GOBIERNO DE ESPAÑA
MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES

CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

Non-equilibrium quantum dynamics & conservation laws

Rafael Molina(IEM-CSIC), Jordi Mur-Petit(Oxford), Armando Relaño(UCM), Dieter Jaksch(Oxford)

Nature Communications 9, 2006 (2018)

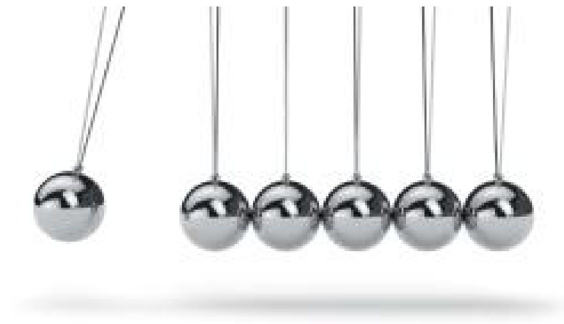
UNIÓN EUROPEA
Fondo Europeo de Desarrollo Regional

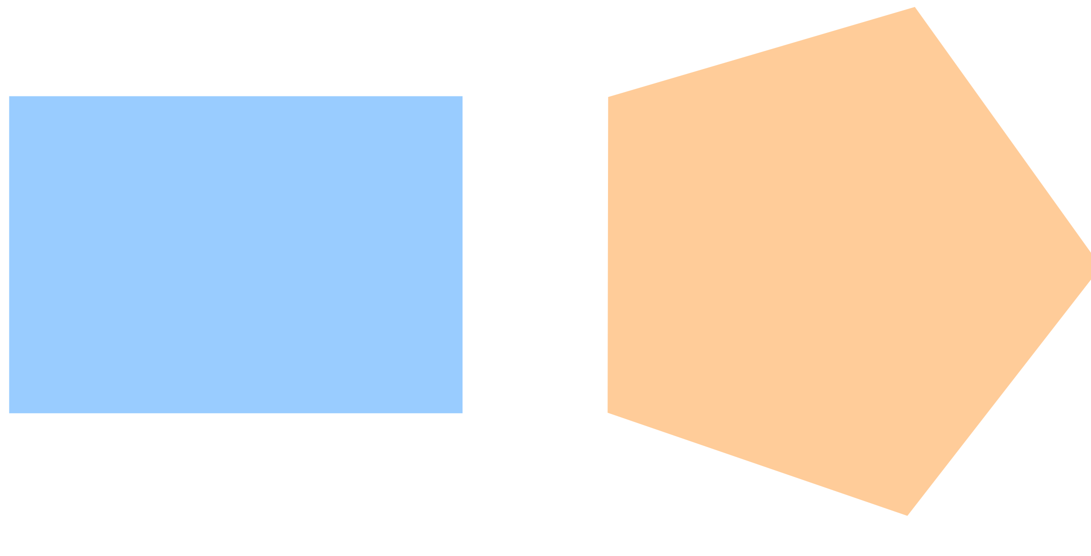
Outline

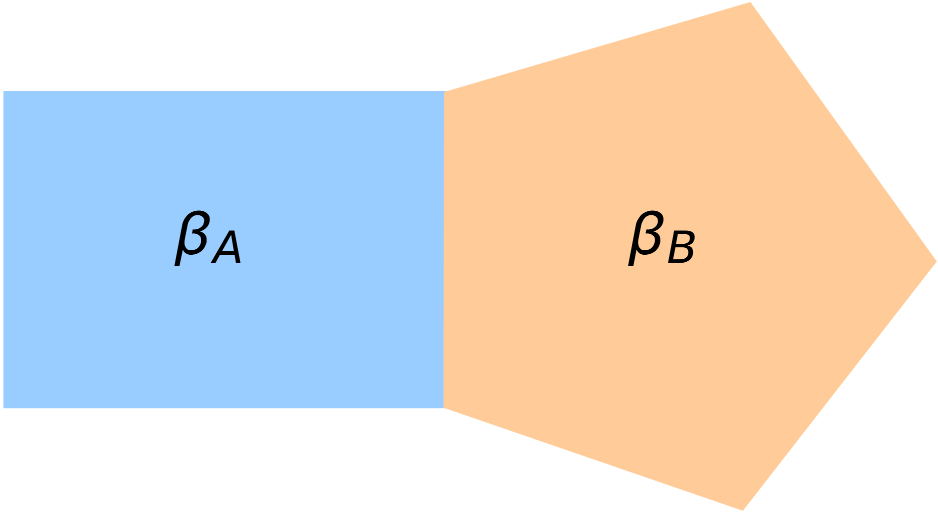
▶ Motivation: Integrability vs. relaxation

● QFRs for systems with charges

● Outlook







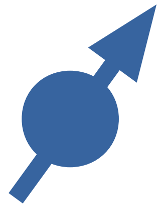
Equilibrium:

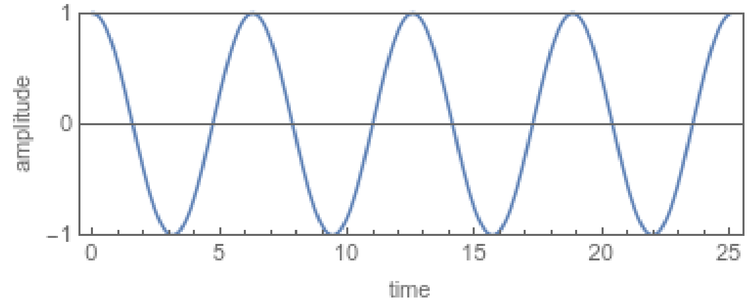
- $\exists \beta_A, \beta_B$
- $\beta_A = \beta_B$
- $\rho_\beta \propto \exp(-\beta H)$ (Gibbs)

0^{th} Law: "*Temperature exists*"

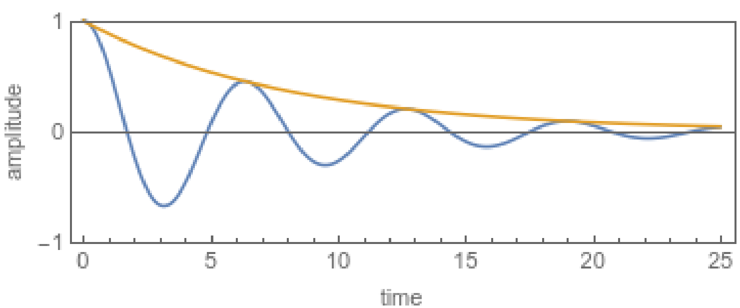
Relaxation in quantum systems?

Single isolated spin:
Bloch oscillations

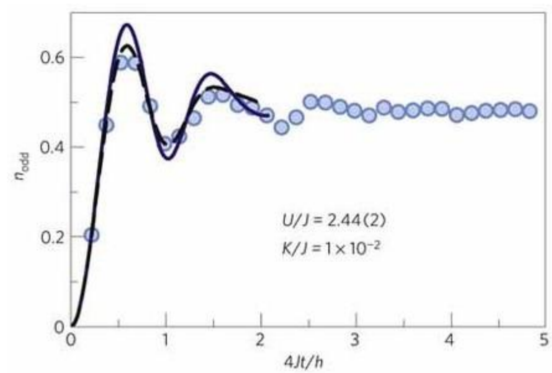




Coupling to environment:
Noise



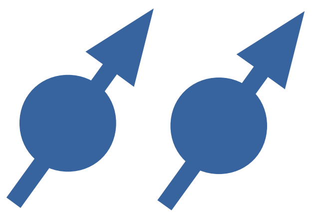
Many-body (closed):
ETH



Trotzky et al., Nat. Phys. (2012)

Known exceptions to relaxation

Decoherence free subspaces



$$|0\rangle|0\rangle \longrightarrow |0\rangle|0\rangle$$

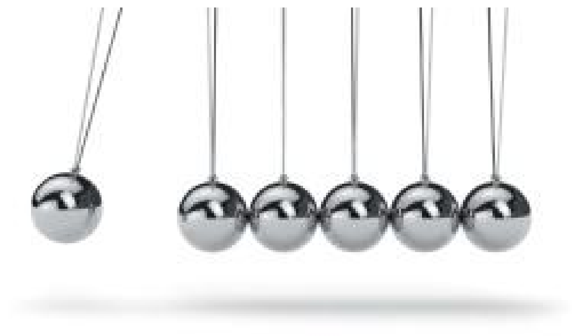
$$|0\rangle|1\rangle \longrightarrow e^{i\phi}|0\rangle|1\rangle$$

