# Exotic ‘superuniversal’ quantum phase transitions in two dimensions

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Talk @ Frustrated Magnetism

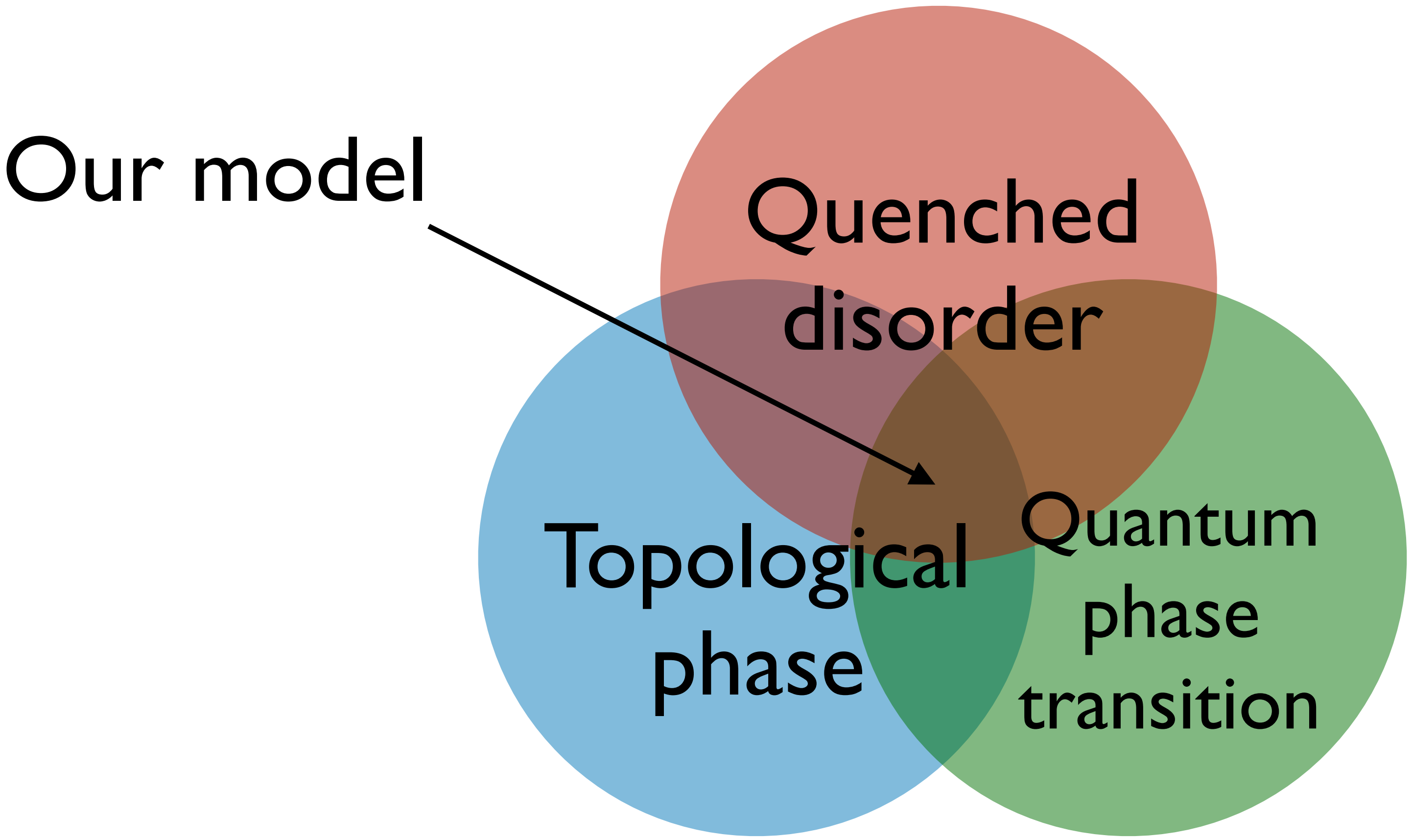
# Collaborators




Sid (Oxford)   Drew (UT Austin)   Romain (Amherst)   Snir (Hebrew)

ref.) **BK**, S.A. Parameswaran, A.C. Potter, R. Vasseur, and S. Gazit, “‘Superuniversal’ infinite-randomness deconfinement transitions in two dimensions”, in preparation

# Main Goal

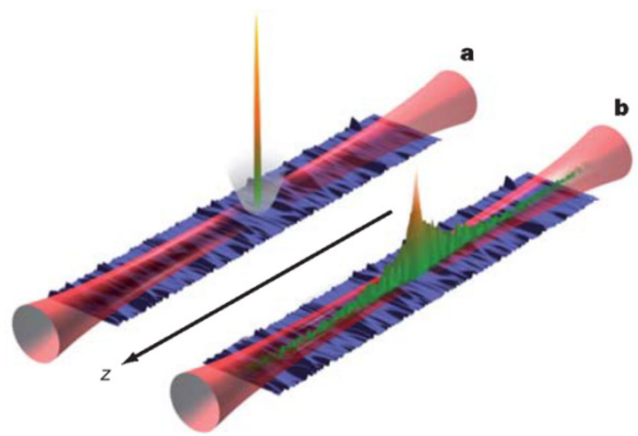



Using a) strong disorder RG & b) quantum Monte Carlo to understand exotic quantum phase transitions

