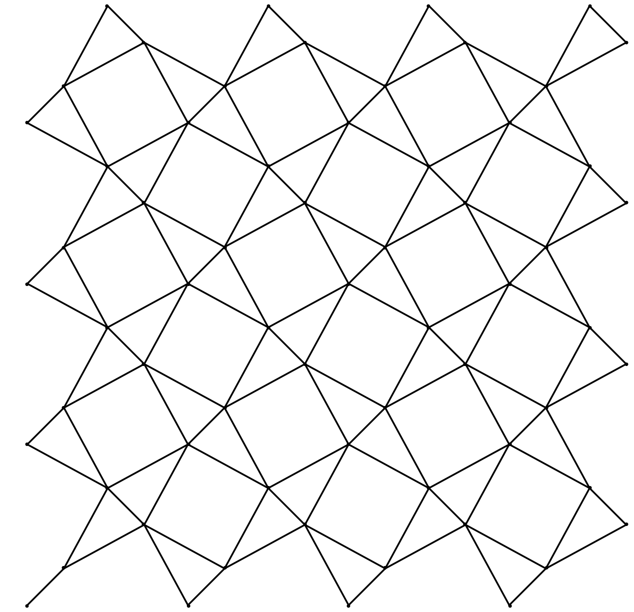


Topological Magnons and the Non-Hermitian Topology of Spontaneous Magnon Decay

Paul McClarty
Max Planck Institute for the Physics of Complex Systems, Dresden

PCS-IBS Frustrated Magnetism Workshop, October 18th 2019, Daejeon

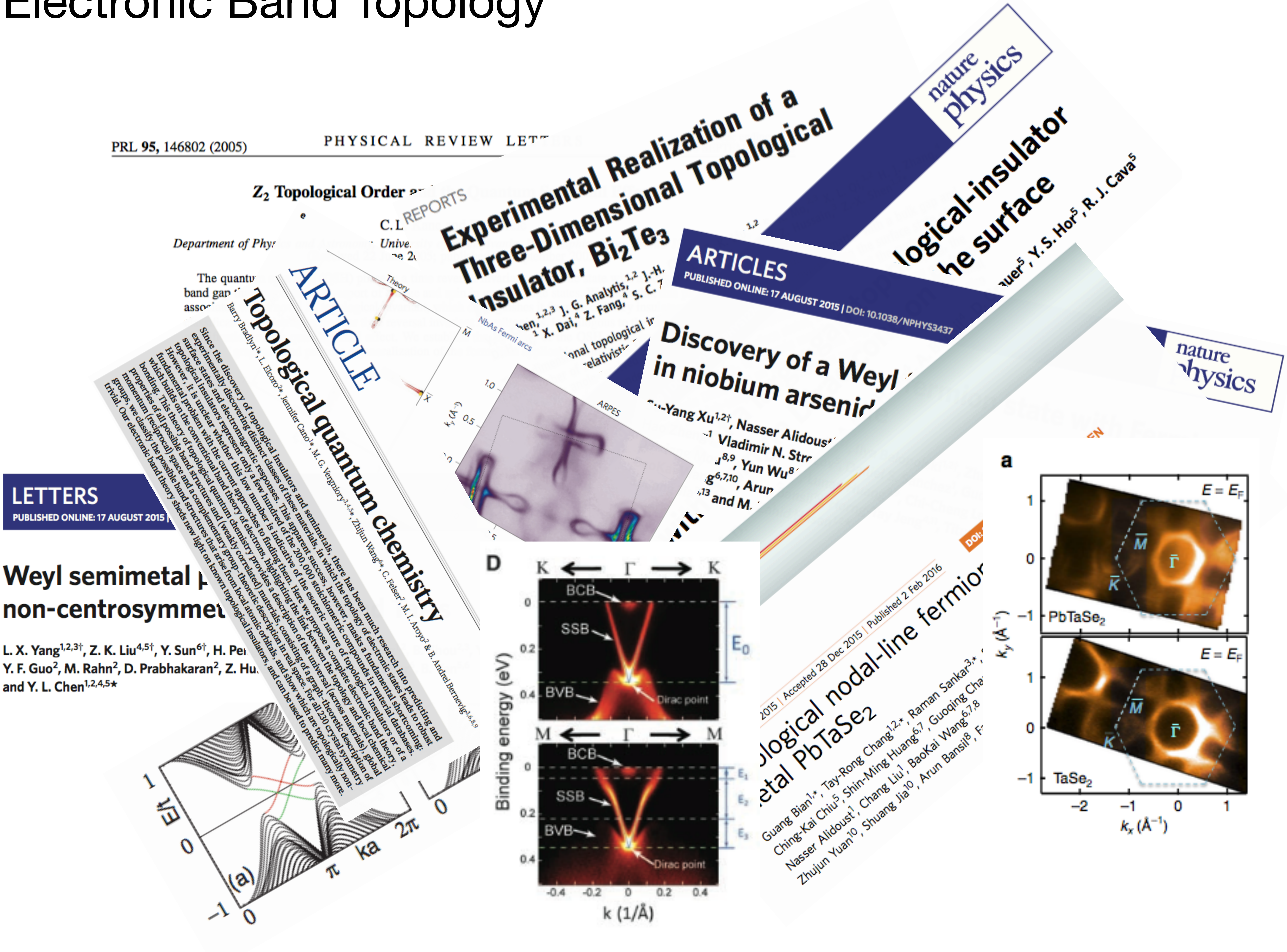
Motivating Questions

Can we achieve further understanding of magnon neutron cross section?

