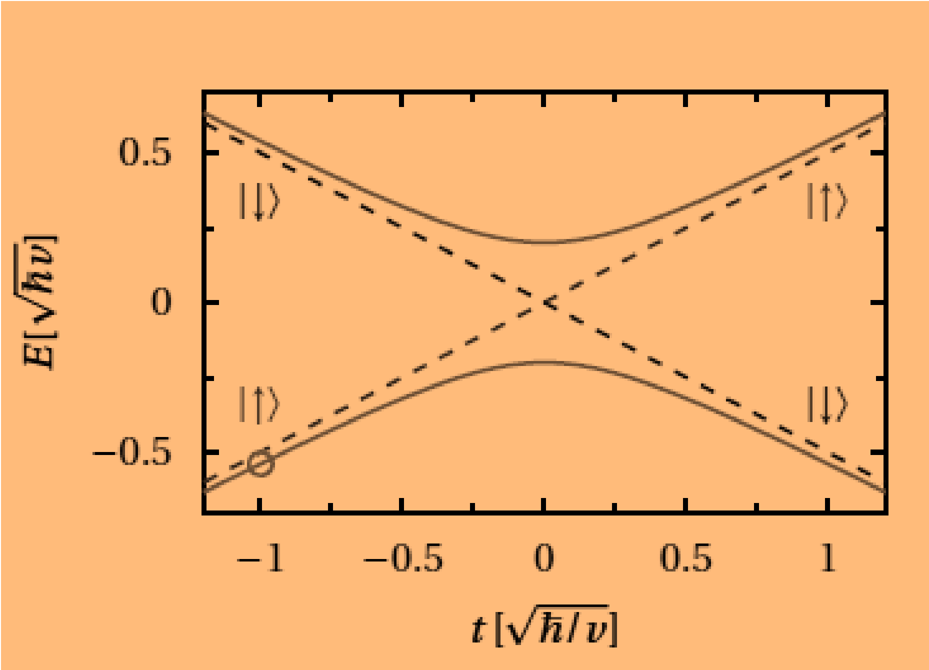
Landau-Zener Transition Driven by a Slow Noise



$$\hat{H} = -\frac{vt}{2}\hat{\sigma}_z + J\hat{\sigma}_x$$

M.E. Raikh (in collaboration with Zhuxi Luo)

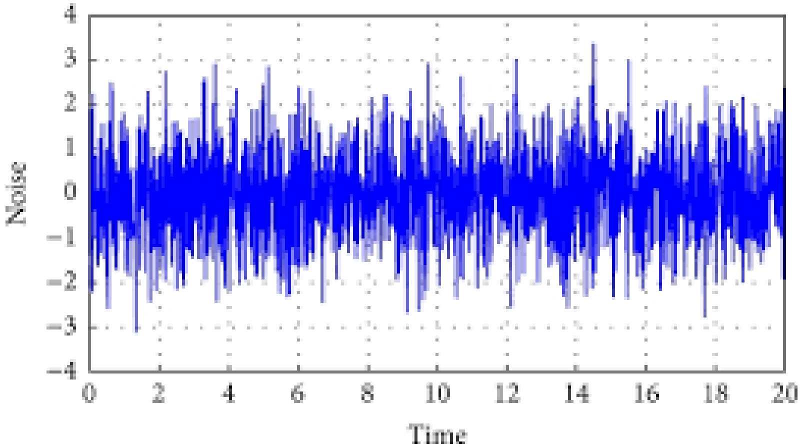
Department of Physics

University of Utah

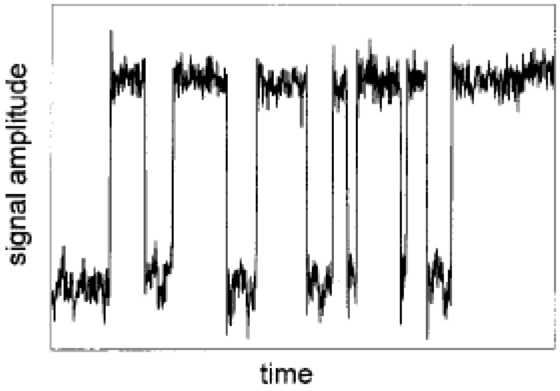
Phys. Rev. B 95, 064305 (2017)

Phys. Rev. B 96, 115437 (2017)

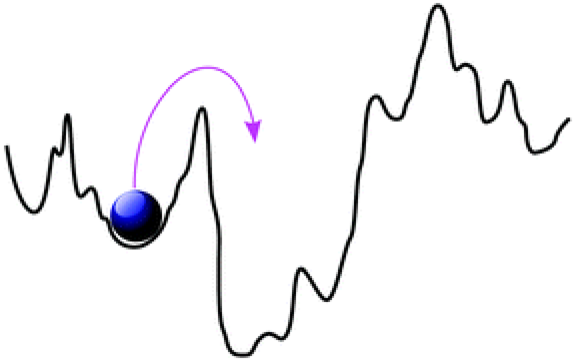
Viewing noise as a disorder in time domain



White noise

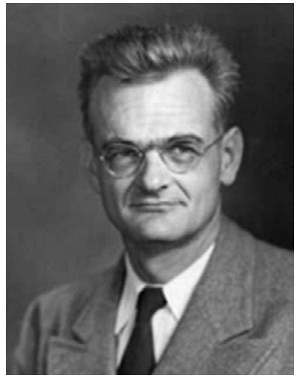


Telegraph noise

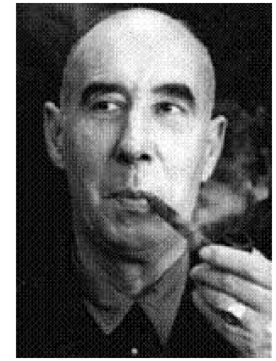


Random potential


(1908-1968)


(1905-1993)


(1906-?)


(1905-1984)

1. **L. D. Landau, Phys. Z. Sovietunion 1, 88 (1932)**
2. **C. Zener, Proc. R. Soc. London A137, 696 (1932)**
3. **E. Majorana, Nuovo Cimento 9, 43 (1932)**
4. **E. C. G. Stueckelberg, Helv. Phys. Acta 5, 369 (1932)**

Zener was aware of Landau’s paper

Rosenkewitsch* states that Landau has obtained the formula $P \sim e^{-\frac{\pi}{2hv}\frac{\Delta^2}{F_1 - F_2}}$ where $\Delta = 2\varepsilon_{12}$, v is the relative velocity, and F_1, F_2 are the “ forces ” acting upon the two states. If the identification $\frac{d}{dt}(\varepsilon_1 - \varepsilon_2) = v(F_1 - F_2)$ can be made, the exponent of Landau's formula is too small by a factor of 2π.

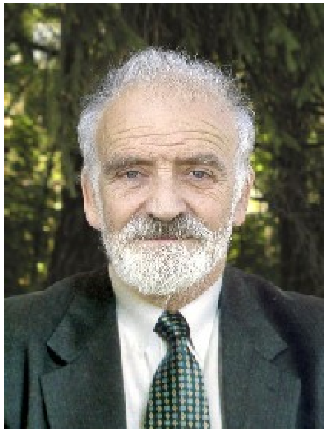
Stueckelberg quotes Landau’s and Zener’s papers

[13]) L. Landau. Sow. Phys. **2**, 46 (1932). Zu ähnlichem Resultat für grosse α^2 kommt auch C. Zener (Proc. Roy. Soc. **137** A, 696 (1932)) durch eine ganz verschiedene Betrachtungsweise.

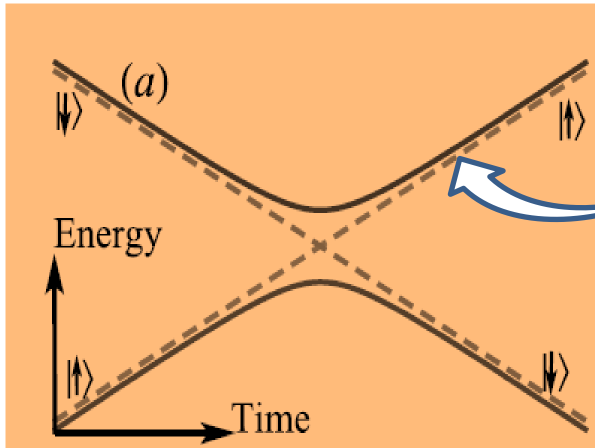
SOVIET PHYSICS JETP VOLUME 11, NUMBER 2 AUGUST, 1960

QUANTUM TRANSITIONS IN THE ADIABATIC APPROXIMATION

A. M. DYKHNE



(1933-2005)



diabatic level

Semiclassical energy levels:

$$E_{\pm}(t) = \pm\sqrt{J^2 + \frac{v^2 t^2}{4}}$$

Energy levels coincide at two points in the complex plane: $t_0 = \pm \frac{2iJ}{v}$

transition time

Probability to stay on the same diabatic level:

$$1 - P_{LZ} = Q_{LZ} = \exp\left(-2\,\mathrm{Im}\int_0^{t_0} dt\,[E_+(t) - E_-(t)] \right) = \exp\left(-2\pi \frac{J^2}{v} \right)$$

Probability to survive

Semiclassical description applies for $Q_{LZ} << 1$

Non-Adiabatic Crossing of Energy Levels

By Clarence Zener, National Research Fellow of U.S.A.

(Communicated by R. H. Fowler, F.R.S.—Received July 19, 1932.)

