Quantum quenches in classical models

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Dynamics of isolated disordered models: evolution, equilibration ? the GGE & FDT effective temperatures

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Question

Does an isolated system reach equilibrium ?

Boosted by recent interest in

— the dynamics after quantum quenches of cold atomic systems

rôle of interactions (integrable *vs.* non-integrable)

— many-body localisation

novel effects of quenched disorder

And, an isolated classical system ?

The (old) ergodicity question revisited

LFC, Lozano & Nessi 17. LFC, Lozano, Nessi, Picco & Tartaglia 18

Quantum : Foini, Gambassi, Konik & LFC 17. de Nardis, Panfil *et al.* 17

Quantum quenches

Definition & questions

- Take an isolated quantum system with Hamiltonian $\hat{H}_0$
- Initialize it in, say, $|\psi_0\rangle$ the ground-state of $\hat{H}_0$ (or any $\hat{\rho}(t_0)$)
- Unitary time-evolution $\hat{U} = e^{-\frac{i}{\hbar}\hat{H}t}$ with a Hamiltonian $\hat{H} \neq \hat{H}_0$.

Does the system reach a steady state ?

Is it described by a thermal equilibrium density matrix $e^{-\beta\hat{H}}$?

Do at least some observables behave as thermal ones?

Does the evolution occur as in equilibrium ?

If not, other kinds of density matrices ?

Classical quenches

Definition & questions

- Take an isolated classical system with Hamiltonian H_0, evolve with H
- Initialize it in, say, ψ_0 a configuration, *e.g.* $\{\vec{q}_i, \vec{p}_i\}$ for a particle system

  ψ_0 could be drawn from a probability distribution, *e.g.* $\mathcal{Z}^{-1} e^{-\beta_0 H_0(\psi_0)}$

Does the system reach a steady state ?

Is it described by a thermal equilibrium probability $e^{-\beta H}$?

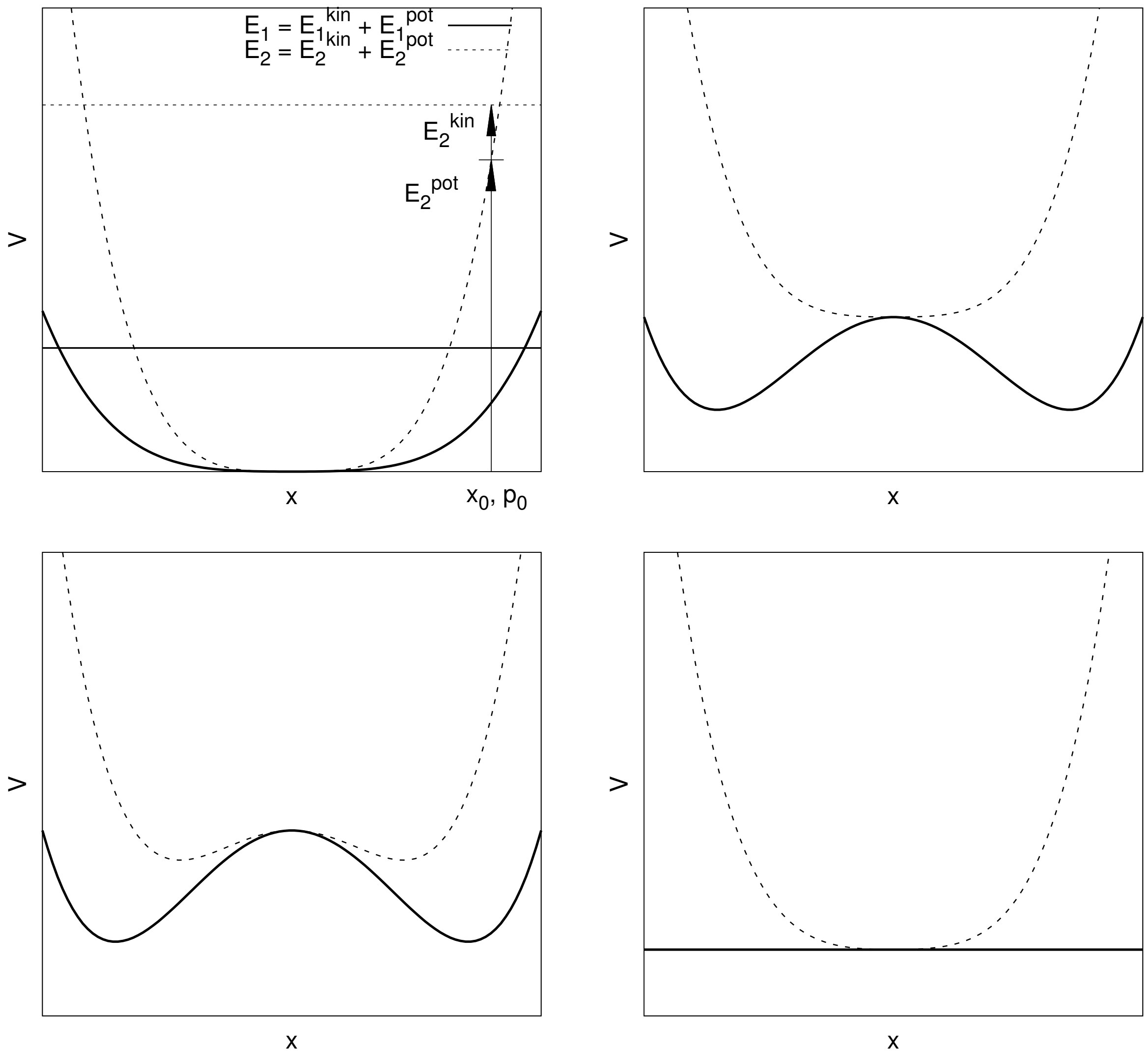
Do at least some observables behave as thermal ones ?

Does the evolution occur as in equilibrium ?

If not, other kinds of probability distributions ?

Quenches

Simple examples (kind of building blocks)



At $t = 0$ change in V

Continuity in variables

$x(0^-) = x(0^+) = x_0$

$p(0^-) = p(0^+) = p_0$

Jump in potential energy

dashed to solid:

energy extraction

solid to dashed:

energy injection

Classical quenches

Models

We chose to study classical disordered many-body models

isolated p spin spherical disordered models

Interesting & very well characterised

equilibrium phases & relaxational dissipative dynamics

rich free-energy landscapes with metastability, flat regions, large and small barriers, etc.

(also interesting in the context of many-body localisation studies)

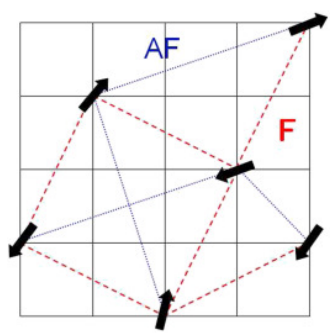
Quenched disorder

Spin Disordered Potential

$V = -\sum_{ij} J_{ij} s_i s_j - \sum_{ijk} J_{ijk} s_i s_j s_k + \dots$

the exchanges $J_{ij}, J_{ijk},$ etc. taken from

a probability distribution (details later)



Real variables $s_i \in \mathbb{R}$

Spherical constraint $\sum_{i=1}^{N} s_i^2 = N$

Connection with the following problem

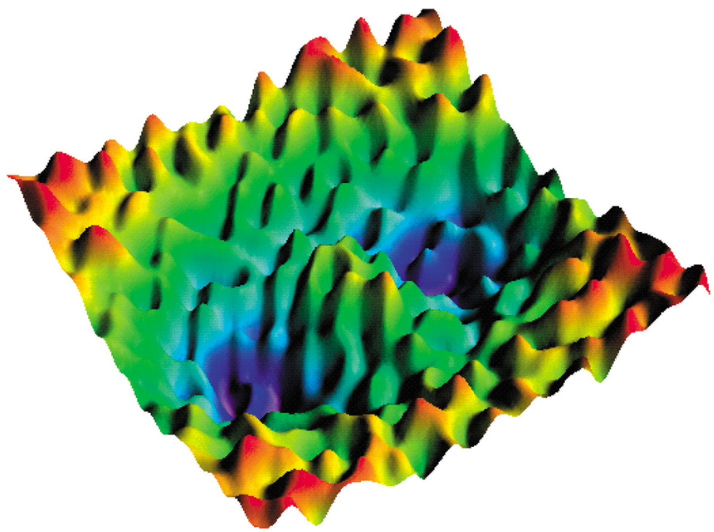
A particle

position $\vec{s} = (s_1, \dots, s_N)$

in an N dimensional space

under a random potential $V(\vec{s})$

Sketch for $N = 2$



but wrapped on the sphere

Classical dynamics

Coordinate-momenta pairs $\{\vec{s}, \vec{p}\}$ and Hamiltonian (const w/Lagrange mult.)

$$H = K(\vec{p}) + V(\vec{s})$$

with the kinetic energy $K(\vec{p}) = \frac{1}{2m} \sum_{i=1}^{N} p_i^2$

Newton-Hamilton equations

$$\boxed{\dot{s}_i = p_i/m \qquad\qquad \dot{p}_i = -dV(\vec{s})/ds_i}$$

The potential energy landscape makes the models behave differently

– N saddles (including min/max) for two body-interactions $V(\vec{s}) = \sum_{i \neq j} J_{ij} s_i s_j$

– $\exp(N\Sigma)$ saddles for more than two body interactions $\sum_{i \neq j \neq k} J_{ijk} s_i s_j s_k$

(With dissipation used to model domain-growth & fragile glasses, respectively)

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The potential energy landscape makes the models behave differently

— Finite energy barriers for two body-interactions $V(\vec{s}) = \sum_{i\neq j} J_{ij} s_i s_j$

— Barriers scale with N for more than two body interactions $\sum_{i\neq j\neq k} J_{ijk} s_i s_j s_k$

(With dissipation used to model domain-growth & fragile glasses, respectively)

The p spin models

$p \geq 3$ clearly non-integrable

Gibbs-Boltzmann equilibrium at β_f expected

unless the system is set on the threshold

$p = 2$ integrable !

Neumann's 1850 model of classical mechanics (thanks to **O. Babelon**)

N constants of motion in involution **K. Uhlenbeck 82**

No Gibbs-Boltzmann equilibrium expected

Generalized Gibbs Ensemble :

$$P(\vec{s}, \vec{p}) = \mathcal{Z}^{-1} \; e^{-\sum_{\mu=1}^{N} \beta_\mu I_\mu(\vec{s},\vec{p})} \quad ?$$

Quantum : Rigol, Dunjko, Olshanii, Muramatsu 07-09

Calabrese, Cardy, Caux, Essler, etc.

