Dirty and frustrated

effects of quenched disorder in magnets with competing interactions

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Refs: PRL **111**, 247201 (2013)
PRB **90**, 094412 (2014)
PRL **119**, 247201 (2017)
PRB **99**, 054416 (2019) …

what is NOT included in this talk

- ❑ Spin glasses (not quite)
- ❑ Dilution into Coulomb spin liquids
- ❑ Dilution into quantum spin liquids
- ❑ Dilution induced ordering in quantum gapped magnets
- ❑ Disorder effects on spin dynamics

Outline

- Spin textures induced by vacancies
- Order from structural disorder: triangular, XY pyrochlore …
- Jammed spin liquid: random bond kagome antiferromagnet

Beginning of Disorder

Jacques Villain

Z. Physik B 33, 31 – 42 (1979)

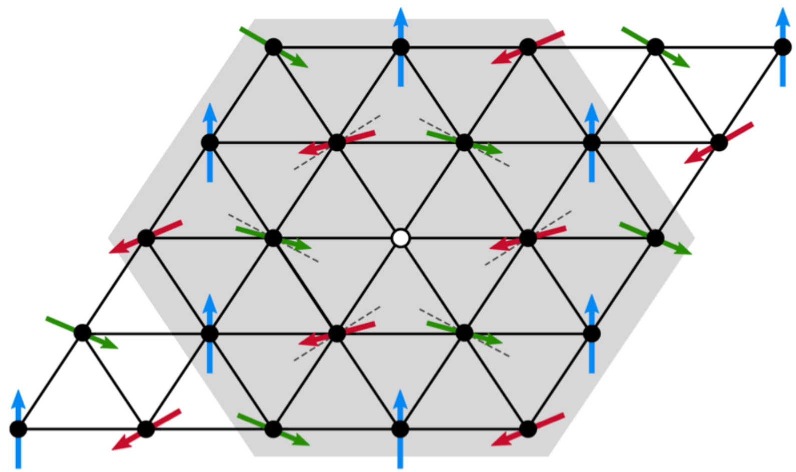
Zeitschrift für Physik B

Insulating Spin Glasses

Jacques Villain
Département de Recherche Fondamentale, Laboratoire de Diffraction Neutronique, Centre d'Etudes Nucléaires de Grenoble, Grenoble, France

Received September 21, 1978

Single vacancy in a noncollinear magnet



$$\delta\theta(\mathbf{r}) \sim 1/r^n$$

- spins tilt from equilibrium directions
- canting angle $\longrightarrow$ strain field in the continuum limit:
- what is the exponent n?
  - $n = D$ (Henley, 2001; Weber & Mila, 2012)
  - $n = D + 1$ (Wollny *et al*, 2011)

Elastic theory of spin texture

- Expansion in small local deviations $\mathbf{m}_i = \mathbf{S}_i - \mathbf{S}_i^{(0)}$ + transformation to the local axes

$$E = \int d^D r \left\{ K\left[(\nabla m^x)^2 + (\nabla m^y)^2 \right] + \lambda m^{y2} \right\}$$

- Laplace's equation

$$\Delta m^x = 0 \ , \quad \Delta m^y - \frac{\lambda}{K} m^y = 0$$

- + the Source term, which encodes information about defect

Elastic theory of spin texture

- Defect antiferromagnetic bond in a collinear ferromagnet

Villain 1979; Parker & Saslow 1988

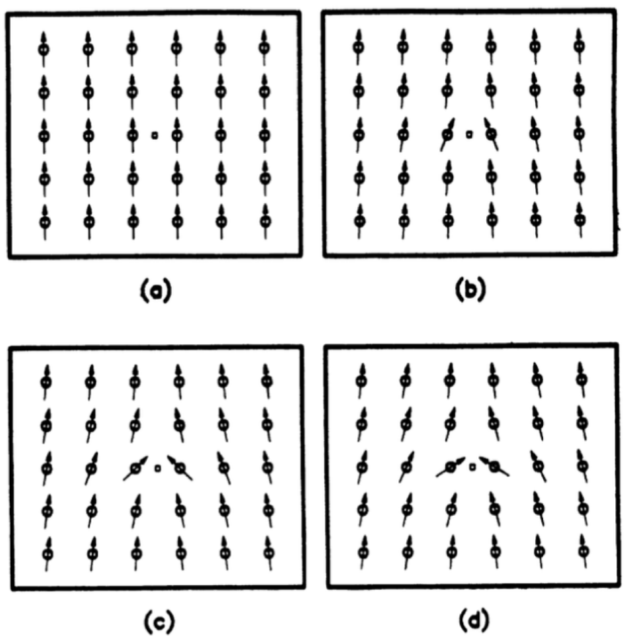


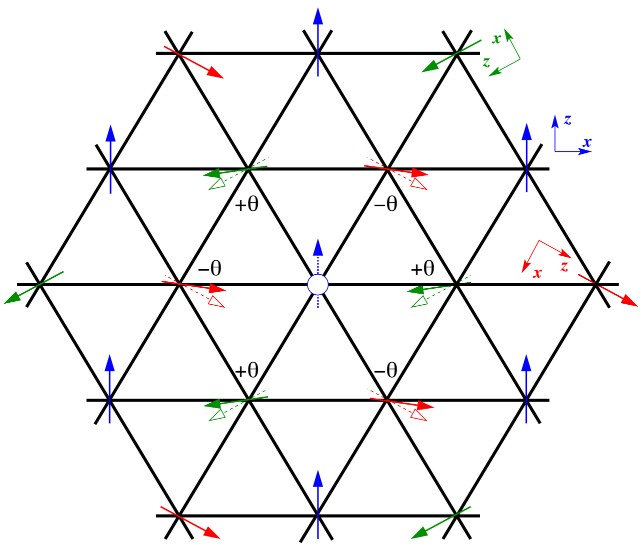
FIG. 7. Spin configurations for a single negative bond J' (near J) and various values of J': (a) $-1.0J$; (b) $-1.1J$; (c) $-1.5J$; (d) $-2.0J$.