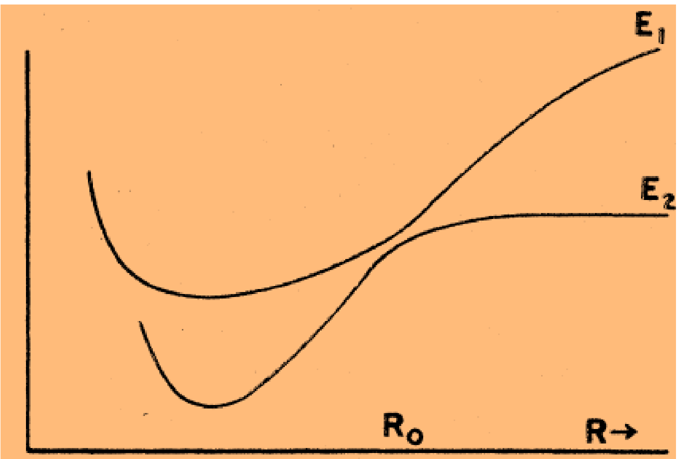


Fig. 1.—Crossing of polar and homopolar states.

$$i\dot{a}_1 = -\frac{vt}{2}a_1 + Ja_2$$

$$i\dot{a}_2 = \frac{vt}{2}a_2 + Ja_1$$

$$\ddot{a}_1 + \left[-\frac{iv}{2} + \left(\frac{vt}{2}\right)^2 + J^2\right]a_1 = 0$$

$$\ddot{a}_2 + \left[\frac{iv}{2} + \left(\frac{vt}{2}\right)^2 + J^2\right]a_2 = 0$$

Asymptotic solution at $t \to \pm\infty$

$$a_1(t) \sim (t)^{\frac{iJ^2}{v}} \exp\left[\frac{ivt^2}{4}\right]$$

Exact result:

$$Q_{LZ} = \frac{|a_1(\infty)|^2}{|a_1(-\infty)|^2} = \left|\exp(i\pi)^{\frac{iJ^2}{v}}\right|^2 = \exp\left(-2\pi\frac{J^2}{v}\right)$$

Why the problem is so delicate?

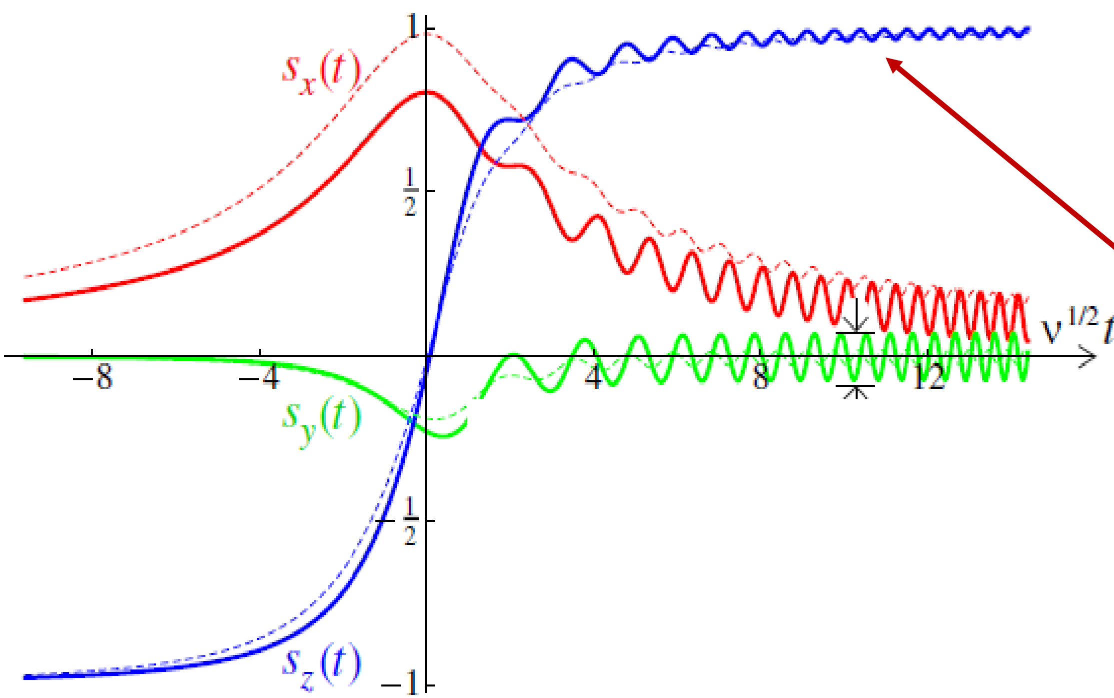
In the language of spin dynamics $|a_1(t)|^2 - |a_2(t)|^2 = S_z(t)$

approach of

$S_z(t)$

to the asymptotic value

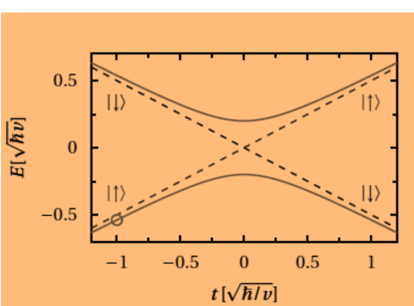
$$S_z(\infty) = 1 - \exp\left(-2\pi \frac{J^2}{v}\right)$$



is accompanied by slowly decaying oscillations

$$S_z(t) \propto \exp\left(\pm \frac{ivt^2}{2}\right)$$

Almost adiabatic transition: $\frac{J}{v} >> \frac{1}{J}$



transition time $\tau_{tr} = \frac{J}{v}$

splitting of energy levels at $t = 0$

Landau-Zener transition today: single passage

PRL **96,** 050402 (2006) PHYSICAL REVIEW LETTERS week ending 10 FEBRUARY 2006

Long-Lived Feshbach Molecules in a Three-Dimensional Optical Lattice

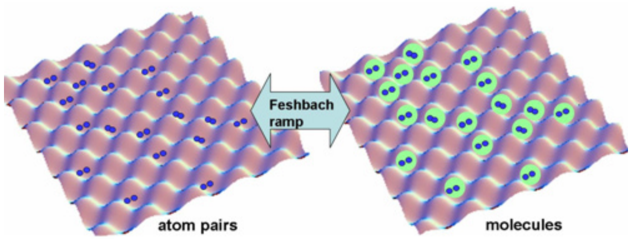
G. Thalhammer,[1] K. Winkler,[1] F. Lang,[1] S. Schmid,[1] R. Grimm,[1,2] and J. Hecker Denschlag[1]

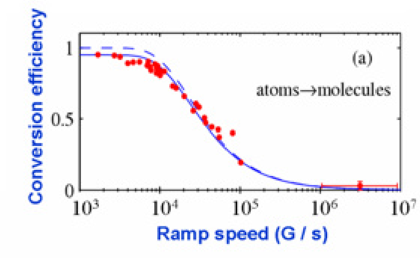
[1]*Institut für Experimentalphysik, Universität Innsbruck, 6020 Innsbruck, Austria*

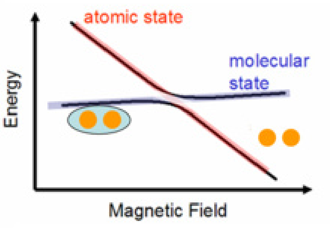
[2]*Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, 6020 Innsbruck, Austria*

(Received 27 October 2005; published 8 February 2006)

We have created and trapped a pure sample of $^{87}Rb_2$ Feshbach molecules in a three-dimensional optical lattice. Compared to previous experiments without a lattice, we find dramatic improvements such as long lifetimes of up to 700 ms and a near unit efficiency for converting tightly confined atom pairs into molecules. The lattice shields the trapped molecules from collisions and, thus, overcomes the problem of inelastic decay by vibrational quenching. Furthermore, we have developed an advanced purification scheme that removes residual atoms, resulting in a lattice in which individual sites are either empty or filled with a single molecule in the vibrational ground state of the lattice.





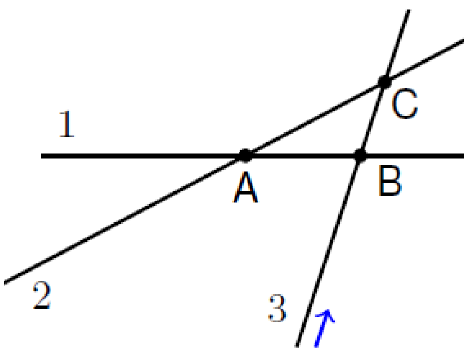


LZ transition is crucial for robust manipulation of coherent quantum states

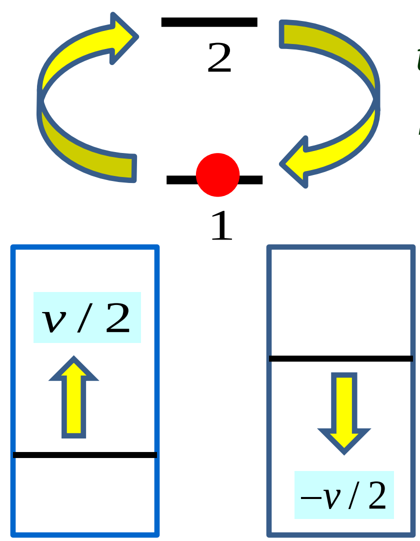
Theoretical developments

LZ transition

Multistate, since 1968

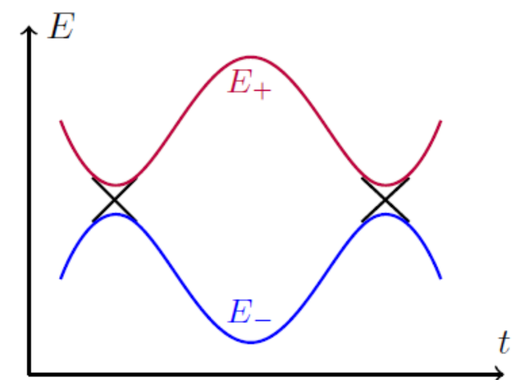


Noise-driven transition, since 1985



two-level system modulates the coupling

Interferometry, since 2005



I. Pioneering paper on multistate LZ transition

SOVIET PHYSICS JETP VOLUME 26, NUMBER 5 MAY, 1968

STATIONARY AND NONSTATIONARY PROBLEMS IN QUANTUM MECHANICS THAT CAN BE SOLVED BY MEANS OF CONTOUR INTEGRATION

Yu. N. DEMKOV and V. I. OSHEROV

Leningrad State University and Institute for Chemical Physics of the U.S.S.R. Academy of Sciences