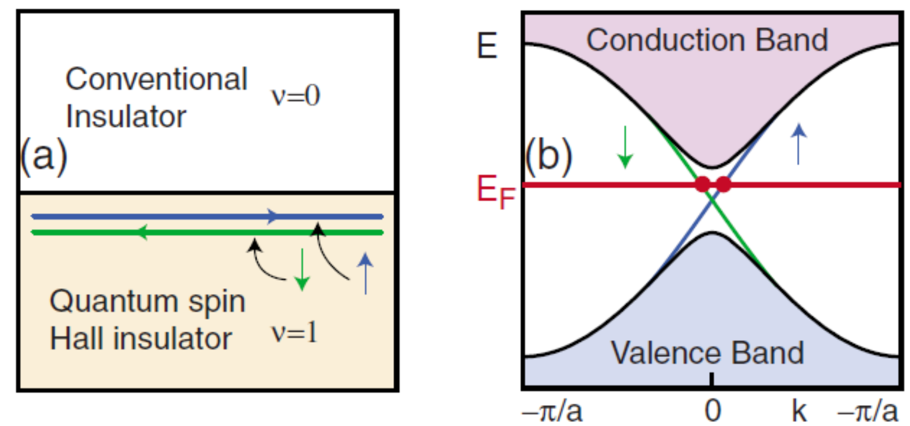
# Introduction

# Quantum Phases of Matter

Hot Quantum

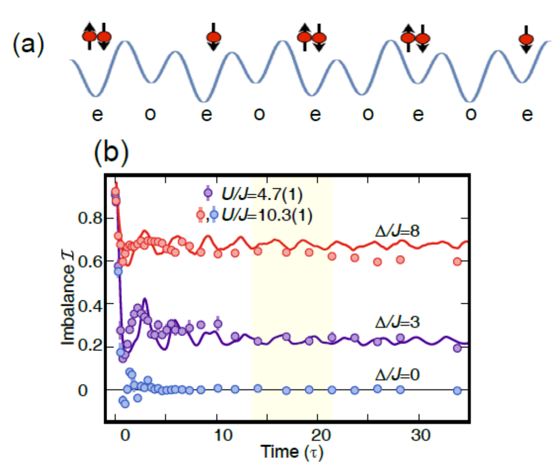



Anderson Localized

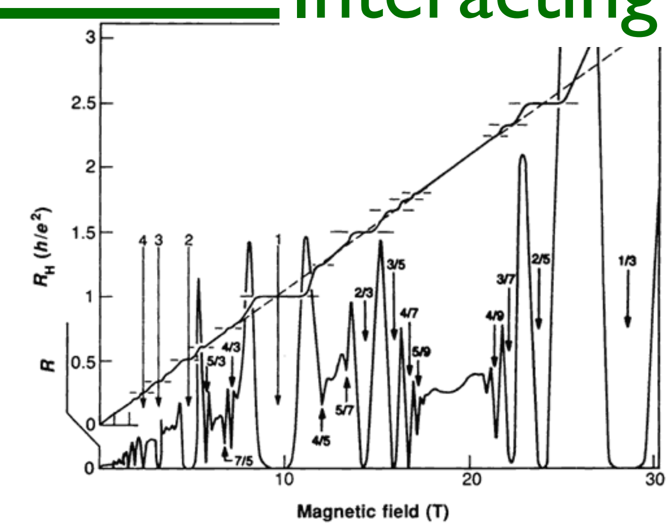



Topological Insulator

Interacting Quantum



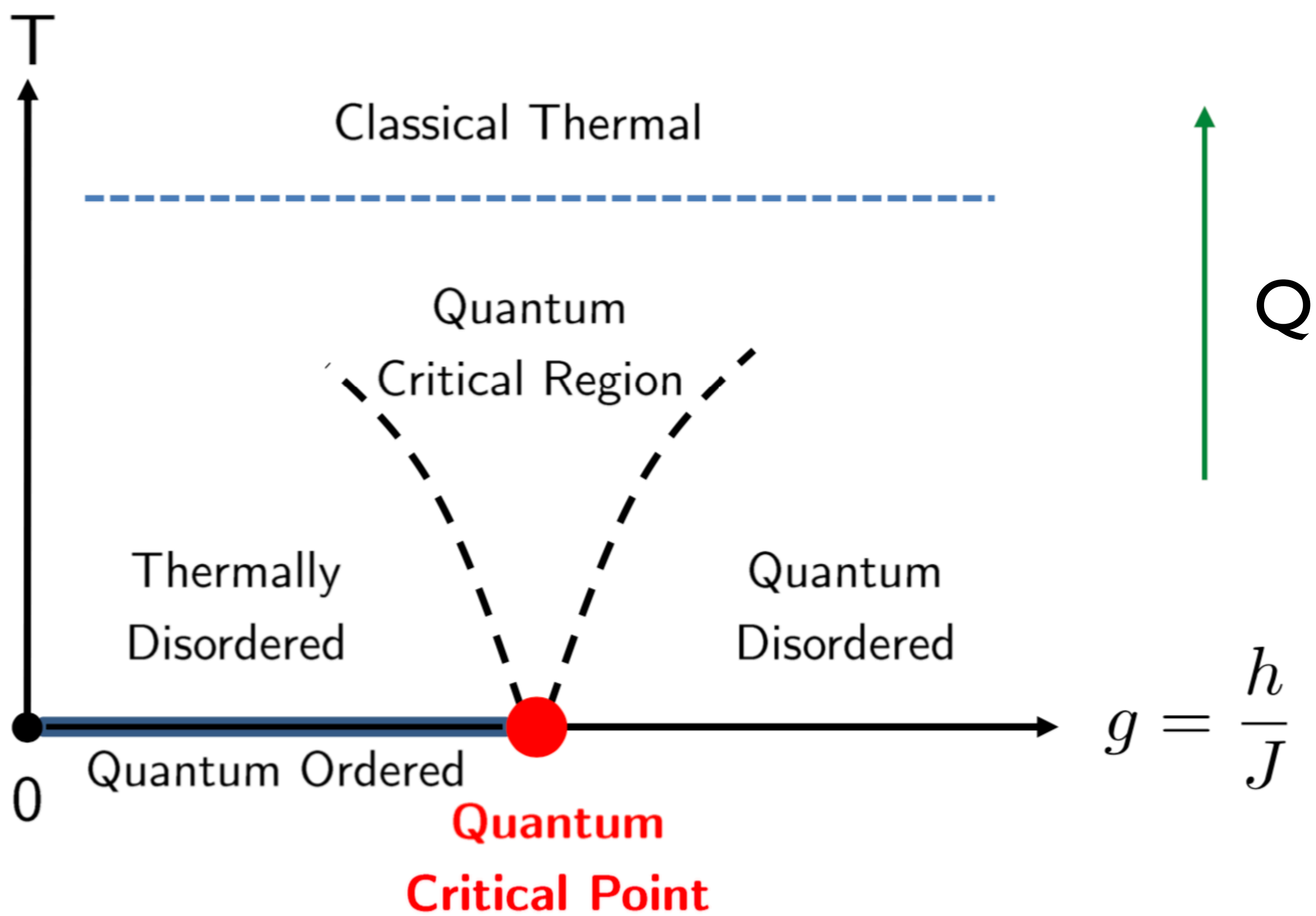

Many-Body Localized




Fractional quantum Hall

# (Quantum) Phase Transition (T>0)

Phase diagram of 1D transverse field Ising model

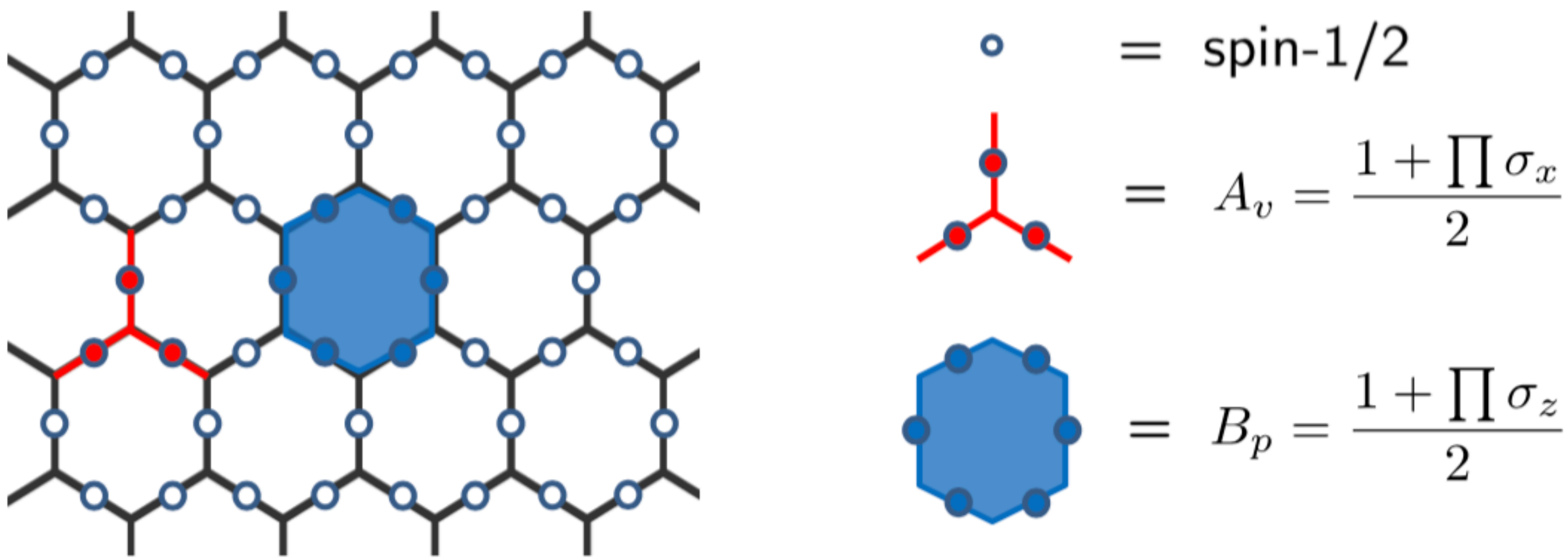



Crossover from
Quantum phase to Classical phase

Add disorder to keep “quantumness” at large T

# Kitaev's
# quantum double model

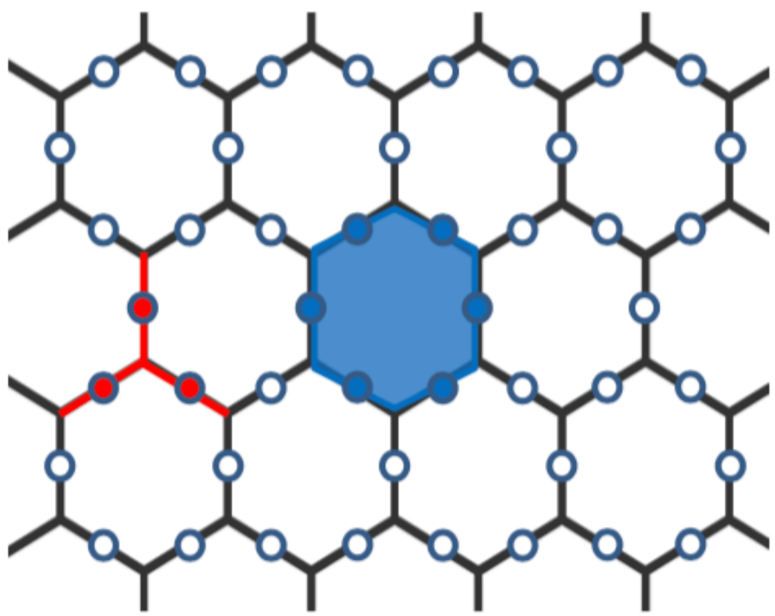
# Toric Code




Hamiltonian: $H = -J_v \sum_v A_v - J_p \sum_p B_p \qquad J_v, J_p > 0$

sum of commuting projectors!

# Toric Code - Ground State(s)

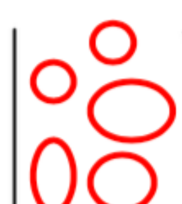


 = spin-1/2

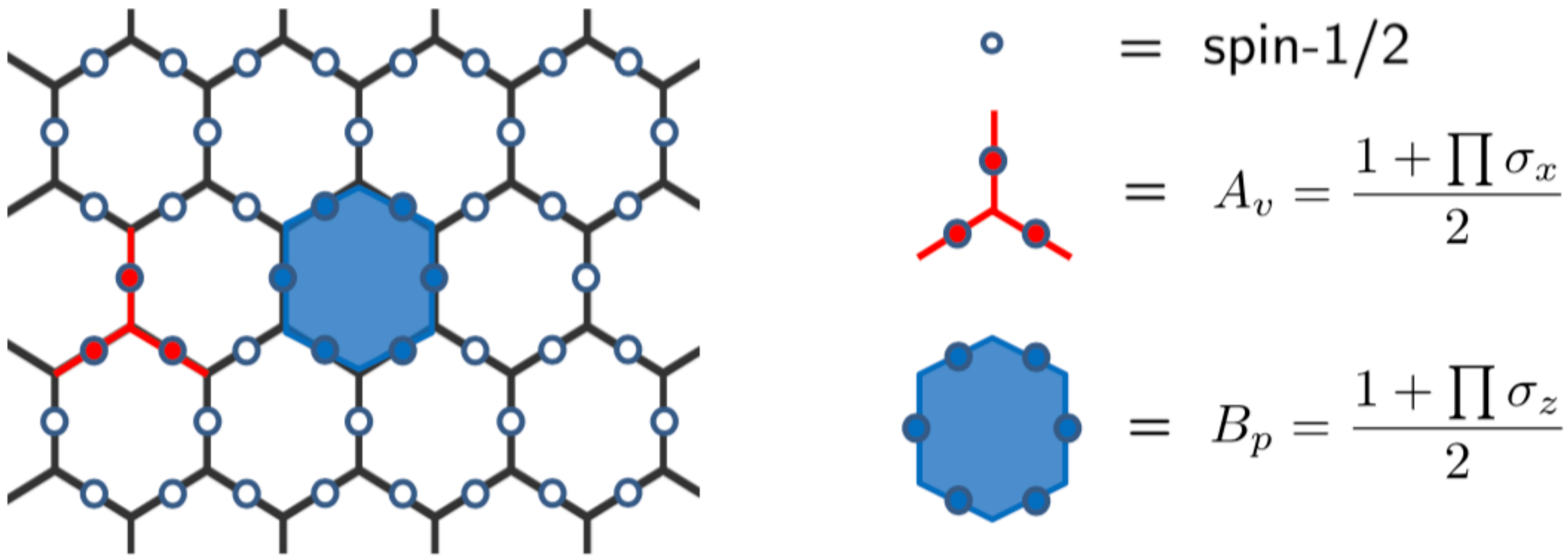
 $= \; A_v = \dfrac{1 + \prod \sigma_x}{2}$

