Floquet topological semimetal with nodal helix:

“The Stark effect in topological semimetals”

Kun Woo Kim

Institute for Basic Science

Quantum transport:

1. Transport of disordered boundary modes.
2. TI nanowire's magneto-conductance.
3. Metal-to-Insulator transition via disorder.

Collaborators:

Gil Refael (Caltech) group.
Junho Seo (KRISS) group.
Hee Chul Park (IBS) group.

Floquet quantum matter:

1. Photocurrent in surface Dirac fermion.
2. Weyl semimetal under strong E-field.
3. Floquet Nodal helix semimetal.

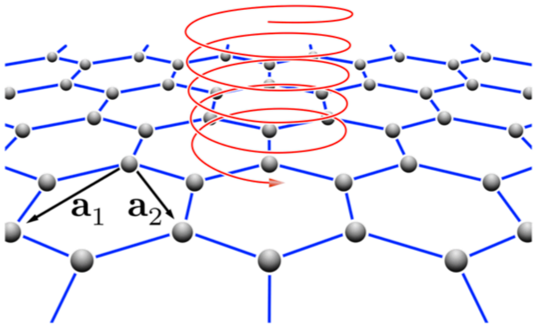
Naoto Nagaosa (RIKEN) group.
Kwon Park (KIAS) group.

Disordered Floquet quantum matter:

1. Anomalous Floquet Anderson insulator.
2. Quantum kicked rotor in 4D.
3. Quantum walk with chiral symmetry in !D.

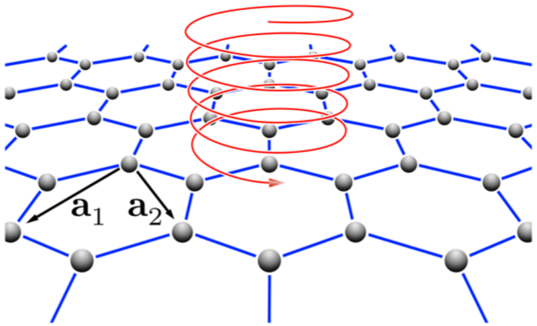
Alexander Altland (Cologne) group.

Floquet topological insulators



Oka and Aoki, PRB 79 081406 (2009)

Floquet topological insulators



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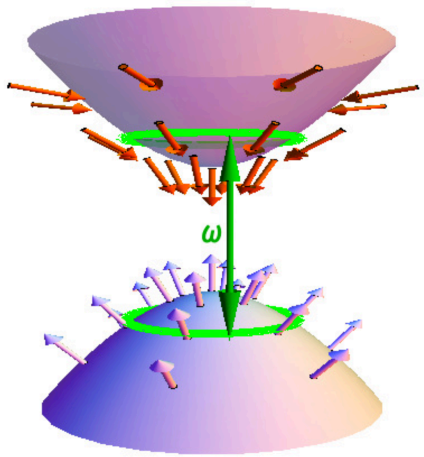

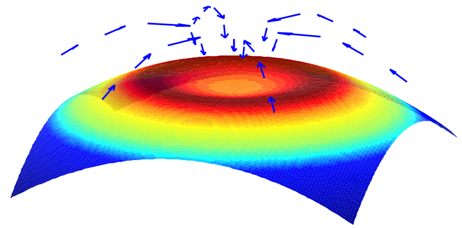

Lindner, Refael, Galitski, Nature Physics, 7, 490 (2011)

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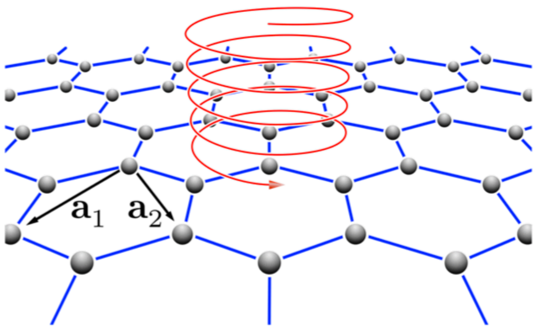

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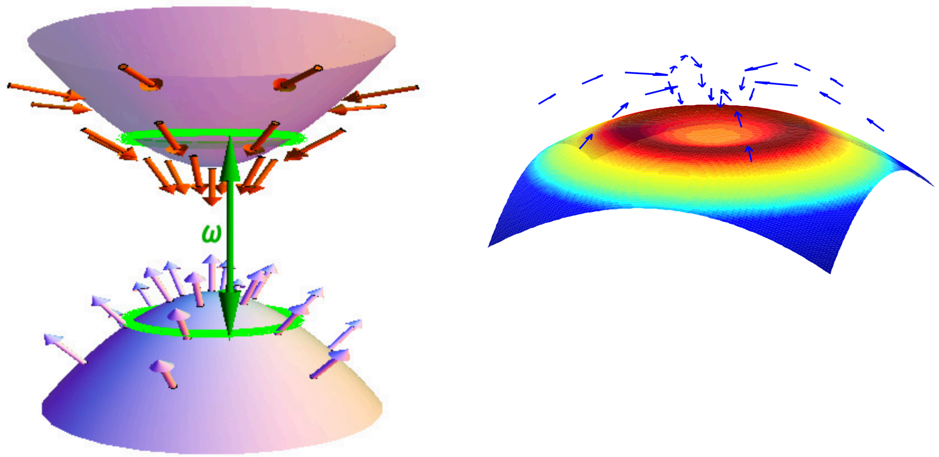

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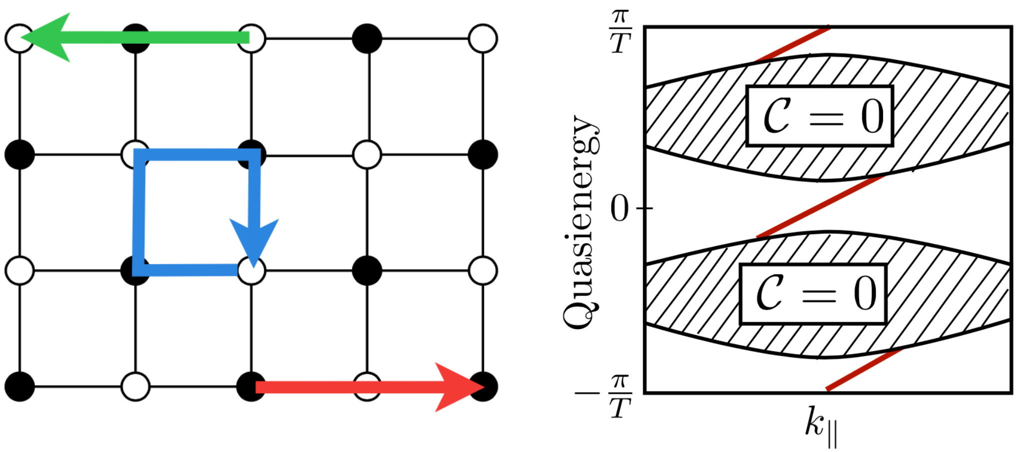

Runder et al. PRX 3, 031005 (2013)

Outline of this talk

1. The Stark effect and Bloch oscillation in solids: theory and experiments

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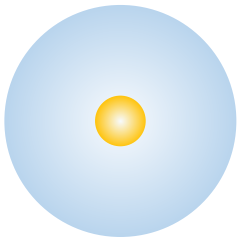
2. Wannier-Stark ladders in topological semimetals:

   - **Weyl semimetal + E-field:** KWK, WR Lee, YB Kim, K Park, Nat. comm. **7**, 13489

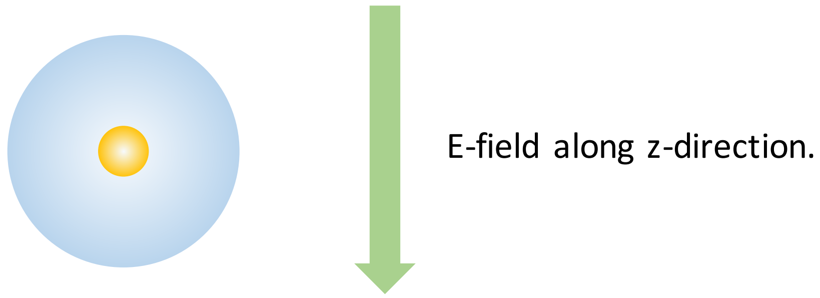
   - **Nodal helix semimetal + E-field**: KWK, HW Kwon, K Park, arXiv:1808.04079

The Stark effect



Consider a hydrogen atom. Due to the rotational symmetry of Coulomb interaction with proton, electronic energy levels are degenerated.

The Stark effect



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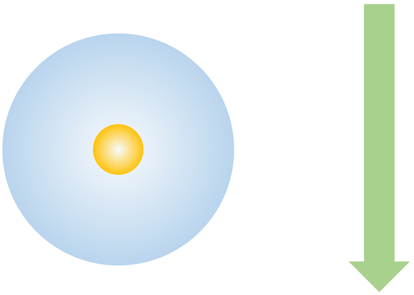
The application of electric field breaks the inversion symmetry and energy level splitting increases with the field strength. (discovered in 1913, Nobel prize in 1919).

The Stark effect



E-field along z-direction.

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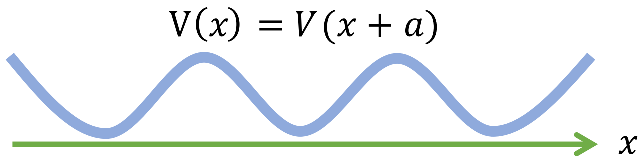
Along with the Zeeman effect, the experimental observation of the Stark effect provided the confirmation of perturbation theory in quantum mechanics.

