Quantum criticality as a new route to delocalization phenomena

International Workshop on Disordered Systems:
From Localization to Thermalization and Topology

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POSTECH

Outline

1. **Introduction**

    - quantum criticality as a new route to delocalization phenomena

2. **Nematic order**

3. **Effective model**

    - effective theory at the critical point, disorder scattering

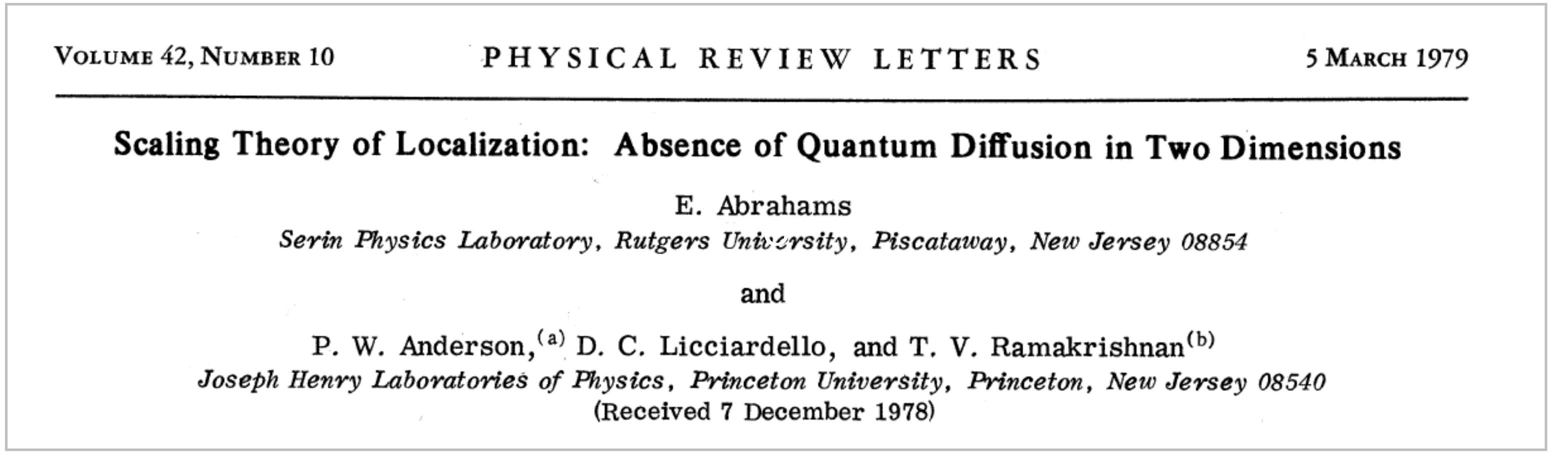
4. **Renormalization group analysis**

    - weak-disorder fixed point, suppression of localization correction

5. **Summary**

Introduction

Scaling function for the localization problem



VOLUME 42, NUMBER 10 PHYSICAL REVIEW LETTERS 5 MARCH 1979

Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions

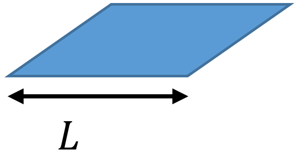
E. Abrahams
Serin Physics Laboratory, Rutgers University, Piscataway, New Jersey 08854

and

P. W. Anderson,[a] D. C. Licciardello, and T. V. Ramakrishnan[b]
Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08540
(Received 7 December 1978)

$g \equiv G/(e^2/\hbar)$
(dimensionless conductance)

L
(system size)



$g(L) = \sigma L^{d-2}$
(Ohm's law)

+

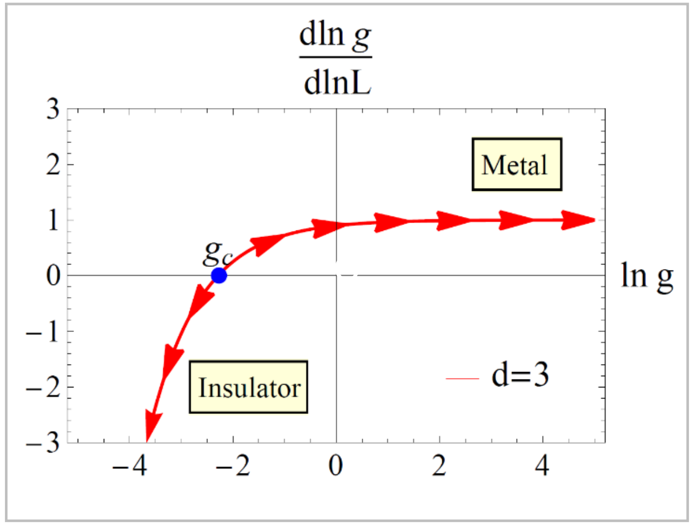
$g(L) \propto exp(-L/\xi)$
(localized states)

$$\frac{d \ln g}{d \ln L} = (d-2) - \frac{1}{\pi^2 g}$$

Localization in two dimensions

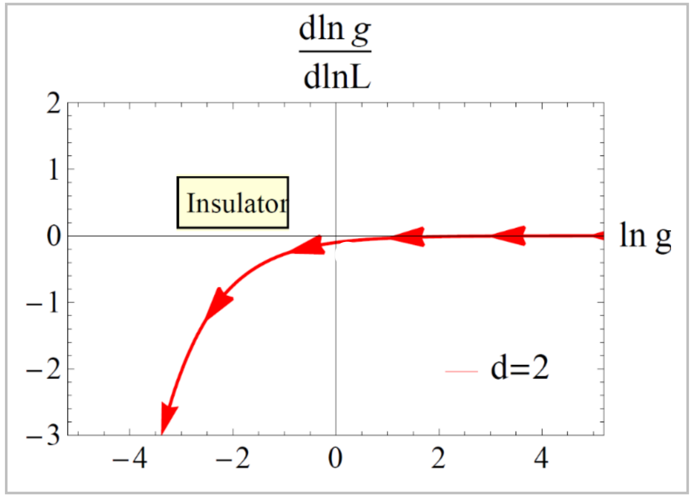
In three dimensions $$\frac{d\ln g}{d\ln L} = 1 - \frac{1}{\pi^2 g}$$