dipolar strain field

$$m^x(\mathbf{r}) = \frac{\vec{r}\cdot\vec{\mu}}{r^D} \sim \frac{1}{r^{D-1}}$$

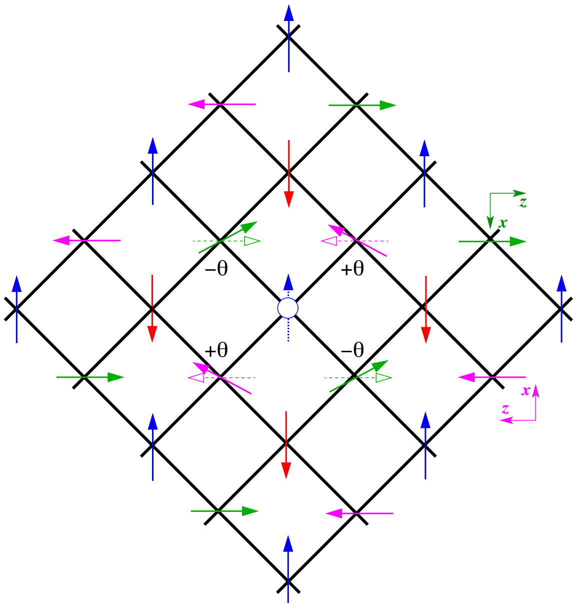
Lattice multipoles from point impurities