Wannier-Stark ladder from the Schrodinger equation



In 1960 G.H. Wannier predicted the Stark effect for a Bloch electron in a constant electric field.

$$\left[-\frac{1}{2m}\frac{\partial^2}{\partial x^2} + V(x) \qquad \right]\psi(x) = \epsilon\psi(x)$$

Wannier, Phys. Rev. 117, 432 (1960)

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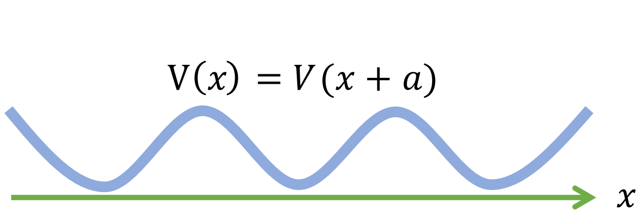

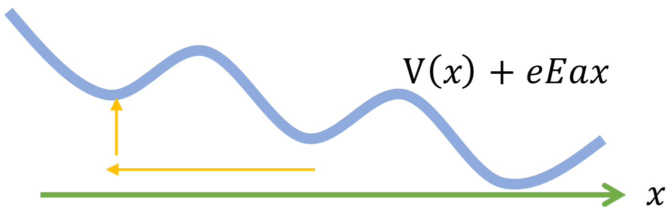

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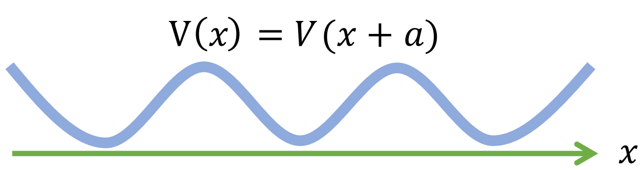

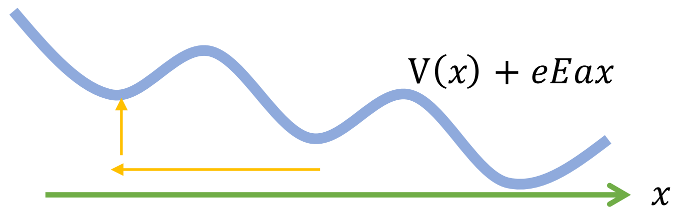

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If "ϵ" is an eigenvalue of the Schrodinger equation with eigenfunction $\psi(x)$, there is a series of ladder like eigenvalues "$\epsilon + maeE$" with eigenfunction $\psi(x - ma)$:

$$\left[-\frac{1}{2m}\frac{\partial^2}{\partial x^2} + V(x) + eE(x - ma)\right]\psi(x - ma) = \epsilon\psi(x - ma)$$

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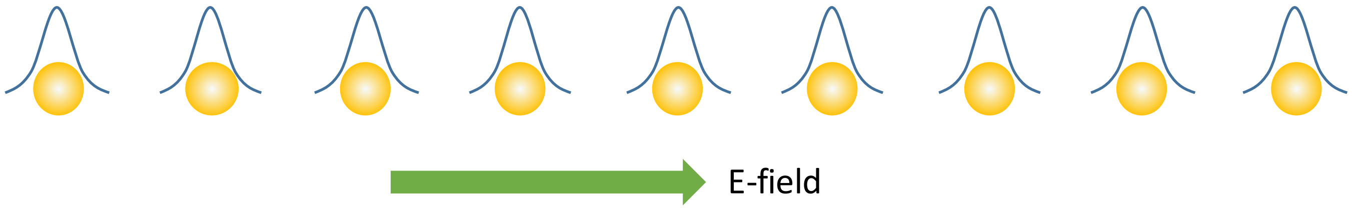
The Stark effect of electrons in lattice: Wannier-Stark ladder

Interestingly, Wannier's solution from the abelian approximation naturally contains the Zak phase:

$$\epsilon = \left(m + \qquad \right) aeE + \frac{a}{2\pi}\int_0^{2\pi/a} \epsilon(k)\,dk$$

Integer m, lattice constant a, a dispersion relation $\epsilon(k)$.



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$$\epsilon = \left(m + \frac{1}{2\pi}\gamma^{Zak}\right) aeE + \frac{a}{2\pi}\int_0^{2\pi/a} \epsilon(k)\, dk$$

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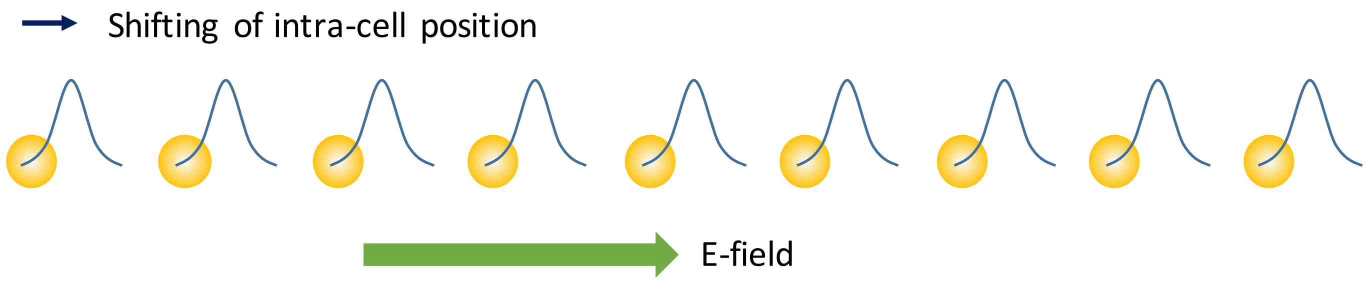
Considering $i\frac{\partial}{\partial k} = \hat{x}$ as a position operator, the Zak phase is a measure of Bloch electron's position within unit cell, or a polarization. It is then naturally translated to a potential energy under an external electric field.



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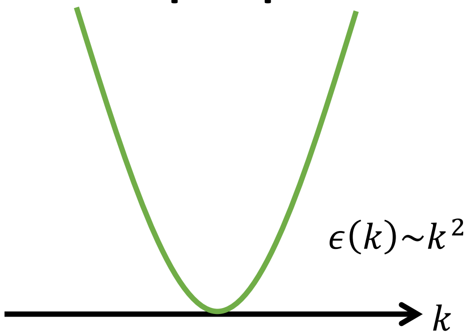
The Stark effect : perspective in momentum space



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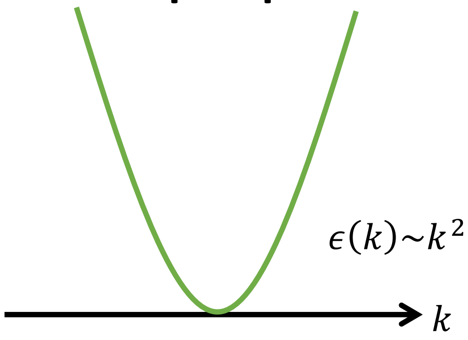

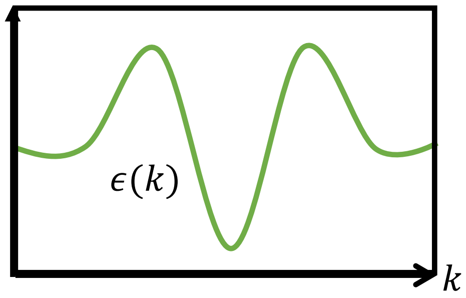

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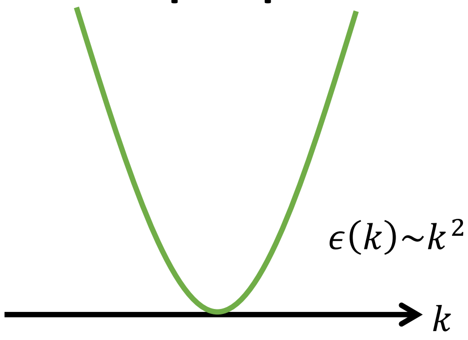

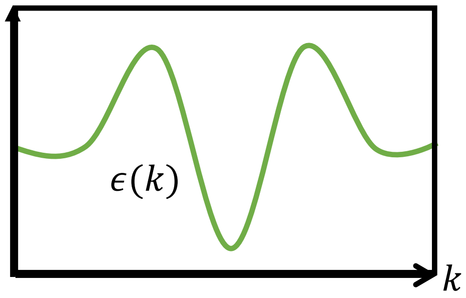

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To travel across the Brillouin zone n-times: $T = \frac{2\pi n}{a} \frac{\hbar}{eE}$.

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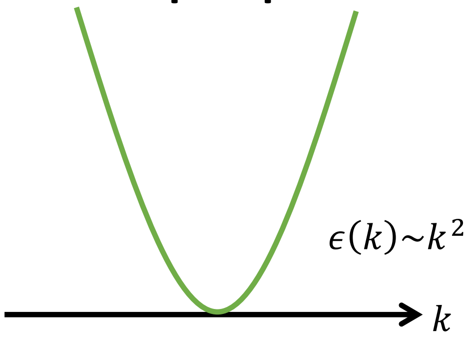

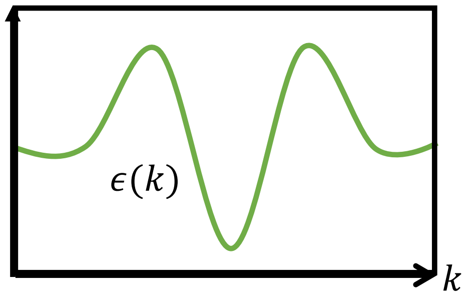

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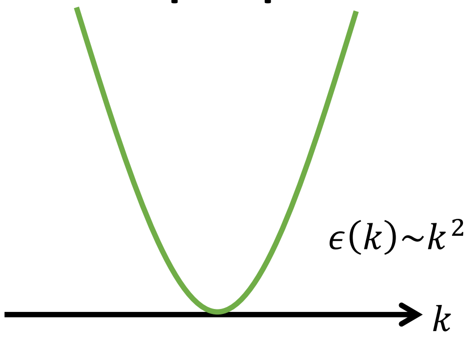

