Macroscopically degenerate ground state in the SU(3) symmetric Heisenberg model on the kagome lattice

Karlo Penc

Wigner Research Centre for Physics
Institute for Solid State Physics and Optics
Budapest

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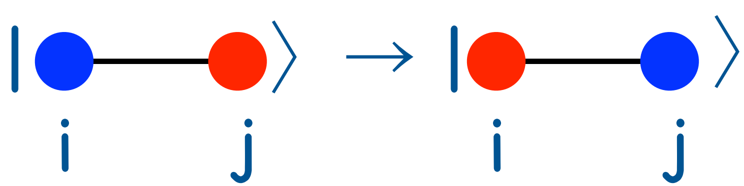
What are the SU(N) symmetric Heisenberg models that we are interested in?

$$\mathcal{H} = \sum_{i,j} \mathcal{P}_{i,j}$$

$\mathcal{P}_{i,j}$ is the transposition operator

$\mathcal{P}_{i,j}$



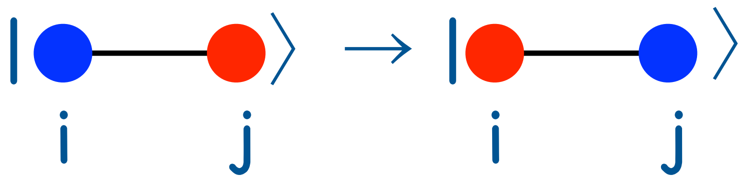
N species on each site that are treated equally.

$$\mathcal{P}_{ij} |\beta_i \alpha_j\rangle = |\alpha_i \beta_j\rangle$$

simplest example:

SU(2) S=1/2 (fundamental representation) [but not the S=1]

What are the SU(N) symmetric Heisenberg models that we are interested in?

$\mathcal{P}_{i,j}$ 

$$\mathbf{S}_1 \cdot \mathbf{S}_2 = S_1^z S_2^z + \frac{1}{2}\left(S_1^+ S_2^- + S_1^- S_2^+\right)$$

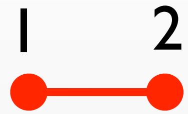
$$\mathbf{S}_1 \cdot \mathbf{S}_2 |\uparrow\uparrow\rangle = \frac{1}{4}|\uparrow\uparrow\rangle \qquad \rightarrow \qquad \left(2\mathbf{S}_1 \cdot \mathbf{S}_2 + \frac{1}{2}\right)|\uparrow\uparrow\rangle = |\uparrow\uparrow\rangle$$

$$\mathbf{S}_1 \cdot \mathbf{S}_2 |\uparrow\downarrow\rangle = -\frac{1}{4}|\uparrow\downarrow\rangle + \frac{1}{2}|\downarrow\uparrow\rangle \qquad \rightarrow \qquad \left(2\mathbf{S}_1 \cdot \mathbf{S}_2 + \frac{1}{2}\right)|\uparrow\downarrow\rangle = |\downarrow\uparrow\rangle$$

For the $S = 1/2$ fundamental representation of the SU(2):

$$\left(2\mathbf{S}_1 \cdot \mathbf{S}_2 + \frac{1}{2}\right) = \mathcal{P}_{1,2}$$

SU(2) vs. SU(3) - two sites



$$\mathcal{H} = \mathcal{P}_{12}$$

$$\mathcal{P}_{12}(|\alpha\beta\rangle - |\beta\alpha\rangle) = -(|\alpha\beta\rangle - |\beta\alpha\rangle)$$ E=−1, odd wave function

$$\mathcal{P}_{12}(|\alpha\beta\rangle + |\beta\alpha\rangle) = +(|\alpha\beta\rangle + |\beta\alpha\rangle)$$ E=+1, even wave function

Addition of two S=½ SU(2) spins:

$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$

using Young diagrams:

$$2 \times 2 = 1 + 3$$

$$\square \otimes \square = \begin{smallmatrix}\square\\\square\end{smallmatrix} \oplus \square\square$$

$\square$ $\uparrow$ and $\downarrow$ spins

$\begin{smallmatrix}\square\\\square\end{smallmatrix}$ $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ singlet
odd (anti-symmetrical)

$\square\square$ $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$ triplet
even (symmetrical)

Addition of two SU(3) spins:

$$\mathbf{3} \times \mathbf{3} = \bar{\mathbf{3}} + \mathbf{6}$$

$$\square \otimes \square = \begin{smallmatrix}\square\\\square\end{smallmatrix} \oplus \square\square$$

$\square$ $|a\rangle$, $|b\rangle$, and $|c\rangle$.

$\begin{smallmatrix}\square\\\square\end{smallmatrix}$ $|ab\rangle - |ba\rangle$, $|ab\rangle - |ba\rangle$, $|ab\rangle - |ba\rangle$
odd (anti-symmetrical).

$\square\square$ $|aa\rangle$, $|bb\rangle$, $|cc\rangle$, $|ab\rangle + |ba\rangle$,
$|ac\rangle + |ca\rangle$, and $|bc\rangle + |cb\rangle$
even (symmetrical)

SU(3) irreps on 3 sites

Addition of three SU(3) spins (27 states):

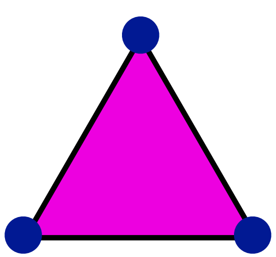
$$\mathbf{3} \times \mathbf{3} \times \mathbf{3} = \mathbf{1} + 2 \times \mathbf{8} + \mathbf{10}$$

$$\ydiagram{1} \otimes \ydiagram{1} \otimes \ydiagram{1} = \ydiagram{1,1,1} \oplus 2\times \ydiagram{2,1} \oplus \ydiagram{3}$$

SU(3) singlet

spins fully antisymmetrized

$$\ydiagram{1,1,1} = |abc\rangle + |bca\rangle + |cab\rangle - |acb\rangle - |acb\rangle - |acb\rangle$$

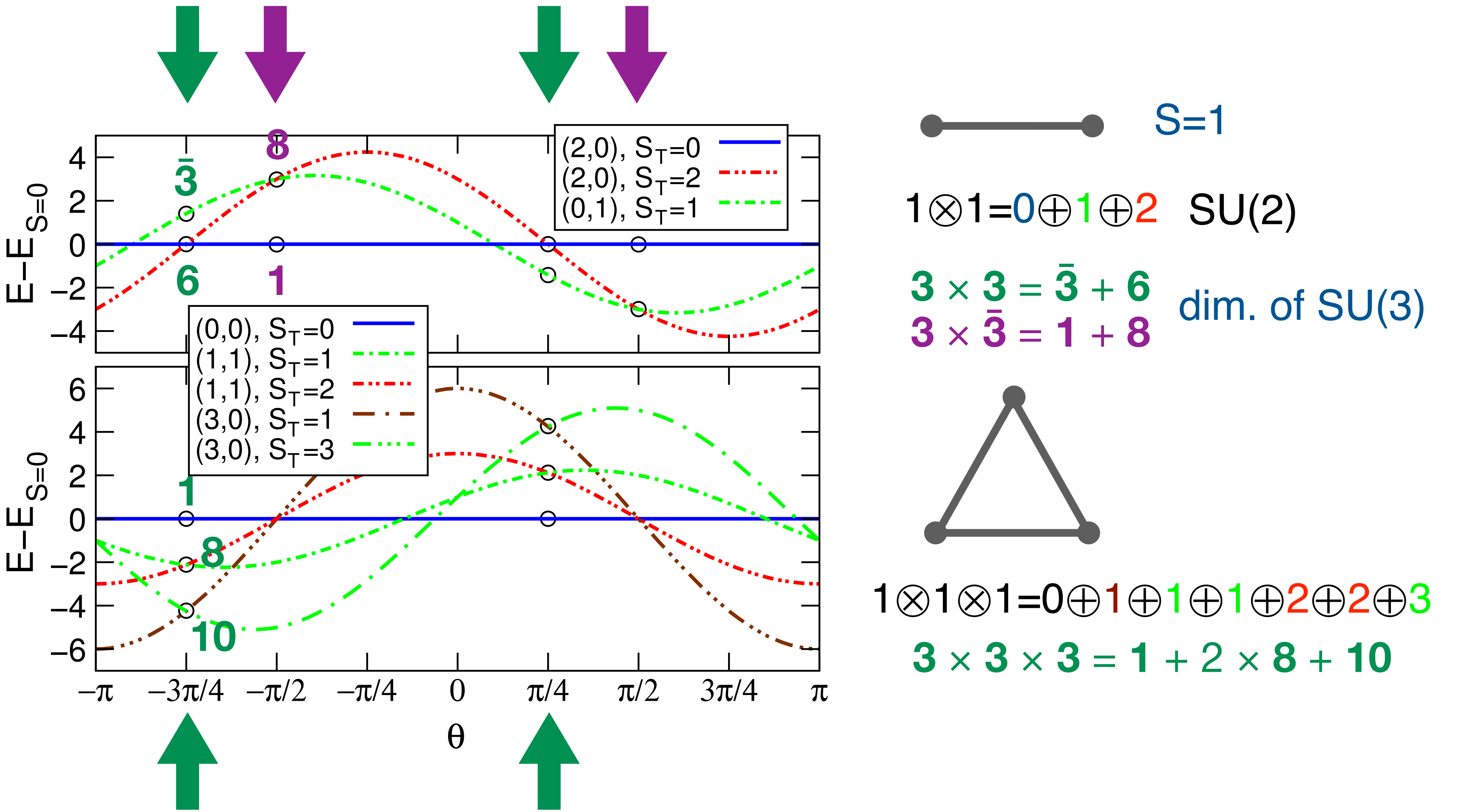


in the SU(3) singlet the spins are fully entangled:
we cannot write it in a product form

SU(3) in S=1 spin model

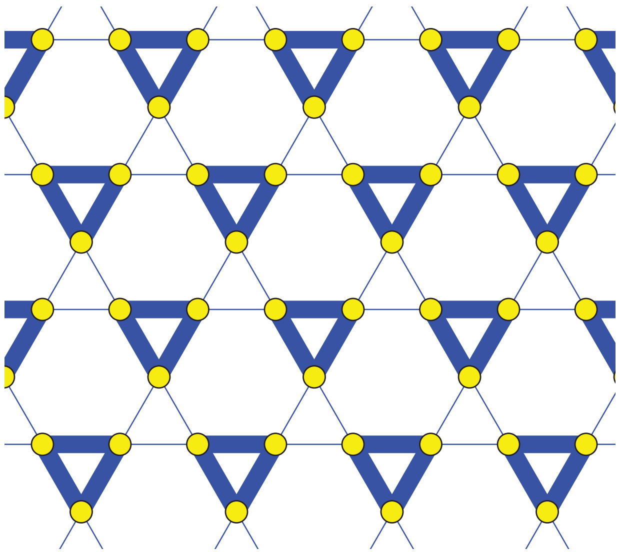
$$\mathcal{H} = J \sum_{i,j} \left[\cos\vartheta \mathbf{S}_i \mathbf{S}_j + \sin\vartheta \left(\mathbf{S}_i \mathbf{S}_j\right)^2 \right]$$

degeneracy as a signature of increased symmetry



What do we know about SU(3) Kagome ?

The trimerized/simplex solid state/simplex valence-bond crystal for the fundamental **3** irrep model and S=1 Kagome (BLBQ, including the pure Heisenberg point)



D. P. Arovas, Phys. Rev. B **77**, 104404 (2008).

SU(3)
Large-N expansion: Hermele & Gurarie, Phys. Rev. B **84**, 174441 (2011);

iPEPS and ED: Corboz, Penc, Mila, & Läuchli, Phys. Rev. B 86, 041106(R) (2012)

S=1
H. J. Changlani, A. M. Läuchli, Trimerized ground state of the spin-1 Heisenberg antiferromagnet on the kagome lattice, Phys. Rev. B **91**, 100407 (2015)

T. Liu, W. Li, A. Weichselbaum; J von Delft, Jan, G. Su, Simplex valence-bond crystal in the spin-1 kagome Heisenberg antiferromagnet, Phys. Rev. B **91**, 060403(R) (2015)

Trimerized phase in the S = 1 Kagome antiferromagnet with ring exchange

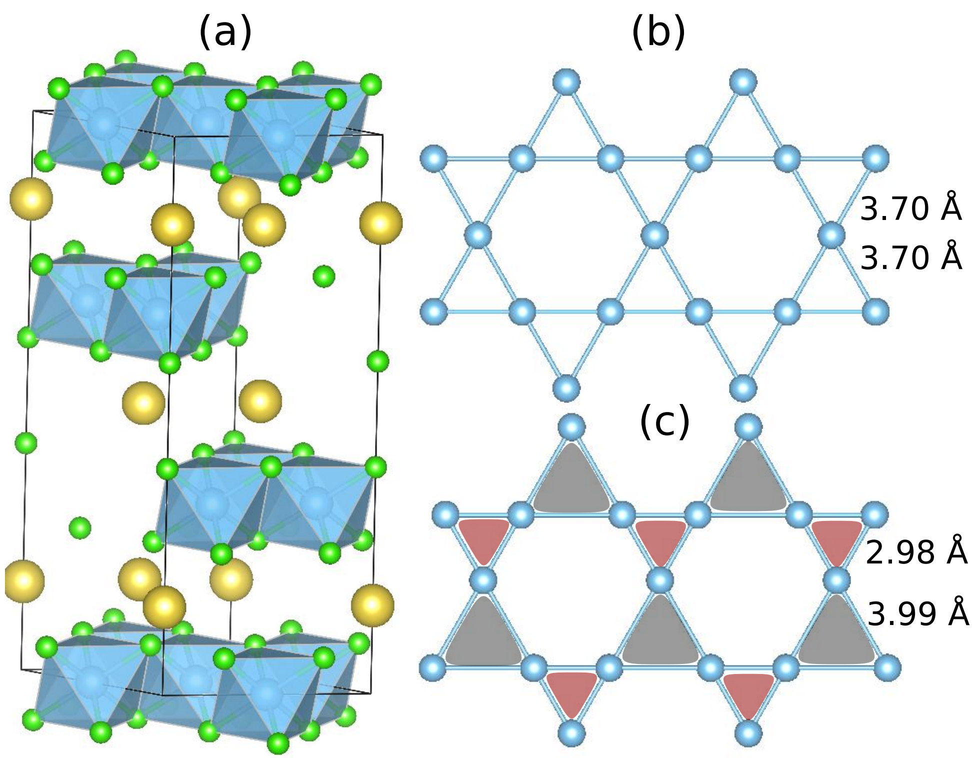
arXiv:1909.02020

Spin–lattice coupling and the emergence of the trimerized phase in the S = 1 Kagome antiferromagnet $Na_2Ti_3Cl_8$

Arpita Paul, Chia-Min Chung, Turan Birol, and Hitesh J. Changlani

layers of edge-sharing $TiCl_6$ octahedra

$$\mathcal{H} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_{bq} \sum_{\langle ij \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j\right)^2 + \frac{J_R}{2} \sum_{\Delta = i,j,k} \left(\left(\mathbf{S}_i \cdot \mathbf{S}_j\right)\left(\mathbf{S}_i \cdot \mathbf{S}_k\right) + \left(\mathbf{S}_i \cdot \mathbf{S}_k\right)\left(\mathbf{S}_i \cdot \mathbf{S}_j\right) \right)$$

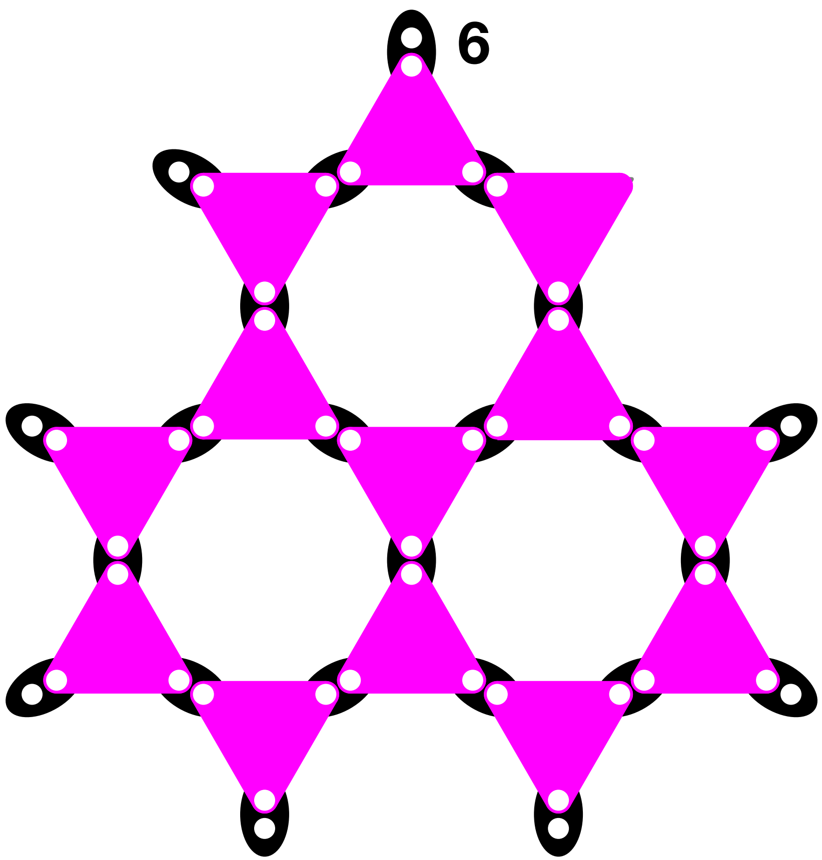


Simplex solid in SU(3) Kagome

D. P. Arovas, Phys. Rev. B **77**, 104404 (2008).

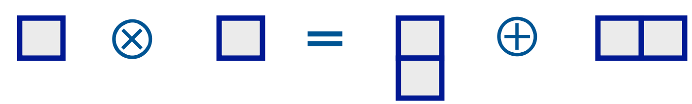
SU(N) singlet on N sites, represented by $b^{\dagger}_{\alpha}(i)$ Schwinger bosons:

$$\epsilon^{\alpha_1 \cdots \alpha_N} b^{\dagger}_{\alpha_1}(i_1) \cdots b^{\dagger}_{\alpha_N}(i_N)|0\rangle,$$



Addition of two SU(3) spins:

$$\mathbf{3} \times \mathbf{3} = \bar{\mathbf{3}} + \mathbf{6}$$



Each site hosts the symmetric, 6 dimensional irrep because of the bosons (like in the S=1 AKLT wave function case).