- Triangular AF: octupole



$$\delta\theta(\mathbf{r}) \sim 1/r^3$$

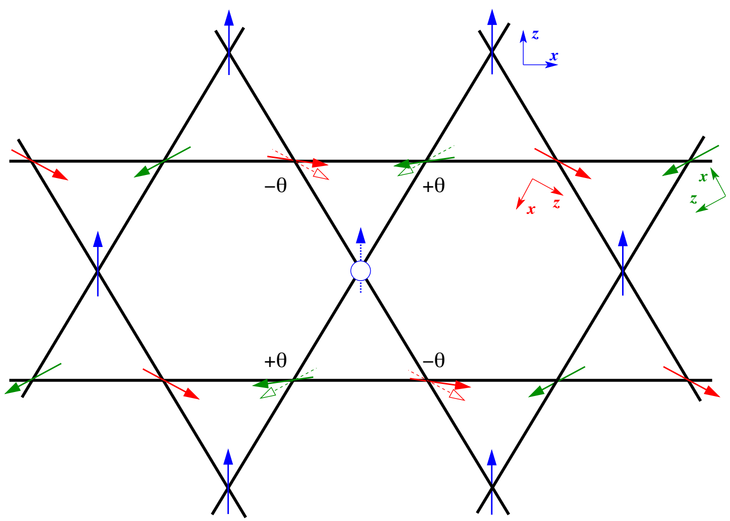
- Frustrated SAF: quadrupole



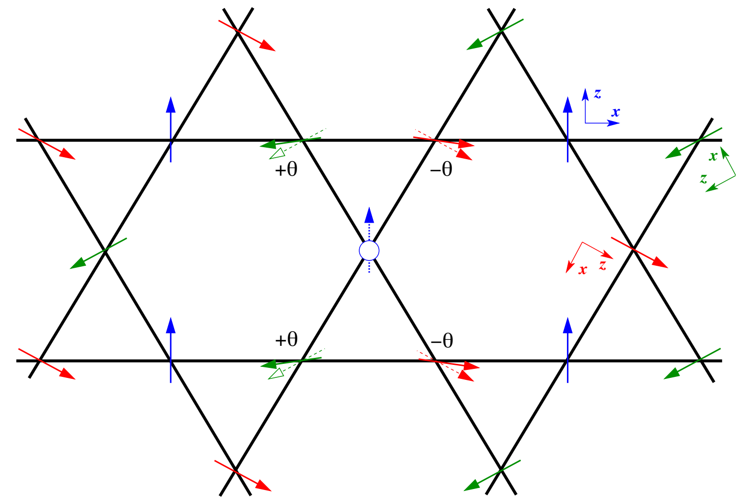
$$\delta\theta(\mathbf{r}) \sim 1/r^2$$

Lattice multipoles from point impurities

- $q = 0$ Kagome AF: quadrupole
- $\sqrt{3} \times \sqrt{3}$ Kagome AF: dipole



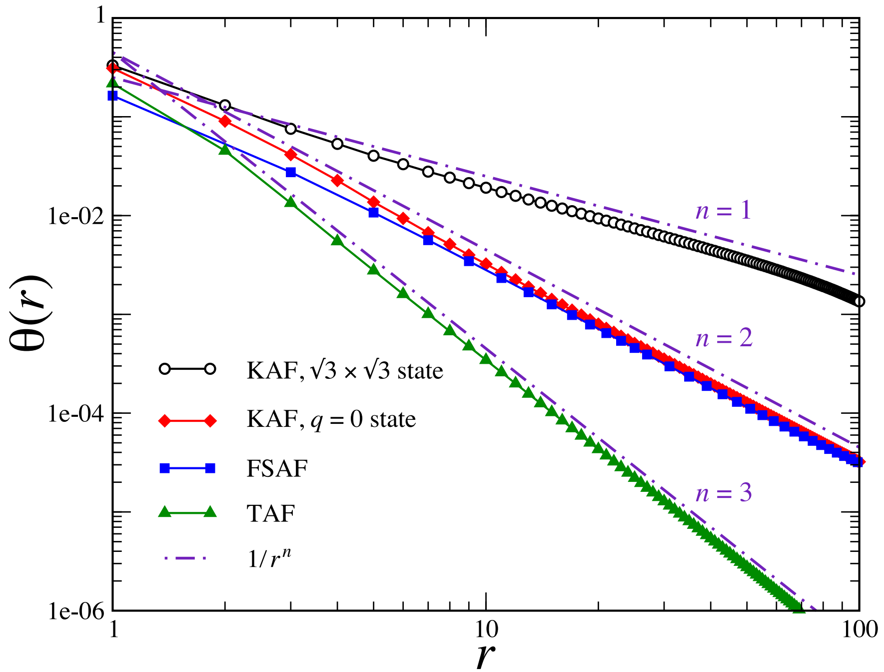
$\delta\theta(\mathbf{r}) \sim 1/r^2$



$\delta\theta(\mathbf{r}) \sim 1/r$

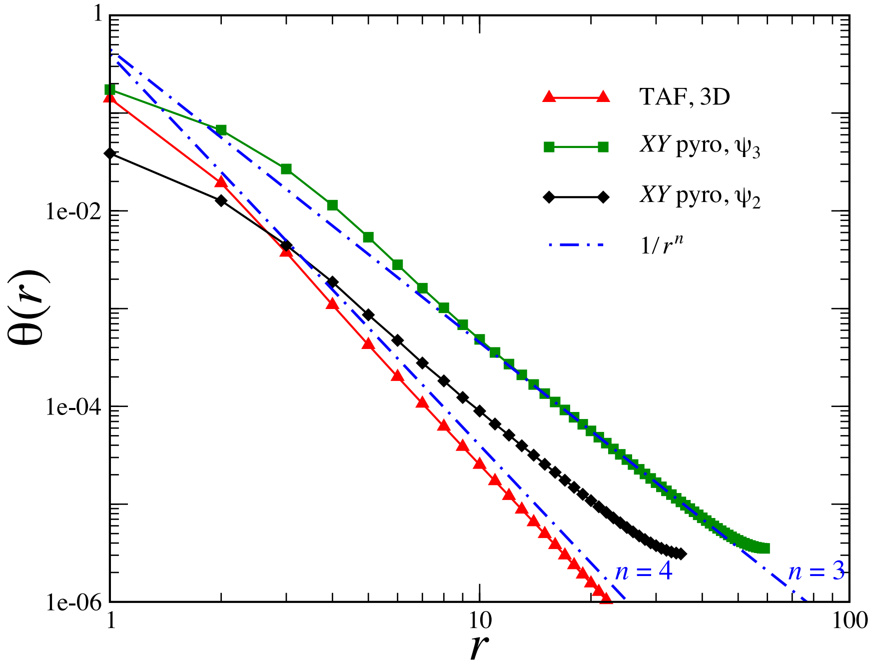
$$\delta\theta^{(m)}(\mathbf{r}) \sim 1/r^{D+m-2}$$

Strain fields in 2D



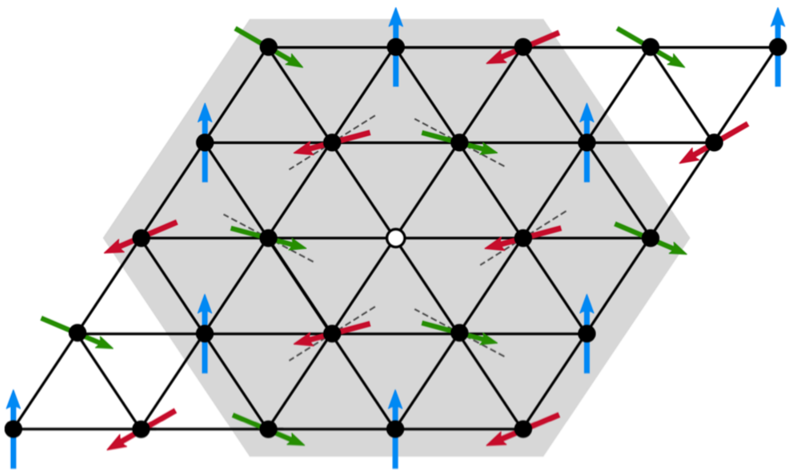
- Classical version of the Anderson's principle: Excitation spectrum (low-energy response) depends on
  (i) symmetry of the Hamiltonian AND
  (ii) broken symmetry in the ground state

Strain fields in 3D



- Impact on thermal transport
- Artificially decorated frustrated lattices (e.g. cold gases)

Order from structural disorder



- Lifting degeneracy between classical ground states
- Perturbation expansion in real space: few nearest-neighbor shells

○ Quenched disorder

$$\Delta E = -\frac{\delta J^2}{2H_{loc}} \sum_{\langle ij \rangle} \sin^2 \theta_{ij} \simeq +(\mathbf{S}_i \cdot \mathbf{S}_j)^2$$

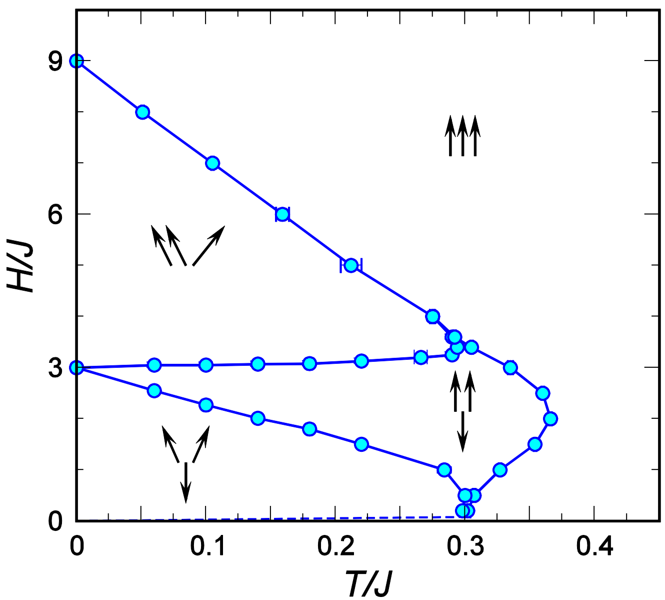
○ Quantum (thermal) fluctuations

$$\Delta E = -\frac{(JS)^2}{8H_{loc}} \sum_{\langle ij \rangle} \cos^2 \theta_{ij} \simeq -(\mathbf{S}_i \cdot \mathbf{S}_j)^2$$