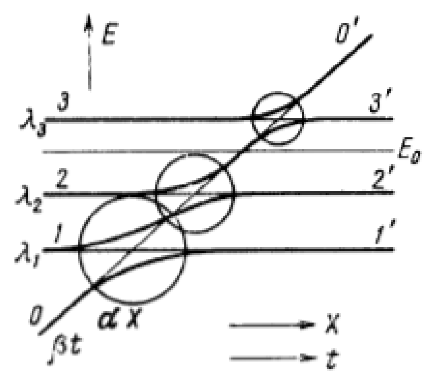
Submitted June 22, 1966

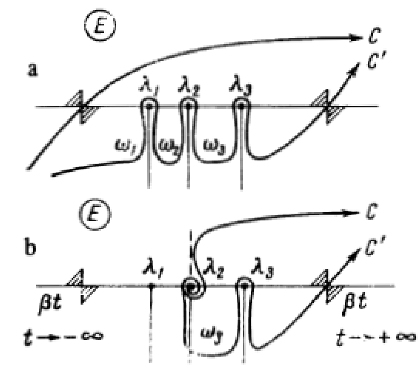
It is shown that if the energy operator consists of a time-independent part H_0 and a perturbation which depends linearly on time and is a projection operator onto a state $|\varphi\rangle$, the exact solution of the Schrödinger equation can be expressed as a contour integral. The S-matrix for such a problem possesses the triangular property and decomposes into elementary Landau-Zener factors, each of which mixes only a pair of states. Similar results are derived for the corresponding stationary problem. Some generalizations are considered, as well as examples, and the connection with previous solutions of the problem of electron detachment and of ionization in atomic and ionic collisions.

$$H = \begin{pmatrix} h_{00} + \beta t, & h_{01}, & h_{02} \dots \\ h_{10}, & h_{11}, & h_{12} \dots \\ \dots & \dots & \dots \end{pmatrix}$$

$$w(E) = \prod_{\lambda_n < E} p_n = \exp\left(-2\pi\beta^{-1} \sum_{\lambda_n < E} h_n^2 \right)$$

The net survival probability is a product of partial probabilities





PHYSICAL REVIEW A, VOLUME 61, 032705

Multipath interference in a multistate Landau-Zener-type model

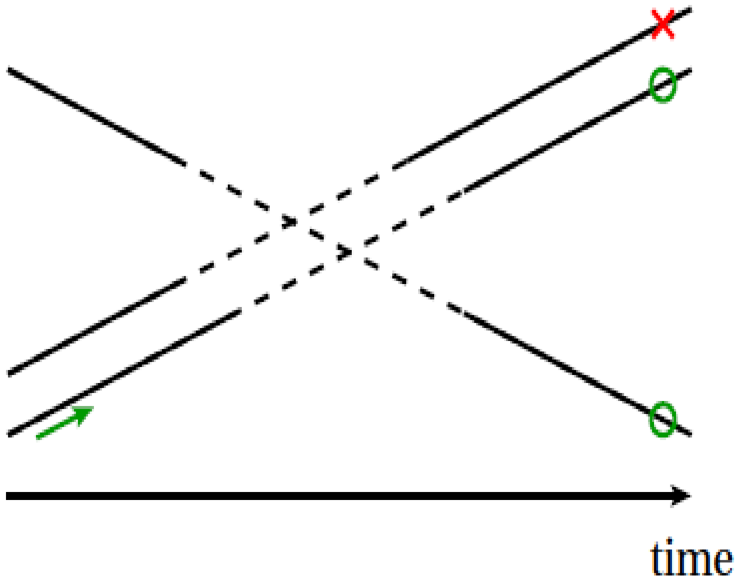
Yu. N. Demkov and V. N. Ostrovsky

Institute of Physics, The University of St. Petersburg, 198904 St. Petersburg, Russia

(Received 1 June 1999; published 10 February 2000)

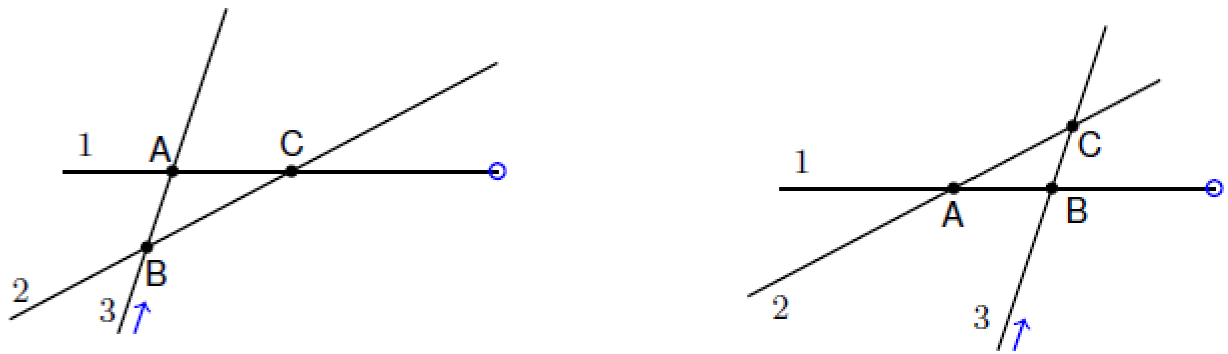
Main result on multistate LZ transition

“No-go” theorem



Upper level is not populated in the limit $t \to \infty$

Consider $3 \to 1$ in the following two situations. Semi-classically, one can view the system as staying on some diabatic level unless met with crossings.



Left: $3 \xrightarrow{B} 2 \xrightarrow{C} 1$ or $3 \xrightarrow{B} 3 \xrightarrow{A} 1 \xrightarrow{C} 1$, complications arise due to interference. This intuitively explains why there is no general solution.

Right: $3 \xrightarrow{B} 1$. The other path would require going backwards in time.

We see a simple re-ordering of the levels would dramatically change the situation. However, simplification is possible if one (a) considers $3 \to 3$ or $1 \to 1$, or (b) turns off the some of the couplings.

PHYSICAL REVIEW B **66**, 205303 (2002)

Multiparticle Landau-Zener problem: Application to quantum dots

N. A. Sinitsyn

Department of Physics, Texas A&M University, College Station, Texas 77843-4242

We propose a simple ansatz that allows us to generate new exactly solvable multistate Landau-Zener models. It is based on a system of two decoupled two-level atoms whose levels vary with time and cross at some moments. Then we consider multiparticle systems with Heisenberg equations for annihilation operators having a similar structure as the Shrödinger equation for amplitudes in multistate Landau-Zener models and show that the corresponding Shrödinger equation in the multiparticle sector belongs to the multistate Landau-Zener class. This observation allows us to generate new exactly solvable models from already known ones. We discuss possible applications of the new solutions in the problem of the driven charge transport in quantum dots.

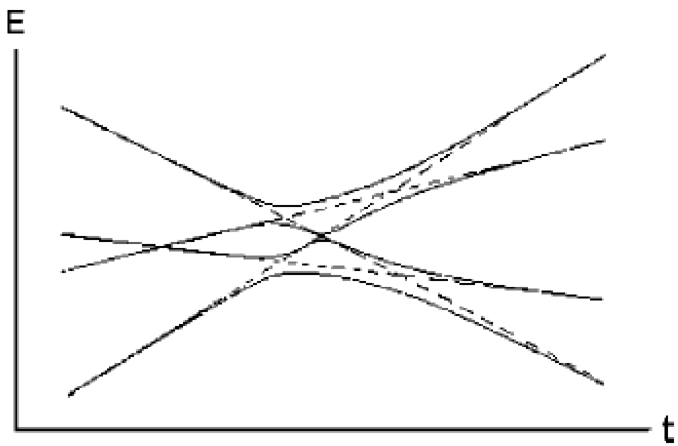


FIG. 1. Time dependence of the adiabatic energies (solid lines) and diagonal elements (dashed lines) of the Hamiltonian (5). The choice of parameters is $\beta_1=5$, $\beta_2=-3$, $\beta_3=0$, $\beta_4=-1.5$, $E_1=3$, $E_2=0.5$, $E_3=-2$, $E_4=-1.5$, $g=1$, $\gamma=1.5$.

All these models provide very simple results. For example, transition probabilities in the Demkov-Osherov model coincide with those taken from successive application of the two state Landau-Zener formula. The same is true for the generalized bow-tie model. Finally in all models the transition probabilities are simple polynomials of $z_k=\exp(-\pi|g_k|^2)$. This fact gives a strong feeling that there should be a common symmetry in the background of all these models. Our results demonstrate the same properties and we know that the reason for this was the symmetry that makes the Hamiltonian equivalent in some sense to the one for a much simpler problem.

***No interference* effects in exactly solvable multilevel LZ problem**

PHYSICAL REVIEW A **70**, 052708 (2004)

Landau-Zener transitions in a multilevel system: An exact result

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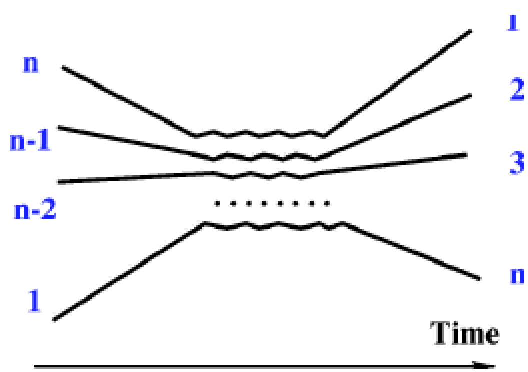
(Received 7 July 2004; published 17 November 2004)

We study the S matrix for the transitions at an avoided crossing of several energy levels, which is a multilevel generalization of the Landau-Zener problem. We demonstrate that, by extending the Schrödinger evolution to complex time, one can obtain an exact answer for some of the transition amplitudes. Similar to the Landau-Zener case, our result covers both the adiabatic (slow evolution) and the diabatic (fast evolution) regimes. The form of the exact transition amplitude coincides with that obtained in a sequential pairwise level crossing approximation, in accord with the conjecture of Brundobler and Elser [J. Phys. A **26**, 1211 (1993)].

$$S_{1\to 1} = b_1 = \exp(\pi\alpha_1) = \exp\left(-\pi\sum_{j\neq 1}\frac{|\Delta_{i1}|^2}{\nu_j-\nu_1}\right).$$

