Spin-1 Redux: Some New Methods (on old results)

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Spin-1 Chain: Rich Source of Concepts and Techniques

1983: Haldane gap proposed

1987: Affleck-Kennedy-Lieb-Tasaki (AKLT) model proposed

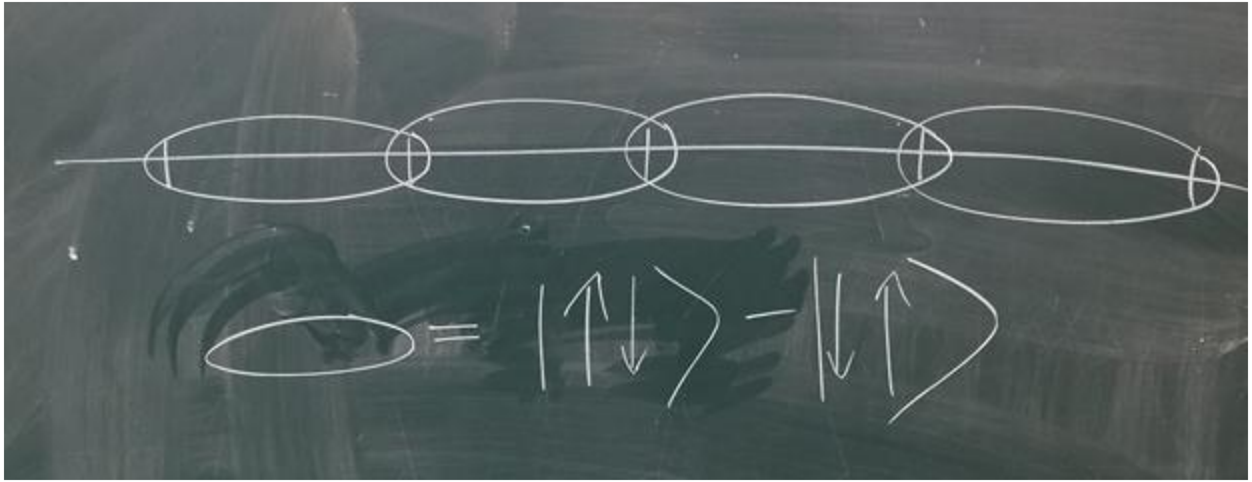
1990's: Matrix Product States (MPS) form proposed; DMRG technique

2010's: SPT re-interpretation; dynamics DMRG & MPS

2015: NP

2019: ??

A Simple Picture why Haldane Gap Exists



Break spin-1 into two spin-½'s

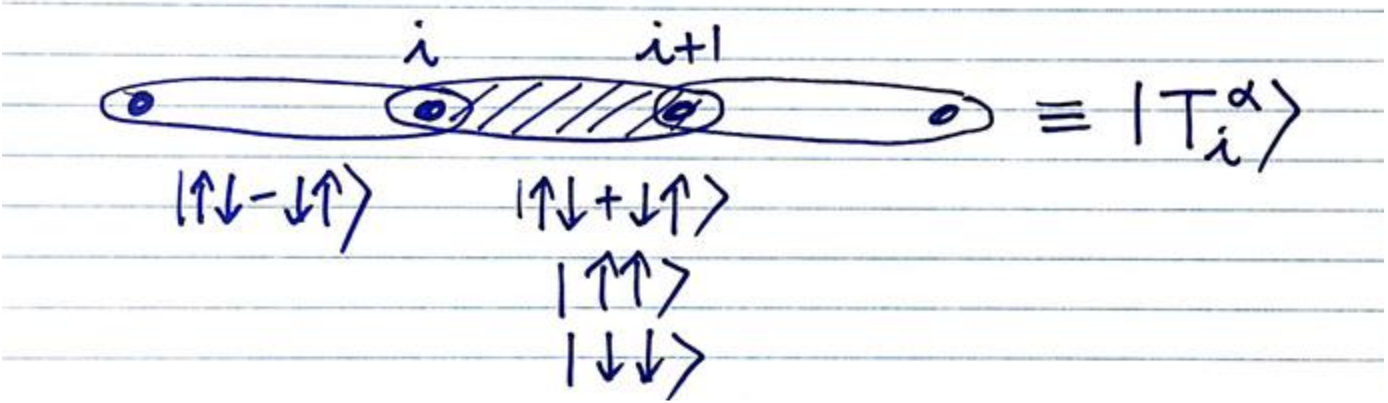
Each bond forms a spin-singlet (out of two spin-½'s)