The initial conditions

- We chose initial states drawn from canonical equilibrium with Hamiltonian H_0 at inverse temperature β'
- The models have phase transitions at a finite β_c

  The high temperature phase is a disordered one, a paramagnet (PM)

  The low temperature phase is different in the two-body and more than two-body interaction models :

  – two ferromagnetic (FM)-like equilibrium states for two-body ($p = 2$)

  – $\mathcal{O}(e^{N\Sigma})$ metastable states, like in a glass, in the $p \geq 3$ case
- Initial conditions: disordered (PM) or confined (FM/metastable) TAP

The quench

Spin Disordered Potential

$V = -\sum_{ij} J_{ij} s_i s_j - \sum_{ijkl} J_{ijkl} s_i s_j s_k s_l$

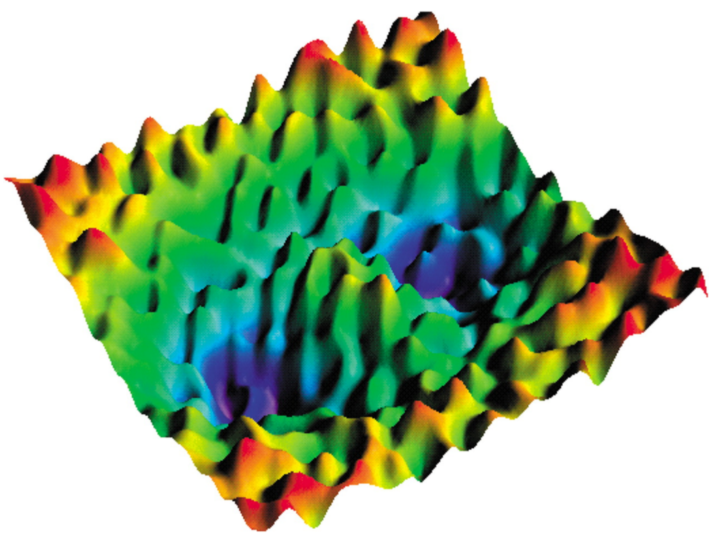
with exchanges J_{ij}, J_{ijkl}, etc. taken

from a Gaussian pdf

zero mean $[J_{i_1 \dots i_p}] = 0$ and

$[J^2_{i_1 \dots i_p}] = p! J_0^2 / (2N^{p-1})$

Initial energy scale J_0

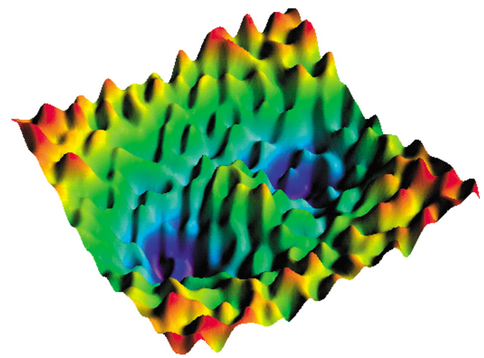


At time $t = 0$

Same configuration $\dot{s}_i(0), s_i(0)$

quench $\boxed{J^0_{i_1 \dots i_p} \mapsto J_{i_1 \dots i_p}}$

Final energy scale J



The rugged landscape is

stretched/contracted and pulled up/down

On the sphere

Dynamic equations

Conservative dynamics

In the $N \to \infty$ limit exact causal Schwinger-Dyson equations

$$(m\partial_t^2 - z_t)R(t,t_w) = \int dt' \; \Sigma(t,t')R(t',t_w) + \delta(t-t_w)$$

$$(m\partial_t^2 - z_t)C(t,t_w) = \int dt' \; \left[\Sigma(t,t')C(t',t_w) + D(t,t')R(t_w,t')\right]$$

$$\boxed{+\frac{\beta' J_0}{J}\sum_{a=1}^{n} D_a(t,0)C_a(t_w,0)}$$

$$\boxed{(m\partial_t^2 - z_t)C_a(t,0) = \int dt' \, \Sigma(t,t')C_a(t',0) + \frac{\beta' J_0}{J}\sum_{a=1}^{n} D_b(t,0)Q_{ab}}$$

$a = 1, \ldots, n \to 0$, replica method to deal with $e^{-\beta' H_0}$ and fix Q_{ab}

Dynamic equations

Conservative dynamics

In the $N \to \infty$ limit exact causal Schwinger-Dyson equations

with the post-quench self-energy and vertex

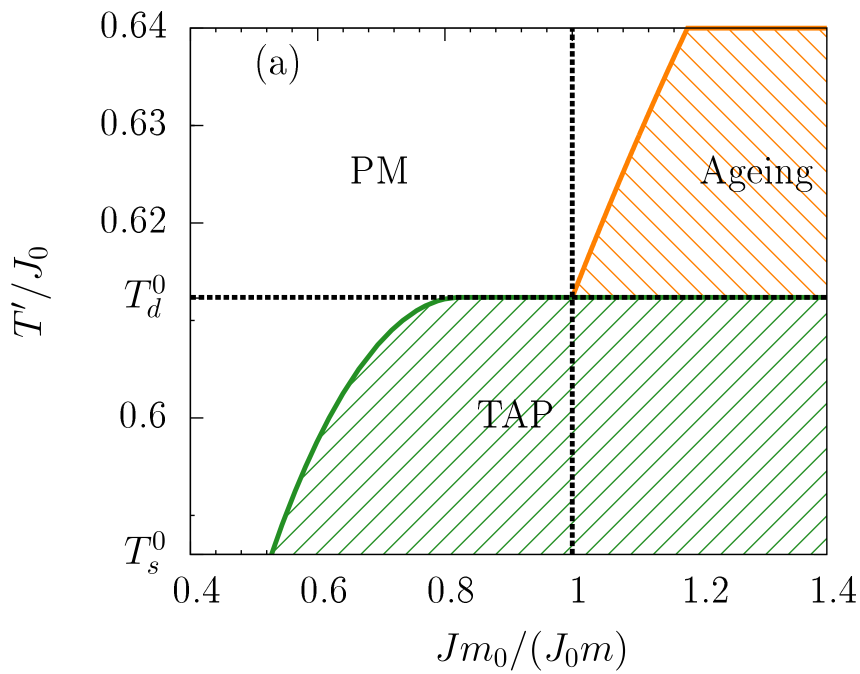
$$\begin{aligned} D(t,t_w) &= \frac{J^2 p}{2}\, C^{p-1}(t,t_w) \\ D_a(t,0) &= \frac{J^2 p}{2}\, C_a^{p-1}(t,0) \\ \Sigma(t,t_w) &= \frac{J^2 p(p-1)}{2}\, C^{p-2}(t,t_w)\, R(t,t_w) \end{aligned}$$

and the Lagrange multiplier z_t fixed by $C(t,t) = 1$

Solvable numerically & analytically at long times

Three body model

Dynamic phase diagram



Initial conditions

Energy injection

Energy extraction

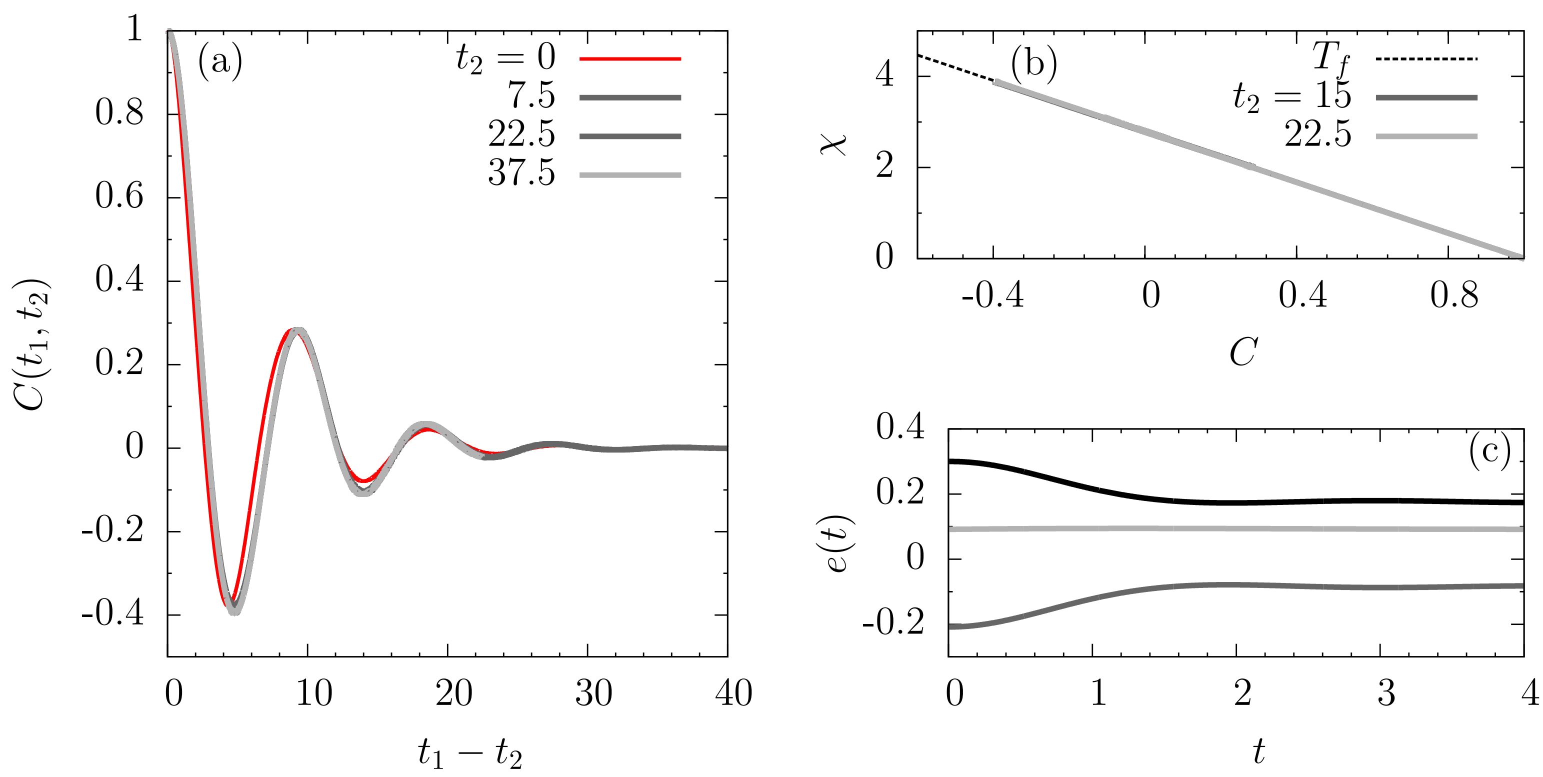
In PM, quenches go to GB equilibrium at $\beta_f(e_f)$ with e_f the final energy

