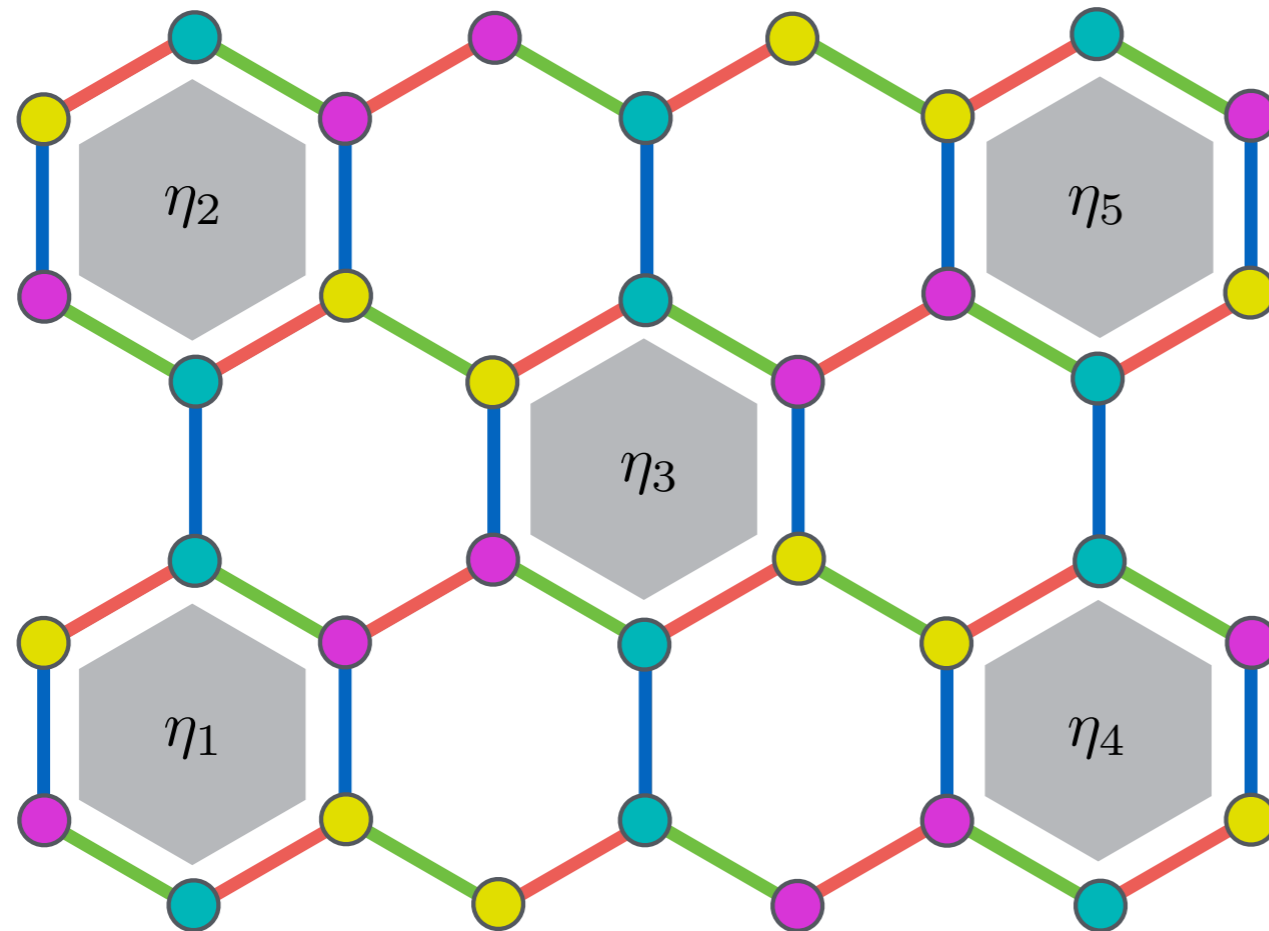


# Spin Dynamics and Plaquette Ordering in the Honeycomb Gamma Model



Phys. Rev. Lett. **122**, 257204 (2019)

Gia-Wei Chern



IBSPCS-KIAS International Workshop - Frustrated Magnetism

Daejeon, Korea, October 18, 2019

# Acknowledgment

- Preetha Saha (UVA)
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  - Zhengtao Wang (UTK)
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- DOE and the Center for Computational Material Spectroscopy and Design at Brookhaven National Lab



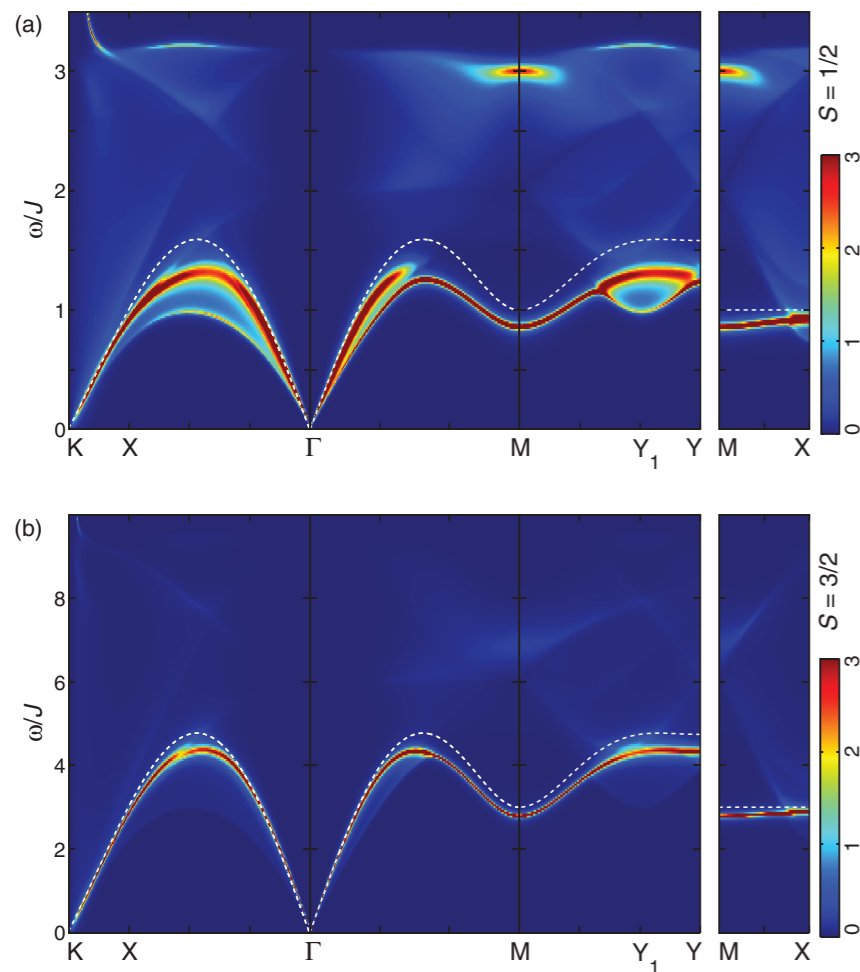


# Outline

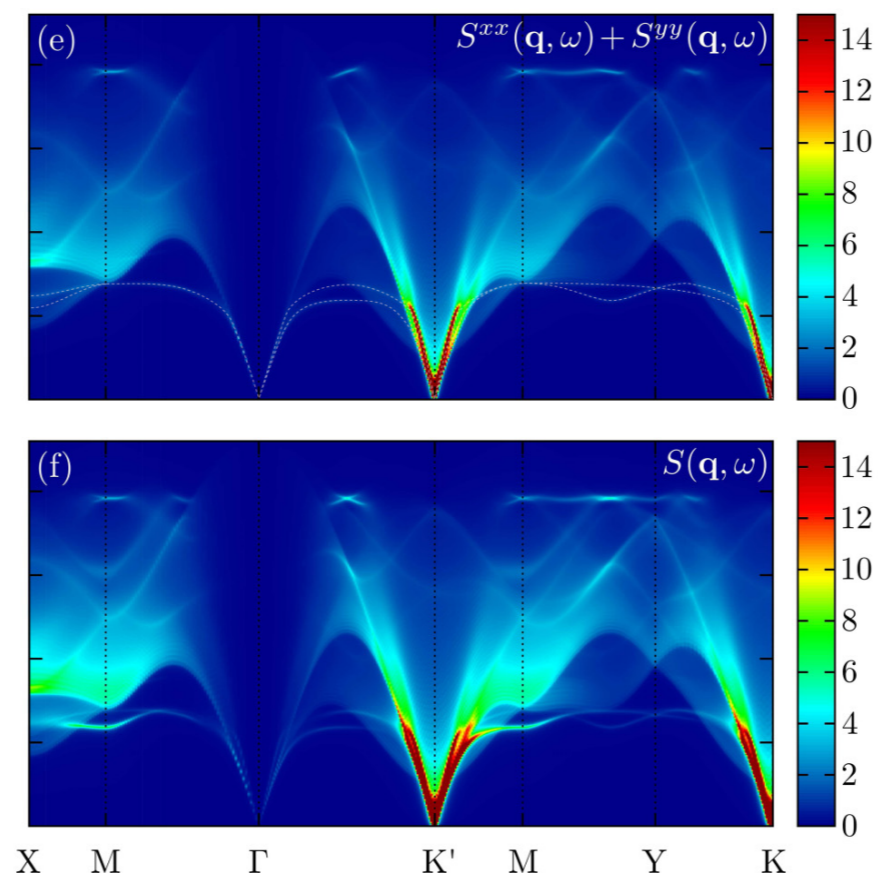
- Numerical methods for dynamical properties of magnetic systems
- Plaquette ordering in the honeycomb Gamma model
  1. Thermal order by disorder in frustrated magnets
  2. Hidden plaquette order in the Gamma model

# Dynamical Structure Factor

- Holstein-Primarkoff  
1/S expansion

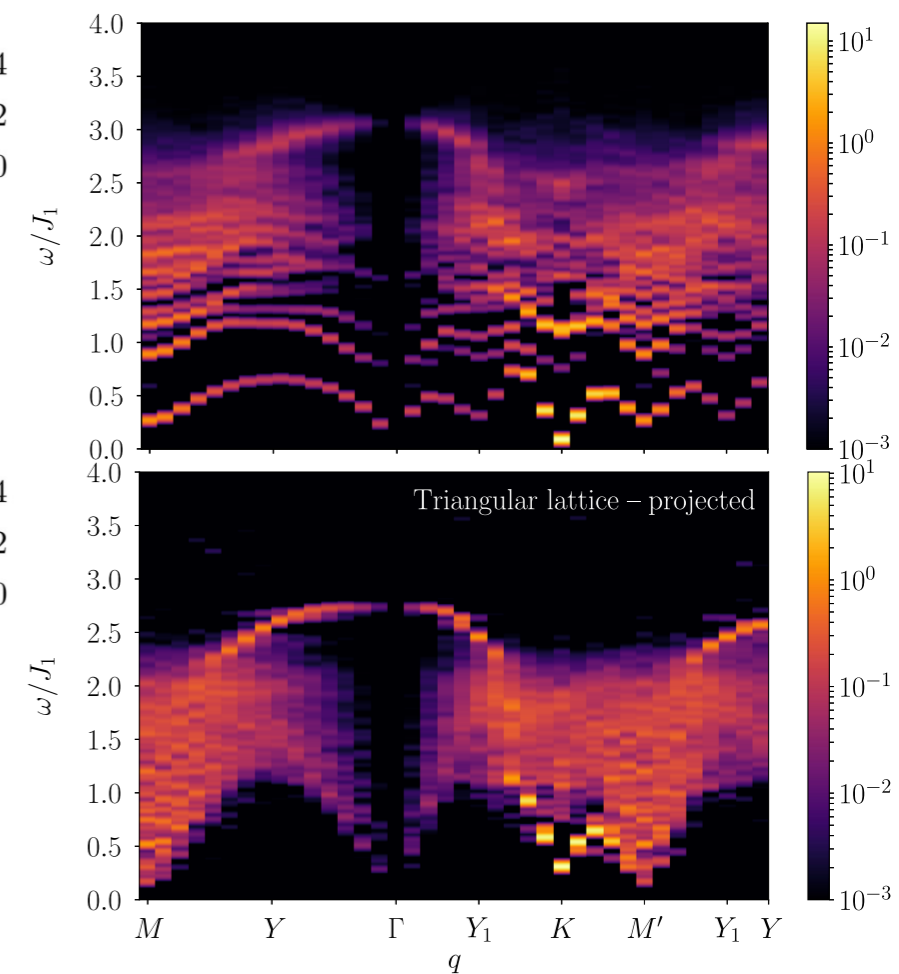


- Schwinger boson



Ghioldi *et al.* PRB **98**, 184403 (2018)

- Variational Monte Carlo

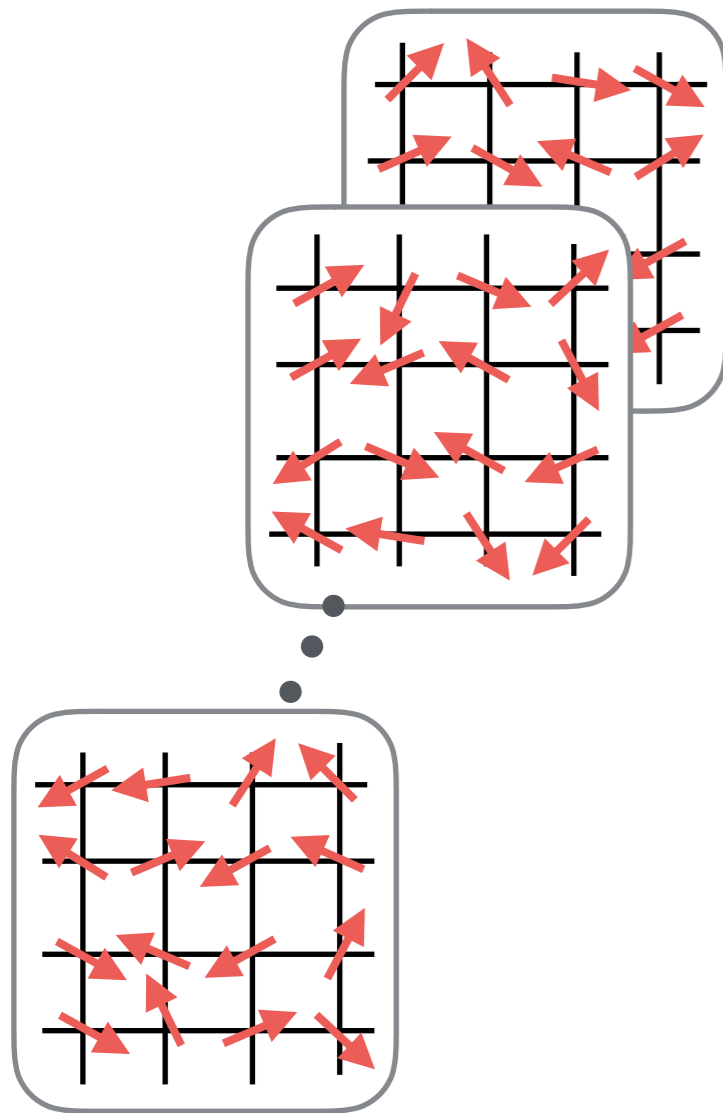


Ferrari and Beca, PRX **9**, 031026 (2019)

Mourigal, Fuhrman, Chernyshev, Zhitomirsky,  
PRB **88**, 094407 (2013)

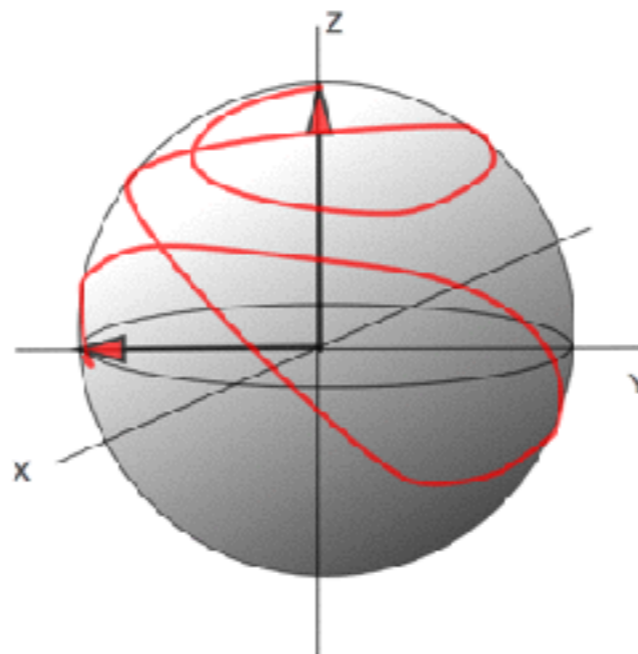
# Classical Spin Liquid / Spin Glass?

- Monte Carlo simulations

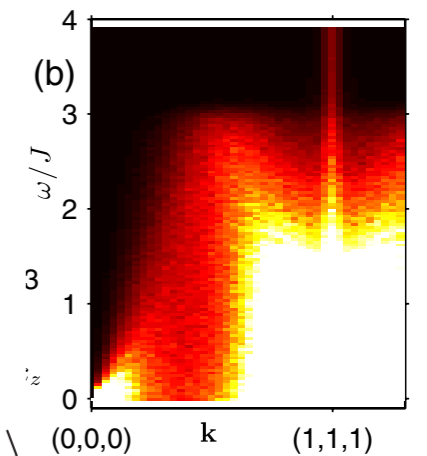
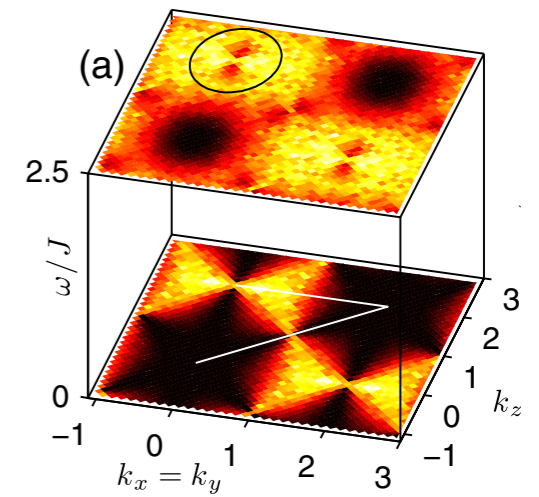


- Landau-Lifshitz equation:

$$\frac{d\mathbf{S}_i}{dt} = -J\mathbf{S}_i \times \sum_j \mathbf{S}_j$$



- Space-time Fourier transform



$$C^{\gamma\gamma}(\mathbf{r}, t) = \langle S_{\mathbf{r}}^{\gamma}(t) S_{\mathbf{0}}^{\gamma}(0) \rangle$$

$$S^{\gamma\gamma}(\mathbf{k}, \omega) = \frac{1}{2\pi} \sum_{\mathbf{r}} \int_{-\infty}^{\infty} e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} C^{\gamma\gamma}(\mathbf{r}, t) dt,$$

**Anomalous Spin Diffusion in Classical Heisenberg Magnets**

Gerhard Müller

*Department of Physics, The University of Rhode Island, Kingston, Rhode Island 02881*

(Received 4 April 1988)

PRL 101, 117207 (2008)

PHYSICAL REVIEW LETTERS

week ending  
12 SEPTEMBER 2008**Propagation and Ghosts in the Classical Kagome Antiferromagnet**J. Robert,<sup>1</sup> B. Canals,<sup>2</sup> V. Simonet,<sup>2</sup> and R. Ballou<sup>2</sup>

PRL 102, 237206 (2009)

PHYSICAL REVIEW LETTERS

week ending  
12 JUNE 2009**Spin Dynamics in Pyrochlore Heisenberg Antiferromagnets**

P. H. Conlon\* and J. T. Chalker

PHYSICAL REVIEW B 96, 134408 (2017)

**Comprehensive study of the dynamics of a classical Kitaev spin liquid**A. M. Samarakoon,<sup>1,2,\*</sup> A. Banerjee,<sup>3</sup> S.-S. Zhang,<sup>4</sup> Y. Kamiya,<sup>5</sup> S. E. Nagler,<sup>3,6</sup> D. A. Tennant,<sup>7</sup>  
S.-H. Lee,<sup>2</sup> and C. D. Batista<sup>1,3,4</sup>

PHYSICAL REVIEW LETTERS 122, 167203 (2019)

**Dynamical Structure Factor of the Three-Dimensional Quantum Spin  
Liquid Candidate NaCaNi<sub>2</sub>F<sub>7</sub>**Shu Zhang,<sup>1,2</sup> Hitesh J. Changlani,<sup>3,4,1,2</sup> Kemp W. Plumb,<sup>5</sup> Oleg Tchernyshyov,<sup>1,2</sup> and Roderich Moessner<sup>6</sup>