Net spin S= ½+½=1 per site

gap = (triplet)-(singlet) energy

Excitations of AKLT state

Turn local singlet into local triplet (=triplon)



This local operation (in triplon picture) is nonlocal for spins

(~soliton/kink excitation):

$$|T_i^\alpha\rangle = \sum_{j\le i} 2|S_j^\alpha\rangle \qquad |S_j^\alpha\rangle \equiv S_j^\alpha|A\rangle$$

Magnon/triplon Dichotomy

Magnon waves = Triplon waves (well known in literature)

$$|\mathcal{T}_k^\alpha\rangle = \sum_i e^{ikx_i} |\mathcal{T}_i^\alpha\rangle$$

$$|S_k^\alpha\rangle \equiv \sum_i e^{ikx_i} |S_i^\alpha\rangle = \frac{1}{2}(1 - e^{ik}) |\mathcal{T}_k^\alpha\rangle$$

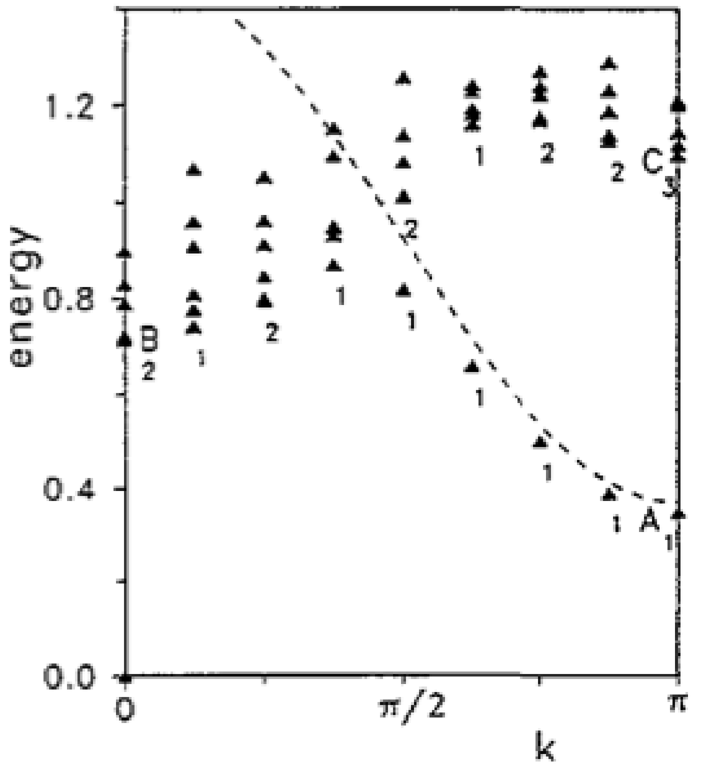
Only one type of excitation, with energies calculated in SMA (Arovas, Auerbach, Haldane, 1988)

$$\boxed{\omega_1(k) = \frac{\langle \mathcal{T}_k^\alpha | H_A | \mathcal{T}_k^\alpha \rangle}{\langle \mathcal{T}_k^\alpha | \mathcal{T}_k^\alpha \rangle} = \frac{5}{27}(5 + 3\cos k)}$$

Sanity check I:

Exact excitation spectrum agrees with SMA one-magnon spectrum quite well

A large portion of excitations around k=0 remains; two-magnons?

