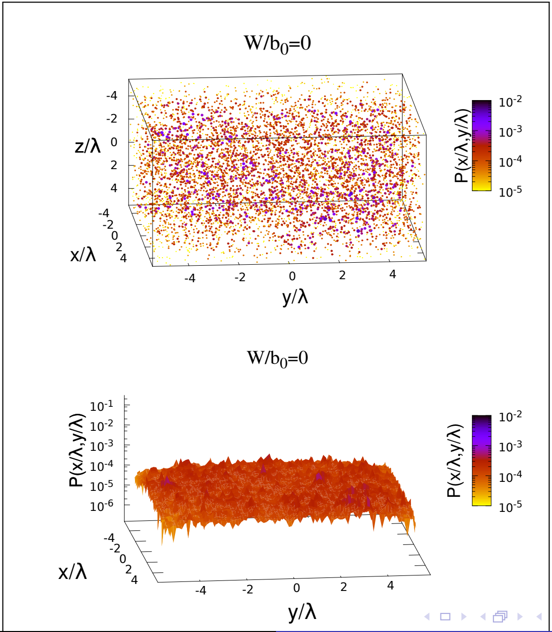


FIG. 1. Scattering setup. The sample is a cubic lattice of linear length L. To each of the $M=L^2$ sites of the layer $n_x=1$ semi-infinite single mode leads are attached.

Extended Subradiant States



Literature

Absence of Localization

PRL 112, 023905 (2014) PHYSICAL REVIEW LETTERS week ending 17 JANUARY 2014

Absence of Anderson Localization of Light in a Random Ensemble of Point Scatterers

S. E. Skipetrov[1,*] and I. M. Sokolov[2,†]
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(Received 19 March 2013; published 16 January 2014)

As discovered by Philip Anderson in 1958, strong disorder can block propagation of waves and lead to the localization of wavelike excitations in space. Anderson localization of light is particularly exciting in view of its possible applications for random lasing or quantum information processing. We show that, surprisingly, Anderson localization of light cannot be achieved in a random three-dimensional ensemble of point scattering centers that is the simplest and widespread model to study the multiple scattering of waves. Localization is recovered if the vector character of light is neglected. This shows that, at least for point scatterers, the polarization of light plays an important role in the Anderson localization problem.

"Localization only in the scalar model for high density"

Magnetic field and localization

PHYSICAL REVIEW LETTERS **121,** 093601 (2018)

Localization Transition for Light Scattering by Cold Atoms in an External Magnetic Field

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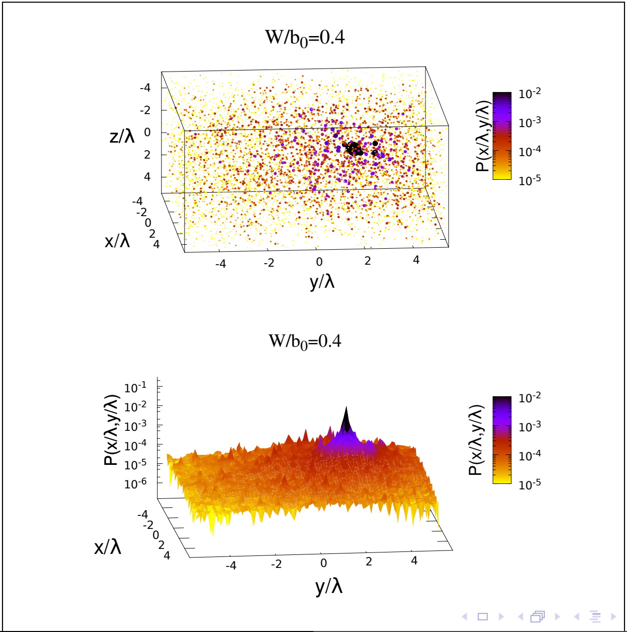
(Received 11 May 2018; published 29 August 2018)

We establish a localization phase diagram for light in a random three-dimensional (3D) ensemble of motionless two-level atoms with a threefold degenerate upper level, in a strong static magnetic field. Localized modes appear in a narrow spectral band when the number density of atoms ρ exceeds a critical value $\rho_c \simeq 0.1 k_0^3$, where k_0 is the wave number of light in the free space. A critical exponent of the localization transition taking place upon varying the frequency of light at a constant $\rho > \rho_c$ is estimated to be $\nu = 1.57 \pm 0.07$. This classifies the transition as an Anderson localization transition of 3D orthogonal universality class.

Localized Subradiant States



Mobility Edge in the Imaginary axis

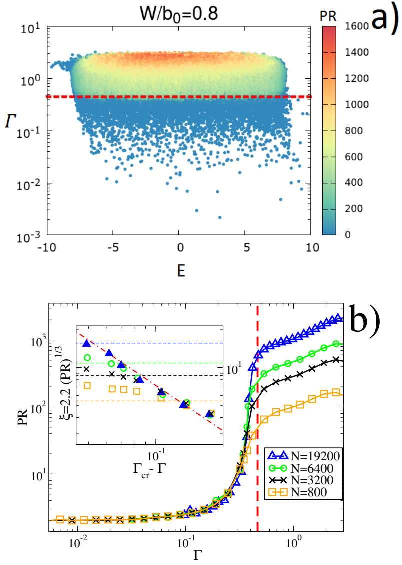


FIG. 4: (Color online) *Mobility edge in the imaginary axis*. Upper

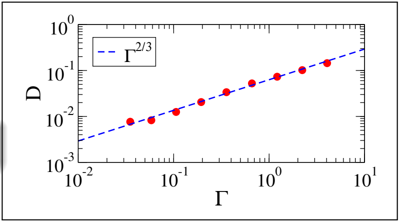
Discussion

Critical Decay Width

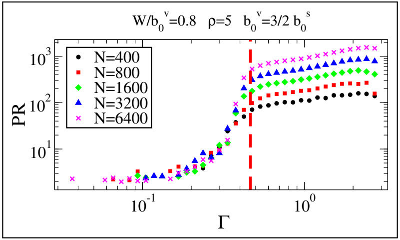
$$\Gamma_{cr} \approx (W/b_0 - 0.053)/1.61$$

- Critical decay width at which PR diverges
- Divergence is consistent with a power law: $(\Gamma_{cr} - \Gamma)^{\nu}$ with $\nu \approx 1.2$
- Opetical thickness b_0 determines strength of the coupling.

Mean Level Spacing and Decay Width



VECTOR MODEL



Conclusions and Perspectives

1. Interplay of cooperative effectes can lead to interesting effects: Mobility edge in the imaginary axis. (see G. L. C., M. Angeli, R. Kaiser, arXiv:1702.04506.)
2. Criticality and Experimental realization

THANK YOU!!!