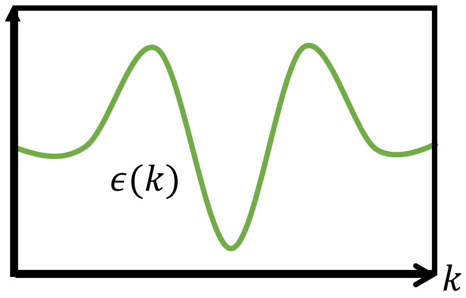

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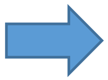
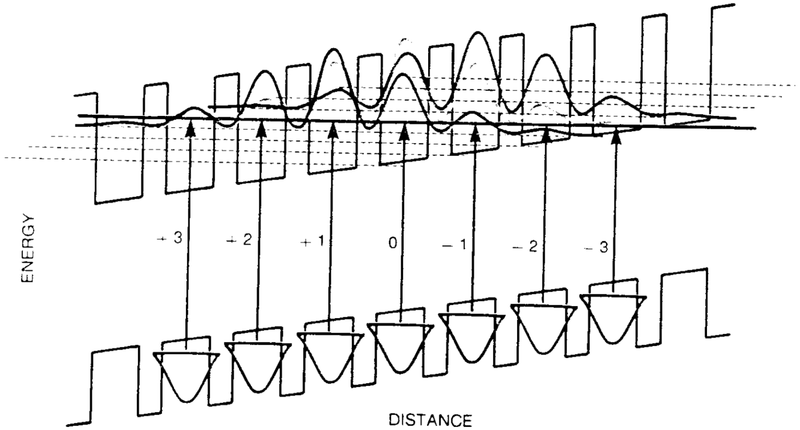
A Bloch electron shouldn't be scattered during the travel: $T < \tau_s$  $n \frac{h}{\tau_s} < eaE$

Experimental limitation: a lattice unit cell of 5Å, E-field 10 (kV/cm), $v_F = 10^5 (m/s)$ yields the length of Bloch oscillation: ~1 (um), which is one order larger than a typical mean free path 100 (nm).

Wannier-Stark ladder in experiments: semicondutor supperlattice



A semiconductor superlattice structure is used to simulate artificial semiconductor with much larger lattice spacing (~100Å). With electric field strength (~10kV/cm), the Wannier-Stark ladder was observed.

Mendez et al. PRL 60, 2426 (1988);

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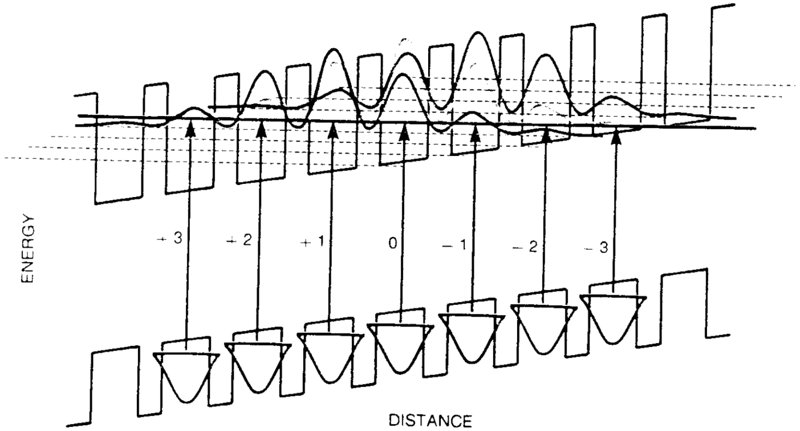

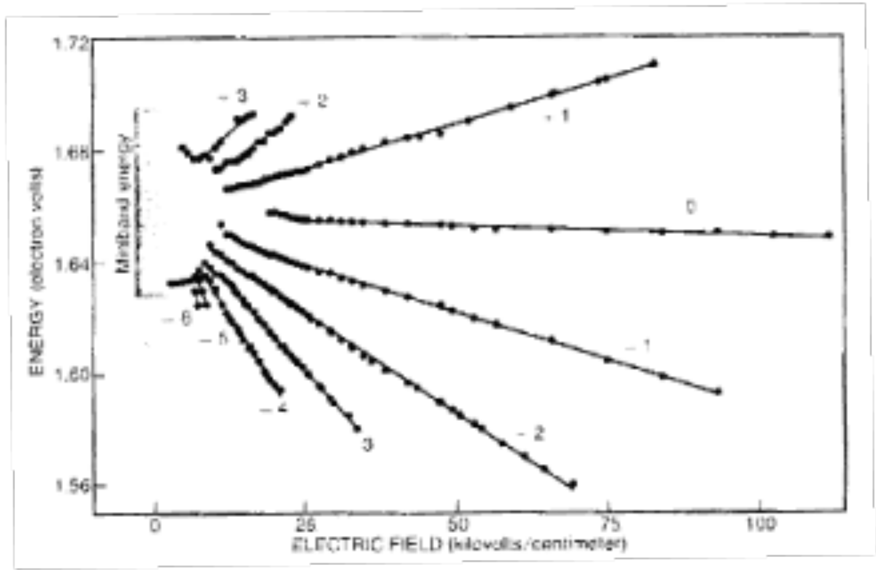

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For a given E-field strength, the number of ladder indicates the localization length of the Bloch oscillation. The stronger the E-field, the more localized is the Bloch electron.

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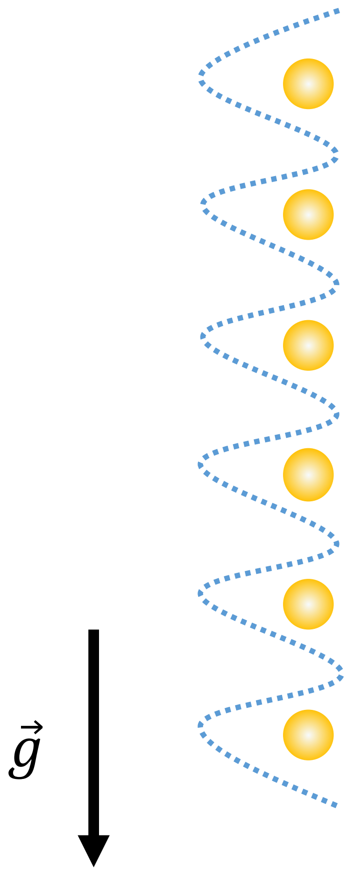
Wannier-Stark ladder in experiments: cold atoms in optical lattice



Standing wave potential with gravity to realize the Stark Hamiltonian:

$$\mathrm{V(x)} = \cos(kx) + mgx\,.$$

Wilkinson et al. PRL **76**, 4512

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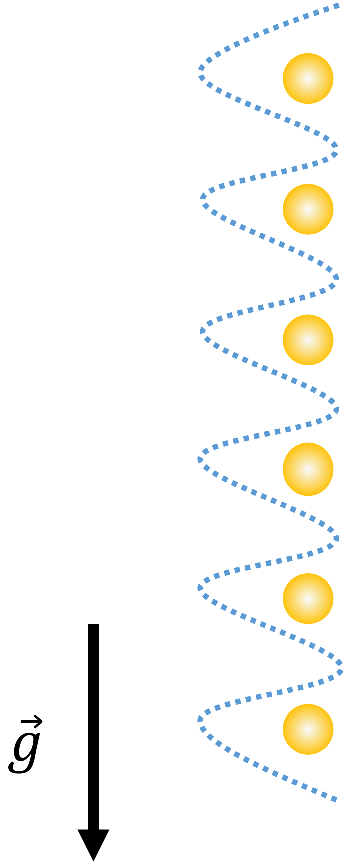

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Two counter propagating waves

$$V(x,t) = \cos(w_1 t + k_1 x) + \cos(w_2 t - k_2 x)\,,$$
$$= 2\ cos\left(\overline{w}t + \frac{1}{2}\Delta kx\right)\cos\left(\frac{1}{2}\Delta wt + \bar{k}x\right),$$

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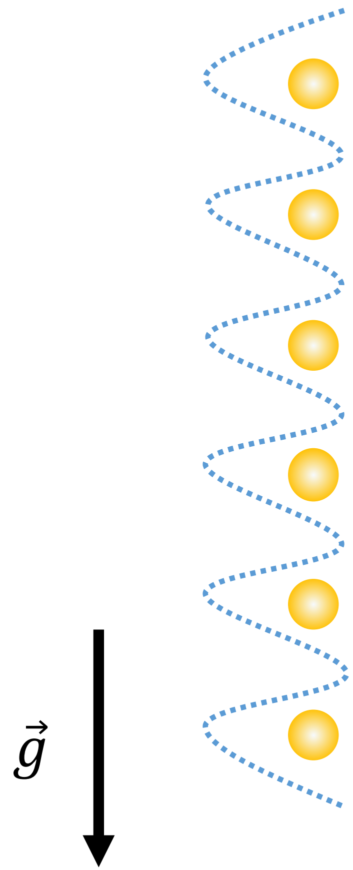

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$$\begin{aligned} V(x,t) &= \cos(w_1 t + k_1 x) + \cos(w_2 t - k_2 x)\,, \\ &= 2\,cos\left(\overline{w}t + \tfrac{1}{2}\Delta kx\right)\cos\left(\tfrac{1}{2}\Delta wt + \bar{k}x\right), \\ &\sim \cos\left(\bar{k}\left[x - \frac{1}{2}at^2\right]\right). \end{aligned}$$

The linearly increasing phase difference, $\Delta w = w_2 - w_1 = at\bar{k}$, is simulating the gravity and it thus provides an external "electric" field.