Following metastable states, GB-like equilibration at β_f determined by e_f

Out of equilibrium relaxation with ageing effects when $e_f = e_{\rm th}$

Three body model

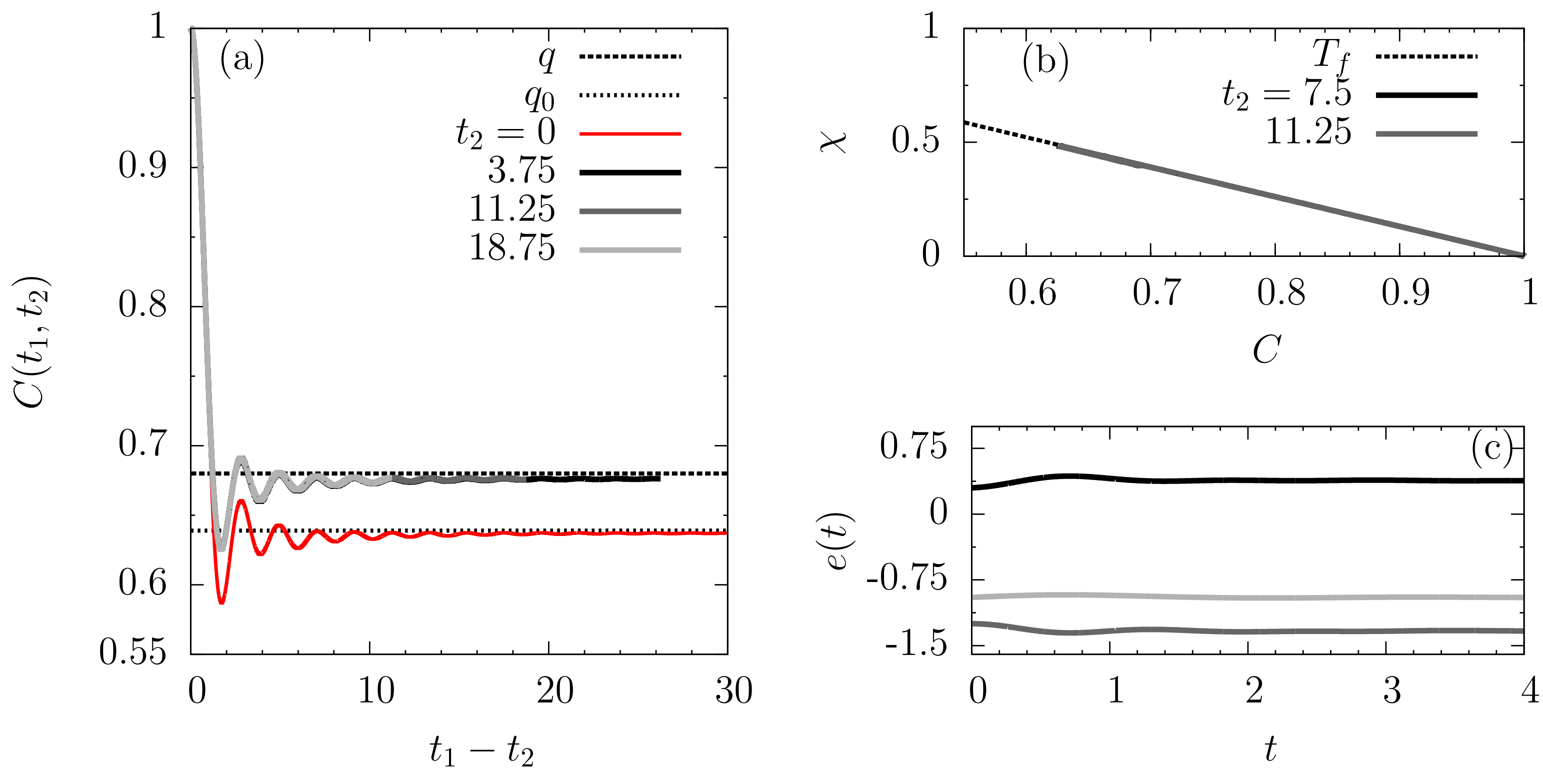
e.g., from equilibrium within a TAP state to the PM



GB equilibration at the temperature of a PM $\quad T_f = e_f + \sqrt{J^2 + e_f^2}$

Three body model

Initial & final configurations in a metastable (TAP) state



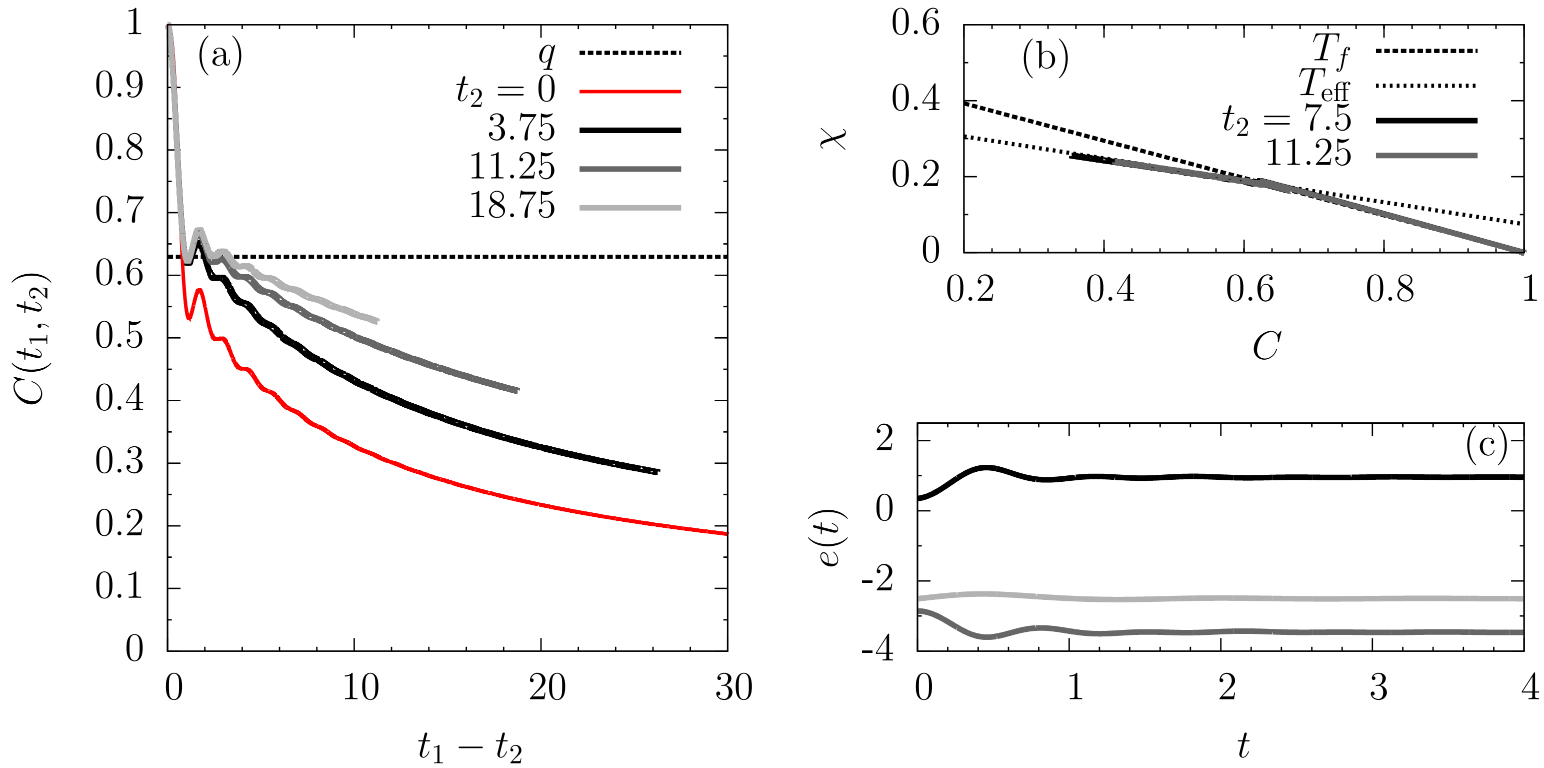
$C(t_1, 0) \to q_0 > 0$ Fidelity

$\lim_{t_1 - t_2 \gg t_0} \lim_{t_2 \gg t_0} C(t_1, t_2) = q > 0$ Decorrelation

Following metastable states, equilibration at β_f fixed by $e_f = e_f^{\mathrm{kin}} + e_f^{\mathrm{pot}}$

Three body model

Energy extraction from PM to threshold

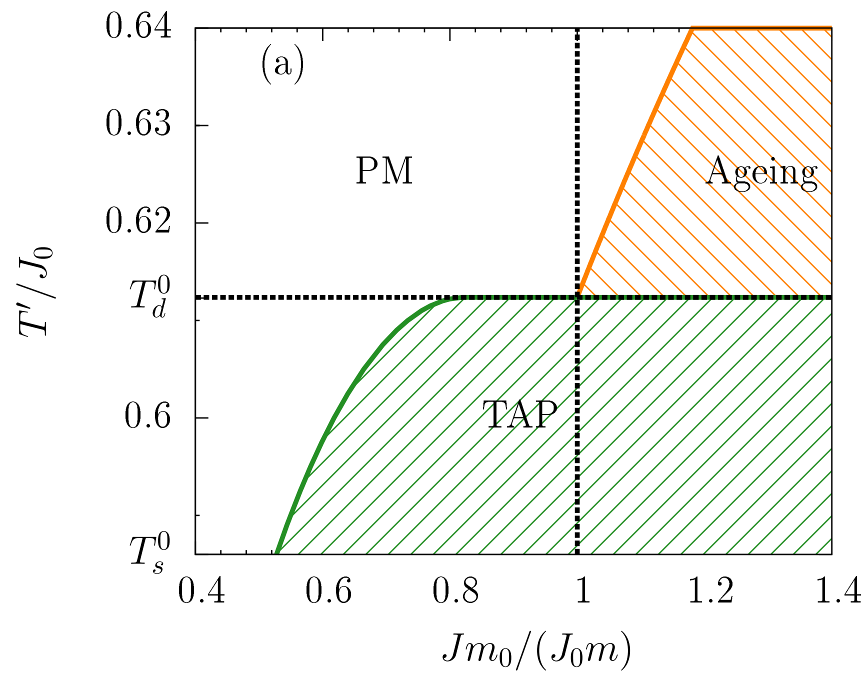


Similar to the relaxational case. Two temperature behaviour, fast and slow decay.

Out of equilibrium relaxation when quench parameters tuned so that $e_f = e_{\rm th}$

Three body model

Dynamic phase diagram - recap



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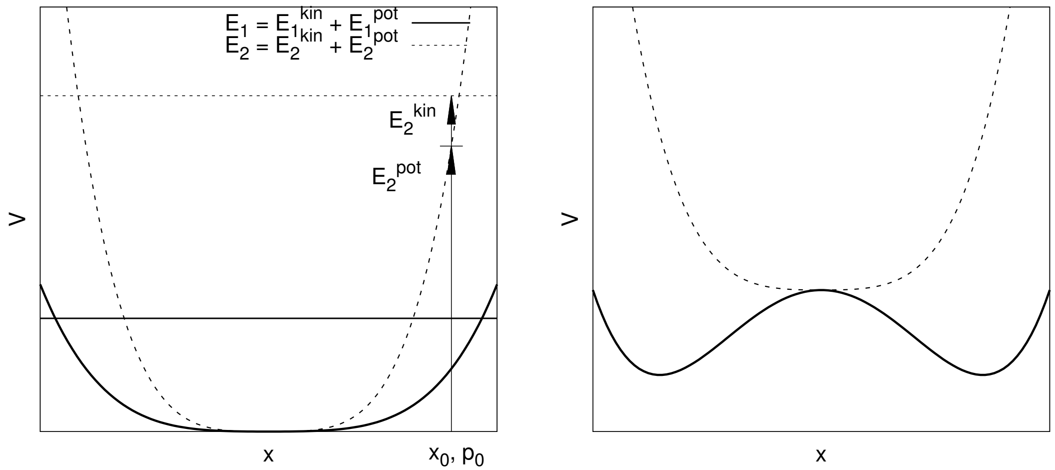
Out of equilibrium relaxation with ageing effects when $e_f = e_{\rm th}$