$$|1\rangle|0\rangle \longrightarrow e^{i\phi}|1\rangle|0\rangle$$

$$|1\rangle|1\rangle \longrightarrow e^{2i\phi}|1\rangle|1\rangle$$

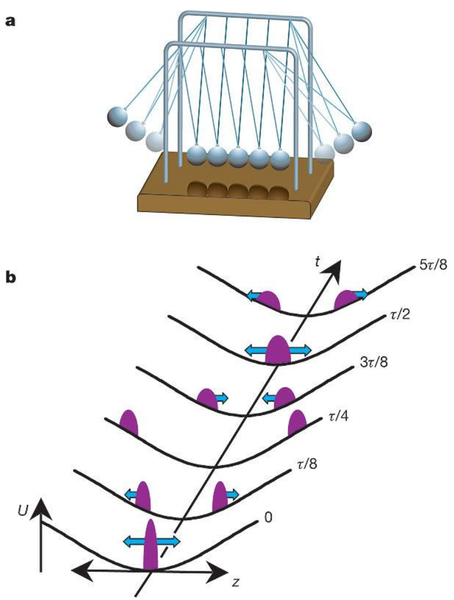
Reservoir engineering

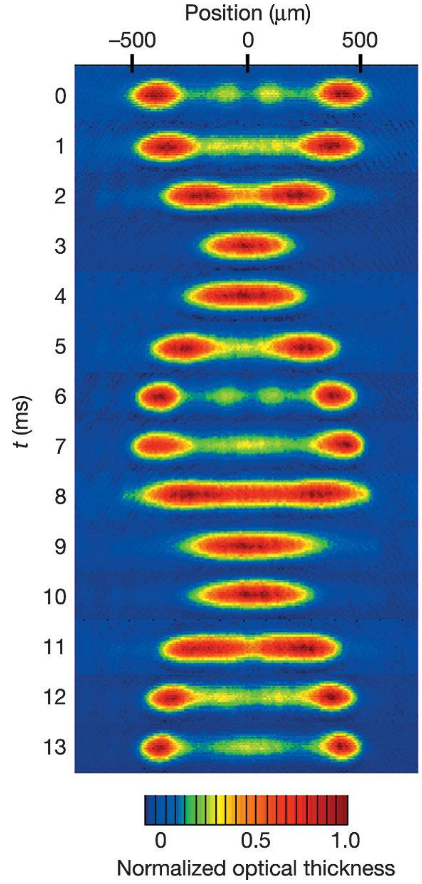
Integrable systems



Integrable systems

Strongly interacting bosons in 1D

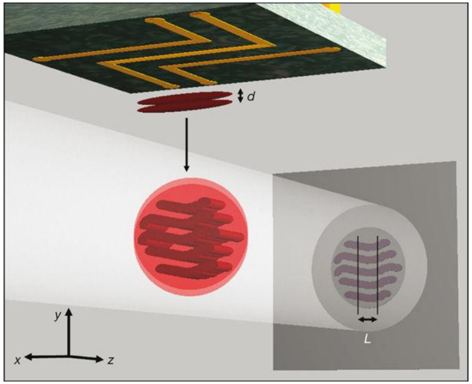


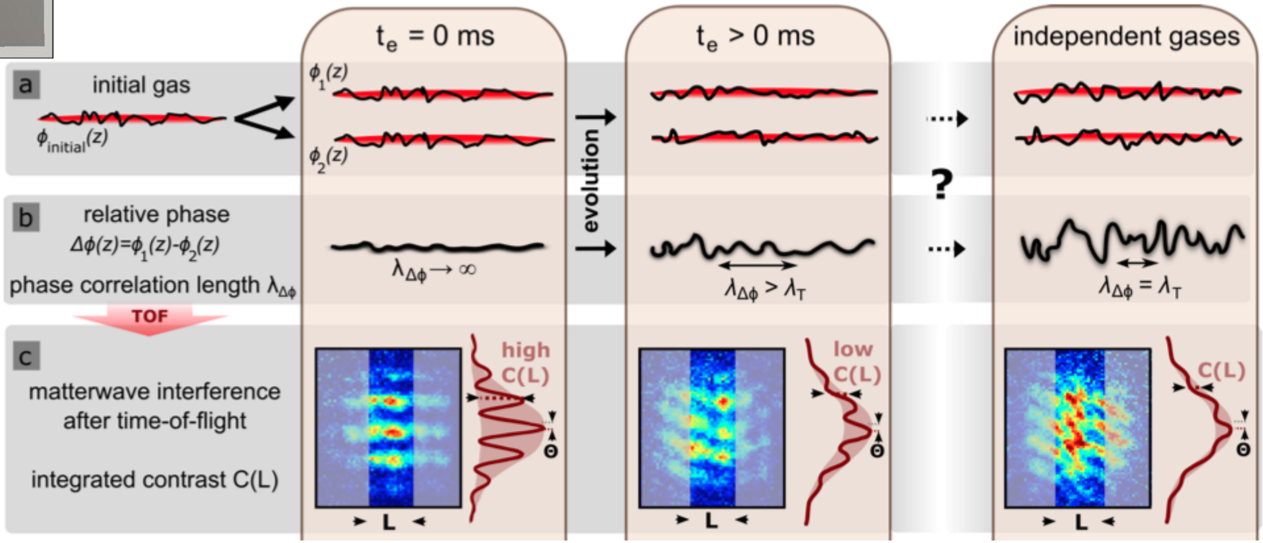


Kinoshita et al., Nature (2006)

Relaxation in an integrable system

Splitting a strongly interacting 1D Bose gas: Relaxation?



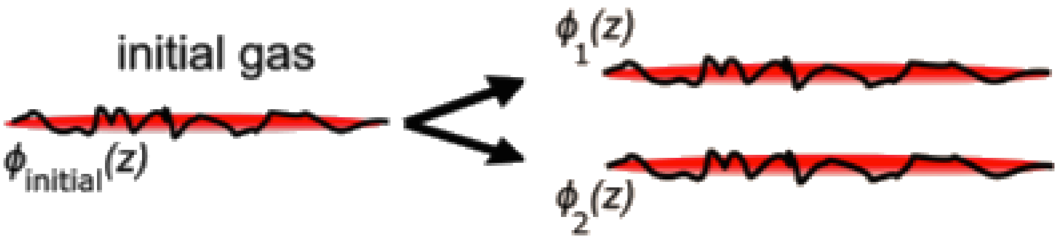


Hofferberth et al., Nature (2007); Gring et al., Science (2012); Langen et al., Science (2015) (Schmiedmayer lab)

Relaxation in an integrable system

Split a 1D gas non-adiabatically

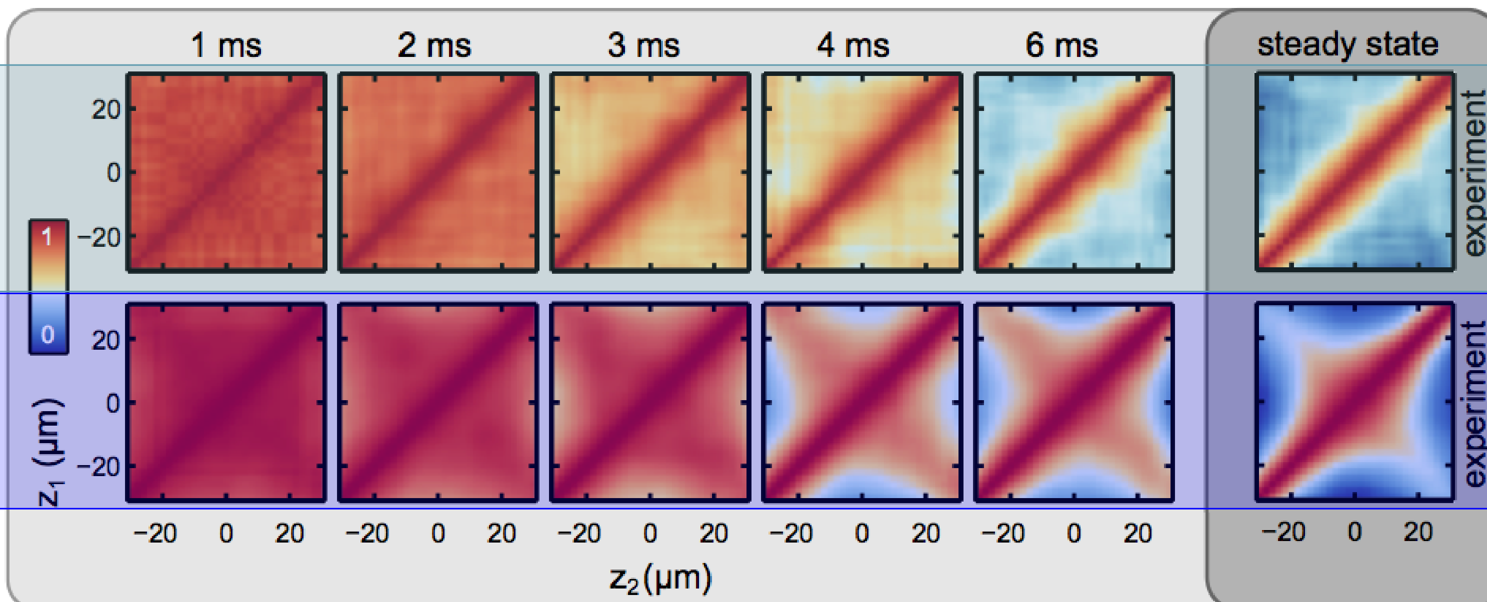
Gring et al., Science (2012)
Langen et al., Science (2015)



Two-point correlation function vs. splitting rate:

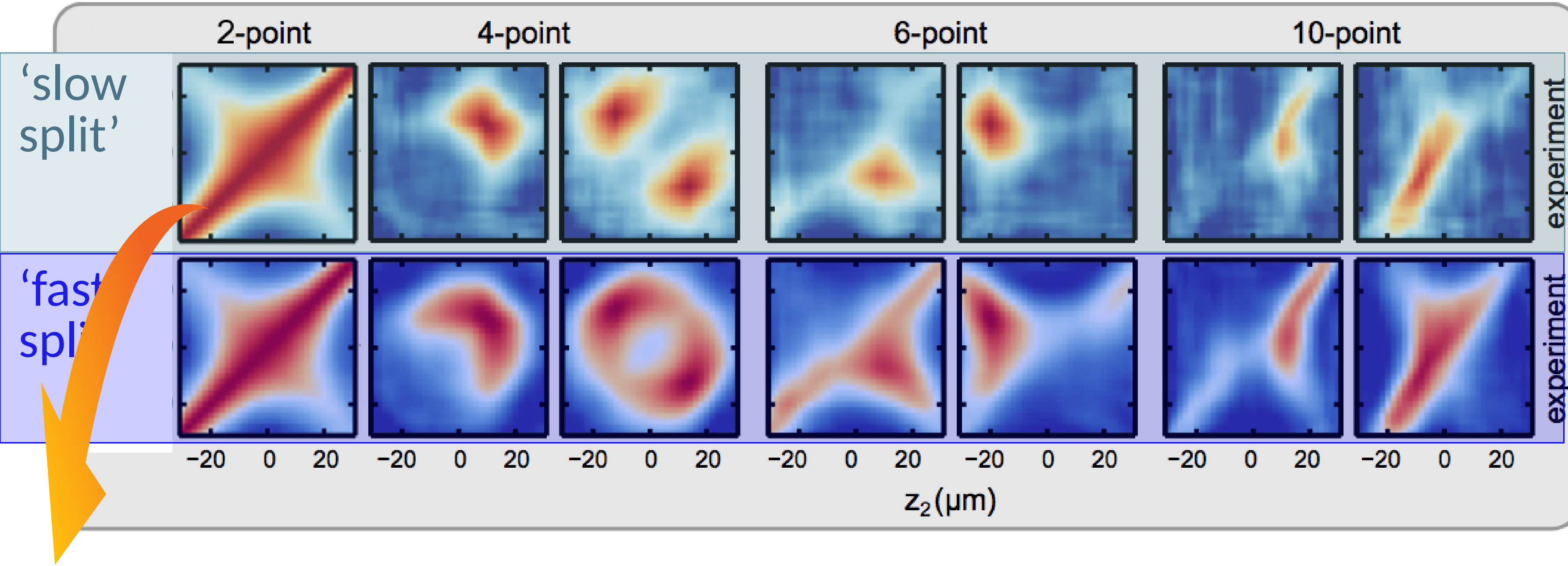
‘slow split’

‘fast split’



Relaxation in an integrable system

$$C(z_1, \ldots, z_N) = \langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_1^\dagger(z_N)\Psi_2(z_N)\rangle$$