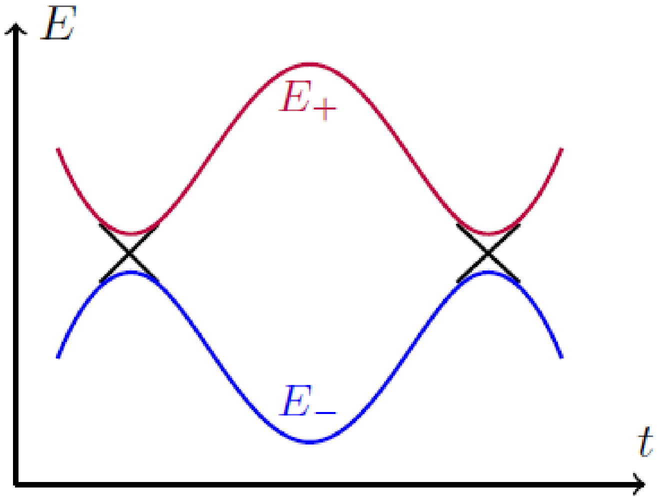


Similarly, for the $i=n$ component, after going through analytical continuation over the contour (a), we obtain

$$S_{n\to n} = \exp(-\pi\alpha_n) = \exp\left(-\pi\sum_{j\neq n}\frac{|\Delta_{in}|^2}{\nu_n-\nu_j}\right)$$

FIG. 1. Time evolution of the adiabatic (frozen) energy levels of the Hamiltonian (1). The transitions analyzed in this work are $|1\rangle_{t=-\infty}\to|1\rangle_{t=+\infty}$ and $|n\rangle_{t=-\infty}\to|n\rangle_{t=+\infty}$.

Mutiple Passage



To finally arrive at E_+, one can either follow the blue-red-red path or the blue-blue-red path. Both contribute $Q_{LZ}(1-Q_{LZ})$.

The adiabatic-impulse approximation

- Between crossings, adiabatic:

$$U = \begin{pmatrix} e^{i\int_{t_i}^{t_f} E_-\ dt} & 0 \\ 0 & e^{i\int_{t_i}^{t_f} E_+\ dt} \end{pmatrix}$$

- At crossings, instantaneous:

$$N = \begin{pmatrix} \sqrt{1-Q_{LZ}}e^{-i\varphi_s} & -\sqrt{Q_{LZ}} \\ \sqrt{Q_{LZ}} & \sqrt{1-Q_{LZ}}e^{i\varphi_s} \end{pmatrix}$$

$$\varphi_s = -\frac{\pi}{4} + \frac{J^2}{v}\left(\ln\frac{J^2}{v} - 1\right) + \arg\Gamma\left(1 - i\frac{J}{\sqrt{v}}\right).$$

- Interference: $Q_+ = 4Q_{LZ}(1-Q_{LZ})\sin^2\Phi_{st}$.
- Generally after n periods, the evolution matrix is $(NU_2NU_1)^n$.

Physics Reports 492 (2010) 1–30

II. Landau–Zener–Stückelberg interferometry

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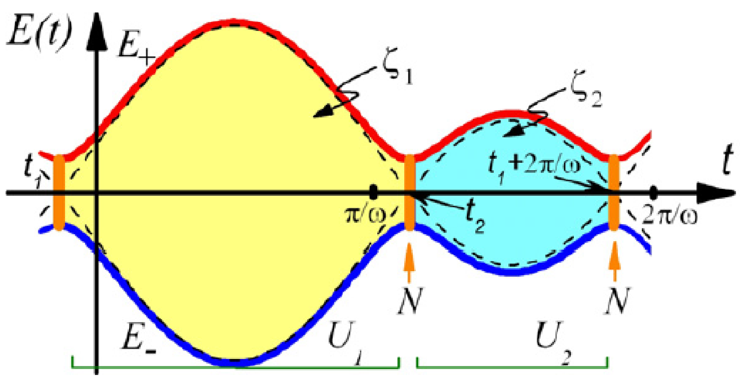
[b] RIKEN Advanced Science Institute, Wako-shi, Saitama, Japan
[c] Department of Physics, The University of Michigan, Ann Arbor, MI, USA

Multiple passage



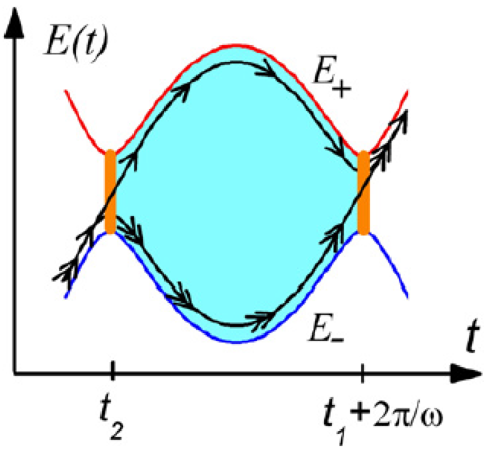


Fig. 2. (Color online) Time evolution of the energy levels during one period. The time-dependent adiabatic energy levels define a two-stage evolution: transitions at the non-adiabatic regions, described by the evolution matrix N, and the adiabatic evolution, described by the matrices $U_{1,2} = \exp(-i\zeta_{1,2}\sigma_z)$. The acquired phases $\zeta_{1,2}$ have a geometrical interpretation: they are equal to the area under the curves, shown by the yellow and blue regions. The diabatic energy levels, $\pm\varepsilon(t)/2$, are shown by the dashed lines.

Fig. 4. (Color online) Double-passage transition. Adiabatic energy levels as in Fig. 2 are plotted. The lines with one and two arrows show the two trajectories where the transition to the upper level happens during the first and the second passages of the avoided level crossing. Their respective transition probabilities are given by $P_{LZ}(1-P_{LZ})$ and $(1-P_{LZ})P_{LZ}$, while the interference is described by Eq. (27).

between crossings $$U(t_f, t_i) = \begin{pmatrix} e^{-i\zeta(t_f,t_i)} & 0 \\ 0 & e^{i\zeta(t_f,t_i)} \end{pmatrix} = e^{-i\zeta(t_f,t_i)\sigma_z}$$

$$N = \begin{pmatrix} \sqrt{1-P_{LZ}}\, e^{-i\tilde{\varphi}_S} & -\sqrt{P_{LZ}} \\ \sqrt{P_{LZ}} & \sqrt{1-P_{LZ}}\, e^{i\tilde{\varphi}_S} \end{pmatrix}$$

$$\zeta(t_f, t_i) = \frac{1}{2}\int_{t_i}^{t_f} \Omega(t)\mathrm{d}t$$

$$\Omega(t) = \sqrt{\Delta^2 + \varepsilon(t)^2}$$

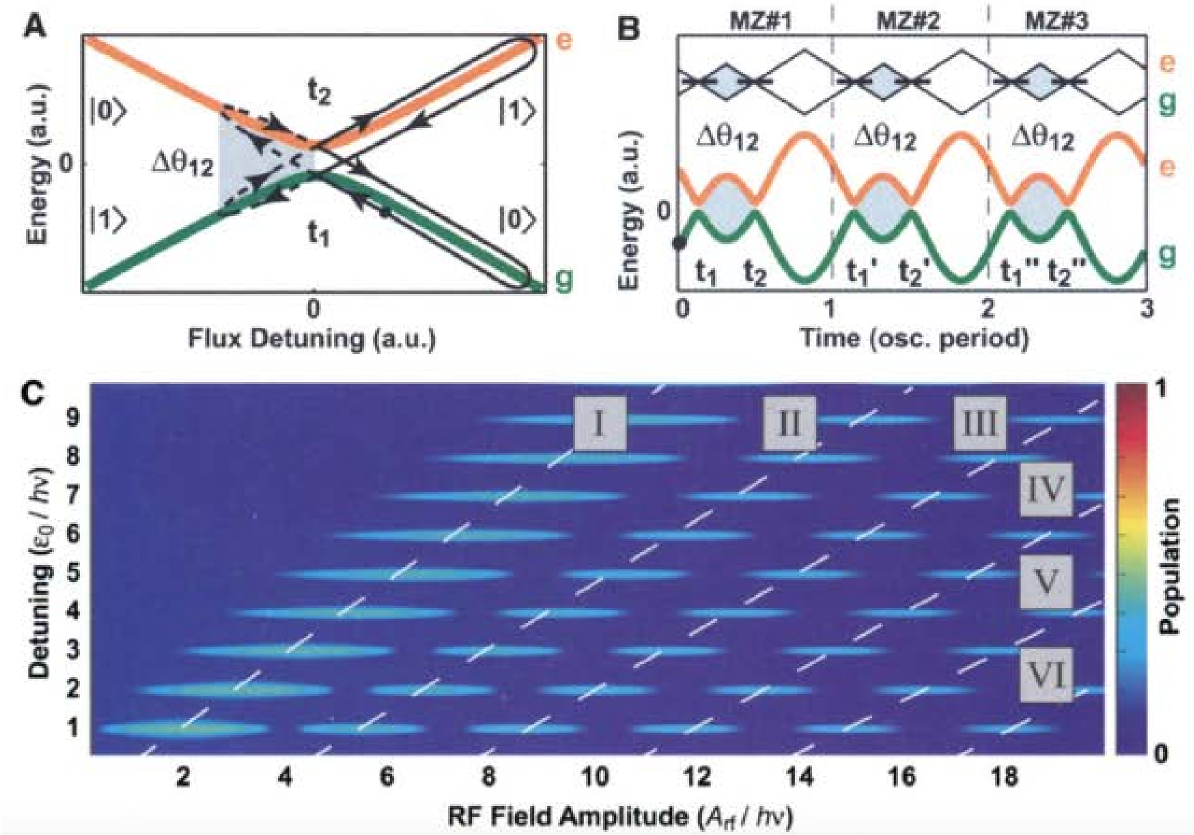
evolution matrice after n full periods

$$U\left(t, t_1 + \frac{2\pi n}{\omega}\right)(NU_2NU_1)^n$$

Interferometry: Experiment

W. D. Oliver, Y. Yu, J. C. Lee, K. K. Berggren, L. S. Levitov, T. P. Orlando, Science Vol. 310, 5754 (2005).

A time-dependent magnetic flux $f(t) = f^{dc} + f^{ac}(t)$ threads the superconducting loop and controls the qubit.