✓ Metal-insulator transition at $g_c = \pi^2$



In two dimensions $$\frac{d\ln g}{d\ln L} = -\frac{1}{\pi^2 g}$$

✓ Insulating phase only

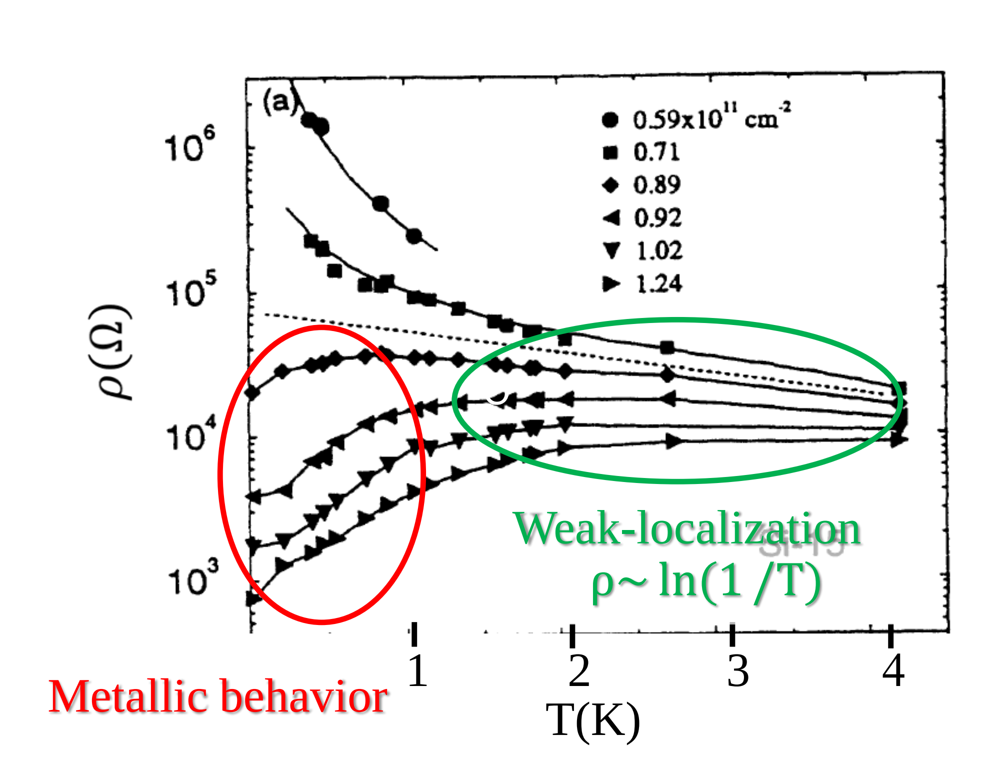


✓ This picture is based on non-interacting systems. How about in interacting systems?

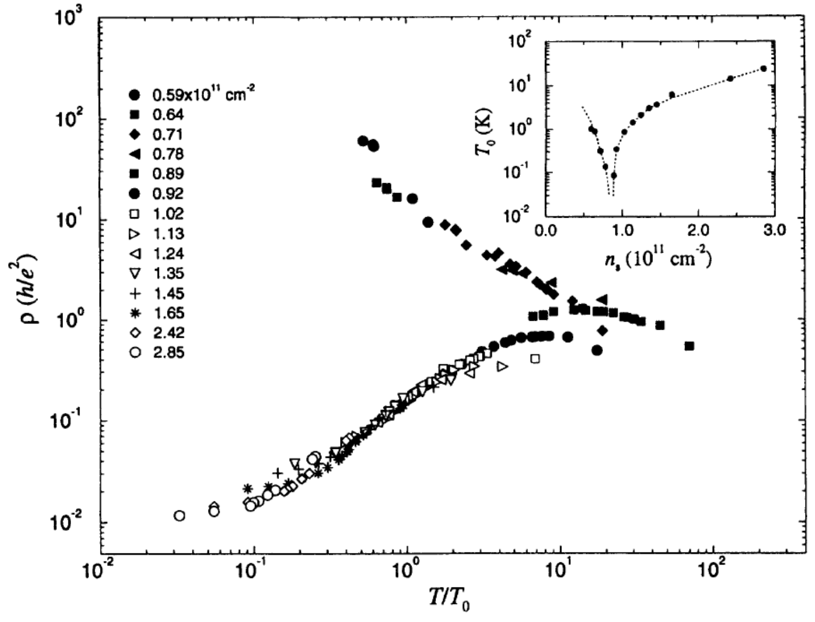
Discovery of a metal-insulator transition in two dimensions

In Si-MOSFET, a metal-insulator transition by varying a carrier density

S.V. Kravchenko, G.V. Kravchenko, J.E. Furneaux, V.M. Pudalov, and M.D'Iorio, Phys. Rev. B (1994)



Scaling behavior → phase transition



Spin-triplet interaction: a cause for metal-insulator transition

Non-linear sigma model: diffusion modes

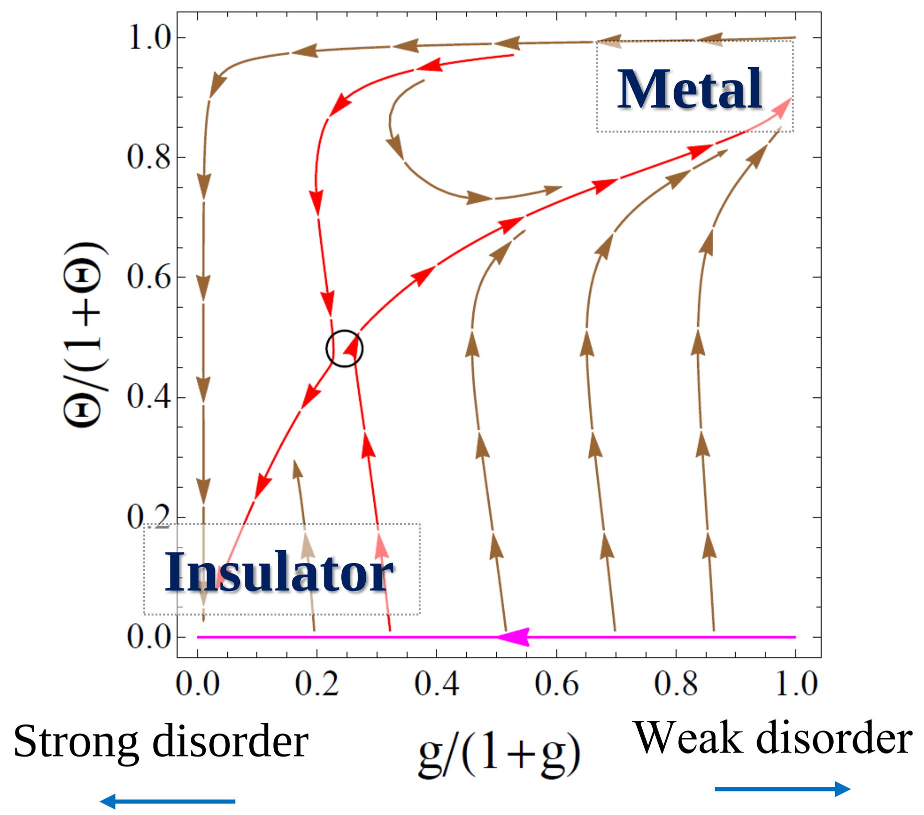
+ scattering due to spin-triplet interaction