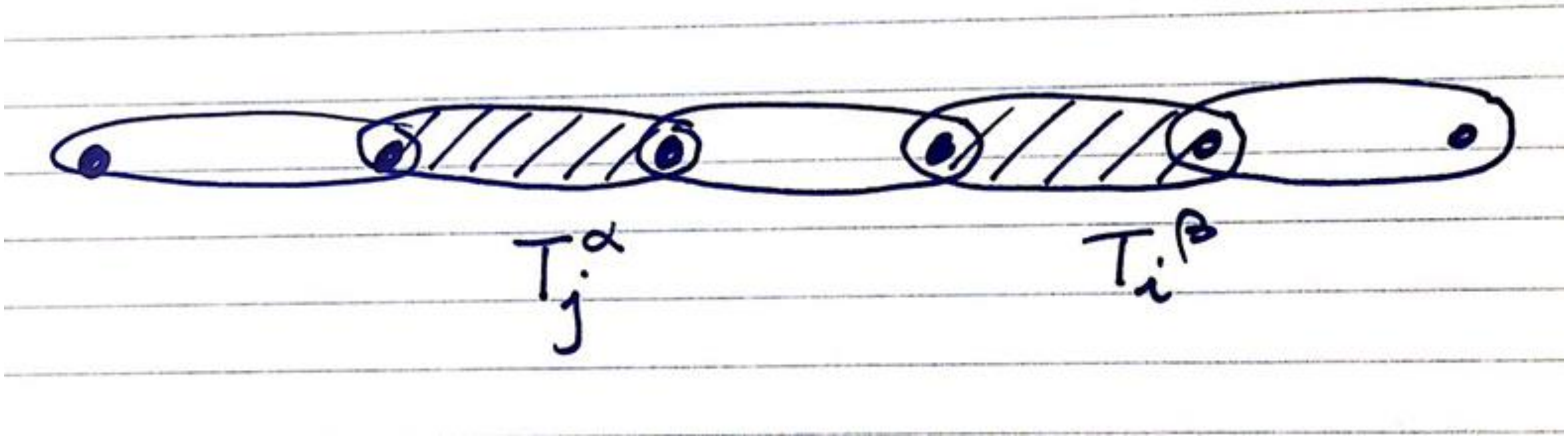


Fath&Solyom, 93

(2Magnon = 2Triplon) Dichotomy?

$$|S_{k_1}^{\alpha_1} S_{k_2}^{\alpha_2}\rangle = \sum_{i,j} e^{ik_1x_i + ik_2x_j}\, |S_i^{\alpha_1} S_j^{\alpha_2}\rangle \qquad |S_i^{\alpha} S_j^{\beta}\rangle = S_i^{\alpha} S_j^{\bar{\beta}} |A\rangle$$

$$|\mathcal{T}_{k_1}^{\alpha_1} \mathcal{T}_{k_2}^{\alpha_2}\rangle = \sum_{i,j} e^{ik_1x_i + ik_2x_j}\, |\mathcal{T}_i^{\alpha_1} \mathcal{T}_j^{\alpha_2}\rangle$$



Our Investigation gives ...

- ❖ | 2-magnon> = |2-triplon> !

$$
\begin{aligned}
|S^{\pm}_{k_1} S^{\pm}_{k_2}\rangle &= R(k_1,k_2)\,|\mathcal{T}^{\pm}_{k_1}\mathcal{T}^{\pm}_{k_2}\rangle \\
|S^{z}_{k_1} S^{z}_{k_2}\rangle &= \frac{1}{4}R(k_1,k_2)\Big(|\mathcal{T}^{0}_{k_1}\mathcal{T}^{0}_{k_2}\rangle + N\delta_{k_1,-k_2}\,|A\rangle\Big) \\
|S^{\pm}_{k_1} S^{\mp}_{k_2}\rangle &= -R(k_1,k_2)\Big(|\mathcal{T}^{\pm}_{k_1}\mathcal{T}^{\mp}_{k_2}\rangle - \frac{1}{2}N\delta_{k_1,-k_2}\,|A\rangle\Big) \\
|S^{\pm}_{k_1} S^{z}_{k_2}\rangle &= \mp\frac{1}{2}R(k_1,k_2)\,|\mathcal{T}^{\pm}_{k_1}\mathcal{T}^{0}_{k_2}\rangle \\
|S^{z}_{k_1} S^{\pm}_{k_2}\rangle &= \mp\frac{1}{2}R(k_1,k_2)\,|\mathcal{T}^{0}_{k_1}\mathcal{T}^{\pm}_{k_2}\rangle\,.
\end{aligned}
\tag{3.6}
$$