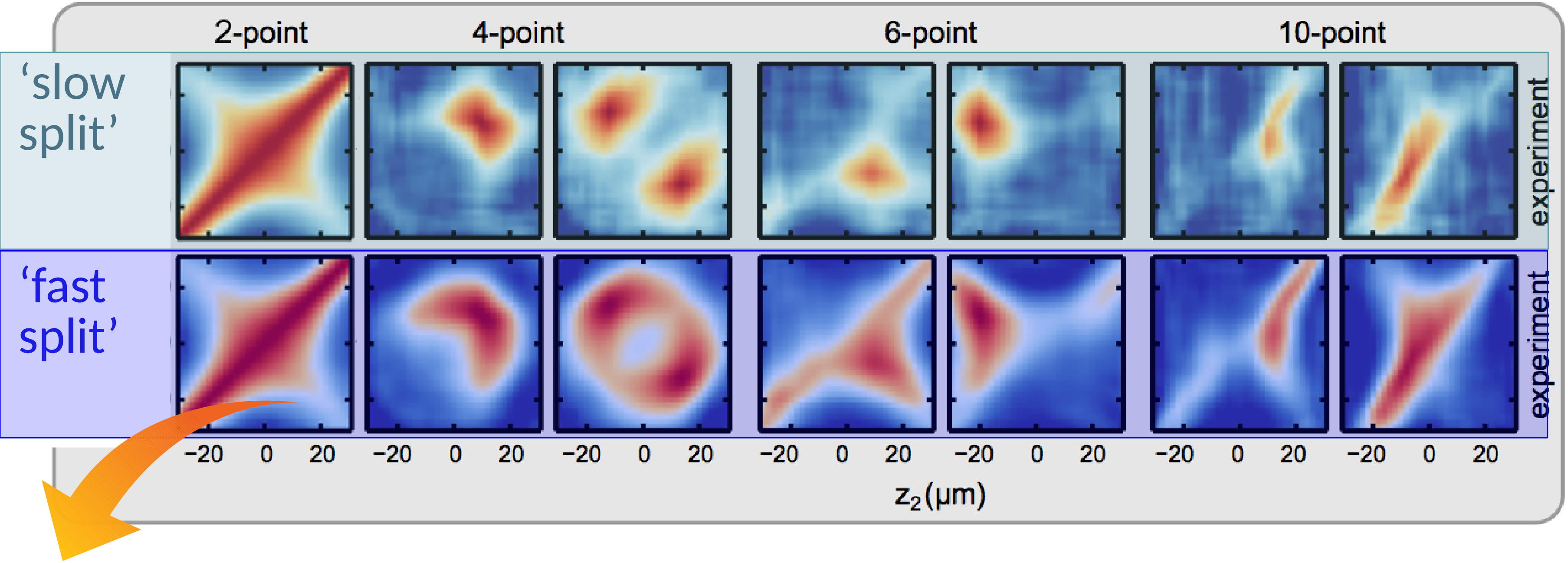
Slow split: Correlations match **Gibbs ensemble** with effective *T*=1/β

$$\hat{\rho}(t \to \infty) \to \hat{\rho}_\beta = \exp(-\beta H)/Z$$

Gring et al., Science (2012)

Beyond Gibbs

$$C(z_1, \ldots, z_N) = \langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_1^\dagger(z_N)\Psi_2(z_N)\rangle$$



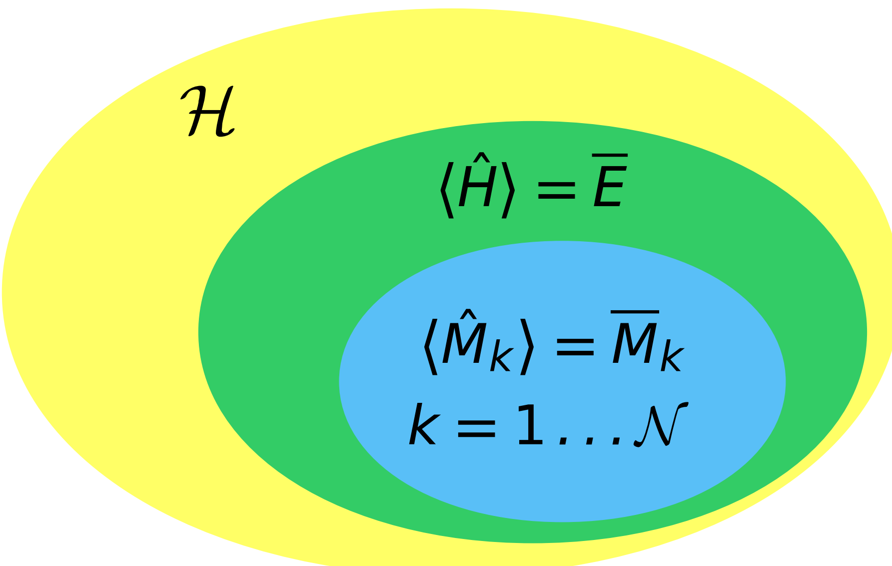
Fast split: Up to 10 different 'temperatures' to match!

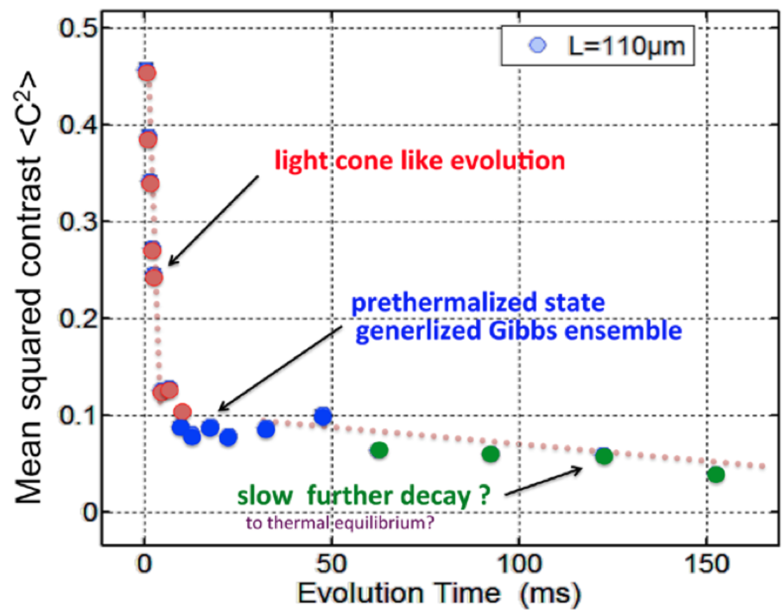
$$\hat{\rho}(t \to \infty) \to \boxed{\hat{\rho}_{\mathrm{GGE}} = \frac{1}{Z}\exp(-\beta\hat{H} - \sum_k \beta_k \hat{M}_k)}\ ,\quad [\hat{M}_k, \hat{H}] = 0$$

Langen et al., Science **348** 207 (2015)

Pre-thermalised states

$$\hat{\rho}(t \to \infty) \to \boxed{\hat{\rho}_{\text{GGE}} = \frac{1}{Z} \exp\left(-\beta \hat{H} - \sum_k \beta_k \hat{M}_k\right)} \ , \ \ [\hat{M}_k, \hat{H}] = 0$$

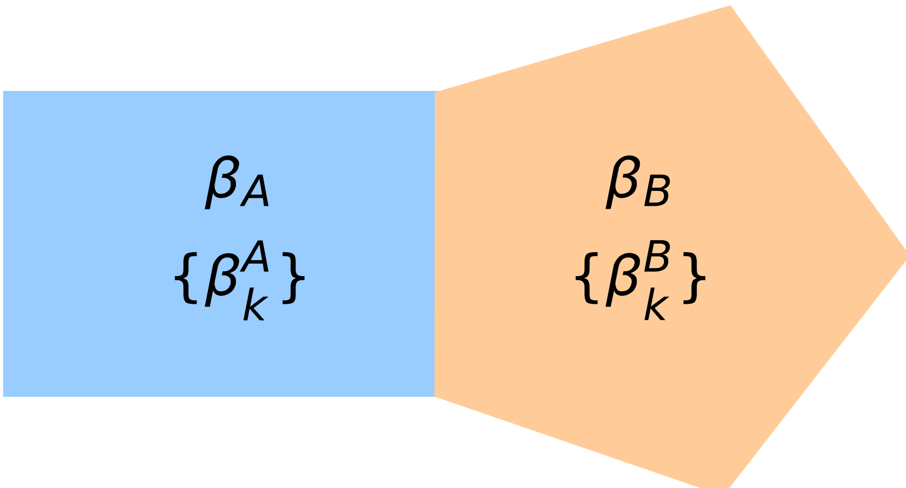


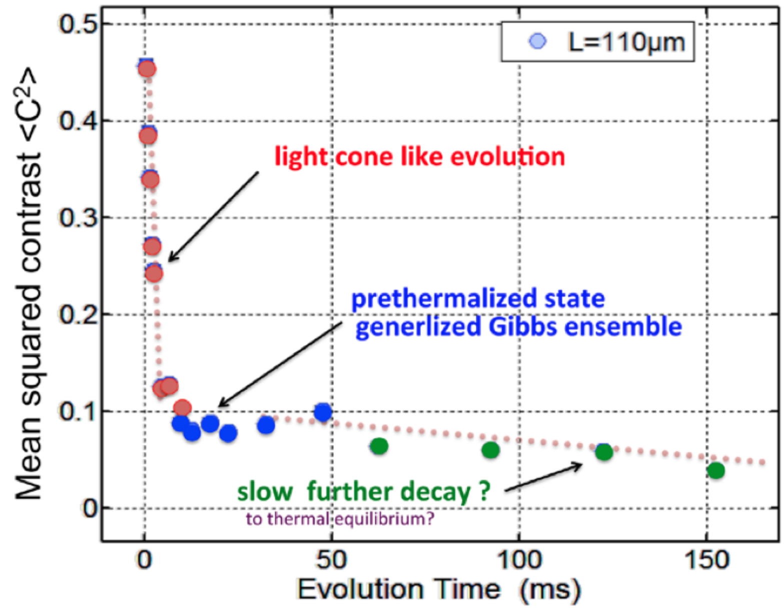


- **Generalized Gibbs ensemble (GGE)**
- Conserved charges prevent relaxation to ‘true’ thermal equilibrium: **→ Pre-thermalised state**

Pre-thermalised states

$$\hat{\rho}(t \to \infty) \to \boxed{\hat{\rho}_{\text{GGE}} = \frac{1}{Z}\exp(-\beta \hat{H} - \sum_k \beta_k \hat{M}_k)} \;,\; [\hat{M}_k, \hat{H}] = 0$$





- **Generalized Gibbs ensemble (GGE)**
- Conserved charges prevent relaxation to 'true' thermal equilibrium: **→ Pre-thermalised state**

Some open questions

$$\hat{\rho}(t \to \infty) \to \boxed{\hat{\rho}_{\mathrm{GGE}} = \frac{1}{Z} \exp(-\beta \hat{H} - \sum_k \beta_k \hat{M}_k)} \;,\; [\hat{M}_k, \hat{H}] = 0$$

- Given $\hat{H}$ how do we identify *all* the ‘relevant’ charge operators $\hat{M}_k$?
- Is it possible to design a general protocol to know if we’re missing any?
- Can we find them from experimental measures? How?
- How do they affect the evolution of a system, e.g., after a quench that breaks integrability?