- quenched disorder favors the least collinear states

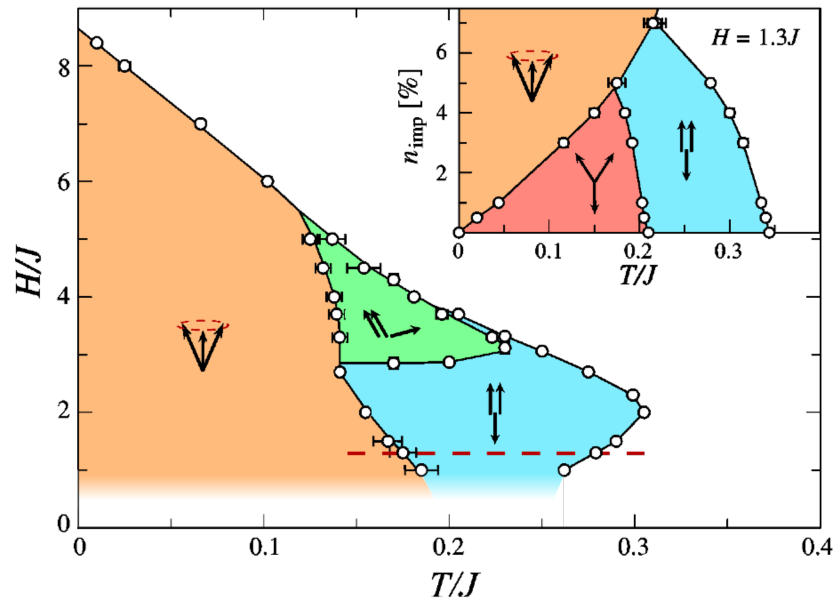
Order by Disorder in Classical TAF

- Pure TAF, no defects



Kawamura & Miyashita (1984)
Gvozdikova *et al* (2011)

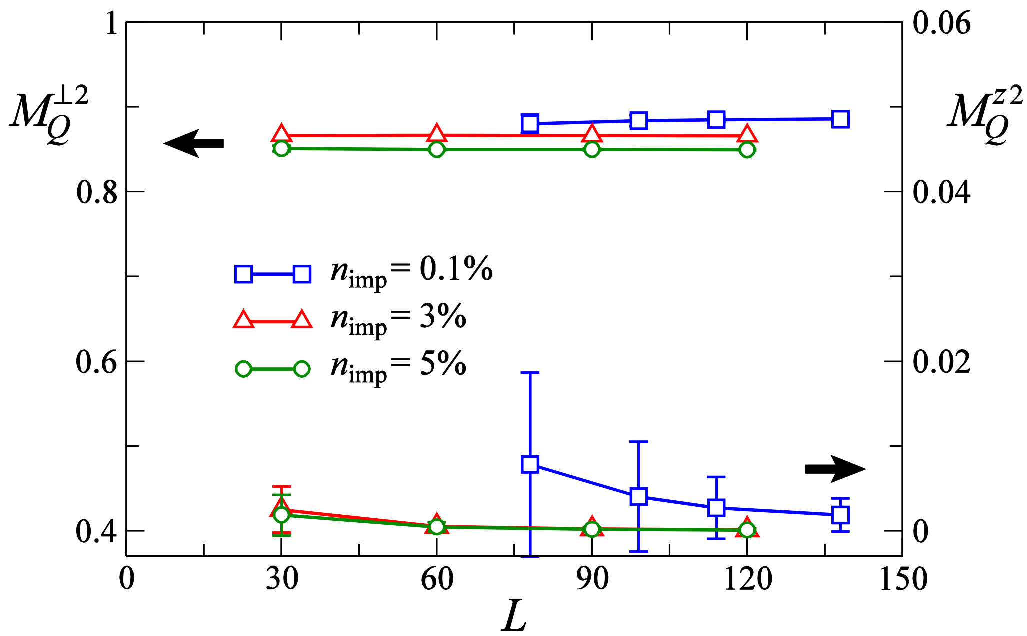
- Diluted TAF, $n_{imp} = 5\%$ vacancies



Maryasin & MZ (2013)

True Long-Range Order at $T = 0$?

- diluted triangular AF at $H = \frac{1}{3} H_s$



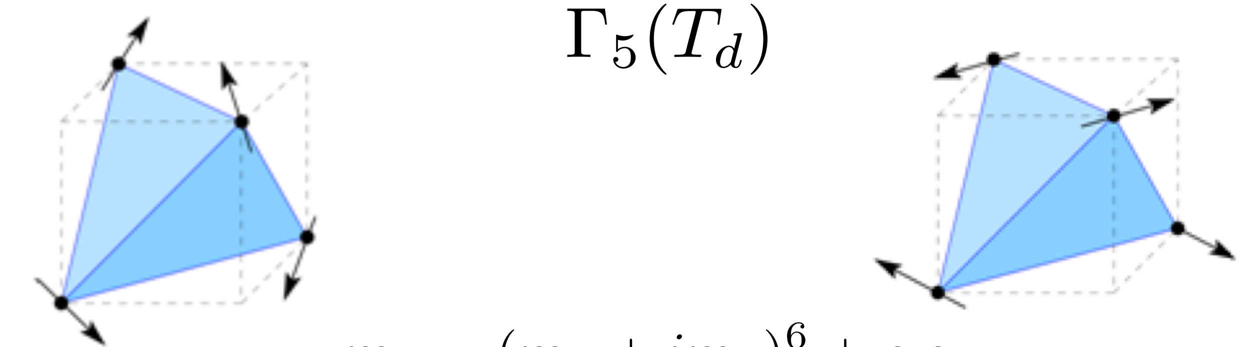
$$\mathbf{M_Q} = \sum_i \langle \mathbf{S}_i \rangle_c e^{i\mathbf{Q}\cdot\mathbf{r}_i}$$