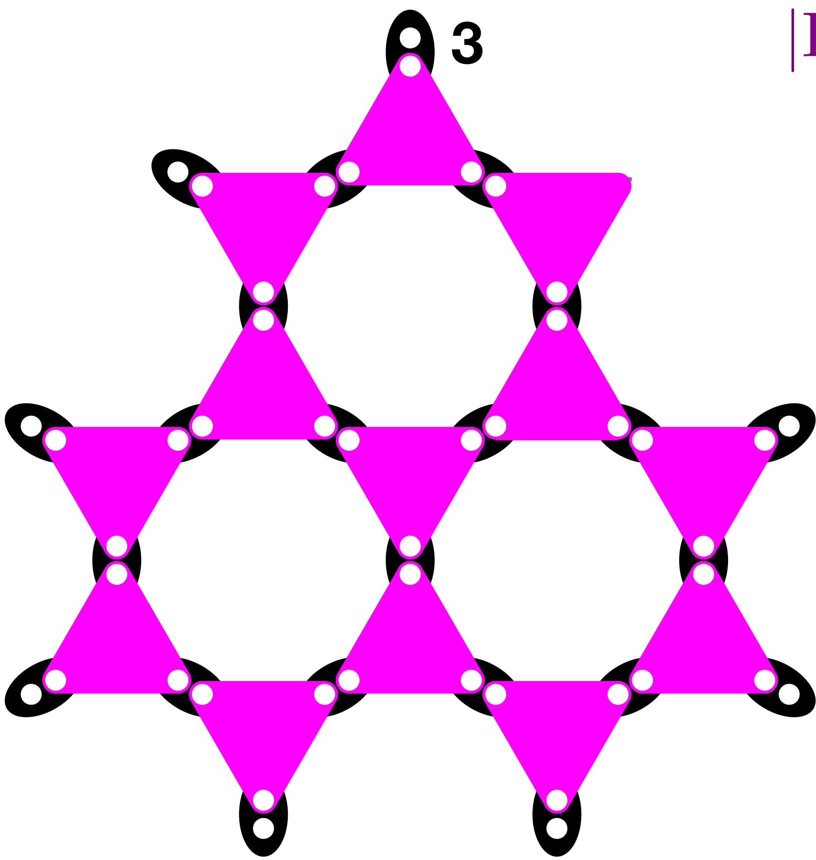
But we can do this with fermions as well !

SU(3) singlet on 3 sites, represented by fermions :

$$|\mathbf{1}(i_1, i_2, i_3)\rangle = \sum_{\alpha,\beta,\gamma} \varepsilon^{\alpha\beta\gamma} f_\alpha^\dagger(i_1) f_\beta^\dagger(i_2) f_\gamma^\dagger(i_3)|0\rangle = \mathcal{F}_{i_1,i_2,i_3}|0\rangle$$

femionic simplex solid wave function:

$$|\mathrm{FSS}\rangle = \prod_{\triangle_i}\prod_{\bigtriangledown_j} \mathcal{F}_{\triangle_i}\mathcal{F}_{\bigtriangledown_j}|0\rangle$$



$$\bar{\mathbf{3}} \times \bar{\mathbf{3}} = \mathbf{3} + \bar{\mathbf{6}}$$

Each site hosts the antisymmetric, 3 dimensional irrep.

Do we know the parent Hamiltonian ?

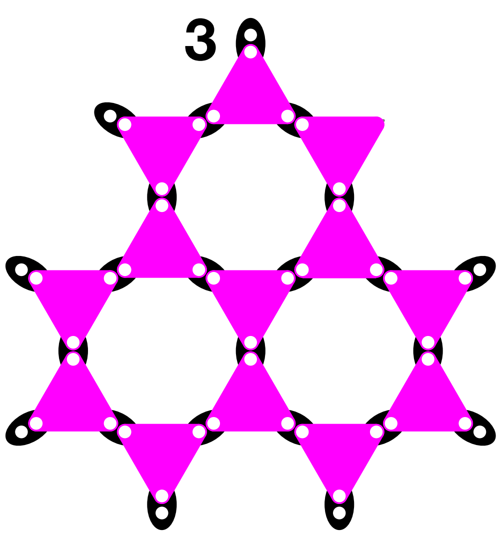
A guess: sum of local projectors, like in the S=1 AKLT model

$$\mathcal{H} = J \sum_{\langle i,j \rangle} \mathcal{P}_{i,j} + K \sum_{\triangle,\bigtriangledown} \left(\mathcal{P}_{i,j,k} + \mathcal{P}_{i,k,j} \right)$$

We may try it on a small system: we generate the FSS, and ask if the condition for being an eigenstate

$$\langle \mathrm{FSS} | \mathcal{H}^2 | \mathrm{FSS} \rangle \langle \mathrm{FSS} | \mathrm{FSS} \rangle = \langle \mathrm{FSS} | \mathcal{H} | \mathrm{FSS} \rangle^2$$

is satisfied with some values of J/K.



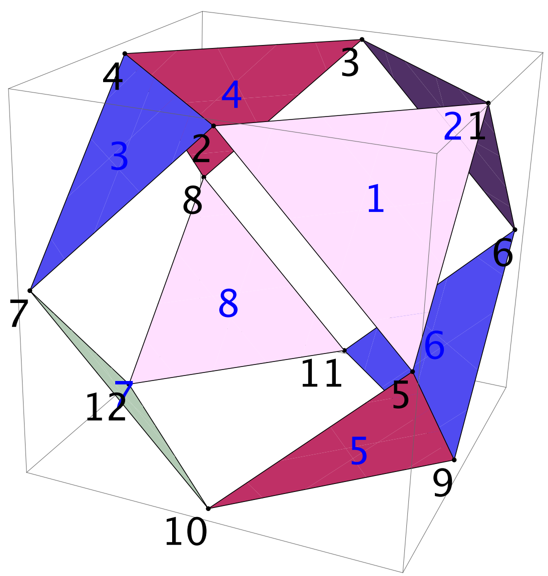
Surprise: it is always satisfied, the FSS is always an eigenstate of H !

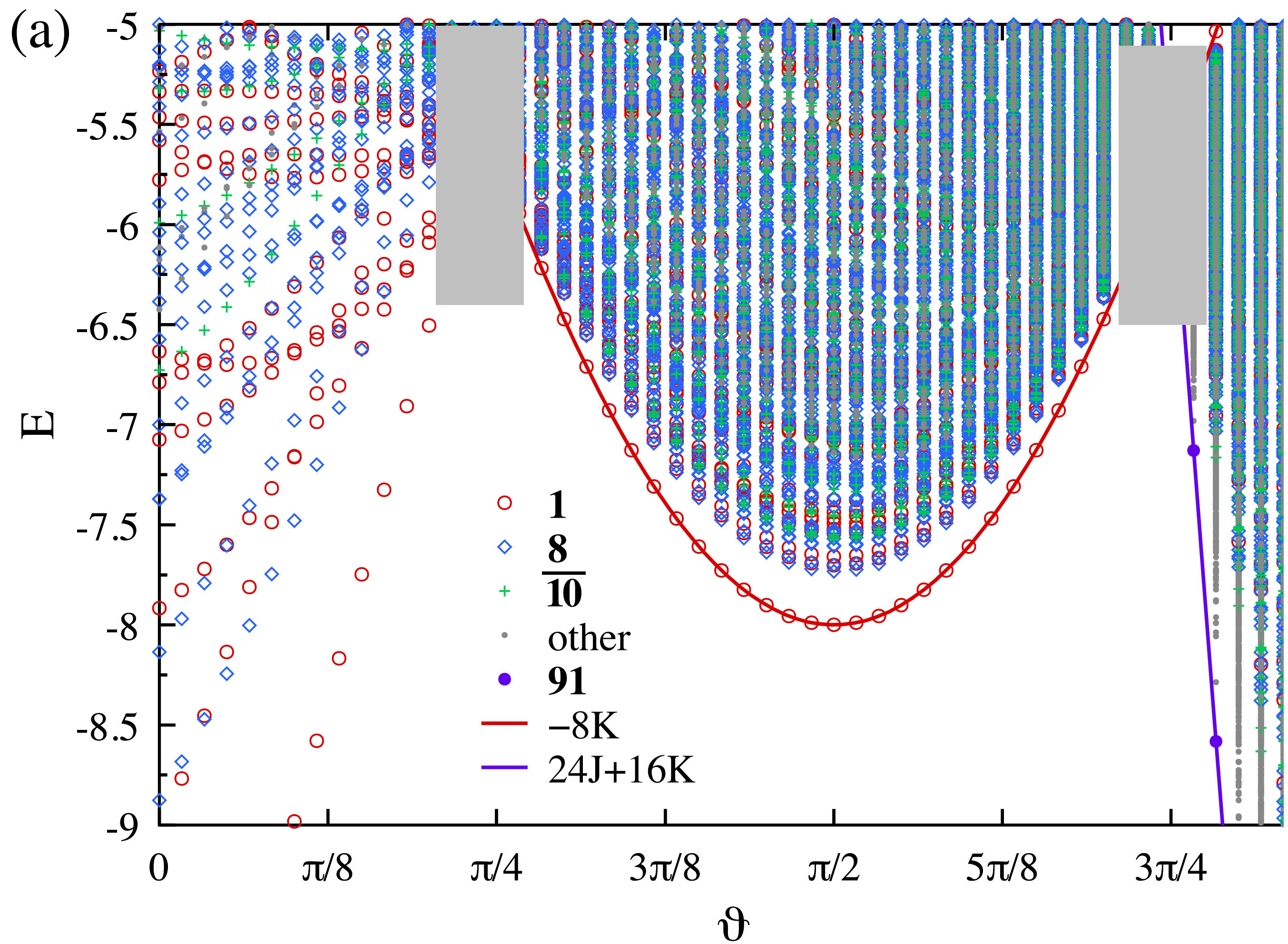
But how does this happen?

full ED for small system (12 sites)

$$\mathcal{H} = J \sum_{\langle i,j \rangle} \mathcal{P}_{i,j} + K \sum_{\triangle,\nabla} (\mathcal{P}_{i,j,k} + \mathcal{P}_{i,k,j})$$

$$J = \cos\vartheta$$
$$K = \sin\vartheta$$

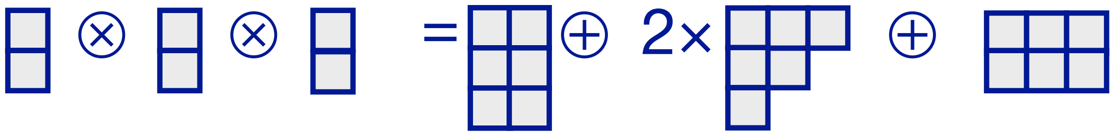




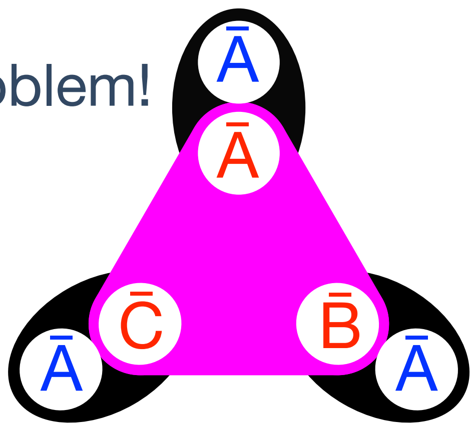
#of states in (4,4,4) sector = 34650, but symmetry group large

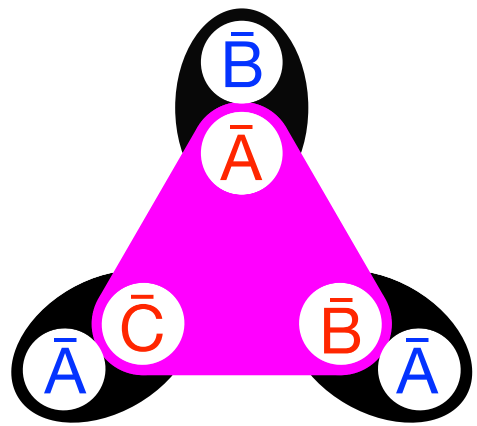
The irreps in a triangle

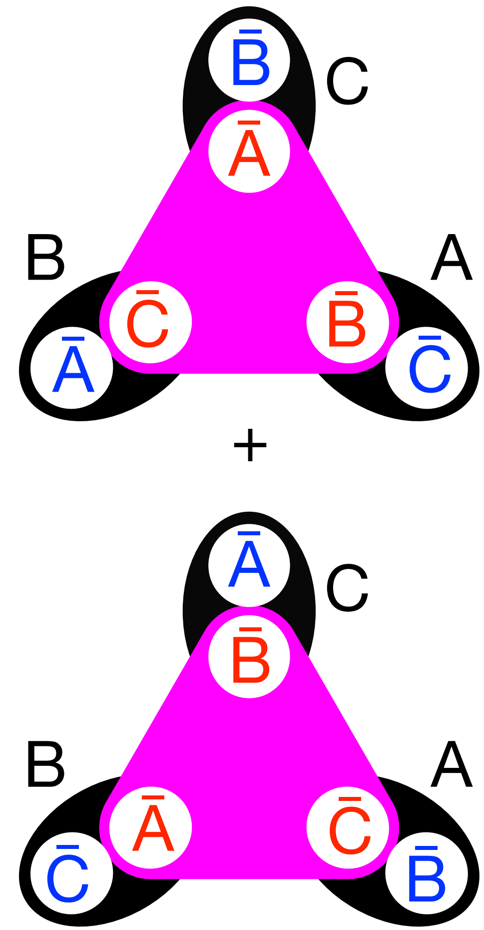
$$\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus 2 \times \mathbf{8} \oplus \overline{\mathbf{10}}$$



problem!







$\mathbf{1} \odot \mathbf{10} = 0$

$\mathbf{1} \odot \mathbf{8} = \mathbf{8}$

$\odot$	$\mathbf{1}$	$\mathbf{8}^R$	$\mathbf{8}^L$	$\overline{\mathbf{10}}$
$\mathbf{1}$		$\mathbf{8}^R$	$\mathbf{8}^L$	
$\mathbf{8}^R$	$\mathbf{8}^R$	$\mathbf{8}^L$	$\mathbf{1} \oplus \mathbf{10}$	$\mathbf{8}^R$
$\mathbf{8}^L$	$\mathbf{8}^L$	$\mathbf{1} \oplus \mathbf{10}$	$\mathbf{8}^R$	$\mathbf{8}^L$
$\overline{\mathbf{10}}$		$\mathbf{8}^R$	$\mathbf{8}^L$	$\mathbf{10}$

The sum cancels because of odd number of antisymmetrizations: $(-1)^3 = -1$

$\mathbf{1} \odot \mathbf{1} = 0$

Comparing the S=1 AKLT chain with FSS

AKLT chain

s=1
s^z=1

s=1
s^z=0

S= 0 or 1

$$\mathcal{H}^{\mathrm{AKLT}} = \sum_{\mathrm{bonds}} |S=2\rangle\langle S=2|$$

$\odot$	$\mathbf{1}$	$\mathbf{8}^R$	$\mathbf{8}^L$	$\overline{\mathbf{10}}$
$\mathbf{1}$		$\mathbf{8}^R$	$\mathbf{8}^L$	
$\mathbf{8}^R$	$\mathbf{8}^R$	$\mathbf{8}^L$	$\mathbf{1} \oplus \mathbf{10}$	$\mathbf{8}^R$
$\mathbf{8}^L$	$\mathbf{8}^L$	$\mathbf{1} \oplus \mathbf{10}$	$\mathbf{8}^R$	$\mathbf{8}^L$
$\overline{\mathbf{10}}$		$\mathbf{8}^R$	$\mathbf{8}^L$	$\mathbf{10}$

Fermionic simplex solid eigenstate of the

$$\mathcal{H}^{\mathrm{FSS}} = \sum_{\triangle,\triangledown} \left(c_{\mathbf{1}} |\mathbf{1}\rangle\langle\mathbf{1}| + c_{\mathbf{10}} |\mathbf{10}\rangle\langle\mathbf{10}| \right)$$

and ground state when $c_1>0$ and $c_{10}>0$.

$$c_{\mathbf{1}} = 3K - 3J$$
$$c_{\mathbf{10}} = 3K + 3J$$

$$\mathcal{H} = J \sum_{\langle i,j\rangle} \mathcal{P}_{i,j} + K \sum_{\triangle,\triangledown} \left(\mathcal{P}_{i,j,k} + \mathcal{P}_{i,k,j} \right)$$

full ED for small system (12 sites)

$$\mathcal{H} = J \sum_{\langle i,j \rangle} \mathcal{P}_{i,j} + K \sum_{\triangle,\nabla} \left(\mathcal{P}_{i,j,k} + \mathcal{P}_{i,k,j}\right)$$