 $= \; B_p = \dfrac{1 + \prod \sigma_z}{2}$

Hamiltonian: $H = -J_v \sum_v A_v - J_p \sum_p B_p \qquad J_v, J_p > 0$

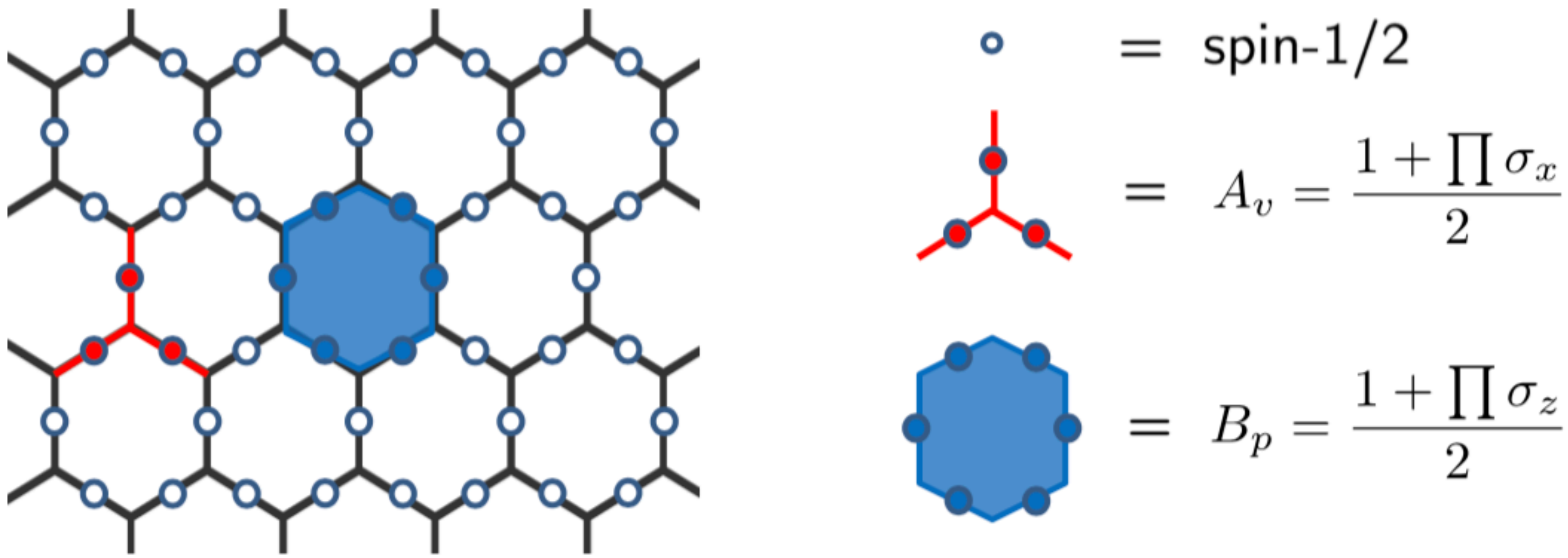
Ground state(s): $A_v = B_p = 1 \qquad |\Psi\rangle = \sum_{\text{loops}} \Big|$  $\Big\rangle$

# Toric Code = $Z_2$ gauge theory




$H = -J_p \sum_p B_p$ with gauge constraints $A_v = 1$

# Toric Code = $Z_2$ gauge theory

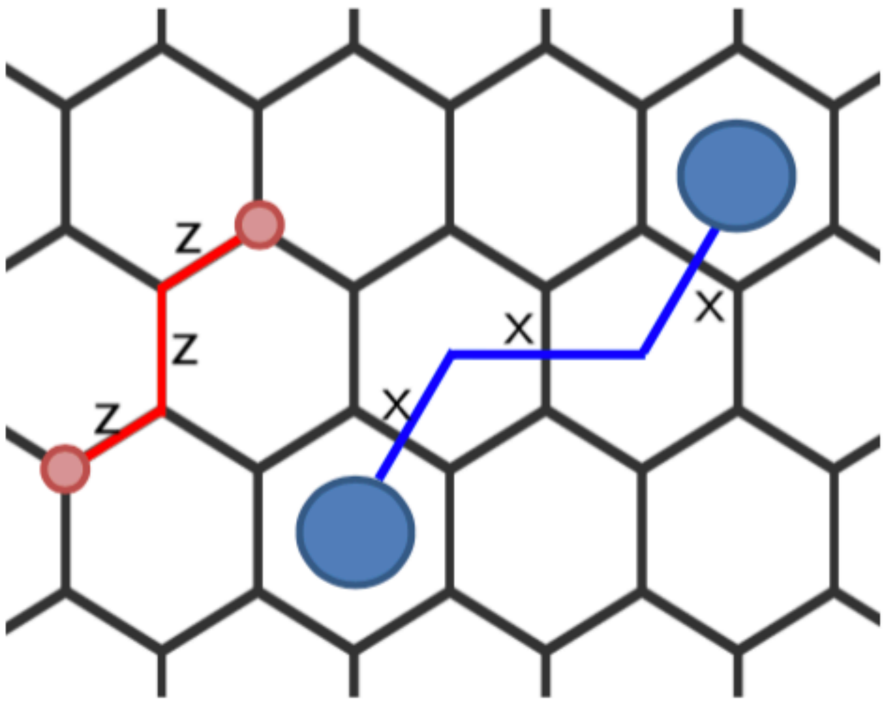


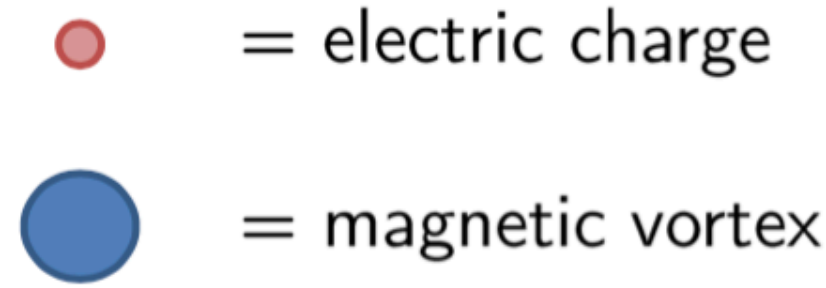

$H = -J_v \sum_v A_v$ with gauge constraints $B_p = 1$

via E&M duality[b]

[b] Buerschaper, Christandl, Kong, & Aguado, Nucl. Phys. B (2013)

# Toric Code - Excitations



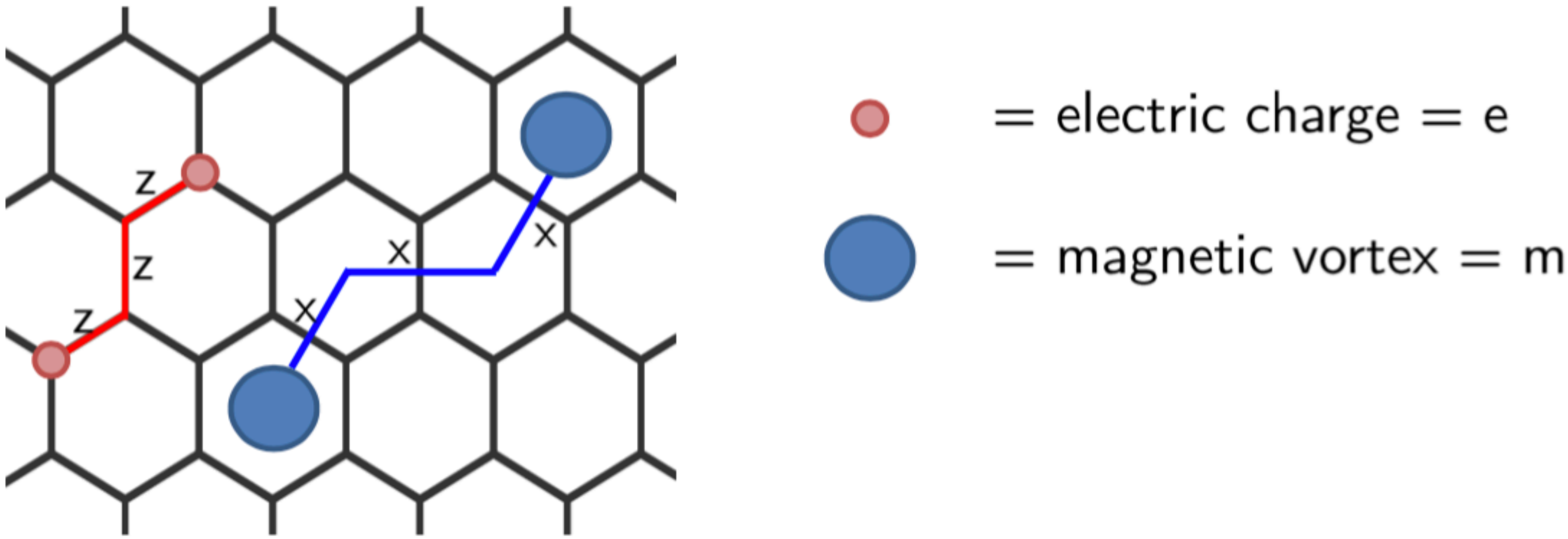




Electric charge lives on a vertex ($A_v = 0$)

Magnetic vortex lives on a plaquette ($B_p = 0$)

These are created by topological "string operators"