A. Punnoose and A.M. Finkel'stein, Science (2005)

$$\frac{d\ln g}{d\ln L} = -\frac{1}{\pi^2 g} + \frac{\Theta}{\pi^2 g} + \frac{1}{\pi^4 g^2}(\Theta - 0.8\Theta^2)$$

$$\frac{d\Theta}{d\ln L} = \frac{1}{\pi^2 g}(2+\Theta) - \frac{4}{\pi^4 g^2}\left(2\Theta + \frac{1}{2}\Theta^2 + 0.08\Theta^3\right)$$

g: dimensionless conductance
Θ: scattering amplitude of diffusion modes



✓ Spin-triplet interaction induces delocalization, resulting in a metallic phase.

Quantum criticality as a new route to delocalization phenomena

- ✓ Can quantum criticality induce delocalization as spin-triplet interaction does?
- ✓ To answer this question, we study 2d nematic quantum critical point in disordered systems.
- ✓ We figure out whether critical fluctuations suppress localization correction or not.

Our approach to the problem

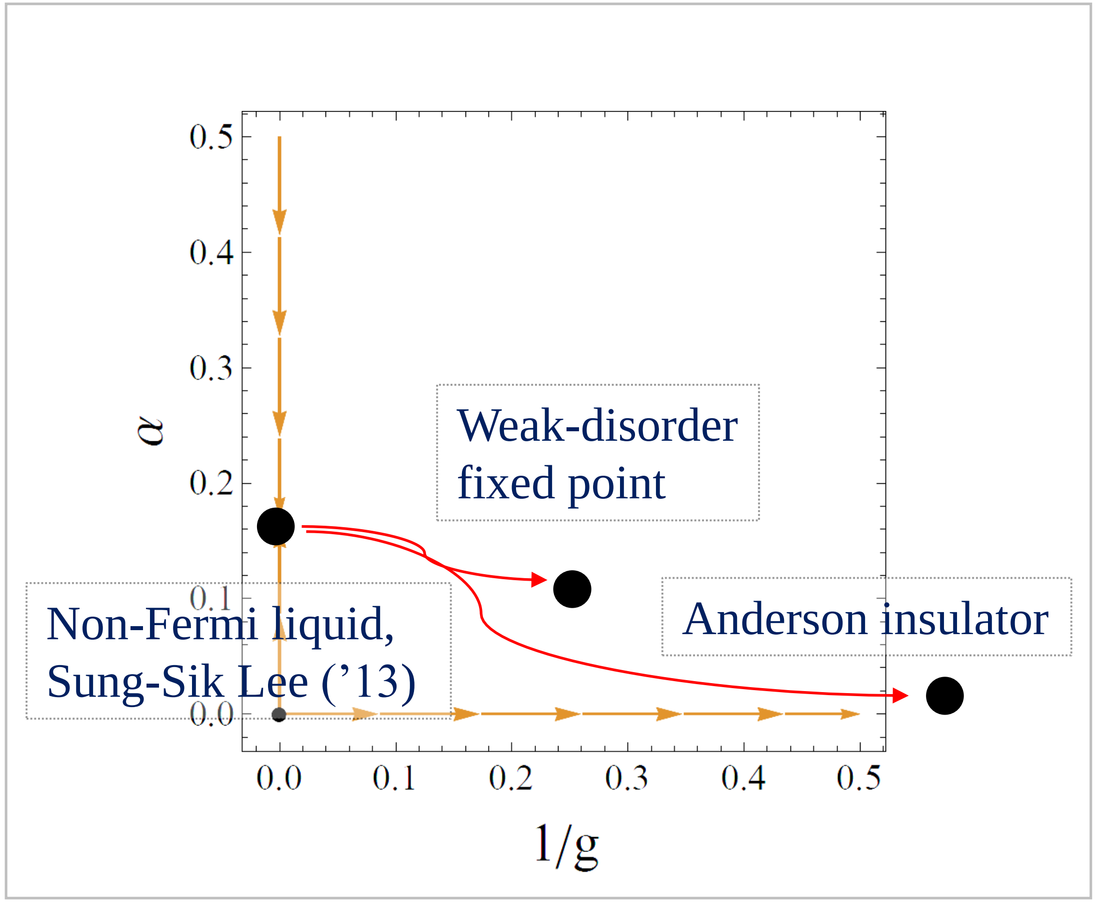
✓ We start from the clean critical point, and add disorder.

✓ We use a renormalization group approach.

$$\frac{\mathrm{dln}\, g}{d\ln L} = -\frac{1}{\pi^2} + F_1(g,\alpha)$$

$$\frac{d\alpha}{d\ln L} = 0.5\alpha - 3\alpha^2 + F_2(g,\alpha)$$

g: dimensionless conductance
α: coupling of electrons and critical fluctuations



✓ In the presence of critical fluctuations, does the system flows to a weak-disorder fixed point or Anderson insulator?