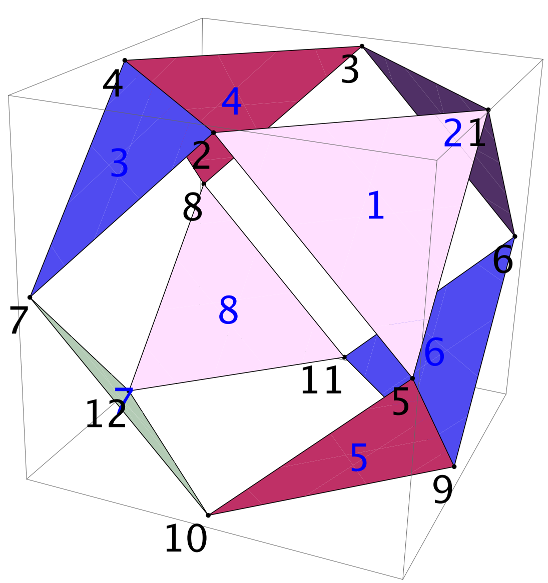
$$\mathcal{H} = \sum_{\triangle,\nabla} |\mathbf{10}\rangle\langle\mathbf{10}| \qquad \mathcal{H} = \sum_{\triangle,\nabla} |\mathbf{1}\rangle\langle\mathbf{1}|$$

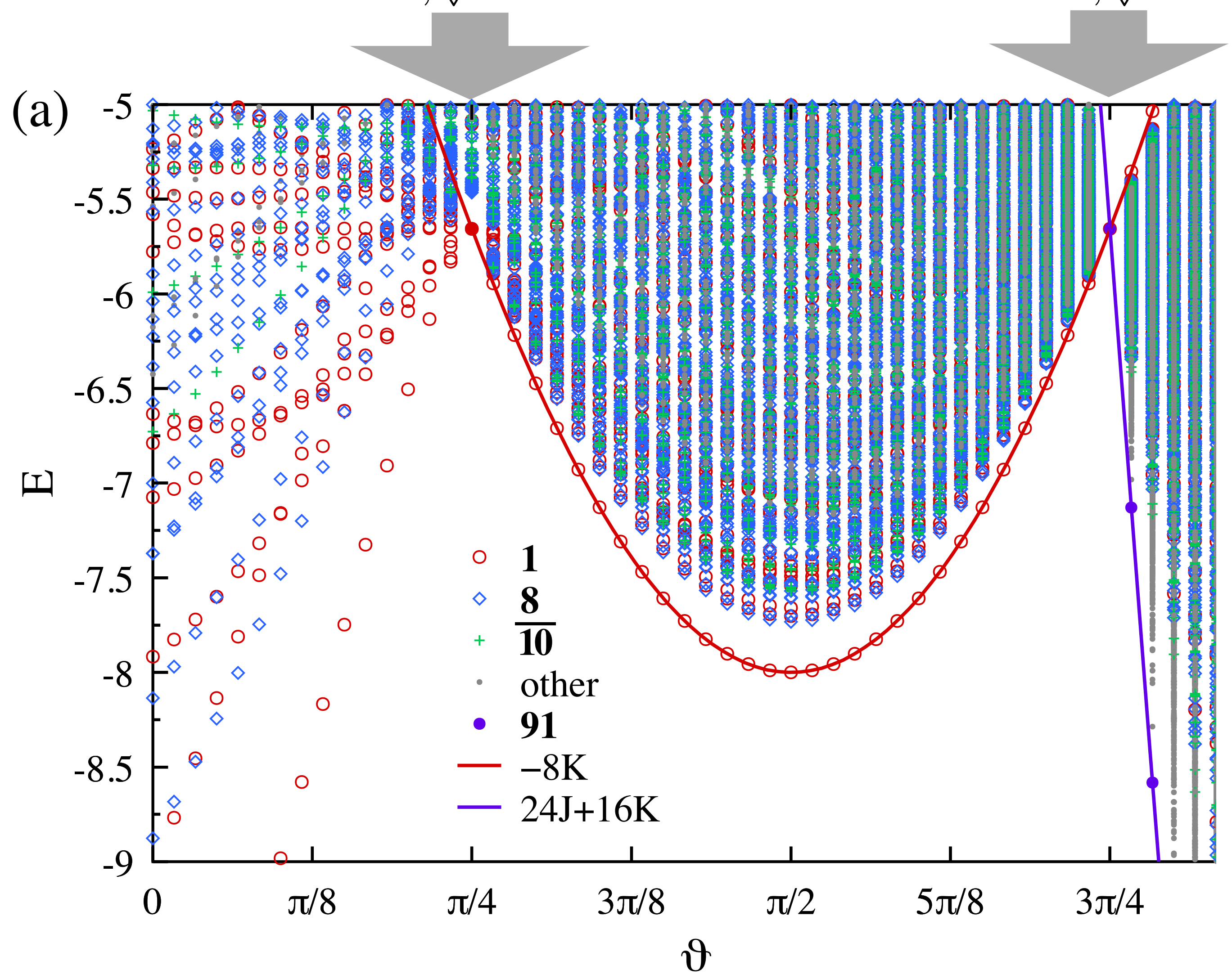
$$J = \cos\vartheta$$

$$K = \sin\vartheta$$

$$c_{\mathbf{1}} = 3(K - J)$$

$$c_{\mathbf{10}} = 3(K + J)$$





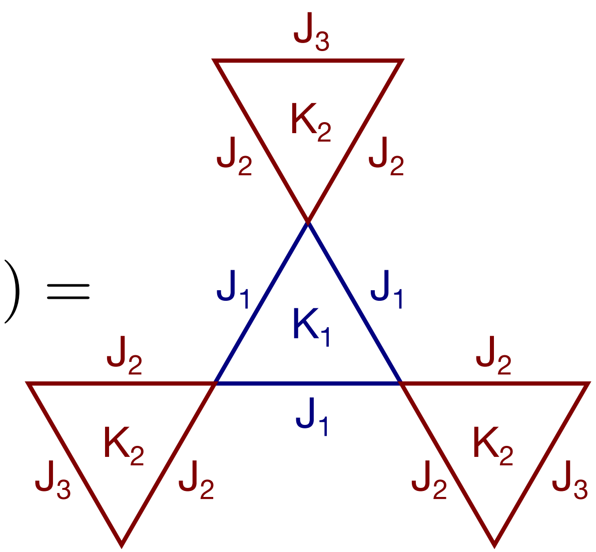
Lower bound on energy

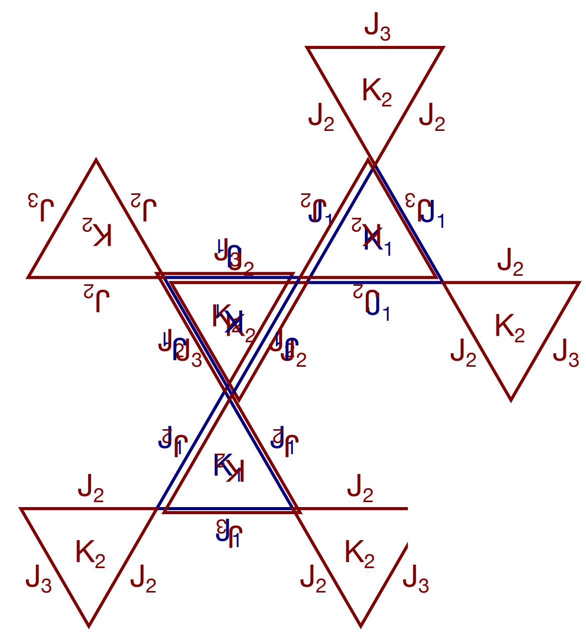
Let us write the lattice Hamiltonian as a sum offer the lattice of a Hamiltonian defined on a (9-site) open cluster:

$$\mathcal{H}(J,K) = \sum_{\text{lattice}} \mathcal{H}_9(J_1,J_2,J_3,K_1,K_2)$$

where

$$\mathcal{H}_9(J_1,J_2,J_3,K_1,K_2) =$$





with a condition

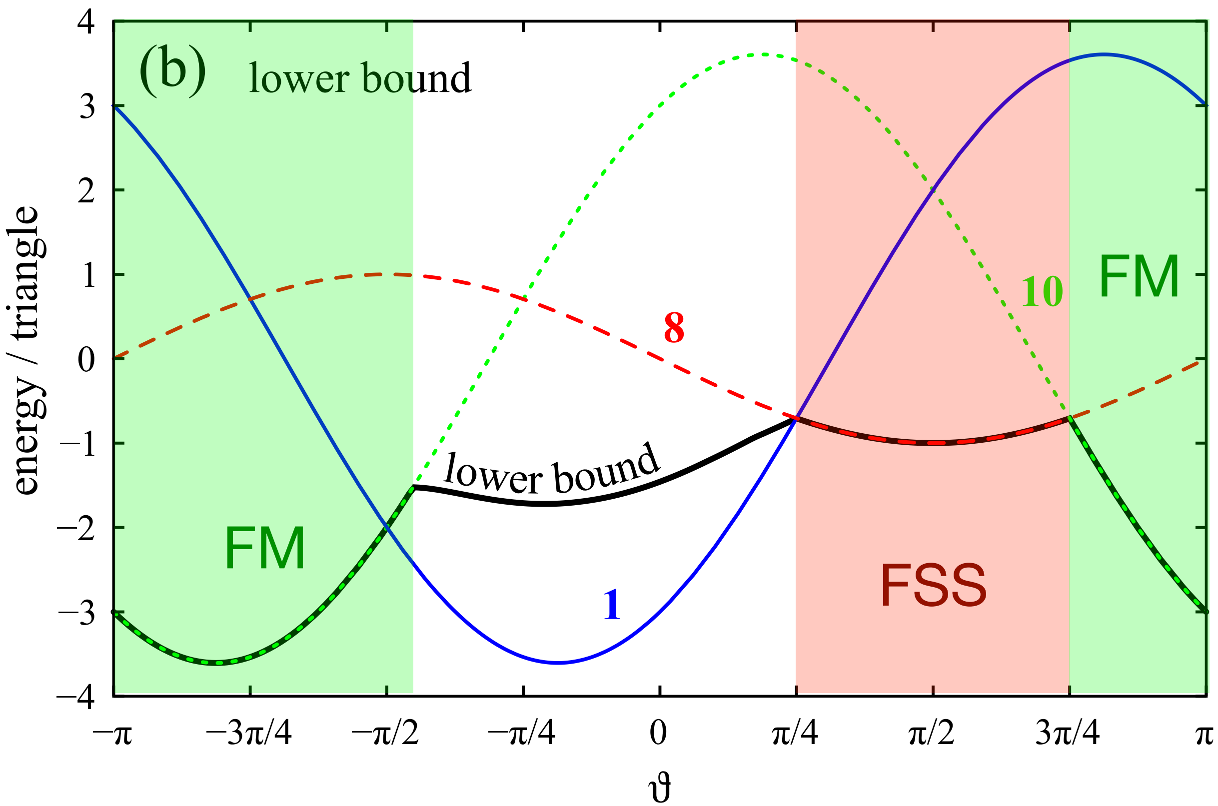
$$J = J_1 + 2J_2 + J_3$$

$$K = K_1 + 3K_2$$

Lower bound on energy

The energy calculated from the ground states of the sub-Hamiltonians will always be lower that the ground state energy of H, as the true ground state of H can be viewed as a variational wavefunction for H9.

$$E_{\mathrm{LB}} = \max_{\substack{J=J_1+2J_2+J_3 \\ K=K_1+3K_2}} E_{\mathrm{GS}}(J_1, J_2, J_3, K_1, K_2)$$



Actually, the energies of a single triangle gives also a lower bond (per triangle)

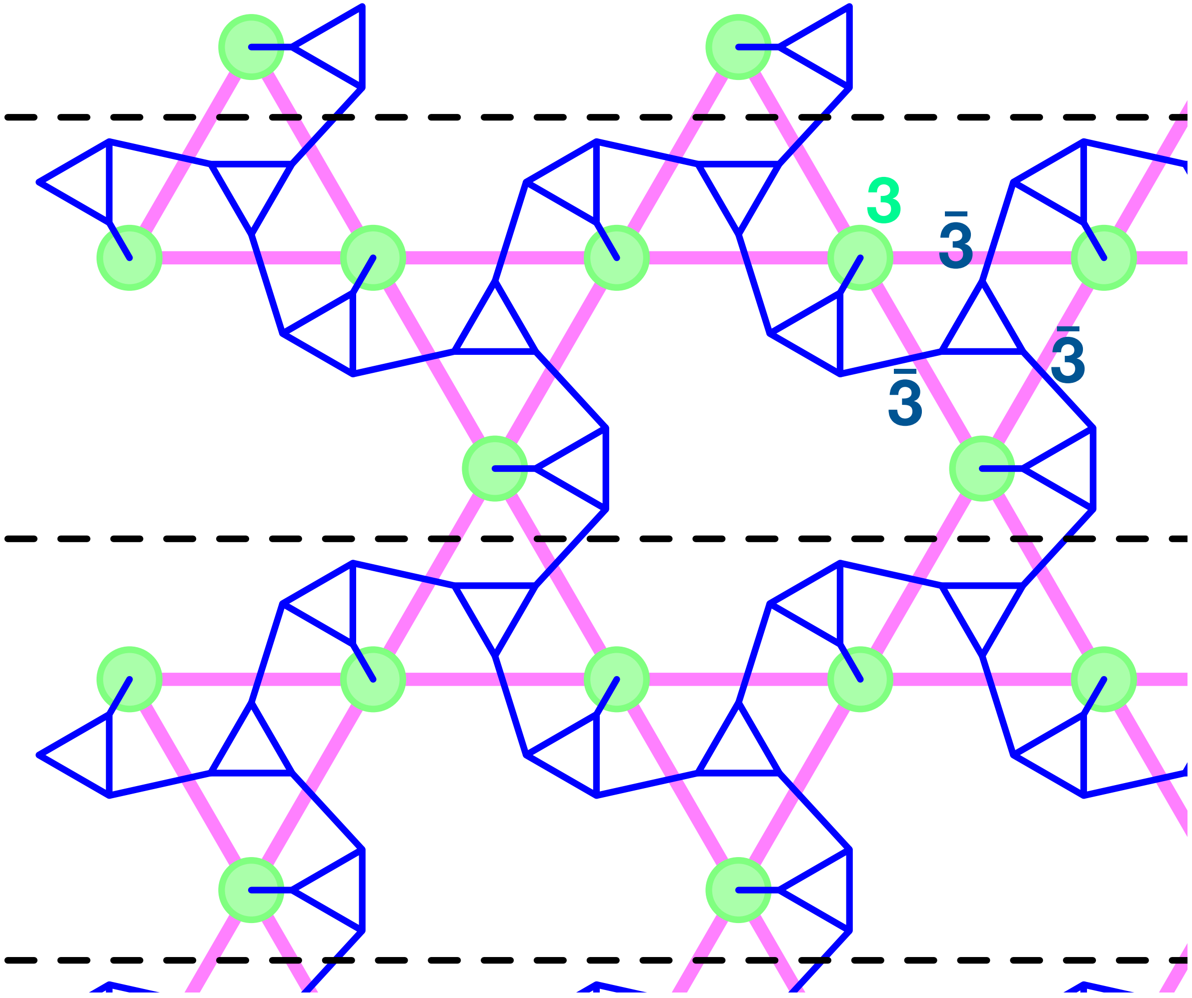
$$\varepsilon_{\mathbf{1}} = -3J + 2K$$

$$\varepsilon_{\mathbf{8}} = -K$$

$$\varepsilon_{\mathbf{10}} = 3J + 2K$$

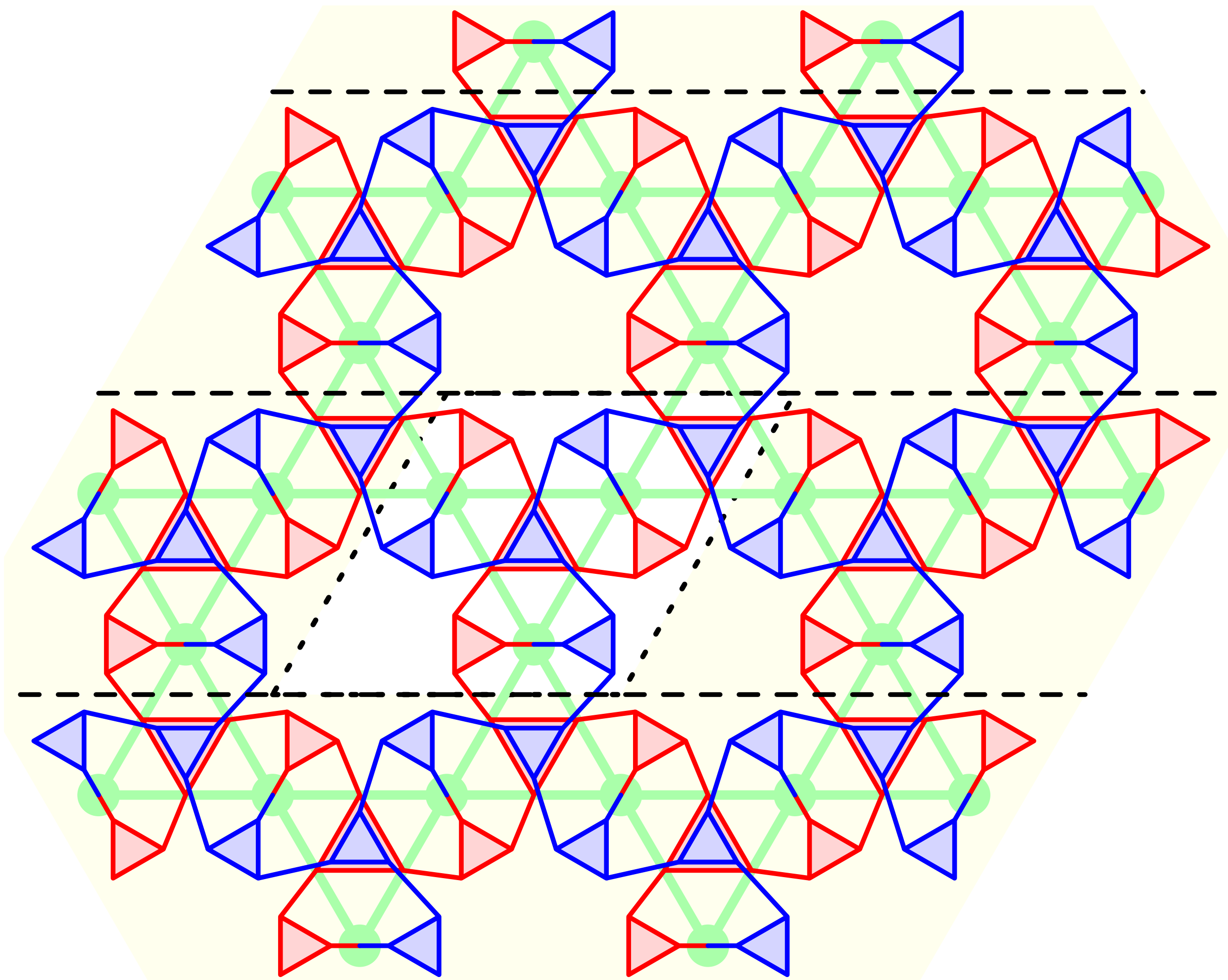
The FM and the FSS saturate the lower bound, they are ground states (beware uniqueness)