Wilkinson et al. PRL **76**, 4512

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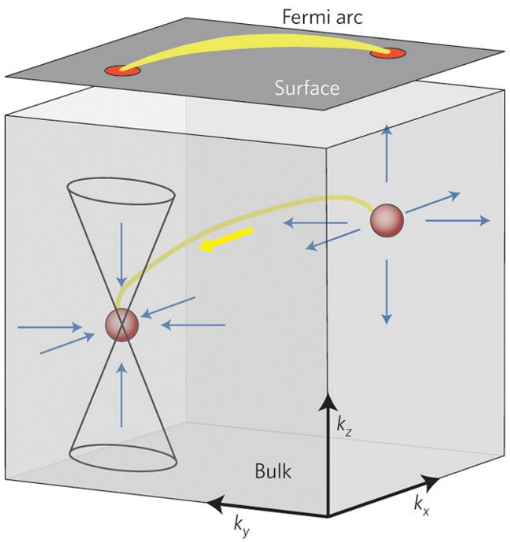
Our working regime: $\frac{h}{\tau_s} < eaE < \text{Bandwidth}$

Weyl semimetal and the Zak phase

Weyl node is located in 3-dimensional space. We can think of the system as a collection of 1-dimensional wires along z-axis in momentum space, $\vec{k}_\perp = (k_x, k_y)$.



Bernevig, Nat. Phys. **11** 698

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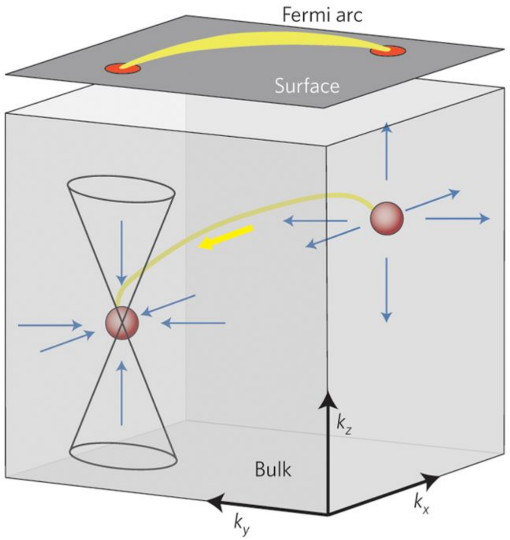

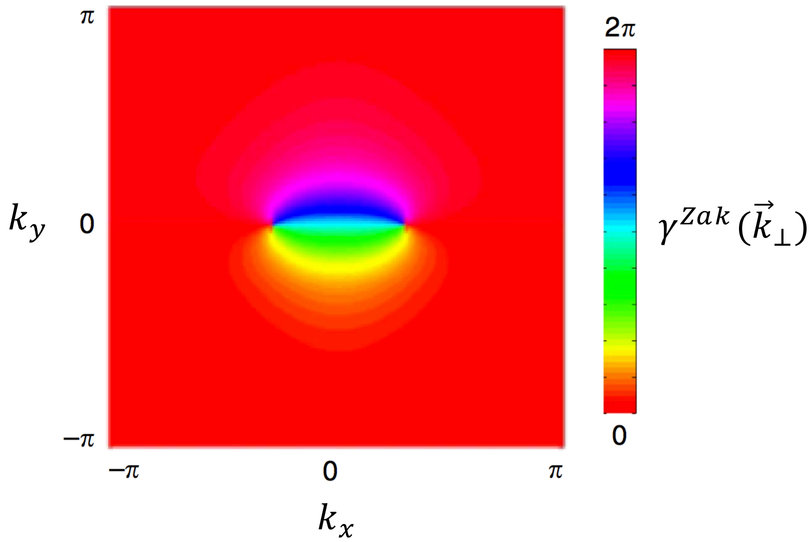

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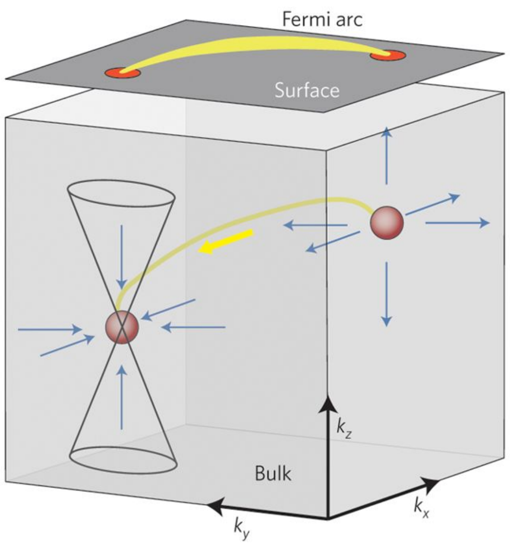

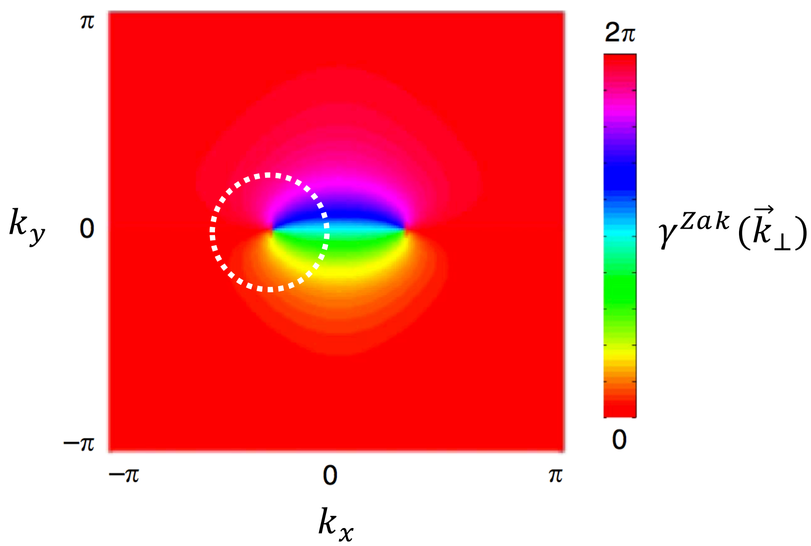

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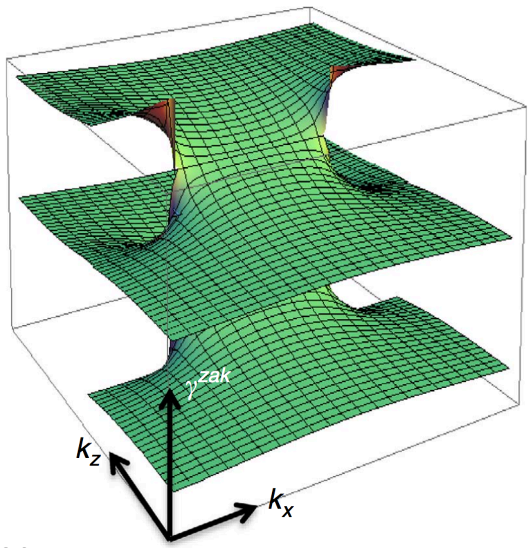
Wannier-Stark ladder in Weyl semimetal

Momentum resolved energy spectrum is:

$$\epsilon(\vec{k}_\perp) = \left(m + \frac{1}{2\pi}\gamma^{Zak}(\vec{k}_\perp)\right) aeE + \frac{a}{2\pi}\int_0^{2\pi/a} \epsilon\left(k_z, \vec{k}_\perp\right) dk_z$$ with integer m.

c



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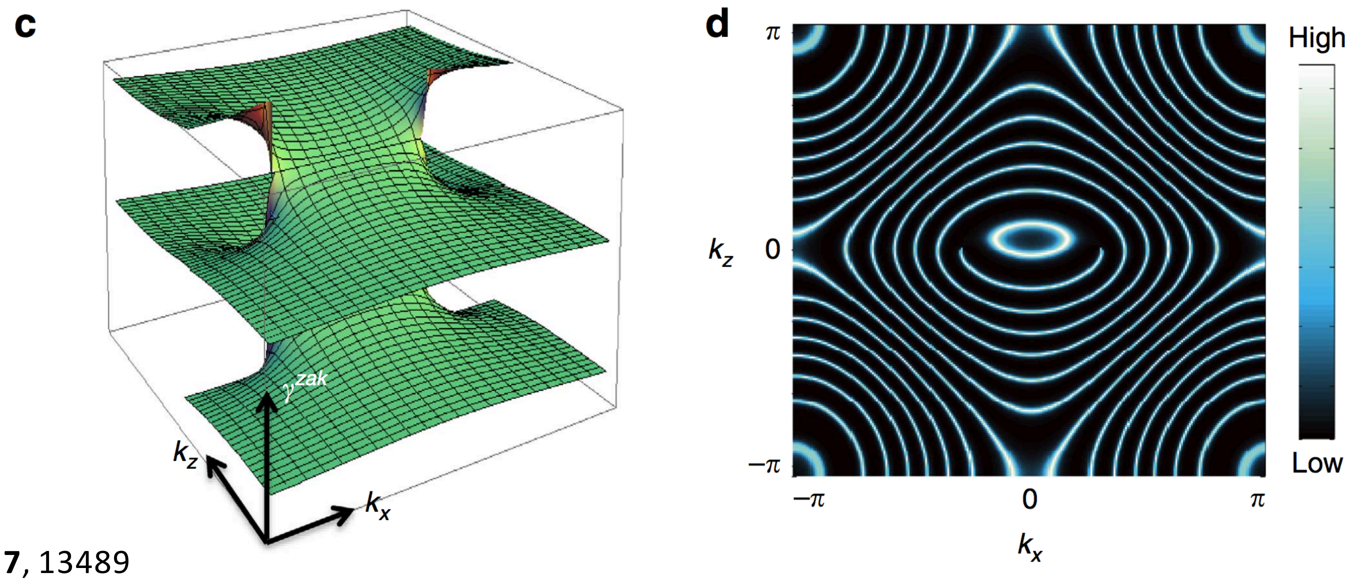
The zero energy cut of spectrum will contain a open line segment from the Zak phase winding.