# Toric Code - Excitations



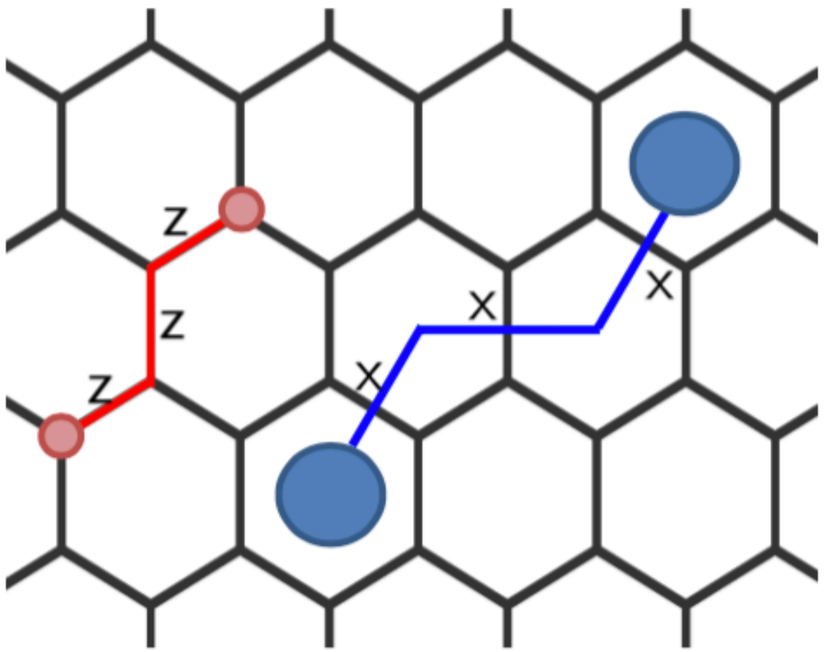

Anyonic excitations: $1,\ e,\ m,$ and $\epsilon = e \times m$

Fusion rules: $e \times e = 1 \quad e \times m = \epsilon$

$m \times m = 1 \quad e \times \epsilon = m$

$\epsilon \times \epsilon = 1 \quad m \times \epsilon = e$

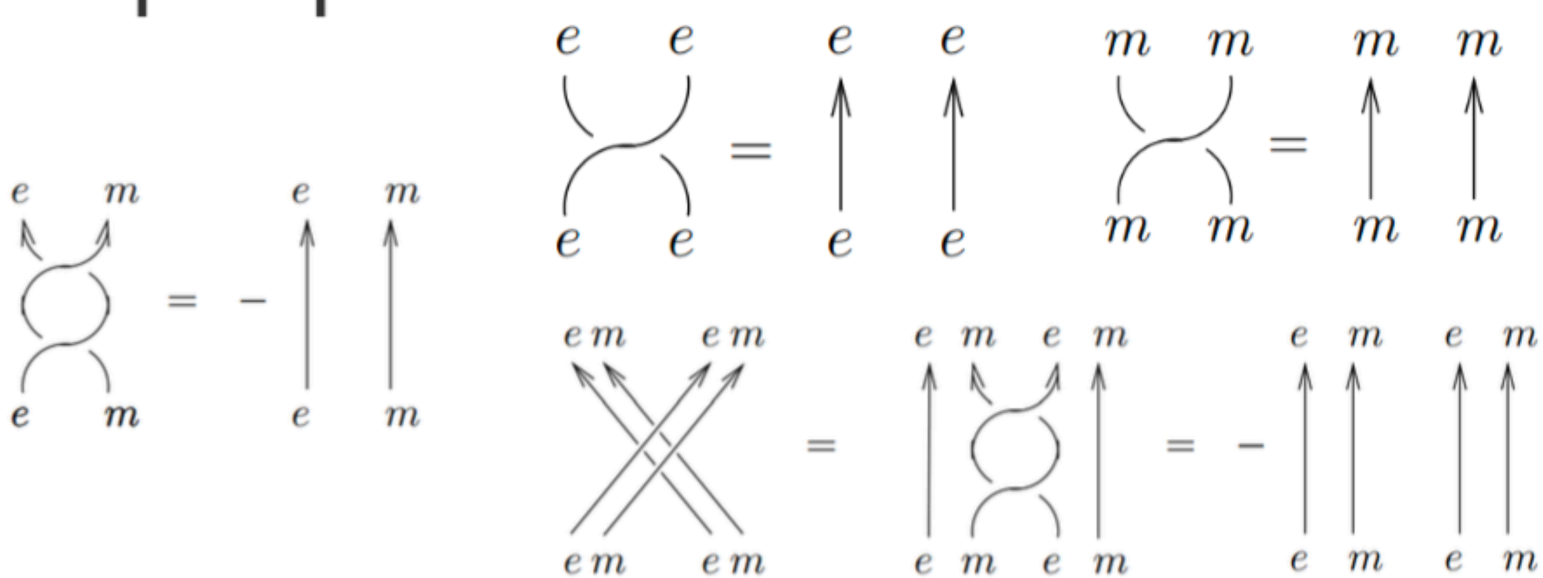
# Toric Code - Excitations




= electric charge = e

= magnetic vortex = m

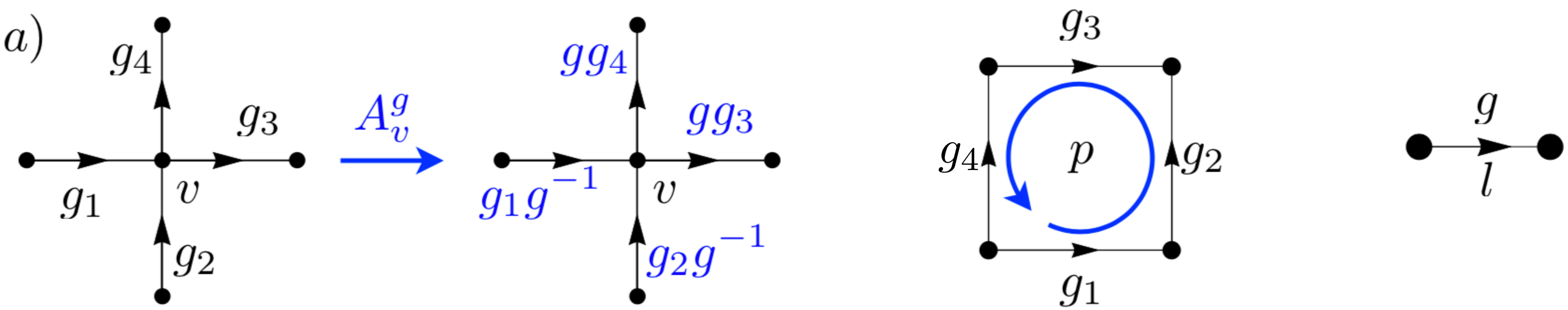
Braidings:



# Quantum Double Model

For any finite group G,

(a)




$$A_v = \sum_g \frac{1}{|G|} A_g^v \qquad B_p^e = \delta_{e, g_1 g_2 g_3^{-1} g_4^{-1}} \qquad C_l = \delta_{g,e}$$

Hamiltonian: $H = -J_v \sum_v A_v - J_p \sum_p B_p \qquad J_v, J_p > 0$

sum of commuting projectors!

# Quantum Double Model

We consider $\boxed{H = -J_v \sum_v A_v \text{ with } B_p = 1}$

In general, the model is different from[b]

$$H = -J_p \sum_p B_p \text{ with } A_v = 1$$

Nonetheless, our model realizes non-abelian anyons (labelled by irreducible representation of G)

[b] Buerschaper, Christandl, Kong, & Aguado, Nucl. Phys. B (2013)

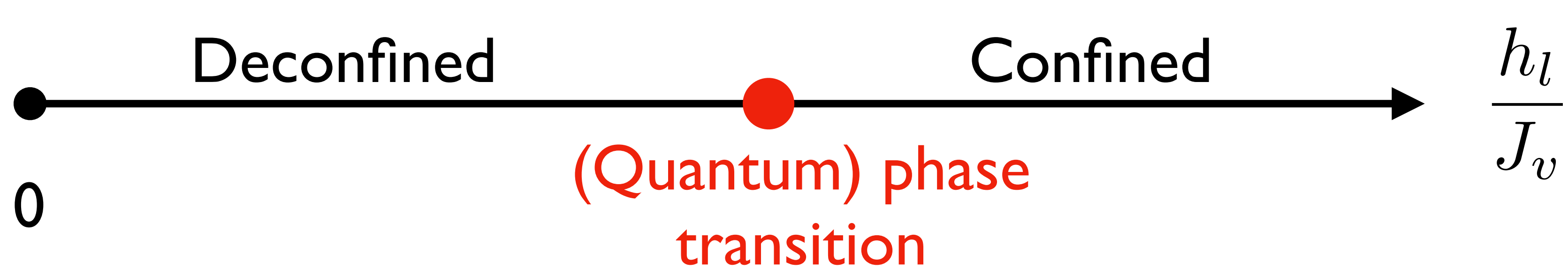
# (Quantum) Phase Transition

Topological-to-trivial (quantum) phase transition by adding "transverse-field" term