$f(t) \approx \Phi_0/2,$

$\delta f = f^{dc} - \Phi_0/2,$

$\pm\varepsilon_0 \propto \pm\delta f.$

$f^{ac} = A_{rf}\cos(2\pi\nu t).$

When $A_{rf} > \varepsilon_0,$

for a full period

$\theta = 2\pi\varepsilon_0/h\nu.$

Constructive interference at $\theta = 2\pi n,$

multi-photon resonances at $\varepsilon_{0,n} = nh\nu.$

Journal of the Physical Society of Japan
Vol. 54, No. 5, May, 1985, pp. 2037–2046

III. Noise-driven LZ transition

Pioneering paper:

Stochastic Theory for Nonadiabatic Level Crossing with Fluctuating Off-Diagonal Coupling

Yosuke KAYANUMA

Department of Physics, Faculty of Science, Tohoku University, Sendai 980

(Received November 22, 1984)

Random coupling: $< g(t)g(t') >= J^2 \exp\left(-\gamma\,|\,t-t'\,|\right)$

$$H = -\frac{vt}{2}\hat{\sigma}_z + g(t)\hat{\sigma}_x$$

General expression:

$$P = \left\langle \left\{ \exp\left[-i\int_{-\infty}^{\infty} H(\tau)\,d\tau \right] \right\}_{2,1} \left\{ \exp\left[-i\int_{-\infty}^{\infty} H(\tau)\,d\tau \right] \right\}_{1,2} \right\rangle$$

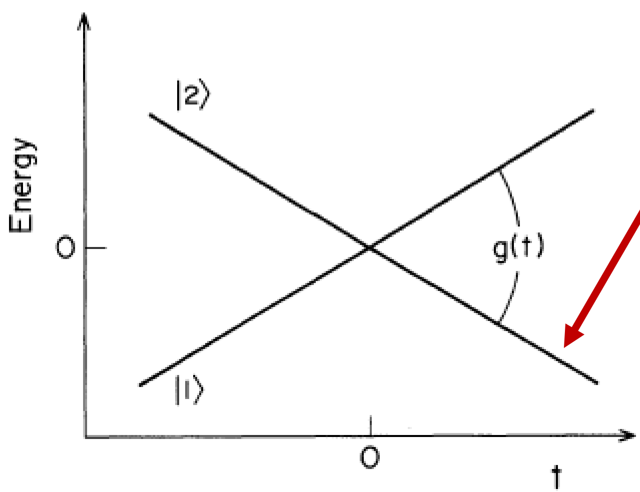


Fig. 1. Level crossing with fluctuating off-diagonal coupling. The energy difference between the diabatic states changes with a constant velocity v and the off-diagonal matrix element $g(t)$ fluctuates around the mean value $\langle g(t) \rangle = 0$.

Expansion in powers of J^2

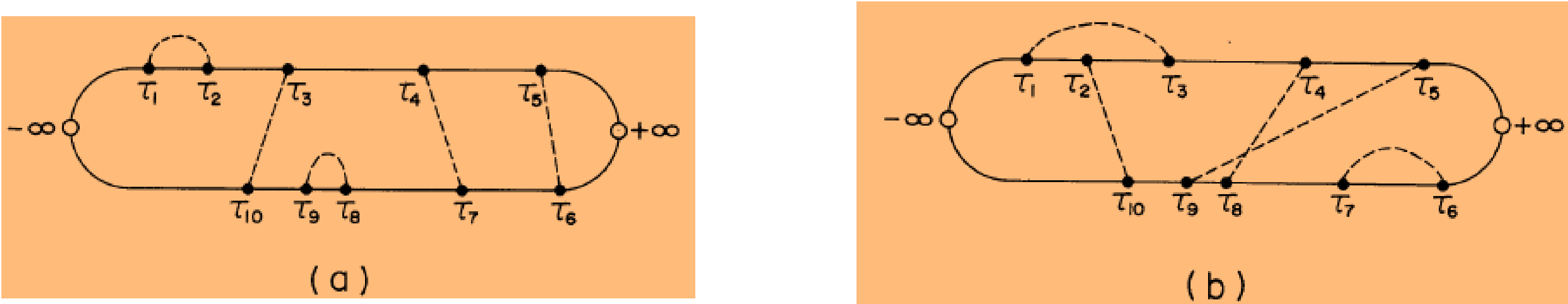


Fig. 2. Diagramatic representation of the time-ordered integral for the perturbation expansion of P. The system starts from $|1\rangle$ at $t=-\infty$, propagates to the right, making a transition at each time-point, and reaches $|2\rangle$ at $t=+\infty$. Figure 2(a) and (b) correspond to two examples of the pairing for a given configuration of the time-points. The two time-points connected by the dashed lines lie within a distance less than $\tau_c = 1/\gamma$.

$$P = -\sum_{n=1}^{\infty}\left(-J^2\right)^n L^{(n)}$$

$$L^{(n)} = \sum_{m=1}^{n}\int_{-\infty}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 \ldots \int_{-\infty}^{\infty} dt_{2m} \ldots \int_{-\infty}^{t_{2n-1}} dt_{2n} \times F^{(n)}\left(t_1,\ldots,t_{2n}\right)\exp\left[i\frac{v}{2}\sum_{j=1}^{2n}(-1)^j t_j^2\right]$$

Fig. 3. Diagramatic representation of the serial time-ordering. The diagram (a) and (b) correspond to the diagram (a) and (b) of Fig. 2, respectively. The diagram (a) is singly connected and (b) is multiply connected. The pattern in this case is $(+1, -1, +1, -1, +1)$.

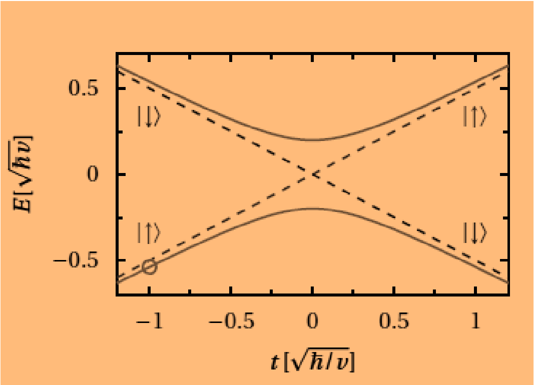
$$F^{(n)}\left(t_1, t_2, \ldots, t_{2n}\right) = \sum_{pairs}\exp\left(-\frac{1}{\tau_c}\sum_{k=1}^{n}\left| t_{i,2k} - t_{i.2k-1}\right|\right)$$

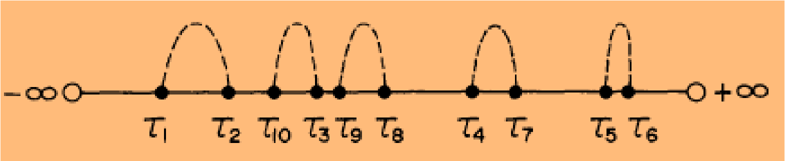
Two limiting cases:

I. Fast noise: correlation time $\tau_c = \gamma^{-1}$ much shorter than the transition time

$$\tau_c << \tau_{tr} = \frac{J}{v}$$

Leading contribution comes from diagrams with ordered times





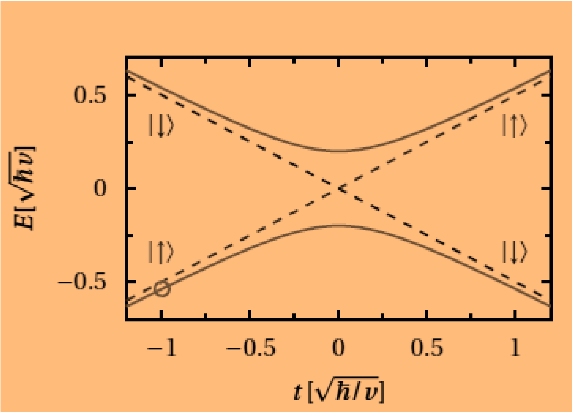
$$F^{(n)}(\tau_1, \tau_2, ..., \tau_{2n}) = \sum_{pairs} \exp\left(-\frac{1}{\tau_c} \sum_{k=1}^{n} | \tau_{i,2k} - \tau_{i.2k-1} | \right)$$

Only a single out of $(2n-1)!!$ pairings contributes

$$L^{(n)} = \frac{1}{2(n!)} \left(\frac{4\pi}{|v|} \right)^n$$

$$P = \frac{1}{2}\left[1 - \exp\left(-\frac{4\pi J^2}{v} \right) \right]$$

Probabilities $\uparrow \rightarrow \uparrow$ and $\uparrow \rightarrow \downarrow$ are almost equal



Derivation for the fast-noise limit in the language of spin dynamics

$$P_{\uparrow\to\uparrow} - P_{\uparrow\to\downarrow} = S_z(\infty)$$

$$b_z = \frac{vt}{2} \qquad b_x = J(t)$$

$$\frac{d\vec{S}}{dt} = \vec{b}\times\vec{S}$$

equation of spin dynamics

$$\frac{dS_z}{dt} = -\int_{-\infty}^{t} dt_1 \cos\left[v\left(\frac{t^2 - t_1^2}{4}\right)\right] b_x(t) b_x(t_1) S_z(t_1)$$

For the fast noise: $< b_x(t) b_x(t_1) >= J_0^2 \exp\left[-\frac{|t - t_1|}{\tau}\right]$

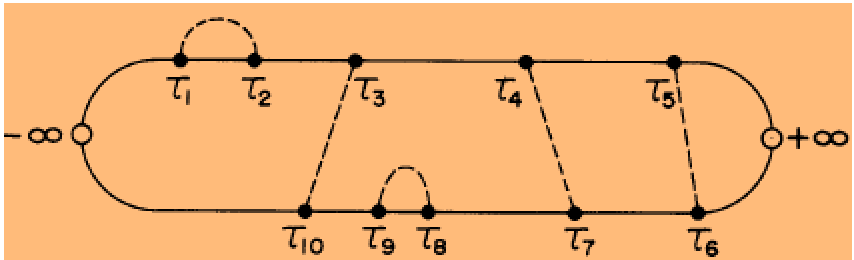
$$\frac{dS_z}{dt} = -\frac{J_0^2\tau}{1+\left(\frac{vt\tau}{2}\right)^2} S_z(t)$$