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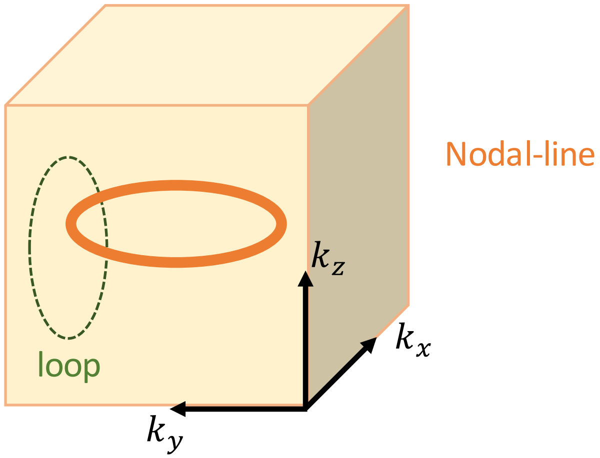
Our working regime: $\frac{h}{\tau_s} < eaE < \mathrm{Bandwidth}$

Nodal-line semimetal and nodal helix in time domain



Nodal-line semimetal is characterized by a Zak phase of a loop enclosing a nodal line.

$$\gamma^{Zak} = \oint \left\langle u_{\vec{k}} \middle| i \frac{\partial}{\partial \vec{k}} u_{\vec{k}} \right\rangle d\vec{k} = \pi$$

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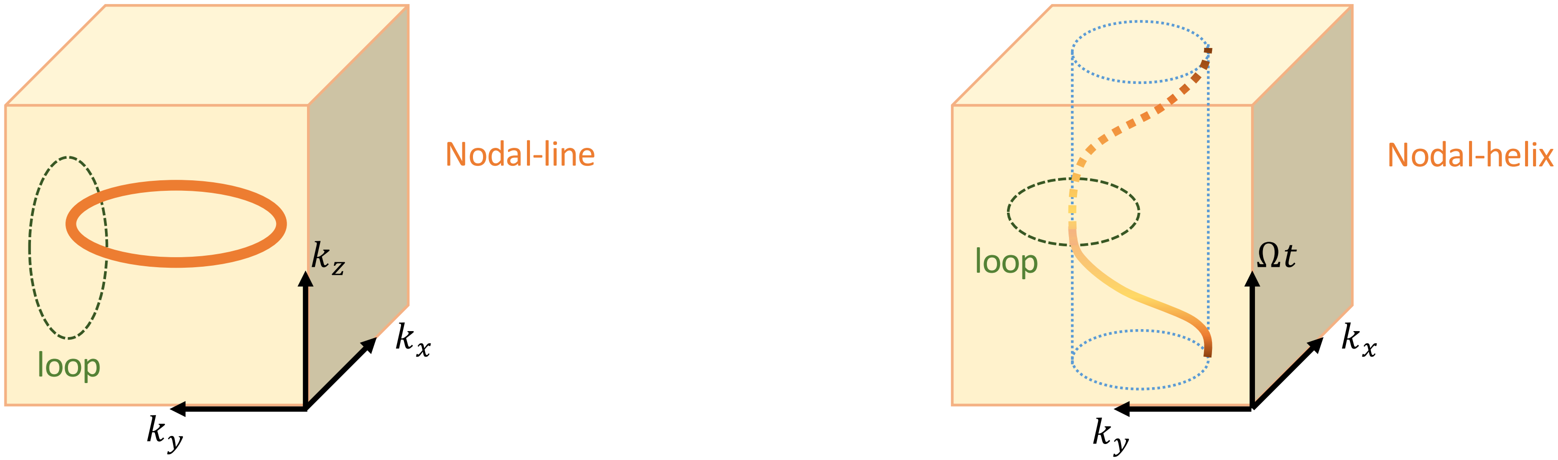

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Consider a gapless 2D Dirac fermion under circularly polarized light. The trace of Dirac point will make a helix in momentum-time domain.

$$H(\vec{k}, t) = [k_x - Acos(\Omega t)]\, \sigma_x + [k_y - Asin(\Omega t)]\, \sigma_y$$

From a Floquet Hamiltonian to a Stark Hamiltonian

Technical slide

Start with a continuum Dirac Hamiltonian: $H(\vec{k},t) = [k_x - Acos(\Omega t)]\ \sigma_x + [k_y - Asin(\Omega t)]\ \sigma_y$

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Writing the Schrodinger equation in the Floquet basis, $\tilde{\phi}_n$:

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$$i\frac{\partial}{\partial t}\psi(t) = H(\vec{k},t)\psi(t) \quad \Rightarrow \quad \sum_m [\widetilde{H}_{nm} - n\Omega\delta_{nm}]\ \tilde{\phi}_m = \epsilon\ \tilde{\phi}_n$$

Where $\widetilde{H}_{nm} = \frac{1}{T}\int_0^T H(t)e^{-i(n-m)\Omega t}dt$.

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Technical side

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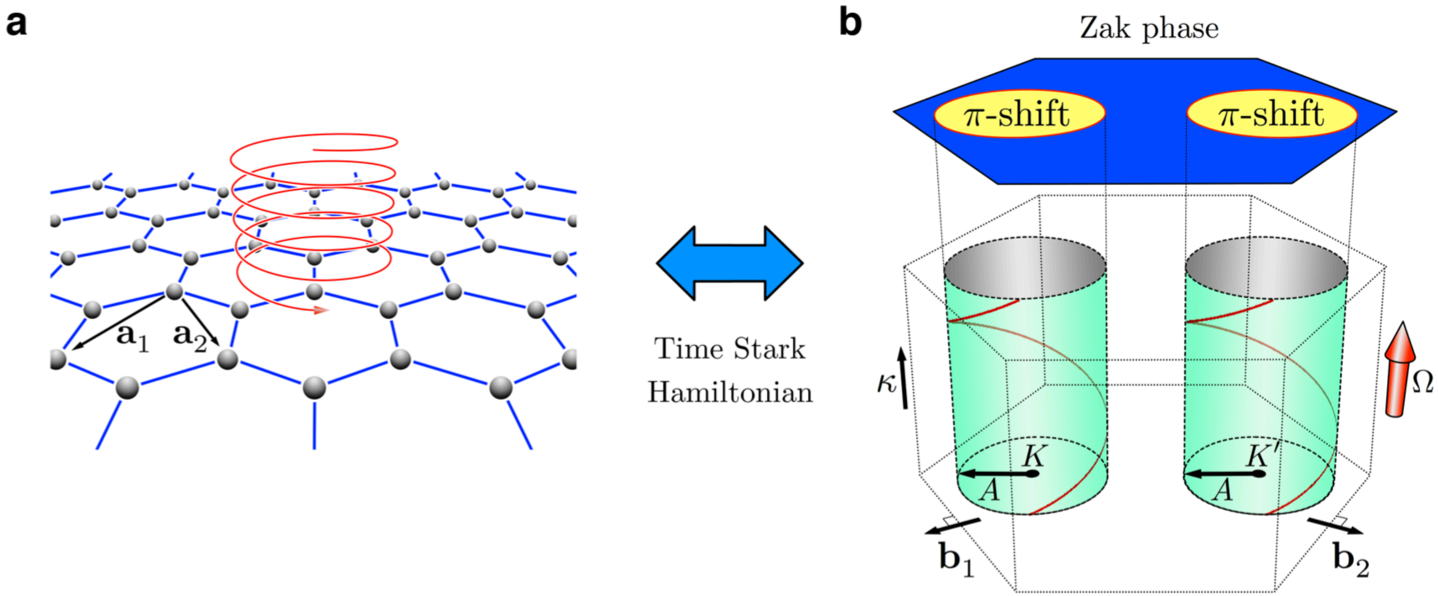
Where $\widetilde{H}_{nm} = \frac{1}{T}\int_0^T H(t)e^{-i(n-m)\Omega t}dt$. The Floquet operator can be thought of as a hopping between Floquet site n and m with a static electric field with strength Ω.

We ended up having 3-dimensional system with E-field from 2-dimensional time-periodic Hamiltonian.

Nodal helix in graphene

With two gapless Dirac fermion in graphene, two nodal helices are generated.



KWK et al., arXiv:1808.04079

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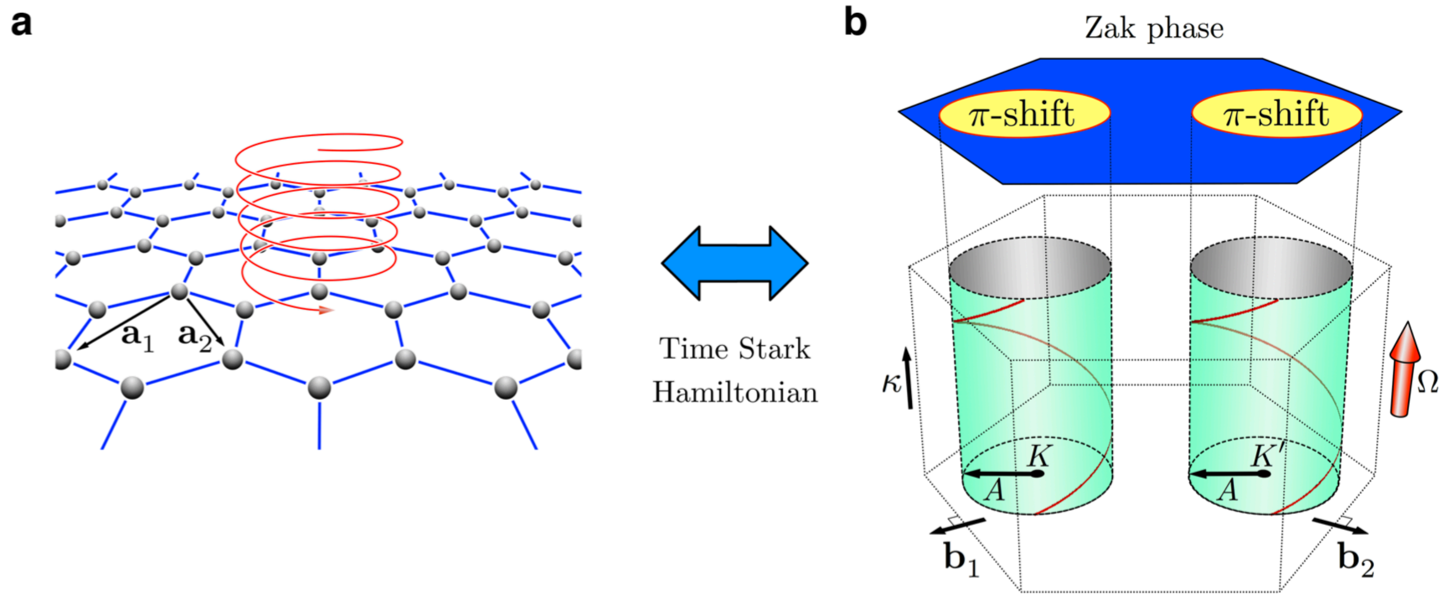
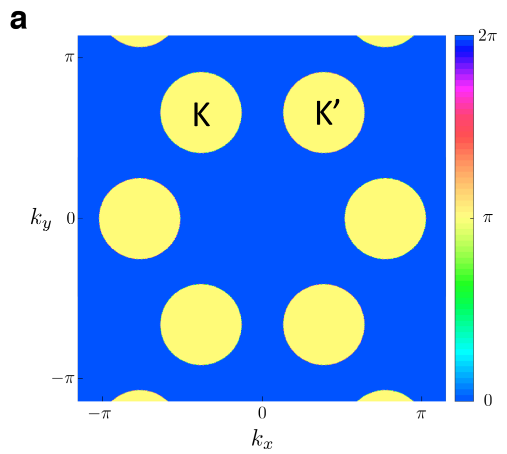