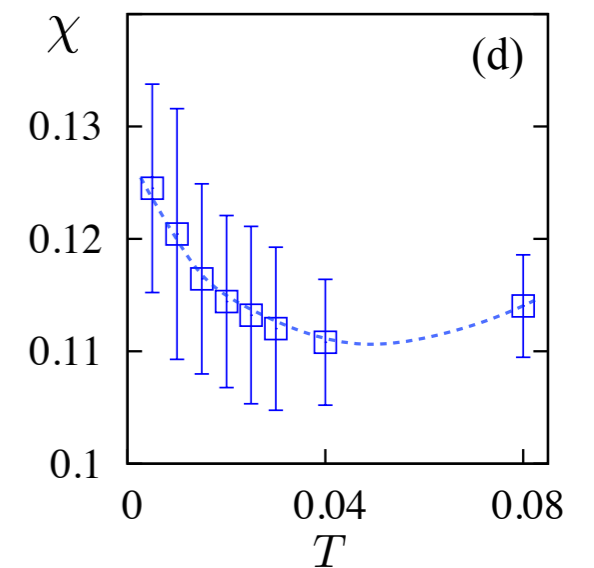
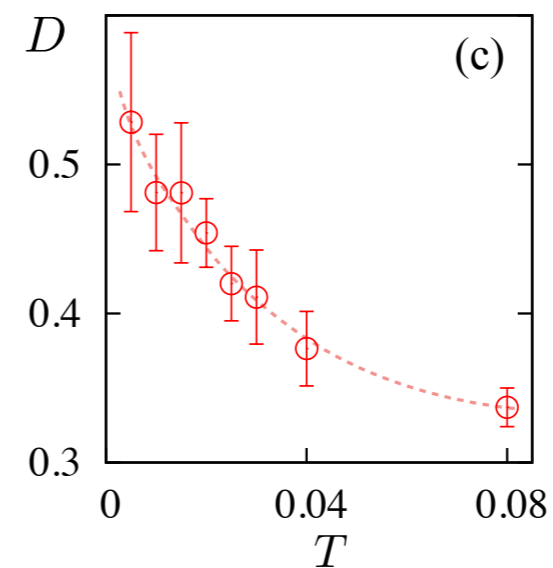
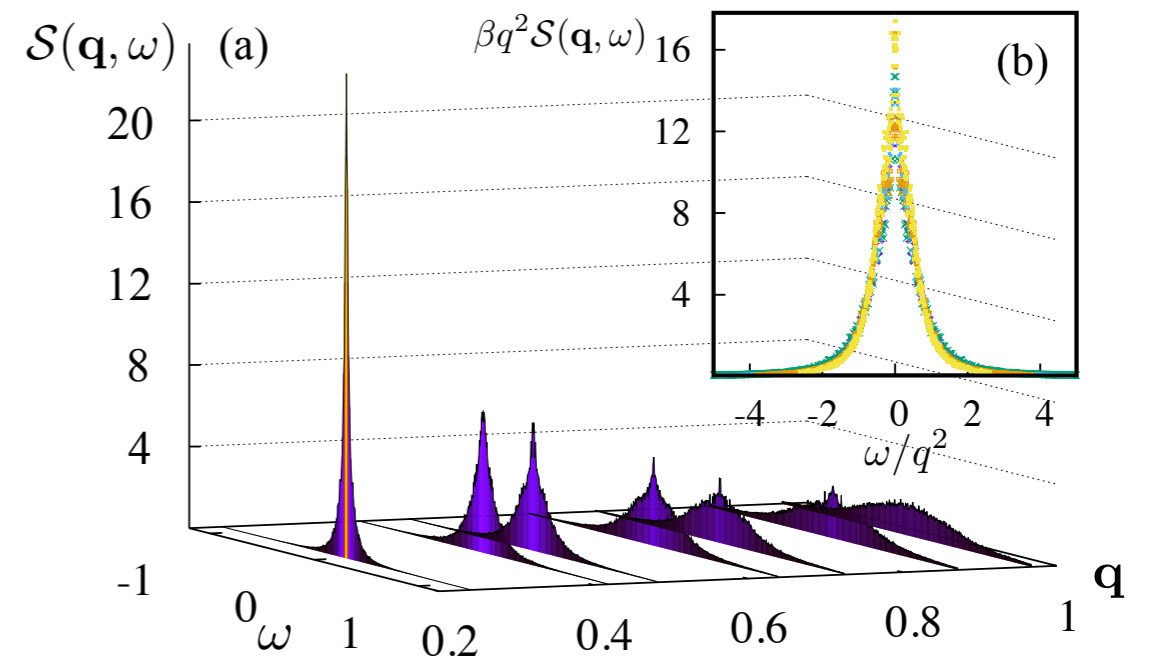
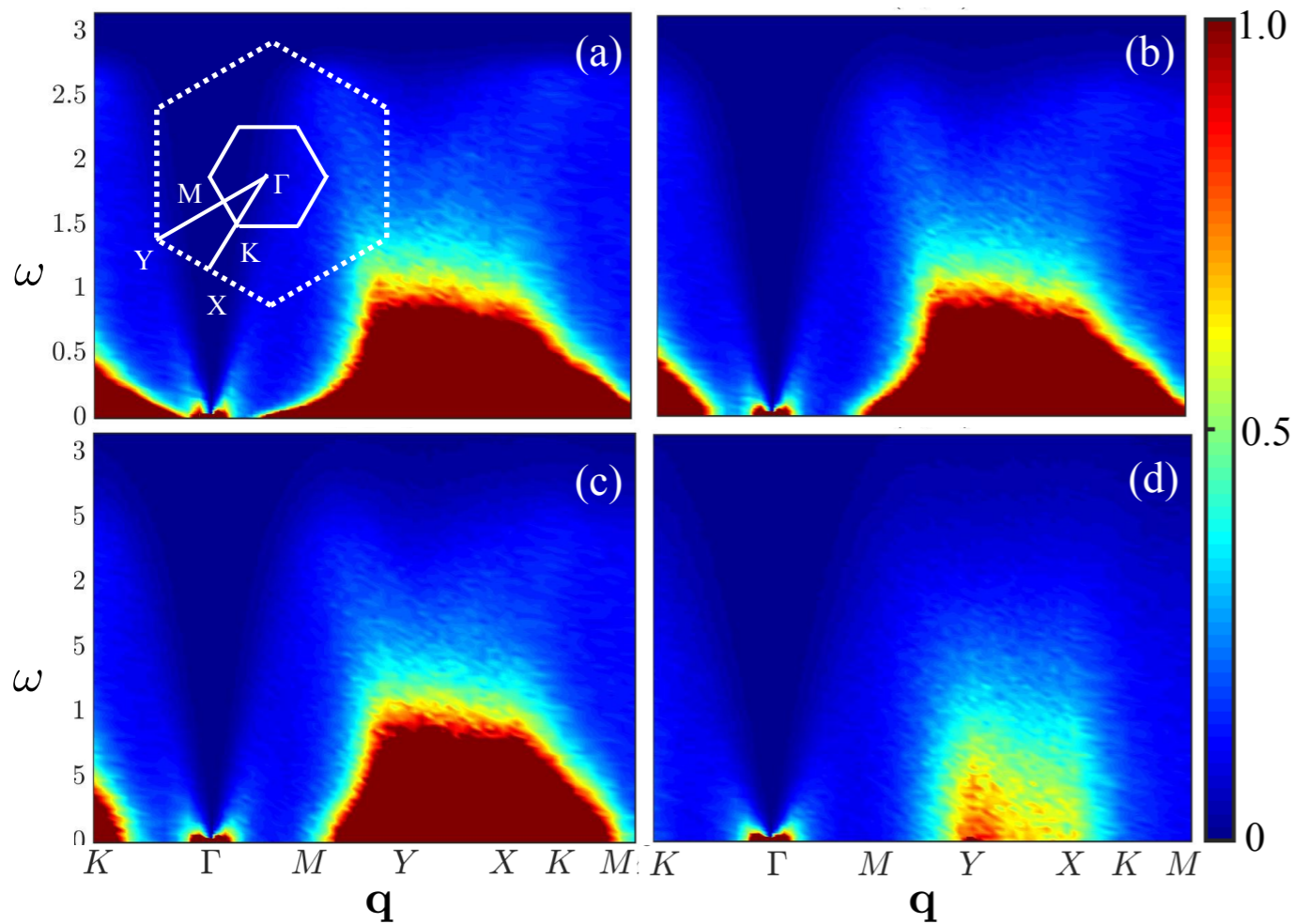
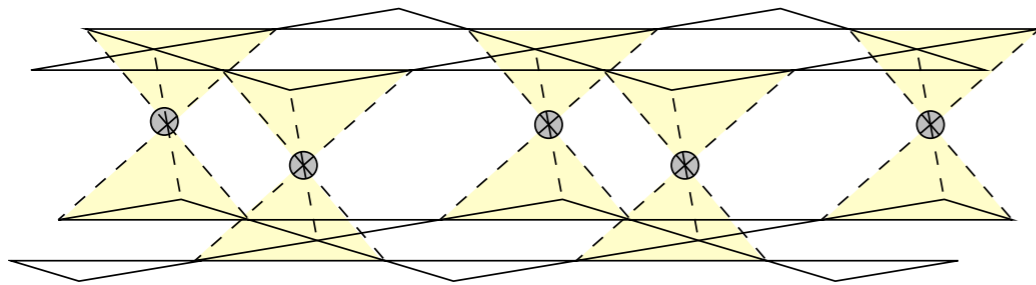
# Spin dynamics of the antiferromagnetic Heisenberg model on a kagome bilayer

Preetha Saha,<sup>1</sup> Depei Zhang,<sup>1</sup> Seung-Hun Lee,<sup>1</sup> and Gia-Wei Chern<sup>1</sup>

<sup>1</sup>Department of Physics, University of Virginia, Charlottesville, VA 22904, USA

(Dated: April 12, 2019)

arXiv:1904.05863v1





ARTICLE

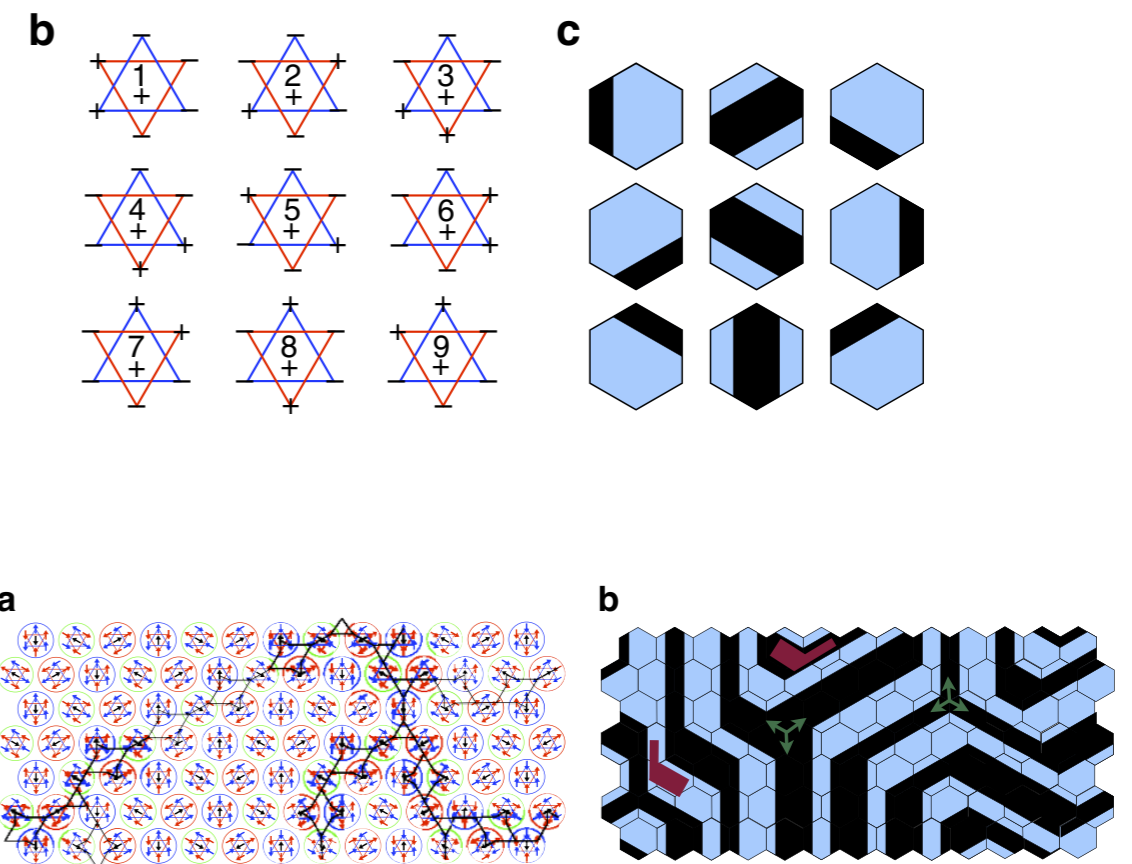
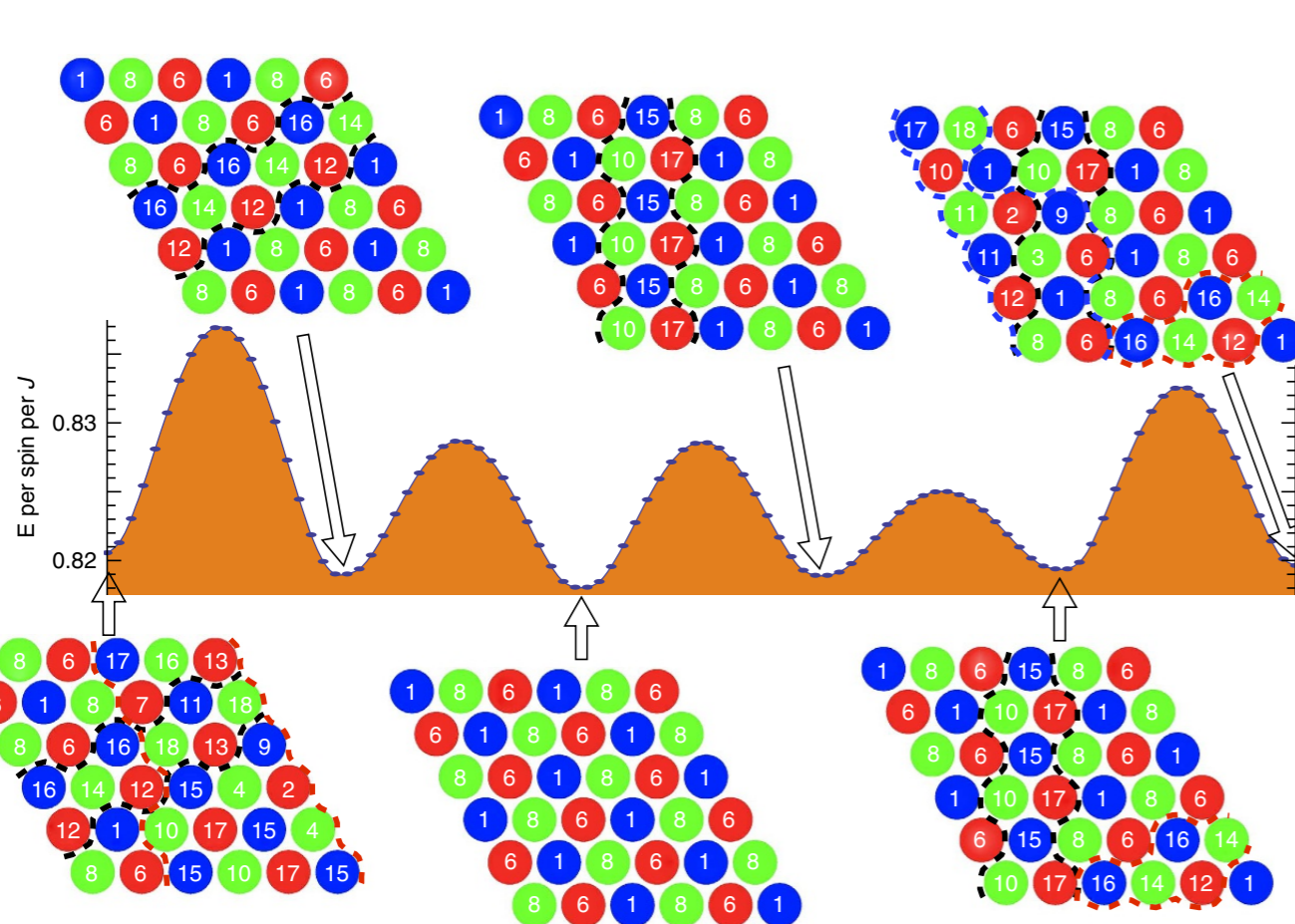
Received 16 Oct 2013 | Accepted 21 Feb 2014 | Published 1 Apr 2014

DOI: 10.1038/ncomms4497

OPEN

# Glassiness and exotic entropy scaling induced by quantum fluctuations in a disorder-free frustrated magnet

I. Klich<sup>1,\*</sup>, S-H Lee<sup>1,\*</sup> & K. Iida<sup>1</sup>



# Dynamics of Phase Separated States in the Double Exchange Model

Jing Luo<sup>1</sup> and Gia-Wei Chern<sup>1</sup>

<sup>1</sup>*Department of Physics, University of Virginia, Charlottesville, VA 22904, USA*

(Dated: April 11, 2019)

arXiv:1904.05252v1

- Double exchange model on square-lattice close to half-filling:

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \left( c_{i\alpha}^\dagger c_{j\alpha} + \text{h.c.} \right) - J \sum_i \mathbf{S}_i \cdot c_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} c_{i\beta},$$

- GPU-enabled Langevin dynamics for preparing initial phase separated states

- Landau-Lifshitz + von Neumann dynamics simulations:

$$\frac{d\mathbf{S}_i}{dt} = -\mathbf{S}_i \times \frac{\partial \langle \mathcal{H} \rangle}{\partial \mathbf{S}_i} = J \mathbf{S}_i \times \boldsymbol{\sigma}_{\alpha\beta} \rho_{i\beta, i\alpha},$$

$$\begin{aligned} \frac{d\rho_{i\alpha, j\beta}}{dt} &= i(t_{ik} \rho_{k\alpha, j\beta} - \rho_{i\alpha, k\beta} t_{kj}) \\ &+ iJ(\mathbf{S}_i \cdot \boldsymbol{\sigma}_{\alpha\gamma} \rho_{i\gamma, j\beta} - \rho_{i\alpha, j\gamma} \boldsymbol{\sigma}_{\gamma\beta} \cdot \mathbf{S}_j). \end{aligned}$$

# Dynamics of Phase Separated States in the Double Exchange Model

Jing Luo<sup>1</sup> and Gia-Wei Chern<sup>1</sup>

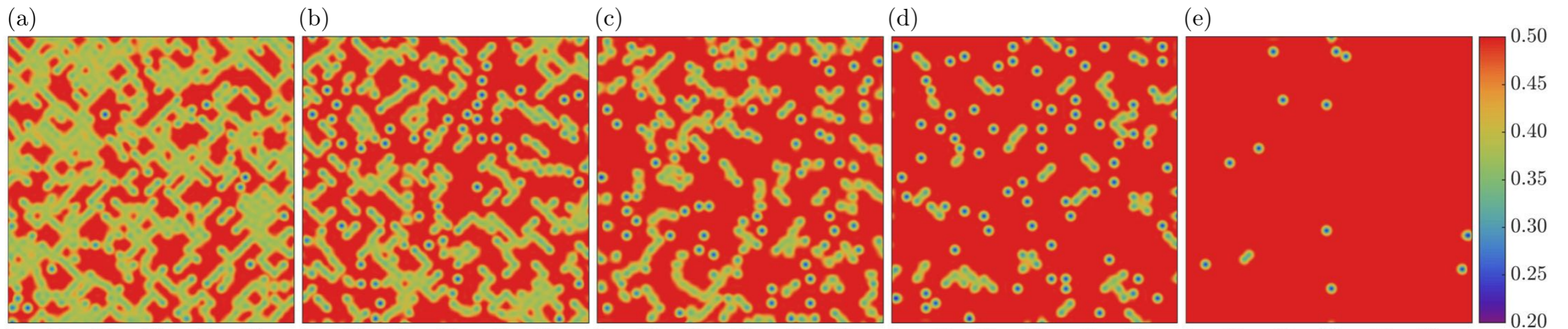
<sup>1</sup>*Department of Physics, University of Virginia, Charlottesville, VA 22904, USA*

(Dated: April 11, 2019)

arXiv:1904.05252v1

- Double exchange model on square-lattice close to half-filling:

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \left( c_{i\alpha}^\dagger c_{j\alpha} + \text{h.c.} \right) - J \sum_i \mathbf{S}_i \cdot c_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} c_{i\beta},$$





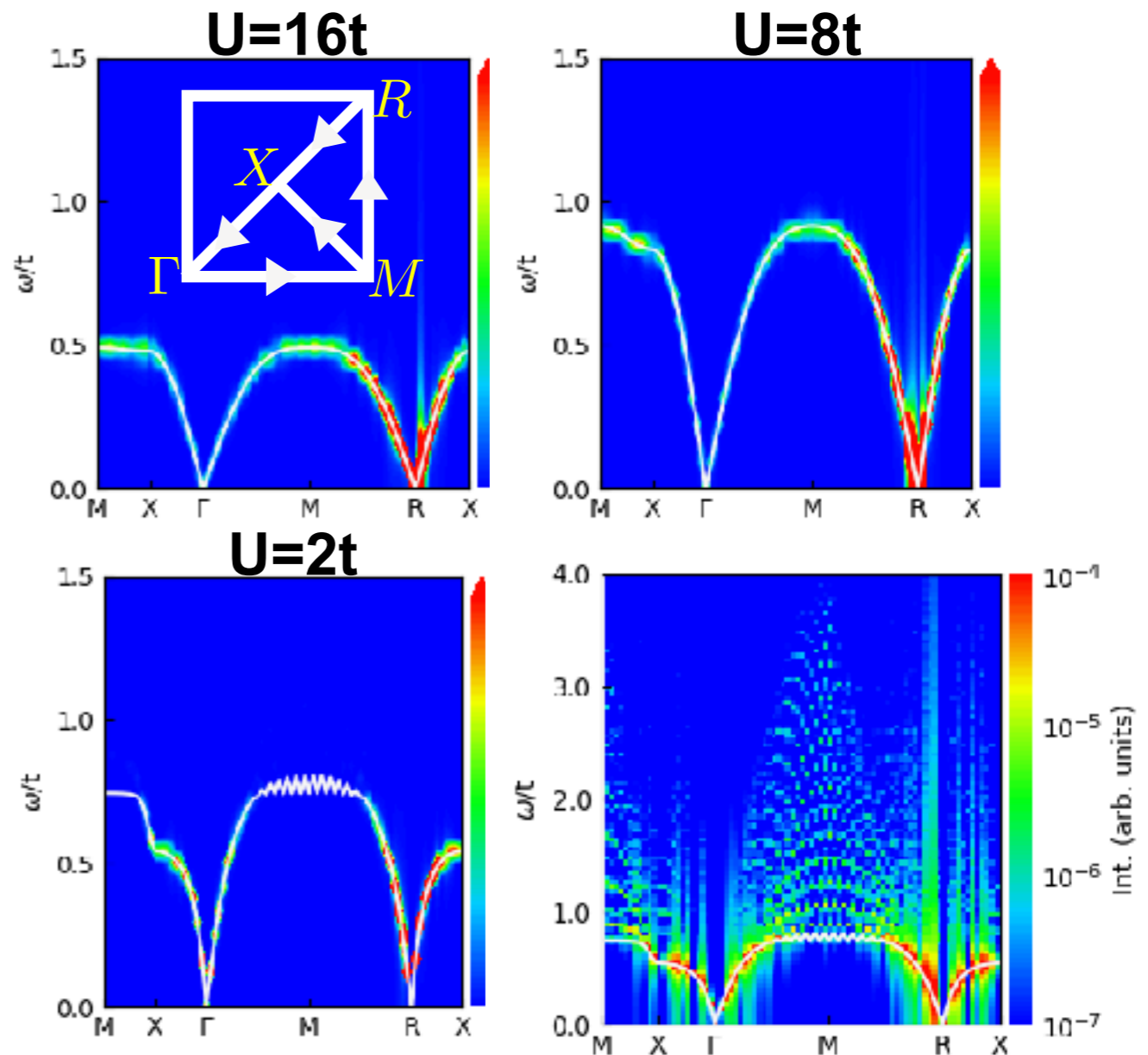
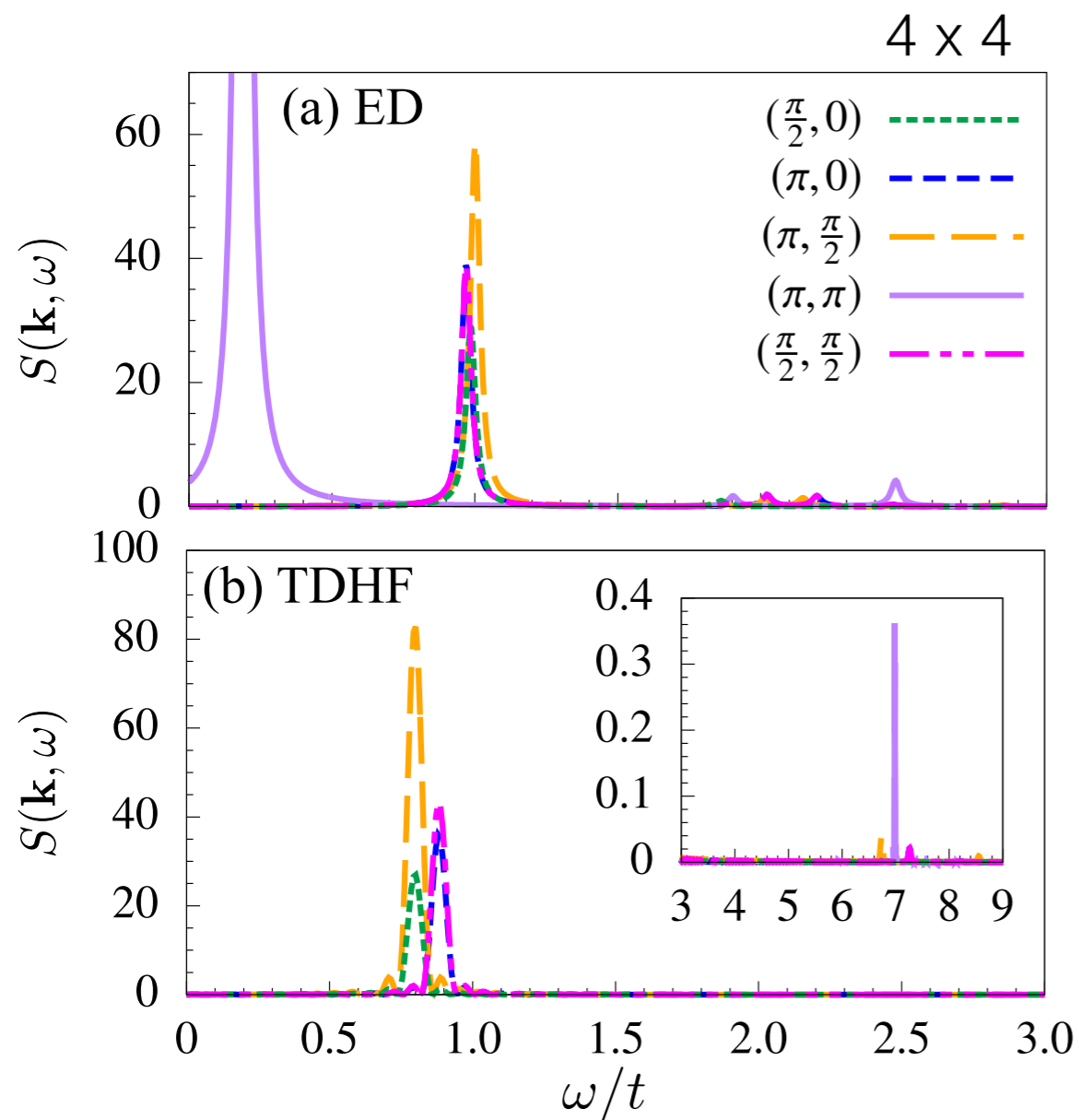
Editors' Suggestion