Nematic order

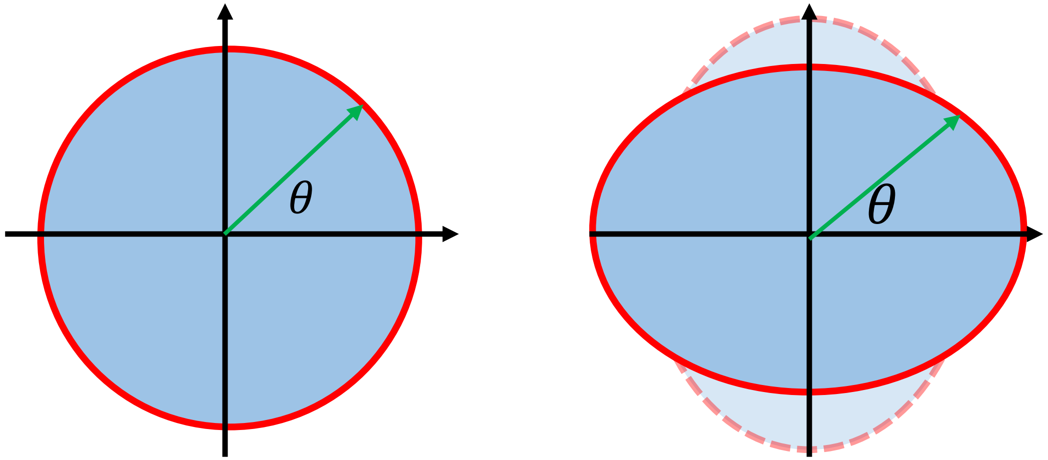
Electronic nematicity

- ✓ Nematic order: lattice symmetry breaking

  driven by electronic degree of freedom

- ✓ Electronic structure change ≫ structural change

- ✓ $O \equiv \sum_{\vec{k}} c_{\vec{k}}^{\dagger} c_{\vec{k}} \exp[\imath 2\theta_{\vec{k}}]$: nematic order parameter



Isotropic phase ($O = 0$) Ordered phase ($O \neq 0$)

- ✓ Pomeranchuk instability in the l=2 channel

$$\delta E \sim \sum_{l} \left(1 + \frac{F_l}{2l+1}\right) C_l (\delta\phi_l)^2$$

F_l: interaction decomposed with the spherical harmonics
$\delta\phi_l$: deformation of Fermi surface

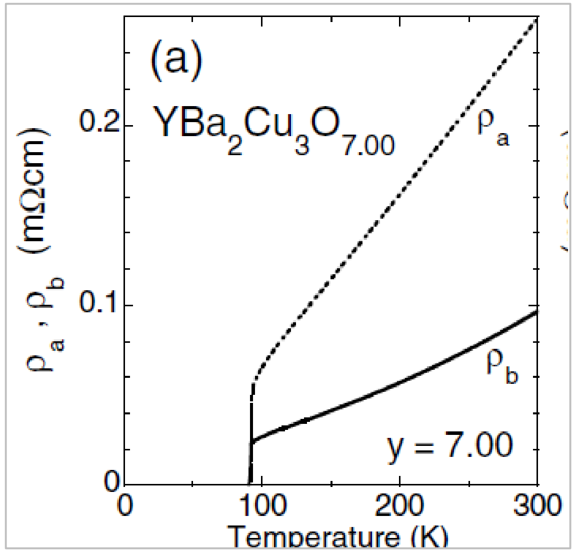
- ✓ If $F_l \leq -(2l+1)$, the Fermi surface is unstable.

Materials

VOLUME 88, NUMBER 13 PHYSICAL REVIEW LETTERS 1 APRIL 2002

Electrical Resistivity Anisotropy from Self-Organized One Dimensionality in High-Temperature Superconductors

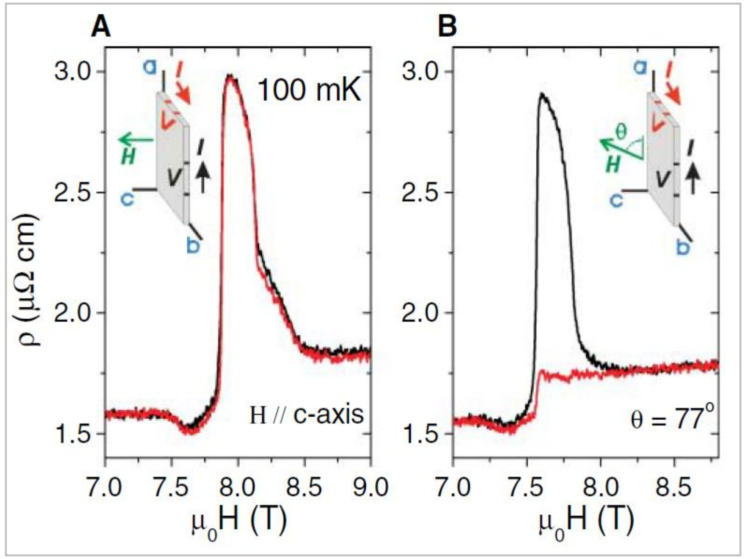
Yoichi Ando, Kouji Segawa, Seiki Komiya, and A. N. Lavrov
Central Research Institute of Electric Power Industry, Komae, Tokyo 201-8511, Japan
(Received 31 July 2001; published 19 March 2002)



Formation of a Nematic Fluid at High Fields in $Sr_3Ru_2O_7$

R. A. Borzi,[1,2,3]* S. A. Grigera,[1,4] J. Farrell,[1] R. S. Perry,[1] S. J. S. Lister,[1] S. L. Lee,[1] D. A. Tennant,[5] Y. Maeno,[6] A. P. Mackenzie[1]*

Science (2007)

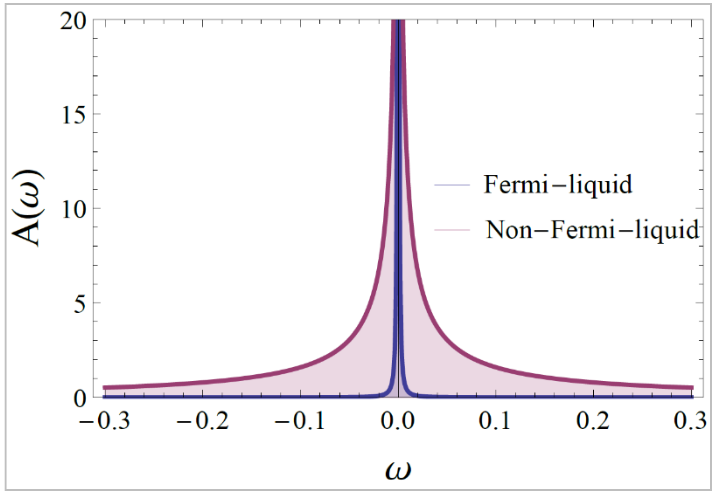


Nematic Electronic Structure in the "Parent" State of the Iron-Based Superconductor $Ca(Fe_{1-x}Co_x)_2As_2$