In the high driving frequency limit, this is well known Floquet quantum spin Hall system by Kitagawa et al. We work this problem in the low frequency limit to preserve the 3-dimensionality, and a strong intensity of light regime to induce a sizable ring of helix in momentum space.

KWK et al., arXiv:1808.04079

Nodal helix in graphene: Zak phase and LDOS at zero energy

a. Zak phase $\gamma^{Zak}(\vec{k}_{\perp})$



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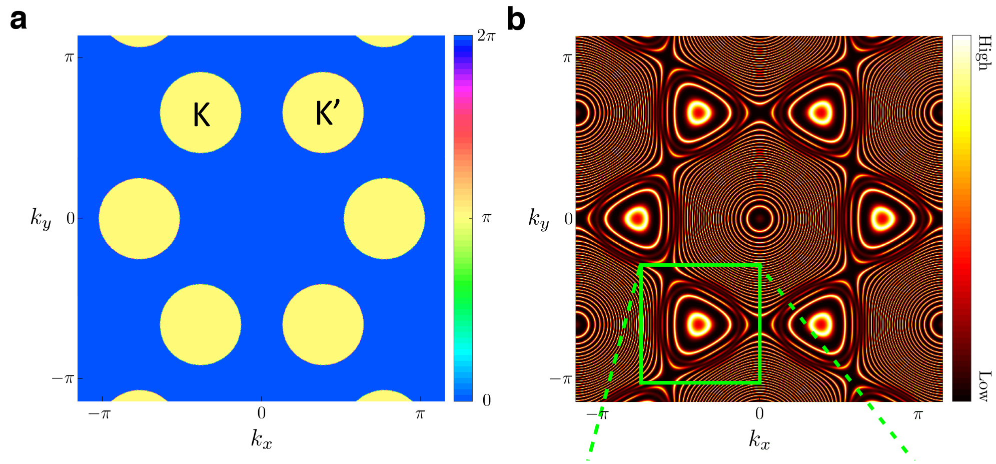


a. Zak phase $\gamma^{Zak}(\vec{k}_\perp)$

b. Momentum resolved LDOS at E=0.

KWK et al., arXiv:1808.04079

Nodal helix in graphene: Zak phase and LDOS at zero energy

A reminder of Wannier-Stark ladder:

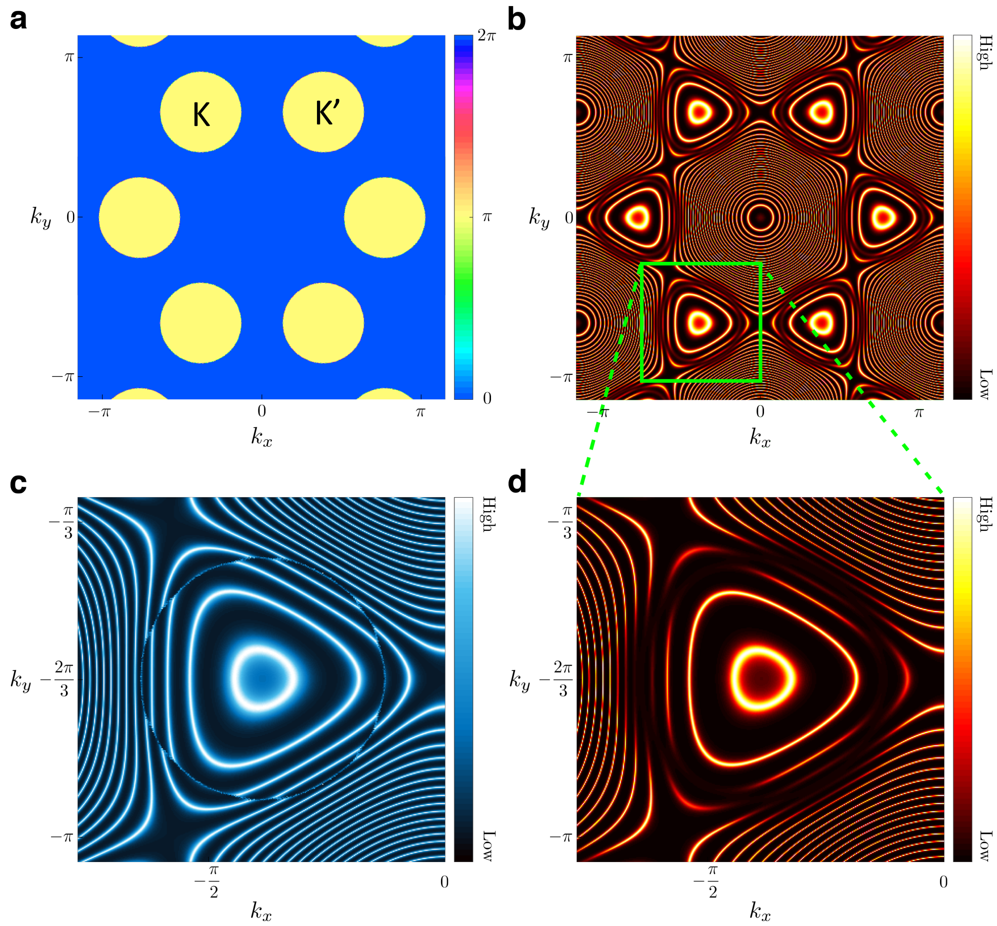
$$\epsilon(\vec{k}_\perp) = \left(m + \frac{1}{2\pi}\gamma^{Zak}(\vec{k}_\perp)\right) aeE + \frac{a}{2\pi}\int_0^{2\pi/a} \epsilon(k_\parallel, \vec{k}_\perp) dk_\parallel$$

a. Zak phase $\gamma^{Zak}(\vec{k}_\perp)$

b. Momentum resolved LDOS at E=0.

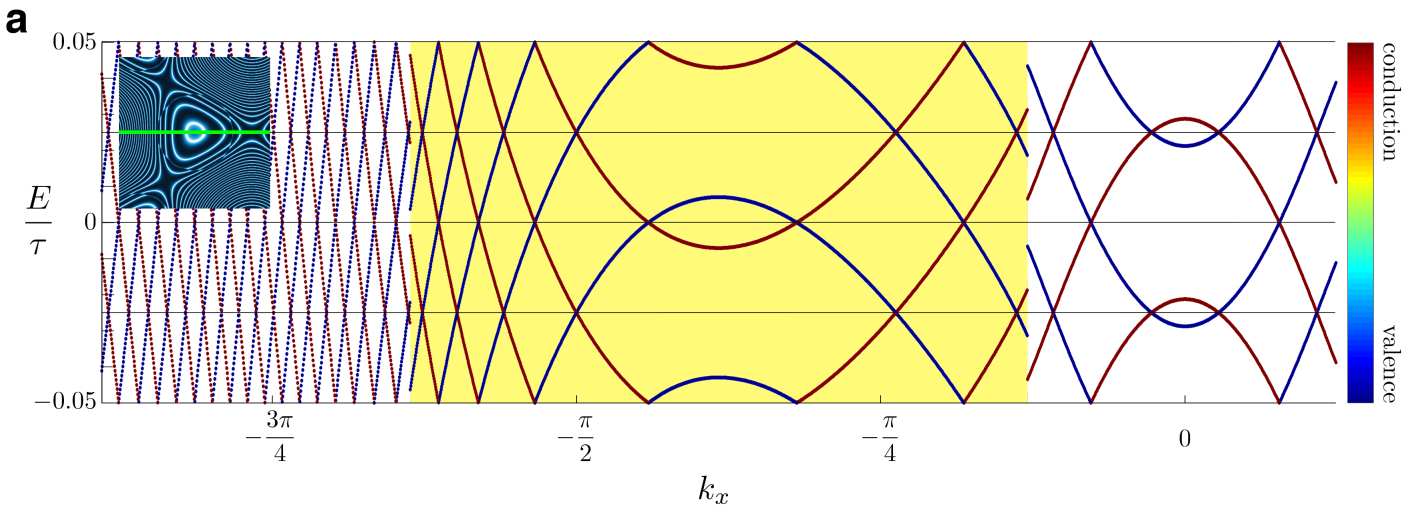
c. LDOS(E=0) from the abelian approximation.

d. Magnified view of **b**.