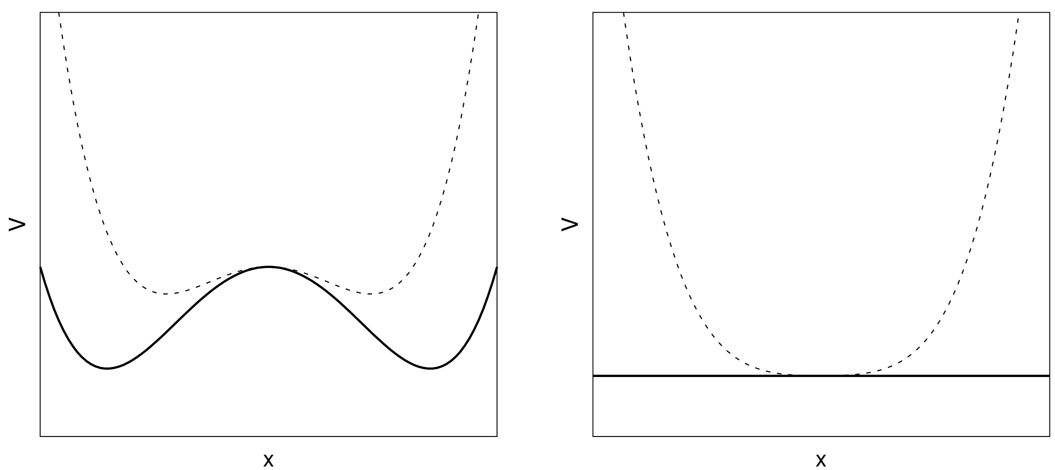
Quenches

Sketches

Solid to dashed: final state PM



Both: following TAP   Dashed to solid : threshold



The p spin models

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Gibbs-Boltzmann equilibrium expected (β_f)

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Cardy, Caux, Calabrese, Essler, etc.

Two body model

Non-linear coupling through the Lagrange multiplier only

Stat-phys notions: Potential energy landscape

The N eigenvectors of the J_{ij} matrix are saddles, the barriers between them are $\mathcal{O}(1)$, the absolute minimum is the alignment of $\vec{s}$ on the eigenvector $\vec{v}_N$ with eigenvalue λ_N at the edge of the spectrum.

Kosterlitz, Thouless & Jones 76 … LFC & Dean 96 … Fyodorov 12-17 …

Mehta, Hauenstein, Niemerg, Simm & Stariolo 14

Classical mechanics/integrable systems **K. Uhlenbeck 82**

Motion of a particle on S_{N-1}, enforced by $\sum_\mu s_\mu^2 = N$

The integrals of motion are $I_\mu = s_\mu^2 + \frac{1}{N} \sum_{\nu(\neq\mu)} \frac{s_\mu^2 p_\nu^2 + s_\nu^2 p_\mu^2 - 2 s_\mu p_\mu s_\nu p_\nu}{\lambda_\nu - \lambda_\mu}$

Two body model

Non-linear coupling through the Lagrange multiplier only

Diagonal in the basis of eigenvectors $\vec{v}_\mu$ of the interaction matrix J_{ij}

Projection of the coordinate (spin) vector on the eigenvectors $s_\mu = \vec{s} \cdot \vec{v}_\mu$

with $\mu = 1, \dots, N$. Newton equations are almost quadratic

$$\boxed{m\ddot{s}_\mu(t) = [z(t) - \lambda_\mu] s_\mu(t)}$$

with $z(t)$ the Lagrange multiplier that enforces the spherical constraint

and λ_μ the eigenvalues (semi-circle law, with support in $[-2J, 2J]$)

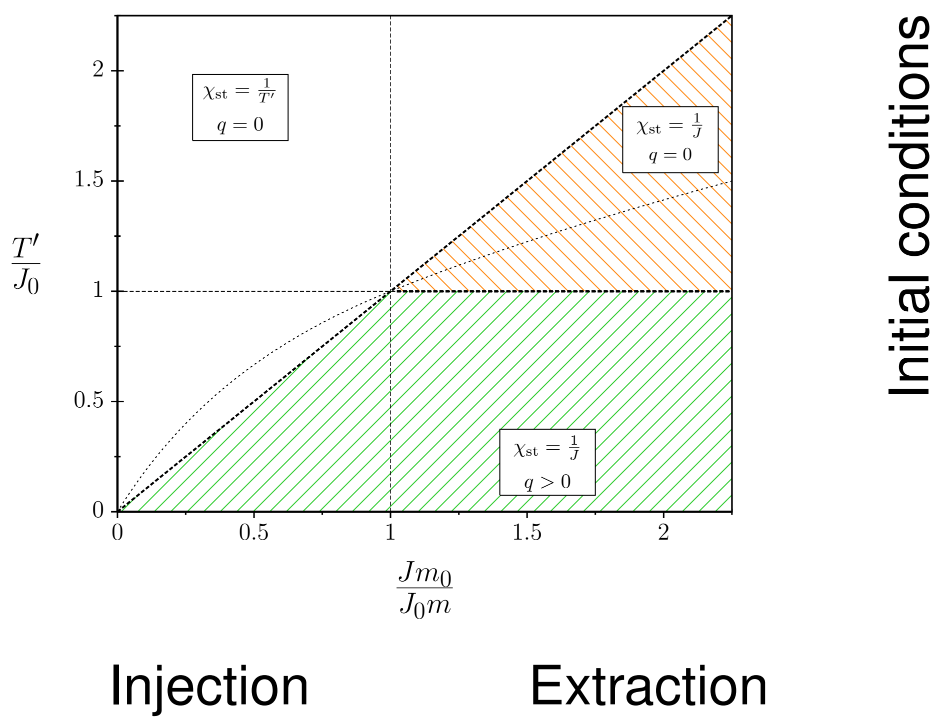
Two methods to solve:

— for $N \to \infty$, closed Schwinger-Dyson equations on $C(t, t_w)$ and $R(t, t_w)$, the global self-correlation and linear response (already shown for general p)

— for finite N, solve Newton equations under the spherical constraint. Similar to **Sotiriadis & Cardy 10** for the quantum O(N) model

Two body model

Richer results !

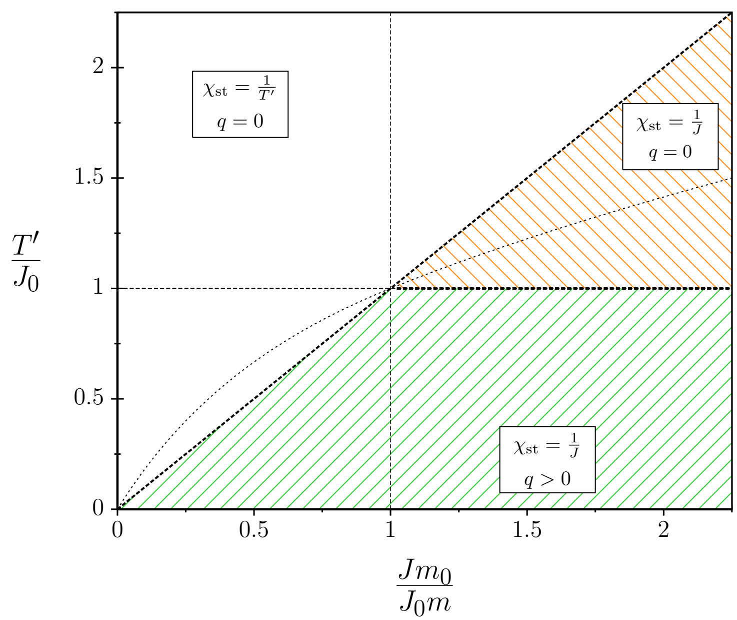


For all parameters $\lim_{t \gg t_{\rm st}} \lim_{N \to \infty} z(t) = z_f + c\, t^{-\alpha}$

Three phases

Two body model

Richer results !



Initial conditions

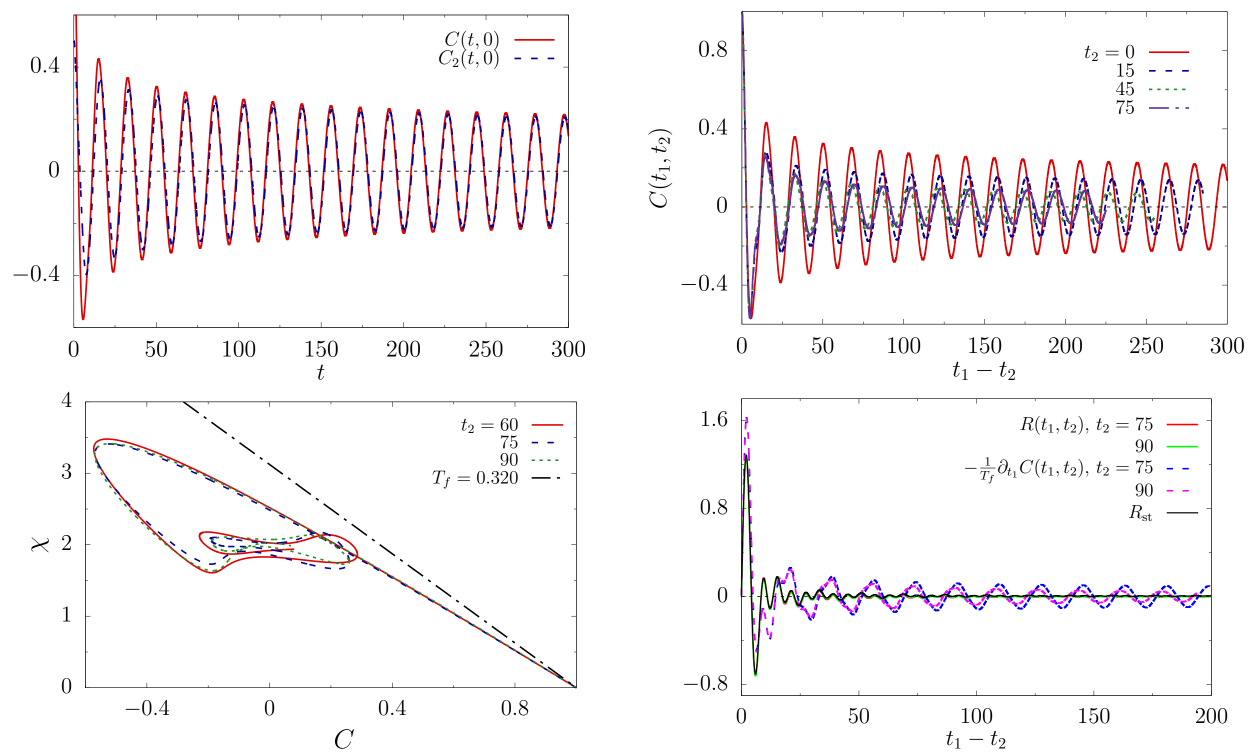
I $\chi_{\rm st} = 1/T'$ $z_f > 2J$ and $\lim_{t \gg t_w} C(t, t_w) = 0$

II $\chi_{\rm st} = 1/J$ $z_f = 2J$ and $\lim_{t \gg t_w} C(t, t_w) = 0$

III $\chi_{\rm st} = 1/J$ $z_f = 2J$ and $\lim_{t \gg t_w} C(t, t_w) = q > 0$

Two body model

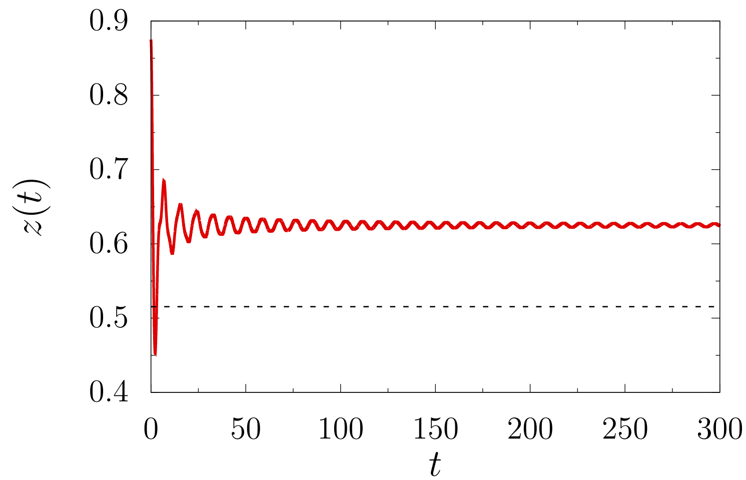
Large energy injection on a condensed state : equilibrium PM ?



