Quantum phase transitions beyond Landau-Ginzburg theory in one dimension revisited

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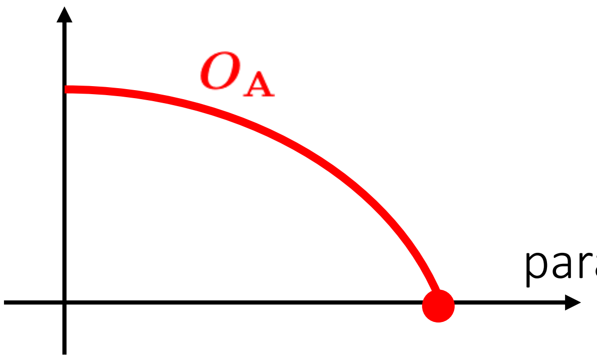
C. Mudry, AF, T. Morimoto, and T. Hikihara, Phys. Rev. B 99, 205153 (2019)

Plan of the talk

A 1D baby version of “deconfined quantum criticality”

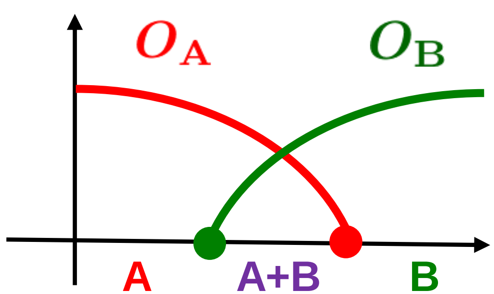
- Introduction
- 1D spin-1/2 J_1-J_2 XYZ model
  - Model & symmetries
  - Theoretical analysis
    - Bosonization (the XXZ model; SU(2)+pert.)
  - Numerical analysis
  - Domain wall
  - Effective theory (alternative derivation)
- summary

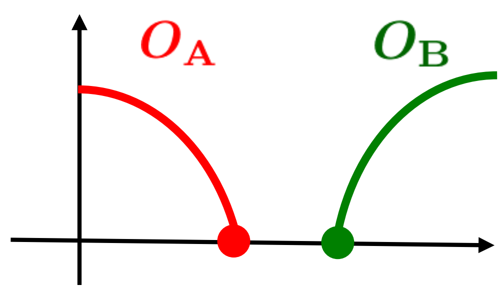
◆ Ginzburg-Landau theory

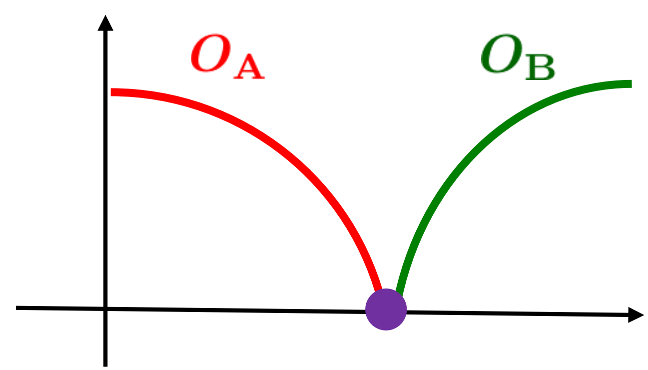


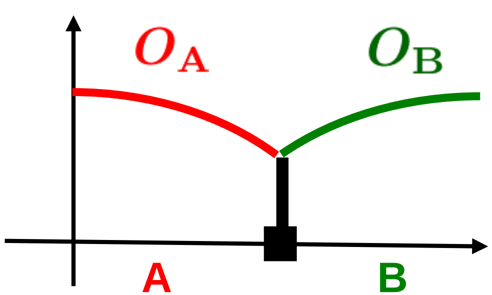
The GL theory can describe continuous order-disorder transitions with spontaneous sym. breaking

Phase diagrams with two ordered phases (w/ different SSBs)









Coexistence phase,
Separated trans.,
1st order trans.
are “allowed”

Direct continuous transition is “not allowed” (need fine tuning)

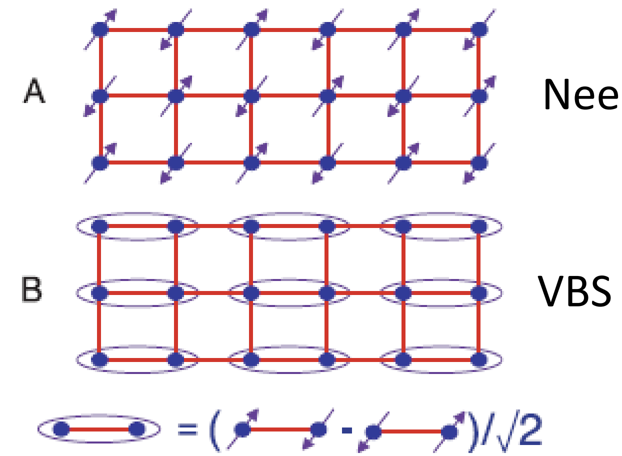
Quantum criticality beyond LGW paradigm

Deconfined Quantum Critical Points

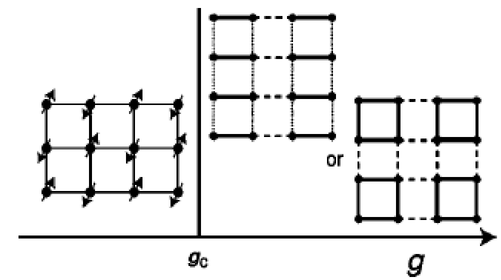
T. Senthil,[1*] Ashvin Vishwanath,[1] Leon Balents,[2] Subir Sachdev,[3] Matthew P. A. Fisher[4]

$$H = J\sum_{\langle rr'\rangle} \vec{S}_r \cdot \vec{S}_{r'} + \ldots$$

Continuous quantum phase transition
between Neel and Valence-Bond-Solid phases



$$S_n = \frac{1}{2g}\int d\tau \int d^2r \left[\frac{1}{c^2}\left(\frac{\partial \hat{n}}{\partial \tau}\right)^2 + (\nabla_r \hat{n})^2\right] + iS\sum_r (-1)^r A_r$$



$$Q = \frac{1}{4\pi}\int d^2r \, \hat{n} \cdot \partial_x \hat{n} \times \partial_y \hat{n}$$

tunneling between sectors w/different Skyrmion numbers

➡ Berry phases $\pm i, \pm 1$ (Haldane 1988)

$$\hat{n} = z^{\dagger}\vec{\sigma} z$$

$$L_z = \sum_{\alpha=1}^{N} |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z|^2 + u(|z|^2)^2 + \kappa(\epsilon_{\mu\nu\kappa}\partial_\nu a_\kappa)^2$$

CP^1 model w/non-compact U(1) gauge field

1490 5 MARCH 2004 VOL 303 SCIENCE www.sciencemag.org

Approach from VBS side (Levin & Senthil, PRB 2004)

PHYSICAL REVIEW B 70, 220403(R) (2004)

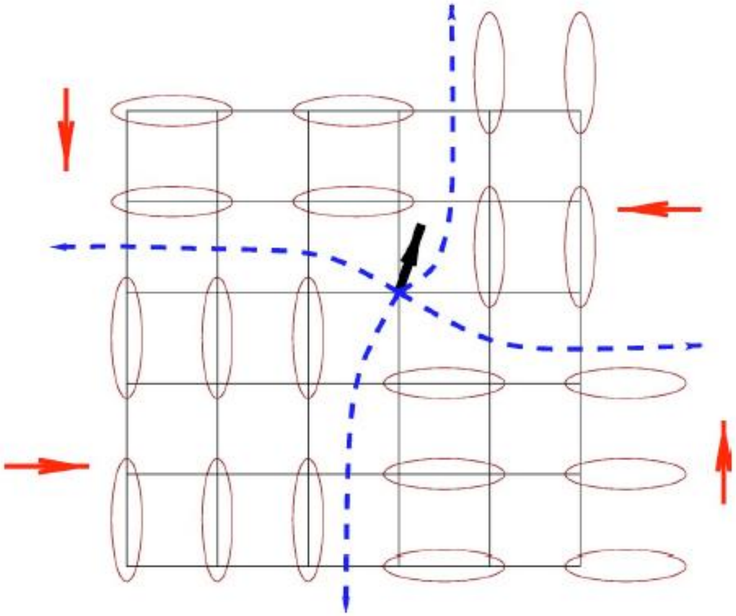


FIG. 4. (Color online) The Z_4 vortex in the columnar VBS state. The blue lines represent the four elementary domain walls. At the core of the vortex there is an unpaired site with a free spin-1/2 moment.

There have been many theoretical studies.
numerics: Sandvik, Prokofiev, Troyer, Z.-Y. Meng, Assaad, …

Recent numerical works on 1D models: Jiang & Motrunich, Roberts, Jiang & Motrunich, Huang, Lu, You, Meng & Xiang, …

The spin-1/2 J_1-J_2 XYZ chain

Hamiltonian

$$H = J_1 \sum_l \left(S_l^x S_{l+1}^x + \Delta_y S_l^y S_{l+1}^y + \Delta_z S_l^z S_{l+1}^z \right) + J_2 \sum_l \left(S_l^x S_{l+2}^x + \Delta_y S_l^y S_{l+2}^y + \Delta_z S_l^z S_{l+2}^z \right)$$

We assume that

$$J_1 > 0, \qquad J_2 > 0, \qquad \Delta_y \geq 0, \qquad \Delta_z \geq 0.$$

Three dimensionless parameters: $J \equiv \frac{J_2}{J_1}, \Delta_y, \Delta_z$

$$0 < \frac{J_2}{J_1} < \frac{1}{2}$$

The nearest-neighbor coupling J_1 is the dominant interaction.
(We do not consider up-up-down-down Ising order.)