## Semiclassical dynamics of spin density waves

Gia-Wei Chern,<sup>1,\*</sup> Kipton Barros,<sup>2</sup> Zhentao Wang,<sup>3</sup> Hidemaro Suwa,<sup>3</sup> and Cristian D. Batista<sup>3,4,†</sup>

- Beyond Mott insulators: 2D Hubbard model

48x48



## Dynamics of the Kitaev-Heisenberg Model

Matthias Gohlke,<sup>1</sup> Ruben Verresen,<sup>1,2</sup> Roderich Moessner,<sup>1</sup> and Frank Pollmann<sup>1,2</sup>

$$H = \sum_{\langle i,j \rangle_\gamma} K_\gamma S_i^\gamma S_j^\gamma + J \sum_{\langle i,j \rangle} S_i \cdot S_j.$$

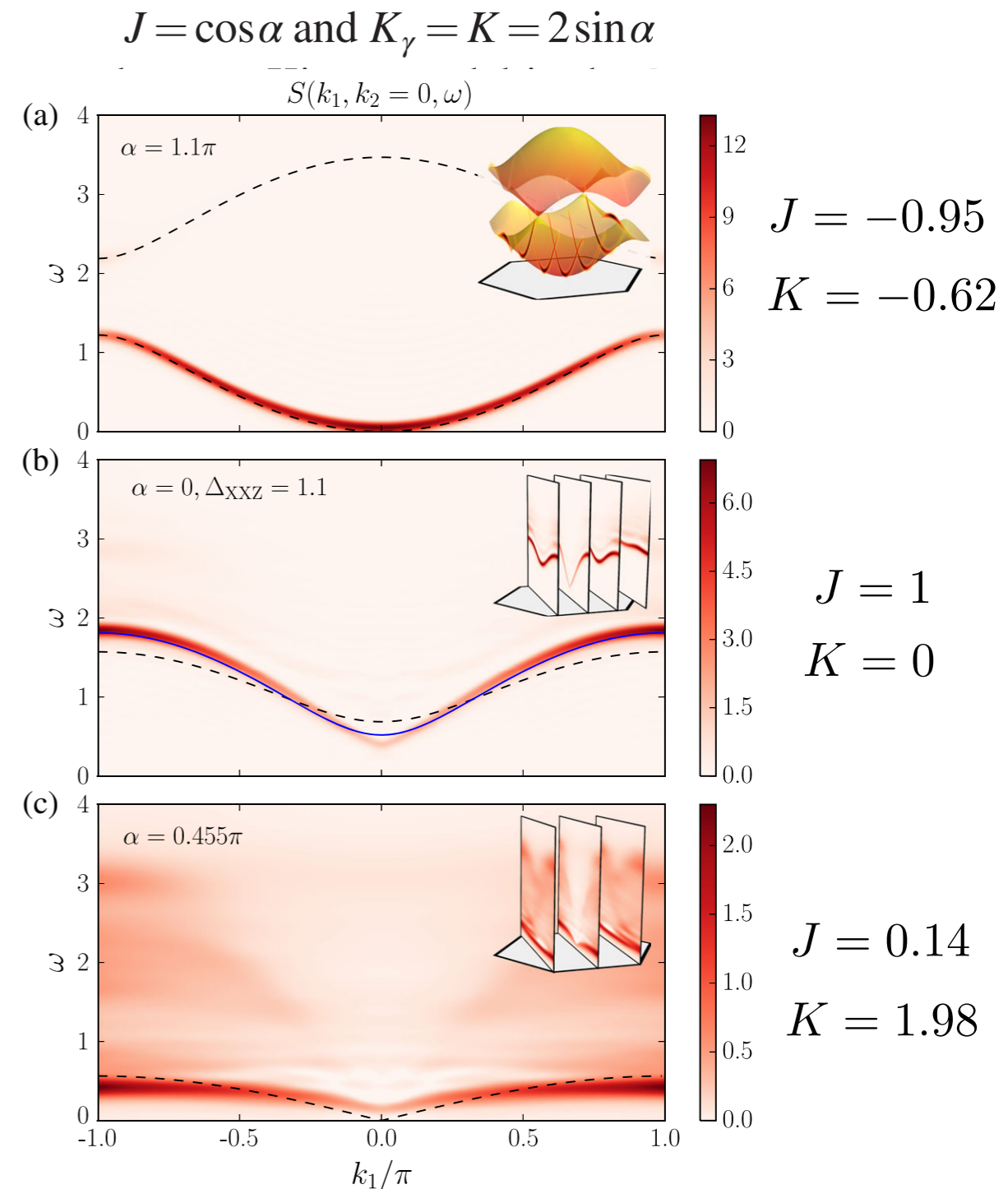
- iDMRG to prepare the ground state  $|\Phi_0\rangle$
- Initial state:  $|\Psi(0)\rangle = S_{\mathbf{r}=0}^+ |\Phi_0\rangle$
- Time-dependent DMRG (TEBD, TDVP, etc)

$$|\Psi(t)\rangle = e^{-i\mathcal{H}t/\hbar} |\Psi(0)\rangle$$

- Space-time Fourier Transform:

$$C^{\gamma\gamma}(\mathbf{r}, t) = \langle S_{\mathbf{r}}^\gamma(t) S_{\mathbf{0}}^\gamma(0) \rangle$$

$$S^{\gamma\gamma}(\mathbf{k}, \omega) = \frac{1}{2\pi} \sum_{\mathbf{r}} \int_{-\infty}^{\infty} e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} C^{\gamma\gamma}(\mathbf{r}, t) dt,$$



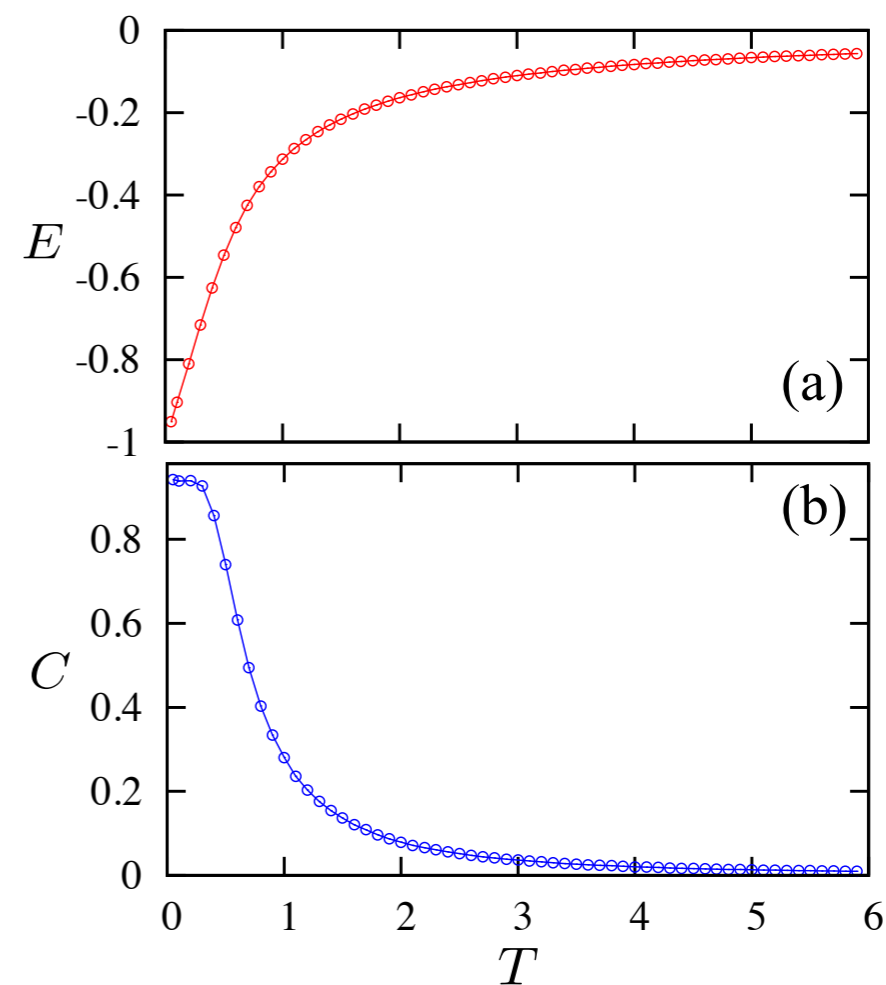
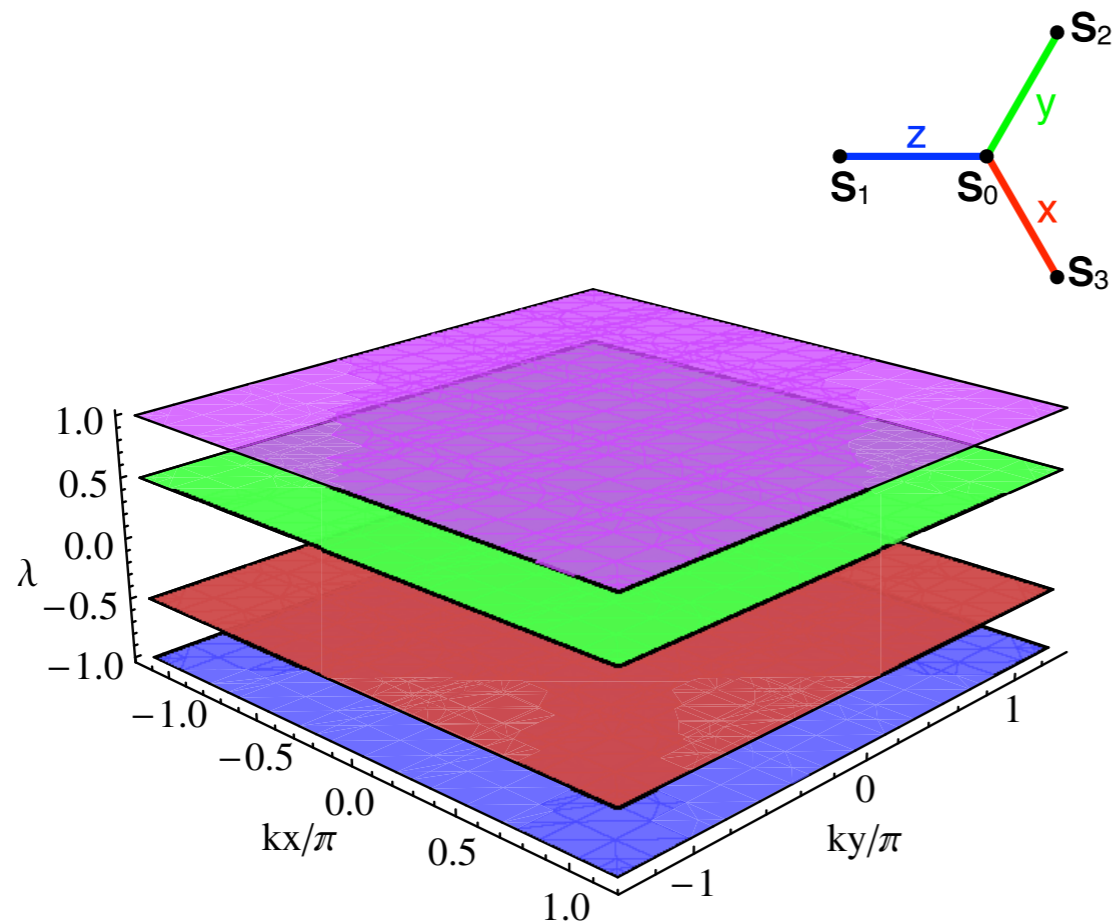
# Classical Spin Liquid Instability Driven By Off-Diagonal Exchange in Strong Spin-Orbit Magnets

Ioannis Rousochatzakis and Natalia B. Perkins

*School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA*

(Received 25 October 2016; published 5 April 2017)

$$\mathcal{H} = \Gamma \sum_{\langle ij \rangle \| x} (S_i^y S_j^z + S_i^z S_j^y) + \Gamma \sum_{\langle ij \rangle \| y} (S_i^z S_j^x + S_i^x S_j^z) + \Gamma \sum_{\langle ij \rangle \| z} (S_i^x S_j^y + S_i^y S_j^x).$$



## Classical Spin Liquid Instability Driven By Off-Diagonal Exchange in Strong Spin-Orbit Magnets

Ioannis Rousochatzakis and Natalia B. Perkins

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(Received 25 October 2016; published 5 April 2017)

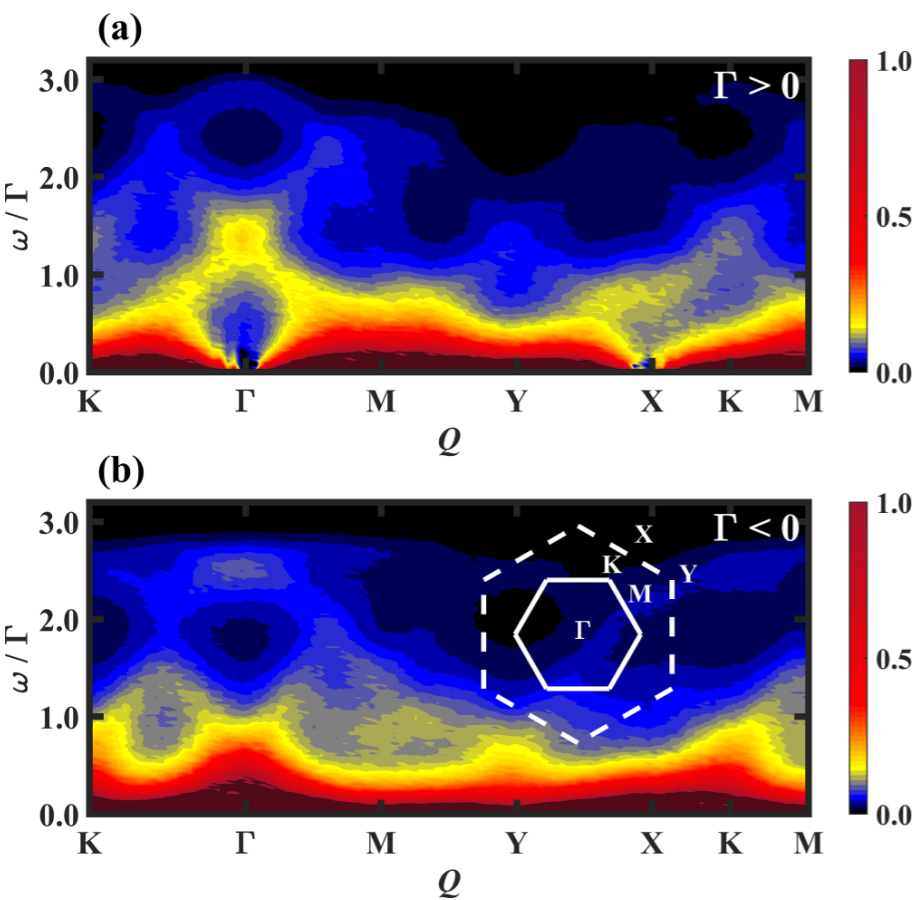
$$\mathcal{H} = \Gamma \sum_{\langle ij \rangle \| x} (S_i^y S_j^z + S_i^z S_j^y) + \Gamma \sum_{\langle ij \rangle \| y} (S_i^z S_j^x + S_i^x S_j^z) + \Gamma \sum_{\langle ij \rangle \| z} (S_i^x S_j^y + S_i^y S_j^x).$$

Spin dynamics & dynamical structure factor  
of this new classical spin liquid?

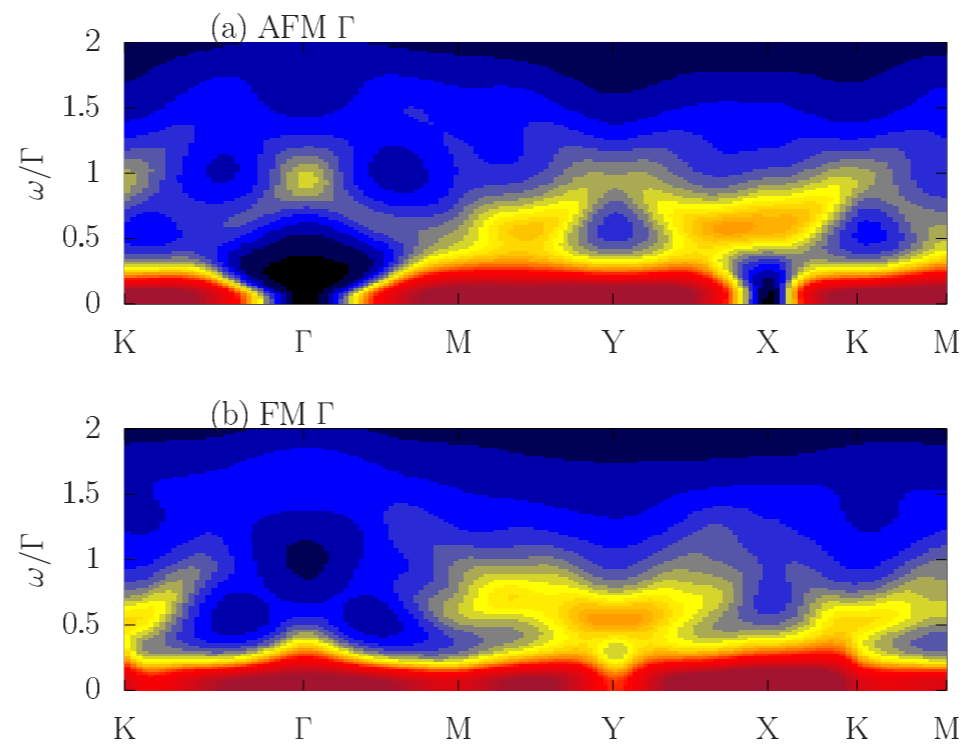
Editors' Suggestion

## Classical and quantum spin dynamics of the honeycomb $\Gamma$ model

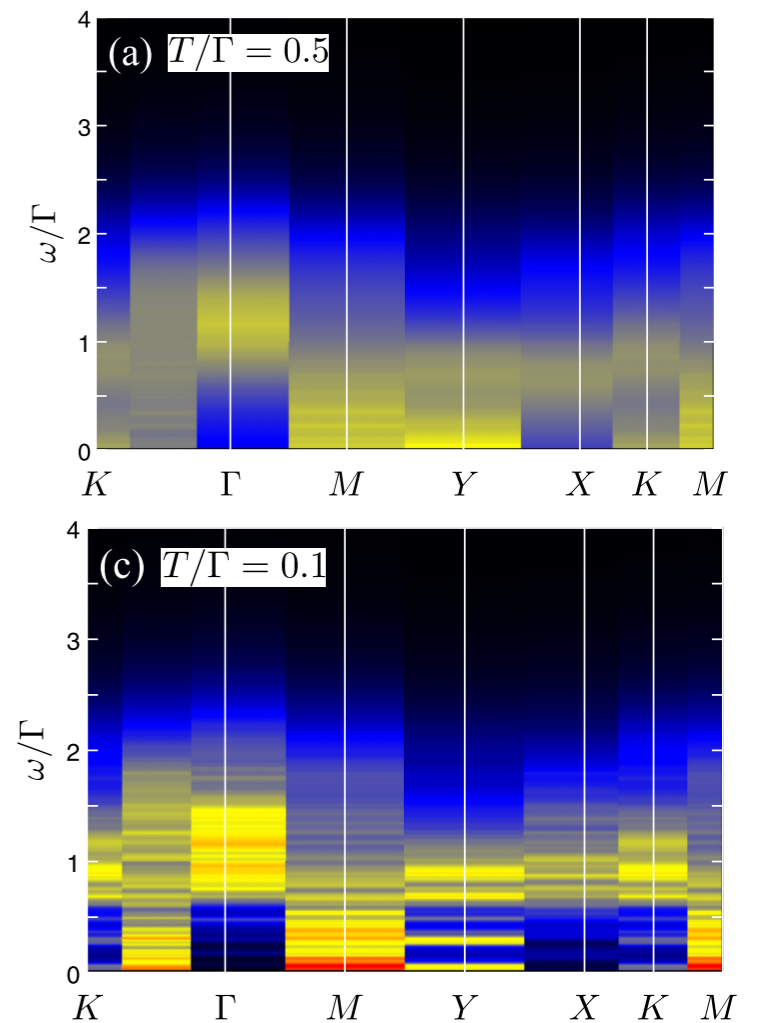
Anjana M. Samarakoon,<sup>1,2</sup> Gideon Wachtel,<sup>3,4</sup> Youhei Yamaji,<sup>5,6</sup> D. A. Tennant,<sup>7,2</sup>  
 Cristian D. Batista,<sup>1,2,8</sup> and Yong Baek Kim<sup>3,9,10</sup>