T.-M. Chuang,[1,2]* M. P. Allan,[1,3]* Jinho Lee,[1,4] Yang Xie,[1] Ni Ni,[5,6] S. L. Bud'ko,[5,6] G. S. Boebinger,[2] P. C. Canfield,[5,6] J. C. Davis[1,3,4,7]†

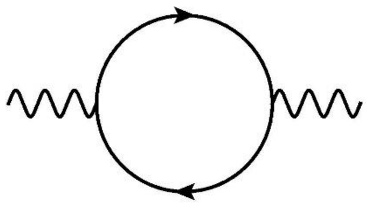
Science (2010)

Nematic quantum critical point



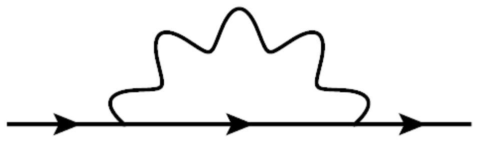
✓ Non-Fermi liquid state at the critical point

✓ Bosons gets a Landau damping term from soft fermionic excitations.



$$\mathrm{Im}\,\Pi = \gamma \frac{|\omega|}{|\vec{q}|}$$

✓ Due to the Landau damping term, fermions get singular corrections.



$$\mathrm{Im}\,\Sigma = a\,\mathrm{sgn}(\omega)|\omega|^{2/3}$$

cf) $\mathrm{Im}\,\Sigma_{\mathrm{FL}} = a\,\mathrm{sgn}(\omega)|\omega|^{2}$

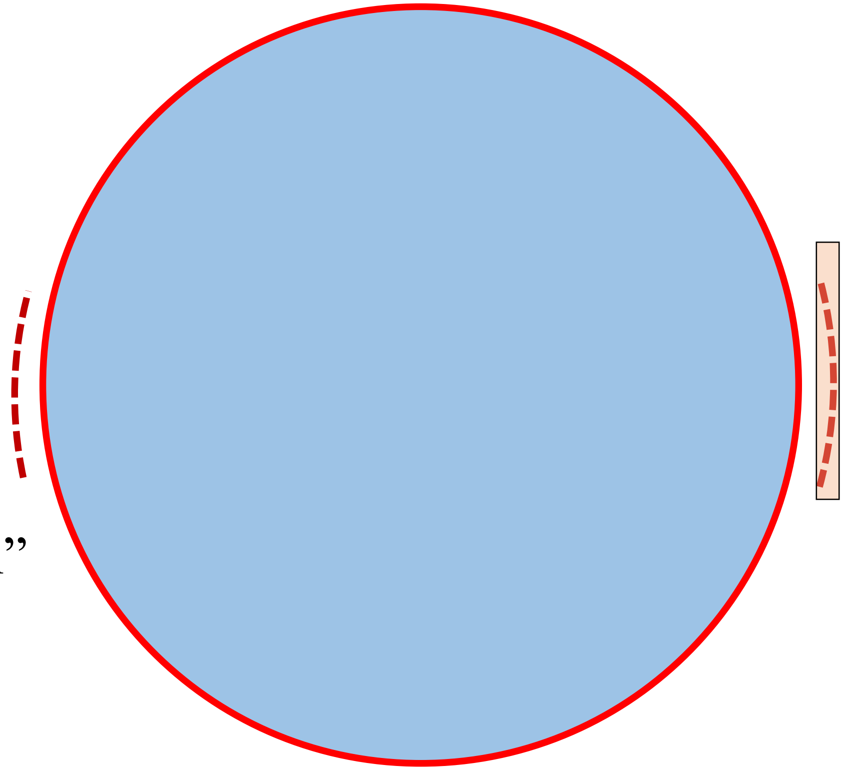
✓ No quasiparticles → we cannot use a non-linear sigma model.

✓ We start from the clean critical point and add disorder.

Effective Model

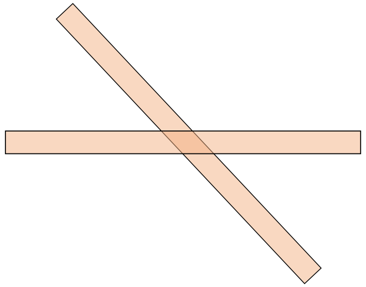
Two-patch model : effective model at the critical point

We can construct an effective theory with two patches.



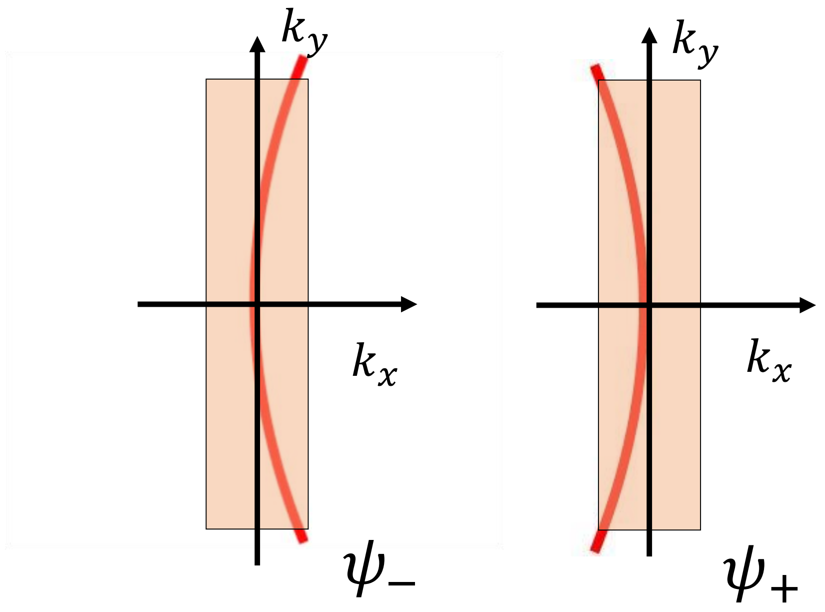
- ✓ “patch”

- ✓ Fermion-boson coupling: $\phi\psi^{\dagger}\psi$
- ✓ Critical bosons scatter fermions along the tangential direction.
- ✓ The overlap is negligible. Different patches are decoupled.



Two-patch model : effective model at the critical point (cont'd)

We can construct an effective theory with two patches.