conical $M_Q^{\perp} \neq 0$ versus collinear $M_Q^{z} \neq 0$ ordering

XY pyrochlore $Er_2Ti_2O_7$

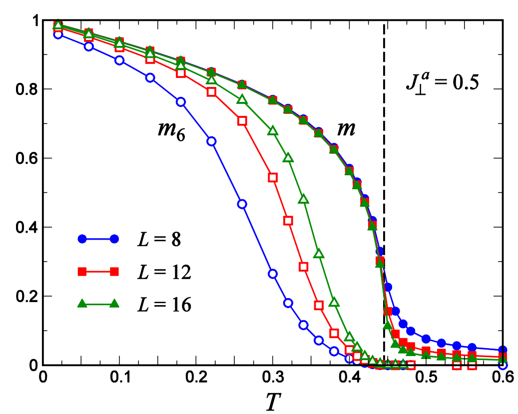
- $q = 0$ noncoplanar antiferromagnetic state in $\mathrm{Er_2Ti_2O_7}$

$\boxed{\psi_2}$ $\Gamma_5(T_d)$ $\boxed{\psi_3}$

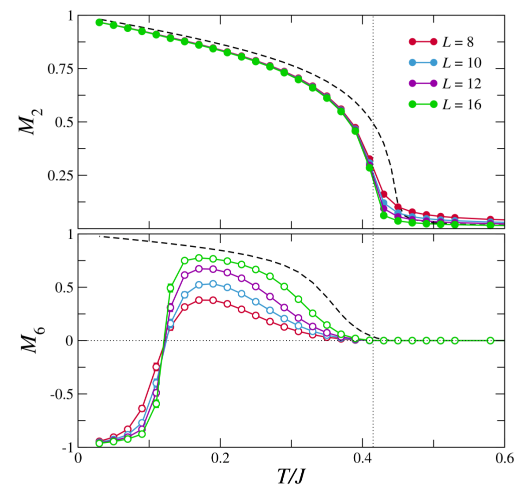


$$m_6 \sim (m_x + i m_y)^6 + c.c.$$

- Pure system, thermal selection



- Nonmagnetic dilution

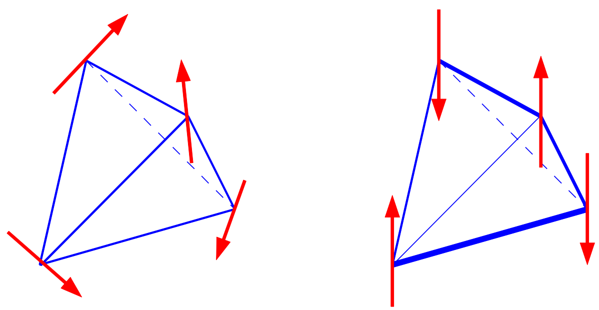


Frustration and Glassiness

- spin glass freezing occurs in many frustrated materials
- intrinsic, disorder-free glassiness
- extrinsic, a little bit of disorder $\longrightarrow$ a little glassiness;

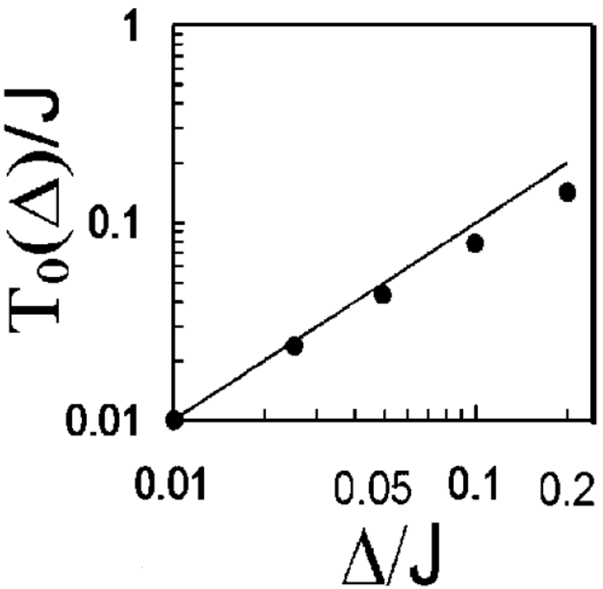
  ex.: weak bond disorder in pyrochlore AF

  - lifts huge degeneracy
  - leads to conventional spin freezing



$$\sum_i \mathbf{S}_i = 0$$

Bellier-Castella *et al* (2001)
Saunders & Chalker (2007)
Andreanov *et al* (2010)



Bond-disordered Kagome AF

- weakly fluctuating exchanges couplings for n.n. bonds

$$\hat{H} = \sum_{<ij>} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j, \quad J_{ij} = 1 + \delta_{ij}, \qquad < \delta_{ij} > = 0$$