Tensor network: the wave function



each triangle represents the antisymmetrizing Levi-Civita symbol

Tensor network: the overlap



graph of contracted Levi-Civita symbols

R. Penrose, Applications of negative dimensional tensors, 1971

Penrose polynomial, defined for plane graphs

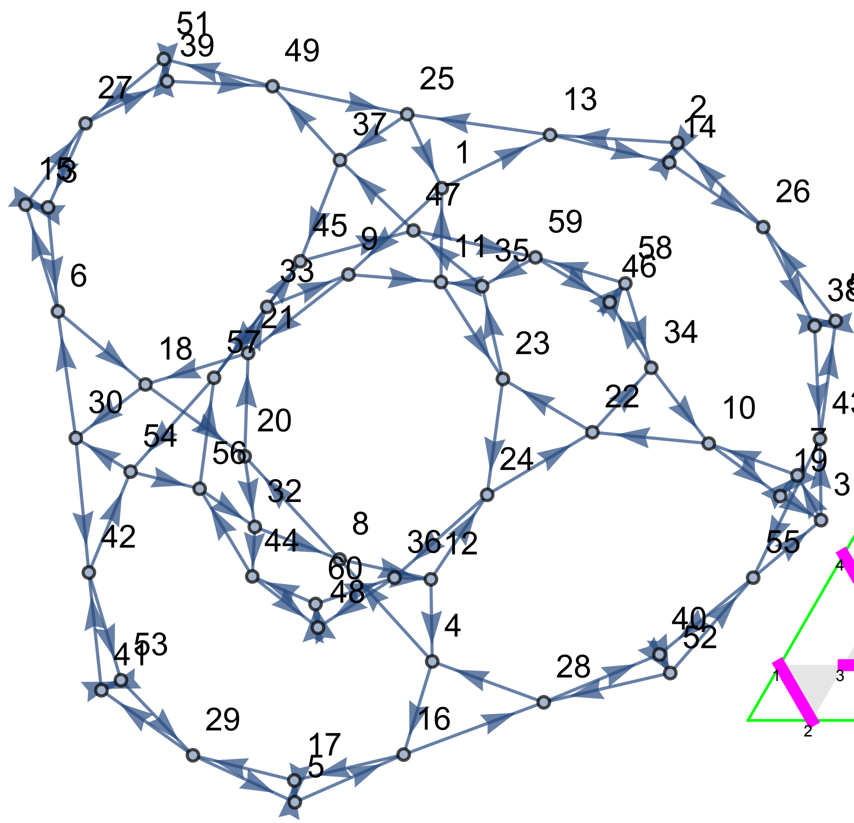
12: 13392
27: 1828256832
36: 2220531642144

gfortran has 128-bit long integer type:-)

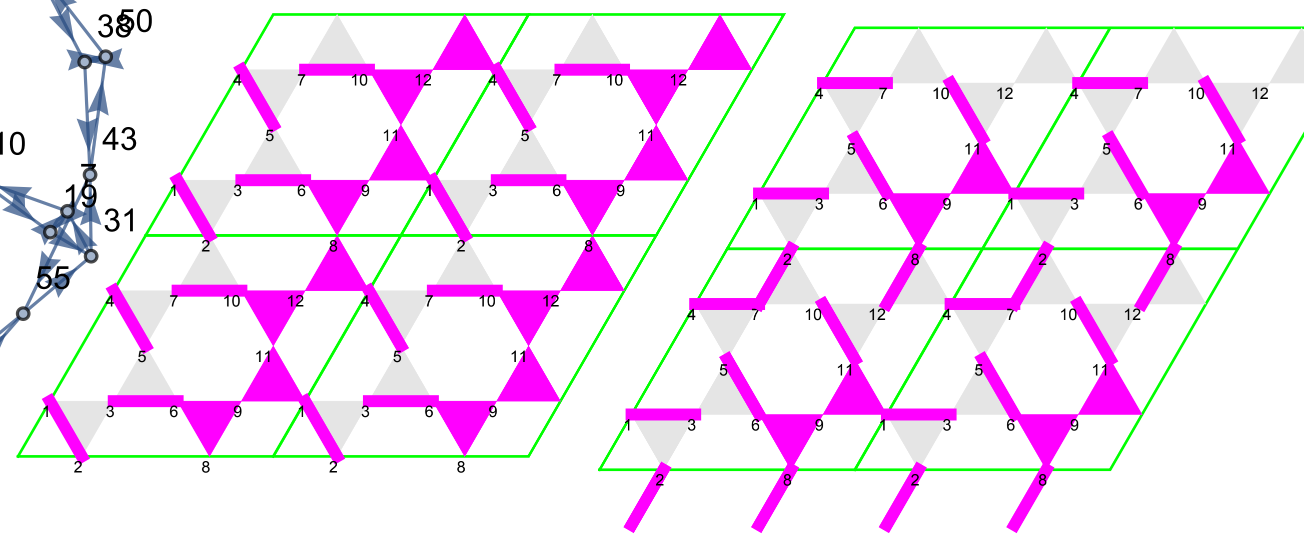
Example for overlap (12 sites)

$$\varepsilon^{1,9,11}\varepsilon^{2,13,14}\varepsilon^{3,6,15}\varepsilon^{4,8,12}\varepsilon^{5,16,17}\varepsilon^{7,10,19}\varepsilon^{18,20,21}\varepsilon^{22,23,24}\varepsilon^{25,37,49}\varepsilon^{26,38,50}$$
$$\varepsilon^{27,39,51}\varepsilon^{28,40,52}\varepsilon^{29,41,53}\varepsilon^{30,42,54}\varepsilon^{31,43,55}\varepsilon^{32,44,56}\varepsilon^{33,45,57}\varepsilon^{34,46,58}\varepsilon^{35,47,59}\varepsilon^{36,48,60}$$
$$\varepsilon_{1,13,25}\varepsilon_{2,14,26}\varepsilon_{3,15,27}\varepsilon_{4,16,28}\varepsilon_{5,17,29}\varepsilon_{6,18,30}\varepsilon_{7,19,31}\varepsilon_{8,20,32}\varepsilon_{9,21,33}\varepsilon_{10,22,34}$$
$$\varepsilon_{11,23,35}\varepsilon_{12,24,36}\varepsilon_{37,45,47}\varepsilon_{38,43,50}\varepsilon_{39,49,51}\varepsilon_{40,52,55}\varepsilon_{41,42,53}\varepsilon_{44,48,60}\varepsilon_{46,58,59}\varepsilon_{54,56,57}$$

$= 49152$



The graphs are "bipartite" (median graph for degree 3 regular bipartite graph)



Penrose graph

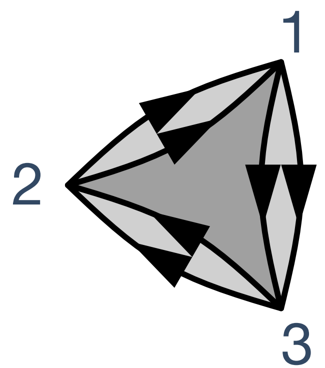
Evaluating Penrose graphs

$$\varepsilon_{i,j,k}\,\varepsilon^{i,j,k} = 6$$

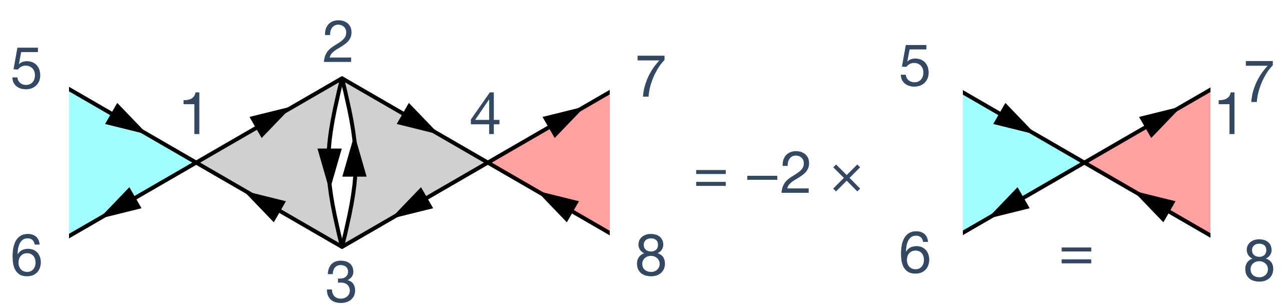
$$\varepsilon_{i,j,k}\,\varepsilon^{i,j,l} = 2\delta_k^l$$

$$\varepsilon_{i,j,k}\,\varepsilon^{i,l,m} = \delta_j^l\delta_k^m - \delta_j^m\delta_k^l$$

implied sum over repeated indices

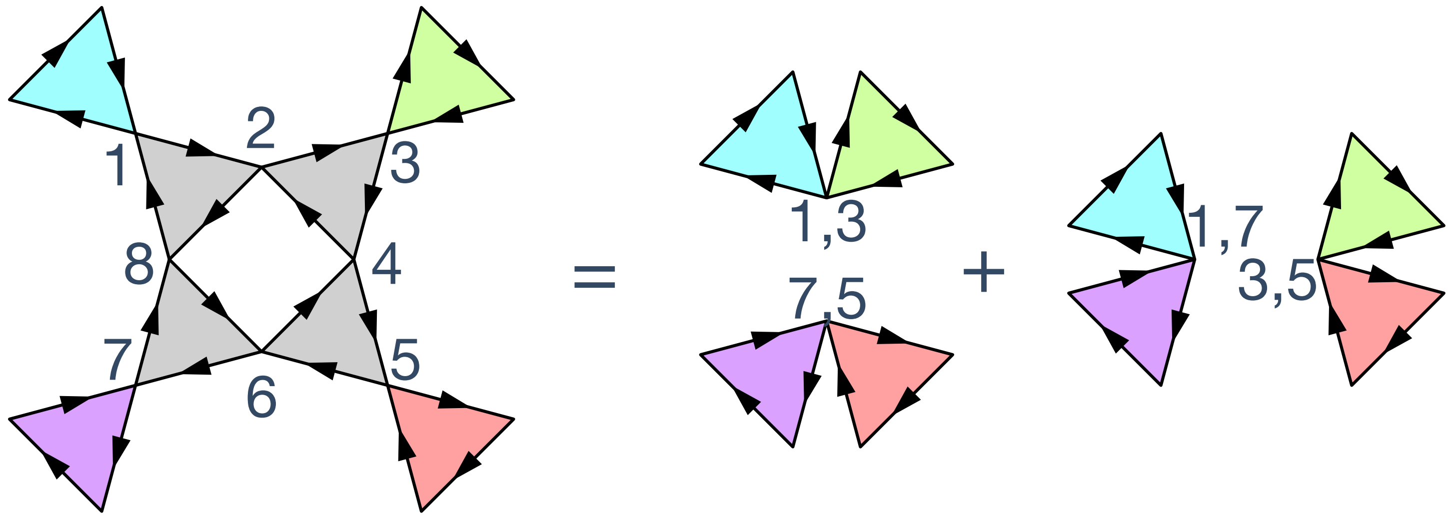


$$\varepsilon_{1,2,3}\,\varepsilon^{1,2,3} = 6$$

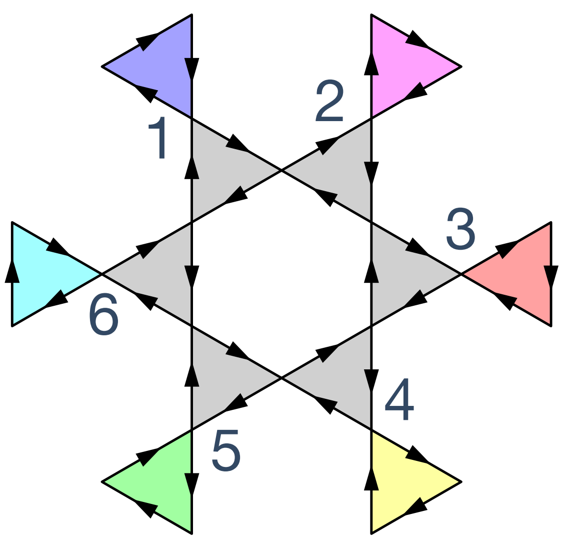


$$\ldots\varepsilon_{5,1,6}\,\varepsilon^{1,2,3}\,\varepsilon_{2,4,3}\,\varepsilon^{4,7,8}\ldots = -\,2 \times \ldots\varepsilon_{5,1,6}\,\varepsilon^{1,7,8}\ldots$$

Evaluating Penrose graphs



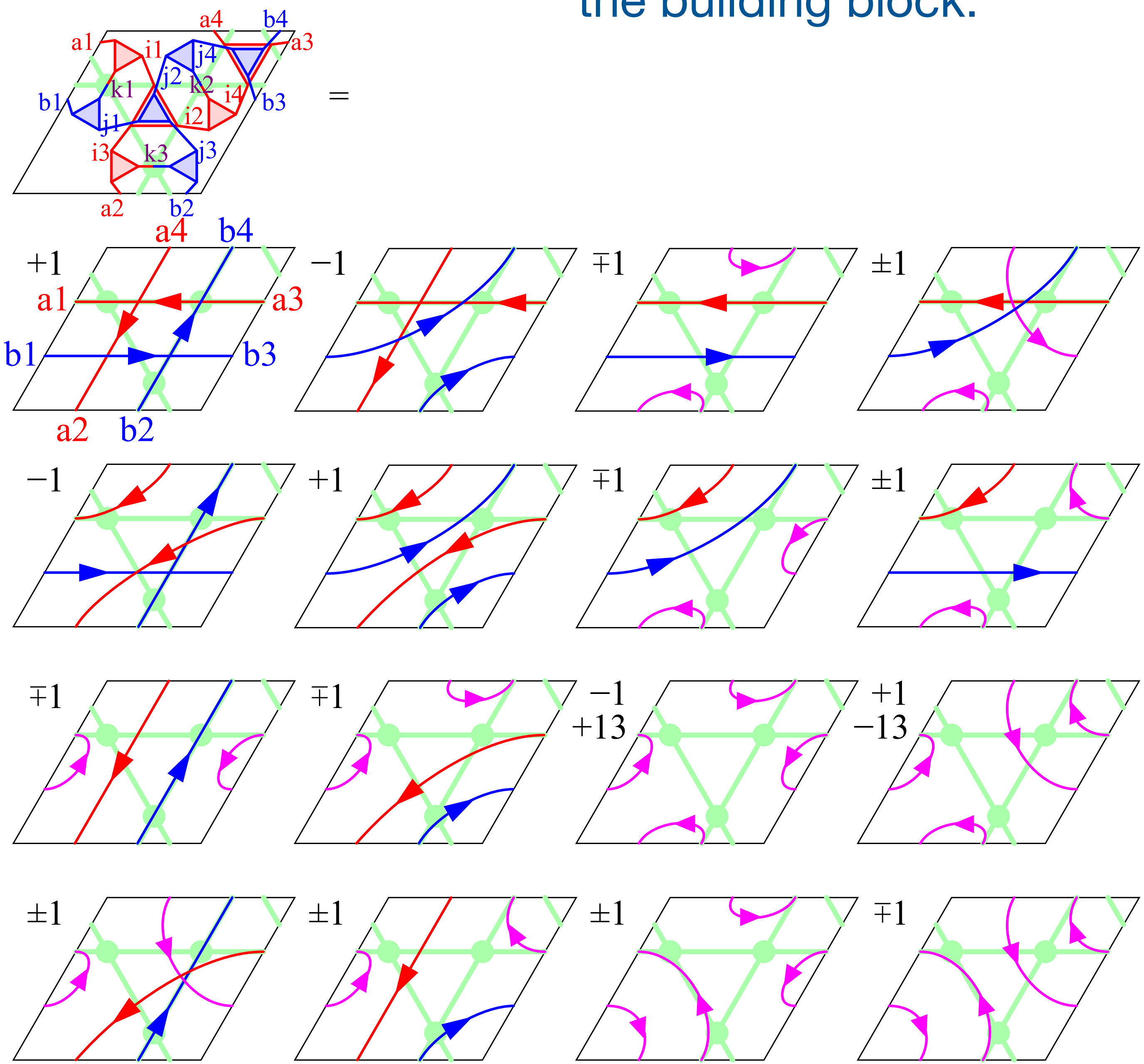
$$\ldots\varepsilon_{1,2,8}\,\varepsilon^{2,3,4}\,\varepsilon_{4,5,6}\,\varepsilon^{6,7,8}\ldots = \delta_1^3\delta_5^7 + \delta_1^7\delta_5^3$$