…

Outline

- ☑ Motivation: Integrability vs. relaxation
- ▶ Thermodynamics and fluctuations
- Generalised QFRs: Dicke model
- Outlook

Fluctuation Relations (classical)

Thermodynamic Laws (1824-)

$$w \geq \Delta F \,, \quad \Delta S \geq 0$$

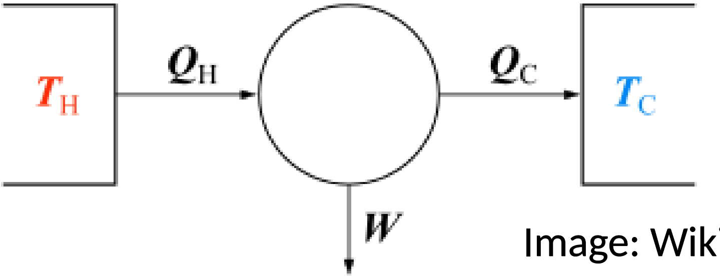


Image: Wikipedia

Fluctuation Relations (classical)

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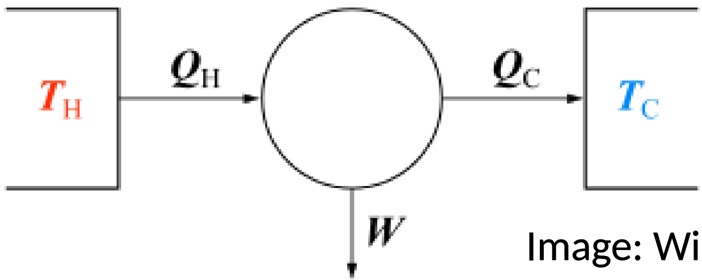


Image: Wikipedia

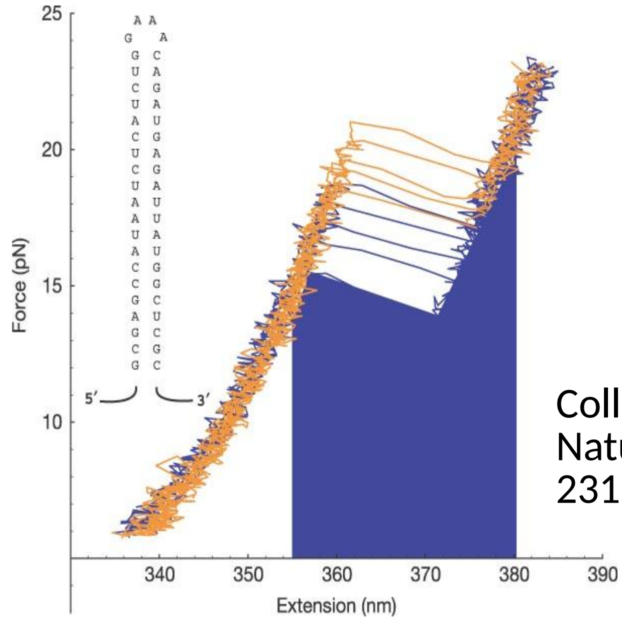
Fluctuation Relations (1993-)

Jarzynski equality

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$

Crooks relation

$$P_f(w) = e^{\beta(w - \Delta F)} P_b(-w)$$



Collin et al., Nature **437** 231 (2005)

Constrain PDF:

$$P(w) \to \langle w \rangle \,, \langle w^2 \rangle \,, \langle w^3 \rangle \,, \ldots$$

- Obtain equilibrium properties from non-equil. measurements

Fluctuation Relations (quantum)

Thermodynamic Laws (1824-)

$$w \geq \Delta F \,, \quad \Delta S \geq 0$$

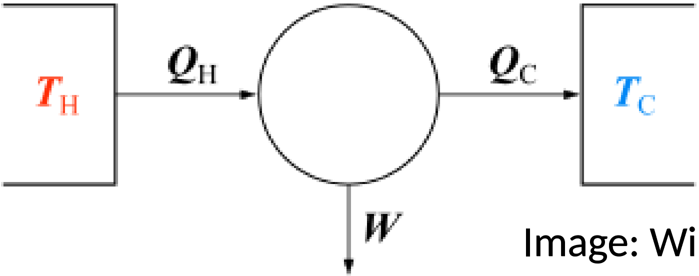


Image: Wikipedia

Fluctuation Relations (1993-)

Quantum FRs (1999-)

Jarzynski equality

Quantum Jarzynski equality (QJE)

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$

Crooks relation

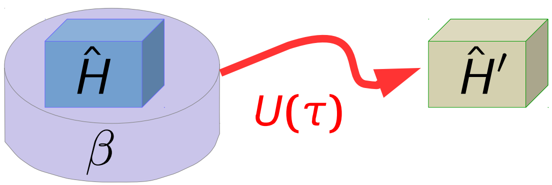
Tasaki-Crooks relation (TCR)

$$P_f(w) = e^{\beta(w - \Delta F)} P_b(-w)$$

QJE: Tasaki (2000), Kurchan (2000), Yukawa (2000), Mukamel (2003), DeRoeck & Maes (2004)
TCR: Tasaki (2000), Monnai (2005)

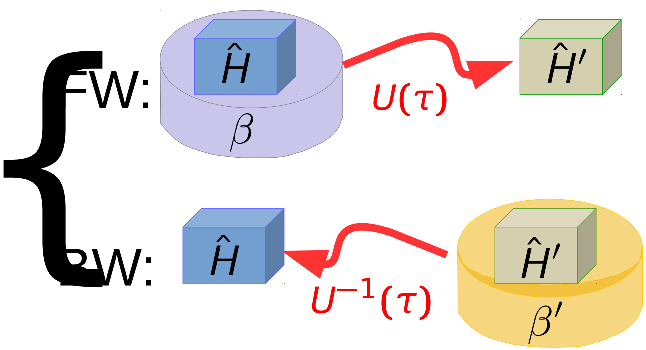
Fluctuation Relations (quantum)

Quantum FRs (1999-)



Quantum Jarzynski equality (QJE)

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$

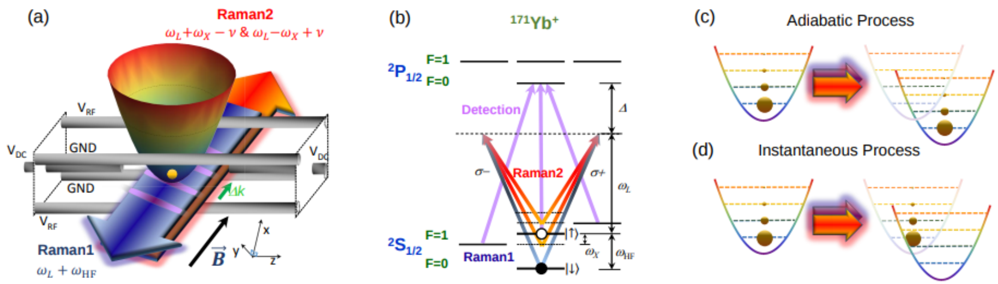


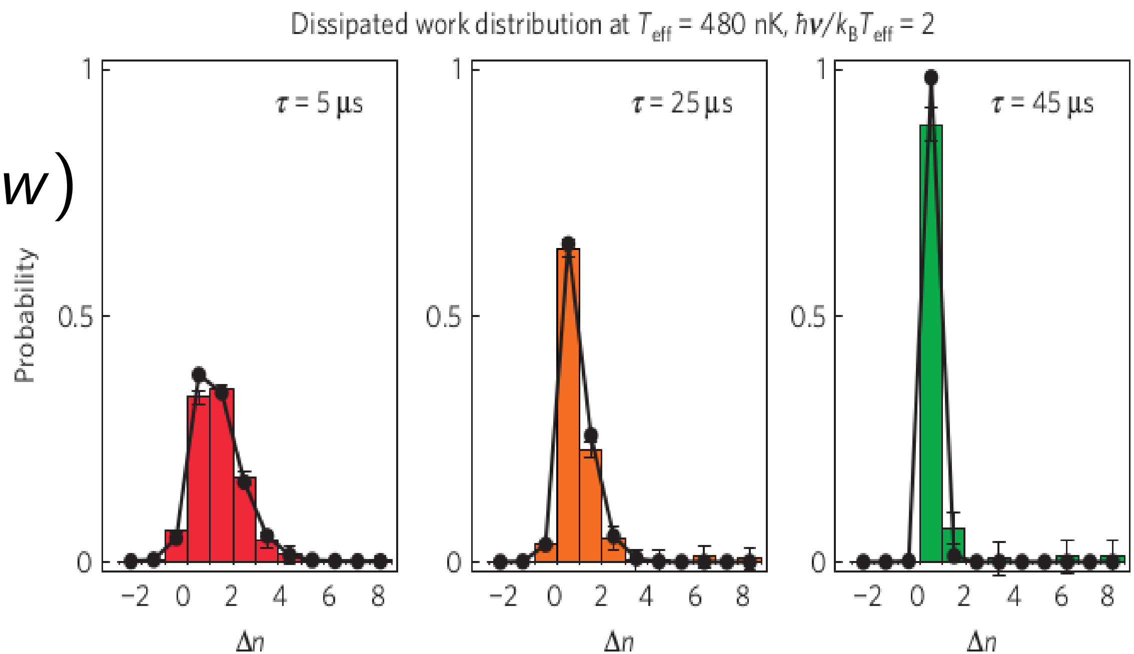
Tasaki-Crooks relation (TCR)

$$P_f(w) = e^{\beta(w-\Delta F)} P_b(-w)$$

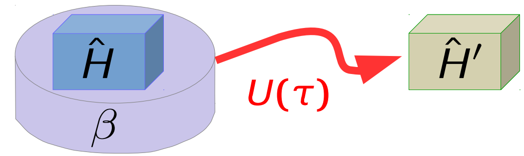
Testing the QJE

An et al., Nat. Phys. **11** 193 (2015) [one trapped ion]





$P(w)$



$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$

Bars: expt. data
Lines: analytical calc.

$$\Delta n \propto w$$

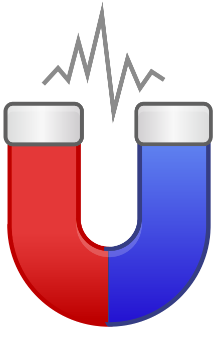
Fast quench ⟷ Slower quench

Outline

☑ Motivation: Integrability vs. relaxation

☑ Thermodynamics and fluctuations

▶ Generalised QFRs: Dicke model

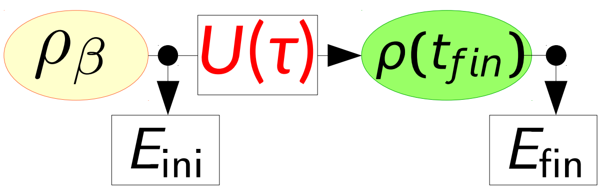


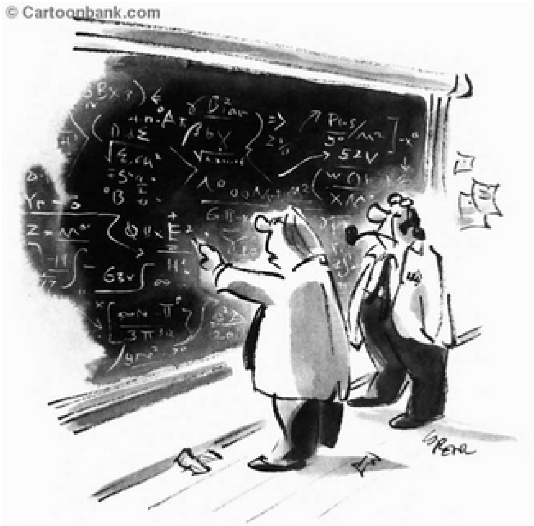
- Outlook

QFRs: The small print

(i) Work defined via
two energy-projection measurements

$$w = E_{\text{fin}} - E_{\text{ini}}$$
$$= \text{Tr}[U\rho_\beta U^{-1} H_{\text{fin}}] - \text{Tr}[\rho_\beta H_{\text{ini}}]$$





"Here's your problem—you forgot the sleaze factor."