- ❖ Two-magnon SMA

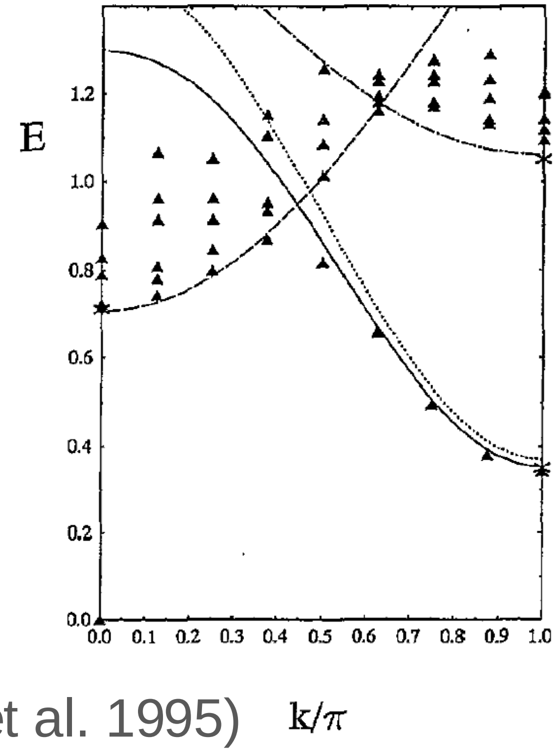
$$\omega_2^{\alpha_1\alpha_2}(k_1,k_2) = \frac{\langle \mathcal{T}^{\alpha_1}_{k_1}\mathcal{T}^{\alpha_2}_{k_2}|H_A|\mathcal{T}^{\alpha_1}_{k_1}\mathcal{T}^{\alpha_2}_{k_2}\rangle}{\langle \mathcal{T}^{\alpha_1}_{k_1}\mathcal{T}^{\alpha_2}_{k_2}|\mathcal{T}^{\alpha_1}_{k_1}\mathcal{T}^{\alpha_2}_{k_2}\rangle}$$

$$\omega_2(k_1,k_2) = \omega_1(k_1) + \omega_1(k_2);$$

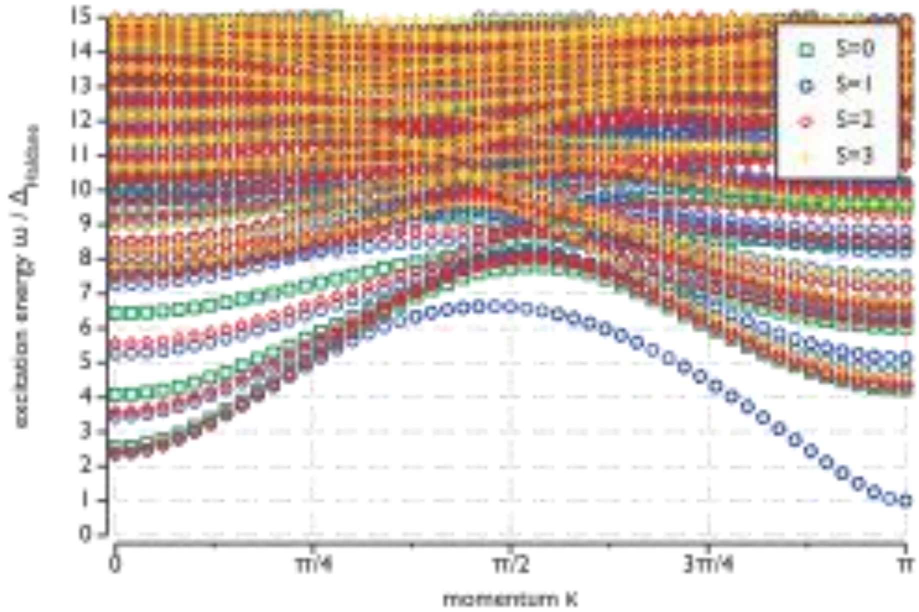
Conclusion: magnons are good quasiparticles!