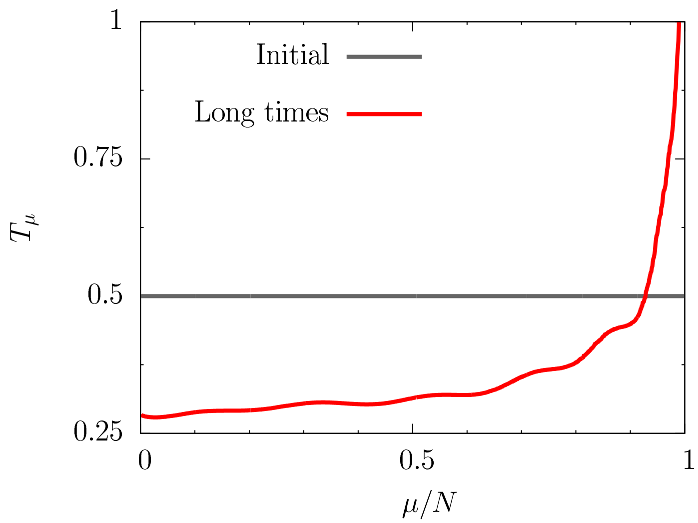
Stationary dynamics but no FDT at a single temperature **no GB equilibrium**

Two body model

Mode temperatures spectrum $\chi_{\rm st} = 1/T'$ phase



$T' = 0.5,\ J = 0.25,\ N = 1024$



Constant asymptotic Lagrange multiplier

e.g. $z(t) \to z_f = T' + J^2/T'$

The time-dependent frequencies

$\Omega^2_\mu(t) \to (z_f - \lambda_\mu)/m \equiv \omega^2_\mu$

The μ modes $s_\mu(t)$ decouple and become $\boxed{\text{independent harmonic oscillators}}$

with conserved energy after $t_{\rm st}$

$e_\mu = e^{\rm kin}_\mu(t) + e^{\rm pot}_\mu(t)$

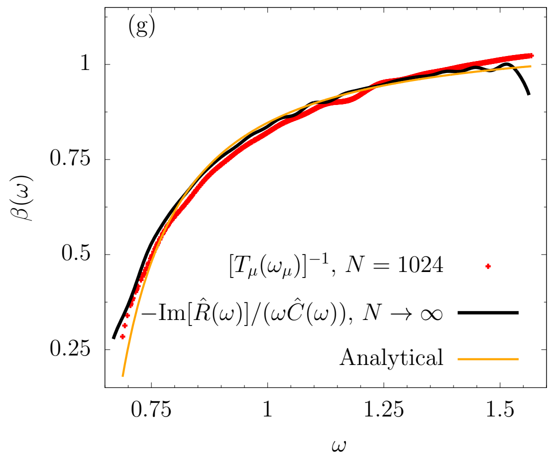
Spectrum of mode temperatures

$\overline{\langle H^{\rm kin}_\mu(t)\rangle} = \overline{\langle H^{\rm pot}_\mu(t)\rangle} = T_\mu$

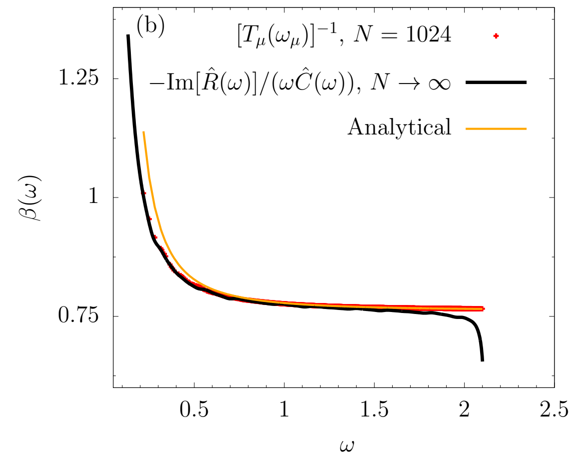
where $\overline{\ldots} = \lim_{\tau\gg 1} \frac{1}{\tau} \int_{t_{\rm st}}^{t_{\rm st}+\tau} dt' \ldots$

Two body model

The T_μs from the FDR at $\omega_\mu = [(z_f - \lambda_\mu)/m]^{1/2}$



Injection



Extraction

A way to measure the mode temperatures with a single measurement

$$\boxed{\beta_{\text{eff}}(\omega_\mu) = -\text{Im}\hat{R}(\omega_\mu)/(\omega_\mu\hat{C}(\omega_\mu)) = \beta_\mu}$$

No “partial equilibration” contradiction from the effective temperature perspective. The modes are uncoupled, they do not exchange energy, and can then have different T_μs

Two body model

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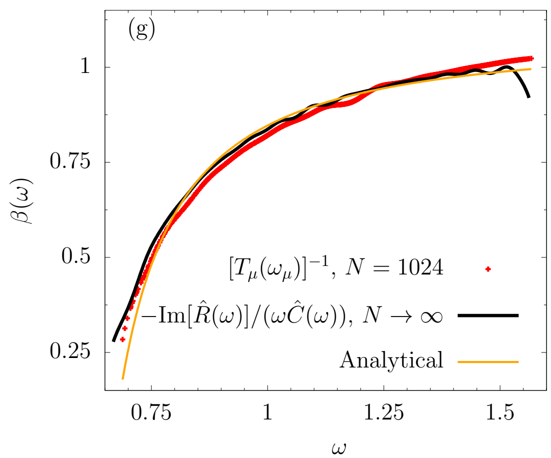

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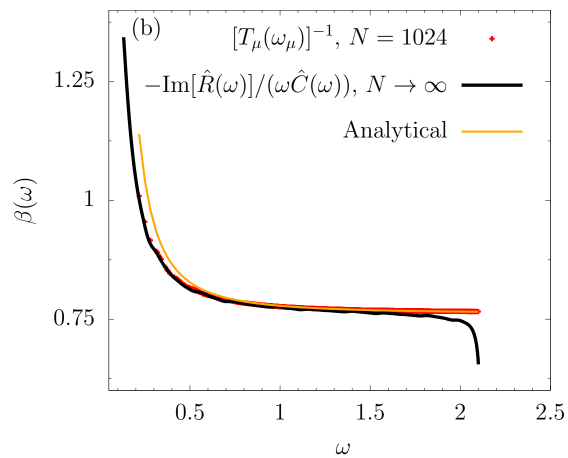

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Idea in **Foini, Gambassi, Konik & LFC 17, de Nardis, Panfil** *et al* **17** for quantum integrable cases, in a classical integrable **LFC, Lozano, Nessi, Picco & Tartaglia 18**

Two body model

Two (or more) possibilities: GB, GGE (or none)

- The system **is not** able to act as a bath on itself and equilibrate to

$$\rho \neq \rho_{\rm GB} = \mathcal{Z}^{-1}\, e^{-\beta_f H}$$

as it is an integrable system.

- **Does it approach a Generalised Gibbs Ensemble (GGE)**

$$\boxed{\rho_{\rm GGE} = \mathcal{Z}^{-1}\, e^{-\sum_{\mu=1}^{N} \beta_\mu^{\rm GGE} I_\mu}}$$

with Uhlenbeck's constants of motion I_μ and $\beta_\mu^{\rm GGE}$ fixed by

$$\boxed{\langle I_\mu \rangle_{\rm GGE} = I_\mu(t=0^+)} \qquad \textbf{?}$$

Quartic integrals to compute, hard but feasible numerically, work in progress to fix β_μ^{GGE}

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as it is an integrable system.

- **Can one use a simpler Generalised Gibbs Ensemble (GGE)**

$$\boxed{\rho_\epsilon = \mathcal{Z}^{-1}\; e^{-\sum_{\mu=1}^{N} \beta_\mu \epsilon_\mu}}$$

with asymptotic mode energies ϵ_μ **and the associated** $\beta_\mu = (k_B T_\mu)^{-1}$

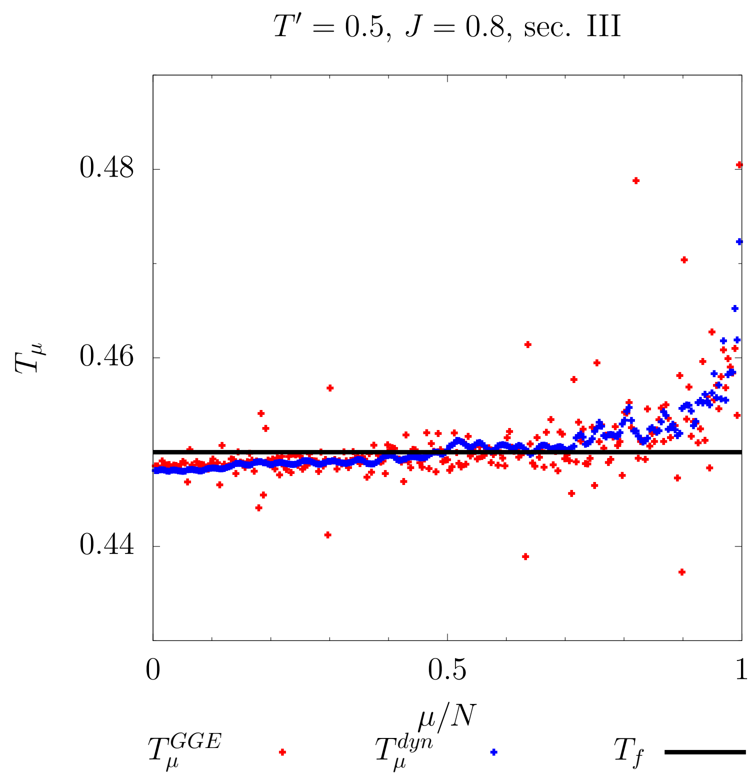
Use Uhlenbeck's constants of motion I_μ **to check whether**

$$\boxed{\langle I_\mu \rangle_{\rho_\epsilon} = I_\mu(t = 0^+)}$$

is consistent with the β_μ **from the asymptotic mode energies**

Two body model

About ρ_ϵ : yes, if following the FM state



and in the other cases ?