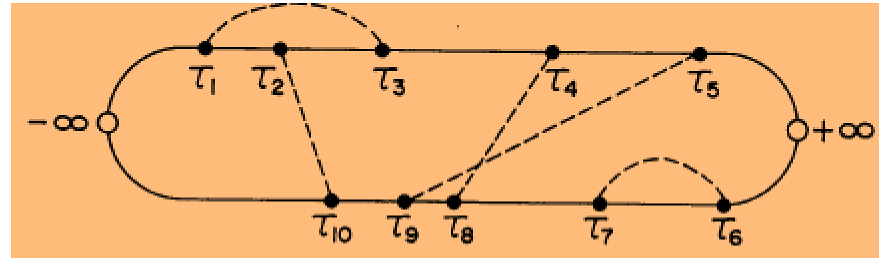
$$S_z(\infty) = S_z(-\infty)\exp\left[-\frac{4\pi J_0^2}{v}\right]$$

exponentially small polarization in rapidly changing magnetic field

II. *Slow noise: very long correlation times* $\frac{\tau_{tr}}{\tau_c} \to 0$

Contributions of all diagrams of the same order are equal





Combinatorial factor:

$$F^{(n)}(\tau_1, \tau_2, \ldots, \tau_{2n}) = (2n-1)!!$$

$$L^{(n)} = \frac{(2n-1)!!}{n!}\left(\frac{2\pi}{|v|}\right)^n$$

$$P_{SF} = \frac{1}{(2\pi)^{1/2} J} \int_{-\infty}^{\infty} dQ \, P_{LZ}(Q) \exp\left(-\frac{Q^2}{2J^2}\right)$$

LZ probability $P_{LZ}(Q) = 1 - \exp\left(-\frac{2\pi Q^2}{|v|}\right)$ ***of non-diagonal matrix element***

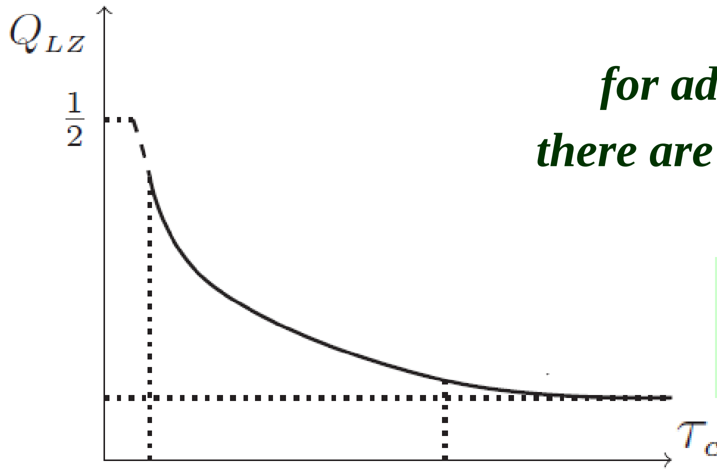
is averaged over the distribution of couplings

In both limits correlation time does not enter into the answer

Our study: Motivation

*1. Analytical results for slow noise do **not** contain correlation time* τ_c

Crossover from "slow" to "fast" noise is not captured by the Kayanuma theory



for adiabatic LZ transition

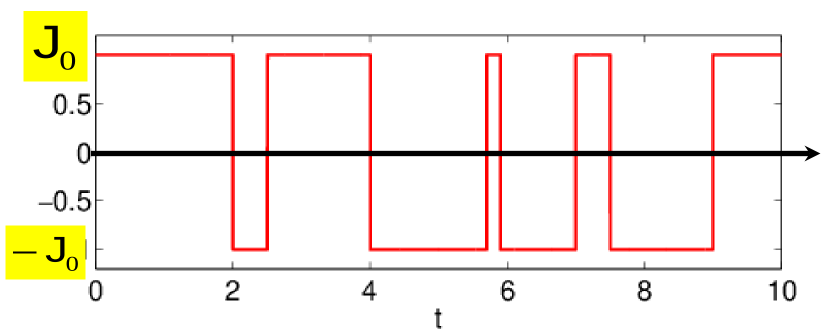
*there are **two** small parameters:* $\frac{v}{J^2}$ *and* $\frac{J}{v\tau_c}$

$$\left(1+\frac{4\pi J^2}{|v|}\right)^{-1/2}$$

limit $\tau_c \to \infty$

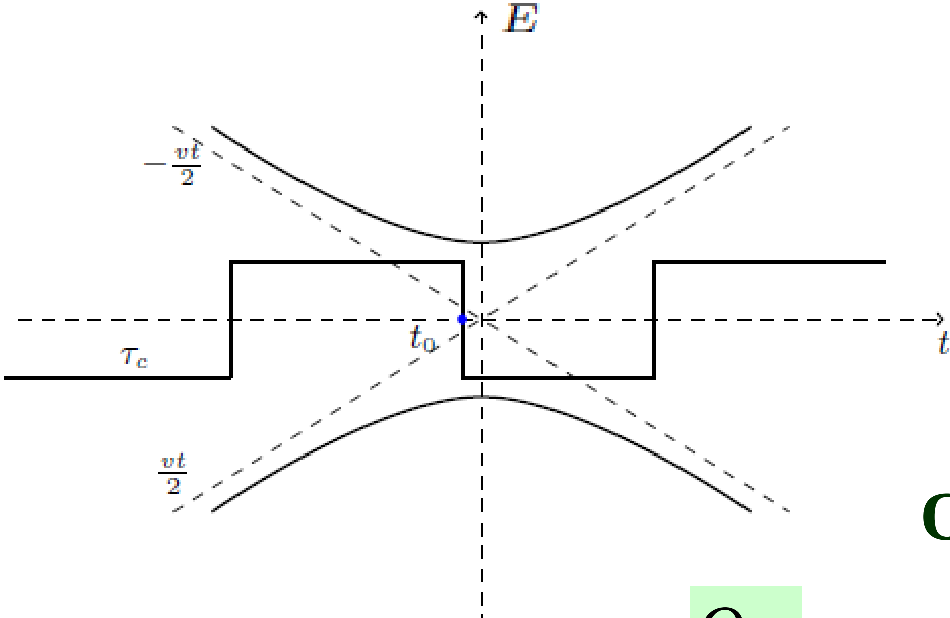
2. For the telegraph noise with correlator $< J(t)J(t') >= J_0^2 \exp\left[-\frac{|t-t'|}{\tau_c}\right]$

$J(t)$ *takes the values* $\pm J_0$ → ***Average*** *LZ probability is **not** affected by noise*



$$1-P_{LZ} = Q_{LZ} = \exp\left(-2\pi\frac{J_0^2}{v}\right)$$

We start from slow telegraph noise: $\tau_{LZ} = \frac{J_0}{v} \ll \tau_c$



$$i\dot{a}_1 = -\frac{vt}{2}a_1 + Ja_2 \qquad i\dot{a}_2 = \frac{vt}{2}a_2 + Ja_1$$

Switching takes place at $t = t_0$

Our prime finding:

Main contribution to Q_{LZ} ***comes from sparse moments*** $t_0 \ll \tau_{LZ} = \frac{J_0}{v}$

For $t < t_0$ ***the solution with right behavior at*** $t \to -\infty$

$$a_1 = D_\nu(z) \qquad a_2 = -i\nu^{1/2}D_{\nu-1}(z)$$

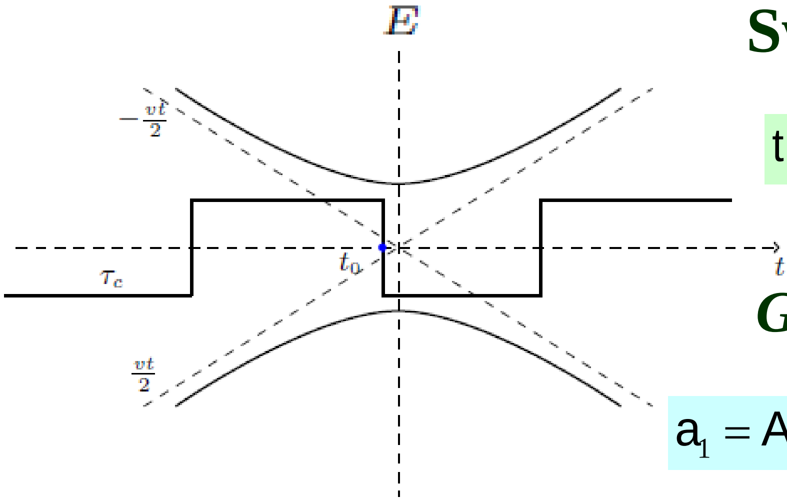
parabolic cylinder function $a_2(-\infty) \to 0$

with index $\nu = -\frac{iJ_0^2}{v}$ ***and argument*** $z = v^{1/2}e^{\pi i/4}t$

Without switching:

$$Q_{LZ} = \frac{|D_\nu(\infty)|^2}{|D_\nu(-\infty)|^2} = \exp(-2\pi|\nu|) \ll 1$$

LZ result



Switching at $t = t_0$

$t < t_0$ $\quad a_1 = D_\nu(z)$ $\quad a_2 = -i\nu^{1/2} D_{\nu-1}(z)$

General solution for $t > t_0$

$$a_1 = AD_\nu(z) + BD_\nu(-z) \qquad a_2 = i\nu^{1/2}\left[AD_{\nu-1}(z) - BD_{\nu-1}(-z)\right]$$