$$\boxed{H = -J_v \sum_v A_v - h_l \sum_l C_l} \quad (J_v, h_l > 0)$$

Gauge constraints

$$\boxed{B_p = 1}$$



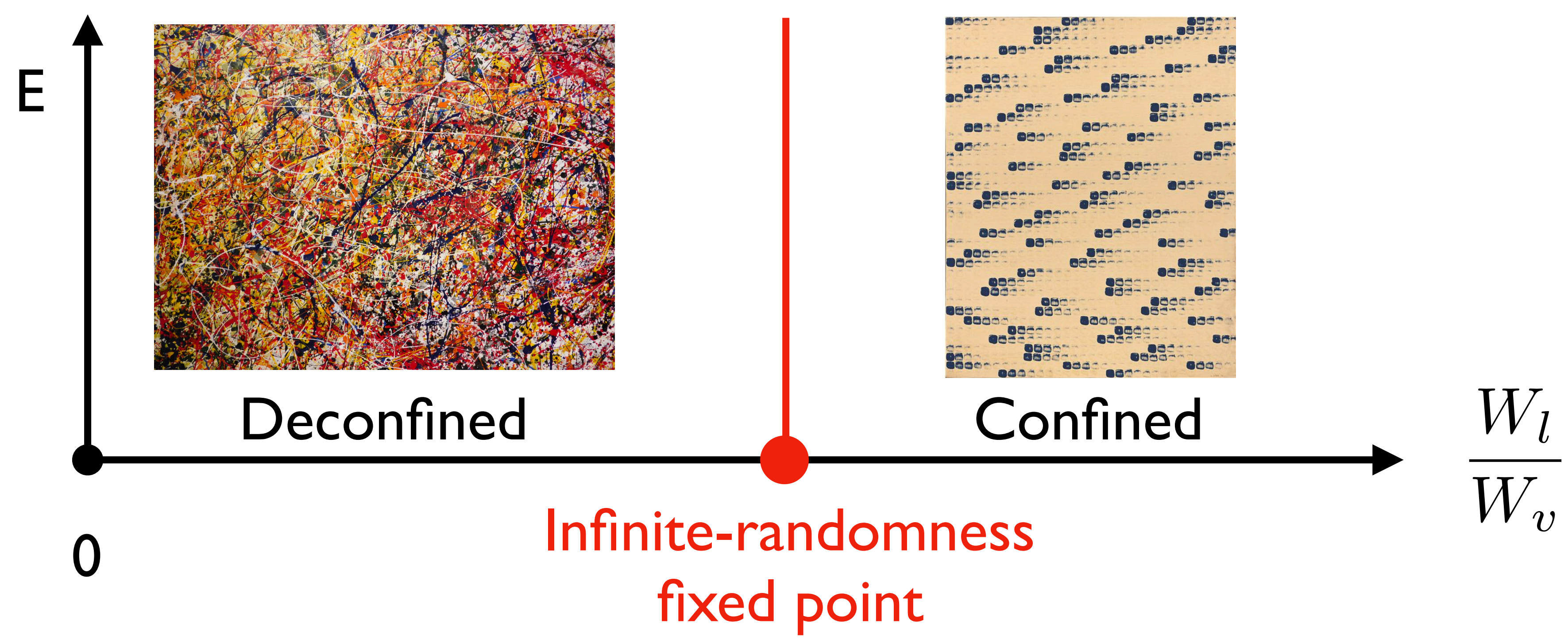
# (Quantum) Phase Transition

Introduce disorder in coupling constants

$$\boxed{H = -\sum_v J_v A_v - \sum_l h_l C_l}$$

$$J_v \in [0, W_v]$$
$$h_l \in [0, W_l]$$

$$\boxed{B_p = 1}$$



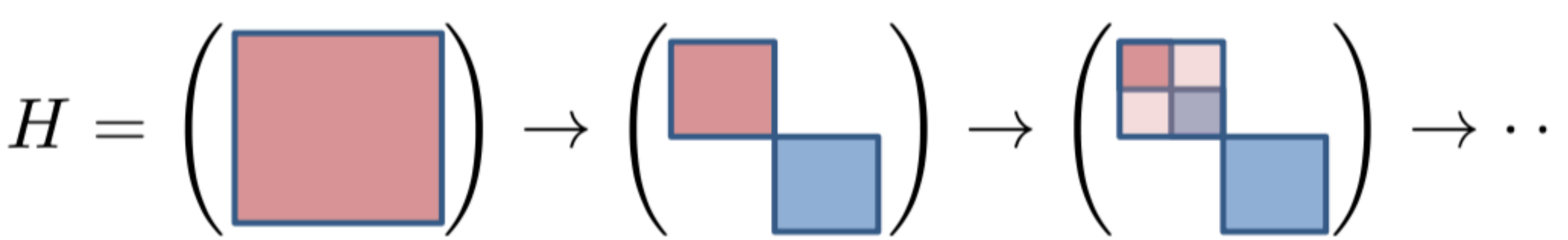
# Strong disorder RG

# Strong Disorder RSRG

1) Pick the *strongest term* in the Hamiltonian

2) Block diagonalize the Hamiltonian (using perturbation theory) and pick the ground-space block

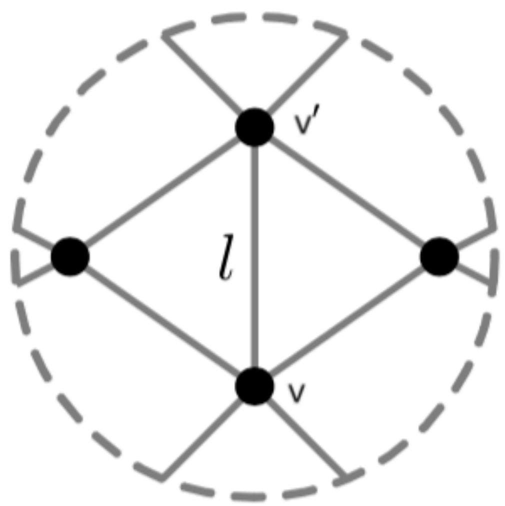
3) Repeat until the renormalized Hamiltonian becomes trivial




Dasgupta & Ma, PRB, (1980); DS Fisher, PRB, (1994).

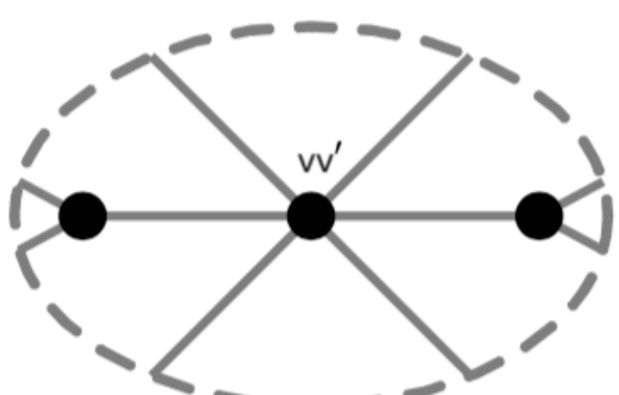
# T-field decimation

1) Pick the *strongest term* in the Hamiltonian




$$H = \cdots \Big( -h_l\, C_l \qquad \Big) \cdots$$

2) Block diagonalize the Hamiltonian

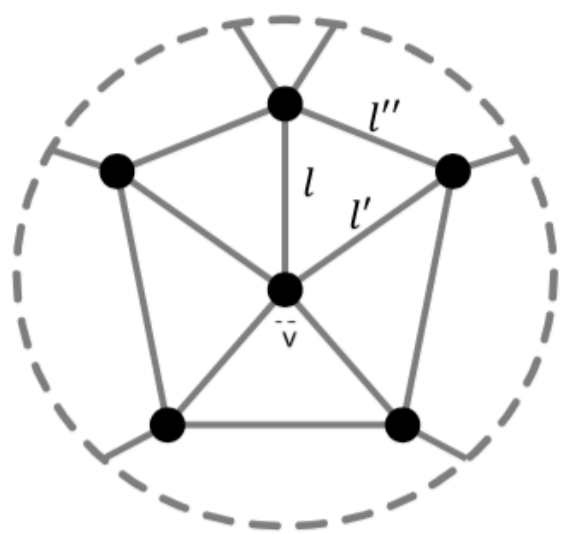



$$H = \cdots \Big( -\frac{2}{|G|} \frac{J_v J_{v'}}{h_l} A_{vv'} \Big) \cdots$$

where $0 < \dfrac{J_v J_{v'}}{h_l} \ll h_l$

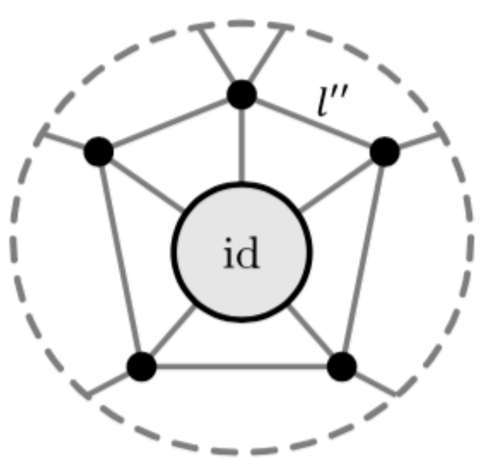
# Vertex term decimation

1) Pick the *strongest term* in the Hamiltonian

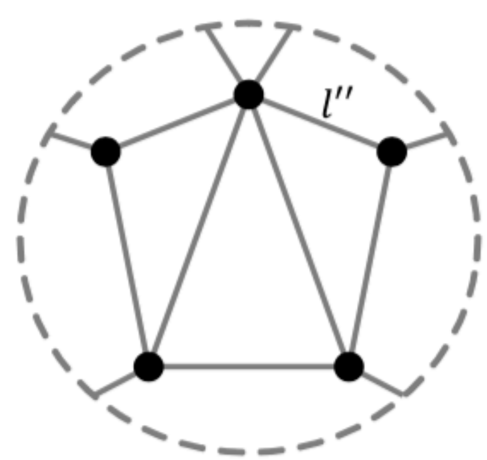



$$H = \cdots \left( - J_v A_v \right) \cdots$$

2) Block diagonalize the Hamiltonian




$\cong$




$$H = \cdots \left( - \frac{2}{|G|} \frac{h_l h_{l'}}{J_v} C_{l''} \right) \cdots$$

where $0 < \frac{h_l h_{l'}}{J_v} \ll J_v$

# Strong Disorder RSRG

Key observations of SD RSRG:

1. Number of states is decreased

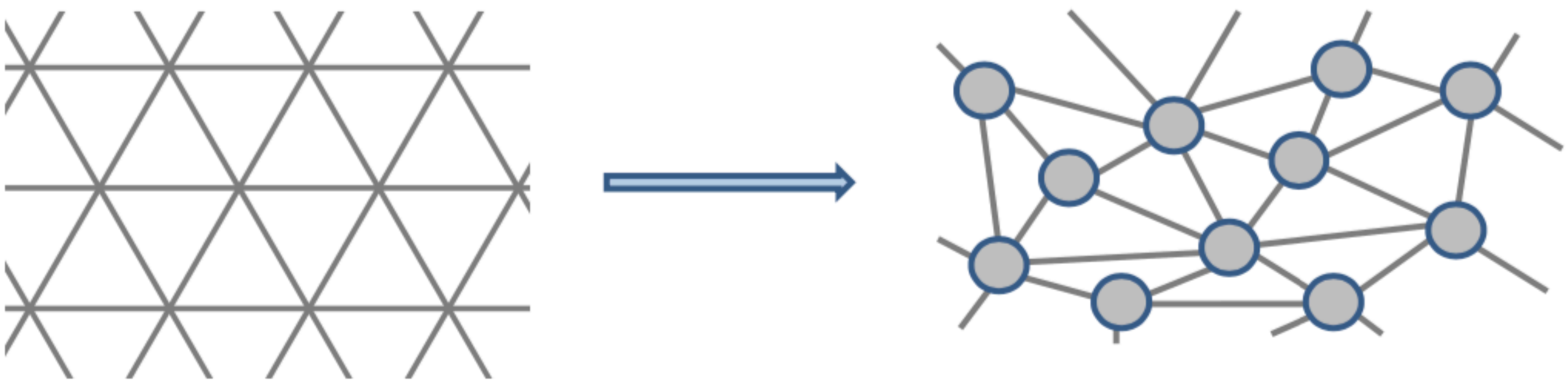
2. Resulting Hamiltonian is self-similar