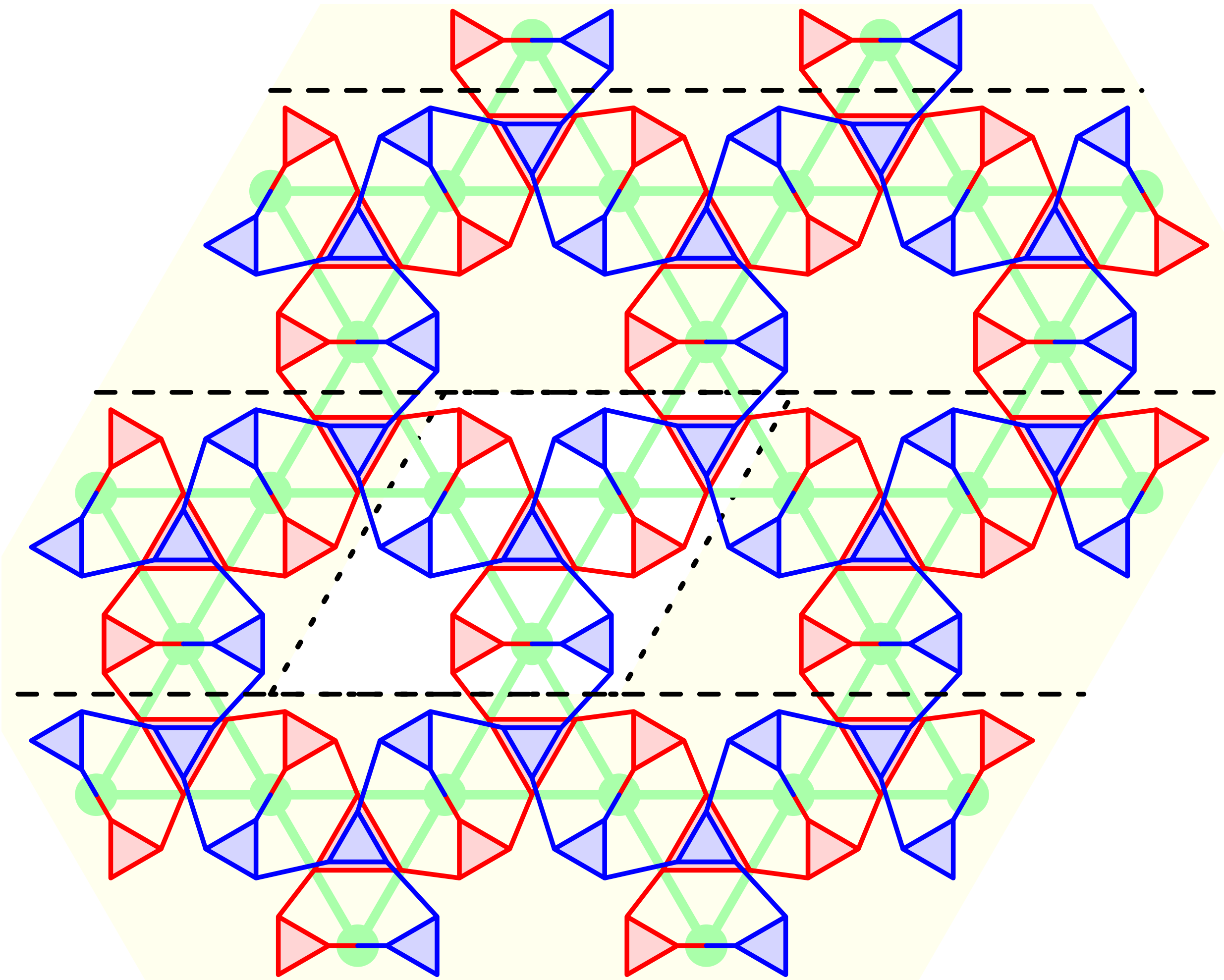
$$= -\,\delta_1^2\delta_3^6\delta_5^4 - \delta_1^6\delta_3^4\delta_5^2 - \delta_1^4\delta_3^2\delta_5^6 + \delta_1^4\delta_3^6\delta_5^2$$

Evaluating using tensor network

the building block:



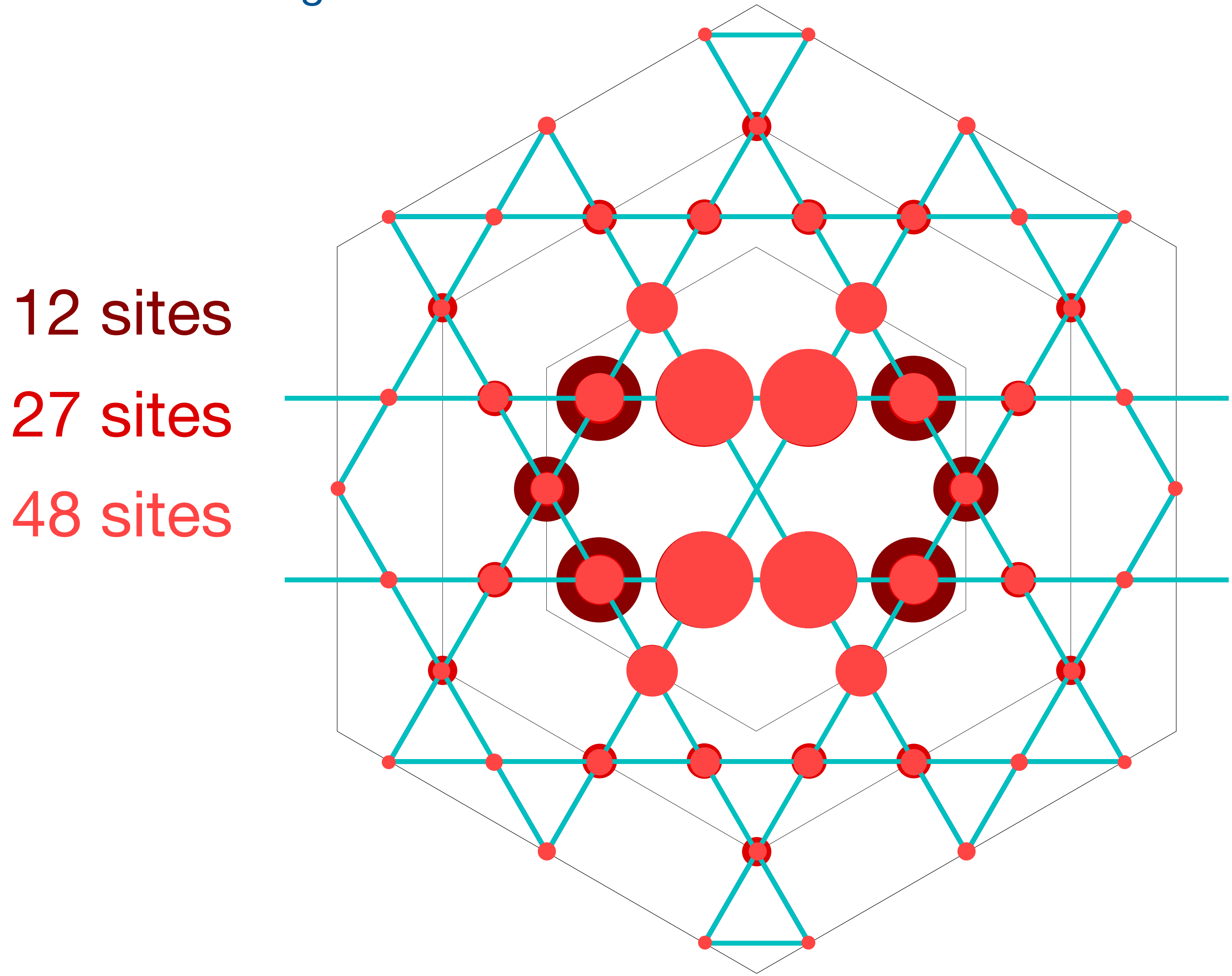
Tensor network: the overlap



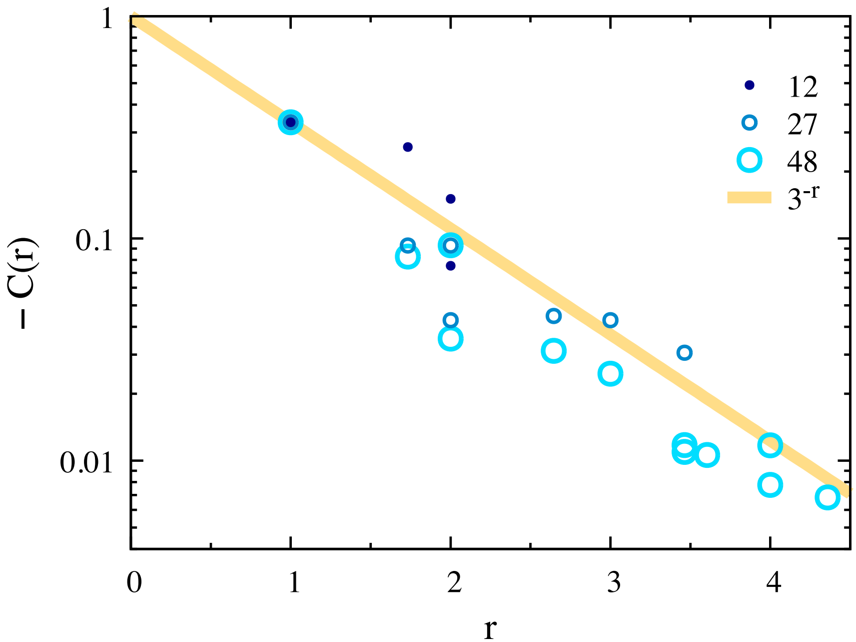
is simply a product
of matrices

Spin-spin correlation function

calculated using "tensor network"

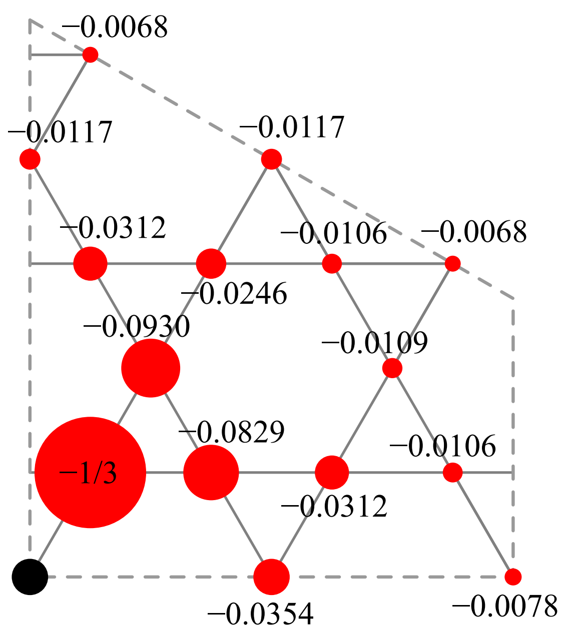


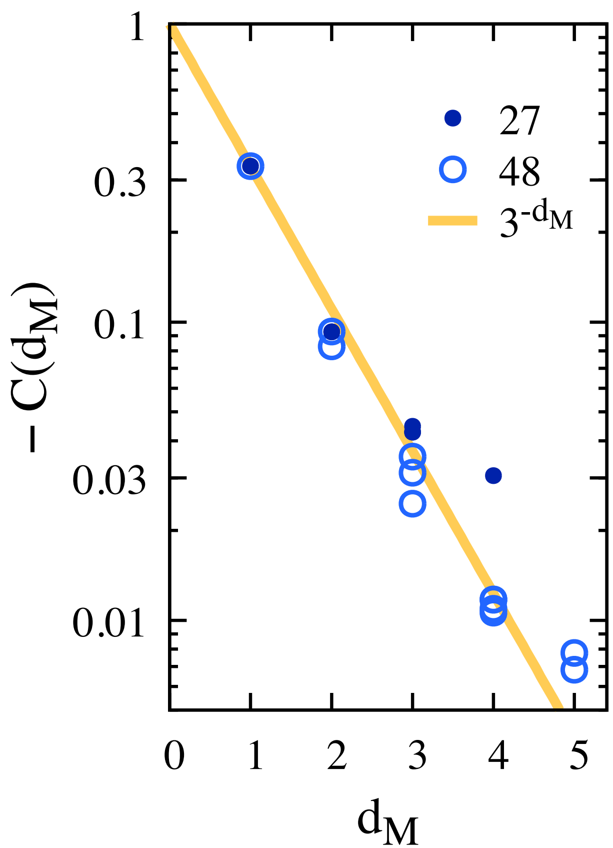
Spin-spin correlation function



decays exponentially,

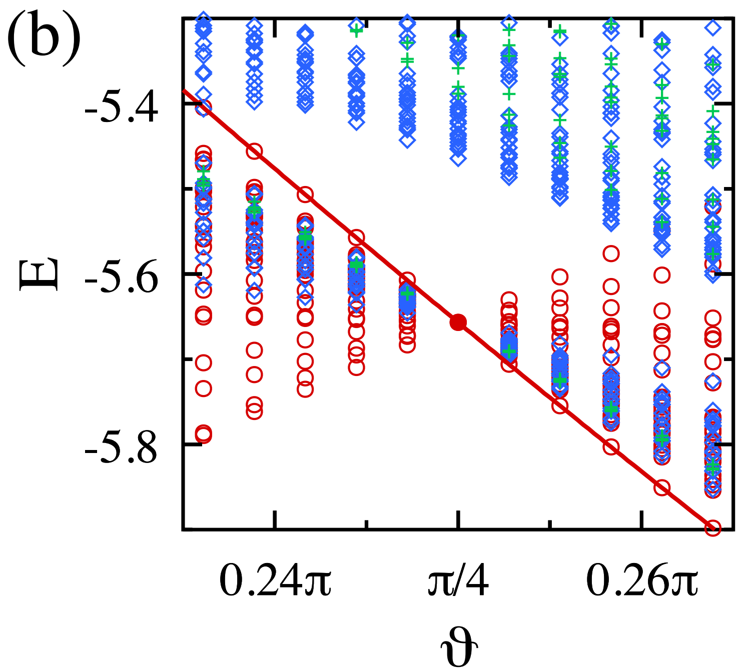
$$\langle \mathrm{FSS}|P_{0,r}|\mathrm{FSS}\rangle = C(r) \approx 3^{-r}$$

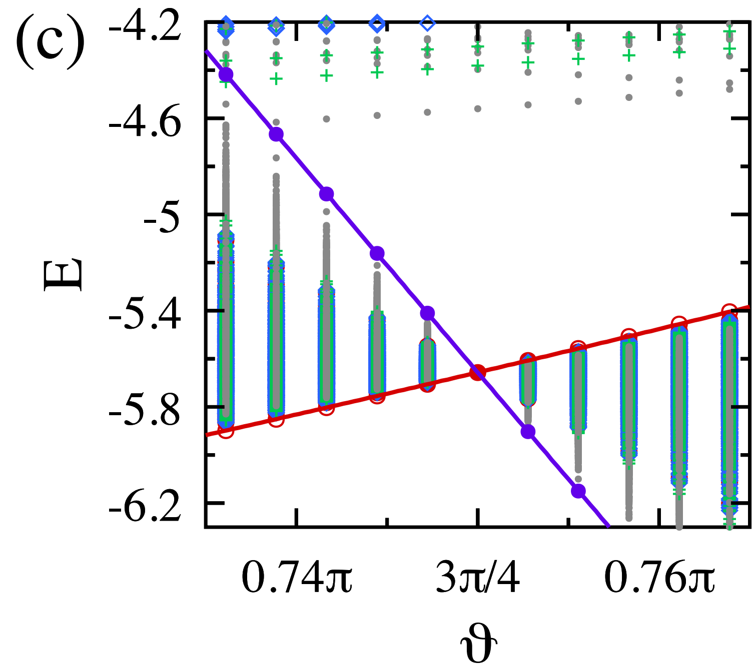




Manhattan distance

full ED for small system (12 sites) - degenerate GS



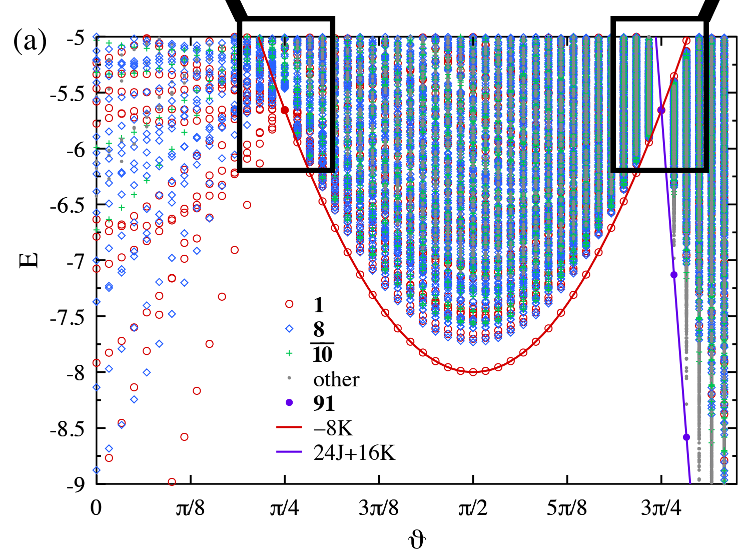


$$\mathcal{H} = \sum_{\triangle,\triangledown} |\mathbf{10}\rangle\langle\mathbf{10}|$$

$$J = K$$

$$\mathcal{H} = \sum_{\triangle,\triangledown} |\mathbf{1}\rangle\langle\mathbf{1}|$$