Symmetries

$$H = J_1 \sum_l \left(S_l^x S_{l+1}^x + \Delta_y S_l^y S_{l+1}^y + \Delta_z S_l^z S_{l+1}^z \right) + J_2 \sum_l \left(S_l^x S_{l+2}^x + \Delta_y S_l^y S_{l+2}^y + \Delta_z S_l^z S_{l+2}^z \right)$$

- π-rotation about the x, y, z axes in the spin space $\mathbb{Z}_2 \times \mathbb{Z}_2$

  R_π^x: $\left(S_l^x, S_l^y, S_l^z\right) \to \left(S_l^x, -S_l^y, -S_l^z\right)$

  R_π^y: $\left(S_l^x, S_l^y, S_l^z\right) \to \left(-S_l^x, S_l^y, -S_l^z\right)$

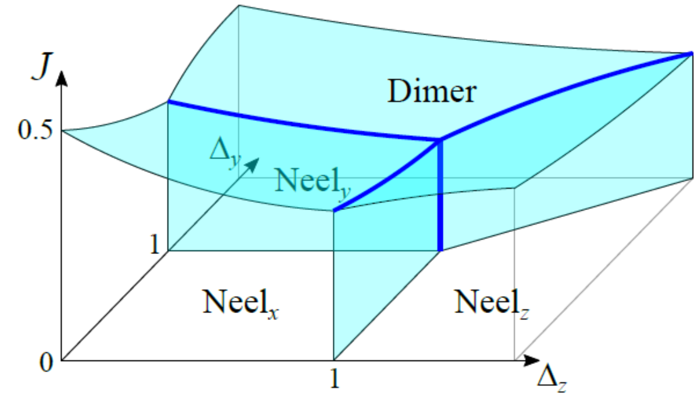
  R_π^z: $\left(S_l^x, S_l^y, S_l^z\right) \to \left(-S_l^x, -S_l^y, S_l^z\right)$

- translation T: $\left(S_l^x, S_l^y, S_l^z\right) \to \left(S_{l+1}^x, S_{l+1}^y, S_{l+1}^z\right)$

- inversion P: $\left(S_l^x, S_l^y, S_l^z\right) \to \left(S_{-l}^x, S_{-l}^y, S_{-l}^z\right)$

- time reversal Θ: $\left(S_l^x, S_l^y, S_l^z\right) \to \left(-S_l^x, -S_l^y, -S_l^z\right)$

Z_2 Ordered phases



- Neel$_x$ phase $\langle S_l^x \rangle = (-1)^l n_x\ ,\ n_x \neq 0$

  The symmetries $R_\pi^y, R_\pi^z, T, \Theta$ are spontaneously broken.

- Neel$_y$ phase $\langle S_l^y \rangle = (-1)^l n_y\ ,\ n_y \neq 0$

  The symmetries $R_\pi^x, R_\pi^z, T, \Theta$ are spontaneously broken.

- Neel$_z$ phase $\langle S_l^z \rangle = (-1)^l n_z\ ,\ n_z \neq 0$

  The symmetries $R_\pi^x, R_\pi^y, T, \Theta$ are spontaneously broken.

- Valence Bond Solid (VBS) or Dimer phase $\langle \vec{S}_l \cdot \vec{S}_{l+1} \rangle = e + (-1)^l d\ ,\ d \neq 0$

  The symmetries P, T are spontaneously broken.

Continuous phase transitions between the dimer and Neel phases are not allowed within the Landau-Ginzburg-Wilson theory.

J_1-J_2 XXZ model U(1) symmetry

Haldane PRB (1982)

$\Delta_y = 1$ $S^x_l S^x_{l'} + S^y_l S^y_{l'} + \Delta_z S^z_l S^z_{l'}$

Jordan-Wigner transformation

$$S^z_l =: c^\dagger_l c_l - \frac{1}{2} \equiv n_l, \qquad S^+_l \equiv S^x_l + \mathrm{i}S^y_l =: c^\dagger_l \exp\left(\mathrm{i}\pi \sum_{n<l} c^\dagger_n c_n\right)$$

$$H_{\mathrm{XXZ}} \equiv J_1 \sum_l \left[\frac{1}{2}\left(c^\dagger_{l+1} c_l + c^\dagger_l c_{l+1}\right) + \Delta_z n_l n_{l+1}\right] + J_2 \sum_l \left[\left(c^\dagger_{l+2} c_l + c^\dagger_l c_{l+2}\right) n_{l+1} + \Delta_z n_l n_{l+2}\right]$$

Introduce left- and right-moving low-energy fermions

$$c_l \approx \sqrt{\mathfrak{a}}\left[e^{+\mathrm{i}\pi x/(2\mathfrak{a})} \psi_{\mathrm{L}}(x) + e^{-\mathrm{i}\pi x/(2\mathfrak{a})} \psi_{\mathrm{R}}(x)\right]$$

$$\mathcal{H}_{\mathrm{XXZ}} = \mathrm{i}v\left(\psi^\dagger_{\mathrm{L}} \partial_x \psi_{\mathrm{L}} - \psi^\dagger_{\mathrm{R}} \partial_x \psi_{\mathrm{R}}\right) + g_+\left(:\psi^\dagger_{\mathrm{L}} \psi_{\mathrm{L}}: + :\psi^\dagger_{\mathrm{R}} \psi_{\mathrm{R}}:\right)^2$$
$$+ g_-\left(:\psi^\dagger_{\mathrm{L}} \psi_{\mathrm{L}}: - :\psi^\dagger_{\mathrm{R}} \psi_{\mathrm{R}}:\right)^2 + g_{\mathrm{u}}\left(:\psi^\dagger_{\mathrm{L}} \psi^\dagger_{\mathrm{L}}::\psi_{\mathrm{R}} \psi_{\mathrm{R}}: + :\psi^\dagger_{\mathrm{R}} \psi^\dagger_{\mathrm{R}}::\psi_{\mathrm{L}} \psi_{\mathrm{L}}:\right)$$

Umklapp scattering

Bosonization

$$\psi_{\mathrm{L}}(x) = \frac{e^{-\mathrm{i}\varphi_{\mathrm{L}}(x)}}{\sqrt{2\pi\mathfrak{a}}}, \qquad \psi_{\mathrm{R}}(x) = \frac{e^{+\mathrm{i}\varphi_{\mathrm{R}}(x)}}{\sqrt{2\pi\mathfrak{a}}} \qquad [\varphi_{\mathrm{R}}(x), \varphi_{\mathrm{R}}(y)] = -[\varphi_{\mathrm{L}}(x), \varphi_{\mathrm{L}}(y)] = \mathrm{i}\pi\,\mathrm{sgn}(x-y), \quad [\varphi_{\mathrm{R}}(x), \varphi_{\mathrm{L}}(y)] = \mathrm{i}\pi.$$

$$\mathcal{H}_{\mathrm{XXZ}} = \frac{\tilde{g}_+}{8\pi}\,[\partial_x(\varphi_{\mathrm{L}} + \varphi_{\mathrm{R}})]^2 + \frac{\tilde{g}_-}{8\pi}\,[\partial_x(\varphi_{\mathrm{L}} - \varphi_{\mathrm{R}})]^2 + \tilde{g}_{\mathrm{u}}\,\cos[2(\varphi_{\mathrm{L}} + \varphi_{\mathrm{R}})]$$

$$\tilde{g}_+ = \mathfrak{a}\,J_1\left[1 + \frac{4}{\pi}(\Delta_z + \mathcal{J})\right], \quad \tilde{g}_- = \mathfrak{a}\,J_1\left(1 - \frac{4}{\pi}\mathcal{J}\right), \qquad \tilde{g}_{\mathrm{u}} = \frac{\mathfrak{a}\,J_1}{2\pi^2\mathfrak{a}^2}\,[\Delta_z - \mathcal{J}(2 + \Delta_z)]$$

for small $|\Delta_z| \ll 1$ and $|J| \ll 1$

sine-Gordon model

$$\mathcal{H}_{\mathrm{XXZ}} = \frac{v}{2}\left[\frac{1}{\eta}\,(\partial_x\theta)^2 + \eta\,(\partial_x\phi)^2 + \lambda_\phi\,\cos(\sqrt{8\pi}\,\phi)\right] \qquad \frac{v}{2\eta} := \tilde{g}_-, \qquad 2v\eta := \tilde{g}_+, \qquad \frac{v\lambda_\phi}{2} := \tilde{g}_{\mathrm{u}}.$$

λ_ϕ changes its sign as J increases.

$$\phi(x) := \frac{1}{\sqrt{2\pi}}\,[\varphi_{\mathrm{L}}(x) + \varphi_{\mathrm{R}}(x)], \qquad \theta(x) := \frac{1}{\sqrt{8\pi}}\,[\varphi_{\mathrm{L}}(x) - \varphi_{\mathrm{R}}(x)]$$

$$[\phi(x), \theta(y)] = \mathrm{i}\Theta(y - x) \qquad [\phi(x), \theta(x)] = \mathrm{i}/2$$

Spin operators

$$S_l^z \approx \frac{a}{\sqrt{2\pi}}\partial_x\phi(x) + a_1(-1)^l \sin\left(\sqrt{2\pi}\phi(x)\right) \qquad S_l^+ \approx e^{+i\sqrt{2\pi}\theta(x)}\left[a_2(-1)^l + a_3\sin\left(\sqrt{2\pi}\phi(x)\right)\right]$$

$$S_l^x = a_2\,(-1)^l \cos\left(\sqrt{2\pi}\,\theta(x)\right) + i a_3 \sin\left(\sqrt{2\pi}\,\theta(x)\right)\sin\left(\sqrt{2\pi}\,\phi(x)\right)$$

$$S_l^y = a_2\,(-1)^l \sin\left(\sqrt{2\pi}\,\theta(x)\right) - i a_3 \cos\left(\sqrt{2\pi}\,\theta(x)\right)\sin\left(\sqrt{2\pi}\,\phi(x)\right)$$