No magnon-magnon interaction effect! (9-fold degeneracy)

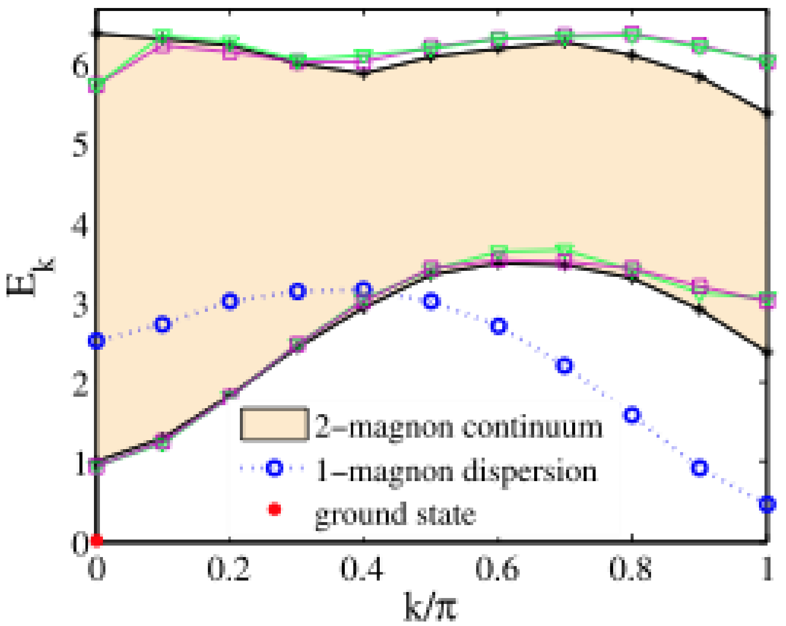
$$\boxed{\begin{aligned}\omega_2(k) &= \omega_1\left(\pi + \frac{k}{2}\right) + \omega_1\left(-\pi + \frac{k}{2}\right)\\ &= \frac{10}{27}\left(5 - 3\cos\frac{k}{2}\right).\end{aligned}}$$



(Mikeska et al. 1995)



(Haegeman et al. 2012)

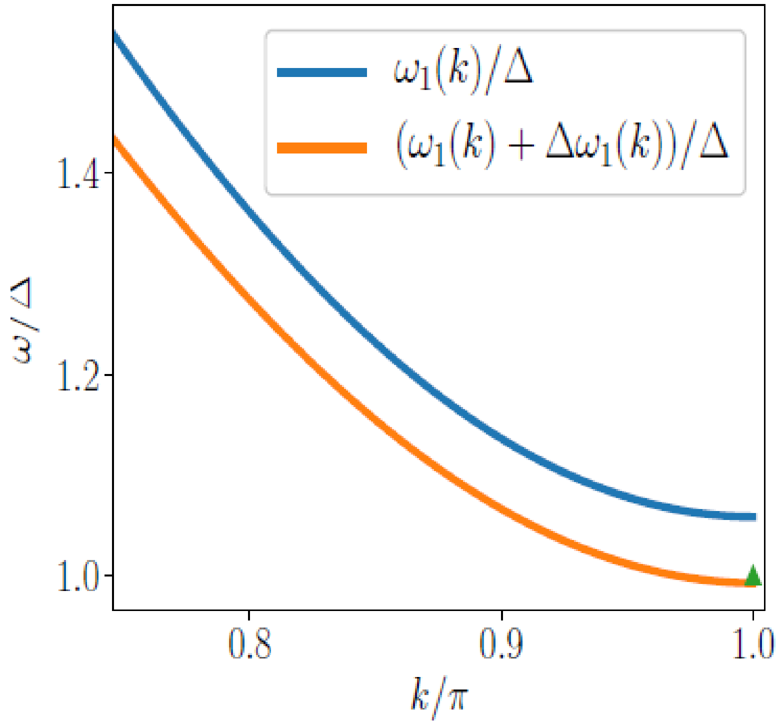


(Ng et al. 2014)

Low-energy Effective Hamiltonian

- Effective Hamiltonian of AKLT may be constructed from one- and two-magnon basis

- A small "mixing" between one and two magnon sector treated in perturbation theory (our PRB paper for details) -> improved energy gap



Critique of the Use of Coherent State Basis in Action

- Action for lattice spin Hamiltonian written in spin coherent-state basis leads to Berry phase + non-linear sigma model description (Haldane, Affleck, etc. In 80s)

- Ground states of spin-1 chain (~AKLT) are difficult to get from action

- Spin coherent-state basis is NOT god-given; any coherent states will do as long as they are (over-)complete and useful

Part II: Constructing action in fluctuating MPS (FMPS) basis

- For spin-1 antiferromagnetic chain, singlet & triplet basis are more intuitive; captures nature of excitations better

- Can we construct new kind of coherent states?
- Can we write down new kind of action?

