Disorder-induced local moments in a SU(2) symmetric Majorana spin liquid

Random-singlet phenomenology of low-temperature susceptibility

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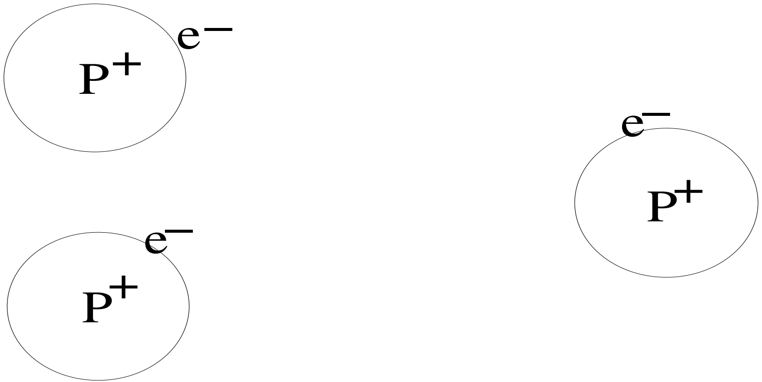
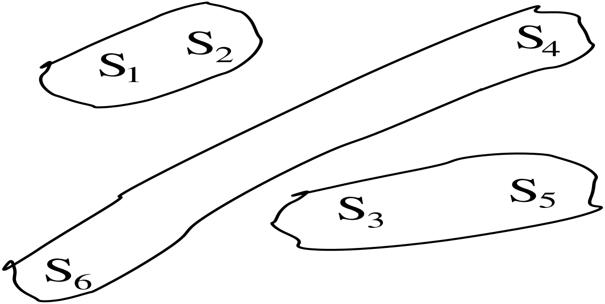
Background: Random singlet physics in Si:P



- Low density of P dopants in Si $\rightarrow$ Half-filled "Hubbard model" on random lattice
  Electrical insulator
- At low energies: Physics of $S = 1/2$ local moments

Random singlet phenomenology of $\chi(T)$



- $\sum_{i,j} J_{ij}\vec{S}_i \cdot \vec{S}_j$ with broad distribution of $J_{ij} > 0$
- RG: Singlet pairs with broad distribution of binding energies
- $N(T)$: Pairs with binding energy $< T$
  $\chi(T) \sim \frac{N(T)}{T} \sim T^{\alpha-1}$
  $1 > \alpha(T) > 0$ «effective» exponent
  Varies slowly with fitting range, depends on concentration of P
  (Bhatt & Lee)

Asymptotically exact?

- In $d = 1$, picture asymptotically exact for the random-exchange antiferromagnetic chain
  $\chi(T) = \frac{\Gamma_T^{-2}}{T}$ as $T \to 0$.
  $(\Gamma_T \equiv \log(J/T))$
  [J: scale of antiferromagnetic exchange]
  Multiplicative log shows up as effective exponent $\alpha(T)$ in fits
  (Dasgupta & Ma, D. S. Fisher)
- For $d > 1$, distributions do not broaden under RG
  (Bhatt & Lee; Motrunich & Huse)
  But random-singlet phenomenology in broad temperature/field range

Recent interest in random-singlet phenomenology in $d > 1$

- Possibility of random-singlet physics in bond disordered VBS phases proximate to spin liquid states
  (Kimchi, Nahum, Senthil 2018)
  partially motivated by properties of triangular lattice S=1/2 magnet $YbMgGaO_4$
- Numerical evidence for bond disordered VBS phase in JQ model (with multispin interactions)
  (Liu, Shao, Lin, Guo, Sandvik 2018)

Apparently wide applicability of phenomenology

- Random singlet phenomenology for $C_v(H,T)$ and $\chi(H,T)$ (or $M(H,T)$) in variety of disordered frustrated magnets
  $H_3LiIr_2O_6$, $LiZn_2Mo_3O_8$, $ZnCu_3(OH)_6Cl_2$ and 1T-TaS_2
  (Kimchi, Sheckleton, McQueen, Lee 2018)
  Argument': When distributions don't broaden indefinitely, also get few large local moments $\rightarrow$
  Curie tail + random-singlet phenomenology

In this talk...