Landau-Lifshitz dynamics



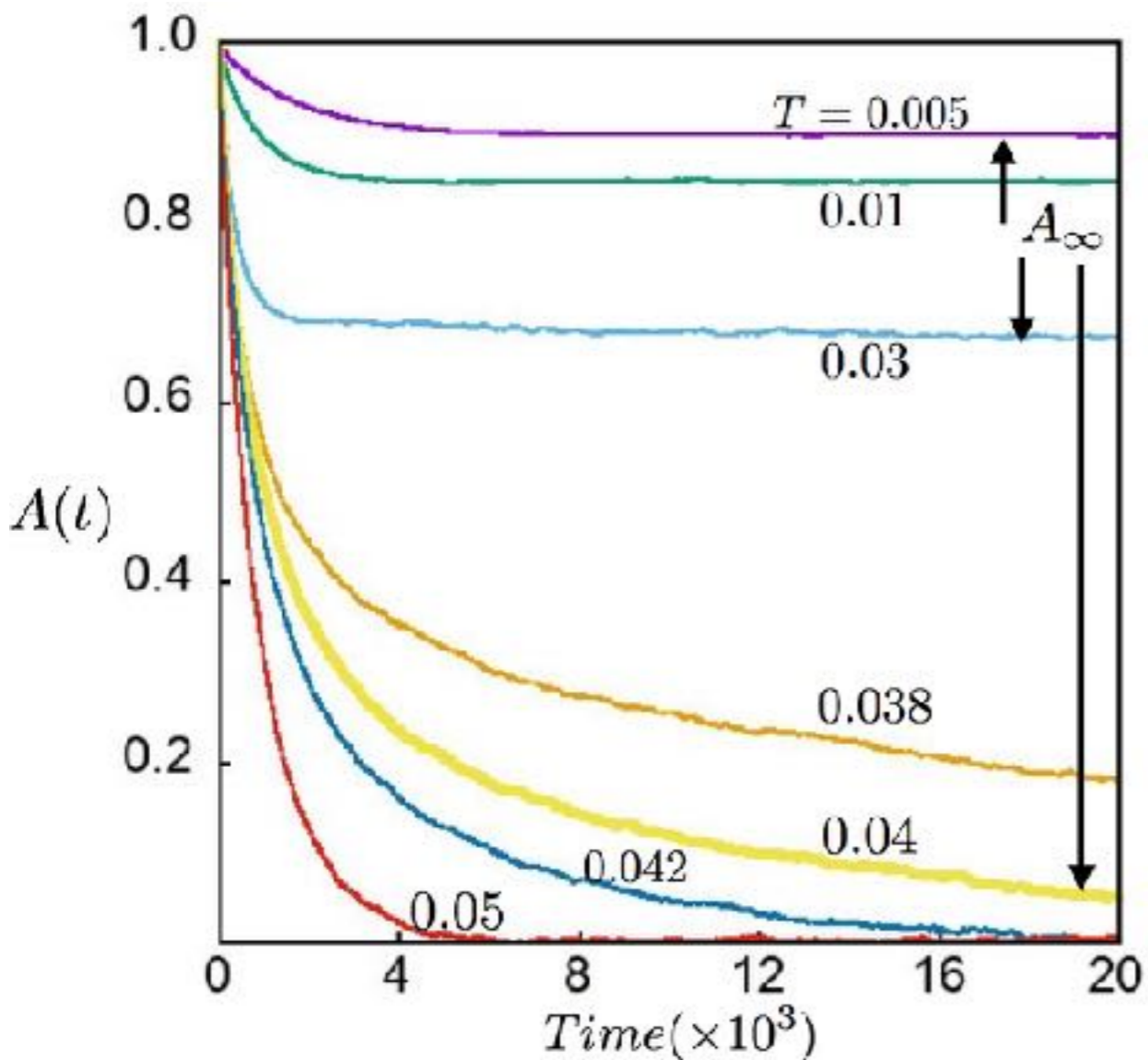
Langevin dynamics with soft spins



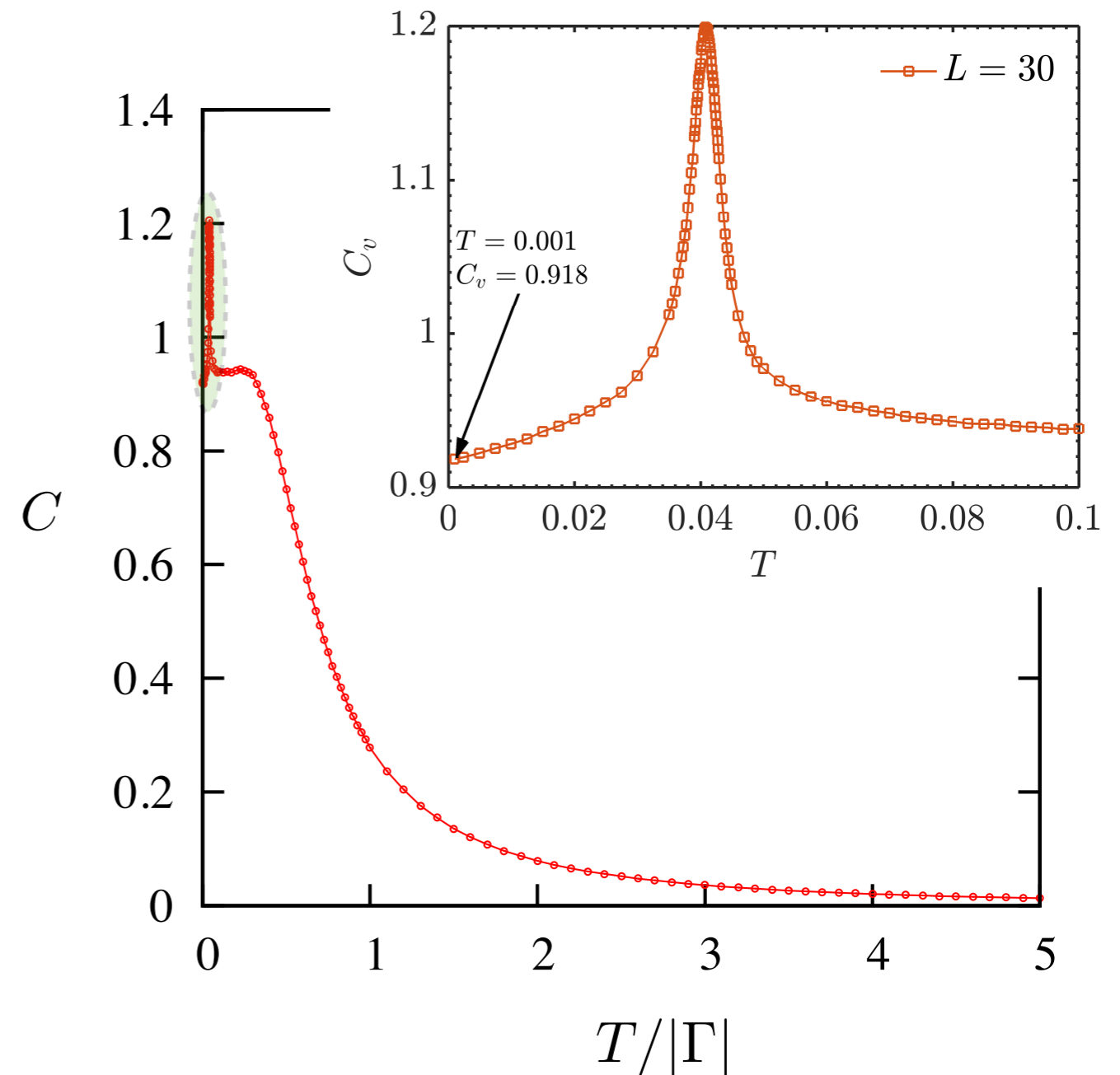
Exact diagonalization  
 spin-1/2

# Unexpected phase transition

- Autocorrelation function:



- Heat capacity vs  $T$ :





# Order-by-disorder transition in frustrated magnets

VOLUME 64, NUMBER 1

PHYSICAL REVIEW LETTERS

1 JANUARY 1990

## Ising Transition in Frustrated Heisenberg Models

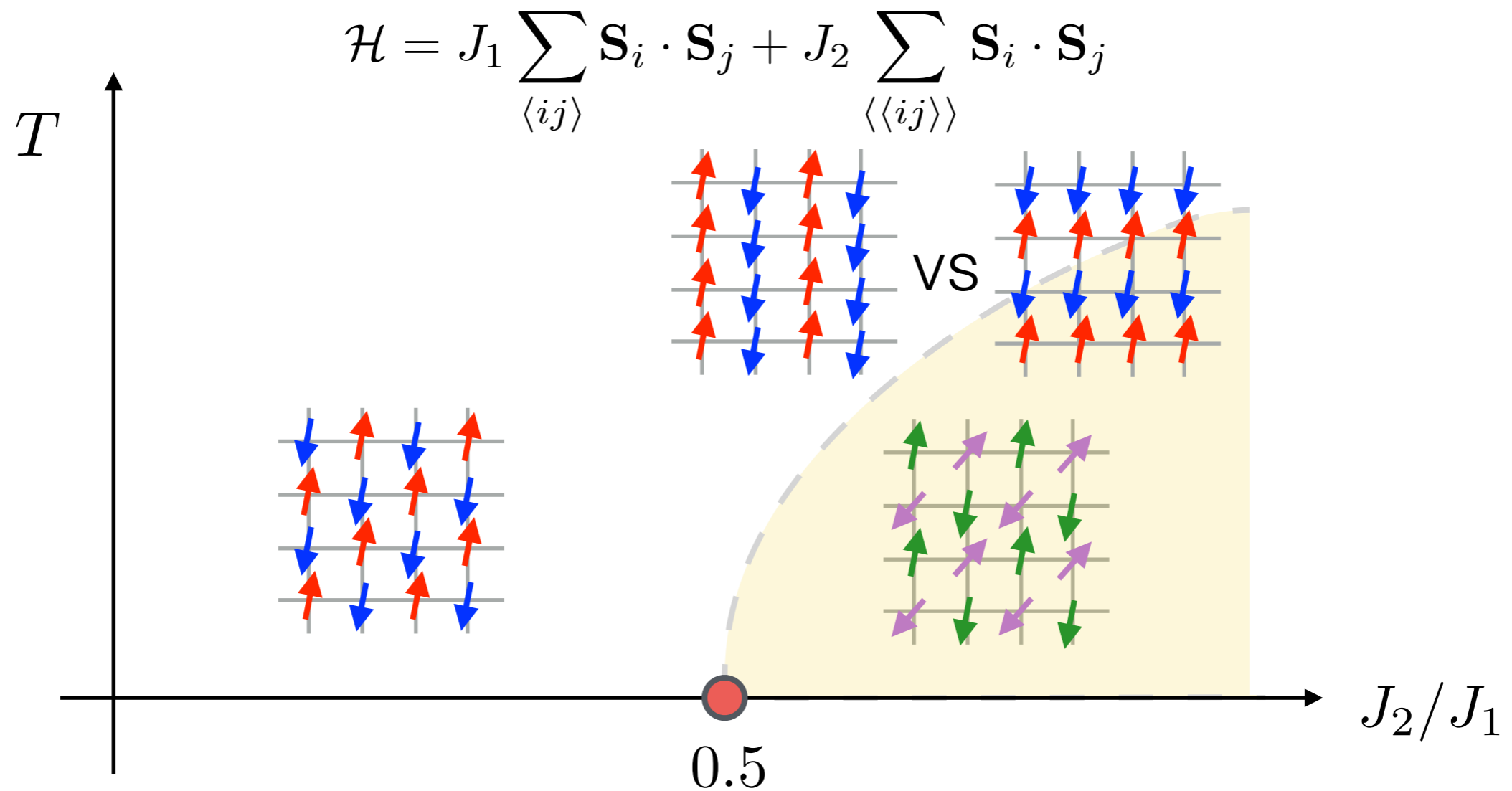
P. Chandra

*Corporate Research Science Laboratories, Exxon Research and Engineering Company, Annandale, New Jersey 08801*

P. Coleman and A. I. Larkin<sup>(a)</sup>

*Serlin Physics Laboratory, Rutgers University, P.O. Box 849, Piscataway, New Jersey 08854*

(Received 5 June 1989)



# Order-by-disorder transition in frustrated magnets

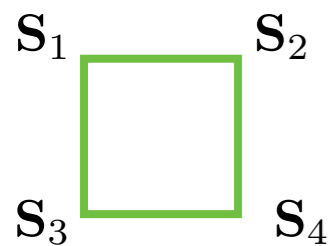
VOLUME 91, NUMBER 17

PHYSICAL REVIEW LETTERS

week ending  
24 OCTOBER 2003

## Ising Transition Driven by Frustration in a 2D Classical Model with Continuous Symmetry

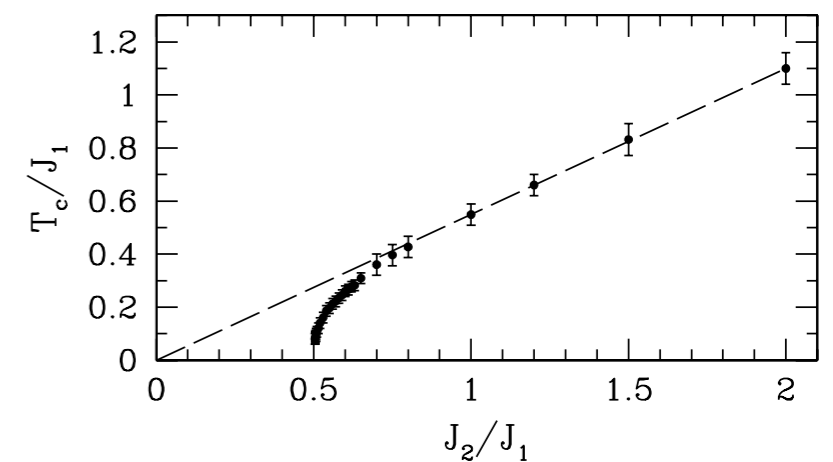
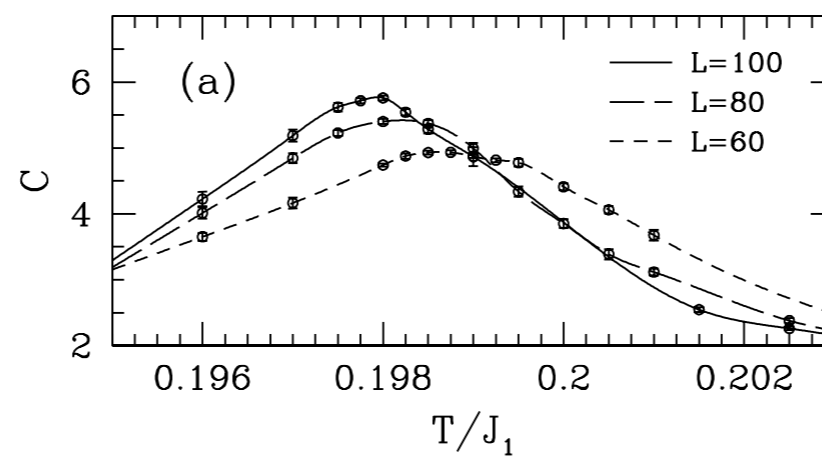
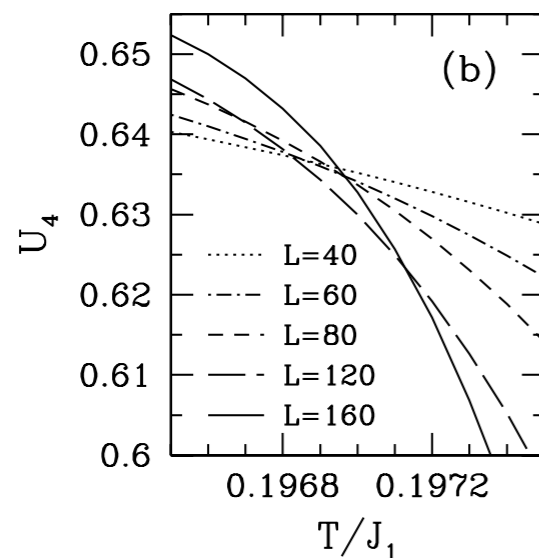
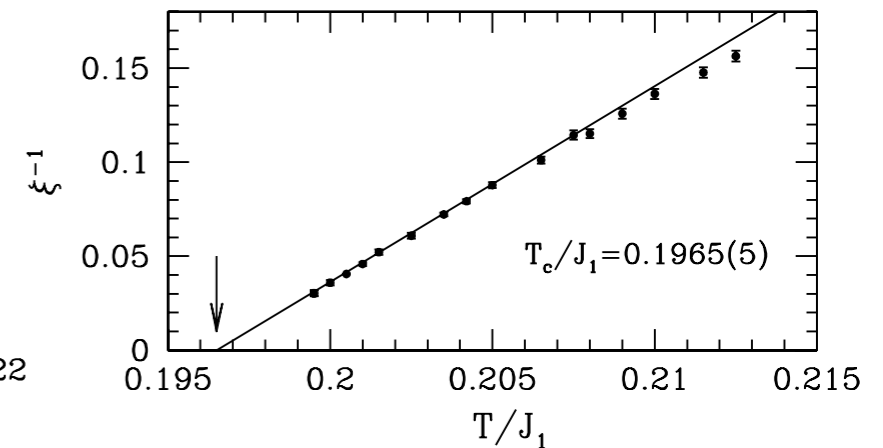
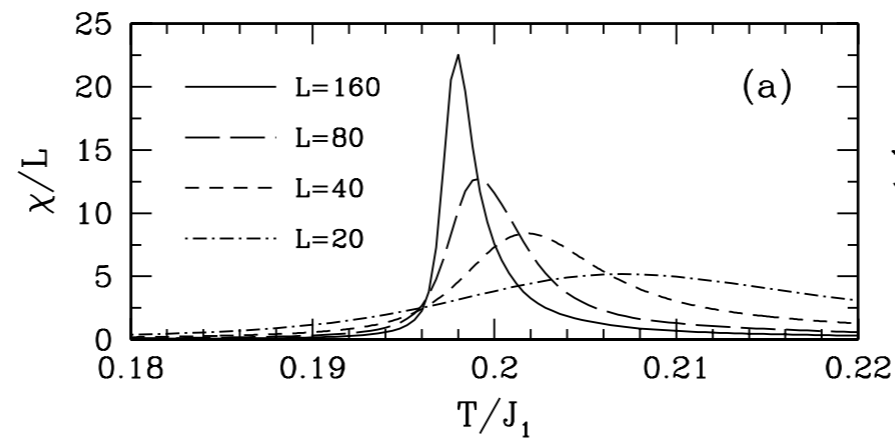
Cédric Weber,<sup>1,2</sup> Luca Capriotti,<sup>3</sup> Grégoire Misguich,<sup>4</sup> Federico Becca,<sup>5</sup> Maged Elhajal,<sup>1</sup> and Frédéric Mila<sup>1</sup>



$$\sigma = \frac{\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{S}_3 \cdot \mathbf{S}_4 - \mathbf{S}_1 \cdot \mathbf{S}_4 - \mathbf{S}_2 \cdot \mathbf{S}_3}{|\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{S}_3 \cdot \mathbf{S}_4 - \mathbf{S}_1 \cdot \mathbf{S}_4 - \mathbf{S}_2 \cdot \mathbf{S}_3|}$$

$$M = \frac{1}{N} \sum_{\square} \sigma_{\square}$$

$$\chi = \frac{N}{T} (\langle M^2 \rangle - \langle |M| \rangle^2)$$

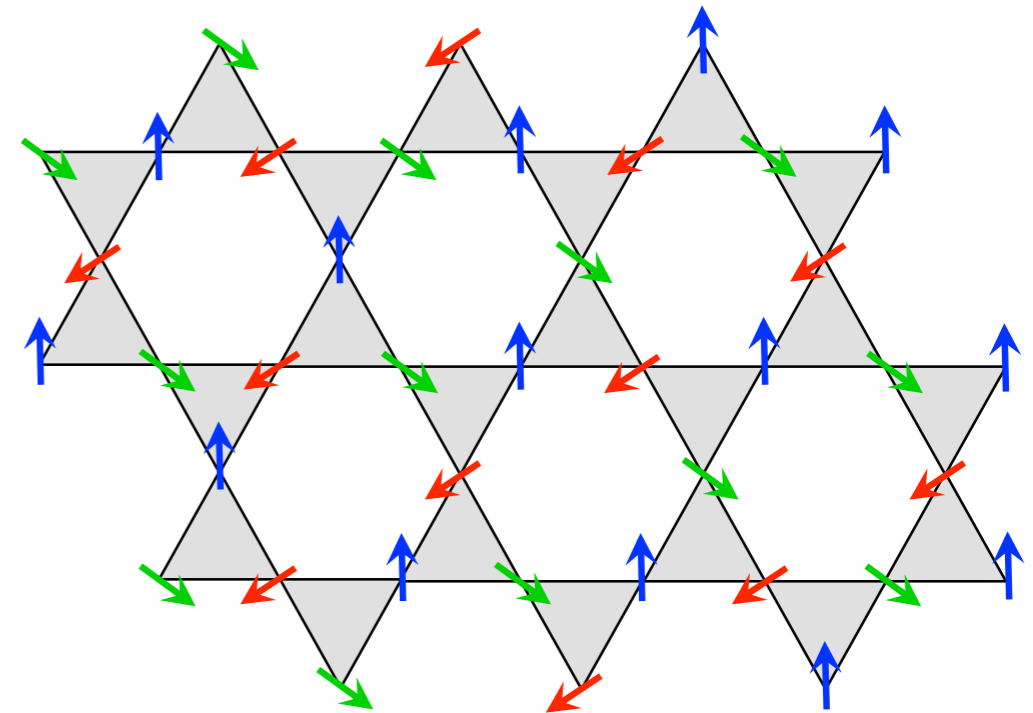
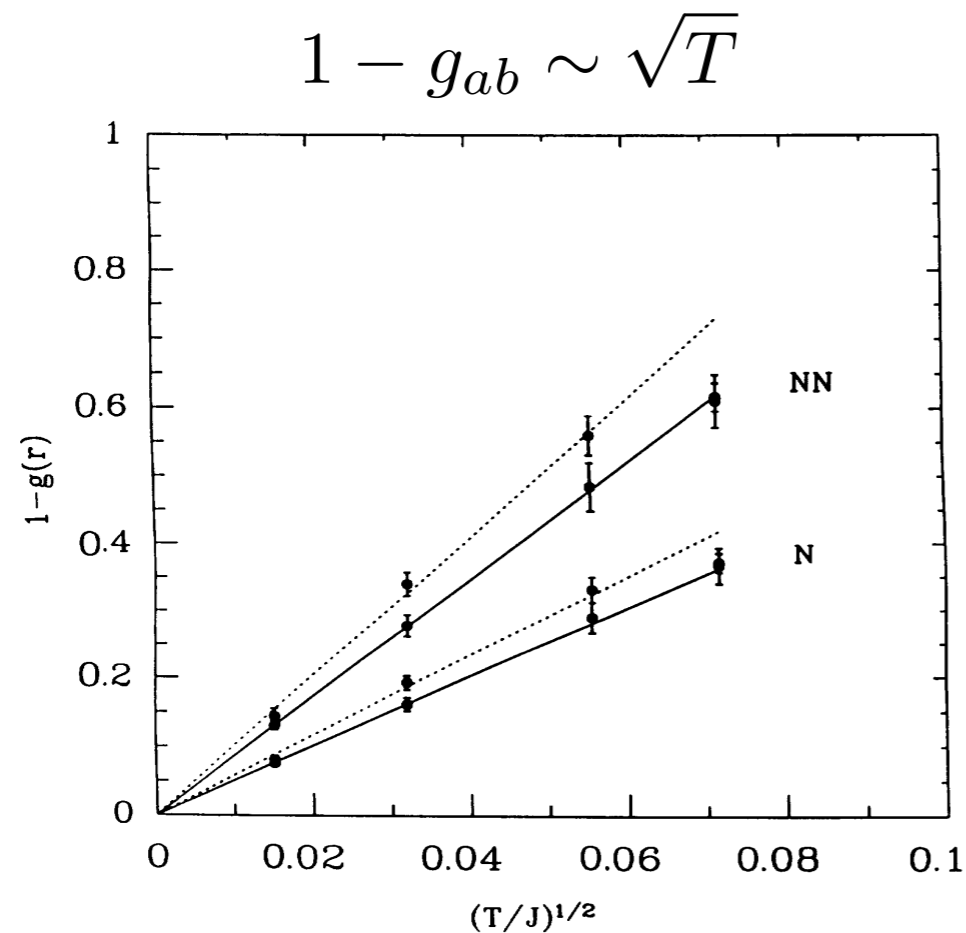