Talkner, Hänggi et al., J. Phys. A **40** F569 (2007)
Talkner & Hänggi, PRE **93** 022131 (2016)

QFRs: The small print

(i) Work defined via
two energy-projection measurements

$$w = E_{\mathrm{fin}} - E_{\mathrm{ini}}$$
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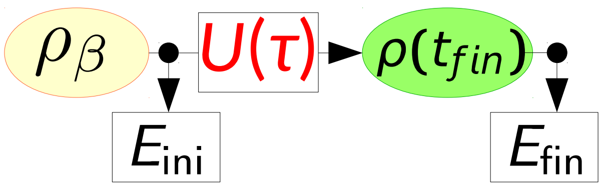
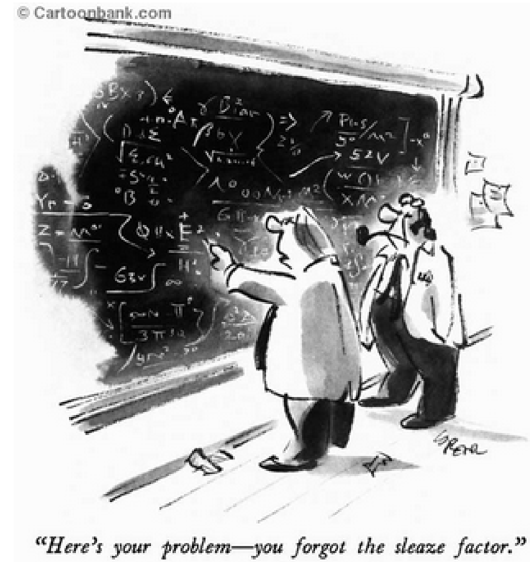
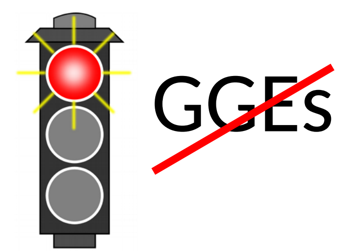




(ii) Initial state: canonical (Gibbs) equilibrium state

$$\rho(t=0) = \rho_\beta = \frac{1}{Z}\exp[-\beta H_{\mathrm{ini}}]$$



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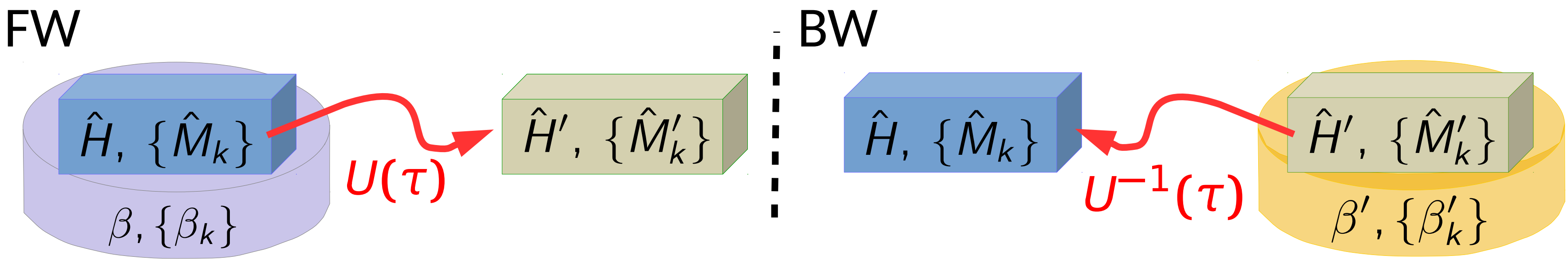
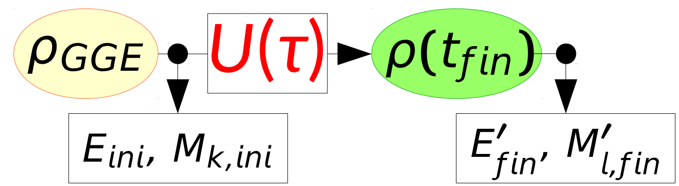
QFRs for GGEs

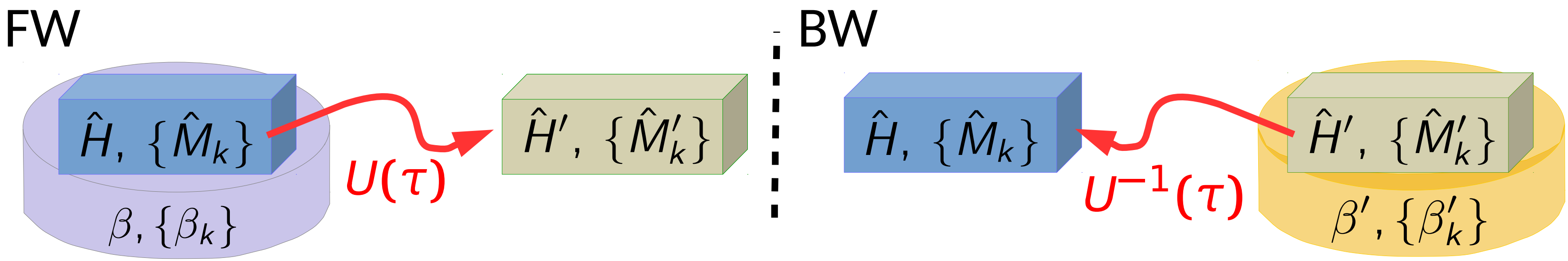


(Use two projective measurements:



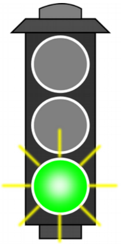
)

QFRs for GGEs



$$\langle e^{-W} \rangle = e^{-\Delta F}$$ Generalised QJE

$$P_{FW}(W) = e^{W-\Delta F} P_{BW}(-W)$$ Generalised TCR



GGEs!

$$A_{\text{ini}} = \beta E_{\text{ini}} + \sum_k \beta_k M_{k,\text{ini}}, \quad A_{\text{fin}} = \beta' E'_{\text{fin}} + \sum_l \beta'_l M'_{l,\text{fin}}$$

$$w = E_{\text{fin}} - E_{\text{ini}} \mapsto W = A_{\text{fin}} - A_{\text{ini}}$$ *Generalised work*

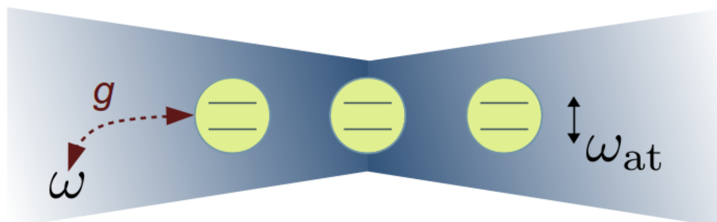
Hickey & Genway, PRE 2014;
Guryanova et al.; Yunger Halpern et al., Nat. Comms. (2016)

J.Mur-Petit, A. Relaño, RAM & D. Jaksch, Nat. Comms. (2018)

Testing ground: Dicke model

$$\hat{H} = \hbar\omega_{\text{com}}\hat{a}^\dagger\hat{a} + \hbar\omega_{\text{at}}\hat{J}_z + \hat{H}_{\text{int}}\,, \qquad \hat{J}_{x,y,z} = \sum_{j=1}^{N}\frac{1}{2}\sigma_{x,y,z}^{(j)}$$

$$\hat{H}_{\text{int}} = \frac{2g}{\sqrt{N}}\left[(1-\alpha)(\hat{J}_+\hat{a} + \hat{J}_-\hat{a}^\dagger) + \alpha(\hat{J}_+\hat{a}^\dagger + \hat{J}_-\hat{a})\right]$$



Two phases $g_{cr} = \sqrt{\omega\omega_{\text{at}}}/2$

- $g > g_{cr}$ → Superradiant
- $g < g_{cr}$ → Subradiant

Two regimes

- ***α*={0, 1}** → Integrable (TCM)

$$\hat{M} = \hat{J} + \hat{J}_z + \hat{a}^\dagger\hat{a}$$

- Otherwise → Not integrable

Testing ground: Dicke model

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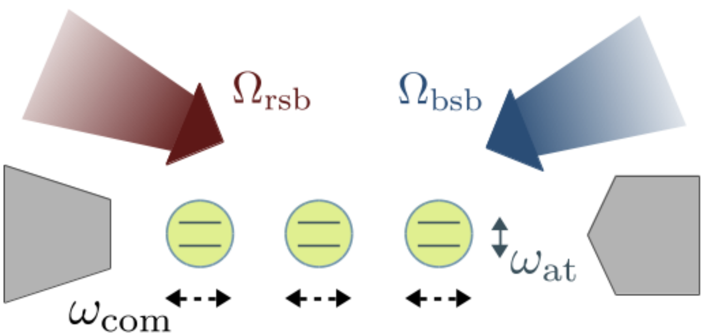


$$g = (\Omega_{\text{rsb}} + \Omega_{\text{bsb}})/2$$

$$\alpha = \Omega_{\text{bsb}}/(\Omega_{\text{rsb}} + \Omega_{\text{bsb}})$$

Two phases $g_{\text{cr}} = \sqrt{\omega\omega_{\text{at}}}/2$

- $g>g_{cr}$ → Superradiant
- $g<g_{cr}$ → Subradiant

Two regimes

- ***α*={0, 1}** → Integrable (TCM)

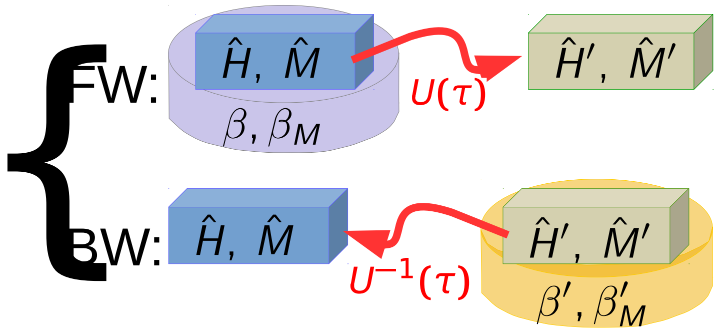
$$\hat{M} = \hat{J} + \hat{J}_z \pm \hat{a}^\dagger\hat{a}$$

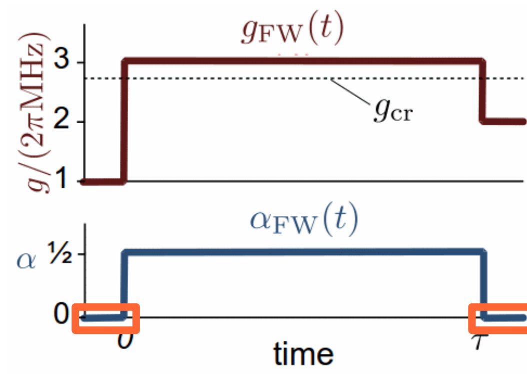
- Otherwise → Not integrable

Dicke model – Results

1. Prepare system in GGE state
2. Non-equilibrium protocol: quench α
3. Repeat many times to collect statistics of work

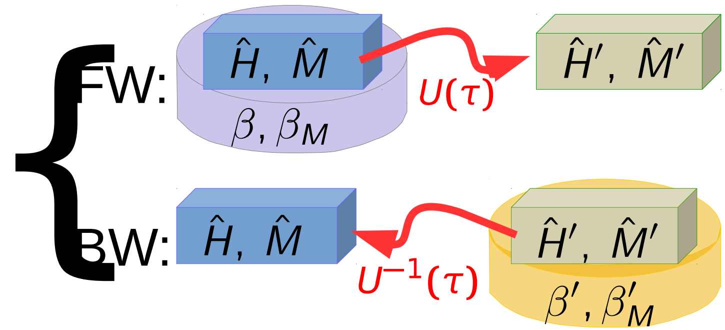
Repeat with time-reversed protocol (BW).