Random-singlet-like susceptibility of diluted SU(2)-symmetric Majorana spin liquid

- Tractable example of a disordered SU(2)-symmetric Majorana spin liquid in $d = 2$
  with $\chi(T) = \frac{\mathcal{C}}{4T} + \frac{N(\Gamma_T)}{4T}$ as $T \to 0$
- $N(\Gamma_T)$ consistent with random-singlet physics
  $N(\Gamma_T) \sim \Gamma_T^{-y}$ for $T^* \ll T \ll J$ (y nonuniversal)
  $N(\Gamma_T) \sim \Gamma_T^{1/3} \exp(-c\Gamma_T^{2/3})$ for $T \ll T^*$
  $N(\Gamma_T)$ factor gives effective exponent $\alpha(T)$

Natural interpretation and question

- $\mathcal{C}$
  Density of "forever-free" moments
  Composite' large-lengthscale objects
- $N(\Gamma_T)$
  Density of singlet-pairs with binding energies smaller than T

Results raise question:
How literally should we take this interpretation?
Alternate strong-disorder RG approach to go beyond solvable limit?

Source of these results...

Free-fermion physics of bipartite random hopping problem with vacancy and flux disorder

- $\chi(T) \propto \kappa(T)$ (compressibility)
  $N(\Gamma_T) \to$
  integrated DOS for single-particle energies $0 < |\epsilon| < J \times 10^{-\Gamma_T}$
  $\mathcal{C} \to$
  density of protected zero modes of single-particle problem

Setting: Honeycomb model of Yao & Lee

$$\mathcal{H} = J \sum_{\langle \vec{r}\vec{r}' \rangle_\lambda} \tau^\lambda_{\vec{r}} \tau^\lambda_{\vec{r}'} \vec{S}_{\vec{r}} \cdot \vec{S}_{\vec{r}'} - \sum_{\vec{r}} \vec{B} \cdot \vec{S}_{\vec{r}} \; . \tag{1}$$

- $\vec{\tau}$: "Orbital degrees of freedom that remain dynamical at low energy
- $\vec{S} = \frac{\vec{\sigma}}{2}$: spin-half moments
- Effective H for $S = 1/2$ antiferromagnet on decorated honeycomb lattice
  Strong AF exchange within each triangle; multi-spin interactions between triangles
  Each $\vec{S}_r$: Total spin on each triangle r.
  Chirality $\tau^z_r = \pm 1$: Two different doublet states of a triangle r

Majorana representation

- $\sigma^z_{\vec{r}} = -ic^x_{\vec{r}} c^y_{\vec{r}}$
  $\tau^z_{\vec{r}} = -ib^x_{\vec{r}} b^y_{\vec{r}}$
  and cyclic permutations
- $c^\lambda_{\vec{r}}$ and $b^\lambda_{\vec{r}}$ are Majorana (real) fermion operators.

Single-site Hilbert space doubled by this representation
(Shastry-Sen, Tsvelik)

Constraint on fermion states

- $D_{\vec{r}} \equiv -ic^x_{\vec{r}} c^y_{\vec{r}} c^z_{\vec{r}} b^x_{\vec{r}} b^y_{\vec{r}} b^z_{\vec{r}} = +1$ at each site $\vec{r}$
  Curious fact: $D = -1$ sector also provides faithful representation of $\vec{\sigma}$ and $\vec{\tau}$.
  $\rightarrow$
  No "unphysical" states. Instead: Two copies of physical states at each site
- In $D = +1$ sector: $\sigma^\alpha_{\vec{r}} \tau^\beta_{\vec{r}} = ic^\alpha_{\vec{r}} b^\beta_{\vec{r}}$
  Similar reduction in $D = -1$ sector

Reduction leads to exact solution

- On bond $\langle rr' \rangle \lambda$ (orientation $\lambda = x, y, z$) get term:
  $u_{\langle rr' \rangle \lambda}(i\vec{c}_r \cdot \vec{c}_{r'})$
  where $u_{\langle rr' \rangle \lambda} = -ib_r^\lambda b_{r'}^\lambda$
- Three copies of Kitaev's non-interacting Majorana model, all coupled to same static Z_2 gauge field

Majorana fermion Hamiltonian

$$\mathcal{H} = \frac{J}{2} \sum_{\alpha=x,y,z} \sum_{\langle \vec{r}\vec{r}' \rangle_\lambda} u_{\langle \vec{r}\vec{r}' \rangle_\lambda} (i c^\alpha_{\vec{r}} c^\alpha_{\vec{r}'} + h.c.) + B \sum_{\vec{r}} i c^x_{\vec{r}} c^y_{\vec{r}} \tag{2}$$

where $\vec{B} = B\hat{z}$.