# Hidden Order in a Frustrated System: Properties of the Heisenberg Kagomé Antiferromagnet

J. T. Chalker,<sup>(1),(a)</sup> P. C. W. Holdsworth,<sup>(1),(2)</sup> and E. F. Shender<sup>(1),(3),(b)</sup>

- Coplanar vector:  $\mathbf{n}_\Delta = \mathbf{S}_1 \times \mathbf{S}_2 + \mathbf{S}_2 \times \mathbf{S}_3 + \mathbf{S}_3 \times \mathbf{S}_1$
- nematic correlations:  $g_{ab} = \frac{3}{2} \langle (\mathbf{n}_a \cdot \mathbf{n}_b)^2 \rangle - \frac{1}{2}$



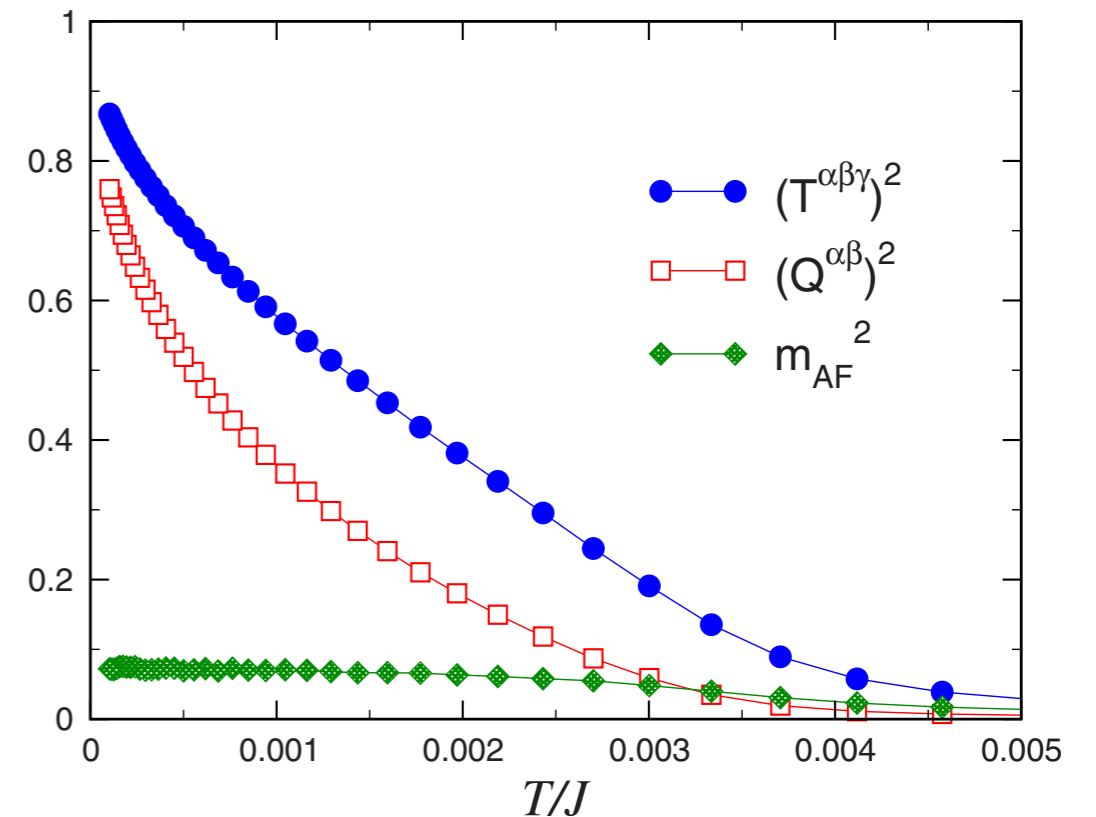
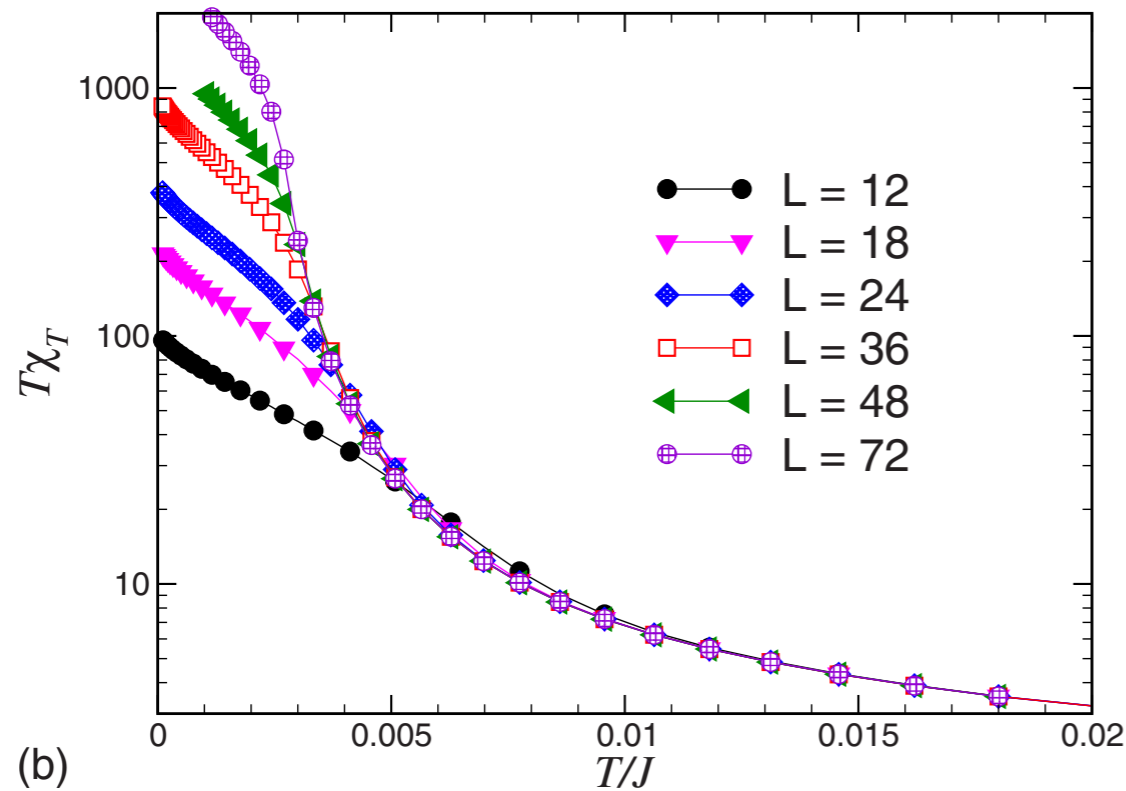


# Octupolar ordering of classical kagome antiferromagnets in two and three dimensions

M. E. Zhitomirsky

- coplanar/nematic order = quadrupole order  $Q^{\alpha\beta} = \frac{1}{N} \sum_i \left( S_i^\alpha S_i^\beta - \frac{1}{3} \delta_{\alpha\beta} \right)$
- Octupolar order:  $T_i^{\alpha\beta\gamma} = S_i^\alpha S_i^\beta S_i^\gamma - \frac{1}{5} S_i^\alpha \delta_{\beta\gamma} - \frac{1}{5} S_i^\beta \delta_{\alpha\gamma} - \frac{1}{5} S_i^\gamma \delta_{\alpha\beta}$

$$\chi_T = \frac{1}{TN} \sum_{i,j} \langle T_i^{\alpha\beta\gamma} T_j^{\alpha\beta\gamma} \rangle$$

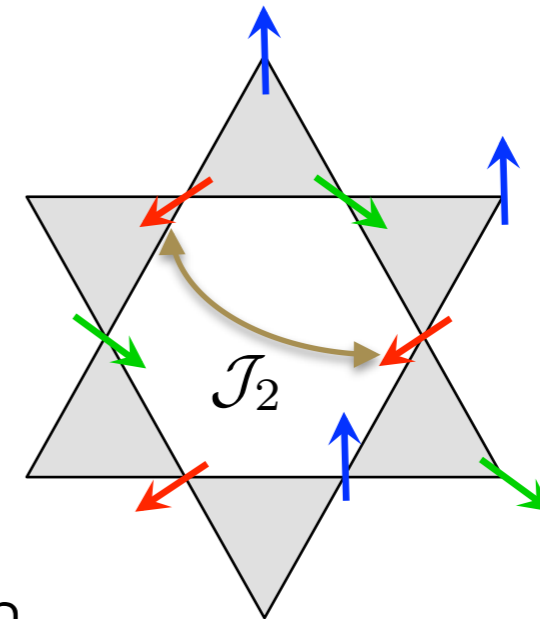


# Dipolar Order by Disorder in the Classical Heisenberg Antiferromagnet on the Kagome Lattice

Gia-Wei Chern<sup>1,2,3</sup> and R. Moessner<sup>1</sup>

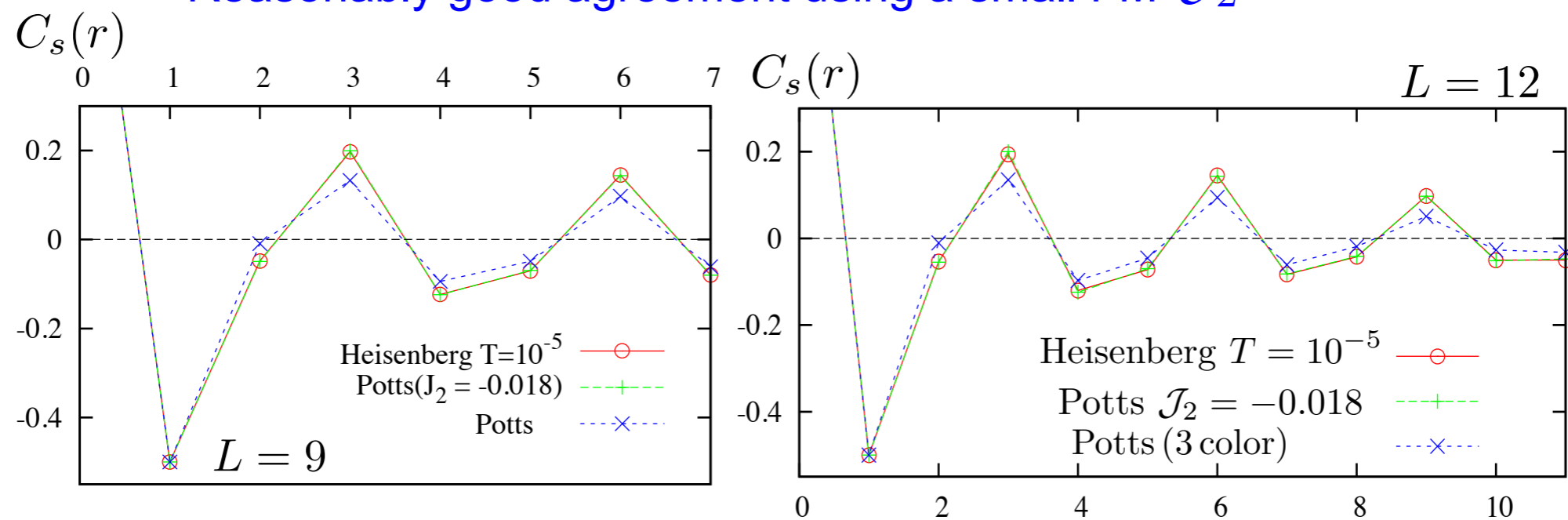
- Effective 3-color (Potts) model with ferromagnetic effective  $J_2$  interaction:

$$\mathcal{S}_{\text{eff}} = \mathcal{J}_2 \sum_{\langle\langle ij \rangle\rangle} \delta_{\sigma_i, \sigma_j}$$



- Try to fit entropic weight with NNN interaction

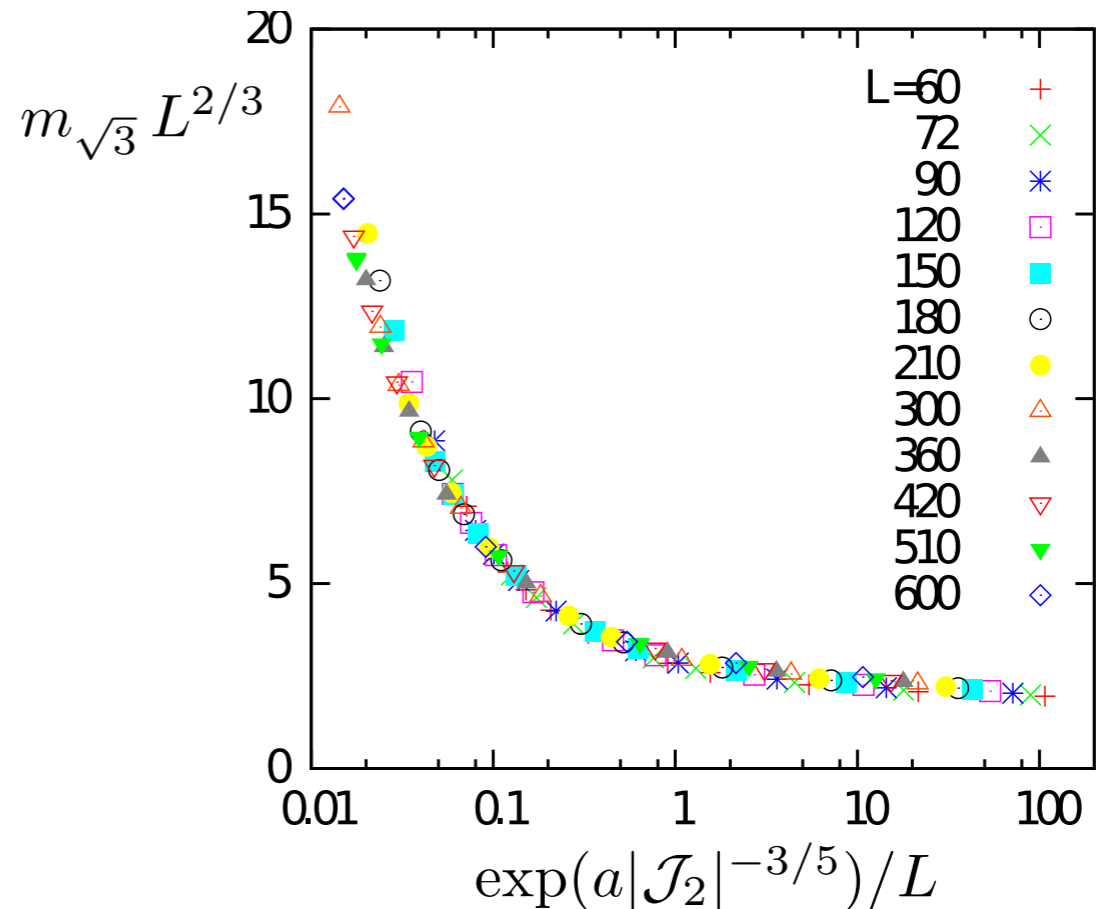
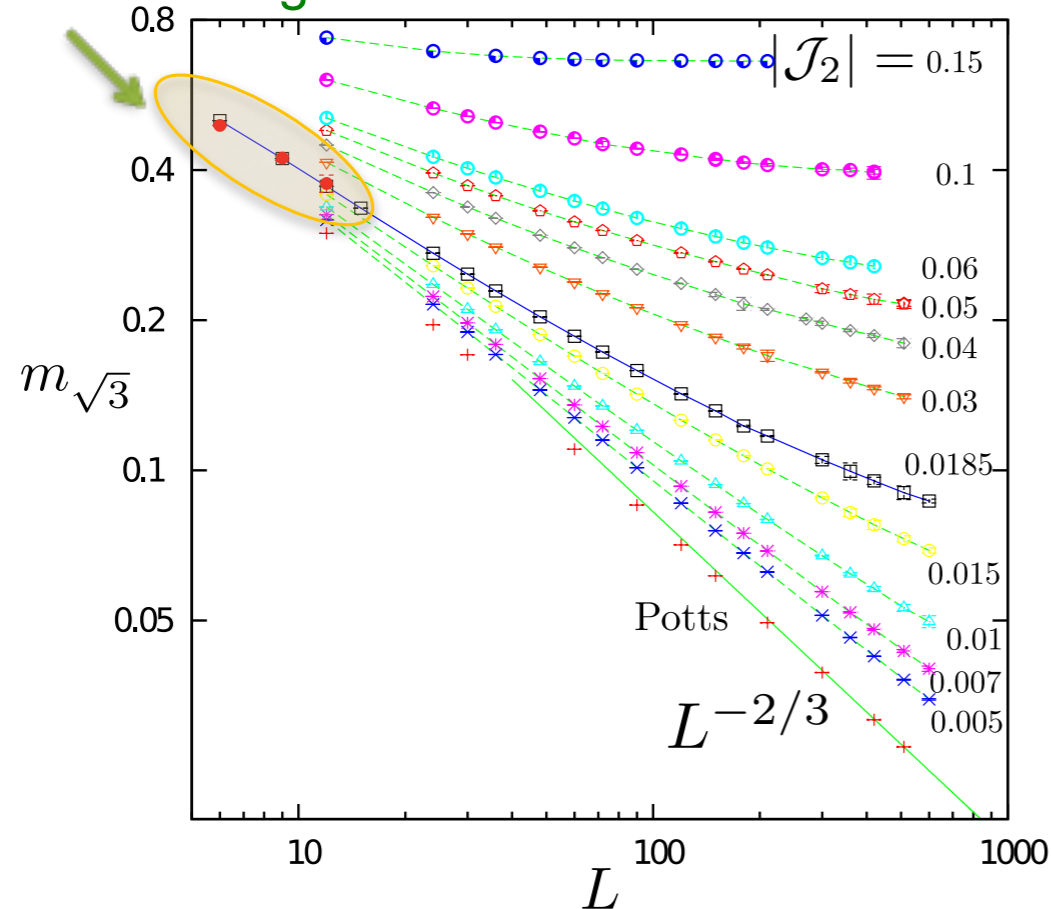
Reasonably good agreement using a small FM  $\mathcal{J}_2 \approx -0.018$



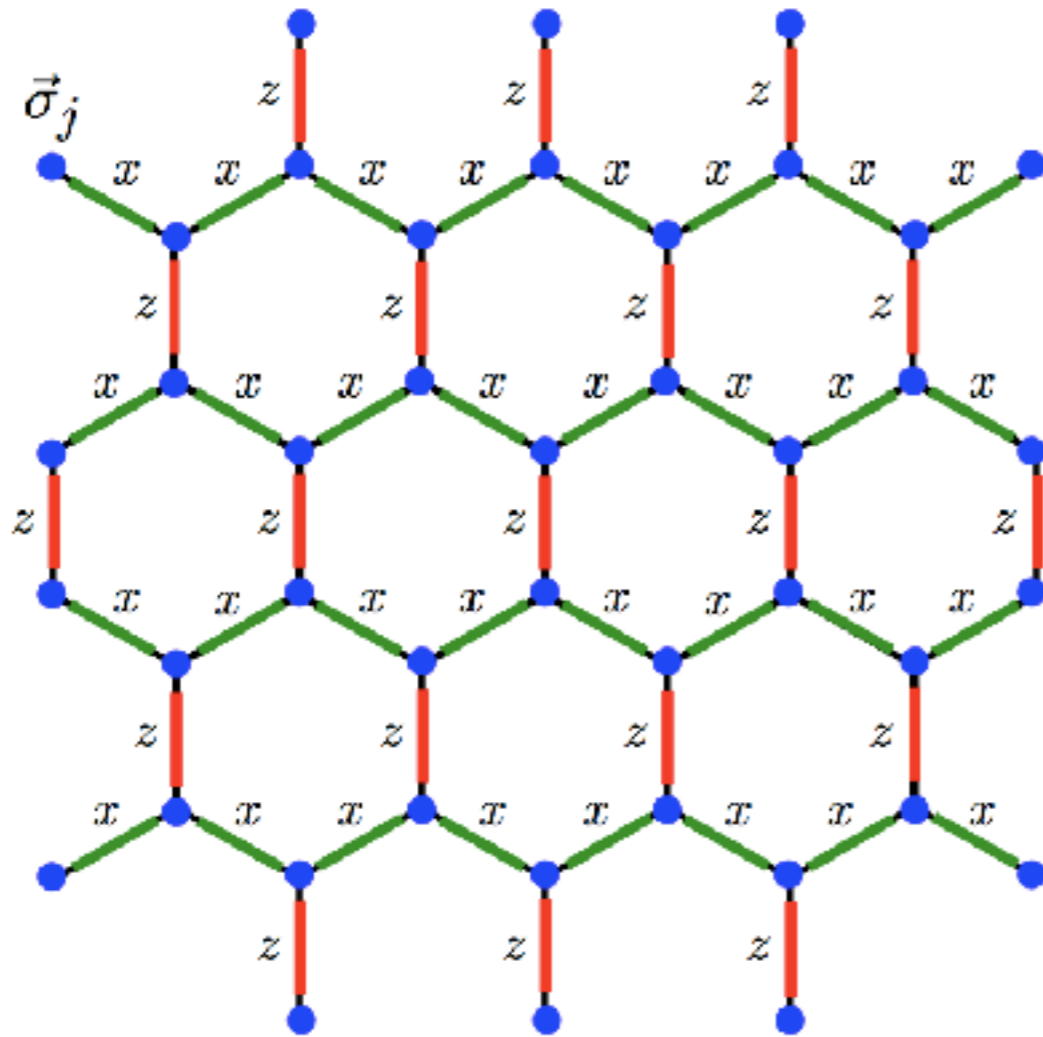
# KT transition

- critical  $\mathcal{J}_2^c = 0$  : a small negative  $\mathcal{J}_2$  increases the stiffness and drive the system into the flat phase
- KT transition: divergence of correlation length  $\xi \sim \exp\left(\frac{\text{const}}{|\mathcal{J}_2|^{3/5}}\right)$
- Finite-size scaling:  $m_{\sqrt{3}} L^{2/3} = \mathcal{F}\left(\frac{\xi(\mathcal{J}_2)}{L}\right)$  (S. Korshunov, '02)

Heisenberg AFM at  $T=10^{-5}$



# Gamma model on honeycomb lattice



- Important interaction in addition to Kitaev and Heisenberg terms in real materials:  $A_2\text{IrO}_3$ ,  $\text{RuCl}_3$
- A new type of highly frustrated magnets