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- **What are the relations between $\beta_\mu^{\rm GGE}$ and β_μ, and I_μ and e_μ ?**

Conclusions

Study of the quenched dynamics of classical isolated disordered models

We showed that they can

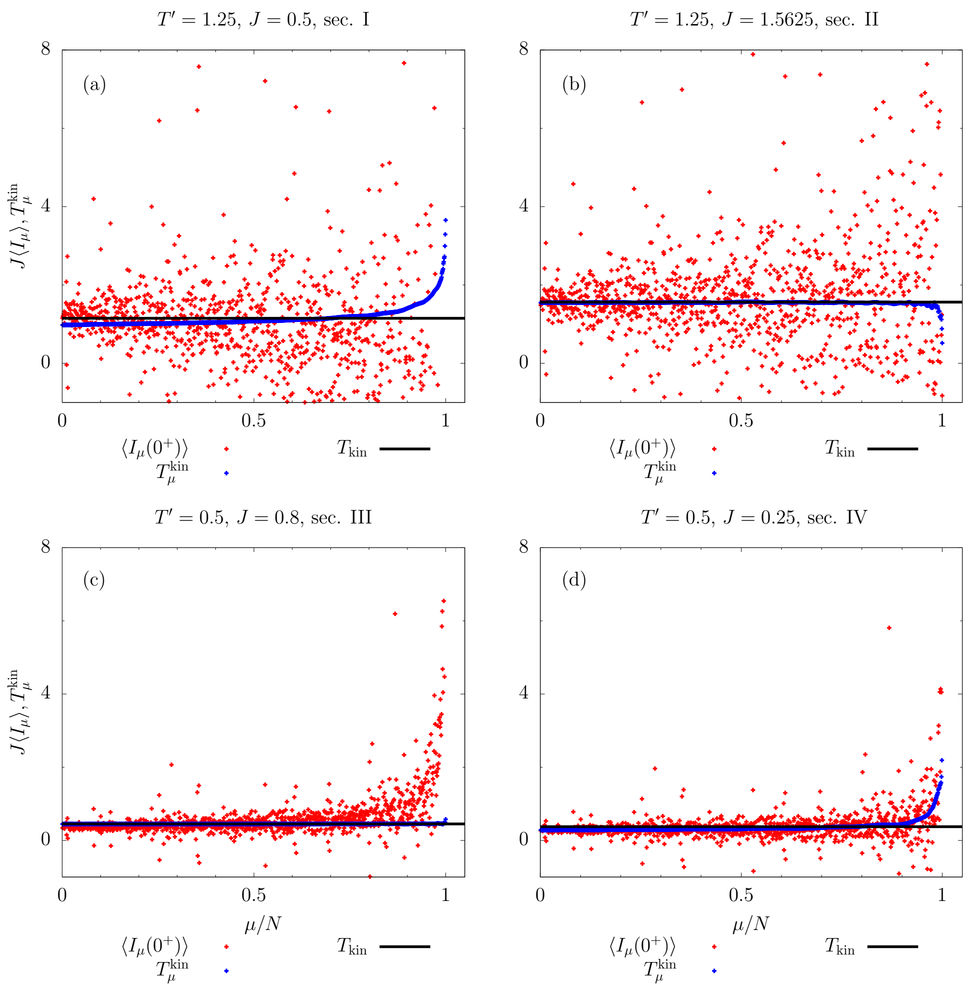
- equilibrate to GB measures
- undergo non-stationary (aging) dynamics
- or do not reach GB measures and (most probably) approach a GGE

depending on the type of model (highly interacting or quasi quadratic) and the kind of quench performed.

Works on the extension of these studies to the quantum models and the better understanding of the approach to a GGE are under way

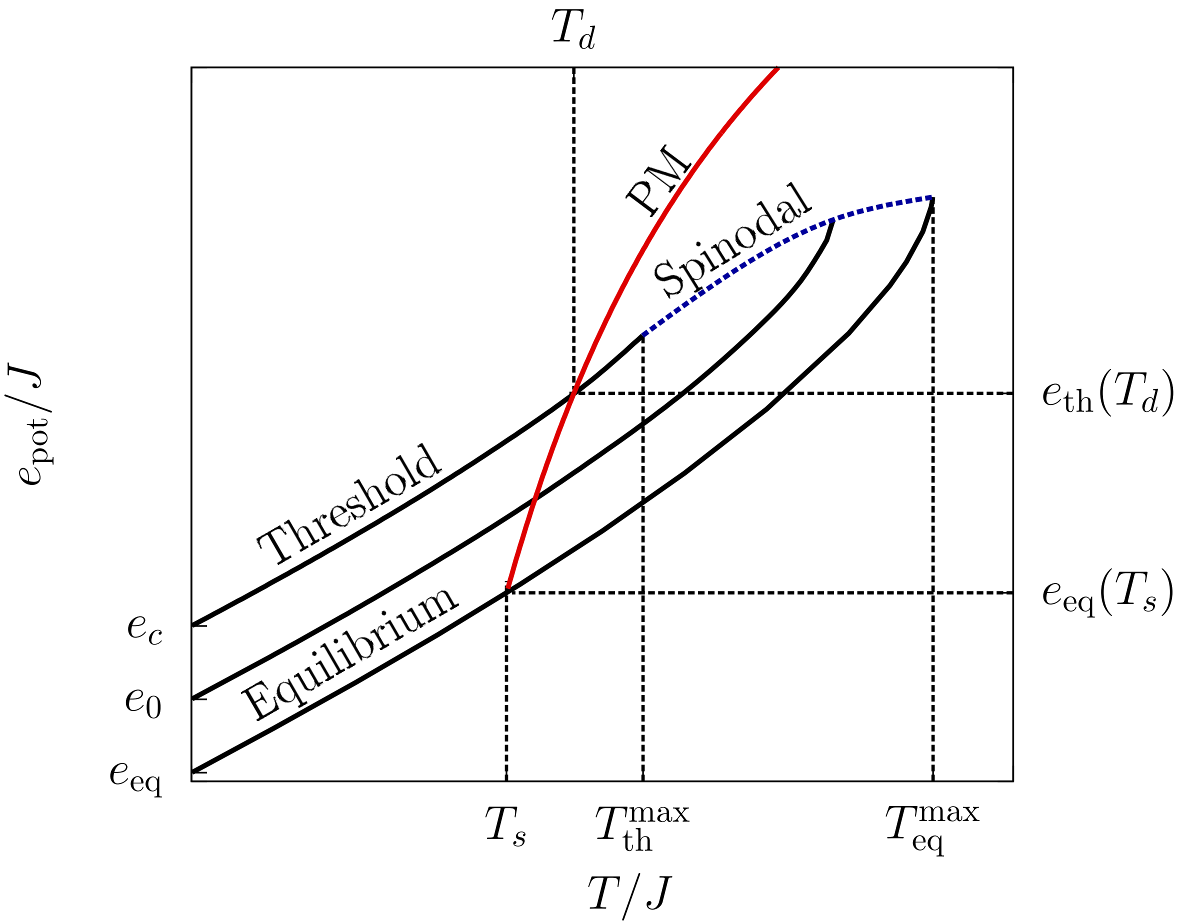
Two body model

Integrals of motion and mode energies - work in progress



Three body interactions

Potential energy landscape in canonical equilibrium



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— for finite N, solve Newton equations under the spherical constraint

Dynamic equations

Conservative dynamics for $p = 2$

In the $N \to \infty$ limit exact causal Schwinger-Dyson equations

$$(m\partial_t^2 - z_t)C(t,t_w) = \int dt' \left[\Sigma(t,t')C(t',t_w) + D(t,t')R(t_w,t')\right]$$

$$\boxed{+\frac{\beta' J_0}{J}\, D(t,0)C(t_w,0) + \text{Other Term}}$$

$$(m\partial_t^2 - z_t)R(t,t_w) = \int dt'\, \Sigma(t,t')R(t',t_w) + \delta(t-t_w)$$

Other equation

with the post-quench self-energy and vertex

$$D(t,t_w) = J^2\, C(t,t_w) \qquad\qquad \Sigma(t,t_w) = J^2\, R(t,t_w)$$

and the Lagrange multiplier z_t fixed by $C(t,t) = 1$ (Technical)

Two body model

An implicit solution for finite N

The projection of the spin configuration on the eigenvector $\vec{v}_\mu$ reads ($m = 1$)

$$s_\mu(t) \quad = \quad s_\mu(0) \sqrt{\frac{\Omega_\mu(0)}{\Omega_\mu(t)}} \ \cos \int_0^t dt' \ \Omega_\mu(t') + \frac{\dot{s}_\mu(0)}{\Omega_\mu(0)\Omega_\mu(t)} \ \sin \int_0^t dt' \ \Omega_\mu(t')$$

The time-dependent frequency $\Omega_\mu(t)$ and Lagrange multiplier $z(t)$ are fixed by

$$\frac{1}{2} \frac{\ddot{\Omega}_\mu(t)}{\Omega_\mu(t)} - \frac{3}{4} \left(\frac{\dot{\Omega}_\mu(t)}{\omega_\mu(t)} \right)^2 + \Omega_\mu^2(t) = z(t) - \lambda_\mu$$

with initial conditions $\dot{\Omega}_\mu(0) = 0$, $\Omega_\mu^2(0) = \lambda_{\max} - \lambda_\mu$ and $z(t) = e_f + \frac{2}{N} \sum_\mu \lambda_\mu \langle s_\mu^2(t) \rangle$

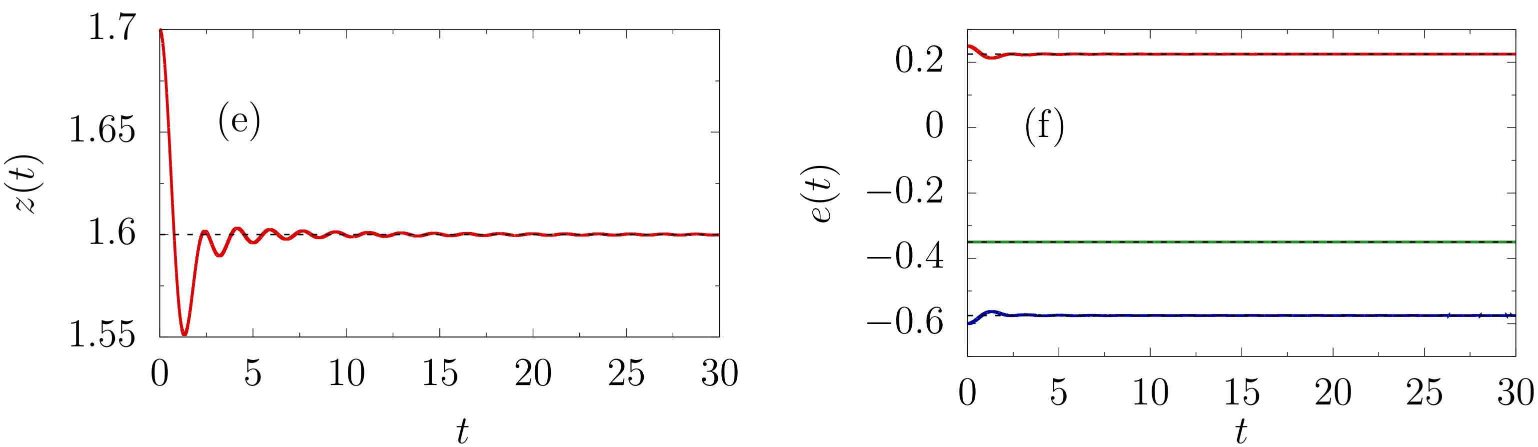
Note that the initial conditions $\{s_\mu(0), \dot{s}_\mu(0)\}$ know about the pre-quench potential and the λ_μ about the post-quench one