Neel order parameters

$$N_x(x) := \cos\left(\sqrt{2\pi}\,\theta(x)\right), \quad N_y(x) := \sin\left(\sqrt{2\pi}\,\theta(x)\right), \quad N_z(x) := \sin\left(\sqrt{2\pi}\,\phi(x)\right)$$

Dimer (VBS) order parameter

$$D(x) := \cos\left(\sqrt{2\pi}\,\phi(x)\right)$$

U(1) symmetry (rotation about S^z axis)

$$(\phi,\theta) \mapsto (\phi, \theta + \delta\theta)$$

Symmetry transformations

$$R_\pi^x : (\phi,\theta) \mapsto (-\phi, -\theta),$$
$$R_\pi^y : (\phi,\theta) \mapsto (-\phi, \sqrt{\pi/2} - \theta),$$
$$R_\pi^z : (\phi,\theta) \mapsto (\phi, \theta + \sqrt{\pi/2}),$$
$$T : (\phi,\theta) \mapsto (\phi + \sqrt{\pi/2}, \theta + \sqrt{\pi/2}),$$
$$P : (\phi,\theta) \mapsto (-\phi + \sqrt{\pi/2}, \theta),$$
$$\Theta : (\phi,\theta) \mapsto (-\phi, \theta + \sqrt{\pi/2}),$$

Spin operators

$$S_l^z \approx \frac{a}{\sqrt{2\pi}}\partial_x\phi(x) + a_1(-1)^l \sin\left(\sqrt{2\pi}\phi(x)\right) \qquad S_l^+ \approx e^{+i\sqrt{2\pi}\theta(x)}\left[a_2(-1)^l + a_3\sin\left(\sqrt{2\pi}\phi(x)\right)\right]$$

$$S_l^x = a_2\,(-1)^l \cos\left(\sqrt{2\pi}\,\theta(x)\right) + ia_3 \sin\left(\sqrt{2\pi}\,\theta(x)\right)\sin\left(\sqrt{2\pi}\,\phi(x)\right)$$

$$S_l^y = a_2\,(-1)^l \sin\left(\sqrt{2\pi}\,\theta(x)\right) - ia_3 \cos\left(\sqrt{2\pi}\,\theta(x)\right)\sin\left(\sqrt{2\pi}\,\phi(x)\right)$$

Neel order parameters

$$N_x(x) := \cos\left(\sqrt{2\pi}\,\theta(x)\right), \quad N_y(x) := \sin\left(\sqrt{2\pi}\,\theta(x)\right), \quad N_z(x) := \sin\left(\sqrt{2\pi}\,\phi(x)\right)$$

Dimer (VBS) order parameter

$$D(x) := \cos\left(\sqrt{2\pi}\,\phi(x)\right)$$

sine-Gordon model

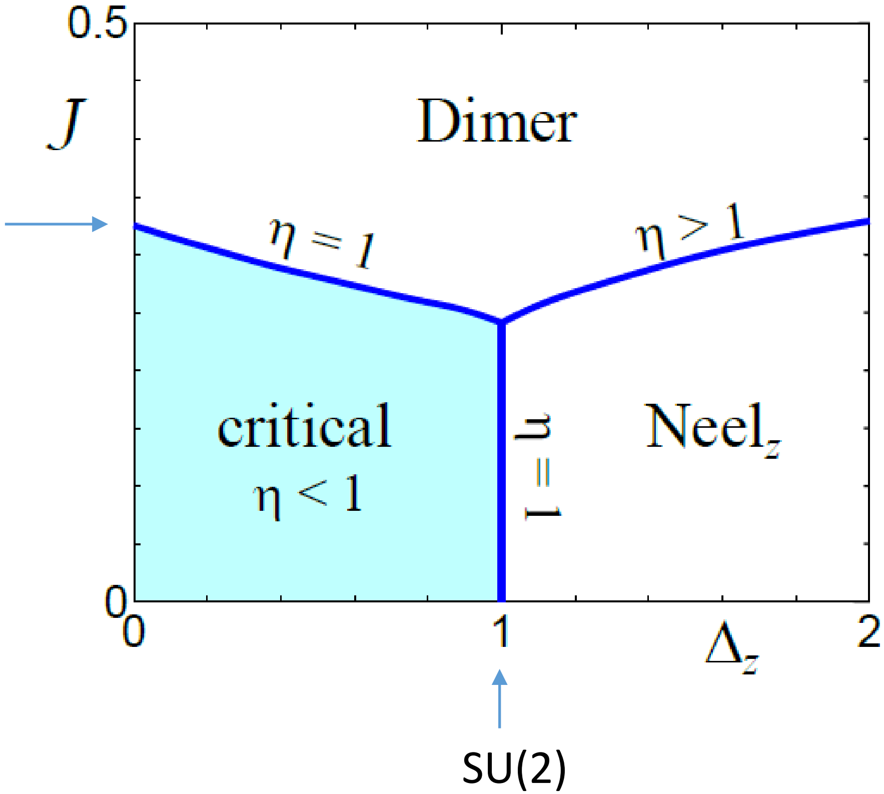
$$\mathcal{H}_{\mathrm{XXZ}} = \frac{v}{2}\left[\frac{1}{\eta}(\partial_x\theta)^2 + \eta\,(\partial_x\phi)^2 + \lambda_\phi\,\cos(\sqrt{8\pi}\,\phi)\right]$$

If $\cos\left(\sqrt{8\pi}\phi\right)$ is relevant,

then $\lambda_\phi > 0 \Longrightarrow$ Neelz $\phi = \pm\sqrt{\pi/8}$

$\lambda_\phi < 0 \Longrightarrow$ Dimer $\phi = 0, \sqrt{\pi/2}$

Phase diagram of the J_1-J_2 XXZ model



Haldane, PRB (1982)
Nomura & Okamoto, J Phys A (1994)

$\lambda_\phi = 0$ at the Neelz-Dimer transition

Gaussian criticality (c=1)

$\mathrm{U}(1)_\phi \times \mathrm{U}(1)_\theta$

↕

$\mathrm{U}(1)_L \times \mathrm{U}(1)_R$

$$\mathcal{H}_{\mathrm{XXZ}} = \frac{v}{2}\left[\frac{1}{\eta}\left(\partial_x\theta\right)^2 + \eta\left(\partial_x\phi\right)^2 + \lambda_\phi \cos(\sqrt{8\pi}\,\phi)\right]$$

Scaling dimension $2/\eta$

Breaking U(1)-symmetry (S^z rotation)

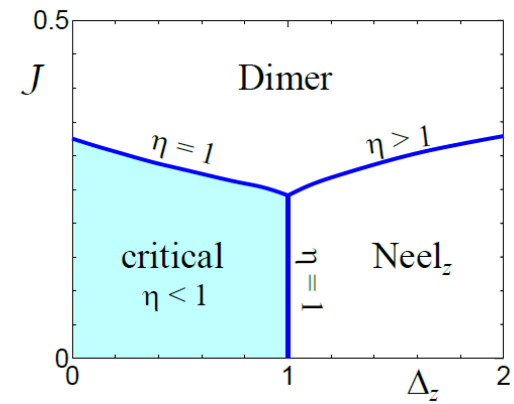
$$(1-\Delta_y)(S_l^x S_{l+1}^x - S_l^y S_{l+1}^y) = \frac{1-\Delta_y}{2}(S_l^+ S_{l+1}^+ + \text{H.c.}) \approx a_2^2\,(\Delta_y - 1)\cos(\sqrt{8\pi}\,\theta)$$

Scaling dimension 2η

$\Delta_y > 1 \implies \theta = \pm\sqrt{\pi/8}$ Neel$_y$ phase

$\Delta_y < 1 \implies \theta = 0, \sqrt{\pi/2}$ Neel$_x$ phase

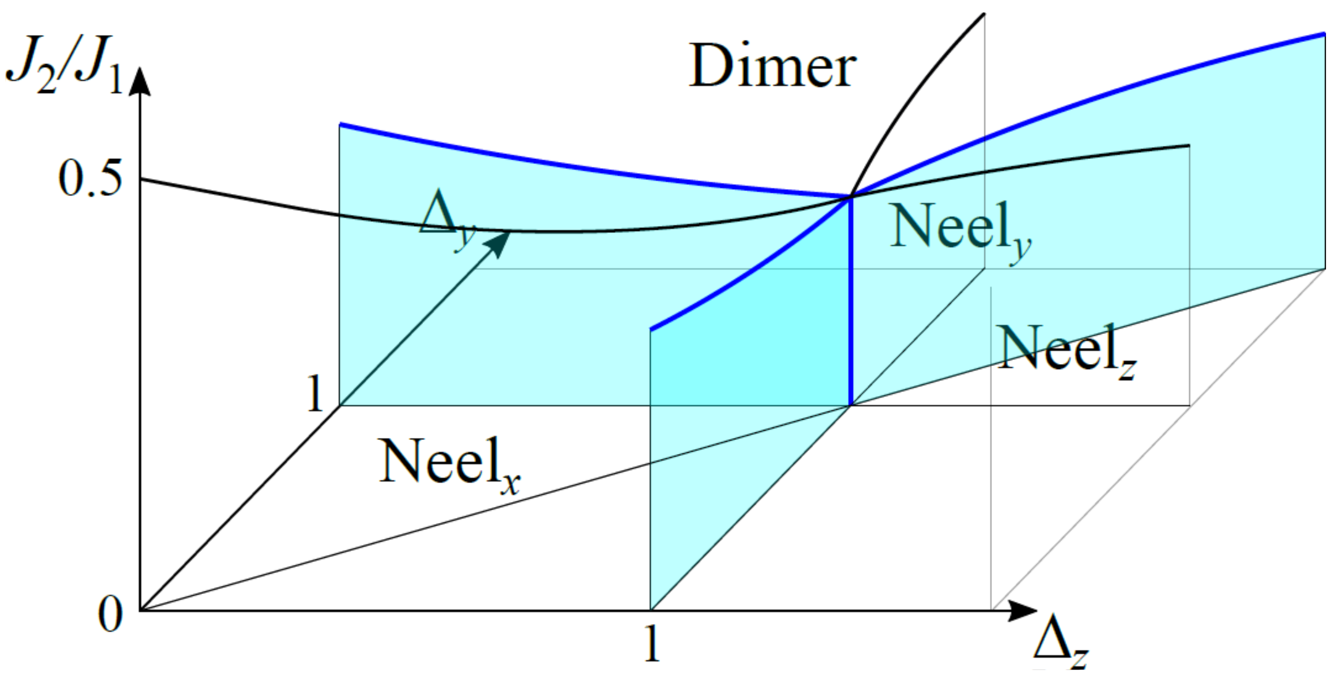
$$N_x(x) := \cos\left(\sqrt{2\pi}\,\theta(x)\right), \quad N_y(x) := \sin\left(\sqrt{2\pi}\,\theta(x)\right),$$