3. "Disorder" is increased

$$\frac{h_i h_{i+1}}{J_i} \ll h_i, h_{i+1}$$

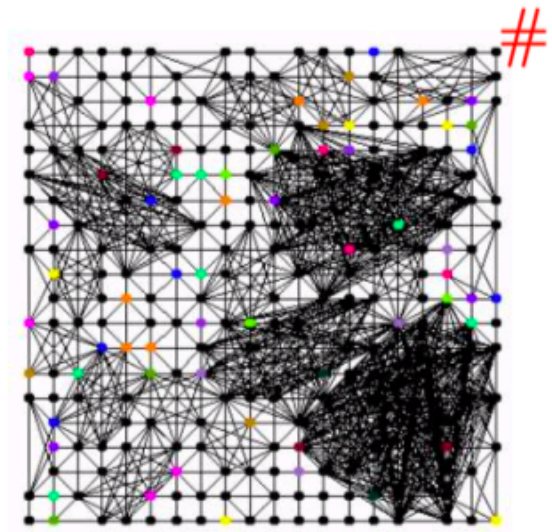
# Strong Disorder RSRG

3) We can keep iterating the RSRG:



# Strong Disorder RSRG

Using Ising-$Z_2$ gauge theory duality, RSRG rules are the same as 2D disordered Ising model




Motrunich, Mau, Huse, DS Fisher, PRB (2000);
# Kovacs & Igloi PRB (2010);
Laumann, Huse, Ludwig, Refael, Trebst, Troyer, PRB (2012)

# Exotic quantum phase transition

Using a tuning parameter $\delta$

the QCP satisfies

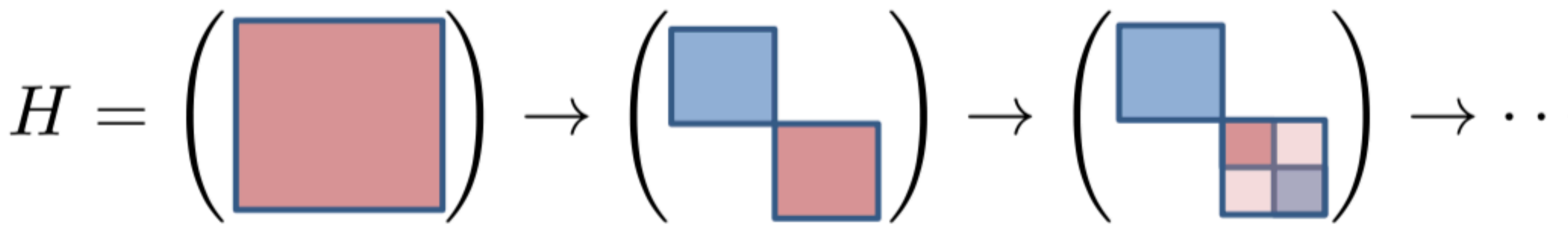
1. "z=∞" critical point: $z \sim \delta^{-\psi\nu}$

2. "Average" correlation function: $\left[G(r)\right]_{\mathrm{dis}} \sim \dfrac{1}{r^{\eta}}$

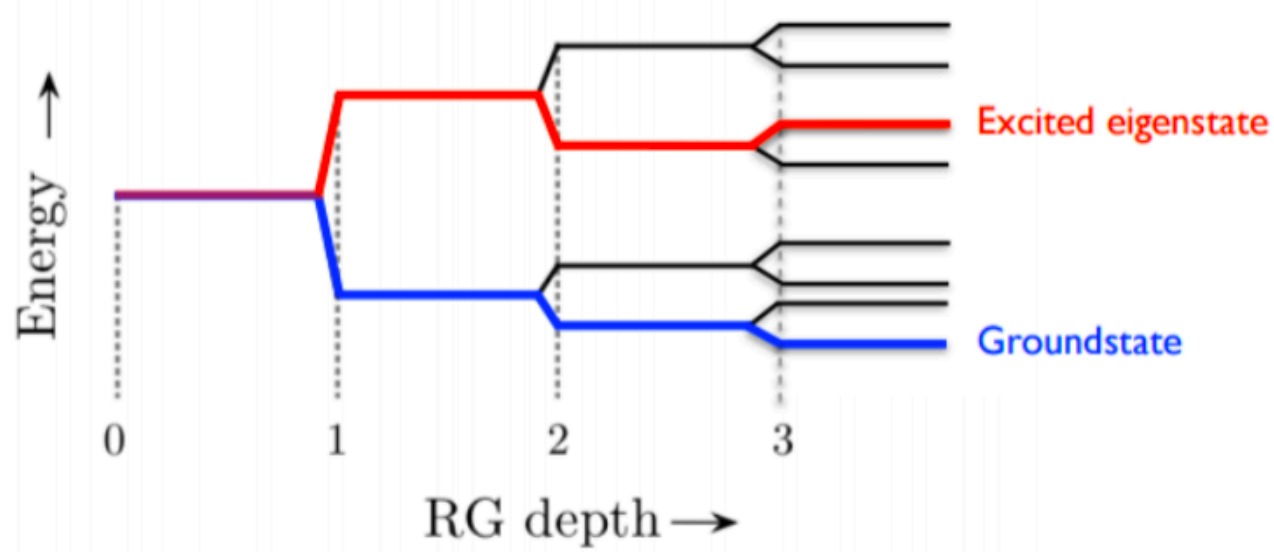
3. "Typical" correlation function: $-\log G_{\mathrm{typ}}(r) \sim r^{\psi}$

# Strong Disorder RSRG-X

Target eXcited state by choosing an *excited subspace* block




The whole many-body spectrum can be constructed




Pekker, Refael, Altman, Demler, Oganesyan, PRX (2014)

Vasseur, Potter, Parameswaran, PRL (2015)

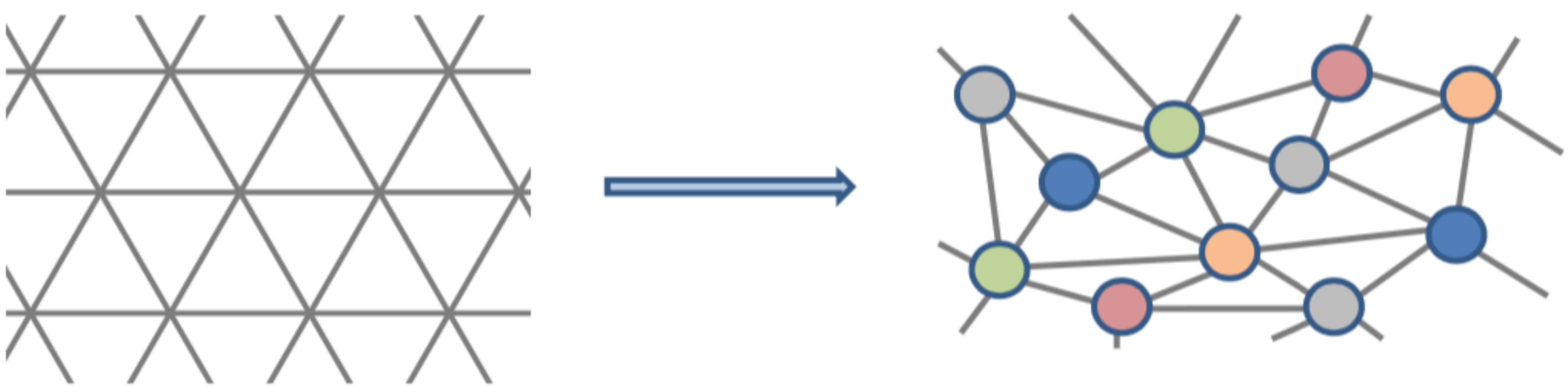
You, Qi, Xu, PRB (2015)

**BK**, Potter, Vasseur, PRB (2016)

# Strong Disorder RSRG-X

For abelian topological order,

We can keep iterating the RSRG-X:



RSRG-X is the same as RSRG except for the sign change in coupling constants

# Quantum Monte Carlo

# Metropolis Monte Carlo

Want to compute the "ensemble" average via "time" average:

$$\langle O \rangle = \sum_C O(C) \frac{e^{-\beta E(C)}}{Z} \approx \frac{1}{T} \sum_{i=1}^{T} O(C_i)$$

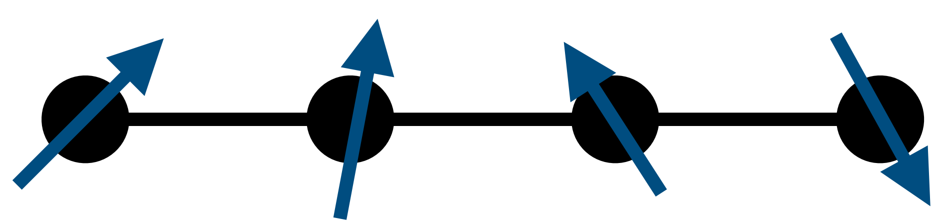
$\{C_i\}$ is generated via Markov moves satisfying