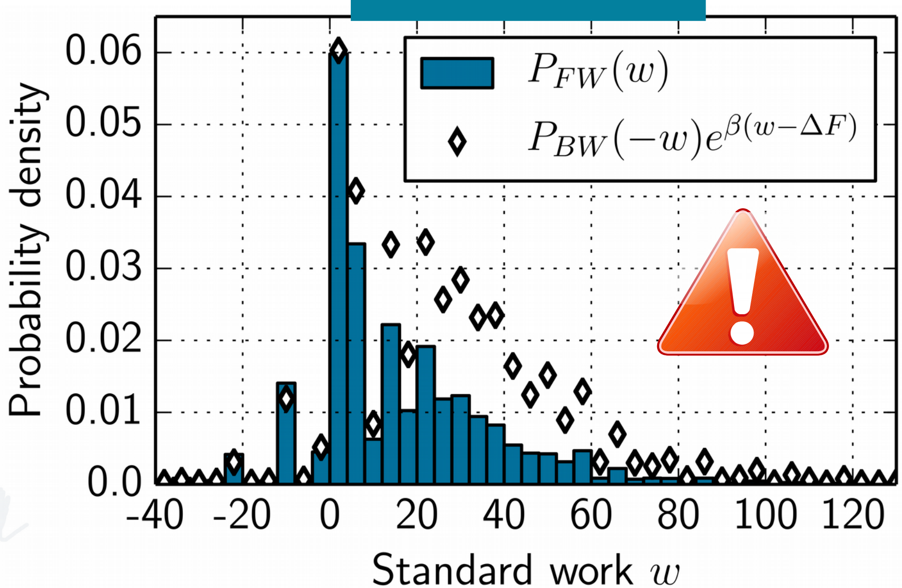


Dicke model: TCR for τ≈1 μs

$$(*)\; P_{FW}(w) = e^{\beta(w-\Delta F_{Gibbs})} P_{BW}(-w)$$



Standard TCR

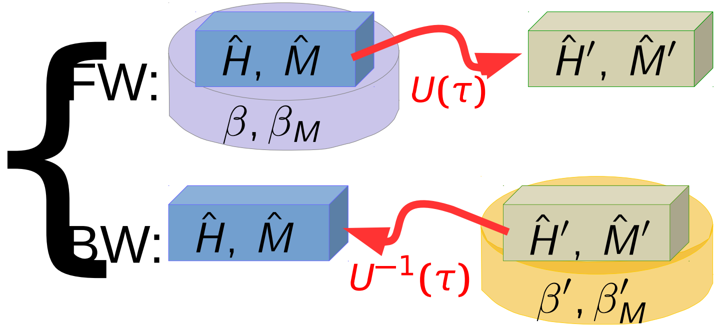


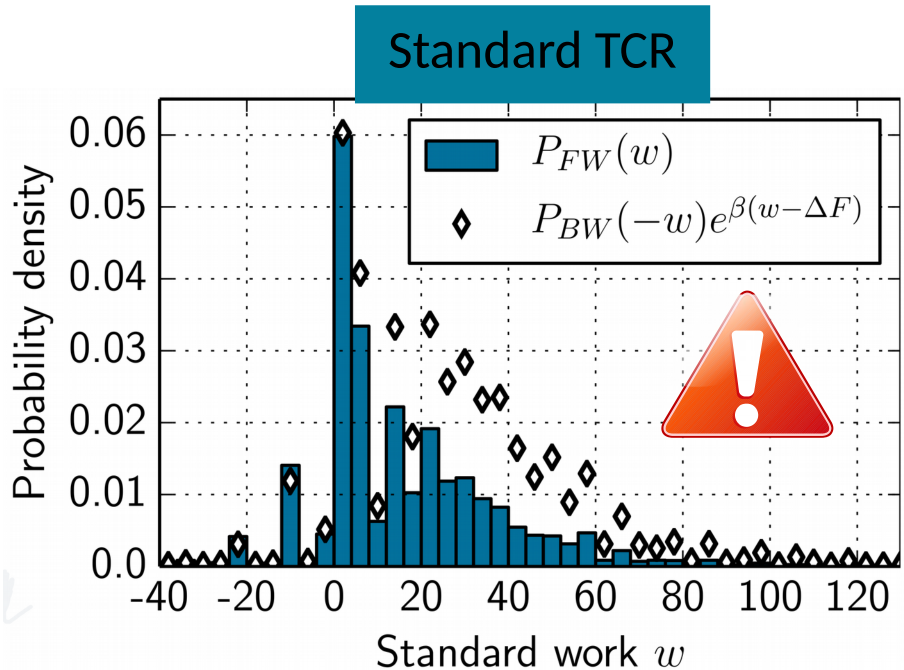
$$\rho_{ini} = \rho_{GGE}(\beta = 0.1, \beta_M = -0.1)$$

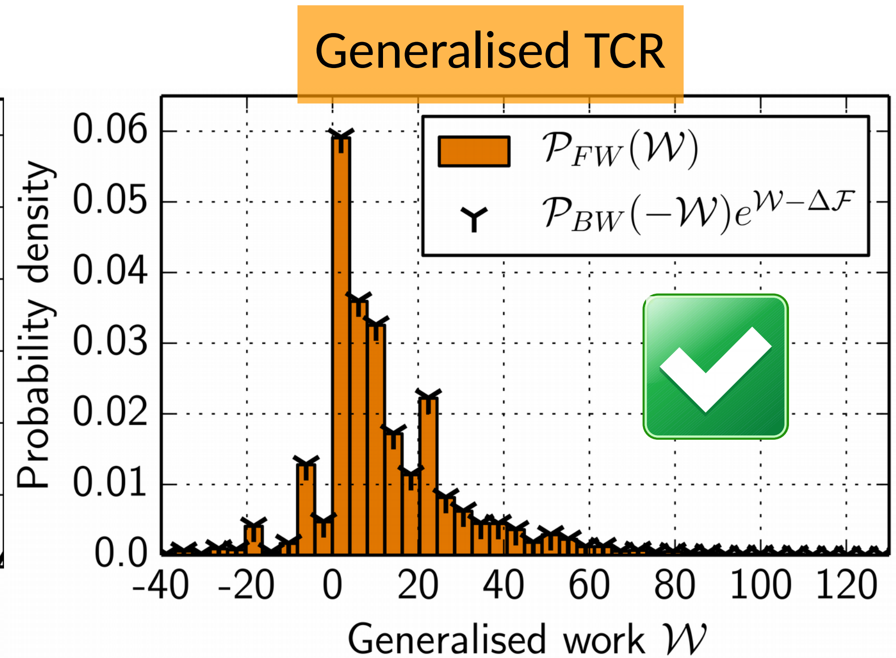
- Detect if ρ_{ini} is missing charges
- Beware wrong estimates of $\beta, \Delta F$

Dicke model: TCR for τ≈1 μs

$(*)\ P_{FW}(w) =$	$e^{\beta(w-\Delta F_{Gibbs})} P_{BW}(-w)$
$P_{FW}(W) =$	$e^{W-\Delta F_{GGE}} P_{BW}(-W)$





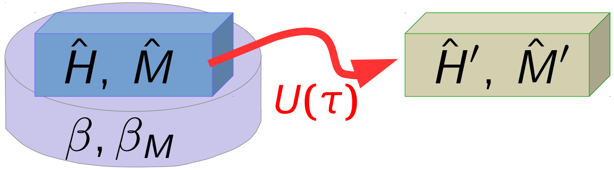


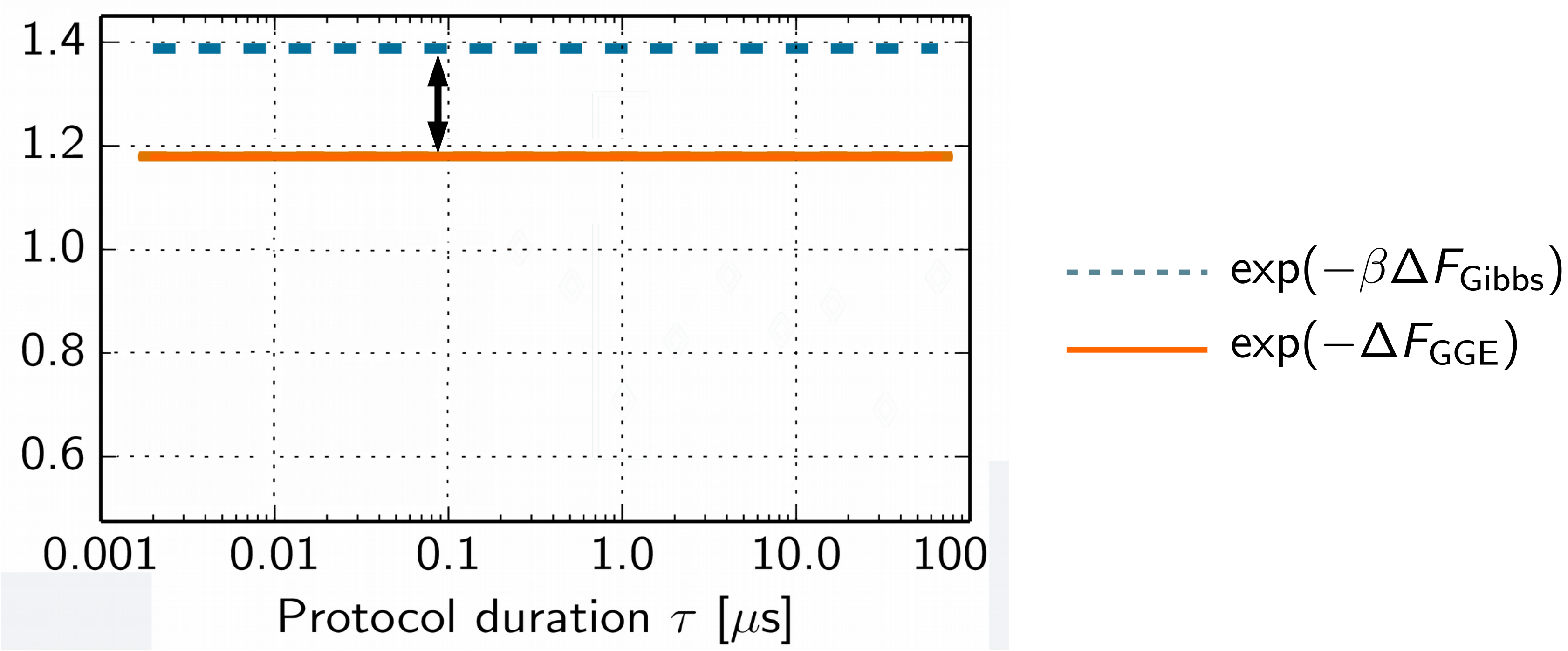
$\rho_{ini} = \rho_{GGE}(\beta = 0.1, \beta_M = -0.1)$