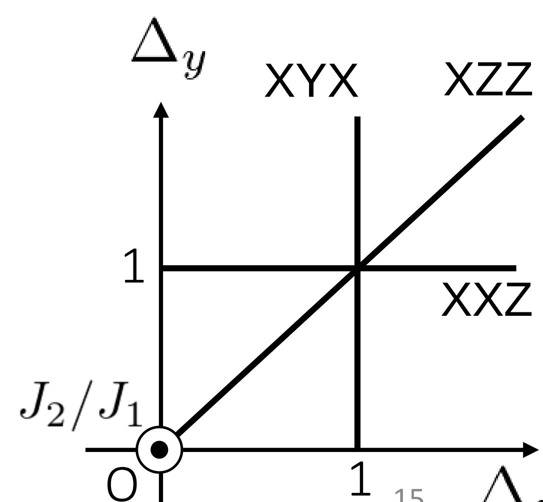


◆ J_1-J_2 XYZ chain

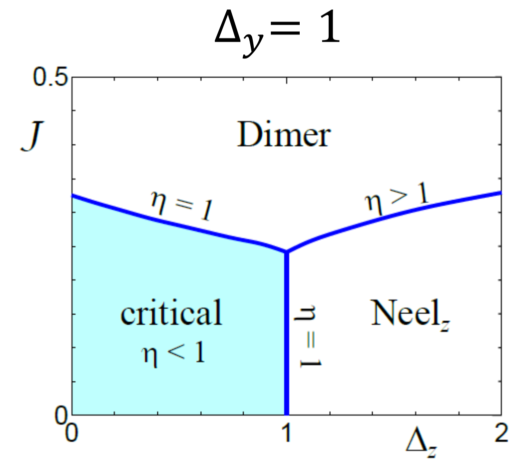
$$H_{\rm XYZ} = J_1 \sum_l \left(S_l^x S_{l+1}^x + \Delta_y S_l^y S_{l+1}^y + \Delta_z S_l^z S_{l+1}^z\right) + J_2 \sum_l \left(S_l^x S_{l+2}^x + \Delta_y S_l^y S_{l+2}^y + \Delta_z S_l^z S_{l+2}^z\right)$$

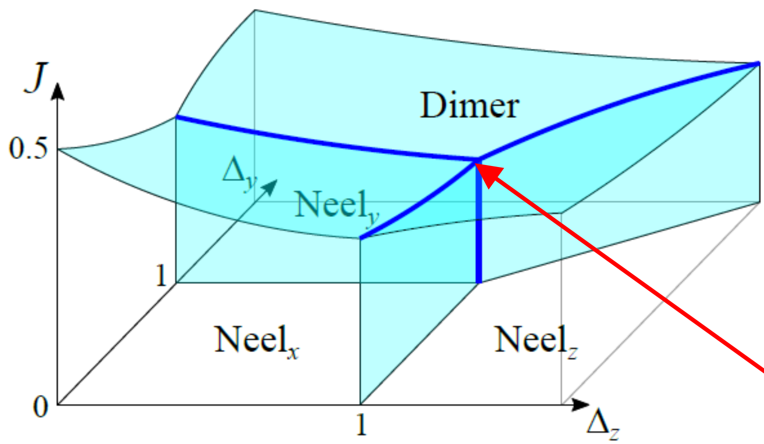
$$J_1 > 0, \quad 0 \le \frac{J_2}{J_1} < \frac{1}{2}, \quad \Delta_y \ge 0, \quad , \Delta_z \ge 0$$





Phase diagram





$\Delta_z = 1$ plane

$\Delta_y = \Delta_z$ plane

SU(2) symmetric point (with no marginal pert.)

Dimer phase: $J > J_c(\Delta_y, \Delta_z)$

Neel phases: $J < J_c(\Delta_y, \Delta_z)$ ⇨ Neelx phase: $\Delta_y < 1, \Delta_z < 1$

Neely phase: $\Delta_y > 1, \Delta_y > \Delta_z$

Neelz phase: $\Delta_z > 1, \Delta_z > \Delta_y$

1-loop RG near the SU(2) symmetric point

SU(2) symmetric point: $\Delta_y = \Delta_z = 1, \qquad J = 0.2411 \ldots$

$$\mathcal{H}_0 \equiv \frac{1}{2}\left[(\partial_x\theta)^2 + (\partial_x\phi)^2\right]$$

SU(2) current operators:

$$J_{\mathrm{L}}^{\pm} := \frac{1}{\mathsf{a}}\, e^{\pm \mathrm{i}\sqrt{2}\phi_{\mathrm{L}}}, \qquad J_{\mathrm{L}}^{z} := \frac{1}{\sqrt{2}}\,\partial_x\phi_{\mathrm{L}}, \qquad J_{\mathrm{M}}^{x} := \frac{J_{\mathrm{M}}^{+} + J_{\mathrm{M}}^{-}}{2}, \qquad J_{\mathrm{M}}^{y} := \frac{J_{\mathrm{M}}^{+} - J_{\mathrm{M}}^{-}}{2\mathrm{i}}$$

$$J_{\mathrm{R}}^{\pm} := \frac{1}{\mathsf{a}}\, e^{\mp \mathrm{i}\sqrt{2}\phi_{\mathrm{R}}}, \qquad J_{\mathrm{R}}^{z} := \frac{1}{\sqrt{2}}\,\partial_x\phi_{\mathrm{R}}, \qquad \mathrm{M} = \mathrm{L}, \mathrm{R}$$

$$\phi_{\mathrm{L}}(x) := \sqrt{\pi}[\phi(x) + \theta(x)], \qquad \phi_{\mathrm{R}}(x) := \sqrt{\pi}[\phi(x) - \theta(x)]. \qquad \langle J_{\mathrm{M}}^{+}(x)\, J_{\mathrm{M}}^{-}(0)\rangle = -1/x^2$$

Current-current interactions

$$\begin{aligned}\mathcal{H}_{JJ} &:= \lambda_x\, J_{\mathrm{L}}^{x}\, J_{\mathrm{R}}^{x} + \lambda_y\, J_{\mathrm{L}}^{y}\, J_{\mathrm{R}}^{y} + \lambda_z\, J_{\mathrm{L}}^{z}\, J_{\mathrm{R}}^{z} \\ &= -\frac{1}{\mathsf{a}^2}\,(\lambda_x - \lambda_y)\cos(\sqrt{8\pi}\,\theta) - \frac{1}{\mathsf{a}^2}\,(\lambda_x + \lambda_y)\cos(\sqrt{8\pi}\,\phi) \\ &\quad - \frac{\pi\lambda_z}{2}\left[(\partial_x\theta)^2 - (\partial_x\phi)^2\right]\end{aligned}$$

$$\mathcal{H}_0 \equiv \frac{1}{2}\left[(\partial_x\theta)^2 + (\partial_x\phi)^2\right]$$

$$\mathcal{H}_{JJ} := \lambda_x\, J_{\mathrm{L}}^x\, J_{\mathrm{R}}^x + \lambda_y\, J_{\mathrm{L}}^y\, J_{\mathrm{R}}^y + \lambda_z\, J_{\mathrm{L}}^z\, J_{\mathrm{R}}^z$$
$$= -\frac{1}{\mathsf{a}^2}\left(\lambda_x - \lambda_y\right)\cos(\sqrt{8\pi}\,\theta) - \frac{1}{\mathsf{a}^2}\left(\lambda_x + \lambda_y\right)\cos(\sqrt{8\pi}\,\phi) - \frac{\pi\lambda_z}{2}\left[(\partial_x\theta)^2 - (\partial_x\phi)^2\right]$$

$$\mathcal{H}_0 + \mathcal{H}_{JJ} = \frac{v}{2}\left[\frac{1}{\eta}\left(\partial_x\theta\right)^2 + \eta\left(\partial_x\phi\right)^2 + \lambda_\phi\,\cos(\sqrt{8\pi}\,\phi)\right] + \frac{A}{\mathsf{a}^2}\left(\Delta_y - 1\right)\cos(\sqrt{8\pi}\,\theta)$$