1. Ergodicity, i.e., *all* configuration is generated

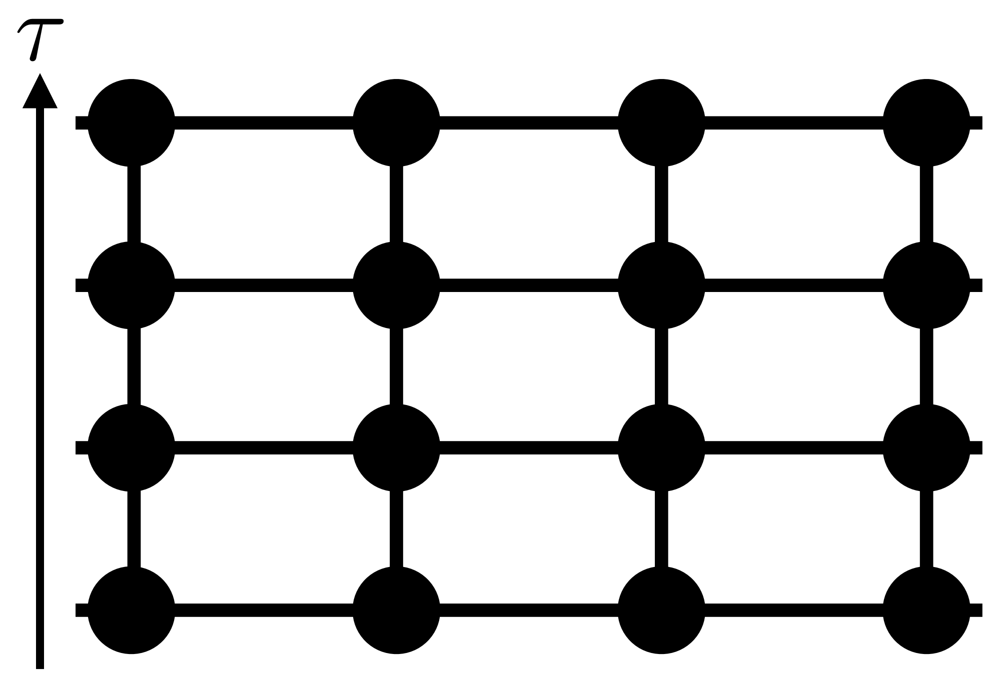
2. Detailed balance, i.e., $P(C'|C)P(C) = P(C|C')P(C')$

# What is "quantum" MC?

*Quantum* Monte Carlo is the classical MC in disguise



$$H = -J\sum_i \delta_{s_i,s_{i+1}} - h\sum_i\sum_{s,s'}\frac{1}{Q}|s\rangle\langle s'|$$




$$H = -J\sum_{x,y}\delta_{s_{x,y},s_{x+1,y}} - \gamma\sum_{x,y}\delta_{s_{x,y},s_{x,y+1}}$$

, where $\gamma = -\log\left(\frac{e^{\Delta\tau h}+Q-1}{e^{\Delta\tau h}-1}\right)$

# Quantum Monte Carlo

QMC on disordered gauge theory is possible, but has no available cluster update

We simulate dual Potts model instead and use Swendsen-Wang cluster update

Moreover, we employ stochastic series expansion (SSE) which is free from Trotter error

# Disordered Potts model via QMC

To identify the critical point, we employ the binder ratio, a dimensionless observable

$$B_{\mathrm{av}} = \frac{1}{2}\left[3 - \frac{\langle M^4 \rangle}{\langle M^2 \rangle^2}\right]_{\mathrm{dis}}$$

which goes to 1 in ferro-, 0 in para-magnetic phase.

@critical point, the binder ratio independent of system size

# Stochastic Series Expansion

Consider $Z = \sum_C W(C) = \mathrm{Tr}\{e^{-\beta H}\}$

$$= \sum_{\alpha_0} \langle \alpha_0 | \sum_{n=0}^{\infty} \frac{\beta^n}{n!} (-H)^n | \alpha_0 \rangle$$

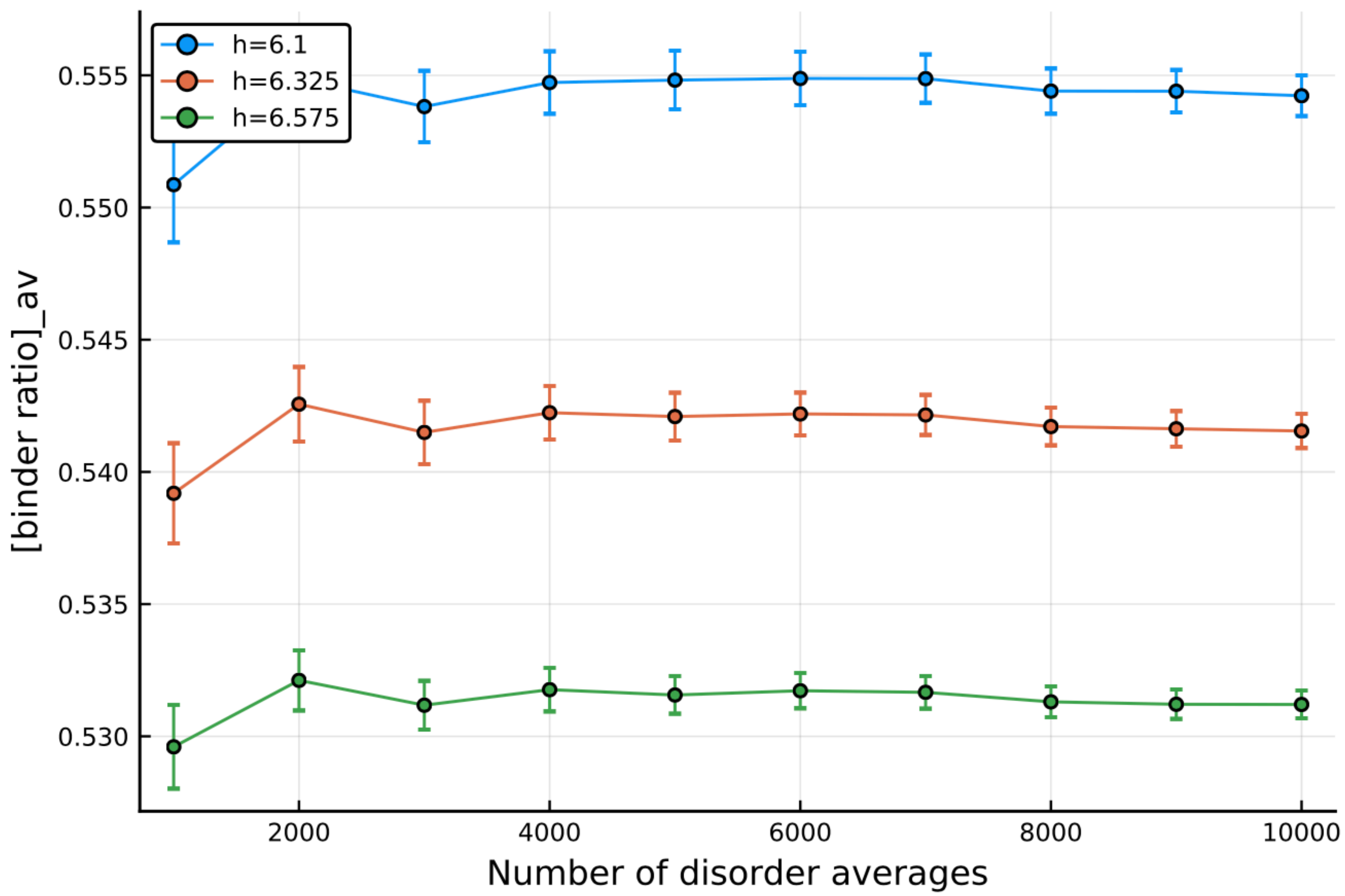
Since H is a sum of local terms, $H = -\sum_I H_I$

$$Z = \sum_{\{\alpha_i\}} \sum_{n=0}^{\infty} \sum_{I_i} \frac{\beta^n}{n!} \prod_{i=1}^{n} \langle \alpha_{i+1} | H_{I_i} | \alpha_i \rangle$$