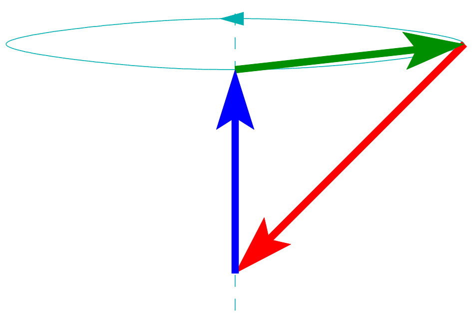
- single triangle

$$J_{12}\mathbf{S}_1 \cdot \mathbf{S}_2 + J_{13}\mathbf{S}_1 \cdot \mathbf{S}_3 + J_{23}\mathbf{S}_2 \cdot \mathbf{S}_3 = \left(\sqrt{\frac{J_{12}J_{13}}{J_{23}}} \mathbf{S}_1 + \ldots \right)^2 + \text{const}$$

- the ground-state constraint

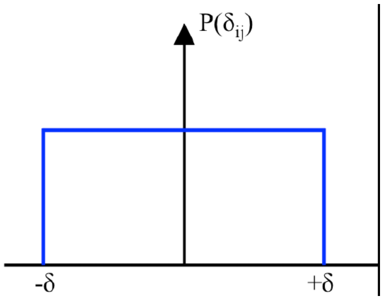
$$\mathbf{L}_\alpha = \sum_i \gamma_{i\alpha} \mathbf{S}_i = 0$$

$$\gamma_{i\alpha} = \sqrt{J_{ij}J_{ik} / J_{jk}}$$



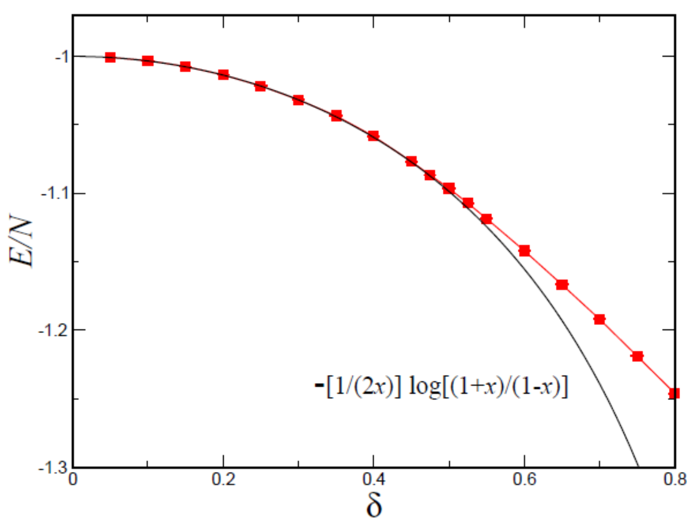
Ground-states construction: numerical

- specific realization: box distribution $J_{ij} = 1 + \delta_{ij}$

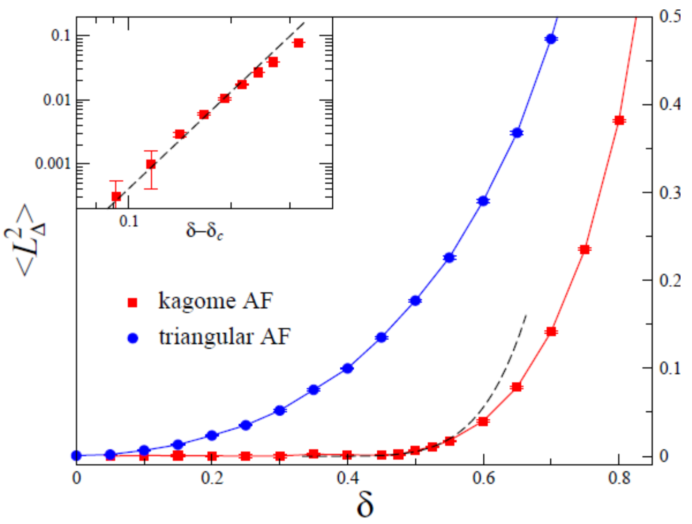


- Monte Carlo search for the lowest energy spin configuration

$$E = \sum_{\triangle} \left[\frac{1}{2} L_{\triangle}^2 - \frac{3}{2} \left\langle \frac{1}{J_{ik}} \right\rangle \right]$$

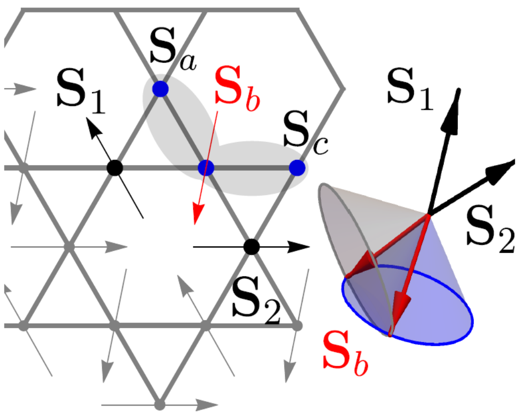


➢ constraint satisfaction until $\delta_c = 1/3$

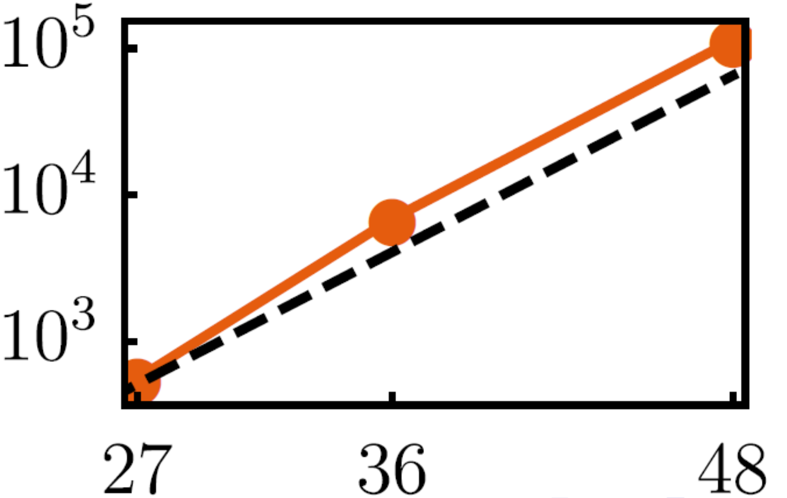


Ground-state construction: analytical

- `transfer matrix' construction
  - two choices at each step
  - total number of degenerate states
    $$N_{\mathrm{st}} = 2^{N/3} \sim 1.2599^{\,N}$$
  - infinite discrete degeneracy with continuous spins