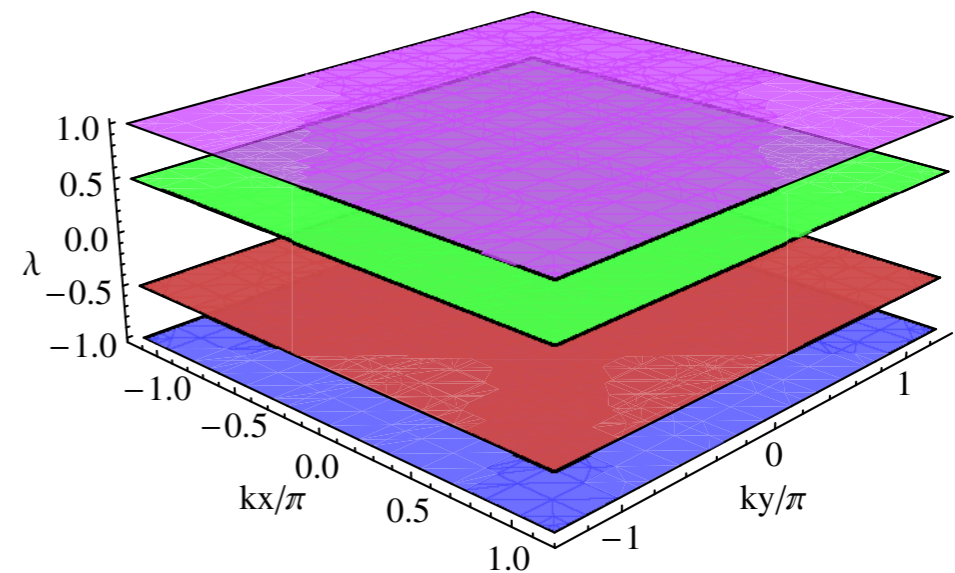
$$\mathcal{H} = \Gamma \sum_{\langle ij \rangle \parallel x} (S_i^y S_j^z + S_i^z S_j^y) + \Gamma \sum_{\langle ij \rangle \parallel y} (S_i^z S_j^x + S_i^x S_j^z) + \Gamma \sum_{\langle ij \rangle \parallel z} (S_i^x S_j^y + S_i^y S_j^x).$$

# Macroscopically degenerate ground states

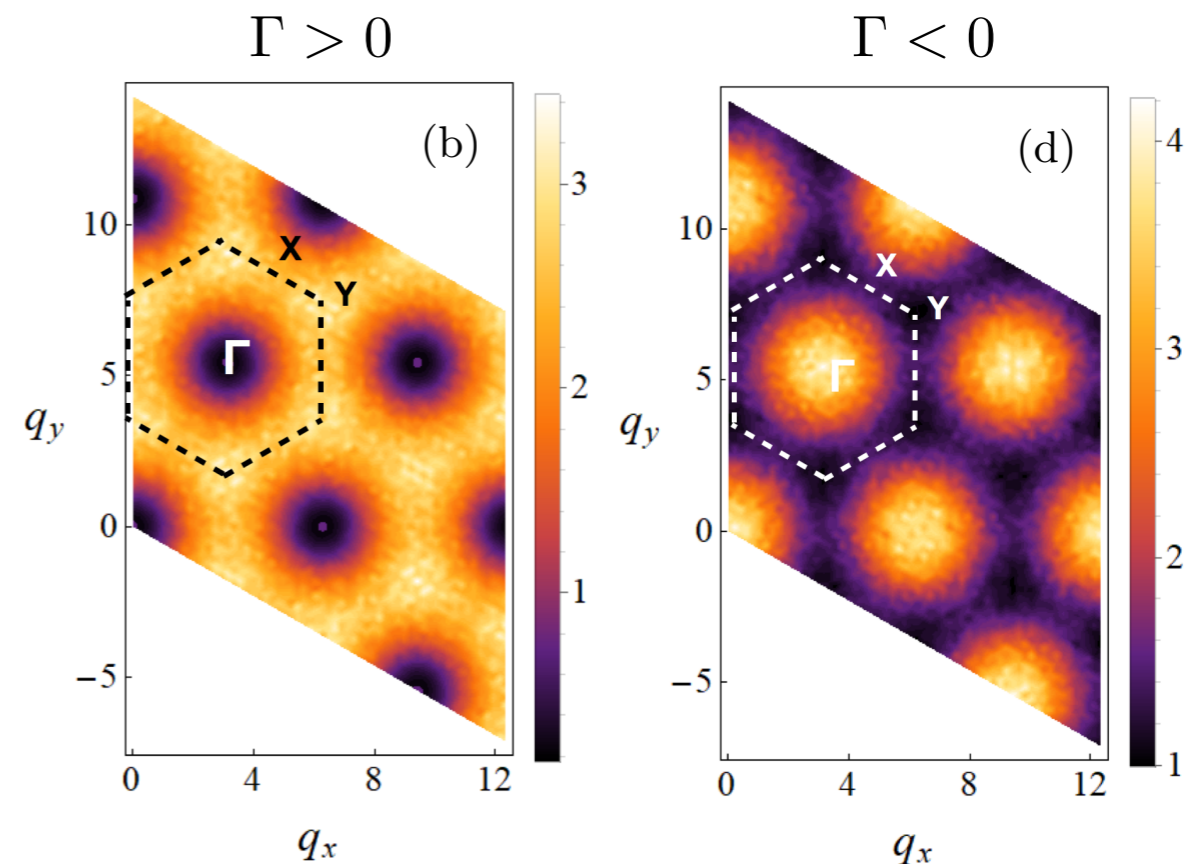
(Rouschatzakis & Perkins, PRL 2017)

- Spectrum under spherical approximation:

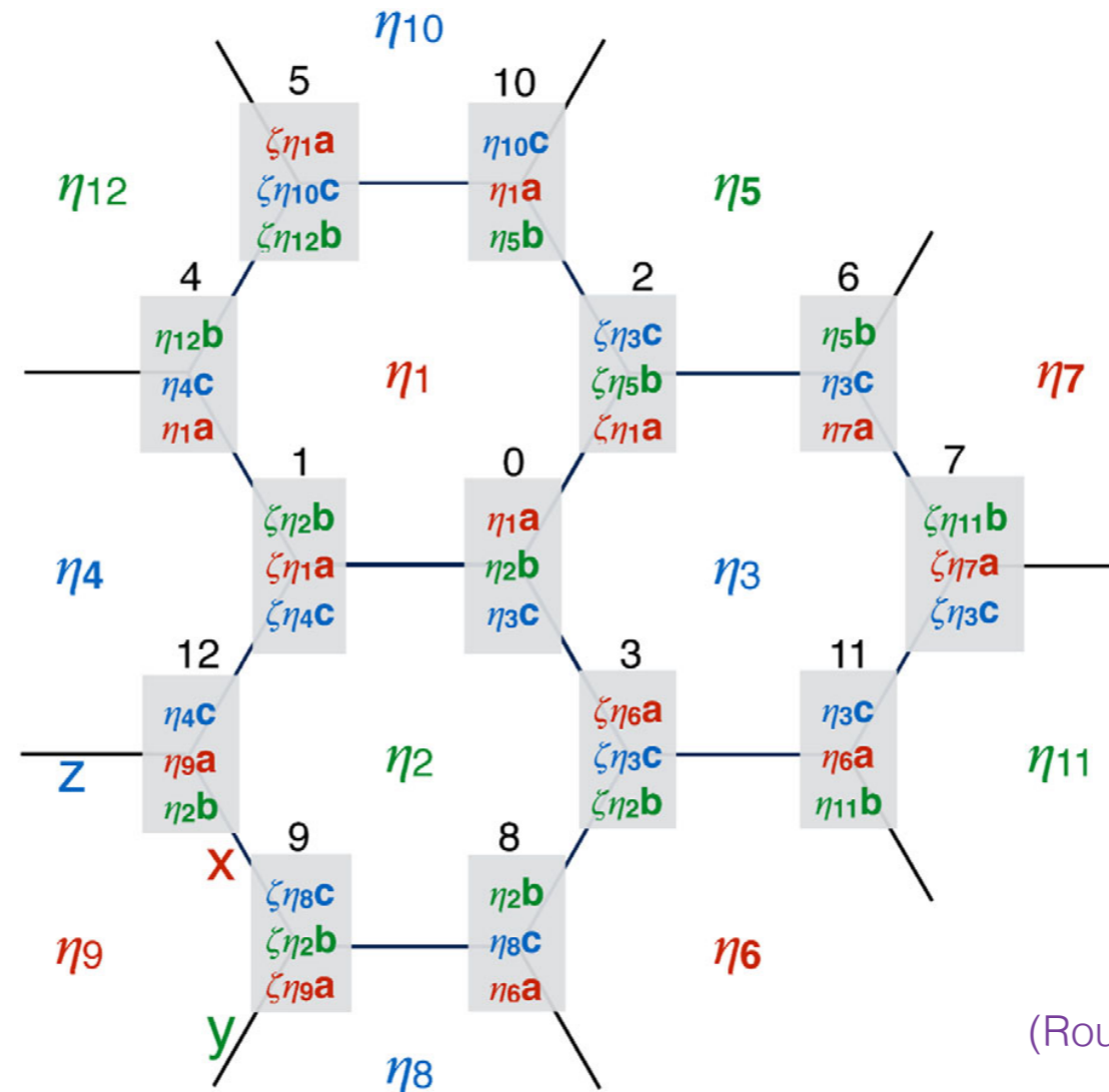
$$\mathcal{H} = \Gamma \sum_{\mathbf{k}} \sum_{ab=1,2} \sum_{\alpha\beta=x,y,z} \mathbb{H}_{\alpha\beta}^{ab}(\mathbf{k}) S_{\alpha}^a(\mathbf{k}) S_{\beta}^b(-\mathbf{k})$$



- Static Structure factor  $S(\mathbf{q})$ :



# Macroscopically degenerate ground states



(Rousochatzakis & Perkins, PRL 2017)

- The degenerate ground states are characterized by continuous variables  $\hat{\mathbf{n}} = (a, b, c)$  and a set of discrete Ising variables  $\{\eta_i\}$

# Macroscopically degenerate classical ground states

$\sqrt{3} \times \sqrt{3}$  order

$$\mathbf{S}_A = (a, b, c)$$

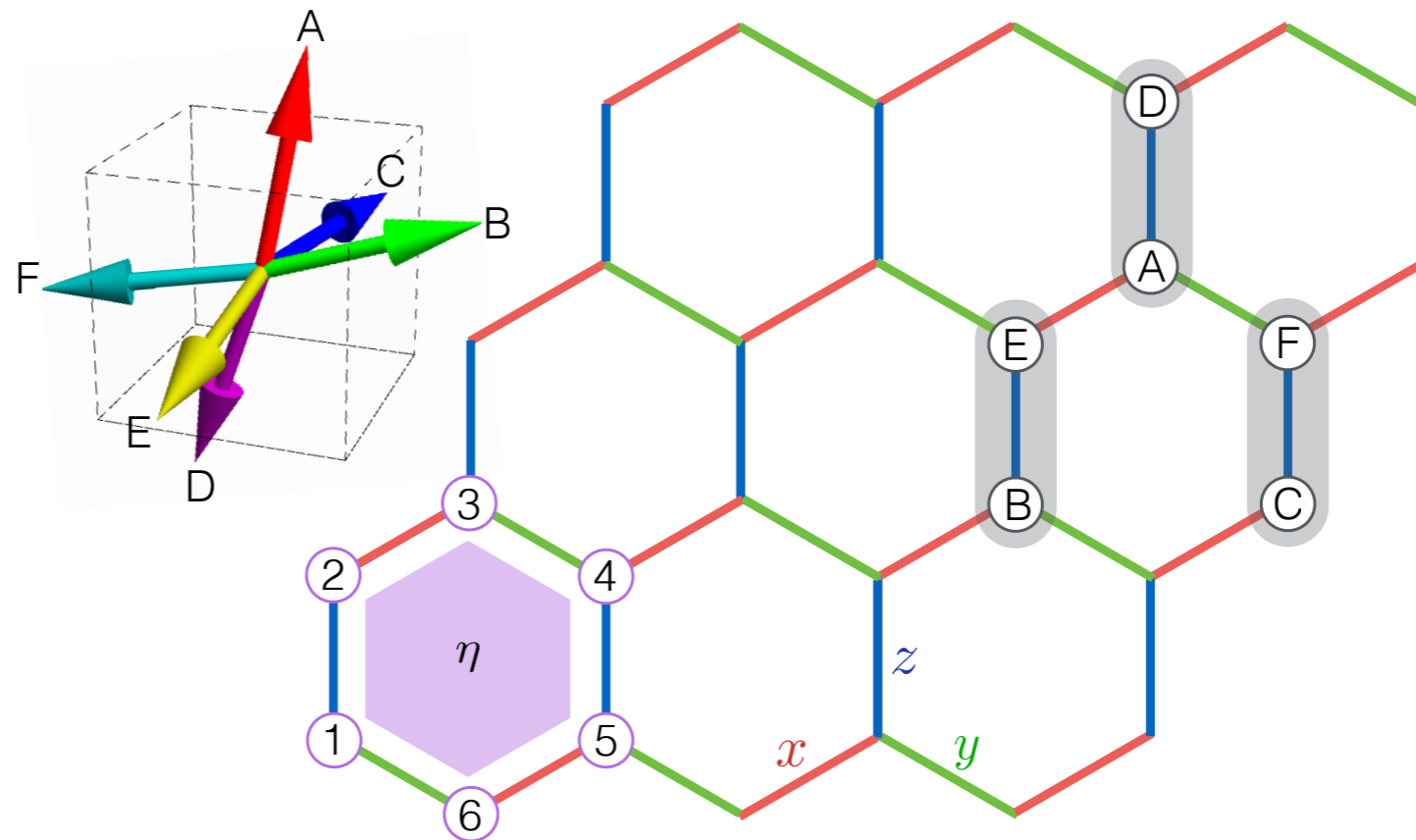
$$\mathbf{S}_B = (c, a, b)$$

$$\mathbf{S}_C = (b, c, a)$$

$$\mathbf{S}_D = \pm(b, a, c)$$

$$\mathbf{S}_E = \pm(a, c, b)$$

$$\mathbf{S}_F = \pm(c, b, a)$$



$$S_1^x \rightarrow \eta S_1^x, \quad S_2^y \rightarrow \eta S_2^y, \quad S_3^z \rightarrow \eta S_3^z,$$

$$S_4^x \rightarrow \eta S_4^x, \quad S_5^y \rightarrow \eta S_5^y, \quad S_6^z \rightarrow \eta S_6^z$$

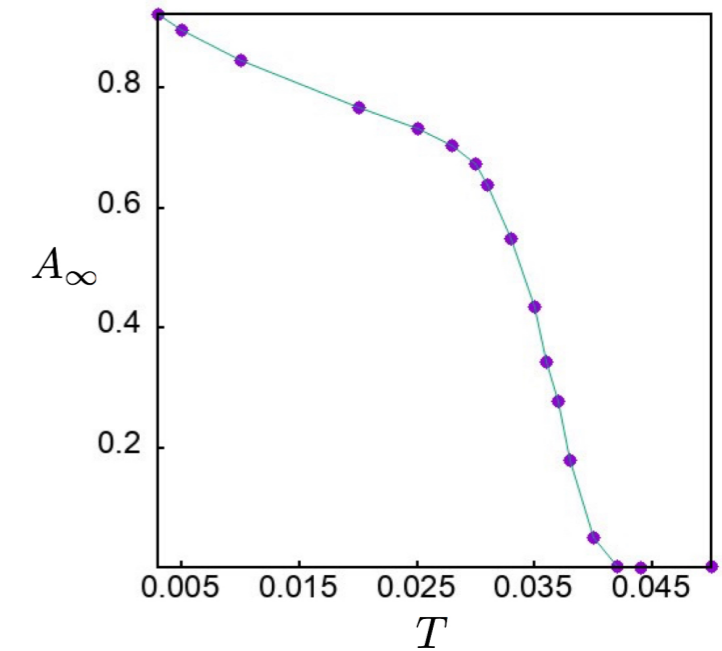
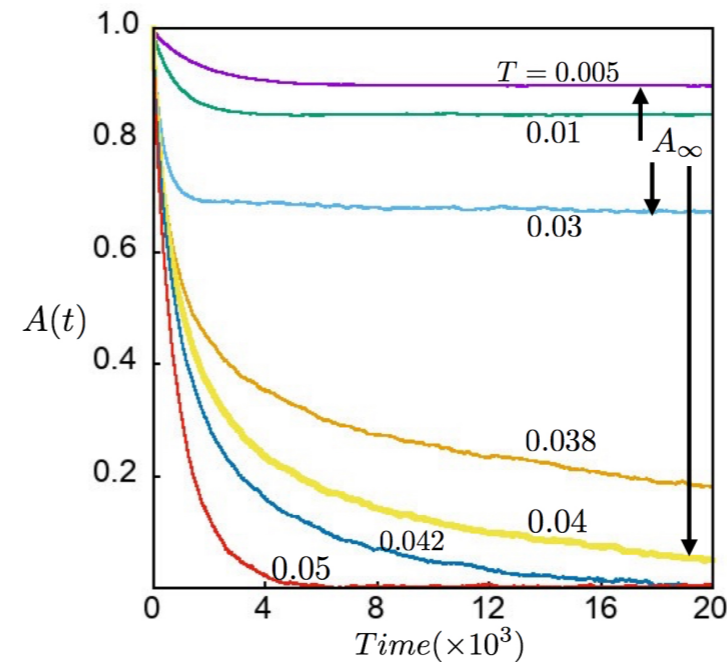


# Thermal order by disorder ?

- Auto-correlation function:

$$A(t) = \frac{1}{N} \sum_i \langle \mathbf{S}_i(t) \cdot \mathbf{S}_i(0) \rangle$$

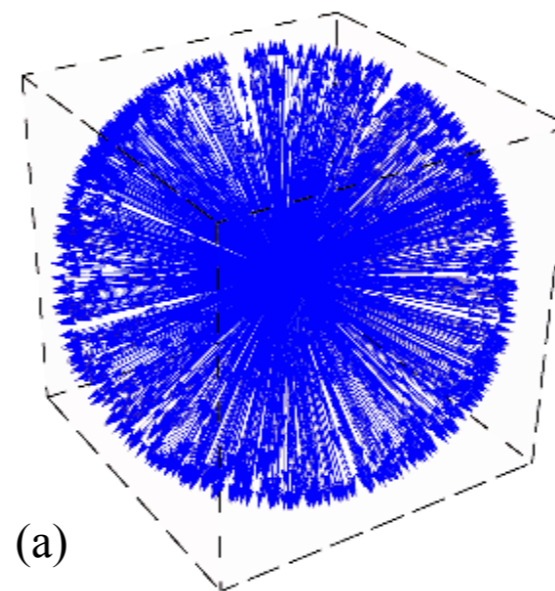
$$\sim A_\infty + (1 - A_\infty) e^{-t/\tau}$$



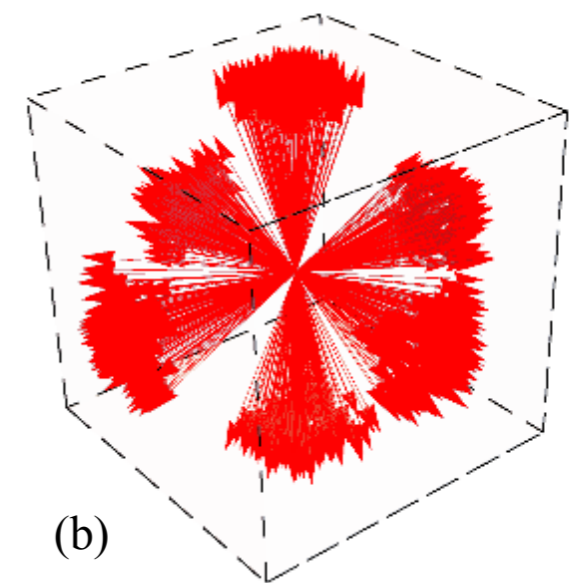
- Snapshots of spins from Monte Carlo simulations

Spins favor cubic directions !

Phase transition ?  
Order parameter ?



(a)  $T > 0.04|\Gamma|$



(b)  $T < 0.04|\Gamma|$

# Flux variable

- flux variable on hexagonal plaquettes:

$$W_\alpha = S_1^x S_2^y S_3^z S_4^x S_5^y S_6^z,$$

- Integrals of motion in the quantum Kitaev model

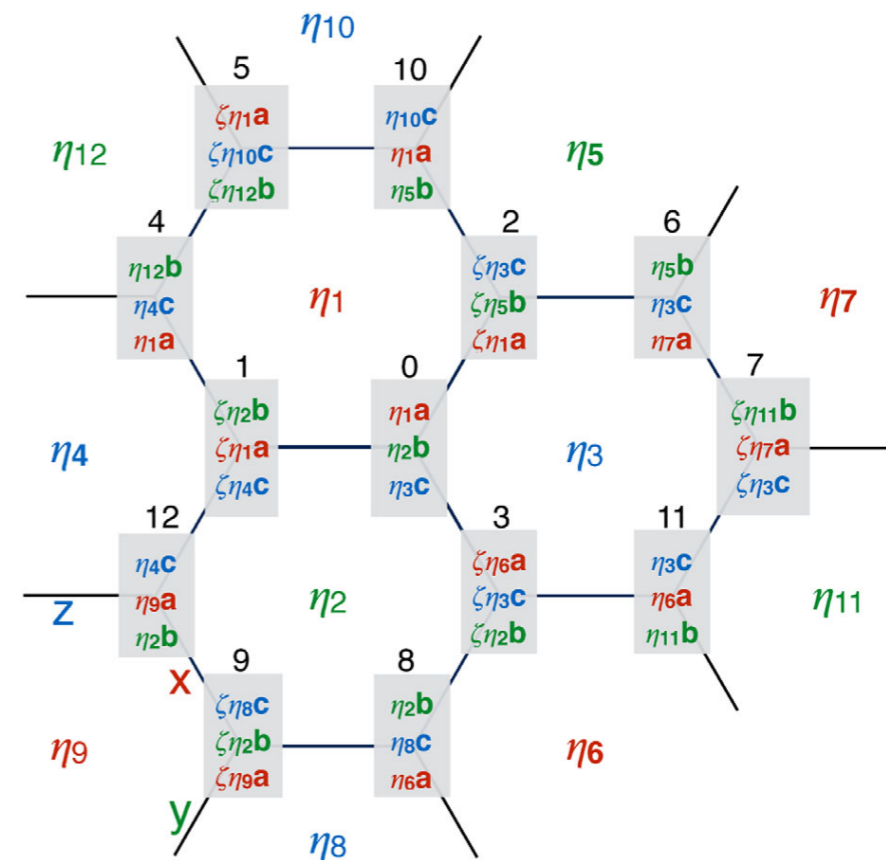
$$[W_\alpha, W_\beta] = 0 \quad [W_\alpha, \mathcal{H}_{\text{Kitaev}}] = 0$$

- In the classical ground states of the Gamma model:

$$W_{h \in A} = W_{\eta_1} = S_0^x S_1^y S_4^z S_5^x S_{10}^y S_2^z / S^6 = \zeta \tilde{a}^6,$$

$$W_{h \in B} = W_{\eta_2} = S_8^x S_9^y S_{12}^z S_1^x S_0^y S_3^z / S^6 = \zeta \tilde{b}^6,$$

$$W_{h \in C} = W_{\eta_3} = S_{11}^x S_3^y S_0^z S_2^x S_6^y S_7^z / S^6 = \zeta \tilde{c}^6,$$