$A > 0$

$$\lambda_x - \lambda_y = A\,(1 - \Delta_y), \quad \lambda_y - \lambda_z = A\,(\Delta_y - \Delta_z), \quad \lambda_z - \lambda_x = A\,(\Delta_z - 1).$$

$$\eta = \sqrt{\frac{1+\pi\lambda_z}{1-\pi\lambda_z}} \approx 1 + \pi\lambda_z$$

1-loop RG

Scaling dimension of $\cos\left(\sqrt{8\pi}\phi\right) = 2/\eta \approx 2 - 2\pi\lambda_z$

Scaling dimension of $\cos\left(\sqrt{8\pi}\theta\right) = 2\eta \approx 2 + 2\pi\lambda_z$

$\Rightarrow$ $$\frac{d}{dl}\left(\lambda_x \pm \lambda_y\right) = \pm 2\pi\lambda_z\left(\lambda_x \pm \lambda_y\right) \qquad dl = d\log L$$

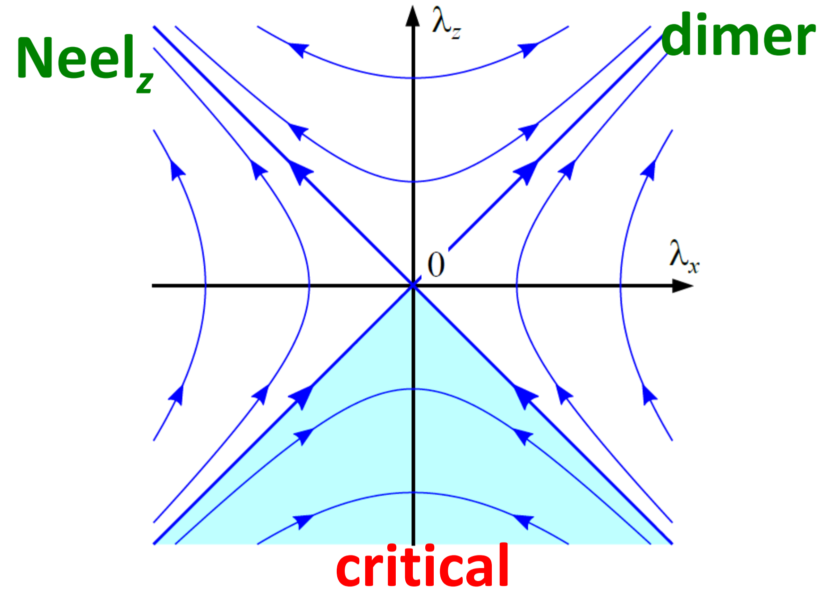
$$\frac{\mathrm{d}\lambda_x}{\mathrm{d}\ell} = 2\pi\,\lambda_y\,\lambda_z, \quad \frac{\mathrm{d}\lambda_y}{\mathrm{d}\ell} = 2\pi\,\lambda_z\,\lambda_x, \quad \frac{\mathrm{d}\lambda_z}{\mathrm{d}\ell} = 2\pi\,\lambda_x\,\lambda_y$$

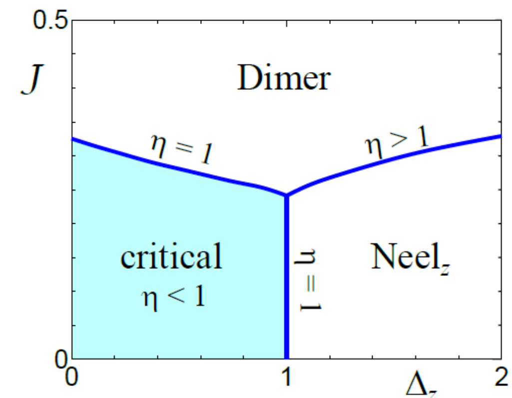
Three lines of fixed points:
(1) $\lambda_x = \lambda_y = 0$, (2) $\lambda_y = \lambda_z = 0$, (3) $\lambda_z = \lambda_x = 0$.

On the $\lambda_x = \lambda_y$ plane (XXZ)

$$\frac{d\lambda_x}{dl} = 2\pi\lambda_x\lambda_z, \qquad \frac{d\lambda_z}{dl} = 2\pi\lambda_x^2$$

critical region: $\lambda_z \leq -|\lambda_x|$





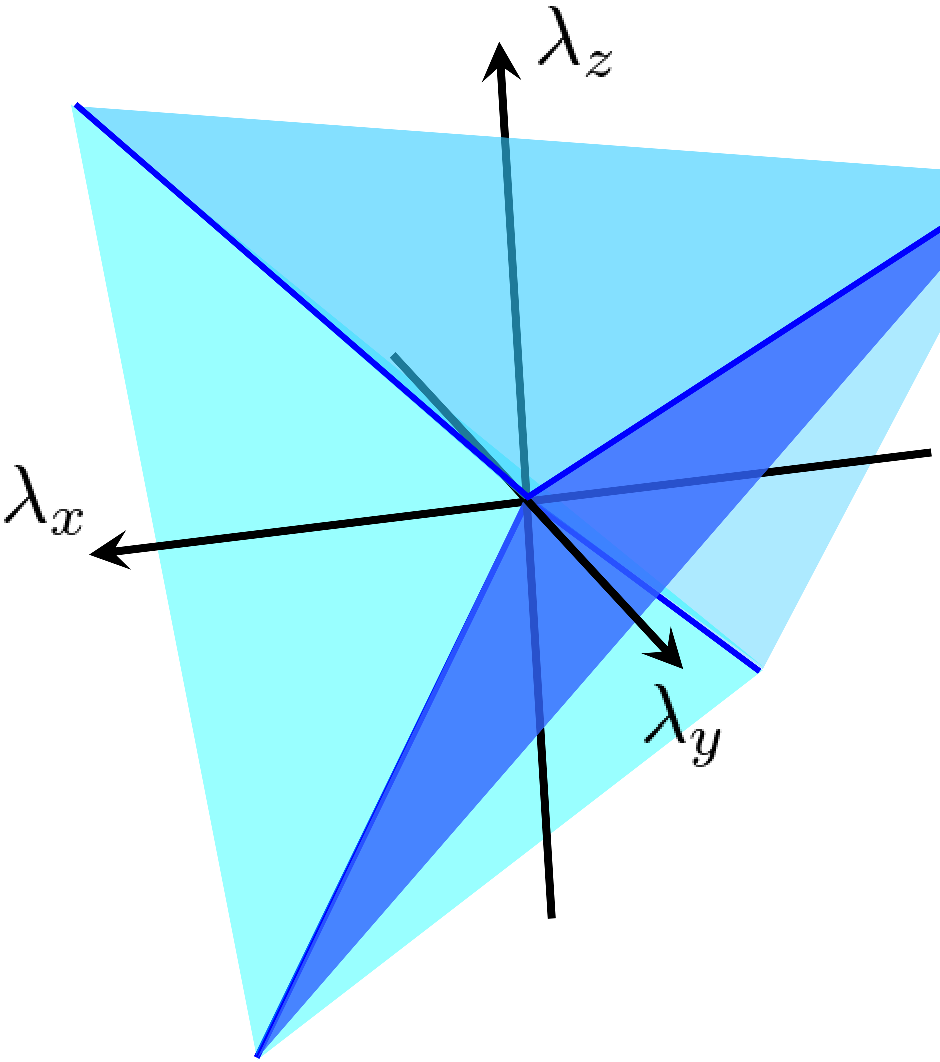
On the $\lambda_x = -\lambda_y$ plane

$$\frac{d\lambda_x}{dl} = -2\pi\lambda_x\lambda_z, \qquad \frac{d\lambda_z}{dl} = -2\pi\lambda_x^2$$

RG flows are reversed from those on the $\lambda_x = \lambda_y$ plane.

critical region: $\lambda_z \geq +|\lambda_x|$

Phase diagram in the $(\lambda_x, \lambda_y, \lambda_z)$ space



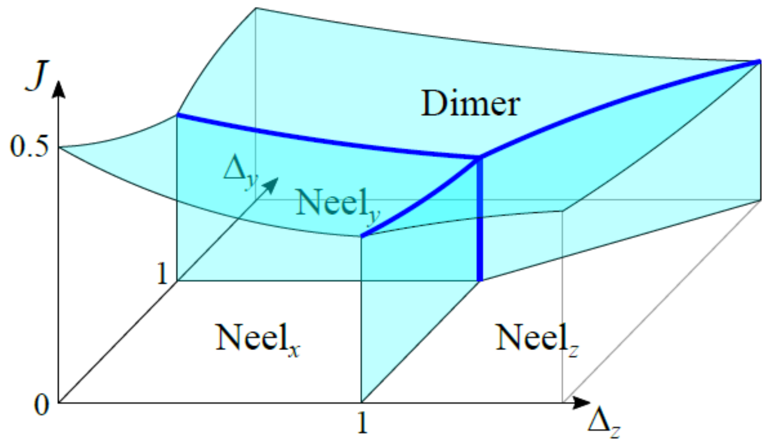
The 6 critical planes

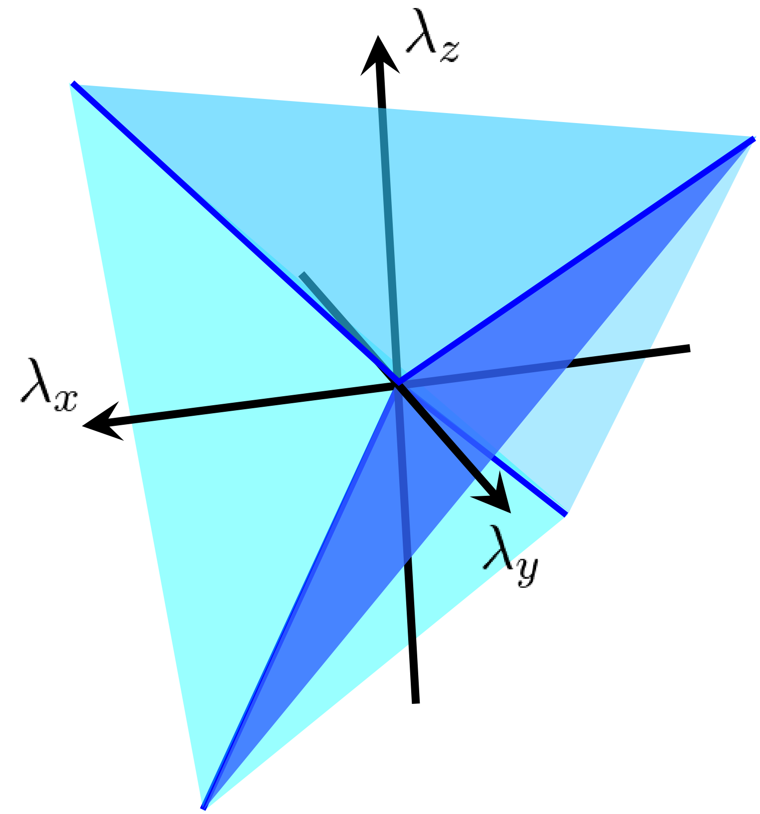
$$\lambda_x = \pm\lambda_y$$
$$\lambda_y = \pm\lambda_z$$
$$\lambda_z = \pm\lambda_x$$