Coherent States in FMPS basis

- In Schwinger boson (SB) notation, four bond-basis states are

$$\boxed{\begin{aligned}\mathcal{S}_{ij}^\dagger &= \frac{1}{\sqrt{2}}(a_i^\dagger b_j^\dagger - b_i^\dagger a_j^\dagger), \quad (\mathcal{T}_{ij}^1)^\dagger = a_i^\dagger a_j^\dagger, \\ (\mathcal{T}_{ij}^0)^\dagger &= \frac{1}{\sqrt{2}}(a_i^\dagger b_j^\dagger + b_i^\dagger a_j^\dagger), \quad (\mathcal{T}_{ij}^{-1})^\dagger = b_i^\dagger b_j^\dagger\end{aligned}}$$

- Heisenberg Hamiltonian becomes
  (systematic switch from spin to bond language)

$$H = \sum_i H_i = \sum_i K(\mathbf{S}_i \cdot \mathbf{S}_{i+1}) \qquad \boxed{H_i = K(1 - \mathcal{S}_{i,i+1}^\dagger \mathcal{S}_{i,i+1})}$$

- Singlet-triplet bond coherent states:

- **Introduce $|N_i\rangle = z1_i\,|S_i\rangle + z2_i\,|T_i^{1}\rangle + z3_i\,|T_i^{0}\rangle + z4_i\,|T_i^{-1}\rangle$ at (i,i+1) bond ($|z1|^2 + |z2|^2 + |z3|^2 + |z4|^2 = 1$)**

$$\boxed{|\mathbf{N}\rangle = \left(\prod_i N_{i,i+1}\right)|v\rangle}$$

- This is an entangled basis state

  (In contrast, ordinary coherent state is a product state)

- Completeness of |N> proven (in preparation)

Path Integral in FMPS basis

$$
\begin{aligned}
&\langle \mathbf{N}(\tau+\Delta\tau)|e^{-\Delta\tau H}|\mathbf{N}(\tau)\rangle \\
&\simeq \langle \mathbf{N}(\tau+\Delta\tau)|(1-\Delta\tau H)|\mathbf{N}(\tau)\rangle \\
&\simeq \langle \mathbf{N}(\tau)|\mathbf{N}(\tau)\rangle e^{\Delta\tau\left[\frac{\langle \partial_\tau \mathbf{N}(\tau)|\mathbf{N}(\tau)\rangle}{\langle \mathbf{N}(\tau)|\mathbf{N}(\tau)\rangle} - \frac{\langle \mathbf{N}(\tau)|H|\mathbf{N}(\tau)\rangle}{\langle \mathbf{N}(\tau)|\mathbf{N}(\tau)\rangle}\right]}
\end{aligned}
$$

- Need to evaluate Berry phase and energy terms, using Independent Bond Approximation (IBA):

<N|N> ~ <N1|N1><N2|N2> ....

Path Integral in FMPS basis

$$\langle \mathbf{N}(\tau+\Delta\tau)|e^{-\Delta\tau H}|\mathbf{N}(\tau)\rangle$$
$$\simeq \langle \mathbf{N}(\tau+\Delta\tau)|(1-\Delta\tau H)|\mathbf{N}(\tau)\rangle$$
$$\simeq \langle \mathbf{N}(\tau)|\mathbf{N}(\tau)\rangle e^{\Delta\tau\left[\frac{\langle \partial_\tau \mathbf{N}(\tau)|\mathbf{N}(\tau)\rangle}{\langle \mathbf{N}(\tau)|\mathbf{N}(\tau)\rangle}-\frac{\langle \mathbf{N}(\tau)|H|\mathbf{N}(\tau)\rangle}{\langle \mathbf{N}(\tau)|\mathbf{N}(\tau)\rangle}\right]}$$

- Need to evaluate Berry phase and energy terms, using Independent Bond Approximation (IBA):

$$\langle N|N\rangle \sim \langle N1|N1\rangle\langle N2|N2\rangle \ldots$$

AKLT-like Ground State as Saddle Point

- ❖ Energy minimum obtains for z1~0.8 - a small mixture of triplet and mostly singlet
- ❖ Not exactly AKLT because we used Heisenberg Hamiltonian
- ❖ Next step: gradient expansion, low-energy action, correlation functions