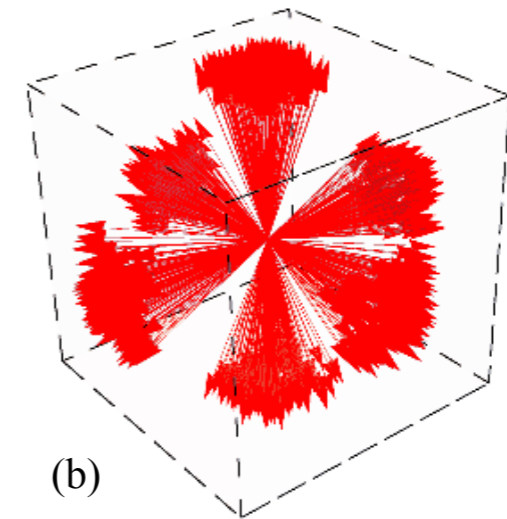
# Phase transition: Plaquette ordering

- Flux variables in the “cubic” phase (a, b, c)  $\sim (1, 0, 0)$ :

$$W_A = a^6 \sim 1$$

$$W_B = b^6 \sim 0$$

$$W_C = c^6 \sim 0$$



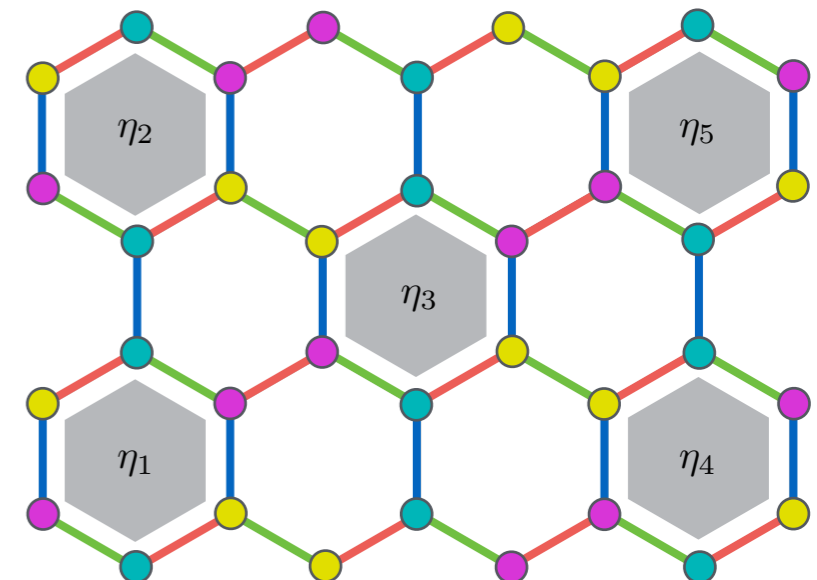
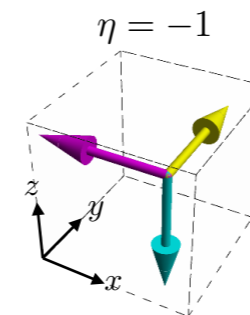
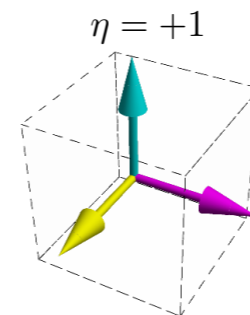
- the flux variables break translation symmetry:

$\sqrt{3} \times \sqrt{3}$  arrangement of the nonzero fluxes

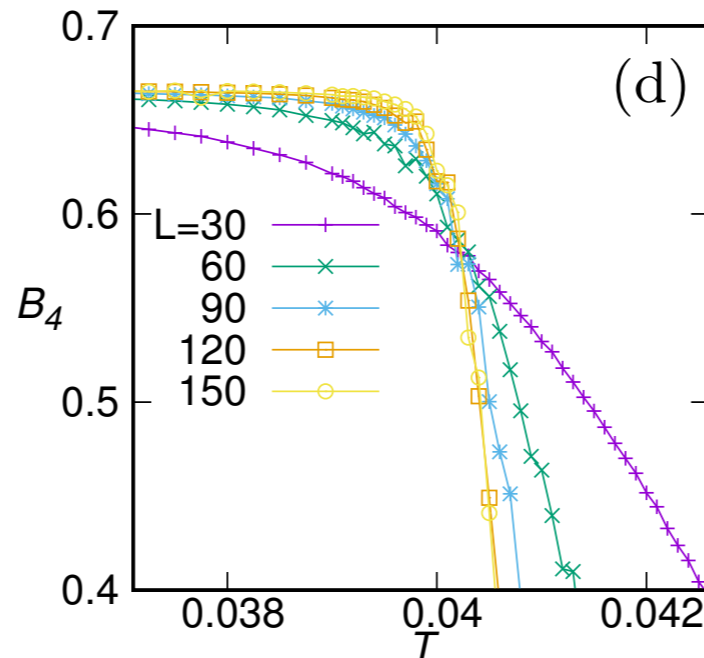
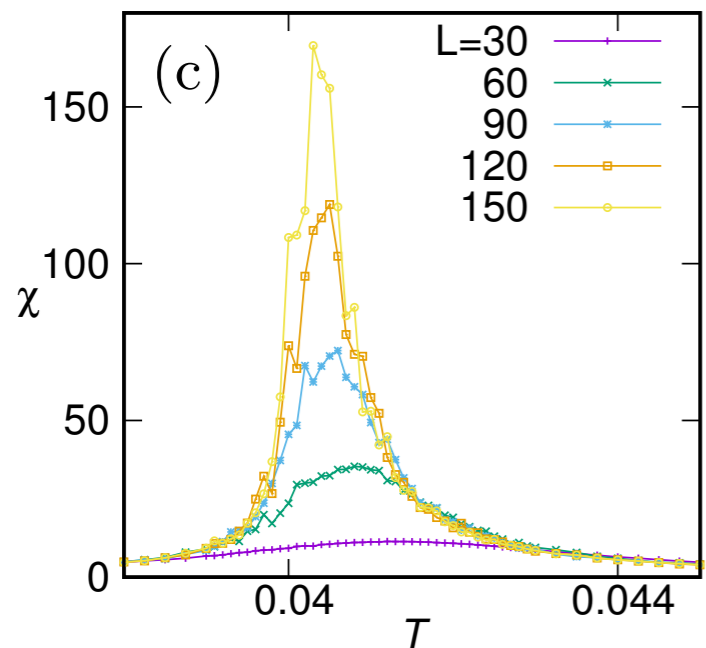
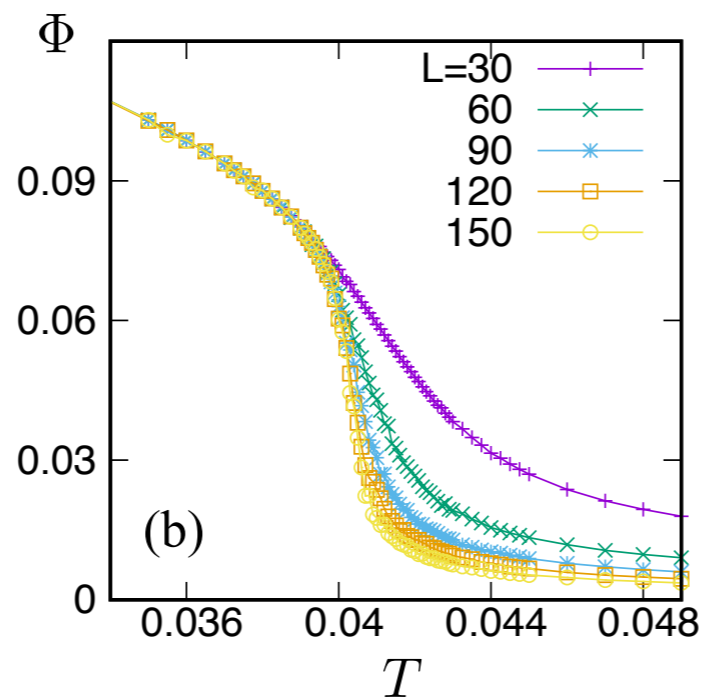
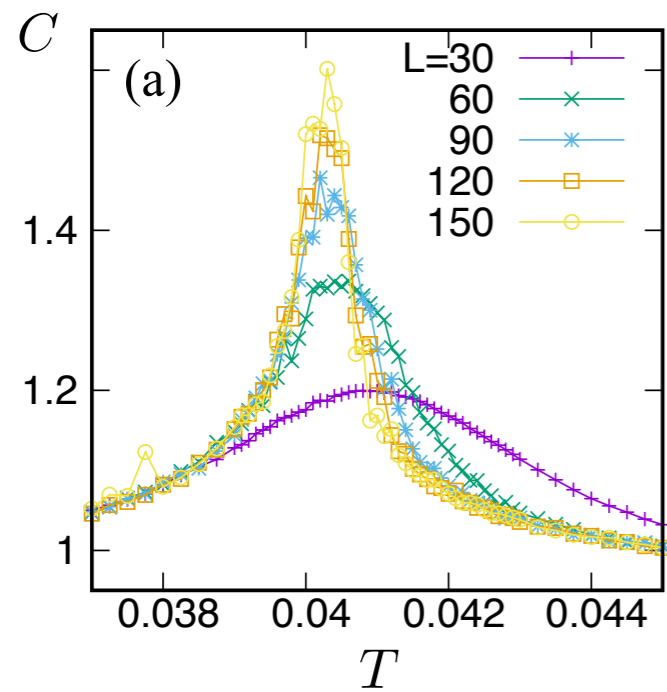
- Order-parameter:

$$\tilde{W}(\mathbf{Q}) = \frac{1}{N} \sum_{\alpha} W_{\alpha} e^{i\mathbf{Q} \cdot \mathbf{r}_{\alpha}},$$

ordering wave vector:  $\mathbf{Q} = \left( \frac{4\pi}{3}, 0 \right)$ .



# Monte Carlo simulations of plaquette ordering



- specific heat

$$C = \frac{\langle \mathcal{H}^2 \rangle - \langle \mathcal{H} \rangle^2}{NT^2}$$

- Order-parameter

$$\Phi = \langle |\tilde{W}(\mathbf{Q})| \rangle,$$

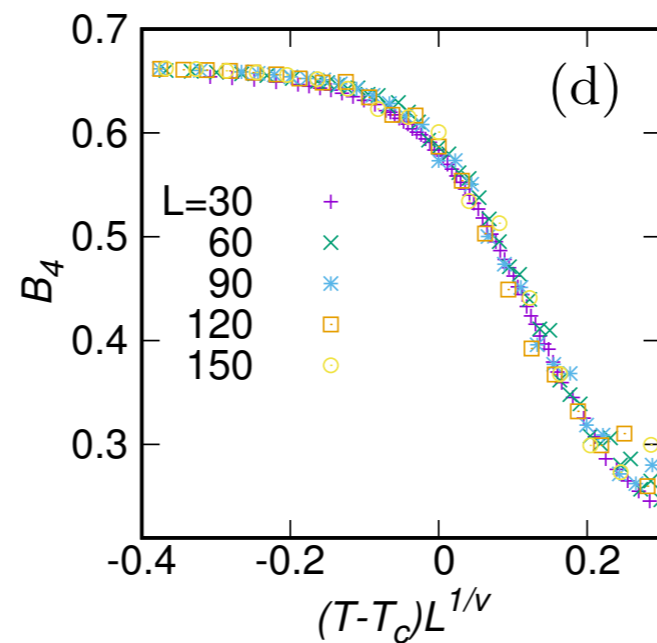
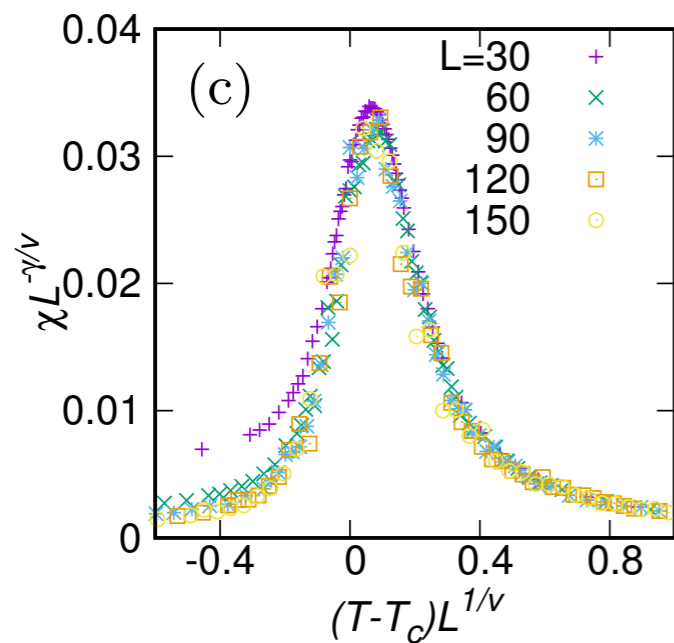
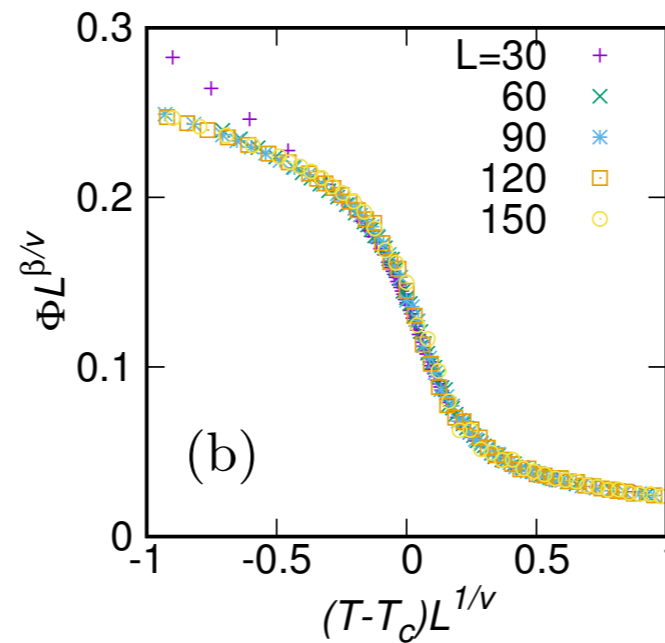
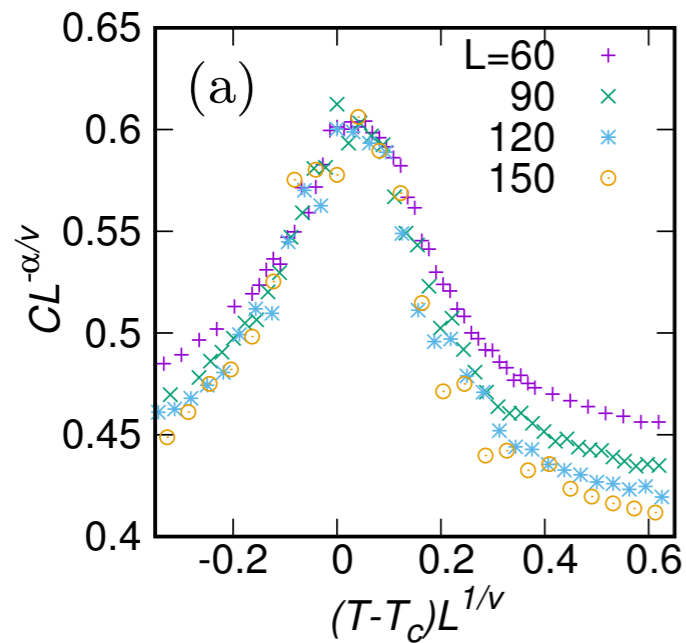
- Susceptibility

$$\chi = N \frac{\langle |\tilde{W}(\mathbf{Q})|^2 \rangle - \langle |\tilde{W}(\mathbf{Q})| \rangle^2}{T}$$

- Binder's cumulant

$$B_4 = 1 - \frac{\langle |\tilde{W}(\mathbf{Q})|^4 \rangle}{3 \langle |\tilde{W}(\mathbf{Q})|^2 \rangle^2}$$

# Finite size scaling analysis



- critical exponents

$$\alpha = 0.167$$

$$\beta = 0.177$$

$$\gamma = 1.47$$

$$\nu = 0.863$$

- 2D 3-state Potts universality class:

$$\alpha = 1/3 = 0.333$$

$$\beta = 1/9 = 0.111$$

$$\gamma = 13/9 = 1.444$$

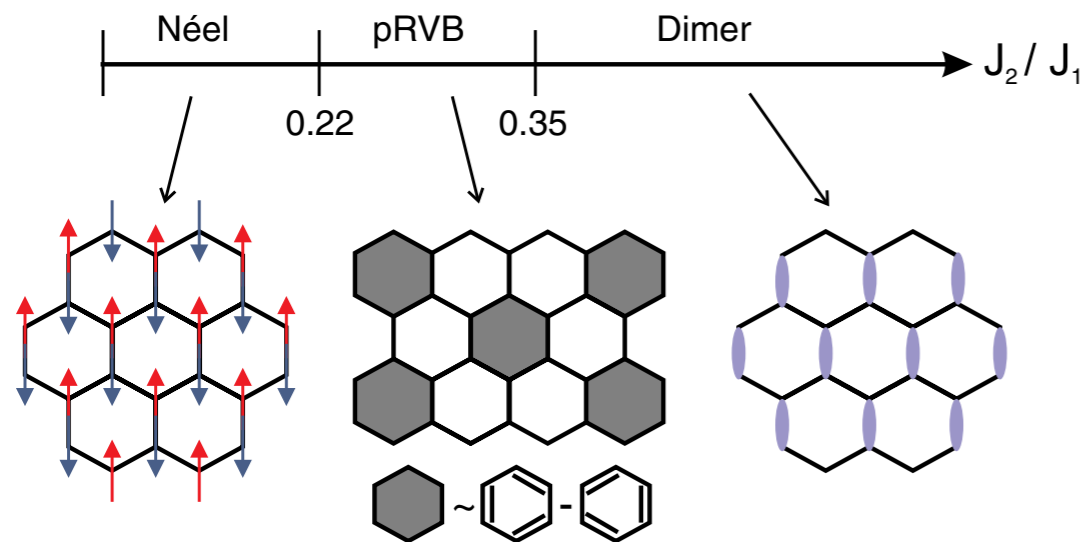
$$\nu = 5/6 = 0.833$$

# Plaquette ordering in quantum models

- J1-J2 antiferromagnetic Heisenberg model on honeycomb lattice:

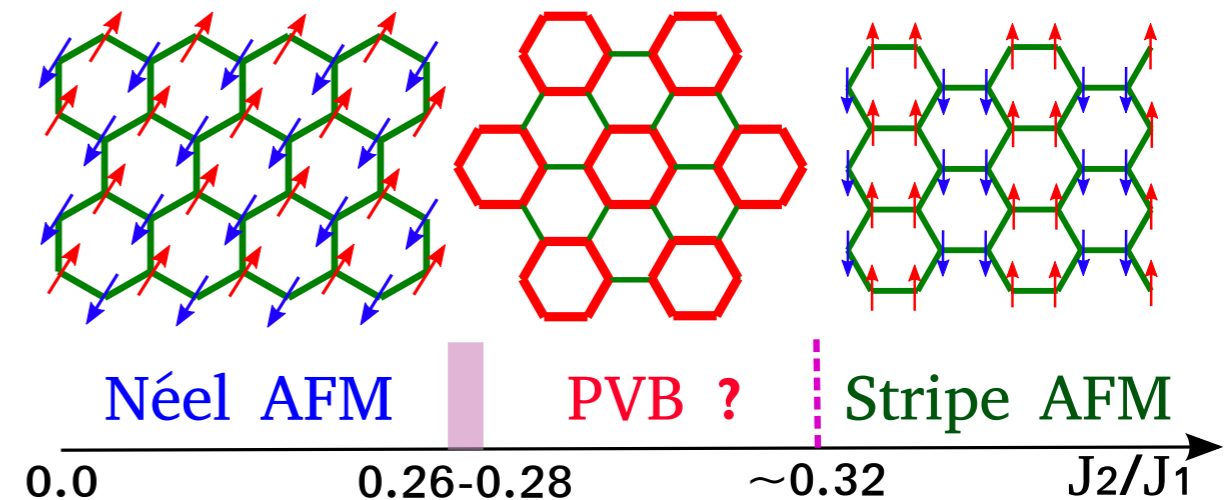
$$\mathcal{H} = J_1 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

- Phase diagram  $S = 1/2$ :



(Ganesh, van den Brink, Nishimoto, PRL 2013)

- Phase diagram  $S = 1$ :

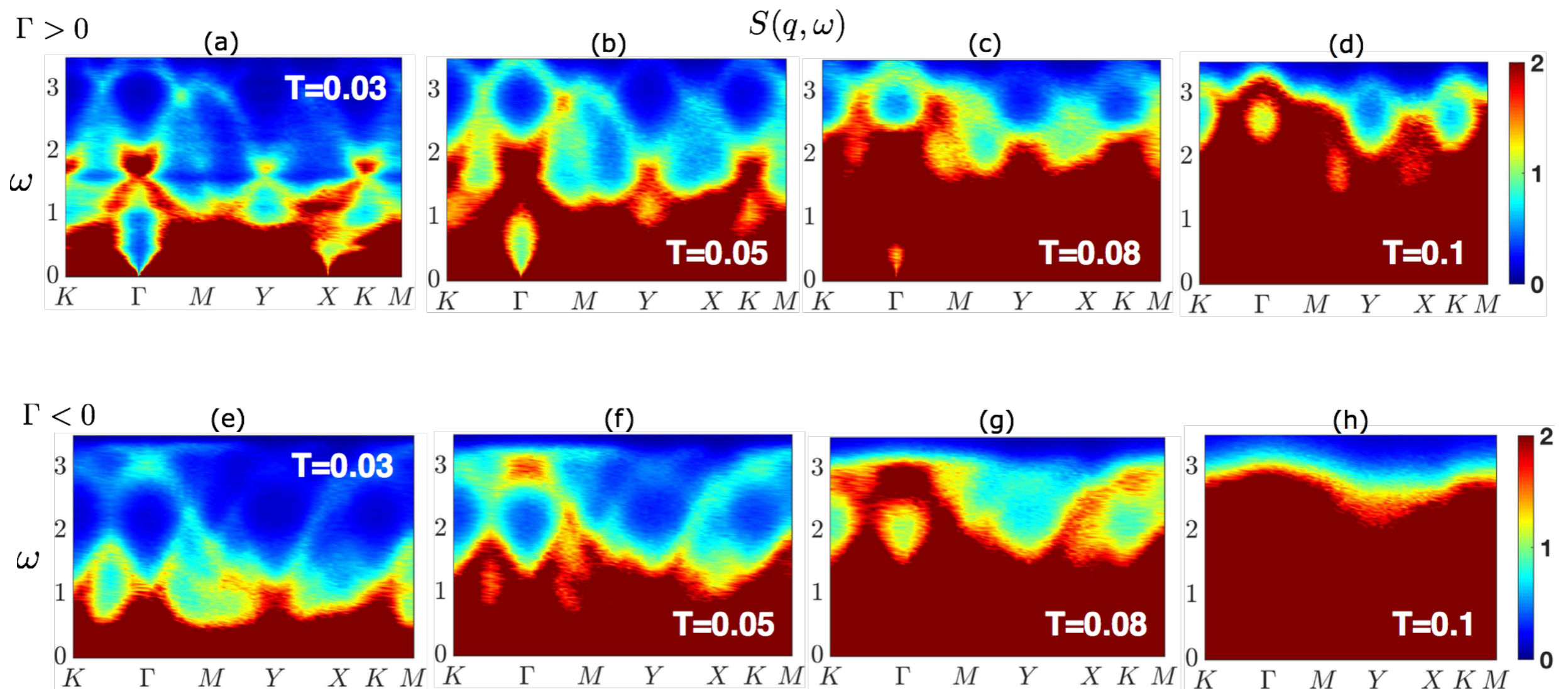


(S-S Gong, Wei Zhu, and D. N. Sheng, PRB 2015)