Similar to **Sotiriadis & Cardy 10** for the quantum O(N) model

Two body model

III Confined states global behaviour as in GB equilibrium at β_f

$$z_f = \lim_{t\to\infty} z(t) = \frac{1}{J}$$



$$e^f_{\rm pot} = \lim_{t\to\infty} e_{\rm pot}(t)$$

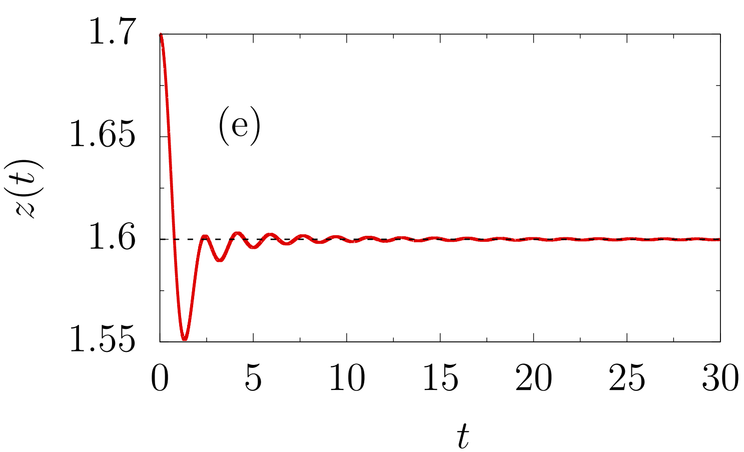
$$e^f_{\rm kin} = \lim_{t\to\infty} e_{\rm kin}(t)$$

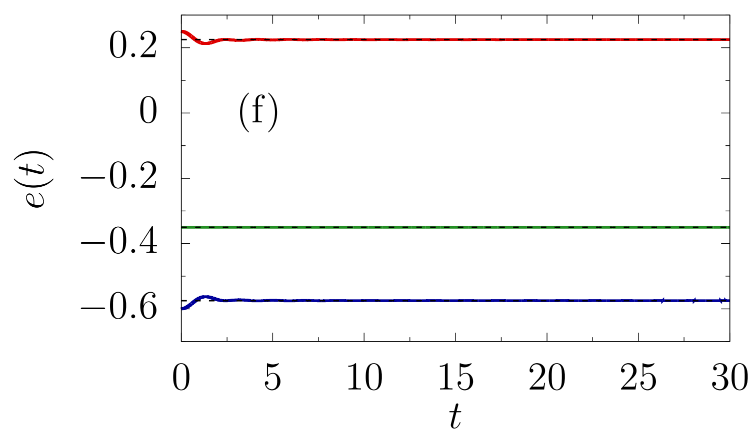
$$e_f = e^f_{\rm kin} + e^f_{\rm pot}$$

Two body model

III Confined states global behaviour as in GB equilibrium at β_f

$$z_f = \lim_{t\to\infty} z(t) = \frac{1}{J}$$





$$e^f_{\rm pot} = \lim_{t\to\infty} e_{\rm pot}(t)$$

$$T_f/2 = e^f_{\rm kin} = \lim_{t\to\infty} e_{\rm kin}(t)$$

$$e_f = e^f_{\rm kin} + e^f_{\rm pot}$$

Two body model

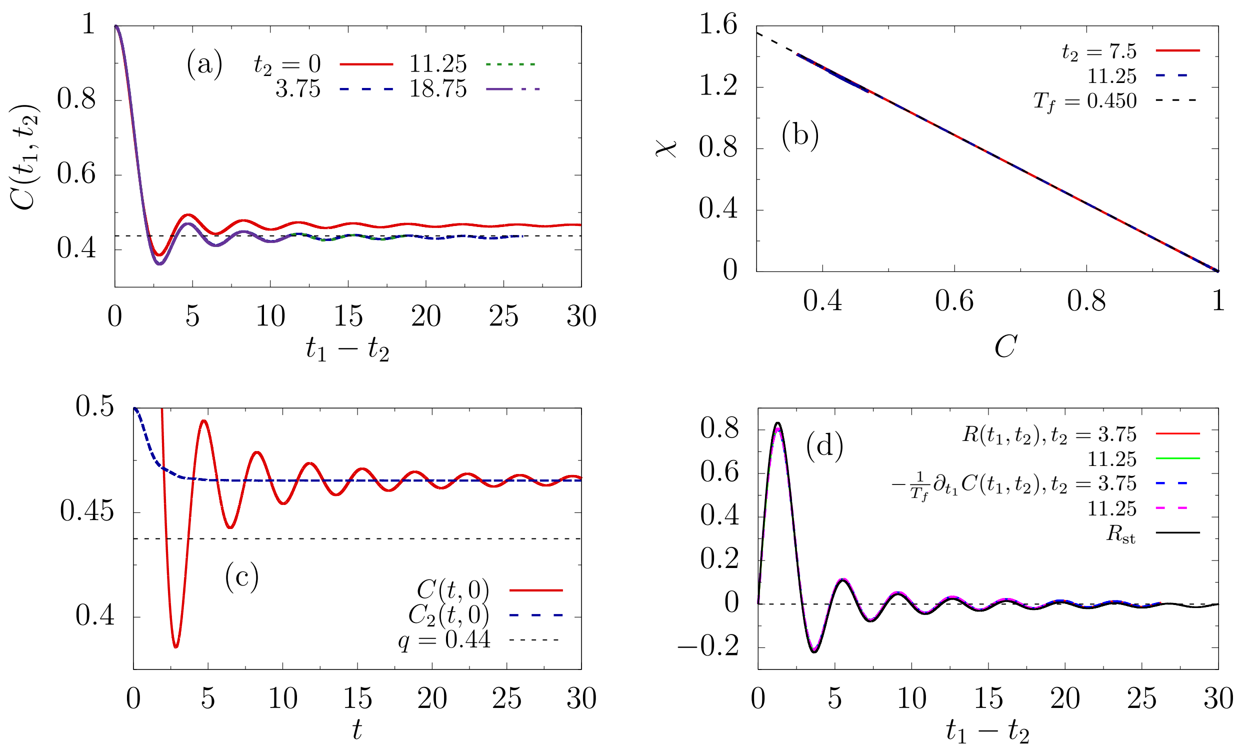
III Confined states global behaviour as in GB equilibrium at β_f

Fidelity

$C(t_1,0) \to q_0$

Integrated linear response

$\chi(t_1,t_2) = \int_{t_2}^{t_1} dt'\, R(t_1,t')$



$\chi = \frac{1}{T_f}(1-C)$

for

$C(t_1,t_2) \geq q$

q and T_f

as in GB equil.

in a confined state

Two body model

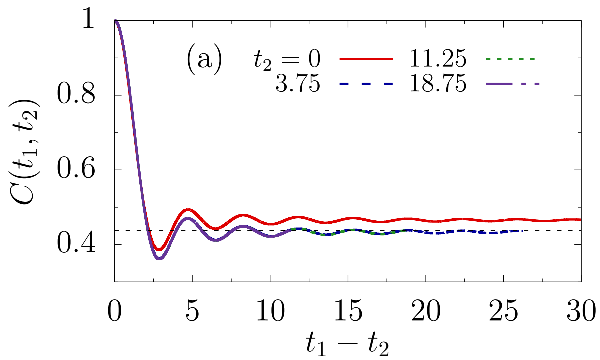
III Confined states global behaviour as in GB equilibrium at β_f

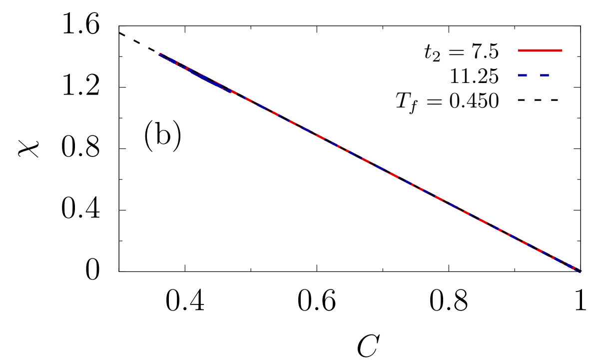
Fidelity

$C(t_1,0) \to q_0$

Integrated linear response

$\chi(t_1,t_2) = \int_{t_2}^{t_1} dt'\, R(t_1,t')$

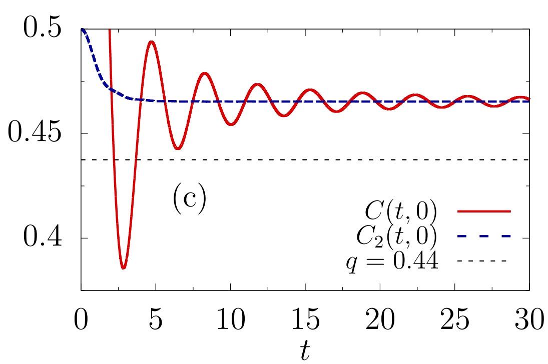


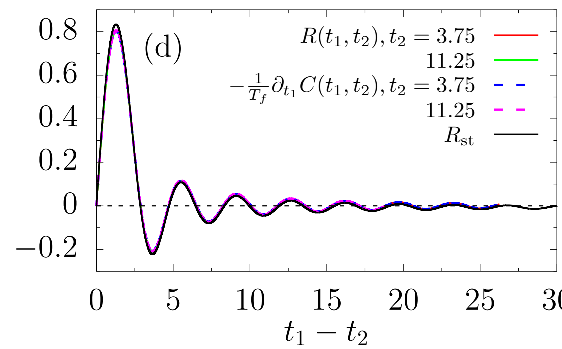


$\chi = \frac{1}{T_f}(1-C)$

for

$C(t_1,t_2) \geq q$



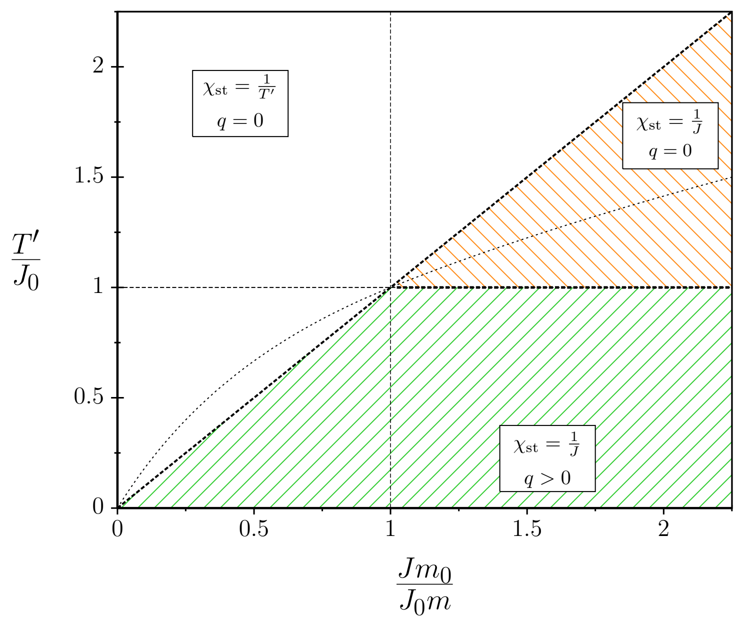


$2e^f_{\rm kin} = T_f$

$2e^f_{\rm pot} = \frac{-1}{T_f}\,(1-q)$

Two body model

Richer results !