Continuity at $t = t_0$

$$D_\nu(z_0) = AD_\nu(z_0) + BD_\nu(-z_0)$$

$$D_{\nu-1}(z_0) = -AD_{\nu-1}(z_0) + BD_{\nu-1}(-z_0)$$

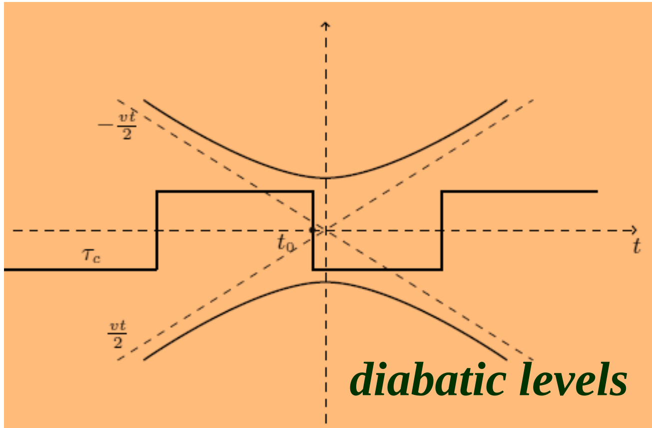
where $z_0 = \nu^{1/2} e^{\pi i/4} t_0$

Solution:

$$A = \frac{\dfrac{D_{\nu-1}(-z_0)}{D_{\nu-1}(z_0)} - \dfrac{D_\nu(-z_0)}{D_\nu(z_0)}}{\dfrac{D_{\nu-1}(-z_0)}{D_{\nu-1}(z_0)} + \dfrac{D_\nu(-z_0)}{D_\nu(z_0)}} \qquad B = \frac{2}{\dfrac{D_{\nu-1}(-z_0)}{D_{\nu-1}(z_0)} + \dfrac{D_\nu(-z_0)}{D_\nu(z_0)}}$$

For switching at $t = 0$ **we have** $B = 1$ ⇨ **complete survival !**

Our prime observation:



Telegraph-noise switching takes place at

$$t_0 << \tau_{LZ} = \frac{J_0}{v}$$

For $t < t_0$ ***the solution with right behavior at*** $t \to -\infty$

$$a_1 = D_\nu(z) \qquad a_2 = -i\nu^{1/2} D_{\nu-1}(z)$$

$$a_2(-\infty) \to 0$$

where $\nu = -\frac{iJ_0^2}{v}$ ***and*** $z = v^{1/2} e^{\pi i/4}$

General solution for $t > t_0$

$$a_1 = AD_\nu(z) + BD_\nu(-z) \qquad a_2 = i\nu^{1/2}\left[AD_{\nu-1}(z) - BD_{\nu-1}(-z)\right]$$

Without switching:

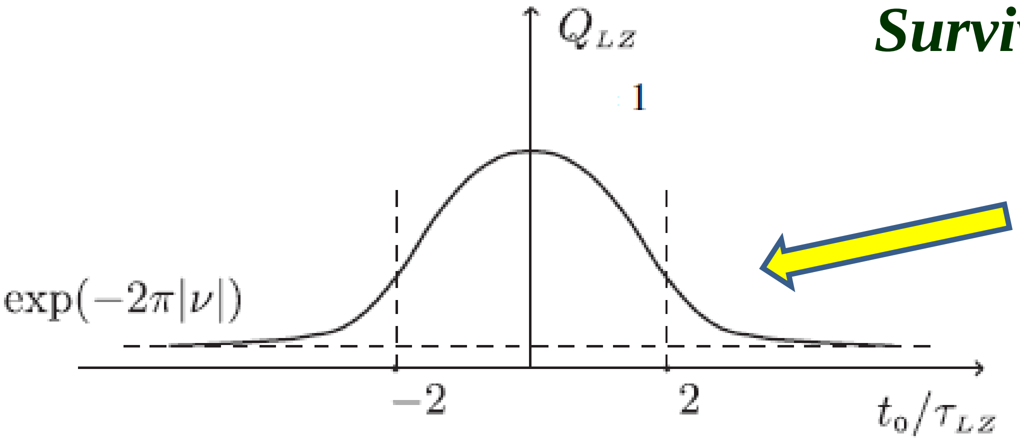
$B = 0 \Rightarrow Q_{LZ} = \frac{|a_1(\infty)|^2}{|a_1(-\infty)|^2} = \exp(-2\pi|\nu|) << 1$

With switching at $t = 0$

$A = 0 \Rightarrow Q_{LZ} = 1$

Rare realizations of noise have an exponentially strong effect

Typical switching moment: $t_0 \sim \tau_c >> \tau_{LZ}$



Survival probability at small t_0

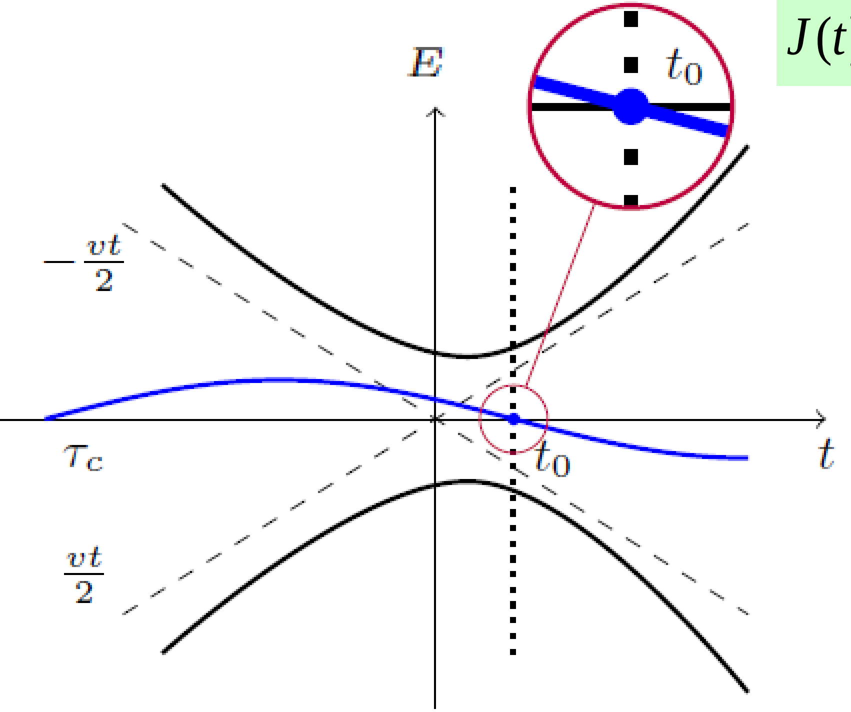
$$Q_{LZ}(t_0) = \frac{4}{4 + \left(\frac{t_0}{\tau_{LZ}}\right)^2}$$

$$\bar{Q}_{LZ} = \exp\left(-2\pi \frac{J_c^2}{v}\right) + 2\pi \frac{\tau_{LZ}}{\tau_c} = \exp\left(-2\pi \frac{J_c^2}{v}\right) + 2\pi \frac{J_c}{v\tau_c}$$

contribution from rare noise realizations

finite- τ_c *correction dominates for strongly adiabatic LZ transition*

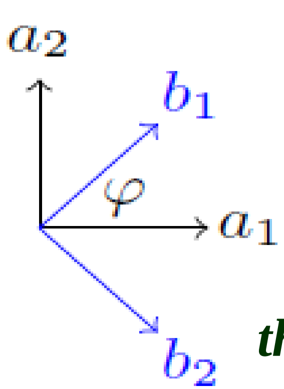
Gaussian noise



$J(t) = (t - t_0) J'$

When $J(t)$ changes continuously, the major contribution to the survival probability comes from realizations for which $J(t)$ passes though zero near the level crossing

With linearized $J(t)$ upon introducing the new variables:



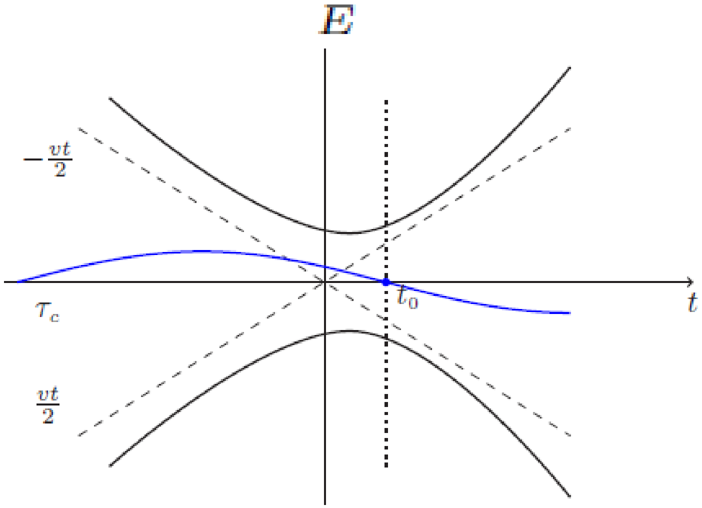
$$b_1 = a_1 \cos\varphi + a_2 \sin\varphi \qquad b_2 = a_1 \sin\varphi - a_2 \cos\varphi$$

where the angle φ is defined as $\tan(2\varphi) = \frac{2J'}{v}$

the Hamiltonian reduces to the effective LZ Hamiltonian $H = \frac{\tilde{v}t}{2}\sigma_z + \tilde{J}\sigma_x$

Effective LZ Hamiltonian:

$$H = \frac{\tilde{v}t}{2}\sigma_z + \tilde{J}\sigma_x$$



renormalized velocity and coupling

$$\tilde{v} = \frac{v}{\cos(2\varphi)} \qquad \tilde{J} = \frac{vt_0 \sin(2\varphi)}{2} \qquad \tan(2\varphi) = \frac{2J'}{v}$$

Effective transition time:

$$\tilde{\tau}_{LZ} = \frac{\tilde{J}}{\tilde{v}} = \frac{J'v}{v^2 + 4(J')^2} t_0$$

Effective survival probability:

$$Q_{LZ}(t_0) = \exp\left[-\frac{2\pi (J')^2 t_0^2}{v} \left(\frac{v^2}{v^2 + 4(J')^2} \right)^{3/2} \right]$$

Since $\tilde{\tau}_{LZ} \le t_0$ ***the condition*** $t_0 << \tau_c$ ***ensures the validity of linearization of*** $J(t)$