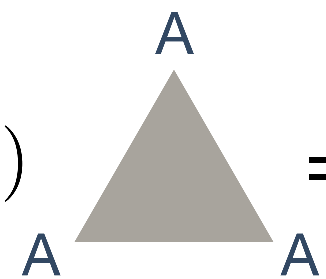
$$J = -K$$

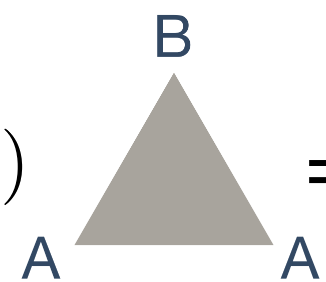


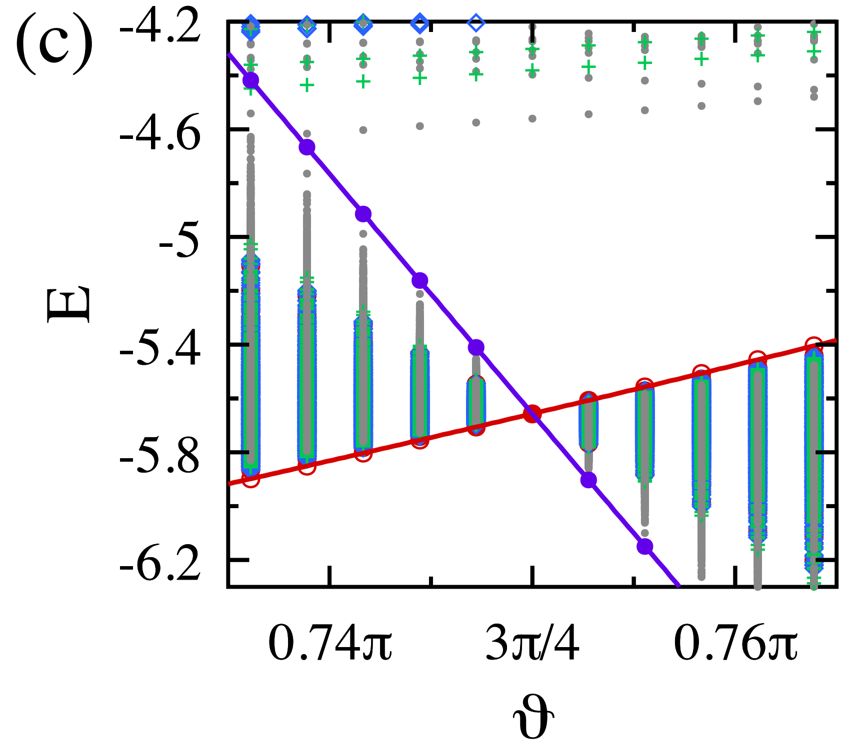
The ϑ=3π/4 (J = –K) case

$$\mathcal{H} = \sum_{\triangle,\bigtriangledown} |\mathbf{1}\rangle\langle\mathbf{1}|$$

$$\mathcal{H}_\triangle = \mathcal{P}_{i,j,k} + \mathcal{P}_{i,k,j} - \mathcal{P}_{i,j} - \mathcal{P}_{i,k} - \mathcal{P}_{j,k}$$

$(\mathcal{H}_\triangle + 1)$  = 0

$(\mathcal{H}_\triangle + 1)$  = 0



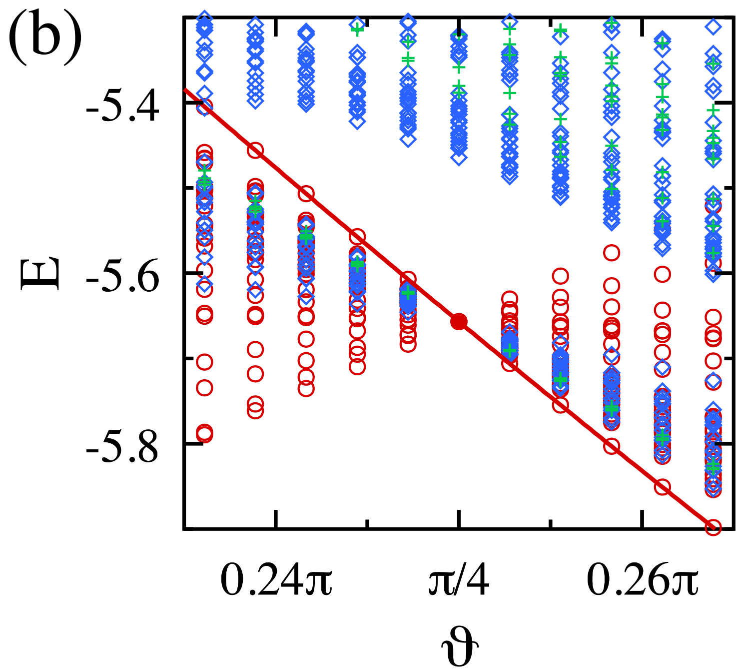
triangles having no more than two colors are degenerate eigenstates

385427 states are degenerate

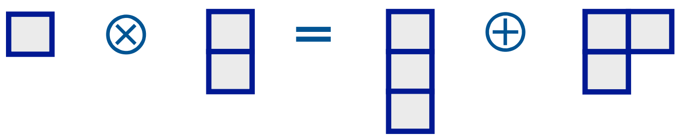
3^{12}=531441 is the total number of states

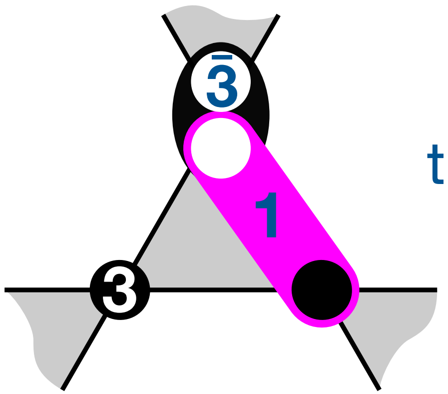
The $\vartheta=\pi/4$ (J = K) case

$$\mathcal{H} = \sum_{\triangle,\bigtriangledown} |\mathbf{10}\rangle\langle\mathbf{10}|$$



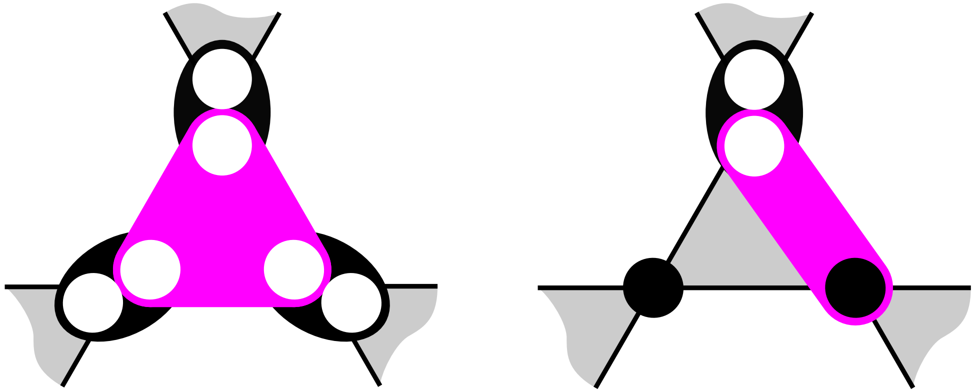
$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$





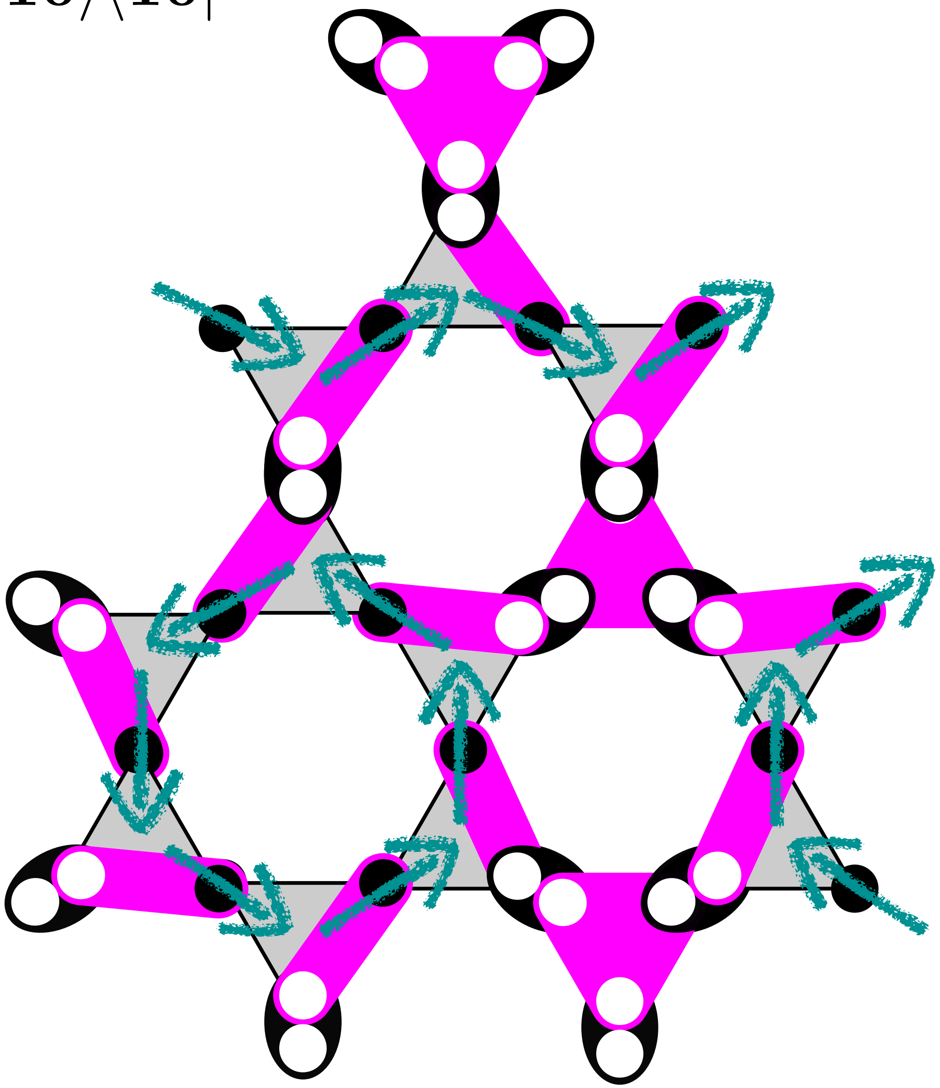
this does not contain **10,** only **1** and **8**

the building blocks are:



The J = K case: Lego time!

$$\mathcal{H} = \sum_{\triangle,\triangledown} |\mathbf{10}\rangle\langle\mathbf{10}|$$



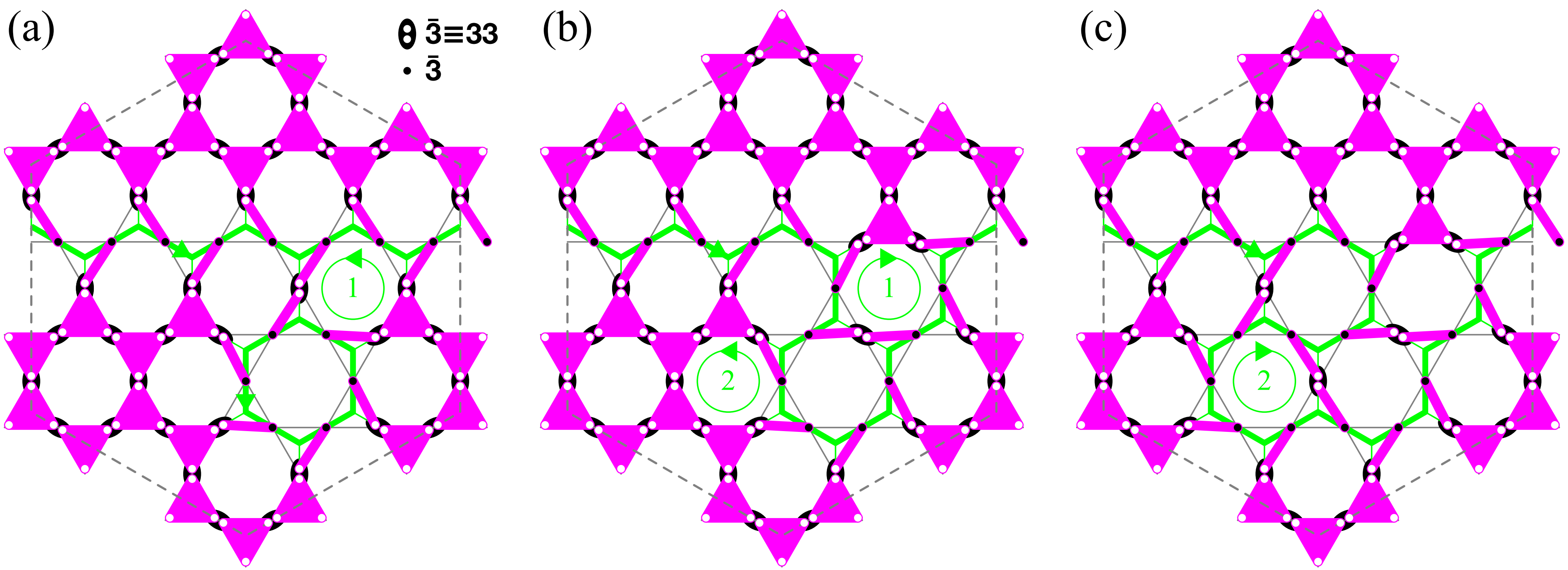
“current conservation” - some kind of a Coulomb liquid ?

On each bond 3 possibilities: 2 directions of arrow and absence of an arrow.