- Convenient: Canonical fermions $f_{\vec{r}} = (c^x_{\vec{r}} - i c^y_{\vec{r}})/2$
- $S^z_{\vec{r}} = \frac{i}{2} c^x_{\vec{r}} c^y_{\vec{r}} = f^\dagger_{\vec{r}} f_{\vec{r}} - 1/2$
- Want to compute: $m^z \equiv \sum_r \langle S^z_{\vec{r}} \rangle / 2L^2$ as function of B and obtain $\chi(T) = \frac{dm^z}{dB}$ at $B = 0$

Calculating susceptibility

- Hamiltonian H for f fermions:
  Tight-binding model with static Z_2 gauge-fields u determining signs of each hopping matrix element $t = u|J|$
- $\chi(T) \rightarrow f$ fermion compressibility $\kappa(T)$ at $\mu \equiv B = 0$.
- c^z Majorana plays no role in susceptibility calculation
- $\chi(T) = \frac{1}{T}\int_{-\Omega}^{+\Omega} d\epsilon \rho_{\text{tot.}}(\epsilon)\frac{e^{\epsilon/T}}{(e^{\epsilon/T}+1)^2}$
  where $\rho_{\text{tot}}(\epsilon)$ is full DOS of H

Projection issues?

- In usual Kitaev model: Projection gives subleading corrections in thermodynamic limit
  (Pedrocchi-Chesi-Loss, Zschocke-Vojta)
- What happens here?
  Again: Only subleading corrections in general.
- For specific boundary conditions: Coefficient of subleading corrections zero

Flux-binding

- Lieb-Loss heuristics:
  Each vacancy binds static π-flux in ground-state sector.
  Numerical verification: Gap to other flux sectors
  (Kitaev)
- At low temperature, χ dominated by this flux-sector

Features of free-fermion H

- Without flux-attachment: site-diluted tight-binding model for graphene)
- In any flux sector, $\rho(\epsilon) = \rho(-\epsilon)$
  "Chiral" (bipartite) symmetry: $\epsilon \to -\epsilon$, $\Psi_B \to -\Psi_B$

$d = 1$ bipartite random-hopping: Dyson form of DOS

- $\rho(\epsilon) \sim 1/[|\epsilon| \log^3(1/|\epsilon|)]$
  Defining: $N(\Gamma_\epsilon) = 2\int_0^{10^{-\Gamma_\epsilon}} \rho(x)dx$,
  $dN/d\Gamma_\epsilon \sim 1/\Gamma_\epsilon^3$ with $\Gamma_\epsilon = \log_{10}(1/|\epsilon|)$
- Controlled strong-disorder RG rederivation:
  Eliminate states at cutoff $\pm\Omega \rightarrow J_i \rightarrow \tilde{J}_i$
  At cutoff scale $\Gamma \equiv \log(1/\Omega)$
  Number of surviving sites: $N(\Gamma) \sim 1/\Gamma^2$
  Fraction $1/\Gamma$ of $\zeta_i \equiv \log(\Omega/\tilde{J}_i)$ have $\zeta_i = 0$
  Distribution of log-couplings becomes infinitely broad at low energies
  $\rightarrow dN/d\Gamma_\epsilon \sim N(\Gamma_\epsilon)/\Gamma_\epsilon$
- Conversely: $\frac{1}{N(\Gamma)} \times dN/d\Gamma \sim 1/\Gamma \rightarrow$ Distributions broaden as $\sim \Gamma$ (Motrunich, KD, Huse)

$d = 2$: Modified Gade-Wegner scaling

- $\rho(E) \sim \frac{1}{|\epsilon|} e^{-b|\ln \epsilon|^{1/x}}$
  equivalently: $dN(\Gamma_\epsilon)/d\Gamma_\epsilon \sim \exp(-b\Gamma_\epsilon^{1/x})$
  $N(\Gamma_\epsilon) \sim \Gamma_\epsilon^{1-\frac{1}{x}} e^{-b\Gamma_\epsilon^{1/x}}$
  Gade & Wegner prediction: $x = 2$
- $\frac{1}{N(\Gamma_\epsilon)} \times dN/d\Gamma_\epsilon \sim 1/\Gamma_\epsilon^{1-1/x} \rightarrow$ broad distributions
  $\rightarrow$ At cutoff scale $\Gamma = \log(1/\Omega)$: width $\sim \Gamma^{1-1/x}$
  Strong disorder effect: $x = 3/2$ (Motrunich, KD, Huse)
  (field-theory confirmation: Mudry, Ryu, Furusaki)
- Distributions broaden as $\sim \Gamma^{1/3}$

What about vacancies?