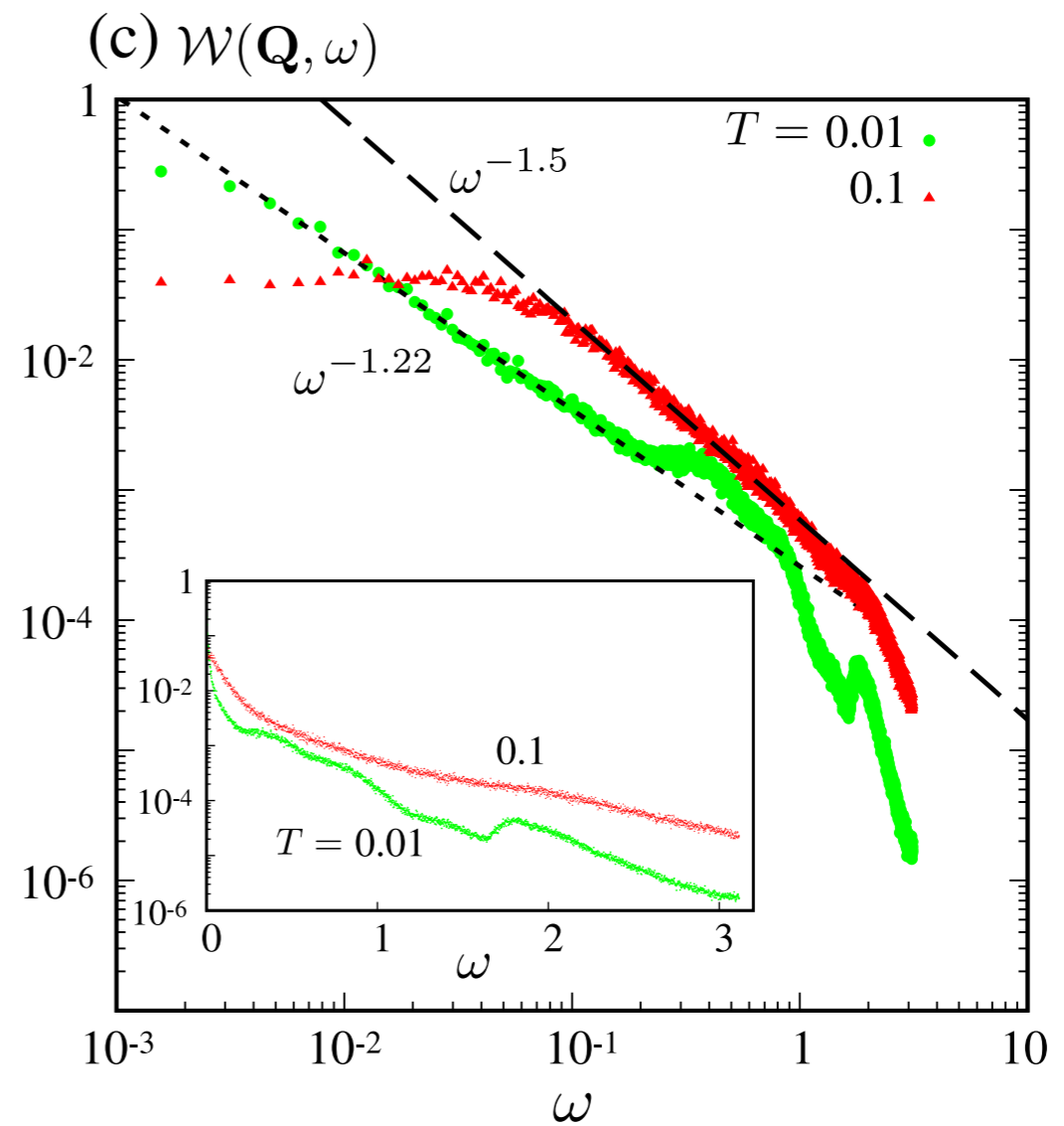
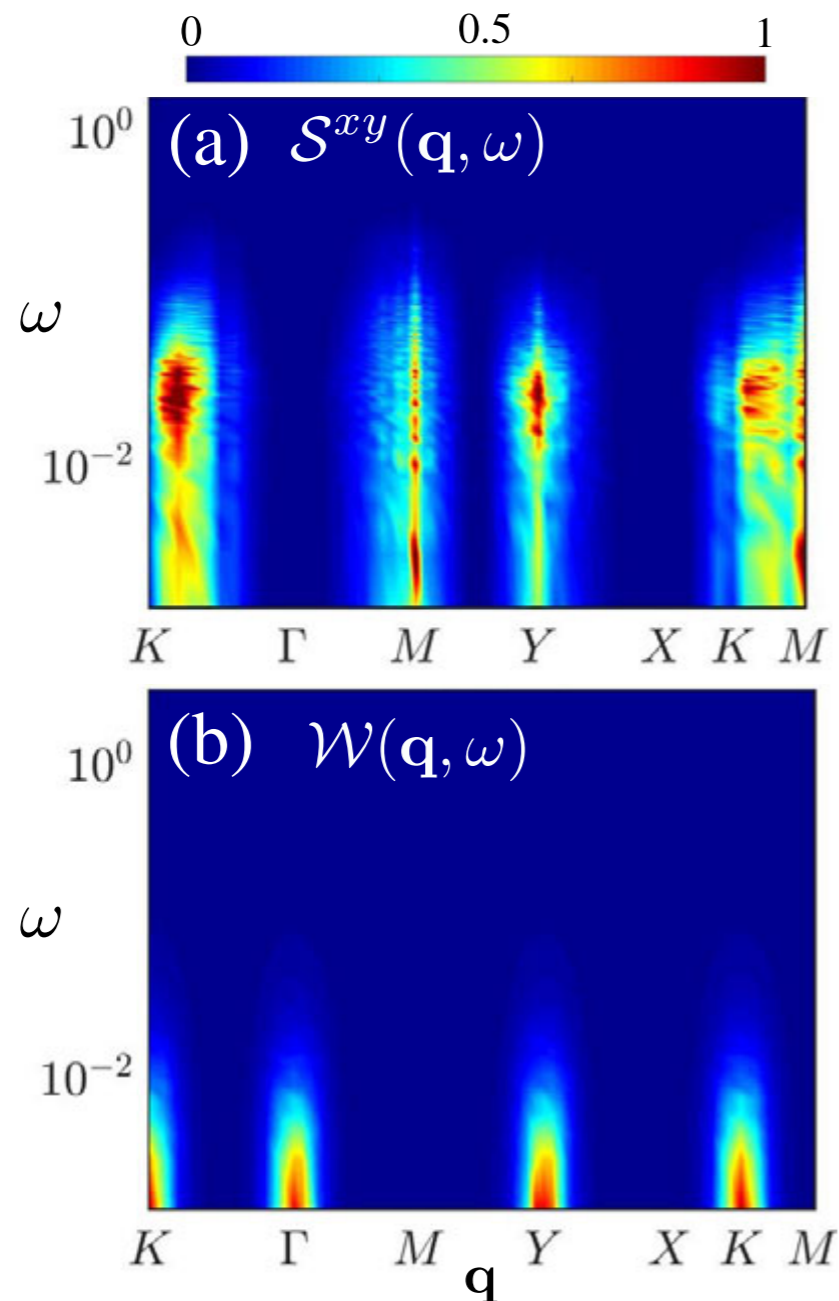


# Dynamical Structure Factor:

- Landau-Lifshitz dynamics:  $\frac{d\mathbf{S}_i}{dt} = -\mathbf{S}_i \times \frac{\partial \mathcal{H}}{\partial \mathbf{S}_i}$ ,
- Dynamical Structure factor:  $S(\mathbf{q}, \omega) = \int \langle \mathbf{S}(\mathbf{q}, t) \cdot \mathbf{S}(-\mathbf{q}, 0) \rangle e^{-i\omega t} dt$



# Dynamical Structure Factor: Off-diagonal





# Quantum order by disorder

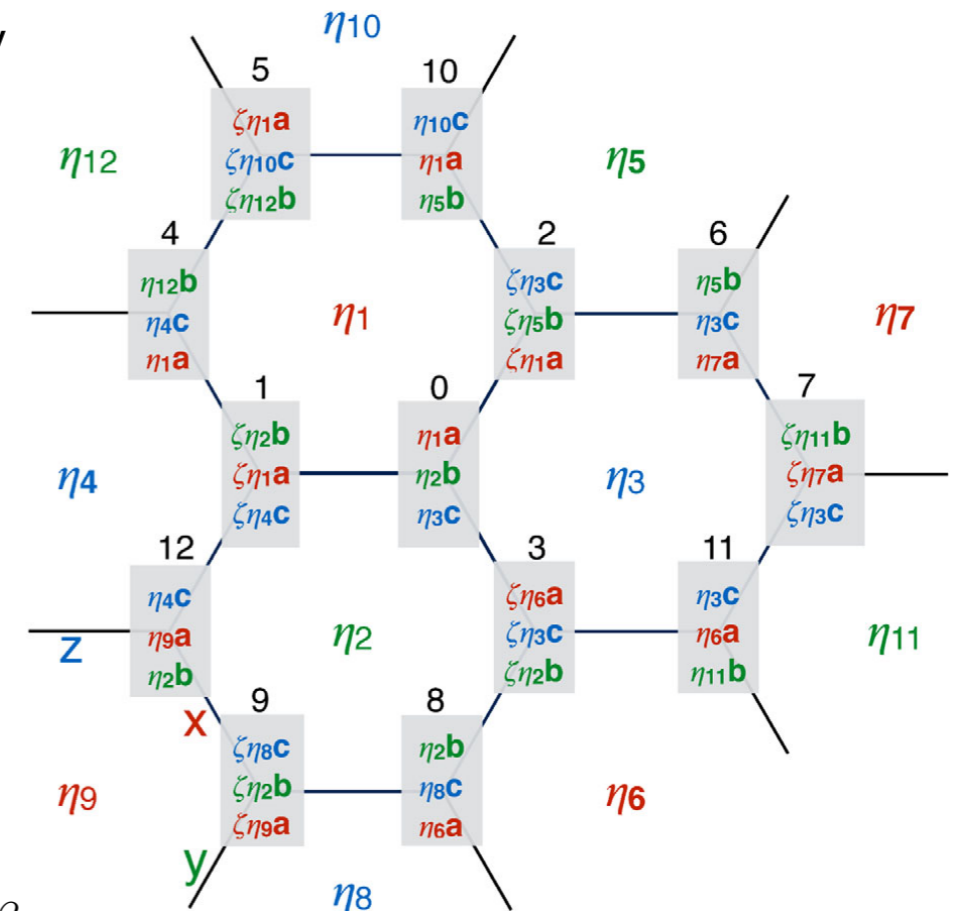
(Rousochatzakis & Perkins, PRL 2017)

- The degenerate ground states are characterized by continuous variables  $\hat{\mathbf{n}} = (a, b, c)$  and a set of discrete Ising variables  $\{\eta_i\}$

- Real-space perturbation calculation:

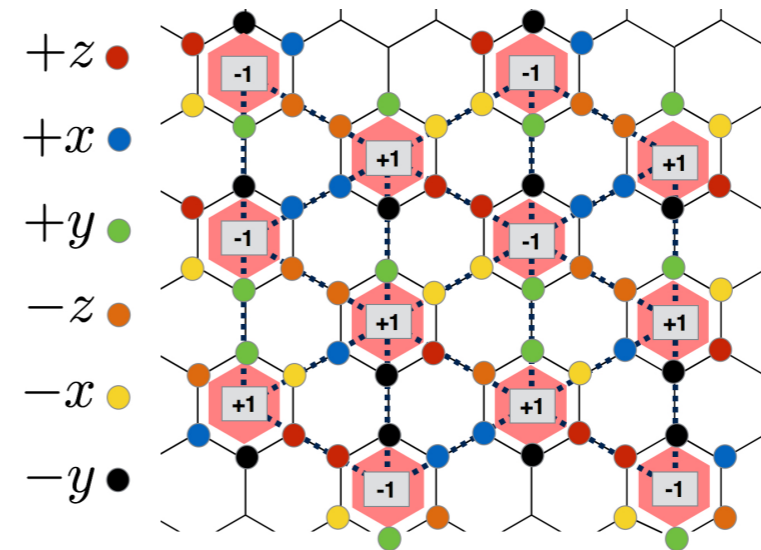
$$E_{\text{eff}} = -\frac{|\Gamma|S}{32}(a^4 + b^4 + c^4) + \frac{\Gamma S a^2}{8} \sum_{\langle \alpha\beta \rangle}^{\text{red}} \eta_\alpha \eta_\beta + \frac{\Gamma S b^2}{8} \sum_{\langle \alpha\beta \rangle}^{\text{green}} \eta_\alpha \eta_\beta + \frac{\Gamma S c^2}{8} \sum_{\langle \alpha\beta \rangle}^{\text{blue}} \eta_\alpha \eta_\beta$$

- (a, b, c) favors cubic direction.

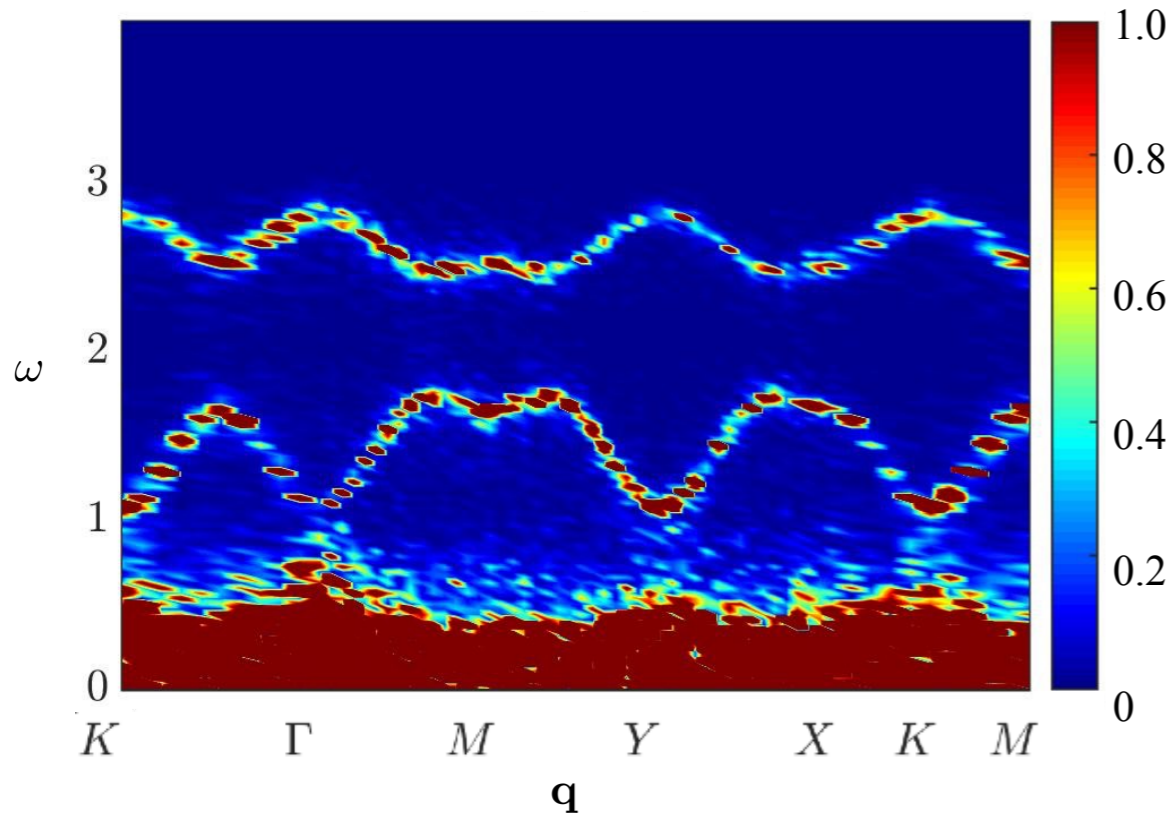


# Quantum Order by disorder

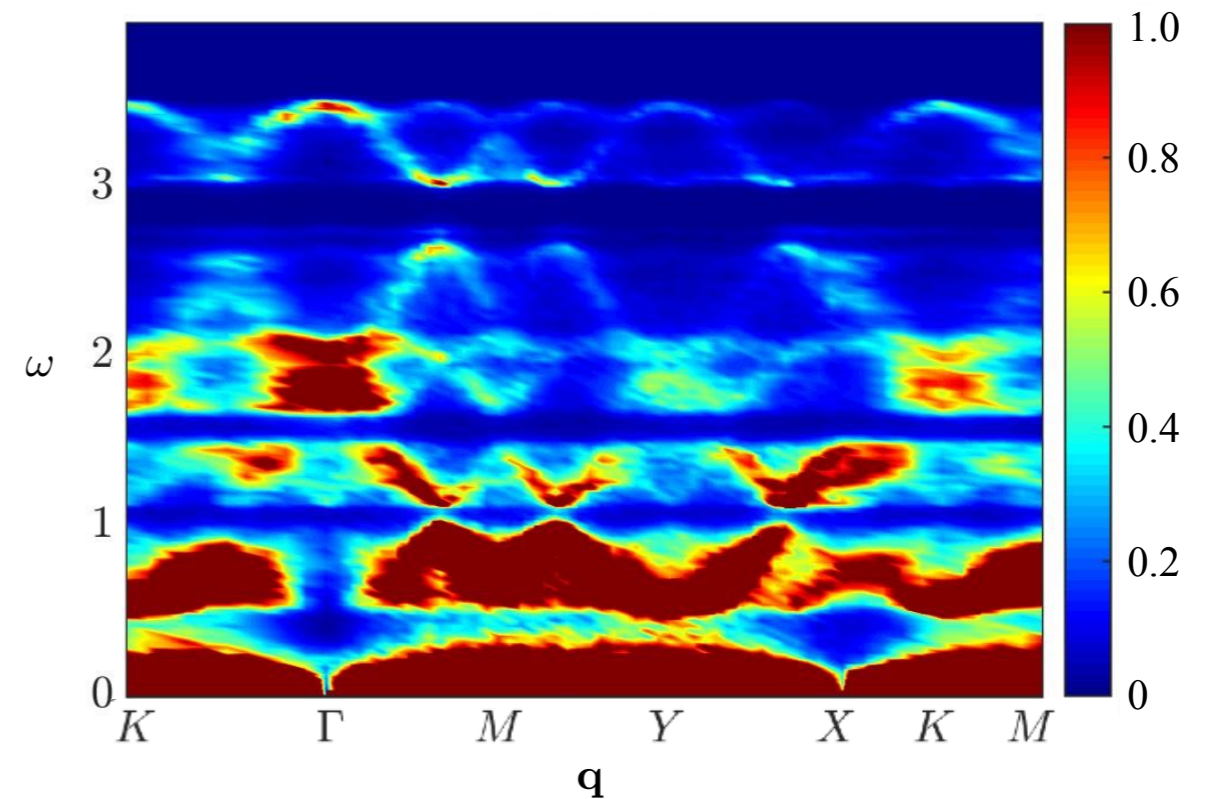
$$\mathcal{H}_{\text{Ising}} = \epsilon \Gamma \sum_{\langle \alpha \beta \rangle}' \eta_{\alpha} \eta_{\beta}$$



$\Gamma < 0$



$\Gamma > 0$



# Quantum spin-1/2 Gamma model?

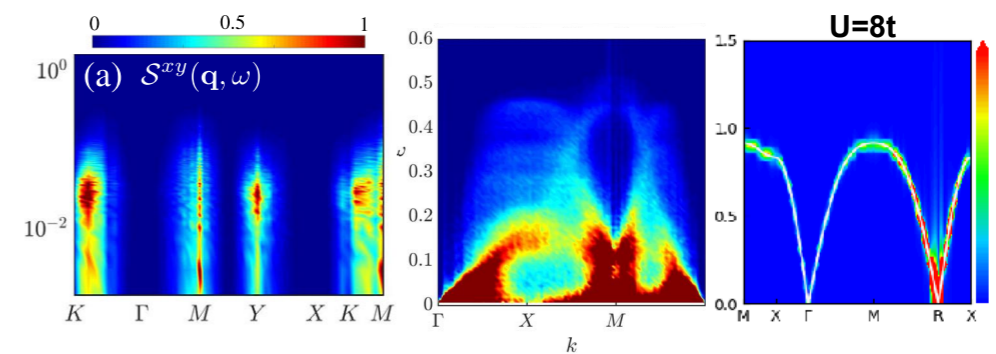
**Title:** Ground State of the Spin-1/2 Honeycomb  $\Gamma$  Model: Zigzag Magnetic Order  
**Authors:** [Liao, Hai-Jun](#); [Huang, Ruizhen](#); [Guo, Yi-Bin](#); [Xie, Zhi-Yuan](#); [Normand, Bruce](#); [Xiang, Tao](#)  
**Publication:** APS March Meeting 2019, abstract id.A37.002  
**Publication Date:** 00/2019  
**Origin:** [APS](#)  
**Bibliographic Code:** [2019APS..MARA37002L](#)

## Abstract

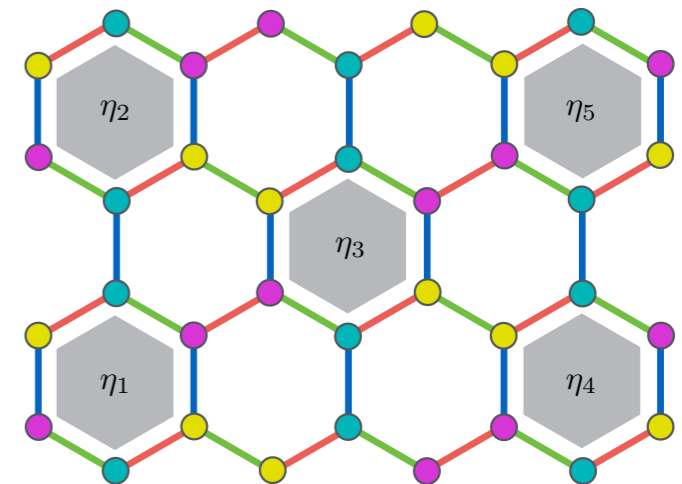
The off-diagonal symmetric interaction,  $\Gamma (S_i^\alpha S_{i+\gamma}^\beta + S_i^\beta S_{i+\gamma}^\alpha)$ , has sprung to prominence as a competing term in the spin Hamiltonians of candidate Kitaev materials. We investigate the quantum ( $S = 1/2$ )  $\Gamma$  model on the honeycomb lattice using the tensor-network method of infinite projected entangled pair states (iPEPS). We demonstrate that the ground state is a zigzag magnetically ordered state, rather than the spin liquid reported on the basis of density-matrix renormalization-group (DMRG) studies. By applying two quasi-one-dimensional numerical treatments, the infinite matrix-product-state (iMPS) and DMRG methods, we show that this contrast is a consequence of the system size considered. Thus the quantum  $\Gamma$  model is quite different from its classical counterpart, which is a classical spin liquid due to its macroscopic ground-state degeneracy.

# Summary

- Numerical methods for dynamical structure factors based on equation-of-motion approach



- Hidden plaquette order and thermal order by disorder in the honeycomb Gamma model



THANK YOU