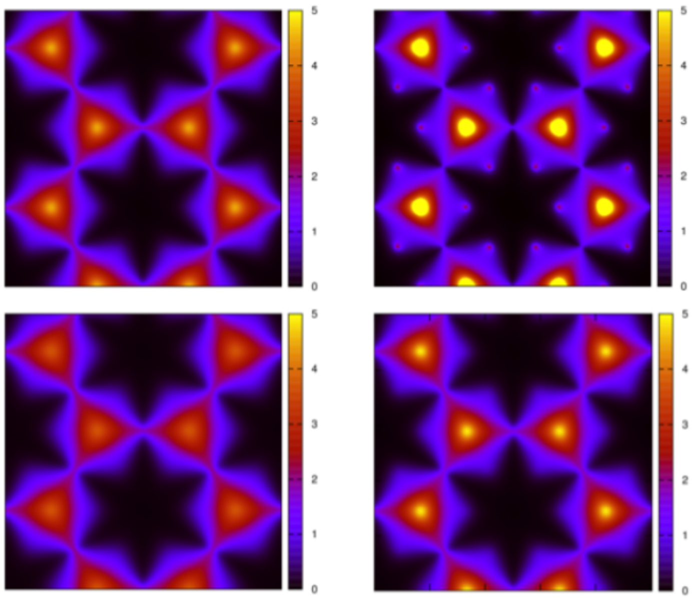
- numerical counting



Ground-state spin correlations

- at zero T, the BD kagome AF behaves as a normal kagome AF at $T^* \sim \delta^2$