Parabolic dispersion

$$
\begin{aligned}
\mathcal{L}_\psi &= \psi_+^\dagger\left[-\imath\omega + k_x + k_y^2\right]\psi_+ \\
&\quad + \psi_-^\dagger\left[-\imath\omega - k_x + k_y^2\right]\psi_-,
\end{aligned}
$$

$$
\mathcal{L}_\phi = \frac{1}{2}\left[\gamma\frac{|\omega|}{|q_y|} + c^2(q_x^2 + q_y^2)\right]|\phi|^2,
$$

Landau damping

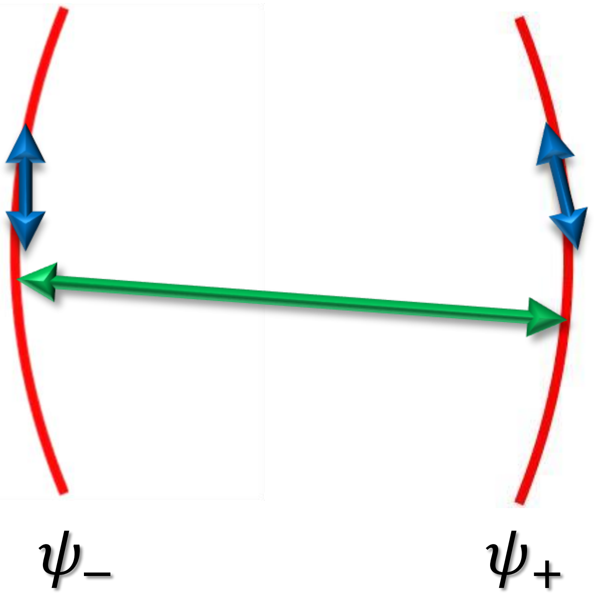
$$
\mathcal{L}_{\psi\phi} = e\phi\left[\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right]
$$

"Yukawa" coupling

Sung-Sik Lee, Phys.Rev B (2013)

Disorder: forward & backscattering

We include two types of disorder scattering: forward and backscattering.

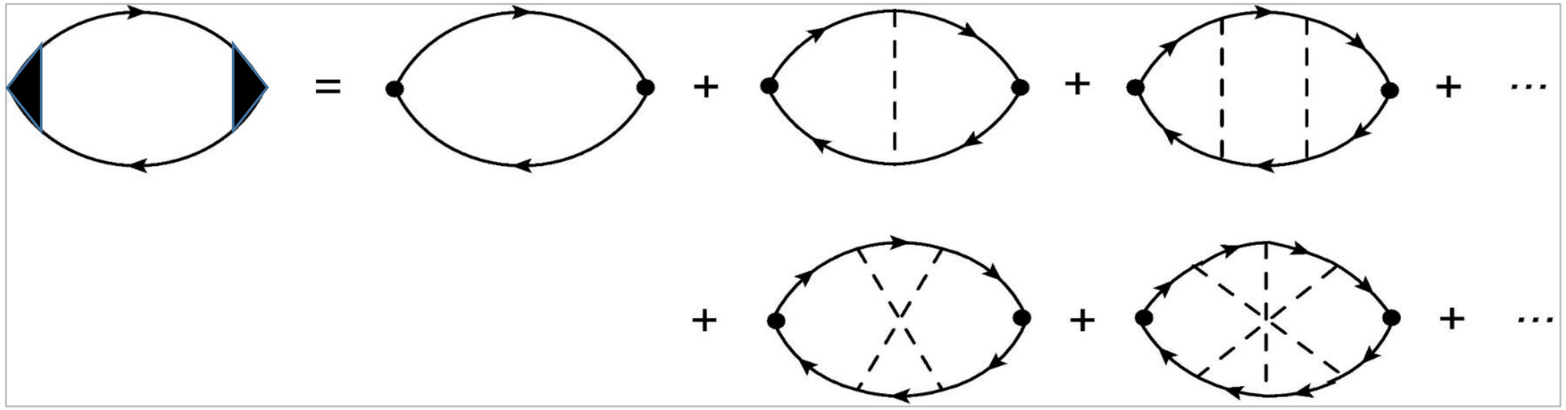


- forward scattering (Γ_0) → Drude conductivity
- Backscattering (Γ_π) → localization correction

✓ We use the replica trick for disorder average.

Disorder: forward & backscattering (cont'd)

Backscattering→ localization correction, forward scattering → Drude conductivity



(Drude conductivity)

(Localization correction)

(1st line) = $\frac{1}{1-\Gamma_0 I}$

✓ $\sigma(L) = \sigma_0 - \delta\sigma, \sigma_0 \sim \frac{1}{\Gamma_0}, \delta\sigma \sim \Gamma_\pi^2$

(2nd line) = $\frac{(\Gamma_\pi)^2 I}{1-\Gamma_\pi I}$

✓ $\Gamma_\pi \to 0$ means that localization correction is suppressed.

Effective action

Parabolic dispersion

$$\mathcal{L}_\psi = \psi_+^\dagger[-\imath\omega + k_x + k_y^2]\psi_+ + \psi_-^\dagger[-\imath\omega - k_x + k_y^2]\psi_-,$$

$$\mathcal{L}_\phi = \frac{1}{2}\left[q_y^2 + \gamma\frac{|\omega|}{|q_y|}\right]|\phi|^2,$$

Landau damping

$$\mathcal{L}_{\psi\phi} = \frac{e}{\sqrt{N_f}}\phi[\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-],$$

Fermion-boson coupling

$$\mathcal{L}_{\psi,dis} = -\frac{\Gamma_0}{2N_f}[\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-][\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-] - \frac{\Gamma_\pi}{2N_f}\psi_-^\dagger\psi_+\psi_+^\dagger\psi_-,$$

Forward scattering

Backscattering

N_f: the fermion's flavor number

Renormalization group analysis

Beta functions up to two-loop order

$$\frac{d\alpha}{d\ln b} = \frac{2}{3}\alpha\left[0.5 - 3\alpha - 4\alpha^2 - \Gamma_0 - 18\Gamma_0^2 - 77\Gamma_0\sqrt{\alpha/N_f} - \Gamma_\pi\right],$$

$$\frac{d\Gamma_0}{d\ln b} = \Gamma_0\left[0.5 + 0.86\Gamma_0 - 48\Gamma_0^2 - \alpha - 1.3\alpha^2 - 1.3\Gamma_0\alpha - 34\Gamma_0\sqrt{\alpha/N_f} + \Gamma_\pi + 3\Gamma_\pi^2/\Gamma_0\right],$$

$$\frac{d\Gamma_\pi}{d\ln b} = \Gamma_\pi\left[0.5 - 3.6\Gamma_0 - 3\Gamma_\pi - 13\alpha\right]$$

$\alpha = e^{4/3}$, e: fermion-boson coupling, Γ_0: forward scattering amplitude

Γ_π: backscattering scattering amplitude, b: a scale parameter ($b \to \infty$ means the low-energy limit)

✓ We consider (i) clean, (ii) non-interacting, (iii) disordered, interacting case one by one.