KWK et al., arXiv:1808.04079

Nodal helix in graphene: dispersion relation

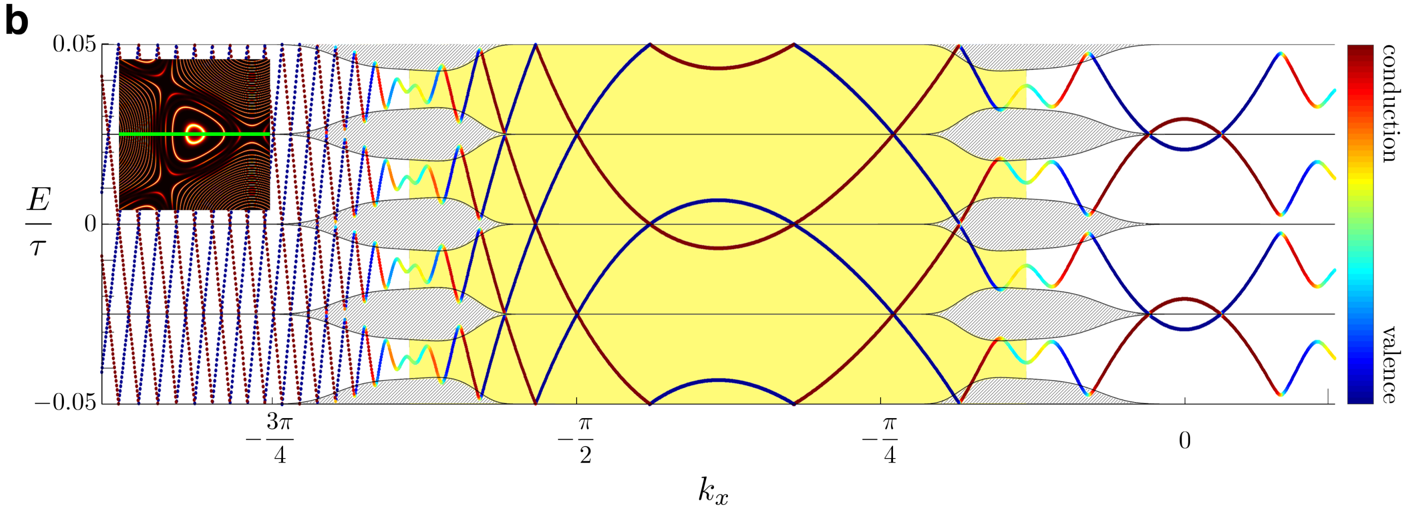


Energy dispersion relation from the abelian approximation shows the Zak phase π shift across the ring marked by yellow shade. By the inter-band tunneling, the discontinuity in spectrum is smoothly connected with energy gap opening.

Here the color code indicates the fraction of eigenfunction belonging to the (conduction/valence) solution from the abelian approximation.

KWK et al., arXiv:1808.04079

Nodal helix in graphene: dispersion relation

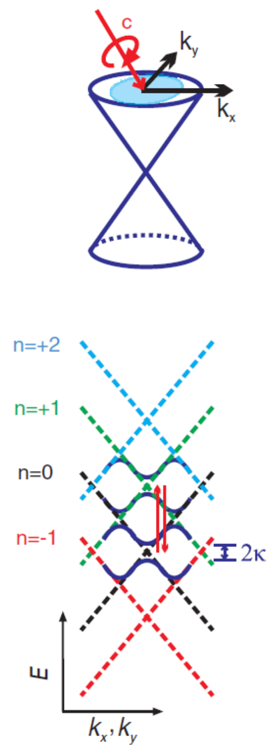


The energy gap is exponentially suppressed with the reduction of driving frequency Ω. In this sense, we argue that the gapless nature is approximately preserved in the Wannier-Stark ladder with an external 'electric' field. Thus, the observation of nodal helix semimetal carrying Zak phase shift π will be possible.

$$\mathcal{E}_{gap} \cong \frac{\Omega A}{\sqrt{2\pi k(A+k)}} e^{-\frac{2(A-k)^4}{k(A+k)\Omega^2}}$$

KWK et al., arXiv:1808.04079

Experiment: irradiated 2D Dirac surface state in 3D topological insulator



Wang et al. Science **342**, 453

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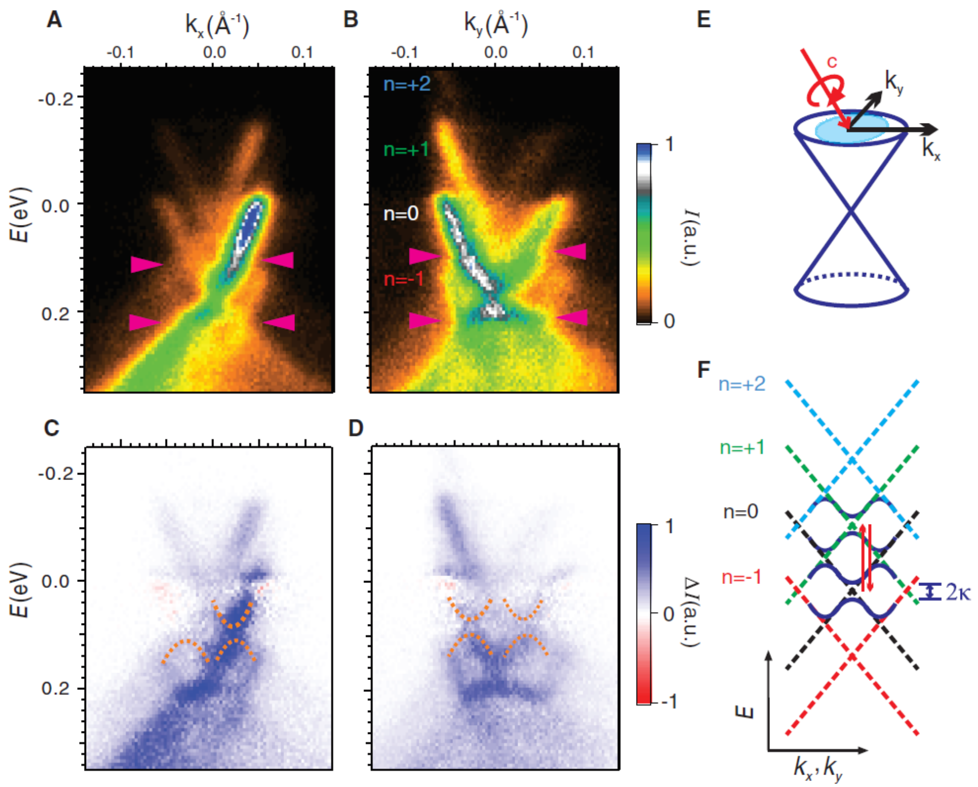

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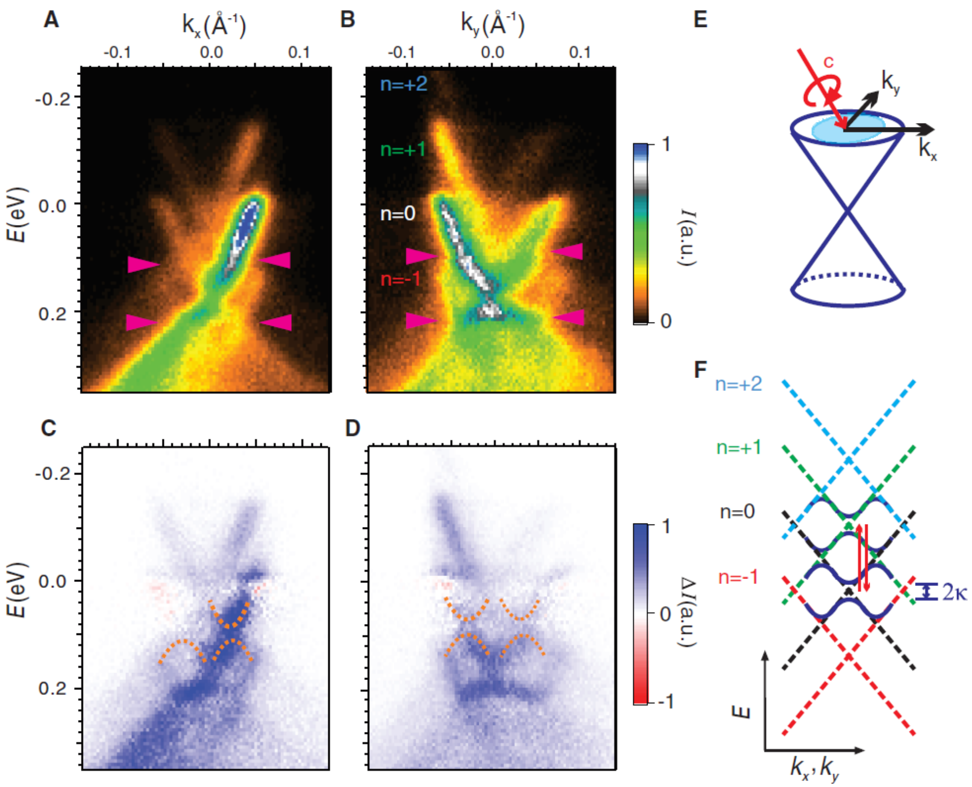


Parameters in this experiment:

Driving frequency = 0.2 (eV)

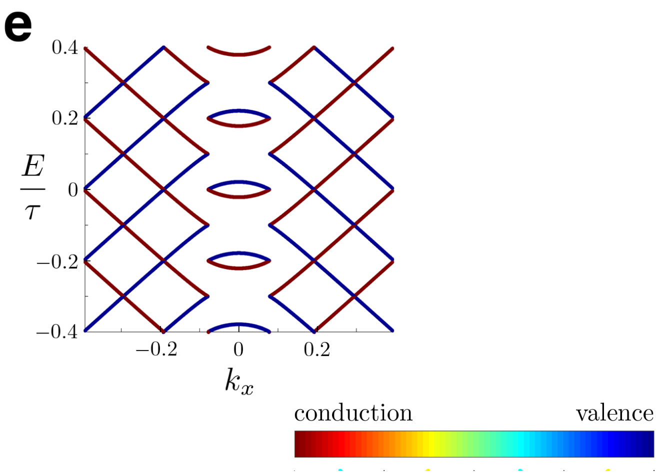
E-field strength = 3.3x10^5(V/cm)

Size of ring k_0 = 0.0165 (1/Å)

We want to point out that this experiment can be interpreted as a series of Wannier-Stark ladder with the Zak phase π change reflected in the spectrum.