Z3 degrees of freedom

topological sectors (definition not obvious because of overlap and non-orthogonality)

The J = K case: singlet states characterized by directed loops on honeycomb lattice

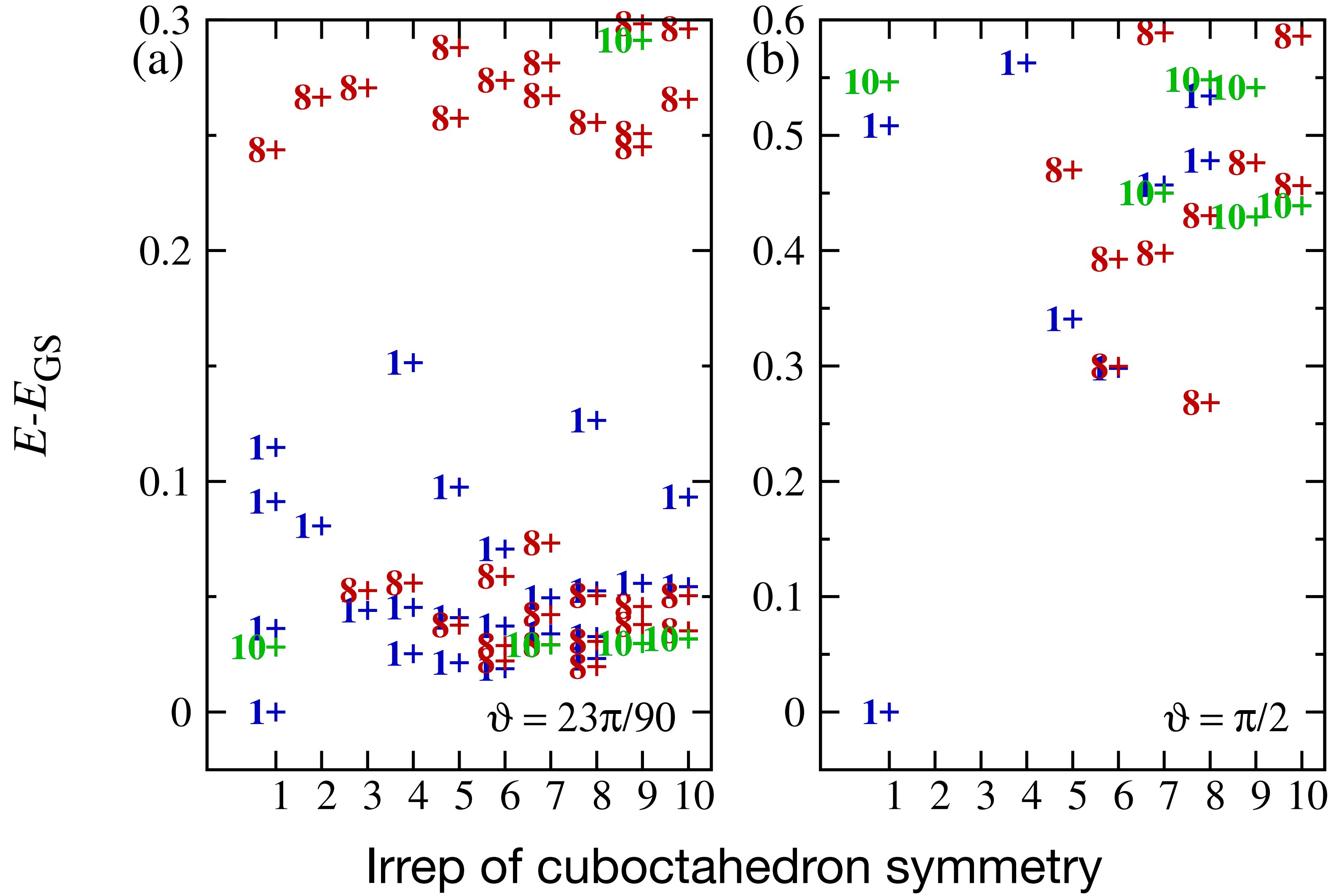


local loops -> extensive number of loops

number of undirected loops = $2\times2\times2^{(\mathrm{Nhex}-1)}$

N	undirected	directed	lin. ind.
12	32	69	48
27	1024	2551	2485
36	8192	22437	

The J = K case: other irreps also appear



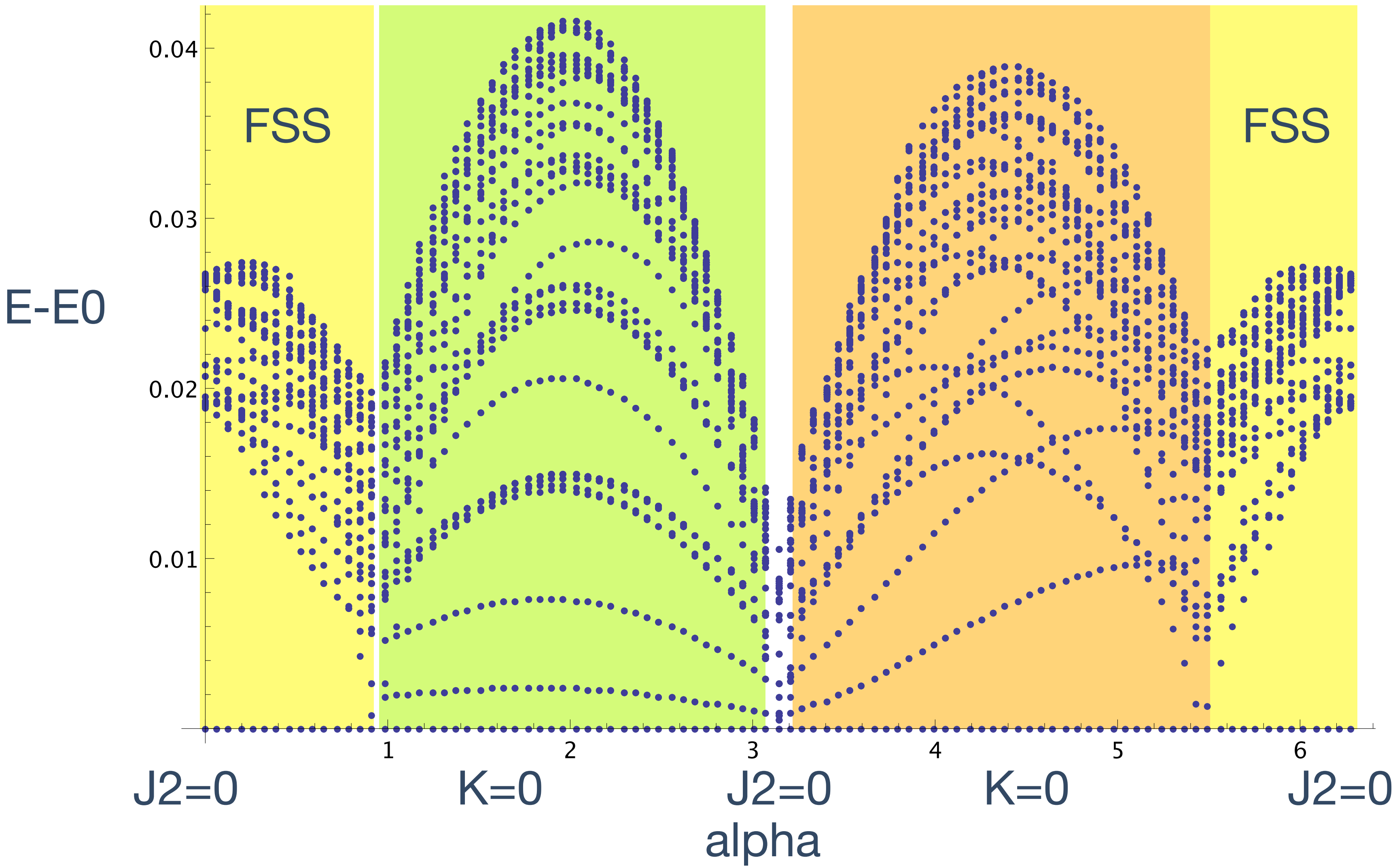
degeneracy at $\vartheta=\pi/4$: 468 = 48×**1** + 40×**8** + 10×$\overline{\mathbf{10}}$

What is the origin of the higher SU(3) irreps?

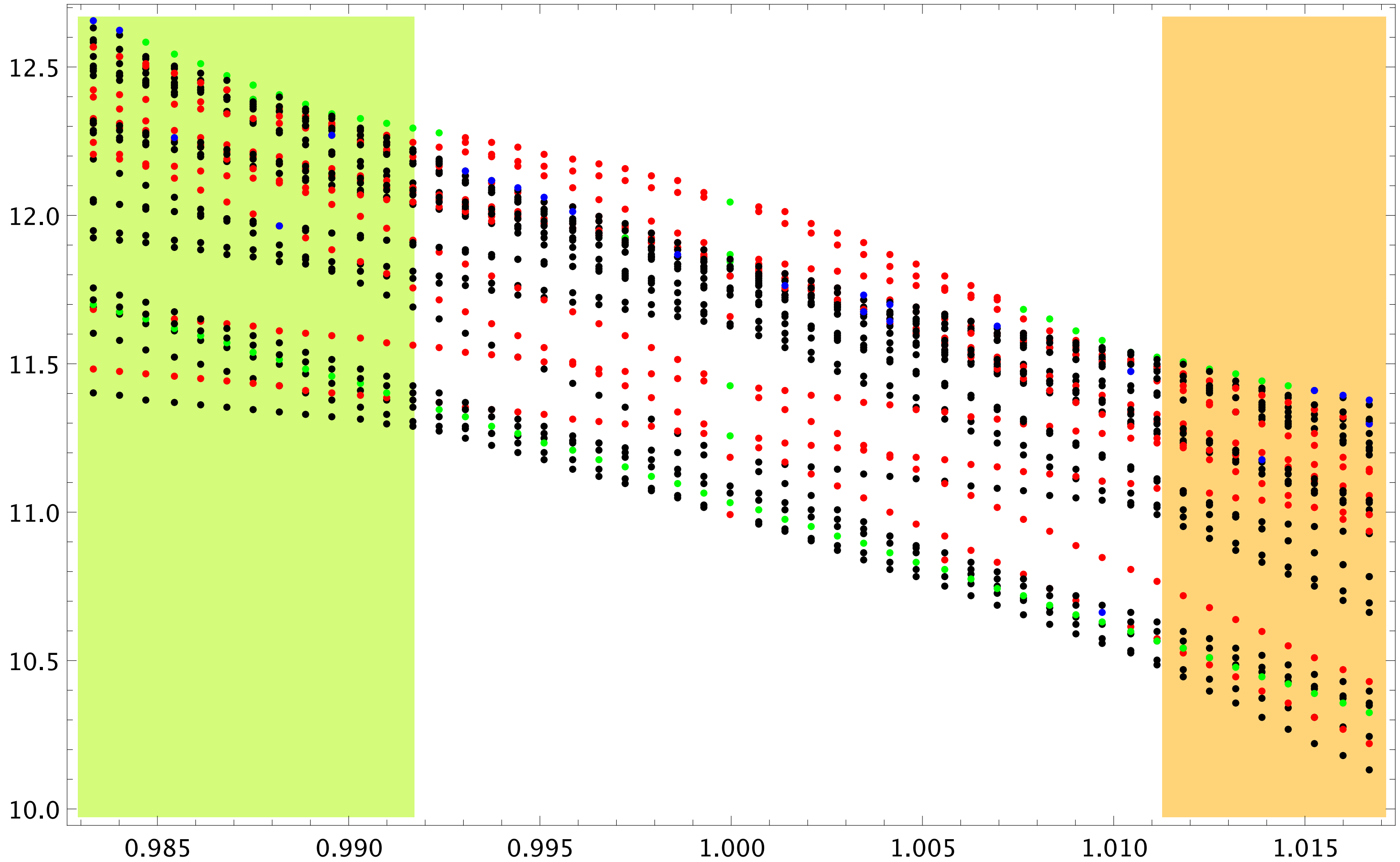
Lifting the degeneracy: K - J2 model

$$K = \cos\alpha$$
$$J_2 = \sin\alpha$$

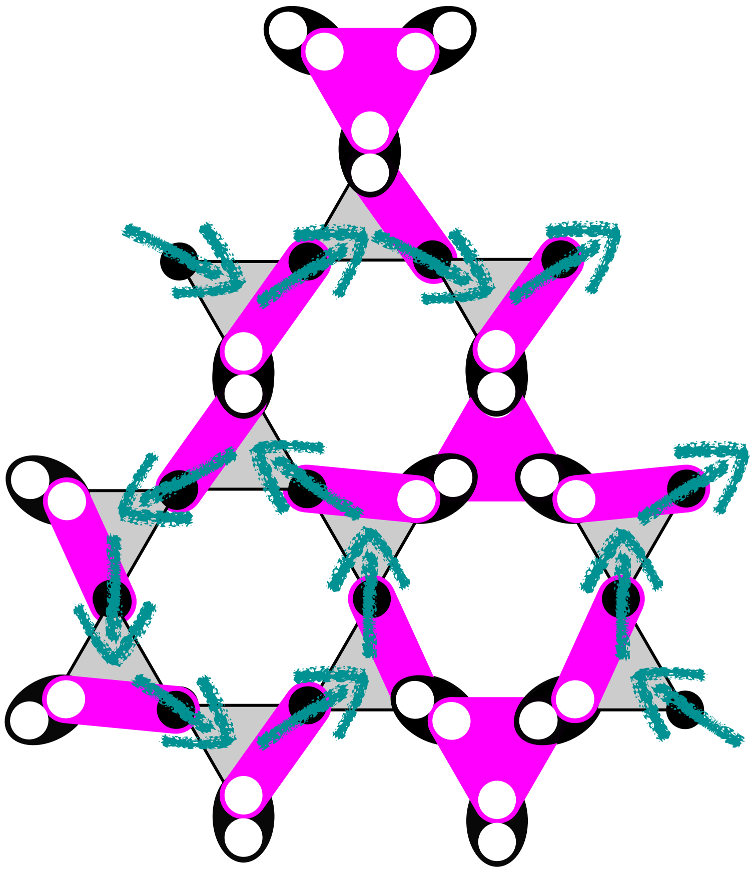
ED in the Hilbert space
spanned by singlets, 27 sites



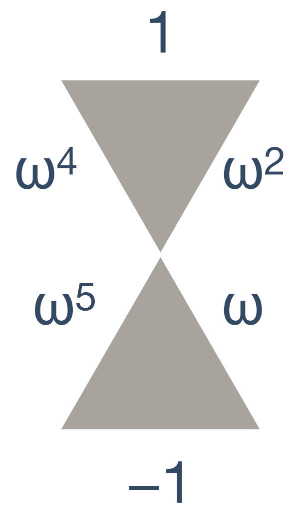
Lifting the degeneracy: K - J2 model



Topological sectors (polarizability)



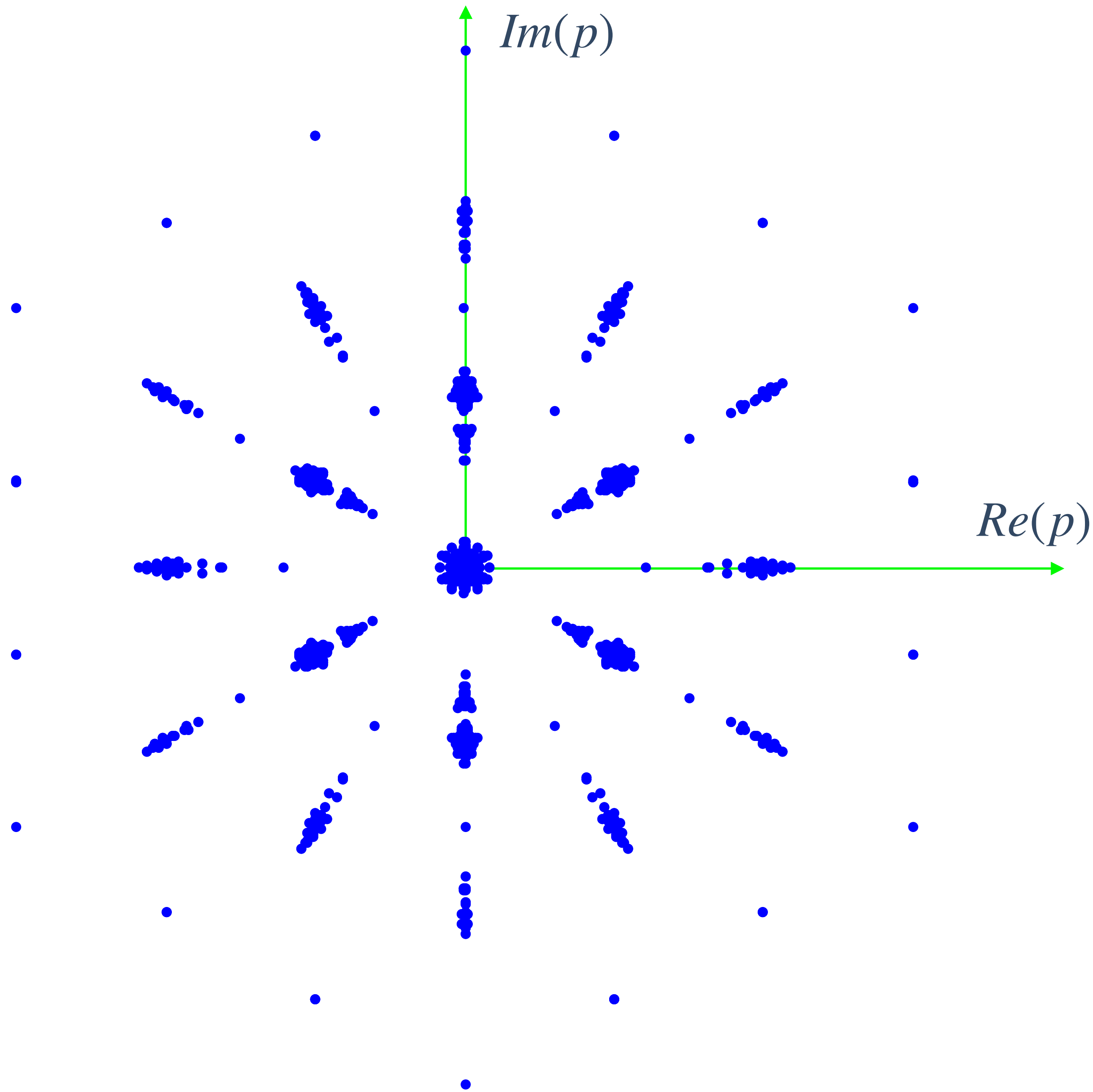
$$\omega = \exp \frac{2\pi i}{6}$$



we calculate the eigenvalues of the polarization operator:

$$p = \sum_{j \in \text{bonds}} \omega^{l(j)} \mathscr{P}_j$$

Topological sectors (polarizability)