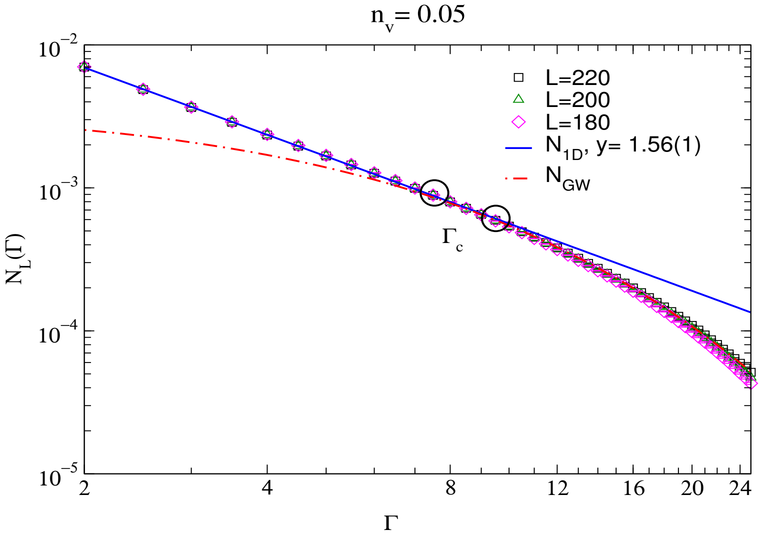
- Usual Kitaev model with vacancies:
  free fermion H has dilution and flux binding
  Numerical result: $N(\Gamma_\epsilon) \sim 1/\Gamma_\epsilon^y$ with $y \approx 0.7$ at not-too-low energies ϵ
  (Willans, Chalker, Moessner)
- surprising violation of modified Gade-Wegner scaling?

Similar surprise in diluted graphene?

Perhaps motivated by Willans-Moessner-Chalker...

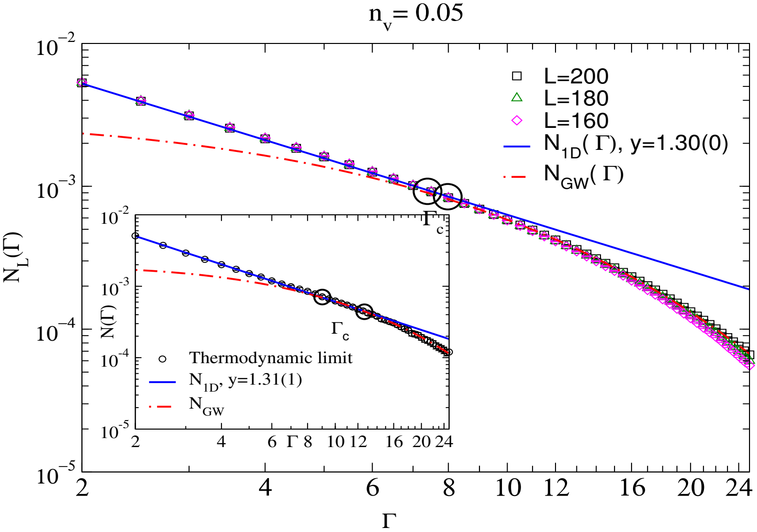
- New field-theoretical prediction: Vacancies change asymptotic universality class
  $N(\Gamma_\epsilon) \sim 1/\Gamma_\epsilon^y$ with $y = 0.5$
  (Ostrovsky, Protopopov, Konig, Gornyi, Mirlin, Skvortsov)
- concurrent numerical evidence: $y \sim 1/2$
  (Hafner, Schindler, Weik, Mayer, Balakrishnan, Narayanan, Bera, Evers)

Actually: surprisingly long crossover...



(Sanyal, KD, Motrunich)

Revisiting calculation of Willans-Chalker-Moessner



Same crossover in diluted system with π flux with flux attached to each vacancy

(Sanyal, KD, Chalker, Moessner, in preparation)

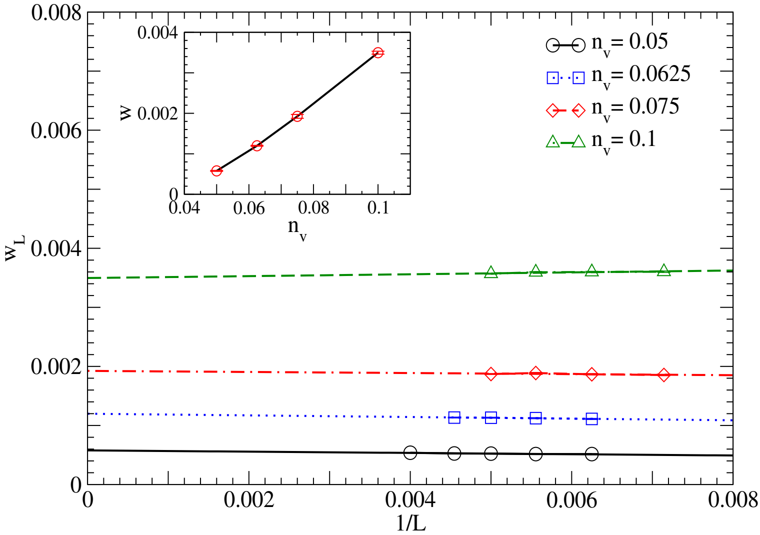
Crossover looks nonuniversal:

- Crossover Γ_c and intermediate asymptotic exponent y nonuniversal:
  depends on n_v and correlations between vacancy positions

But: Zero mode density controls crossover

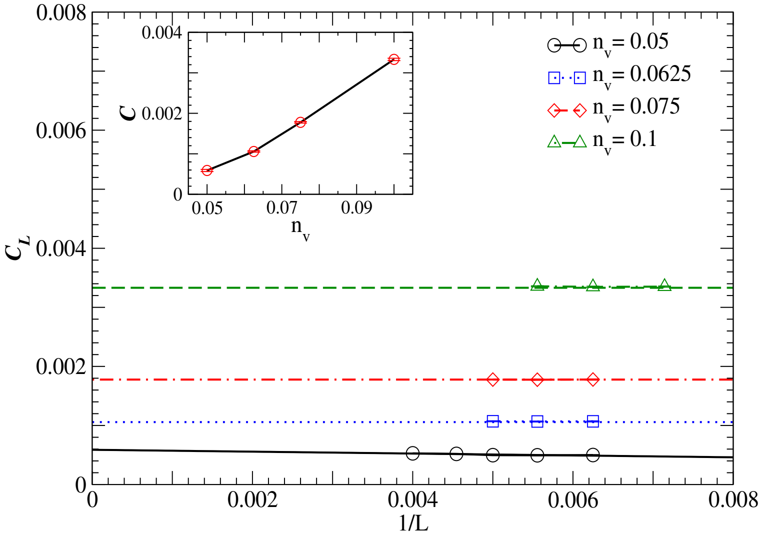
- Other *generic* feature of bipartite random hopping H on diluted lattice:
  Nonzero *density* w of zero modes
  Robust to bond-disorder, flux attachment, boundary conditions
  not associated with lattice imbalance, isolated sites or other trivial effects
- $w(n_v)$, $y(n_v)$, $\Gamma_c(n_v)$ all depend on correlations between vacancy positions
  But $y(w)$ and $\Gamma_c(w)$ are universal
  (Sanyal, KD, Motrunich)

Zero modes in diluted graphene



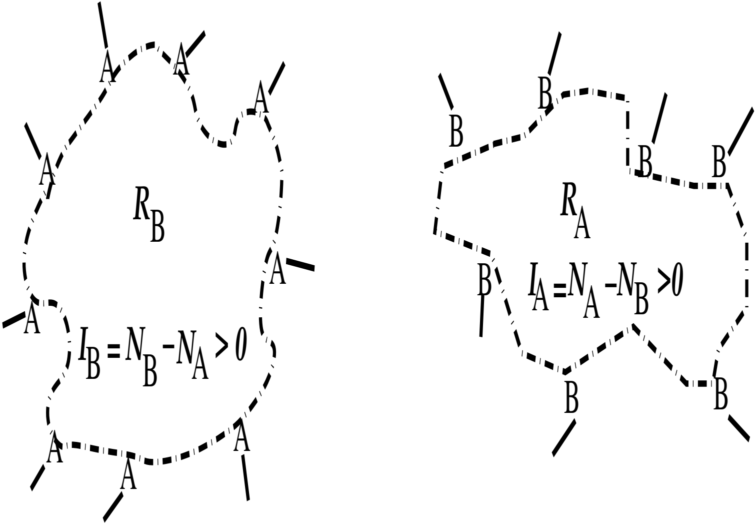
(Sanyal, KD, Motrunich)

Kitaev: Zero modes



(Sanyal, KD, Chalker, Moessner, in preparation)

Zero modes are robust large-lengthscale effect



(Biswas, Islam, KD, in preparation)

Conclusions

- Exactly solvable example of random-singlet physics in SU(2) symmetric Majorana spin liquid state
- key ingredients present:
  Low-temperature susceptibility controlled by flows to strong disorder
  Large-lengthscale cooperative effect controls Curie coefficient associated with emergent free moments

Acknowledgements

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