are boundaries between 4 gapped phases ($\text{Neel}_{x,y,z}$ and dimer).

c=1 Gaussian criticality on critical planes

Phase diagram





$\Delta_y = 1 \iff \lambda_x = \lambda_y$

$\Delta_z = 1 \iff \lambda_z = \lambda_x$

$\Delta_y = \Delta_z \iff \lambda_y = \lambda_z$

$$\lambda_x = b\,(\mathcal{J} - \mathcal{J}_c^\star) + c\left(1 - \frac{\Delta_y + \Delta_z}{2}\right)$$

$$\lambda_y = b\,(\mathcal{J} - \mathcal{J}_c^\star) + c\left(\Delta_y - \frac{\Delta_z + 1}{2}\right)$$

$$\lambda_z = b\,(\mathcal{J} - \mathcal{J}_c^\star) + c\left(\Delta_z - \frac{1 + \Delta_y}{2}\right)$$

Scaling behavior

At a Neel$_\alpha$-dimer transition with $\alpha = x, y, z$ defined by the condition $\mathcal{J} = \mathcal{J}_c$,

$$\langle N_\alpha \rangle \sim \langle D \rangle \sim L^{-1/(2\eta)}, \qquad \begin{cases} \langle N_\alpha \rangle \sim (\mathcal{J}_c - \mathcal{J})^{1/[4(\eta-1)]}\, \Theta(\mathcal{J}_c - \mathcal{J}), \\ \langle D \rangle \sim (\mathcal{J} - \mathcal{J}_c)^{1/[4(\eta-1)]}\, \Theta(\mathcal{J} - \mathcal{J}_c), \end{cases}$$

$$\langle O \rangle = L^{-1/2\eta} F\left(|J - J_c| L^{2-2/\eta}\right) \qquad \frac{1/2\eta}{2 - 2/\eta} = \frac{1}{4(\eta - 1)}$$

At the Neel$_\alpha$-Neel$_\beta$ transition with $\alpha < \beta = x, y, z$ defined by the condition $\Delta_\alpha = \Delta_\beta$,

$$\langle N_\alpha \rangle \sim \langle N_\beta \rangle \sim L^{-\eta/2}, \qquad \begin{cases} \langle N_\alpha \rangle \sim \left(\Delta_\alpha - \Delta_\beta\right)^{\eta/[4(1-\eta)]}\, \Theta\left(\Delta_\alpha - \Delta_\beta\right), \\ \langle N_\beta \rangle \sim \left(\Delta_\beta - \Delta_\alpha\right)^{\eta/[4(1-\eta)]}\, \Theta\left(\Delta_\beta - \Delta_\alpha\right), \end{cases}$$

These express the duality

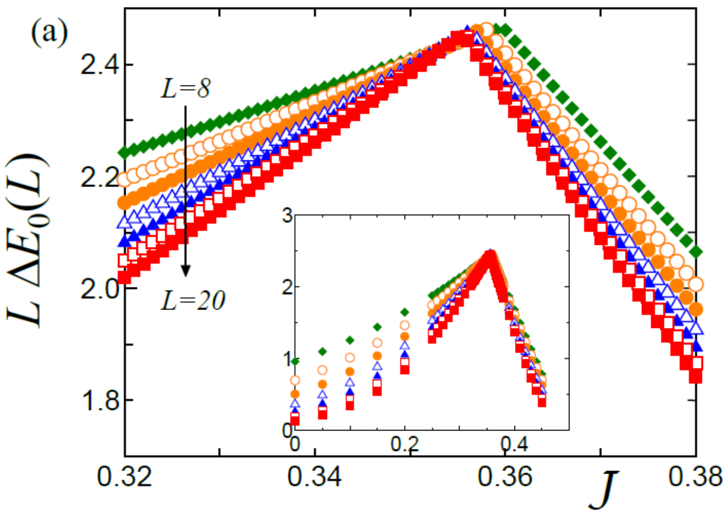
$$\mathcal{J} - \mathcal{J}_c,\ \eta \qquad \longleftrightarrow \qquad \Delta_\beta - \Delta_\alpha,\ 1/\eta.$$

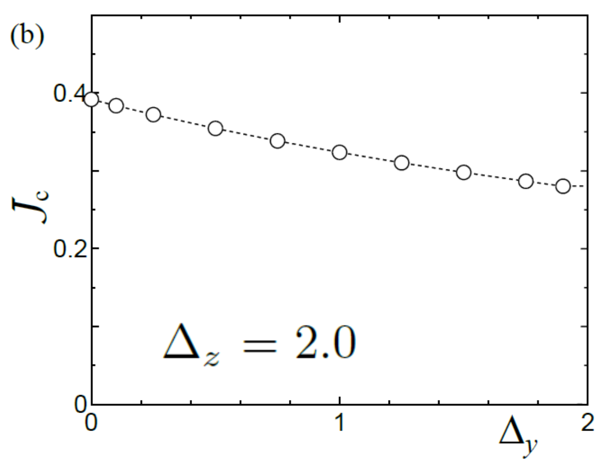
Numerical results

lowest energy gap

$$\Delta E_0(L) := E_1(L) - E_0(L)$$

$\Delta_y = 0.5, \ \Delta_z = 2.0$

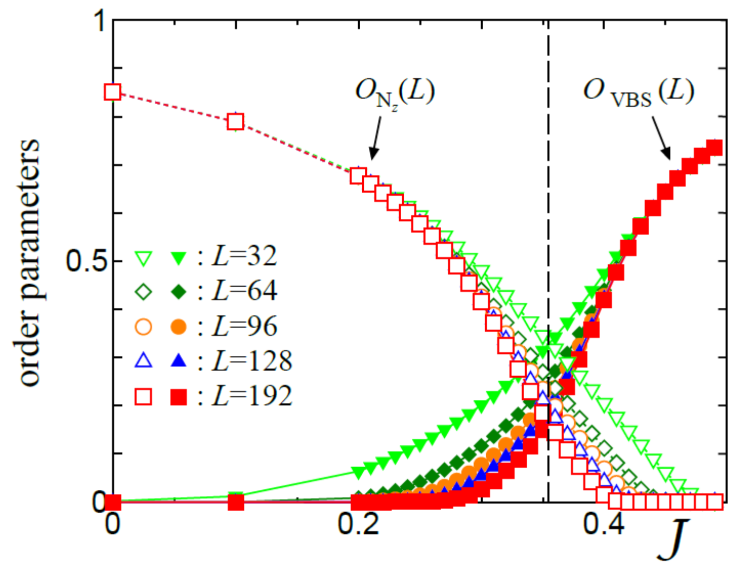


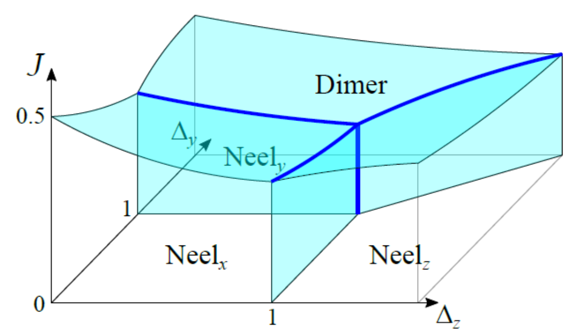


Neelz & VBS (dimer) order parameters

(weak staggerd field, OBC)