Initial conditions

Three Sectors

I $\chi_{\rm st} = 1/T'$ and $\lim_{t \gg t_w} C(t,t_w) = 0$

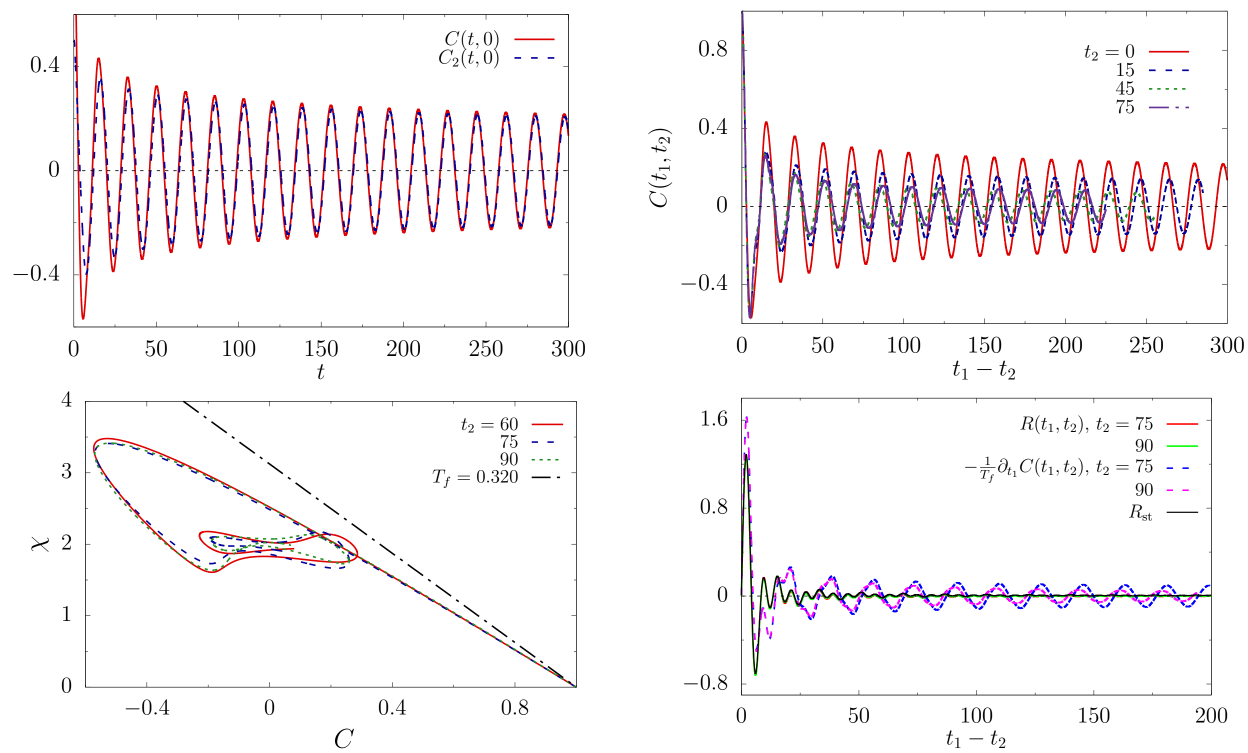
II $\chi_{\rm st} = 1/J$ and $\lim_{t \gg t_w} C(t,t_w) = 0$

GGE ?

III $\chi_{\rm st} = 1/J$ and $\lim_{t \gg t_w} C(t,t_w) > 0$ GB equilibrium ?

Two body model

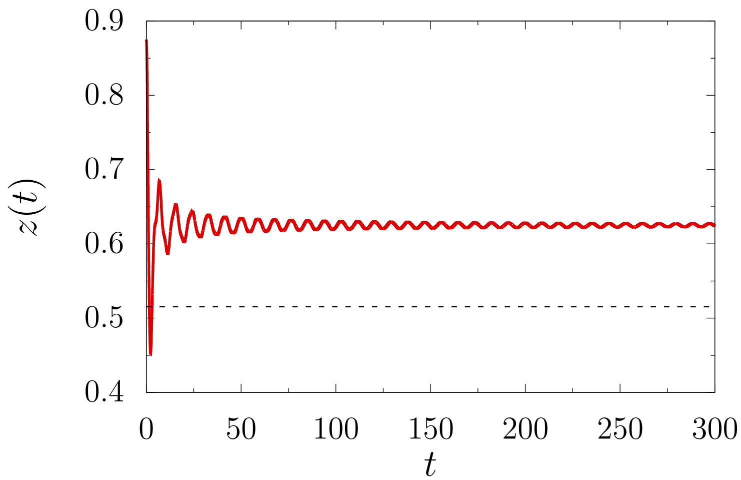
- **Large energy injection on a confined state**



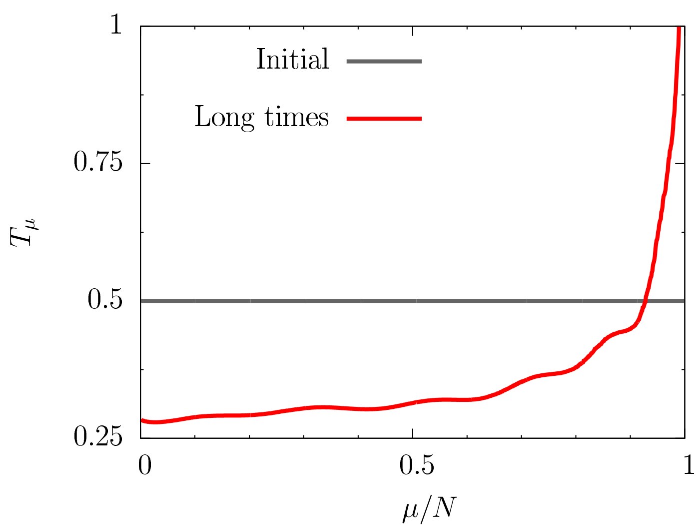
Stationary dynamics but no FDT at a single temperature: no GB equilibrium

Two body model

Large energy injection on a confined state: T_μ spectrum



$T' = 0.5,\ J = 0.25,\ N = 1024$



$z(t) \to z_f = T' + J^2/T'$

The time-dependent frequencies too

$\Omega^2_\mu(t) \to (z_f - \lambda_\mu)/m \equiv \omega^2_\mu$

The μ modes $s_\mu(t)$ decouple and

become independent harmonic oscillators

with conserved energy

$e_\mu = e^{\rm kin}_\mu(t) + e^{\rm pot}_\mu(t)$

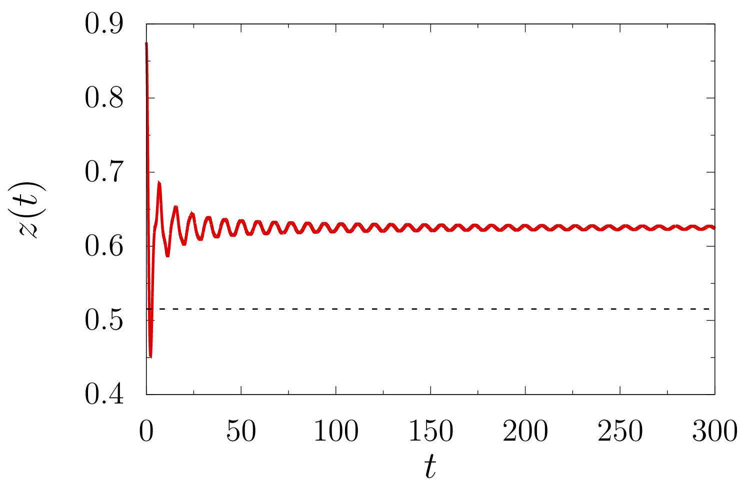
Mode temperatures

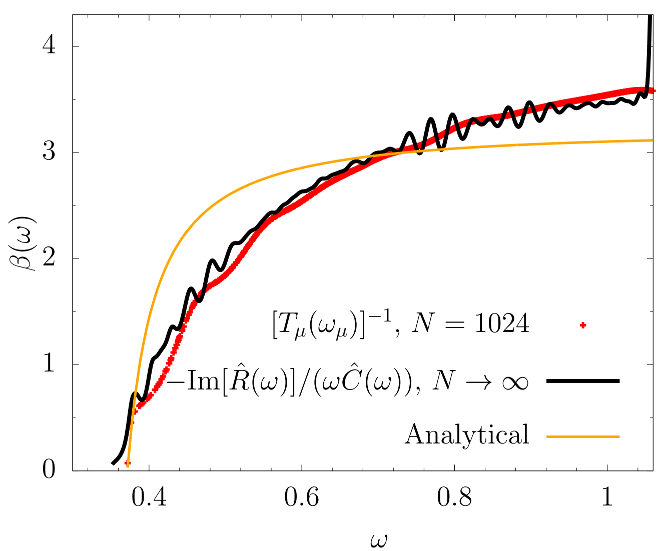
$\overline{\langle H^{\rm kin}_\mu \rangle} = \overline{\langle H^{\rm pot}_\mu \rangle} = T_\mu$

where $\overline{\ldots} = \lim_{\tau \gg 1} \frac{1}{\tau} \int_{t_{\rm st}}^{t_{\rm st}+\tau} dt' \ldots$

Two body model

Large energy injection on a confined state: T_μ from the FDR





$$z(t) \to z_f = T' + J^2/T'$$

The time-dependent frequencies too

$$\Omega^2_\mu(t) \to (z_f - \lambda_\mu)/m \equiv \omega^2_\mu$$

The μ modes $s_\mu(t)$ decouple and

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with conserved energy

$$e_\mu = e^{\text{kin}}_\mu(t) + e^{\text{pot}}_\mu(t)$$

Mode inverse temperatures *vs*

FDR inverse temperature

$$-\mathrm{Im}\hat{R}(\omega)/(\omega\hat{C}(\omega)) = \beta_{\text{eff}}(\omega)$$

Two body model

An integrable model ? Yes, Neumann's model (1850)

Motion of a particle on S_{N-1}, enforced by $\sum_k x_k^2 = N$

The Hamiltonian is

$$H = \frac{1}{4N} \sum_{k \neq l} L_{kl}^2 + \frac{1}{2} \sum a_k x_k^2$$

with $L_{kl} = (x_k p_l - x_l p_k)/\sqrt{m}$

The integrals of motion are $I_k = x_k^2 + \sum_{l(\neq k)} \frac{L_{kl}^2}{a_k - a_l}$

K. Uhlenbeck 1982

Translation from Neumann to $p = 2$ spherical model

$a_k \mapsto -\lambda_\mu$ and $I_\mu = s_\mu^2 + \frac{1}{N} \sum_{\nu(\neq \mu)} \frac{s_\mu^2 p_\nu^2 + s_\nu^2 p_\mu^2 - 2 s_\mu p_\mu s_\nu p_\nu}{\lambda_\nu - \lambda_\mu}$