Case I: clean, interacting system

Setting $\Gamma_0 = \Gamma_\pi = 0$,

$$\frac{d\alpha}{d\ln b} = \frac{2}{3}\alpha\left[0.5 - 3\alpha - 4\alpha^2\right]$$

✓ Here, plus sign means that a coupling increases as an energy scale is lowered.

✓ Fermi liquid state ($\alpha^* = 0$) is unstable.

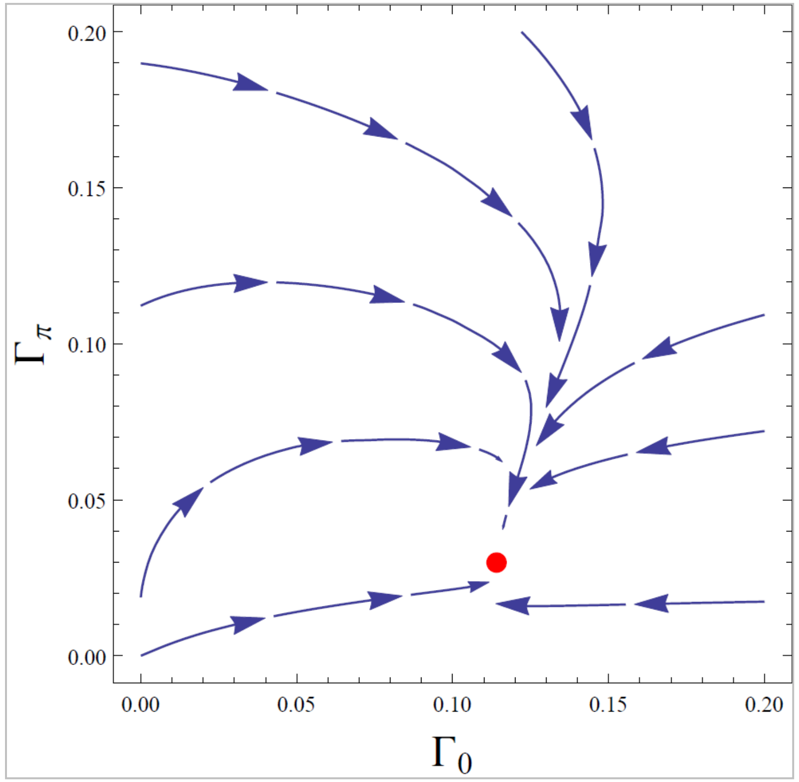
✓ The system goes to non-Fermi liquid state($\alpha^* = 0.14$). *D. Dalidovich, Sung-Sik Lee, Phys.Rev. B (2013)*

Case II: disordered, non-interacting system

Setting $\alpha = 0$,

$$\frac{d\Gamma_0}{d\ln b} = \Gamma_0\left[0.5 + 0.86\Gamma_0 - 48\Gamma_0^2 + \Gamma_\pi + 3\Gamma_\pi^2/\Gamma_0\right],$$

$$\frac{d\Gamma_\pi}{d\ln b} = \Gamma_\pi\left[0.5 - 3.6\Gamma_0 - 3\Gamma_\pi\right]$$



✓ Clean Fermi liquid state($\Gamma^* = \Gamma_\pi^* = 0$) is unstable.

✓ The system goes to disordered Fermi liquid state ($\Gamma_0^* = 0.11, \Gamma_\pi^* = 0.027$).

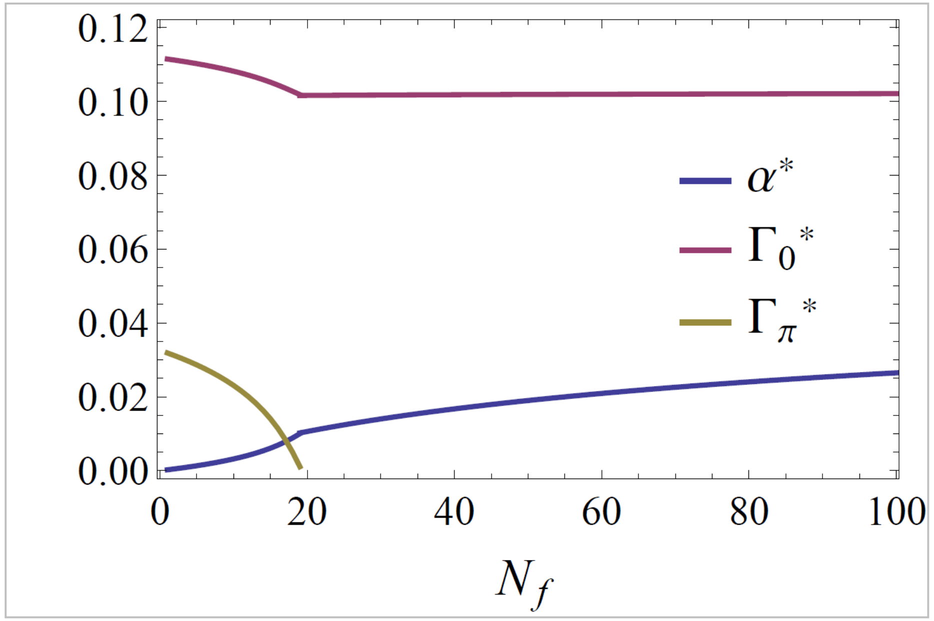
✓ Localization correction is present.

Case III: disordered, interacting system

$$\frac{d\alpha}{d\ln b} = \frac{2}{3}\alpha\left[0.5 - 3\alpha - 4\alpha^2 - \Gamma_0 - \Gamma_\pi - 18\Gamma_0^2 - 77\Gamma_0\sqrt{\alpha/N_f}\right],$$

$$\frac{d\Gamma_0}{d\ln b} = \Gamma_0\left[0.5 + 0.86\Gamma_0 - 48\Gamma_0^2 - \alpha - 1.3\alpha^2 - 1.3\Gamma_0\alpha - 34\Gamma_0\sqrt{\alpha/N_f} + \Gamma_\pi + 3\Gamma_\pi^2/\Gamma_0\right],$$

$$\frac{d\Gamma_\pi}{d\ln b} = \Gamma_\pi\left[0.5 - 3.6\Gamma_0 - 3\Gamma_\pi - 13\alpha\right]$$



✓ The clean Fermi-liquid fixed point ($\alpha^* = \Gamma_0^* = \Gamma_\pi^* = 0$) is unstable.