QJE: Varying protocol duration τ

– std: $\langle \exp(-\beta w) \rangle = \exp(-\beta \Delta F_{\text{Gibbs}})$

– gen: $\langle \exp(-W) \rangle = \exp(-\Delta F_{\text{GGE}})$





$\beta = 0.1,\ \beta_M = 0.3$

QJE: Varying protocol duration τ

- std: $\langle \exp(-\beta w) \rangle = \exp(-\beta \Delta F_{\text{Gibbs}})$
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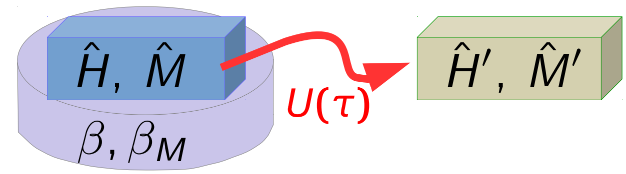

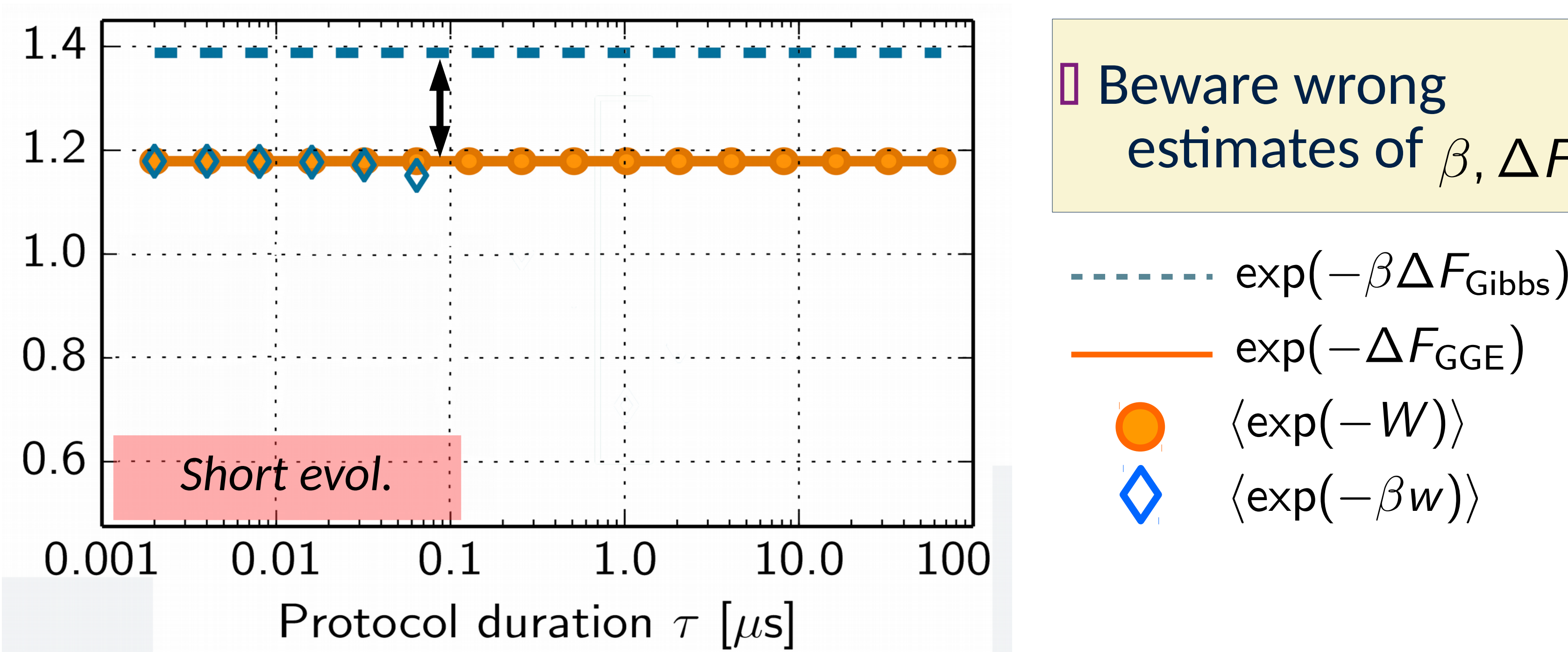

- Beware wrong estimates of $\beta, \Delta F$

$\beta = 0.1,\ \beta_M = 0.3$

QJE: Varying protocol duration τ

– std: $\langle \exp(-\beta w) \rangle = \exp(-\beta \Delta F_{\text{Gibbs}})$

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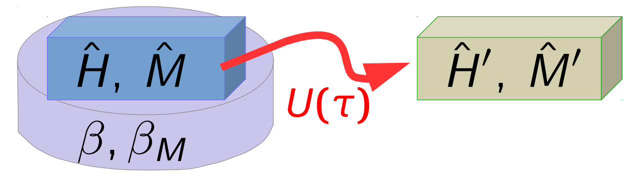
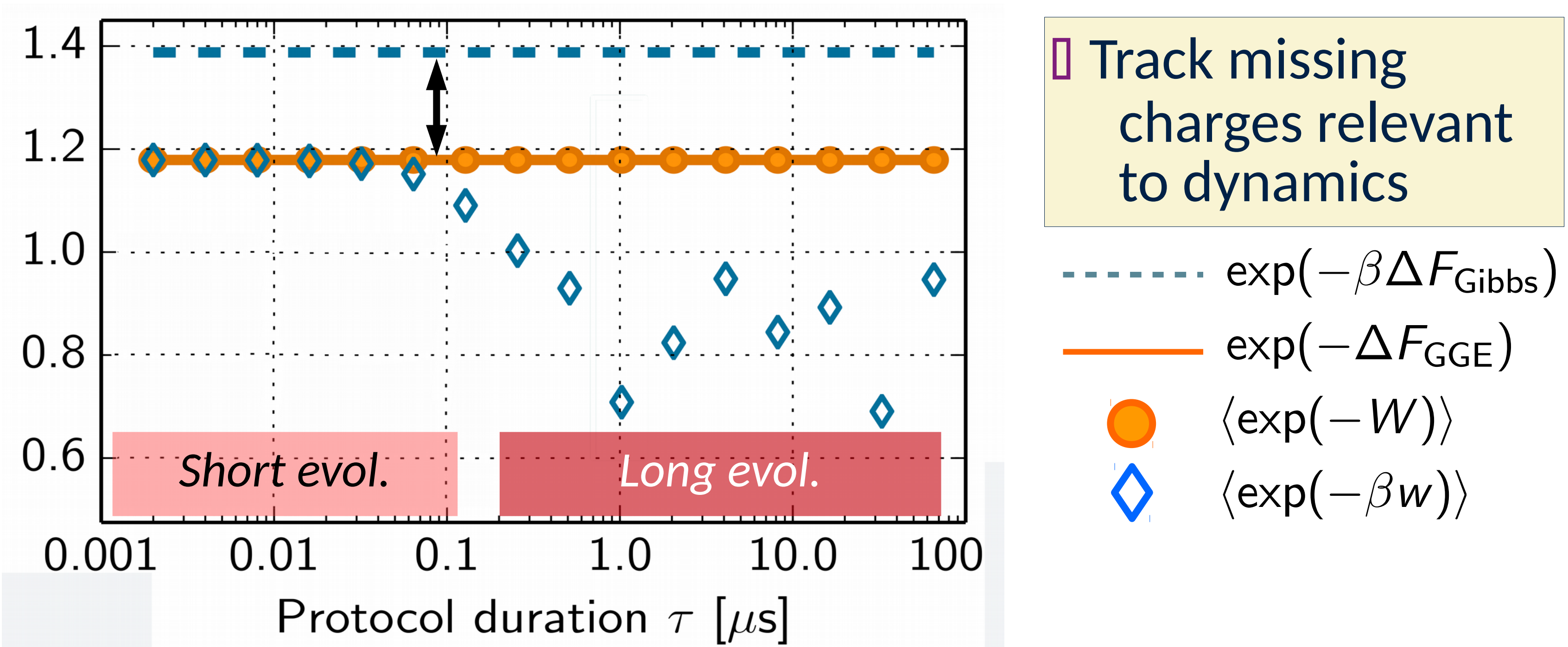




Track missing charges relevant to dynamics

$\beta = 0.1, \ \beta_M = 0.3$

Some open questions

$$\hat{\rho}(t \to \infty) \to \boxed{\hat{\rho}_{\mathrm{GGE}} = \frac{1}{Z}\exp(-\beta\hat{H} - \sum_k \beta_k \hat{M}_k)} \; , \; [\hat{M}_k, \hat{H}] = 0$$

- Given $\hat{H}$ how do we identify *all* the 'relevant' charge operators $\hat{M}_k$?

☑ Is it possible to design a general protocol to know if we're missing any?

☑ Can we find them from experimental measures? How?

☑ How do they affect the evolution of a system, e.g., after a quench that breaks integrability?