$\Delta_y = 0.5, \ \Delta_z = 2.0$





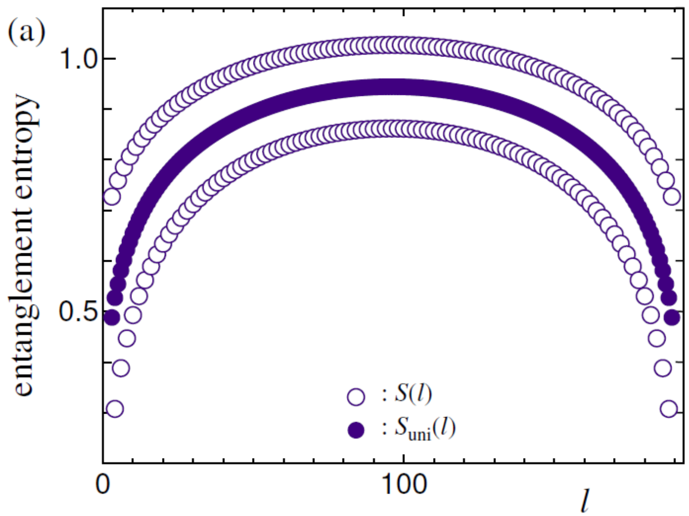
Entanglement entropy

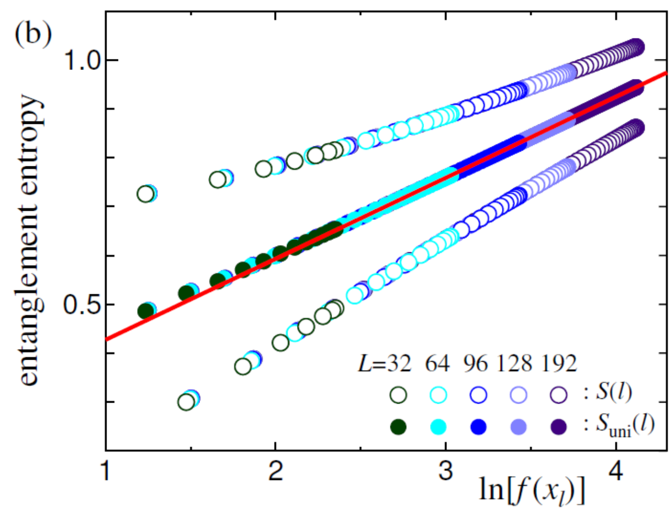
$$\mathcal{S}(l) := -\sum_j \rho_l(j) \ln \rho_l(j)$$

where $\rho_l(j)$ is the jth eigenvalue of the sub-density matrix for the left l-site block in the ground state of the full open chain.

$L = 4N$

$0.993 < c < 1.000$





$$\mathcal{S}(l) = \frac{c}{6} \ln[f(x_l)] + \alpha_{\rm osc} E_{\rm osc}(l) + \mathcal{S}_0$$

$$f(x_l) := \frac{L+1}{\pi} \sin\left(\frac{\pi x_l}{L+1}\right), \qquad x_l := l + \frac{1}{2}$$

$$E_{\rm osc}(l) := E_{\rm bond}(l) - E_{\rm uni}$$

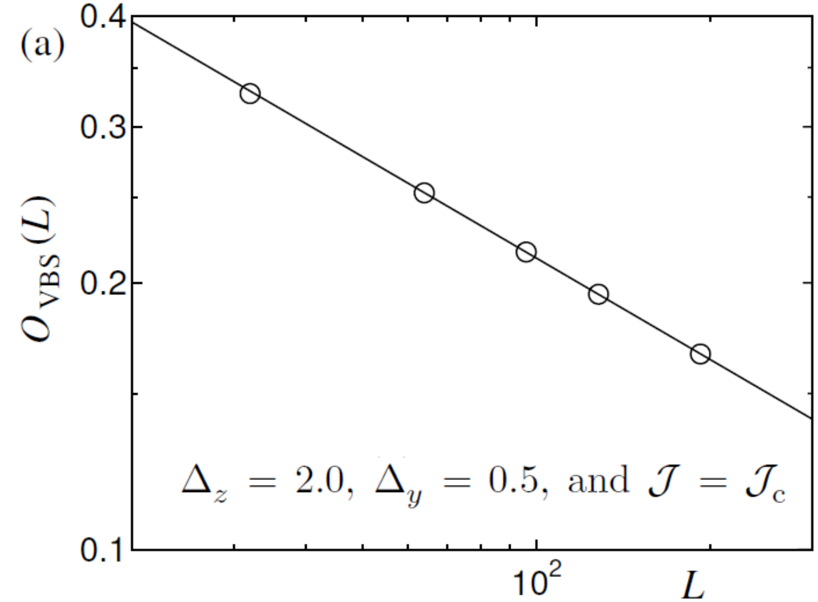
$$E_{\rm uni} := \frac{1}{2}\left[E_{\rm bond}\left(\frac{L}{2}\right) + E_{\rm bond}\left(\frac{L}{2}+1\right)\right]$$

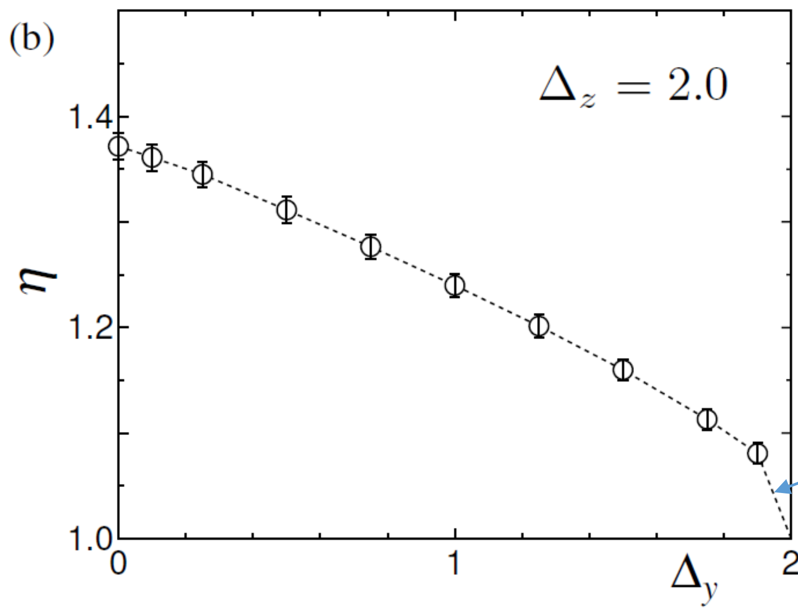
$$E_{\rm bond}(l) := J_1 \left\langle \left(S_l^x S_{l+1}^x + \Delta_y S_l^y S_{l+1}^y + \Delta_z S_l^z S_{l+1}^z\right)\right\rangle_L + \frac{J_2}{2}\left\langle \left(S_{l-1}^x S_{l+1}^x + \Delta_y S_{l-1}^y S_{l+1}^y + \Delta_z S_{l-1}^z S_{l+1}^z + S_l^x S_{l+2}^x + \Delta_y S_l^y S_{l+2}^y + \Delta_z S_l^z S_{l+2}^z\right)\right\rangle_L$$

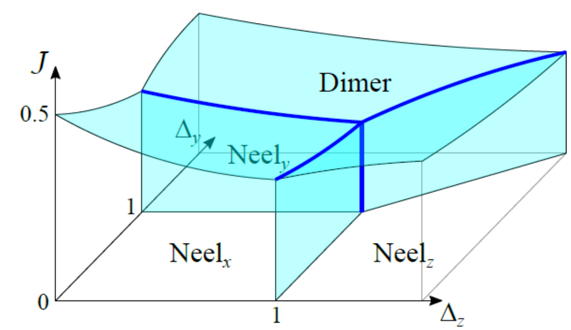
Dimer correlation

$$O_{\rm VBS}(L) := \langle S^x_{\frac{L}{2}}\, S^x_{\frac{L}{2}+1} + S^y_{\frac{L}{2}}\, S^y_{\frac{L}{2}+1} + S^z_{\frac{L}{2}}\, S^z_{\frac{L}{2}+1} \rangle_L - \langle S^x_{\frac{L}{2}-1}\, S^x_{\frac{L}{2}} + S^y_{\frac{L}{2}-1}\, S^y_{\frac{L}{2}} + S^z_{\frac{L}{2}-1}\, S^z_{\frac{L}{2}} \rangle_L$$

$O_{\rm VBS}(L) \sim L^{-\frac{1}{2\eta}}$ at a Dimer-Neel critical point







$\eta - 1 \propto \sqrt{2 - \Delta_y}$

Domain walls

We consider the Neelz-Dimer transition. $N_z(x) := \sin\left(\sqrt{2\pi}\,\phi(x)\right)$ $D(x) := \cos\left(\sqrt{2\pi}\,\phi(x)\right)$

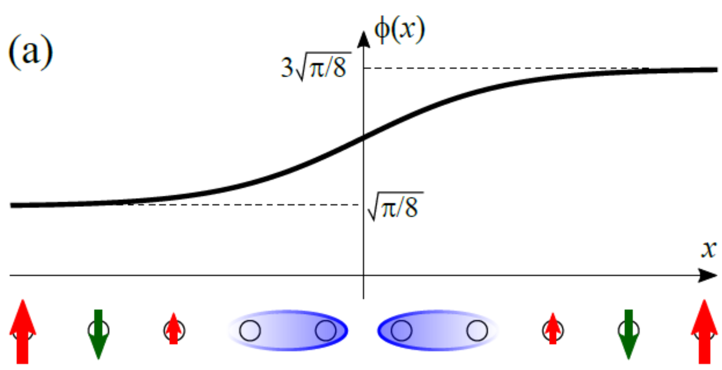
$$\mathcal{H}_{\mathrm{XXZ}} = \frac{v}{2}\left[\frac{1}{\eta}(\partial_x\theta)^2 + \eta(\partial_x\phi)^2 + \lambda_\phi \cos(\sqrt{8\pi}\,\phi)\right]$$

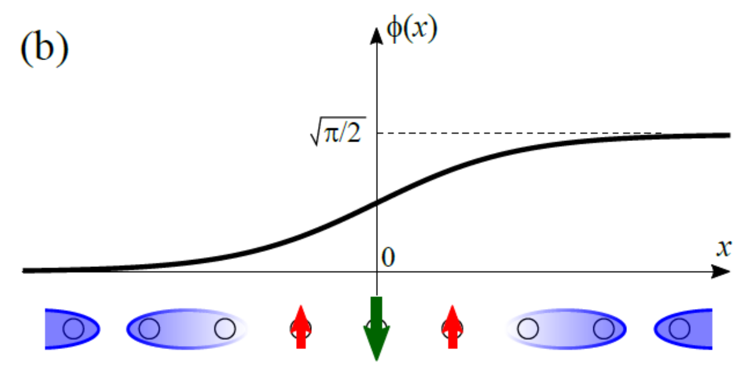
$\eta > 1$ ➡ $\lambda_\theta \cos\left(\sqrt{8\pi}\theta\right)$ is irrelevant and can be ignored.