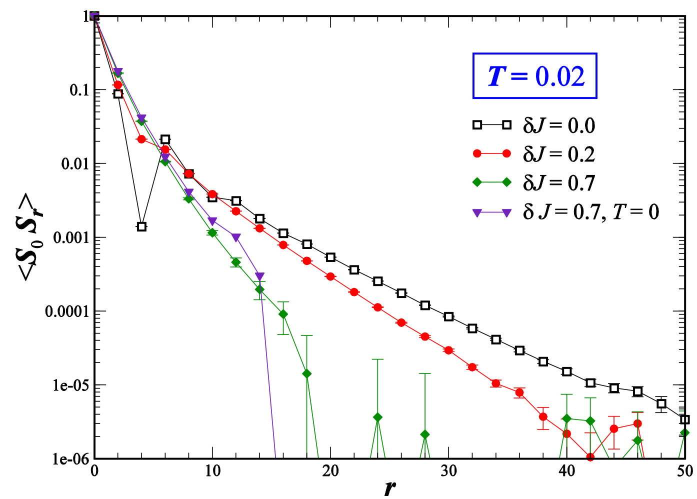
$\delta = 0.2$ $\delta = 0$



$T = 0.005$

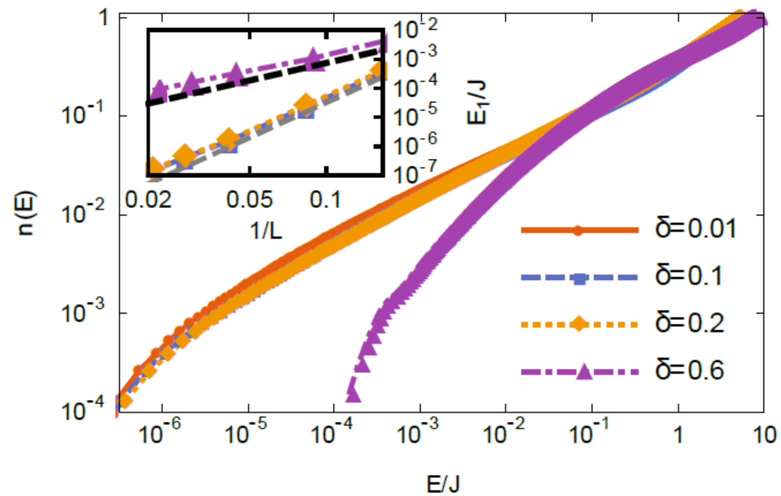
$T = 0.02$

- ➢ algebraic Coulomb correlations are screened by $\xi \sim 1/\delta$

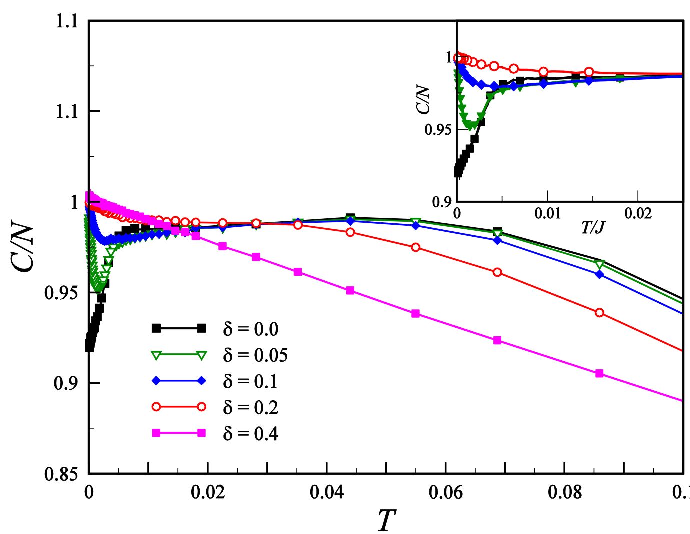


Low-energy excitations

- NO zero-energy modes



- NO order by disorder

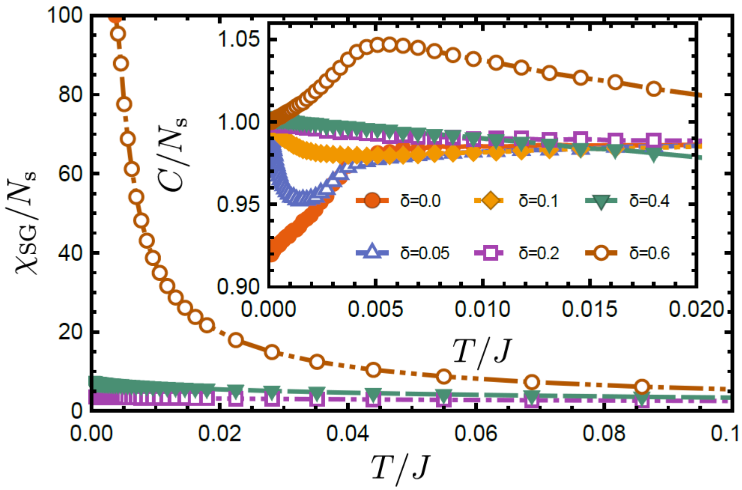


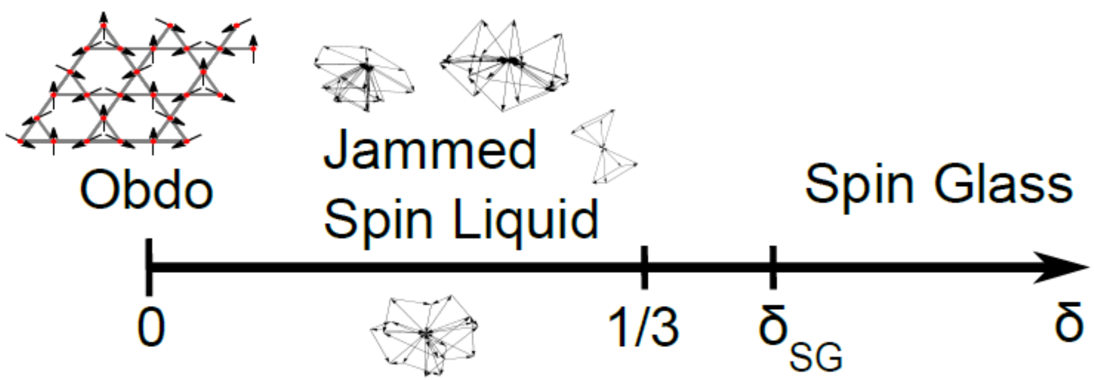
Phase diagram vs disorder strength

- $T = 0$ spin-glass state for strong disorder (large δ > 1/3)

$$Q_{\mathrm{SG}}^{\alpha\beta} = \frac{1}{N}\sum_i S_i^{\alpha(1)} S_i^{\beta(2)}$$

$$\chi_{\mathrm{SG}} = N\sum_{\alpha\beta}\left\langle\left\langle \left|Q_{\mathrm{SG}}^{\alpha\beta}\right|^2 \right\rangle\right\rangle$$





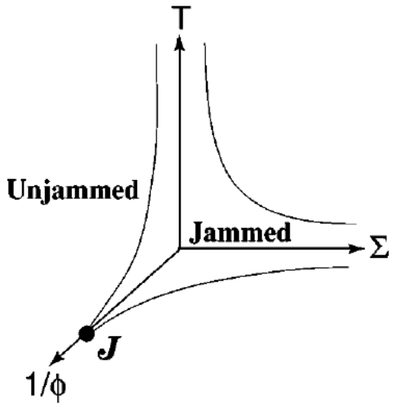
Jamming in Physics

- process by which viscosity of a granular material or a complex fluid increases with increasing particle density

- crowding of the constituent particles prevents them from exploring the full phase space, making a liquid behaving as a solid: finite yield stress

- at the jamming threshold, “random” close packings are *isostatic*, producing marginal or Maxwellian solids:

  \# of the degrees of freedom = # of constraints



Summary

- Quenched disorder produces opposite selection of ground states compared to thermal/quantum fluctuations (always?)

- Controlled impurity doping: multiferroics, thermal transport etc.

- Bond disordered kagome AF remains infinitely degenerate having

  (i) no zero modes

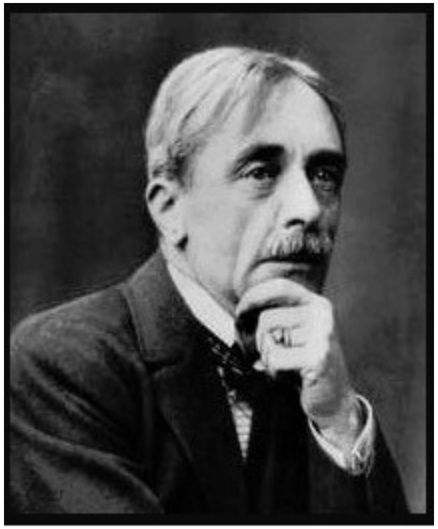
  (ii) no spin-glass correlations

  jammed spin liquid

- open questions:

  - ground-state entropy variations with δ
  - distinction between $\delta < \delta_c$ and $\delta_c < \delta < \delta_{\mathrm{SG}}$ states: flux-flux correlations

Outlook



"Two dangers constantly threaten the world: order and disorder"

(Paul Valery)