Wang et al. Science **342**, 453

A single 2D gapless Dirac fermion: intermediate driving frequency regime



Even with moderate regime of driving frequency, we can see the Wanner-Stark ladder carries a significant amount of conduction(red)/valence(blue) band characters from the abelian approximation.

1st column: Wannier-Stark ladder from the Abelian approximation.

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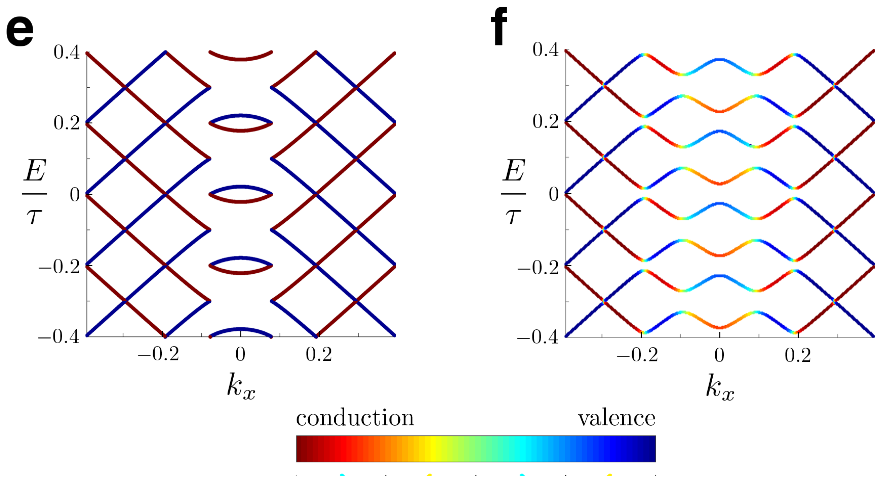

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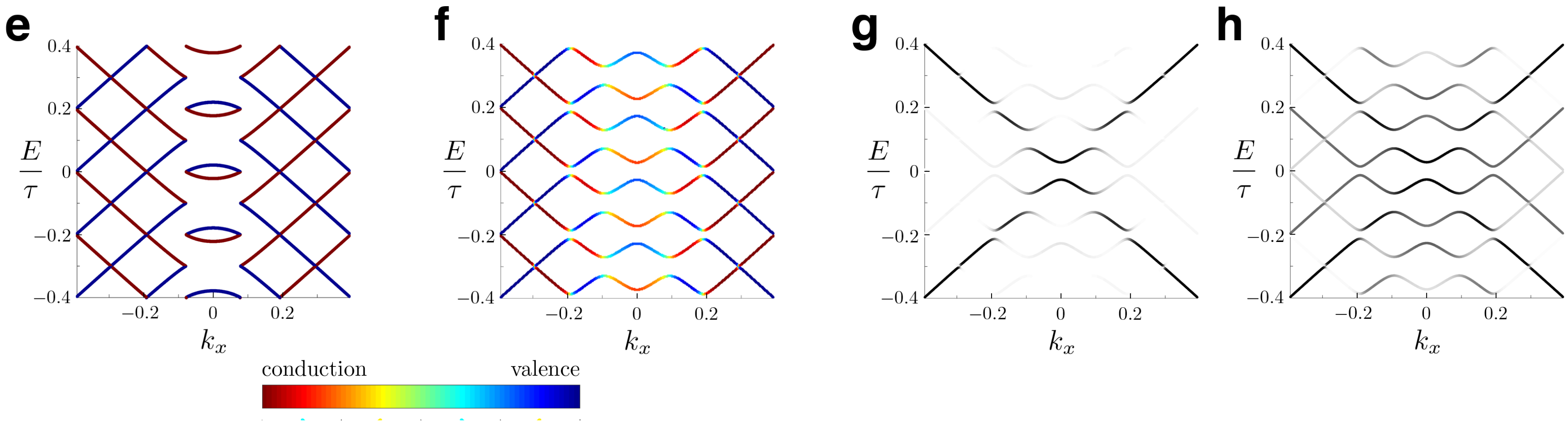

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1st column: Wannier-Stark ladder from the Abelian approximation.
2nd column: Wannier-Stark ladder without approximation.
3rd and 4th column: realistic situation. We limit the number of accessible Floquet bands.

Summary and Outlook

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Thank you for your attention.

Technical slide

A few line of algebra can solve the problem: $$\left[-\frac{1}{2m}\frac{\partial^2}{\partial x^2} + V(x) + eEx\right]\psi(x) = \epsilon\psi(x)$$

Rewrite the equation in the basis of Bloch wave function $\psi(x) = \sum_{nk} B_n(k)\psi_{nk}(x)$:

$$\left[-\frac{1}{2m}\frac{\partial^2}{\partial x^2} + V(x) + eEx\right]\sum_{nk} B_n(k)\psi_{nk}(x) = \epsilon\sum_{nk} B_n(k)\psi_{nk}(x),$$

Then, take an inner product by an Bloch wave function from the left:

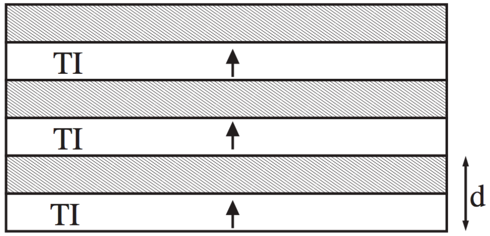
$$\left[\epsilon_n(k) + eEi\frac{\partial}{\partial k} + \sum_m eE\left\langle u_{mk}\middle| i\frac{\partial}{\partial k}u_{nk}\right\rangle\right]B_n(k) = \epsilon B_n(k)$$

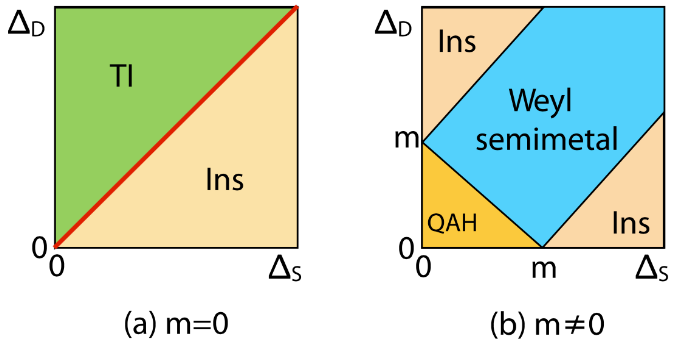
By taking one band only, We can obtain: $$B_n(k) = exp\left(-\frac{i}{eE}\int_0^k \left[\epsilon - \epsilon_n(k') - eE\left\langle u_{nk'}\middle| i\frac{\partial}{\partial k'}u_{nk'}\right\rangle\right]dk'\right)$$

From the periodic condition, energy quantization: $$\epsilon = maeE + \frac{a}{2\pi}\int_0^{2\pi/a}\left[\epsilon_n(k') + eE\left\langle u_{nk'}\middle| i\frac{\partial}{\partial k'}u_{nk'}\right\rangle\right]dk'$$

Weyl semimetal in experiments

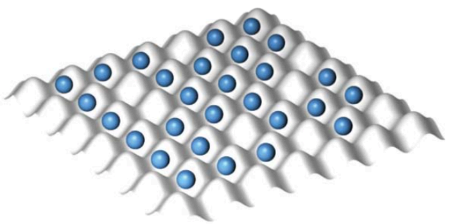
1. Topological insulator multilayer proposal (Burkov and Balents, PRL **107**, 127205)

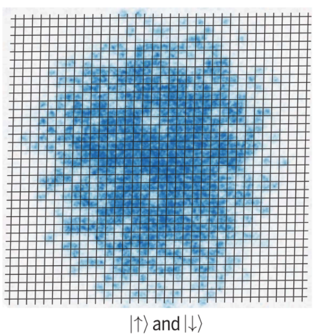




(Δ_S: coupling within TI. Δ_D: coupling between TI's. m: Zeeman coupling on TI surface.)

2. **Cold atoms in optical lattice** (Gross and Bloch, Science **357,** 995)





ill-defined boundary and a limited transport measurement.

Wannier-Stark ladder to characterize Weyl semimetallic phase.

From the Zak phase to the first Chern number

The Zak phase $\gamma^{Zak}(\vec{k}_\perp)$ changes by 2π around projected Weyl node in momentum space $\vec{k}_\perp = (k_x, k_y)$.

This is because, the change of the Zak phase is the first Chern number:

$$\oint \left[\frac{\partial}{\partial \vec{k}_\perp} \Upsilon^{Zak}(\vec{k}_\perp)\right] d\vec{k}_\perp = \oint \frac{\partial}{\partial \vec{k}_\perp}\left[\int_0^{2\pi/a} \left\langle u_{\vec{k}} \middle| i\frac{\partial}{\partial k_z} u_{\vec{k}} \right\rangle dk_z\right] d\vec{k}_\perp$$

$$= \oint \int_0^{2\pi/a} \left[\left\langle \frac{\partial}{\partial \vec{k}_\perp} u_{\vec{k}} \middle| i\frac{\partial}{\partial k_z} u_{\vec{k}} \right\rangle - \left\langle \frac{\partial}{\partial k_z} u_{\vec{k}} \middle| i\frac{\partial}{\partial \vec{k}_\perp} u_{\vec{k}} \right\rangle\right] dk_z\, d\vec{k}_\perp$$

$=$ Winding number from a torus to a sphere.

A Weyl semimetal can be viewed as a collection of 2D Chern insulators in momentum space. And the Fermi arc surface state is a collection of chiral edge states.