Neelz phase: $\lambda_\phi > 0, \phi = \sqrt{\pi/8}\,, 3\sqrt{\pi/8}$

Dimer phase: $\lambda_\phi < 0, \phi = 0, \sqrt{\pi/2}$

domain wall





A competing order is nucleated at a domain wall.

Mean-field theory for J-W fermions

Neelz and Dimer order parameters in terms of left- and right-going JW fermions:

$$n_z(x) = \psi^\dagger_{\mathrm{L}}(x)\,\psi_{\mathrm{R}}(x) + \psi^\dagger_{\mathrm{R}}(x)\,\psi_{\mathrm{L}}(x) = \Psi^\dagger(x)\,\sigma_1\,\Psi(x),$$

$$d(x) = -\mathrm{i}\psi^\dagger_{\mathrm{L}}(x)\,\psi_{\mathrm{R}}(x) + \mathrm{i}\psi^\dagger_{\mathrm{R}}(x)\,\psi_{\mathrm{L}}(x) = \Psi^\dagger(x)\,\sigma_2\,\Psi(x),$$

$$\Psi(x) \equiv \begin{pmatrix} \psi_{\mathrm{L}}(x) \\ \psi_{\mathrm{R}}(x) \end{pmatrix}.$$

$$(n_z, d) \leftrightarrow (\sigma_1, \sigma_2)$$

$$\mathcal{H}_{\mathrm{XXZ}} = \mathrm{i}v\left(\psi^\dagger_{\mathrm{L}}\,\partial_x\psi_{\mathrm{L}} - \psi^\dagger_{\mathrm{R}}\,\partial_x\psi_{\mathrm{R}}\right) + g_+\left(:\psi^\dagger_{\mathrm{L}}\,\psi_{\mathrm{L}}: + :\psi^\dagger_{\mathrm{R}}\,\psi_{\mathrm{R}}:\right)^2$$

$$+\, g_-\left(:\psi^\dagger_{\mathrm{L}}\,\psi_{\mathrm{L}}: - :\psi^\dagger_{\mathrm{R}}\,\psi_{\mathrm{R}}:\right)^2 + g_{\mathrm{u}}\left(:\psi^\dagger_{\mathrm{L}}\,\psi^\dagger_{\mathrm{L}}::\psi_{\mathrm{R}}\,\psi_{\mathrm{R}}: + :\psi^\dagger_{\mathrm{R}}\,\psi^\dagger_{\mathrm{R}}::\psi_{\mathrm{L}}\,\psi_{\mathrm{L}}:\right)$$

$$\mathcal{H}_{\mathrm{MF}}(x) := \mathrm{i}v\left(\Psi^\dagger\sigma_3\partial_x\Psi\right)(x) - g_n\,n_z(x)\,\left(\Psi^\dagger\,\sigma_1\,\Psi\right)(x) - g_d\,d(x)\,\left(\Psi^\dagger\,\sigma_2\,\Psi\right)(x)$$

$$\mathcal{H}_{\mathrm{HS}}(x) := g_n\,n_z^2(x) + g_d\,d^2(x)$$

Dirac mass terms

$$\mathcal{H}_{\mathrm{MF}}(x) := iv\left(\Psi^\dagger \sigma_3 \partial_x \Psi\right)(x) - g_n\, n_z(x)\, \left(\Psi^\dagger\, \sigma_1\, \Psi\right)(x) - g_d\, d(x)\, \left(\Psi^\dagger\, \sigma_2\, \Psi\right)(x)$$

Domain wall in the Neelz phase (d=0)

$n_z(x) = n_z^0\, \tanh(x/\xi)$ ➡ A zero mode localized at x=0: eigenstate of σ_2

Dimer order

Domain wall in the Dimer phase (n_z=0)

$d(x) \;=\; d_0\, \tanh(x/\xi)$ ➡ A zero mode localized at x=0: eigenstate of σ_1

Neelz order

O(2) vector field

$$\boldsymbol{n} := \;(d, n_z) \qquad\qquad \boldsymbol{n}^2 = 1$$

Integrate out fermions

$$\int D\Psi^\dagger D\Psi e^{-\int d\tau \int dx\left(\Psi^\dagger \partial_\tau \Psi + H_{\mathrm{MF}}\right)} \propto e^{-S_{\mathrm{eff}}[\boldsymbol{n}]}$$

$$S_0 = \frac{1}{2g}\int \mathrm{d}\tau \int \mathrm{d}x\, \left[(\partial_\tau \boldsymbol{n})^2 + (\partial_x \boldsymbol{n})^2\right] \Longrightarrow S_0 = \frac{1}{2g}\int \mathrm{d}\tau \int \mathrm{d}x\, \left[(\partial_\tau \varphi)^2 + (\partial_x \varphi)^2\right]$$

$$\boldsymbol{n} = (d, n) = (\cos\varphi, \sin\varphi)$$

$$\mathrm{U}(1) \longrightarrow \mathbb{Z}_4 \longrightarrow \mathbb{Z}_2 \times \mathbb{Z}_2$$

$$S_0 = \frac{1}{2g} \int \mathrm{d}\tau \int \mathrm{d}x \, [(\partial_\tau \varphi)^2 + (\partial_x \varphi)^2] \; \underline{-\lambda_4 \cos(4\varphi)} \; + \underline{\lambda_\varphi \cos(2\varphi)}$$

$$\boldsymbol{n} = (d, n) = (\cos\varphi, \sin\varphi)$$

$\lambda_4 > 0$

Neelz order: $\varphi = \frac{\pi}{2}, \frac{3\pi}{2} \pmod{2\pi}$ $\qquad \lambda_\varphi > 0$

Dimer order: $\varphi = 0, \pi \pmod{2\pi}$ $\qquad \lambda_\varphi < 0$

φ is a 2π periodic field vortices & antivortices

charge ± 2 vortices

$$\mathcal{L}_{\mathrm{vtx}} := \lambda_\vartheta \cos(4\pi\vartheta) \qquad \partial_x \varphi = +\mathrm{i}g \, \partial_\tau \vartheta, \qquad \partial_\tau \varphi = -\mathrm{i}g \, \partial_x \vartheta,$$

In the operator formalism $[\varphi(x), \vartheta(y)] = \mathrm{i}\Theta(y - x)$

$$e^{\mathrm{i}4\pi\vartheta(y)} \varphi(x) e^{-\mathrm{i}4\pi\vartheta(y)} = \varphi(x) + 4\pi \, \Theta(y - x)$$

kink

Note $\Theta(0) = \frac{1}{2}$ at $x = y$.

Effective field theory

$$\mathcal{L}_{\mathbb{Z}_2\times\mathbb{Z}_2} := -\mathrm{i}\partial_x\vartheta\,\partial_\tau\varphi + \frac{g}{2}(\partial_x\vartheta)^2 + \frac{1}{2g}(\partial_x\varphi)^2$$

$$+\lambda_\vartheta\ \cos(4\pi\vartheta) + \lambda_\varphi\ \cos(2\varphi) - \lambda_4\ \cos(4\varphi)$$

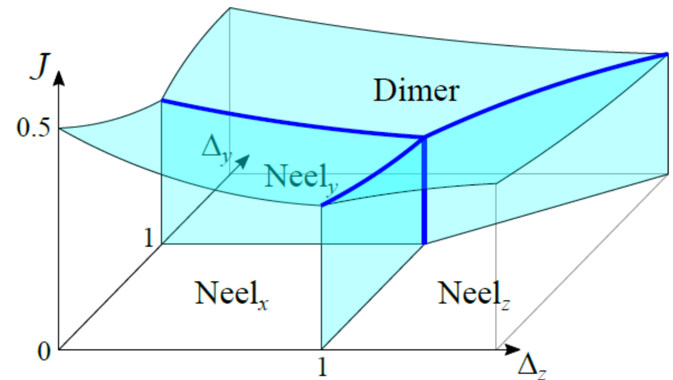
$$(\varphi,\vartheta) = \left(\sqrt{2\pi}\phi, \theta/\sqrt{2\pi}\right)$$

$$\mathcal{H}_0 + \mathcal{H}_{JJ} = \frac{v}{2}\left[\frac{1}{\eta}\left(\partial_x\theta\right)^2 + \eta\left(\partial_x\phi\right)^2 + \lambda_\phi\ \cos(\sqrt{8\pi}\,\phi)\right] + \frac{A}{\mathrm{a}^2}\left(\Delta_y - 1\right)\cos(\sqrt{8\pi}\,\theta)$$

$$S_l^+\,S_{l+1}^+ + S_l^-\,S_{l+1}^-$$

Summary

- 1D J1-J2 XXZ spin chain has four long-range ordered phases for J2/J1<0.5: Dimer, Neelx, Neely, Neelz.



- The transitions between the LRO phases are continuous and in the c=1 Gaussian universality class.

$$H_\eta := \frac{v}{2} \int \mathrm{d}x \left[\frac{1}{\eta} (\partial_x \theta)^2 + \eta (\partial_x \phi)^2 \right]$$

- The perturbative RG analysis of the Gaussian model perturbed by current-current interactions is performed at the SU(2) symmetric point.
- Numerical calculations have confirmed the theory.

- 3D generalization
  Dirac fermion with 6 mass terms (3 Neel & 3 VBS) NLSM with a WZ term
  Neel order is nucleated in the core of a monopole of VBS order parameter, etc.