stochastically sample $\{\alpha_0\}$ and $\{H_{I_i}\}$'s via Markov moves

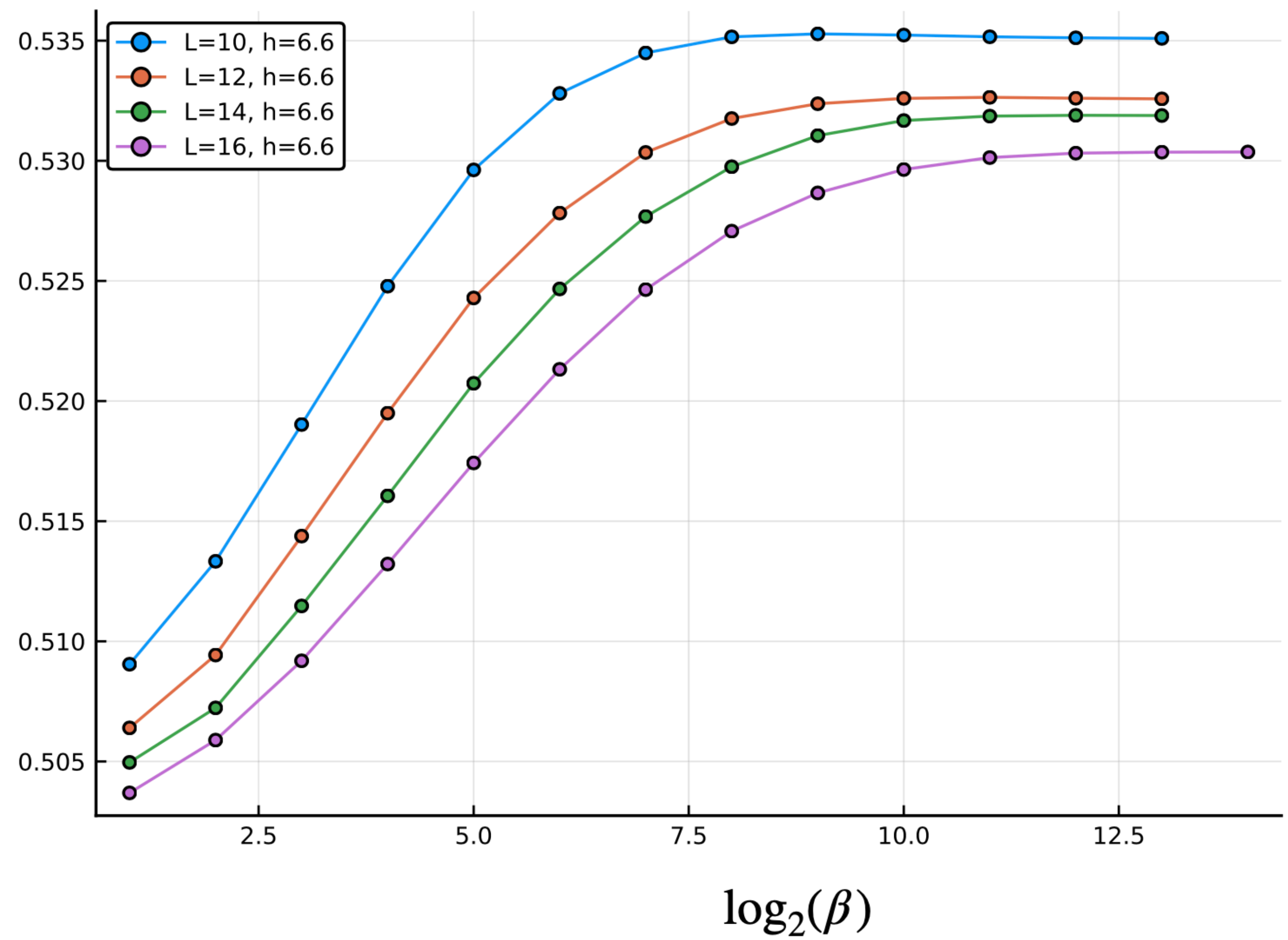
# Difficulties

Disordered systems with random couplings require large number of disorder averages; we use ~10,000

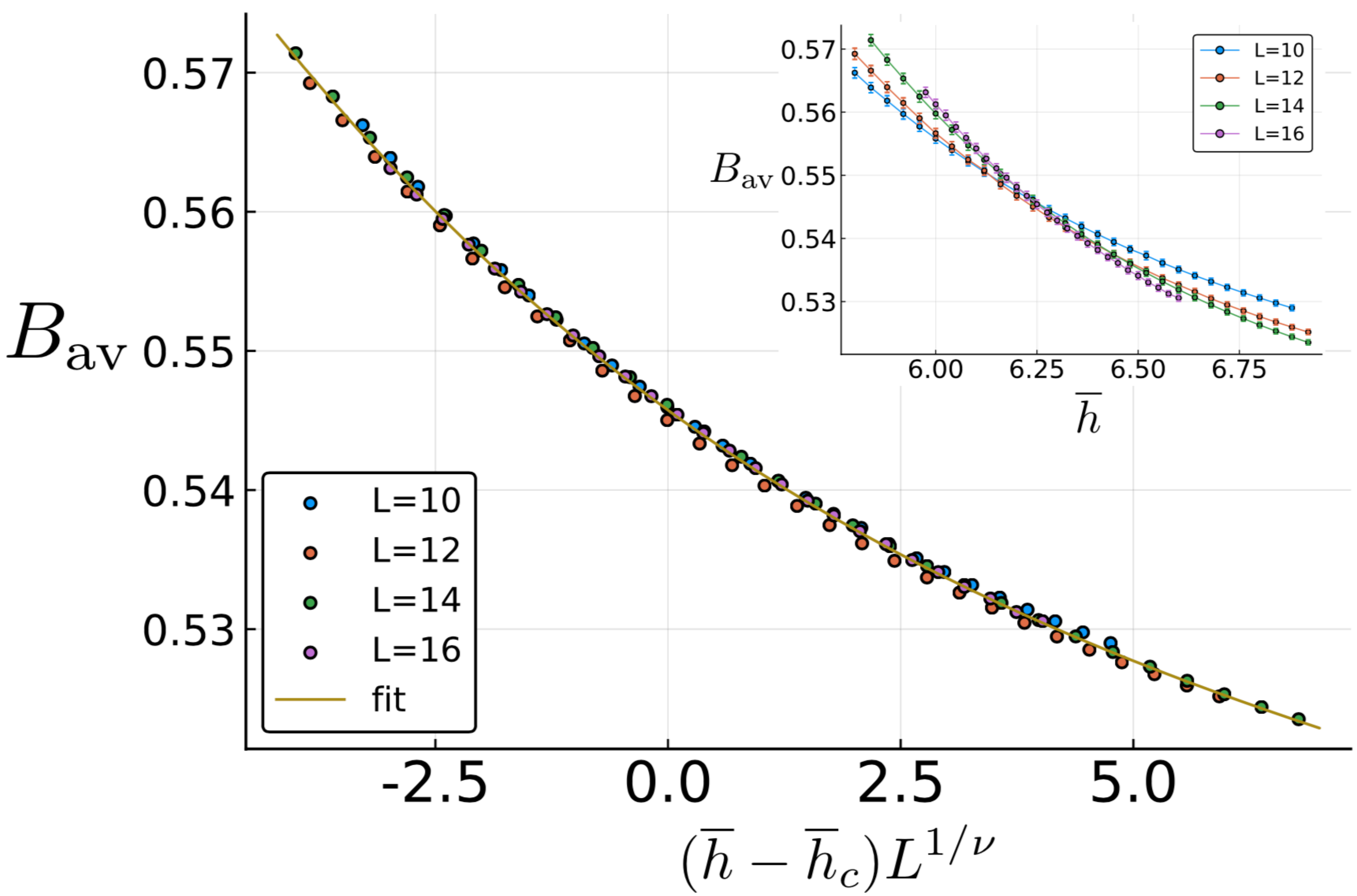


# Difficulties

Critical point is z=∞, i.e., *cannot* use T as a scaling variable; we take a reliable T→0 limit for each L



# Scaling analysis (Q=3)



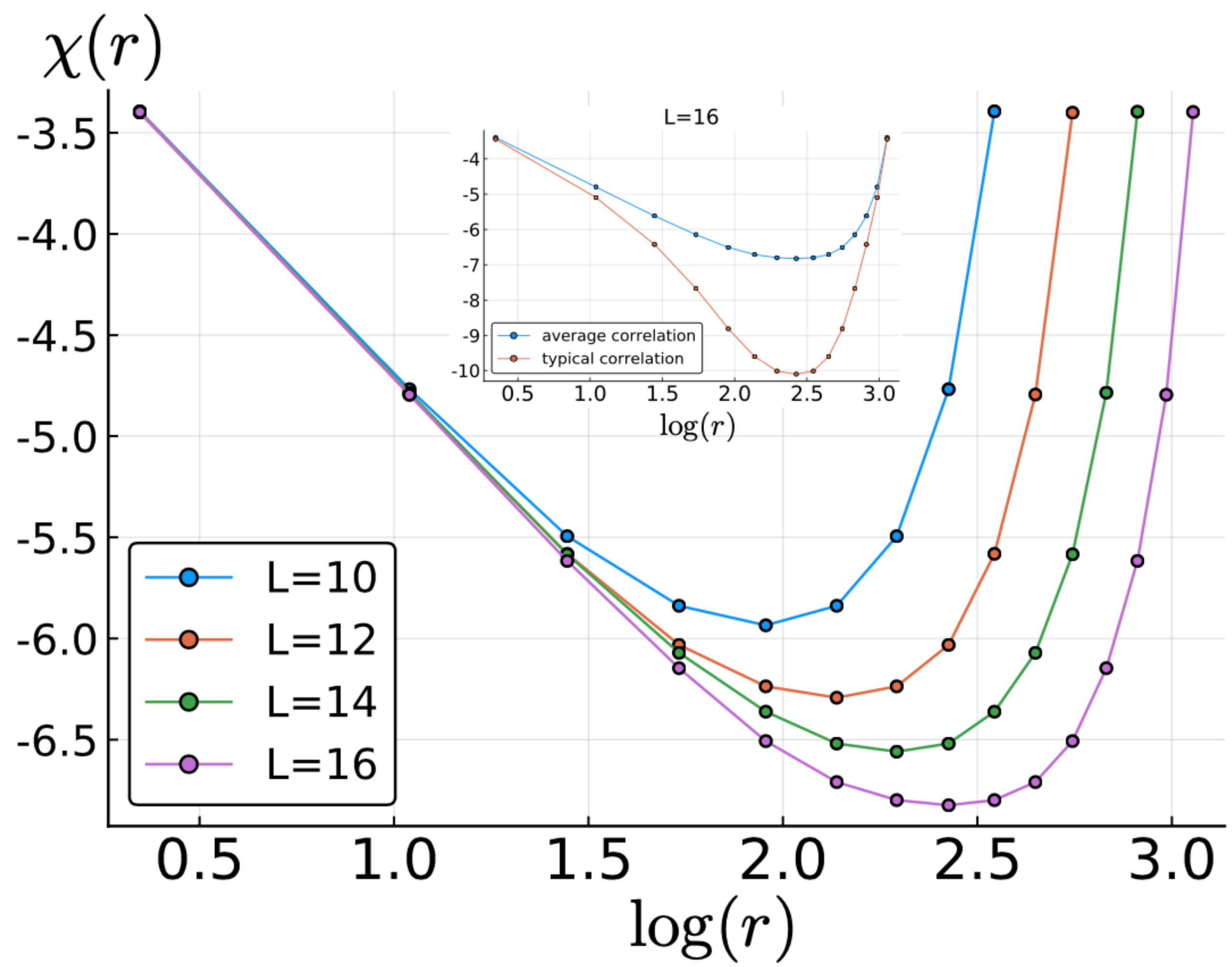

$\nu_{\rm QMC} \approx 1.19$

$\nu_{\rm RSRG} \approx 1.20$

# Correlation function @critical pt

Power law decay of average correlation function

Typical correlation function decays faster



# Conclusion

# Coda

Large class of disordered gauge theories share the common critical point, an infinite randomness fixed point

Provide a strong disorder RSRG picture to understand the infinite randomness fixed point

RSRG results are Independently checked via QMC