Outline

☑ Motivation: Integrability vs. relaxation

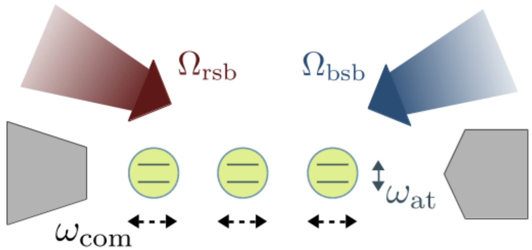
☑ Thermodynamics and fluctuations

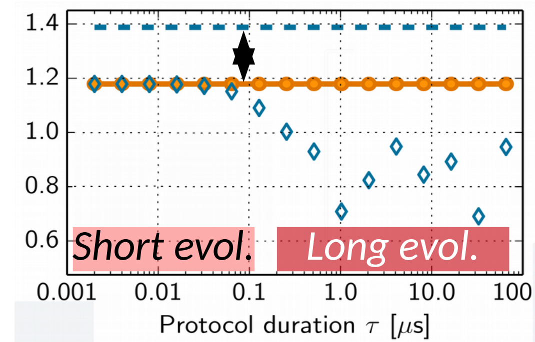
☑ Generalised QFRs: Dicke model

▶ Outlook

Outlook

☑ Dicke





▶ XXZ

$$\hat{H} = \sum_{\langle i,j \rangle} \left[J_x \left(\sigma_{i,x}\sigma_{j,x} + \sigma_{i,y}\sigma_{j,y} \right) + J_z \sigma_{i,z}\sigma_{j,z} \right]$$

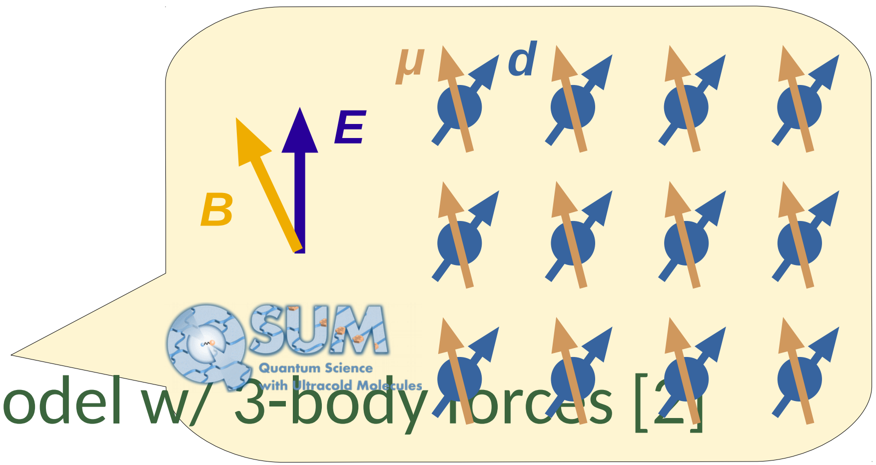
– Conductivity → Local vs. quasi-local vs. non-local charges

– Beyond linear response (Drude weight)

Prosen, PRL **106** 217206 (2011); Ilievski et al., PRL **115** 157201 (2015)

Outlook

- XXZ $$\hat{H} = \sum_{\langle i,j \rangle} \left[J_x \left(\sigma_{i,x}\sigma_{j,x} + \sigma_{i,y}\sigma_{j,y} \right) + J_z \sigma_{i,z}\sigma_{j,z} \right]$$
- Protocols to measure β_k
- Q. phases with electric and magnetic dipoles [1]
- Phase diagram of JCH model w/ 3-body forces [2]
- Q. probe spectroscopy for q. simulators [3]



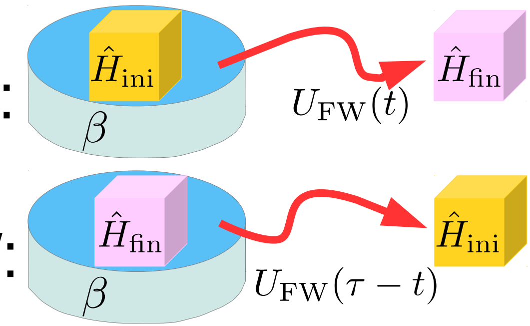
[1] Truppe et al., Nat. Phys. **13** 1173 (2017); Rvachov et al., PRL **119** 143001 (2017); Guttridge et al., PRA **96** 012704 (2017).
[2] Mendonza-Arenas et al., PRA **96** 023821 (2016); Prasad & Martin, arXiv:1710.02424
[3] Usui, Buca, & Jordi Mur-Petit, arXiv:1804.09237

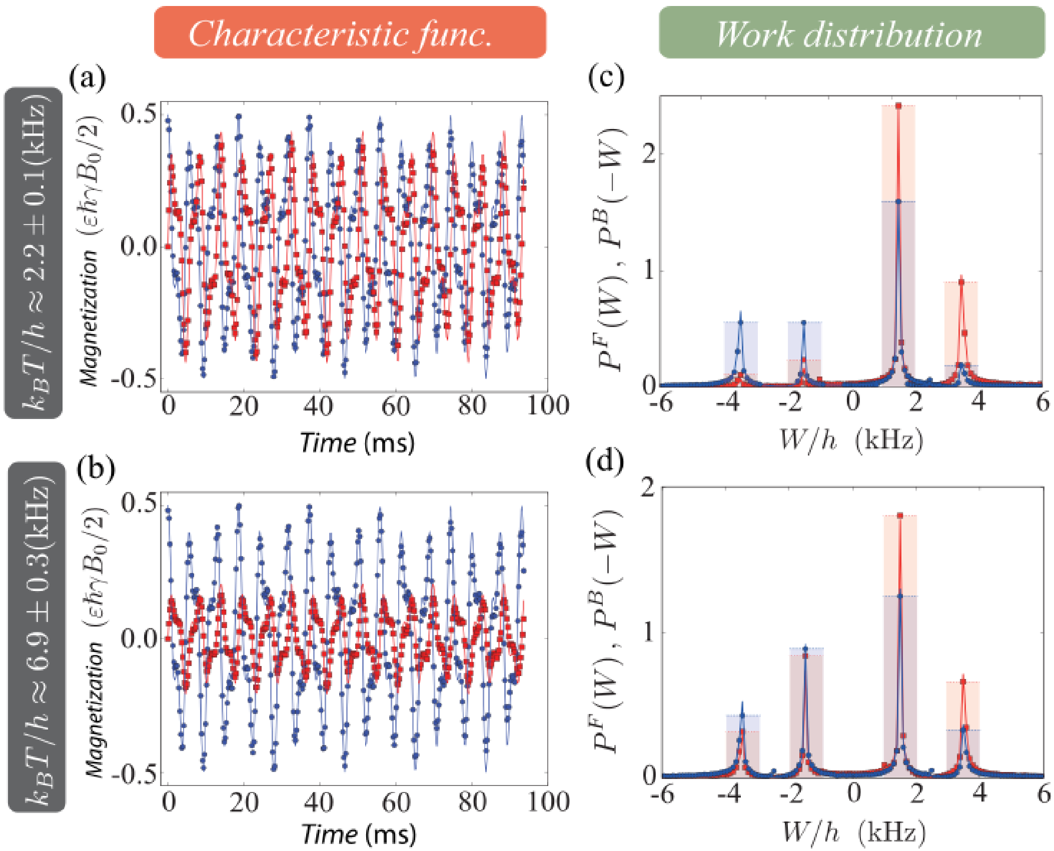
Testing the Quantum: TCR

$$P_f(w) = e^{\beta(w-\Delta F)} P_b(-w)$$

FW:

BW:





Th: Dorner et al., PRL **110** 230601 & Mazzola et al., PRL **110** 230602 (2013)
Expt: Batalhão et al., PRL **113** 140601 (2014) [liquid NMR]

Testing the Quantum: TCR

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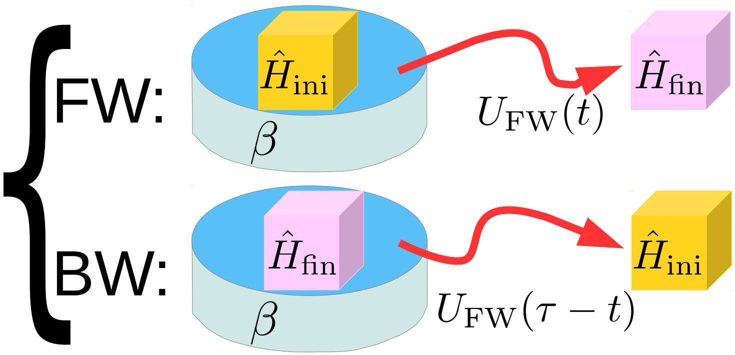

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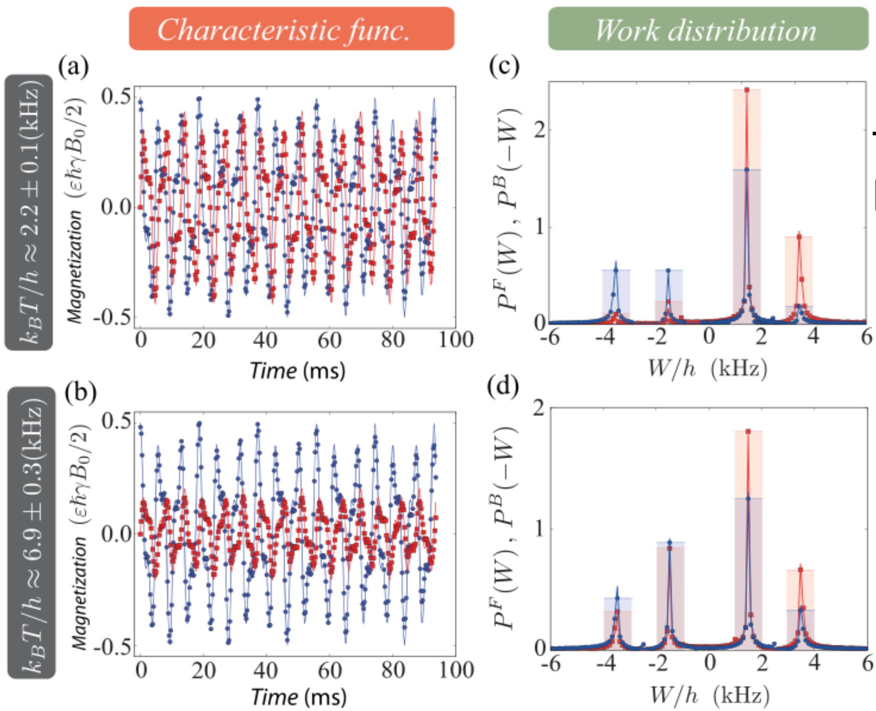

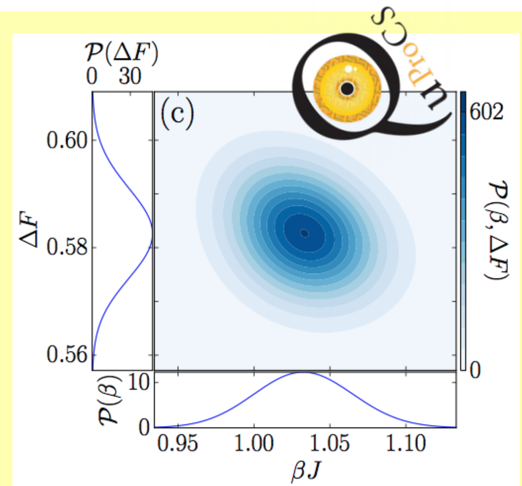

Also: TCR-based protocols for quantum probing (thermometry, correlations…):
T.H. Johnson et al., PRA 93, 053619 (2016); M. Streif et al. PRA 94, 054634 (2016); …