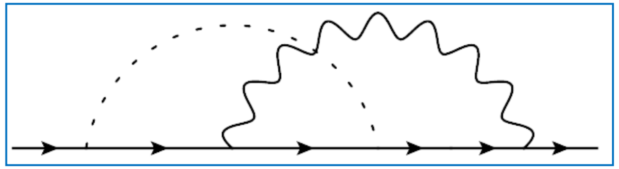
✓ If $N_f < 20$, Γ_π is finite. ✓ If $N_f > 20$, Γ_π vanishes.

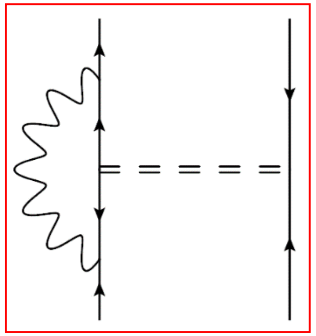
✓ In the system of $N_f < 20(N_f > 20)$, localization correction remains(disappears).

Case III: disordered, interacting system (cont'd)

$$\frac{d\alpha}{d\ln b} = \frac{2}{3}\alpha\left[0.5 - 3\alpha - 4\alpha^2 - 77\Gamma_0\sqrt{\alpha/N_f} + \cdots\right],$$

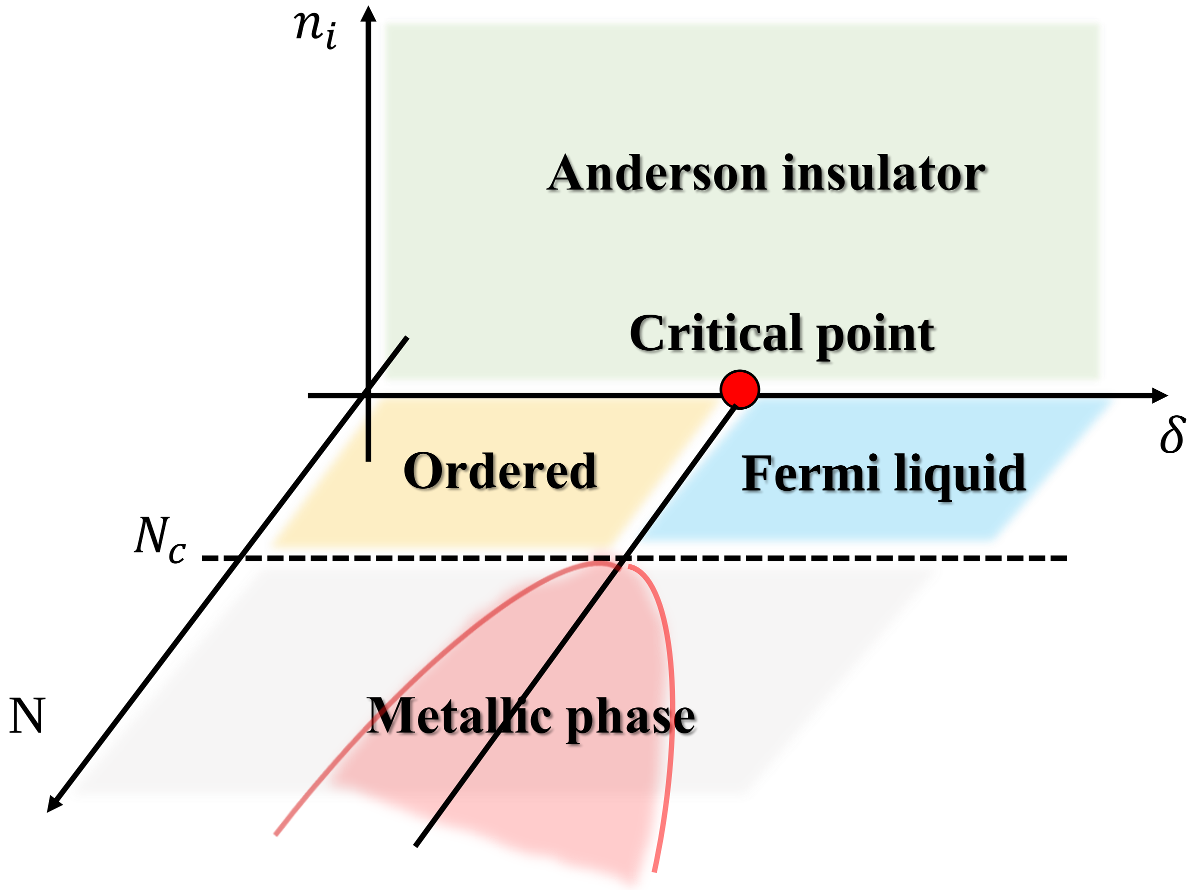
$$\frac{d\Gamma_\pi}{d\ln b} = \Gamma_\pi\left[0.5 - 3.6\Gamma_0 - 3\Gamma_\pi - 13\alpha\right]$$





- ✓ Altshuler-Aronov like correction. This results in a strong screening in α.
- ✓ When N_f is small(large), this correction is large(small). So α becomes small(large).
- ✓ α screens Γ_π strongly. When $\alpha > \alpha_c(\alpha < \alpha_c)$, Γ_π vanishes(survives).

Phase diagram



- ✓ In physical context, N_f is translated as the valley number in a band-structure.
- ✓ When there are a small number of valleys, only Anderson insulator exists.
- ✓ However, if there are a large number of valleys, a metallic phase can appear.

Summary

- ✓ We consider quantum criticality as a new route to delocalization phenomena.
- ✓ We studied nematic quantum critical point in two dimensions, introducing disorder.
- ✓ By performing renormalization group analysis, we find two kinds of fixed points: a weak-disorder fixed point where backscattering is suppressed and another one with backscattering.
- ✓ We interpret the first one as a metallic state and the second one as Anderson insulator.
- ✓ The valley number controls whether the system is in a metallic phase or in an insulating phase.

Thank you!