Upon averaging over $t_0 << \tau_c$

$$\overline{Q}_{LZ} = \int_{-\infty}^{\infty} \frac{dt_0}{\tau_c} Q_{LZ}(t_0) = \left(\frac{v}{2}\right)^{1/2} \frac{1}{|J'|\tau_c} \left(\frac{v^2 + 4(J')^2}{v^2} \right)^{3/4}$$

The result should be averages over the distribution of J'

Gaussian distribution of the slopes J' ***has the width:***

$$\langle J(t)J(t')\rangle = J_c^2 K(t-t') \Rightarrow (J')_c^2 = J_c^2 \frac{\partial^2 K}{\partial^2 t_1^2}\Big|_{t=t'}$$

Averaging of $\bar{Q}_{LZ} = \left(\frac{v}{2}\right)^{1/2} \frac{1}{|J'|\tau_c}\left(\frac{v^2+4(J')^2}{v^2}\right)^{3/4}$ ***over*** J'

Averaging in the domain $J' >> v$

$$\langle \bar{Q}_{LZ} \rangle = \frac{1}{\pi^{1/2}(J')_c}\int_{-\infty}^{\infty} dJ' \, \bar{Q}_{LZ}(J') \exp\left(-\frac{J'^2}{(J')_c^2}\right) = \frac{2}{\pi}\left(\frac{J_c}{v^{1/2}}\right)^{1/2}\left(\frac{1}{v^{1/2}\tau_c}\right)^{3/2}$$

Decay of the survival probability with increasing τ_c

Averaging in the domain $J' << v$

$$\bar{Q}_{LZ} = \left(\frac{v}{2}\right)^{1/2} \frac{1}{|J'|\tau_c} \left(\frac{v^2 + 4(J')^2}{v^2}\right)^{3/4} \approx \left(\frac{v}{2}\right)^{1/2} \frac{1}{|J'|\tau_c} \left[1 + 3\left(\frac{J'}{v}\right)^2\right]$$

Leading contribution:

$$\left\langle \bar{Q}_{LZ} \right\rangle = \left(\frac{v}{\pi}\right)^{1/2} \frac{1}{|(J')_c|\tau_c} \int_{v^{1/2}/\tau_c}^{\infty} \frac{dJ'}{J'} \exp\left(-\frac{J'^2}{(J')_c^2}\right) = \left(\frac{v}{\pi}\right)^{1/2} \frac{1}{|(J')_c|\tau_c} \ln\left(\frac{|(J')_c|\tau_c}{v^{1/2}}\right)$$

minimal J'

exceeds the standard result

$$\left\langle Q_{LZ} \right\rangle = \left(\frac{v}{4\pi}\right)^{1/2} \frac{1}{J_c}$$

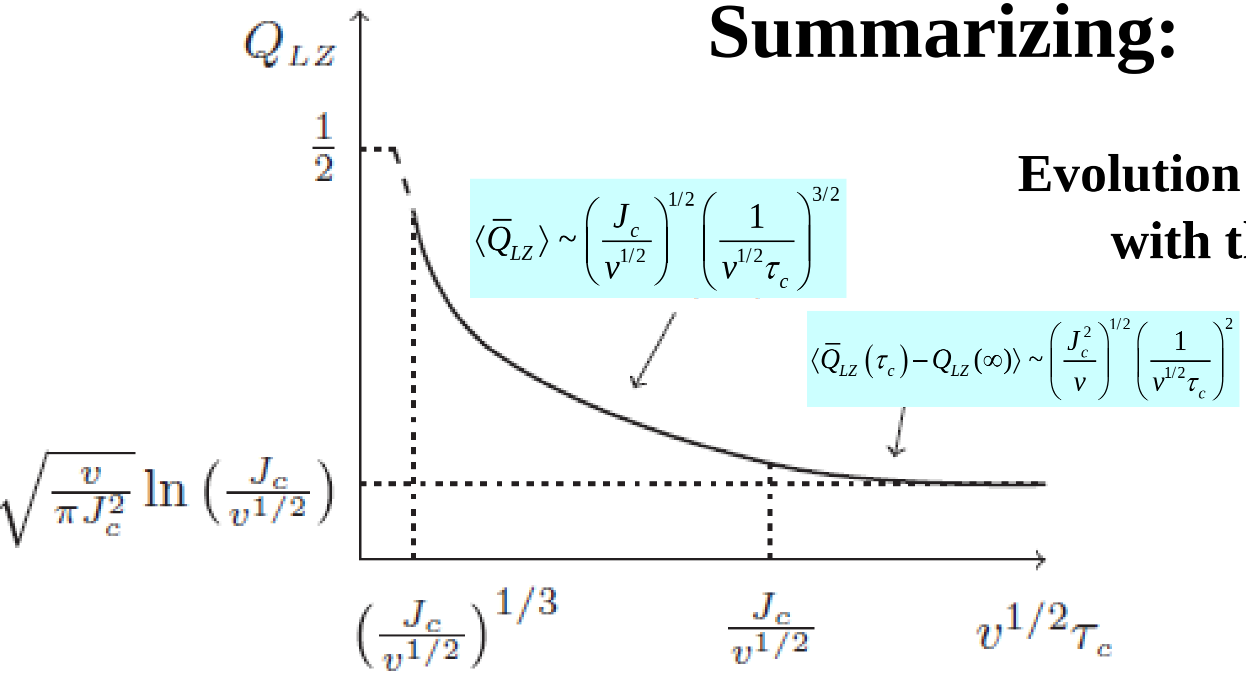
by a big logarithmic factor

Sub-leading correction:

$$\left\langle \bar{Q}_{LZ}(\tau_c) \right\rangle - \left\langle \bar{Q}_{LZ}(\infty) \right\rangle = 3\left(\frac{2}{\pi}\right)^{1/2} \left(\frac{J_c}{v^{1/2}}\right) \left(\frac{1}{v^{1/2}\tau_c}\right)^2$$

Summarizing:

Evolution of the survival probability with the noise correlation time



Fast noise: $Q_{LZ}(\tau_c \to 0) = \frac{1}{2}\left[1 + \exp\left(-\frac{4\pi J_c^2}{v}\right)\right]$

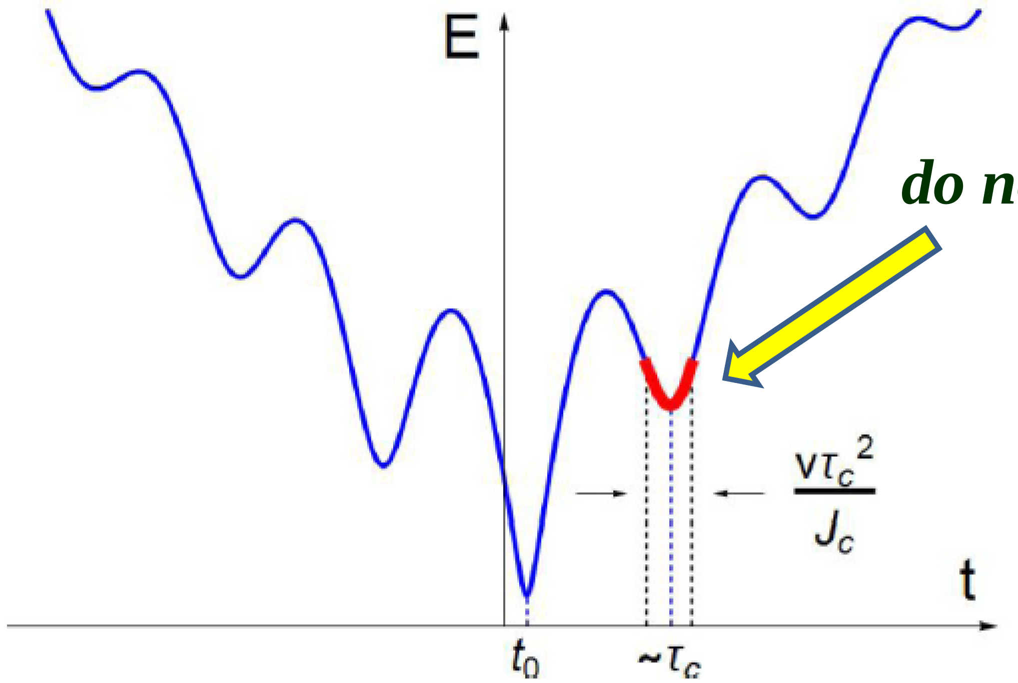
finite- τ_c correction originates from rare realizations when the switching does not take place during anomalously long time

Probability that the switching does not take place during the time τ_{LZ} is $\exp\left(-\frac{\tau_{LZ}}{\tau_c}\right)$

$$\frac{1}{2} - P_{LZ} = \frac{1}{2}\exp\left(-\frac{4\pi J_c^2}{v}\right) + \exp\left(-\frac{J_c}{v\tau_c}\right)$$

contribution from rare realizations

Other moments when coupling passes through zero



do not affect Q_{LZ}

FIG. 5. In the domain of correlation times $J_c^{1/3}/v^{2/3} \ll \tau_c \ll J_c/v$ a crossover between the slow-noise and the fast-noise regimes takes place. In this domain, time-dependent adiabatic levels acquire local minima due to the randomness of $J(t)$. The duration of the Landau-Zener transition (shown with red) in the vicinity of each minimum is much shorter than τ_c. The probability to remain on the same adiabatic level after the transition, given by Eq. (36), is close to 1 so that only the transition in the vicinity of $t = t_0$ is responsible for Q_{LZ}.

Correlator of the telegraph noise

easily made (Itakura and Tokura, 2003). The number k of switches that the fluctuator experiences within a time t follows a Poisson distribution

$$P_k(t) = \frac{(\gamma t)^k}{2^k k!} e^{-\gamma t/2}. \tag{44}$$

The number of switches k determines the number of times the function $\chi(t)$ changes its sign contributing $(-1)^k$ to the correlation function $C(t) \equiv \langle \chi(t)\chi(0) \rangle$. Therefore,

$$C(t) = e^{-\gamma t/2} \sum_{k=0}^{\infty} (-1)^k \frac{(\gamma t)^k}{2^k k!} = e^{-\gamma t}, \quad t \geq 0. \tag{45}$$