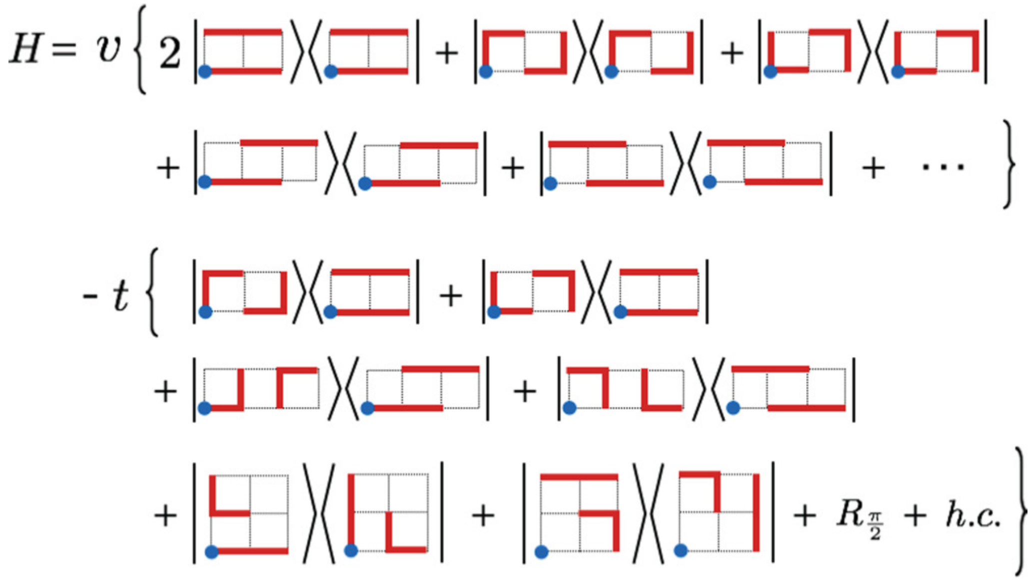
we calculate the eigenvalues of the polarization operator:

$$p = \sum_{j \in \text{bonds}} \omega^{l(j)} \mathscr{P}_j$$

27 sites, 2485 states

Tensor networks: Z_3 topological order

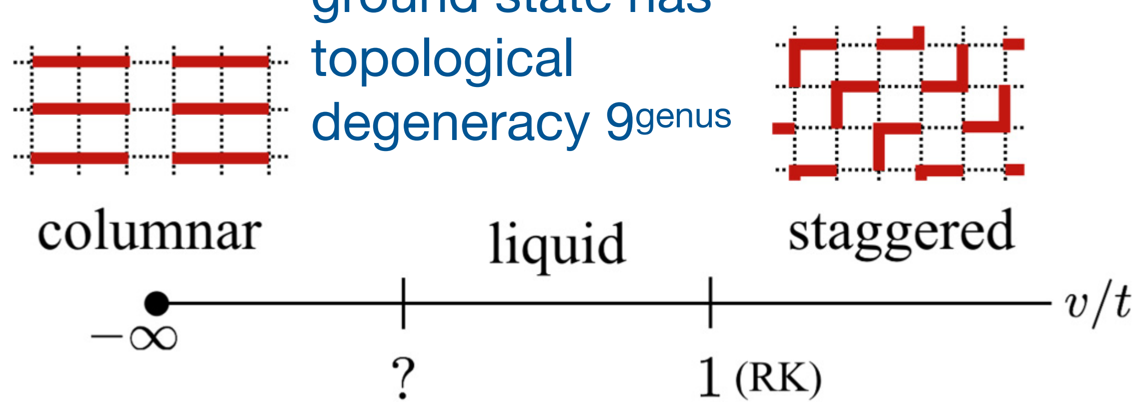
H. Lee, Y. Oh, J. H. Han, and H. Katsura
Resonating valence bond states with trimer motifs
Phys Rev B **95**, 060413(R) (2017)



Trimers are not the singlets of an SU(3) models (antisymmetry missing).

They defined winding numbers, leading to 3 topological sectors along both direction (Z3 vs Z2 in dimer coverings).

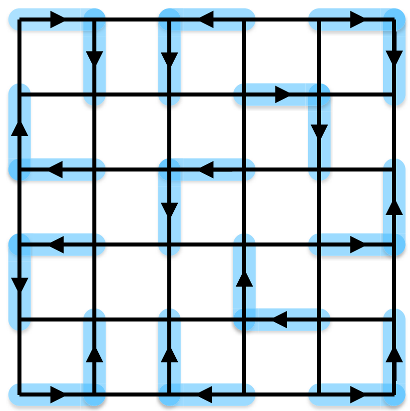
ground state has topological degeneracy 9^{genus}

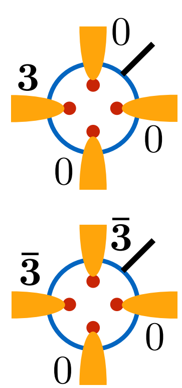


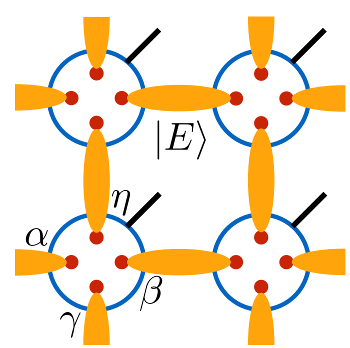
Tensor networks: Z_3 topological order

Xiao-Yu Dong, Ji-Yao Chen, Hong-Hao Tu
SU(3) trimer resonating-valence-bond state on the square lattice
Phys. Rev. B 98, 205117 (2018).

Trimers are now singlets of an SU(3) models (antisymmetry denoted by arrows).







Tensor networks: Z_3 topological order

I. Kurecic, L. Vanderstraeten, N. Schuch, A gapped SU(3) spin liquid with Z_3 topological order, Phys. Rev. B **99**, 045116 (2019)

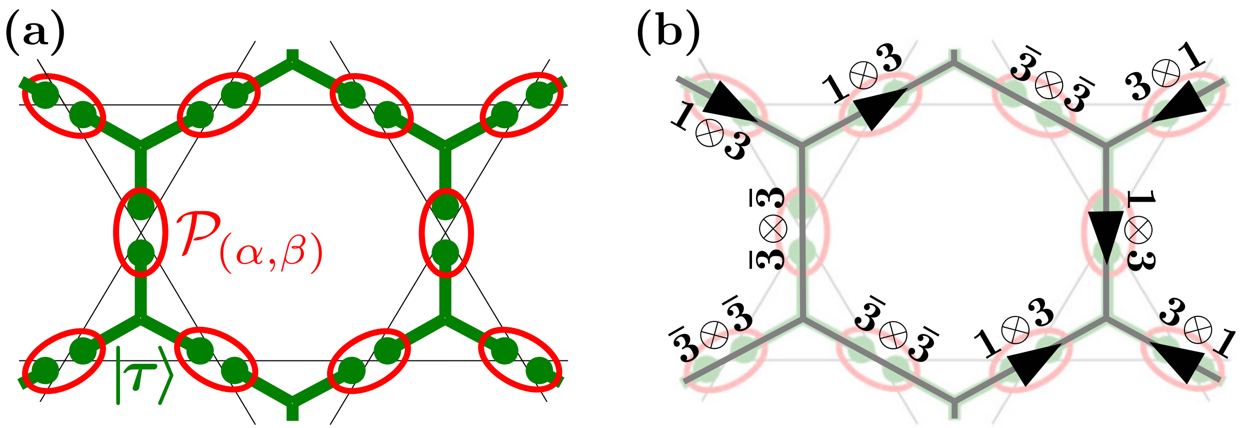
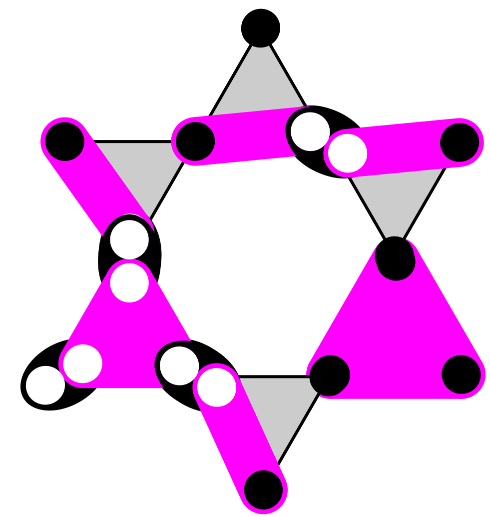
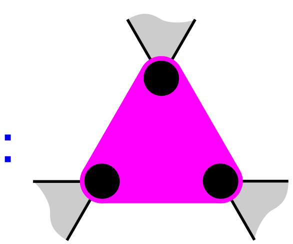


FIG. 1. (a) The model is constructed from trimers $|\tau\rangle$ which are in a singlet state with representation $\mathcal{H}_v \equiv \mathbf{1} \oplus \mathbf{3} \oplus \bar{\mathbf{3}}$ at each site (green dots), to which a map $\mathcal{P}_\bullet$ is applied which selects the physical degrees of freedom from $\mathcal{H}_v \otimes \mathcal{H}_v$. (b) Mapping to a $\mathbb{Z}_3$ topological model: Each site holds a $\mathbb{Z}_3$ degree of freedom: one of two arrows or no arrow. The arrows are pointing towards the $\mathbf{3}$ representation and satisfy a Gauss law across each vertex due to the fusion rules of the SU(3) irreps.



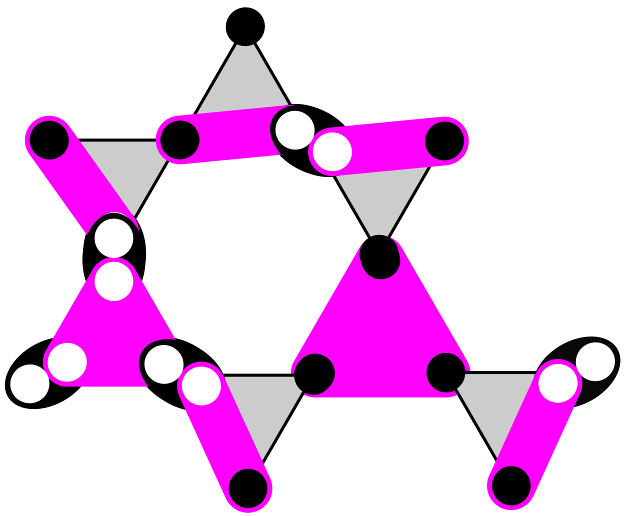
The trimer singlet is new:



parent Hamiltonian has 17 (?) sites, not shown in the papers

Tensor networks: Z_3 topological order

I. Kurecic, L. Vanderstraeten, N. Schuch, A gapped SU(3) spin liquid with Z_3 topological order, Phys. Rev. B **99**, 045116 (2019)

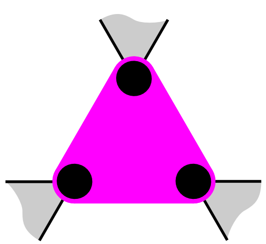


$$N_{\mathrm{sites}} = \frac{3}{2} N_{\bar{\mathbf{3}}\bar{\mathbf{3}}\bar{\mathbf{3}}} + 3 N_{\mathbf{333}} + \frac{3}{2} N_{\bar{\mathbf{3}}\mathbf{3}}$$

$$N_{\mathrm{tris}} = N_{\bar{\mathbf{3}}\bar{\mathbf{3}}\bar{\mathbf{3}}} + N_{\mathbf{333}} + N_{\bar{\mathbf{3}}\mathbf{3}}$$

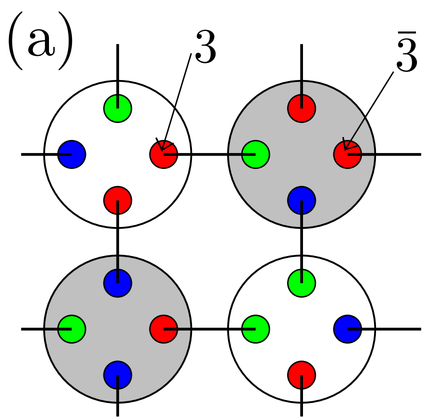
$$3 N_{\mathrm{tris}} = 2 N_{\mathrm{sites}}$$

from these equations: $N_{\mathbf{333}} = 0$

a single  creates an unhappy triangle elsewhere (unless saved by non-orthogonality)

Tensor networks: AKLT state in the SU(3) square lattice

Olivier Gauthé and Didier Poilblanc
Entanglement properties of the two-dimensional SU(3) Affleck-Kennedy-Lieb-Tasaki state
Physical Review B **96**, 121115(R) (2017)



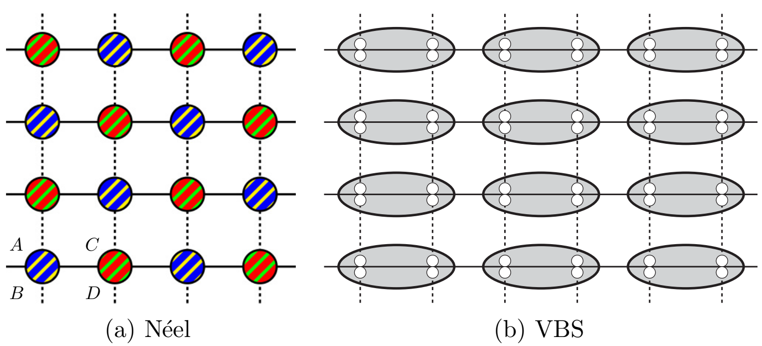
the symmetrical **15** dimensional irrep and its conjugate alternate on the sublattice sites.

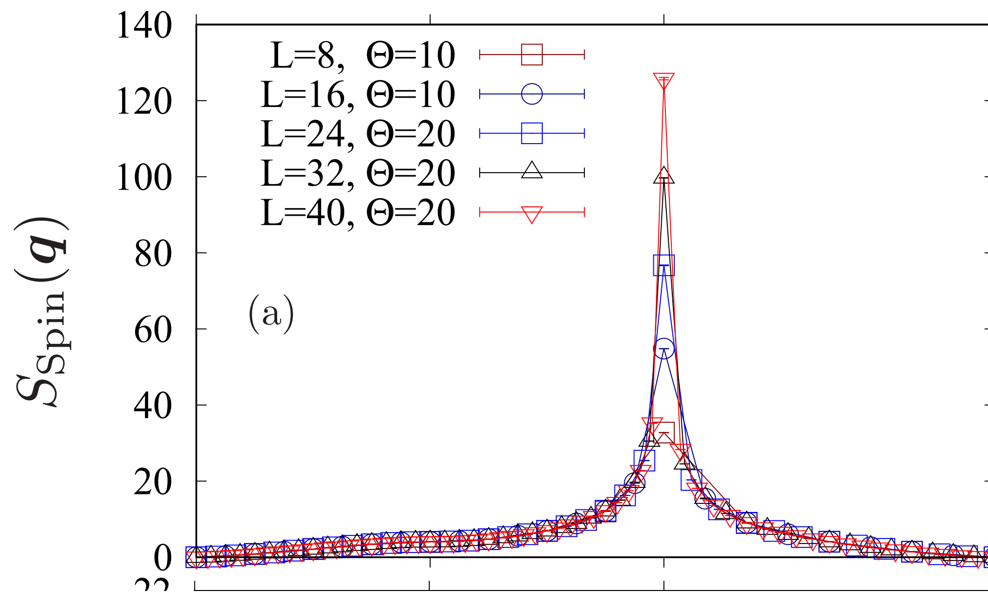
Conclusions

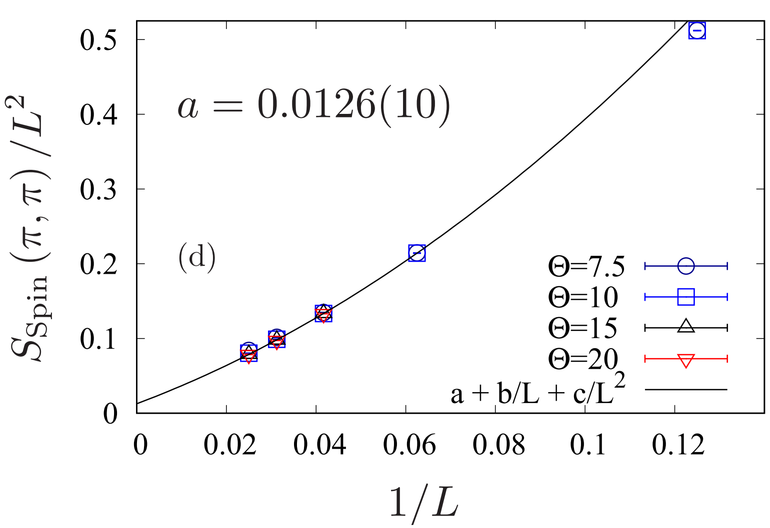
- Designed an exact AKLT-like ground state with a simple parent Hamiltonian.
- For special cases, a macroscopically large number of states become degenerate.
- Gauss law, states characterized by topological (?) quantum numbers (sectors)
- Point separating different phases
- many open questions: Coulomb phase, fractional excitations, origin of non-singlet states,…

Francisco H. Kim, Fakher F. Assaad, Karlo Penc, Frédéric Mila:
Dimensional crossover in the SU(4) Heisenberg model in the six-dimensional antisymmetric self-conjugate representation: Quantum Monte Carlo versus linear flavor-wave theory
Phys. Rev. B **100**, 085103/1-8 (2019) (Received 18 June 2019; published 1 August 2019)

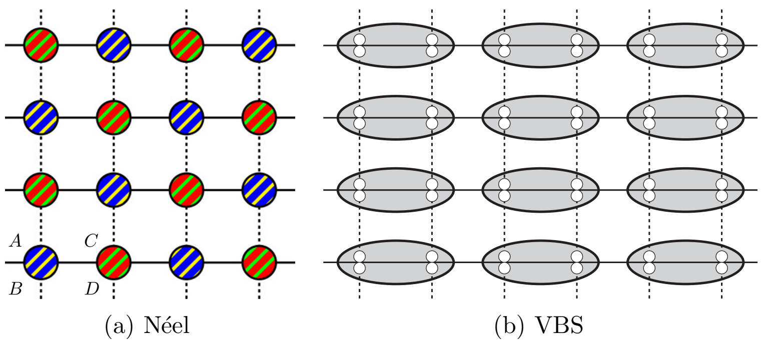
2 competing phases







QMC finite size scaling suggests spin LRO



changing the anisotropy (decreasing the coupling between the chains), there is a phase transition between the VBS and ordered state — **deconfined quantum critical point** ?