Intensities, linewidths.

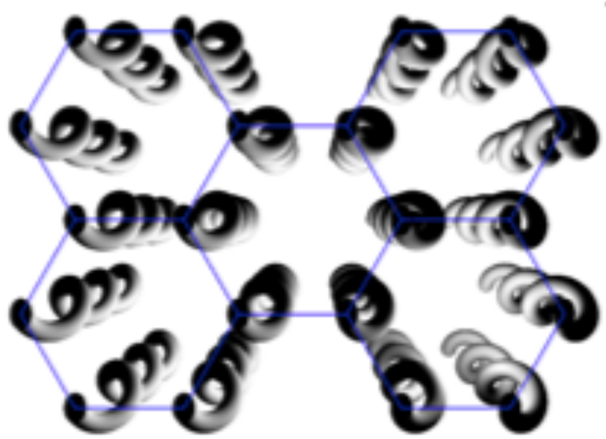
What are the effects of magnon interactions on topological magnon band structures?

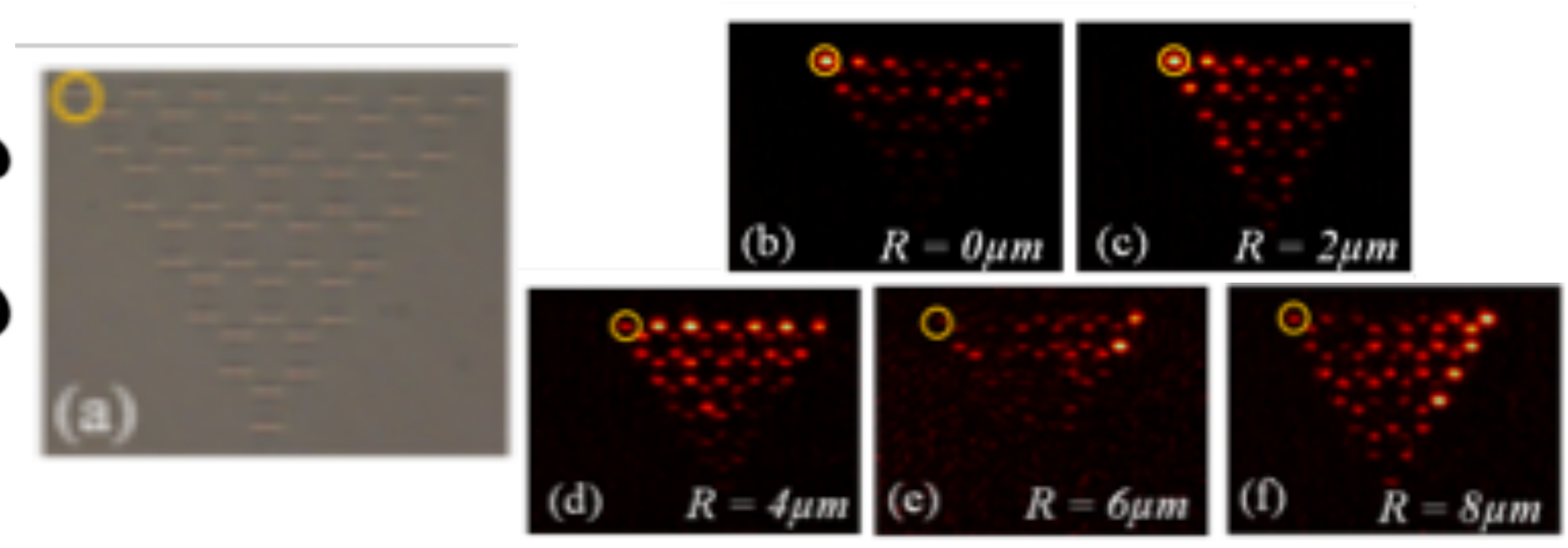
Electronic Band Topology



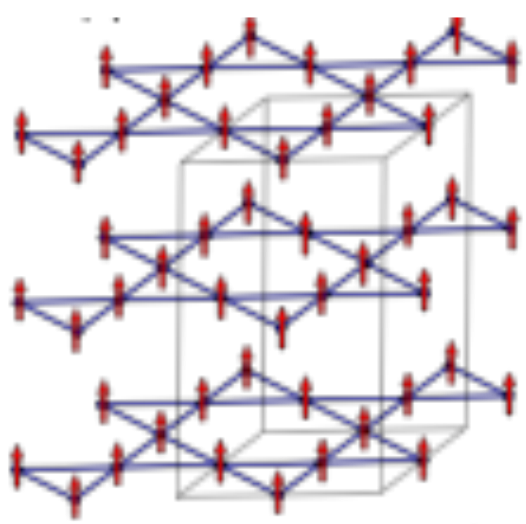
Photonic floquet topological insulator

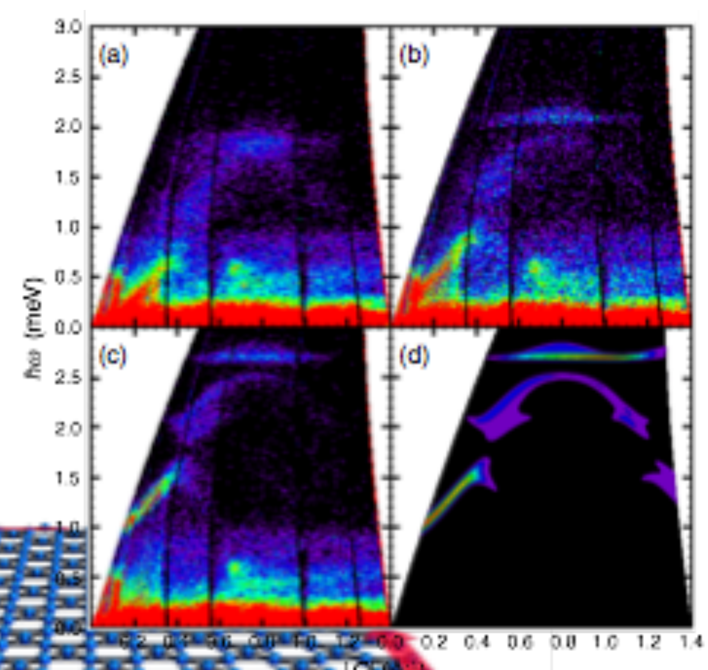
Rechtsman et al.
Nature (2013)



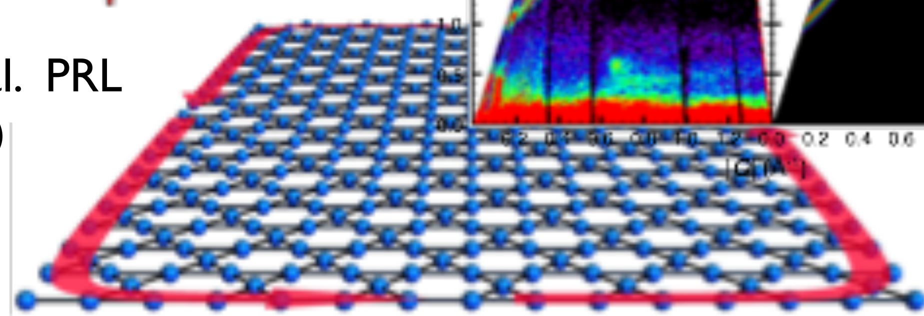


Kagome ferromagnet with DM interactions





Chisnell et al. PRL (2015)



Cu-1,3-benzenedicarboxylate

Bosonic Band Topology

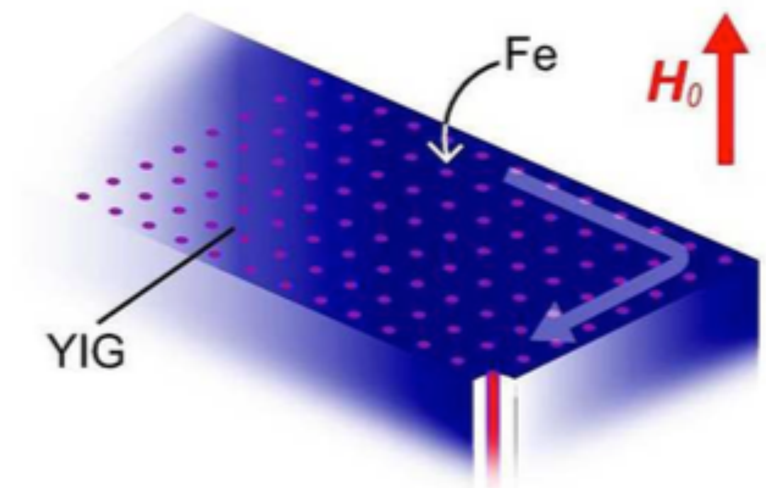
Selected Examples of Topological Magnons

Magnon Analogue of Haldane Model

Honeycomb ferromagnet with DM exchange Owerre (2016)

Dipolar Micromagnetic Array

Shindou, Matsumoto, Murakami, Ohe (2012)



Kitaev Magnets

PAM, X Y Dong, M Gohlke, F Pollmann, R Moessner, K Penc (2018)

Dimer magnets

Shastry-Sutherland Romhanyi, Ganesh, Penc (2015)

PAM et al. (2017)

Dirac/Nodal Line Magnons

Honeycomb Pershoguba (2018)

Cu_3TeO_6 Li et al. (2017)

Weyl Magnons

Breathing Pyrochlore Li, Li, Kim, Balents, Yu, Chen (2016)

AIAO Pyrochlore Jian, Nie (2018)

Differences between Magnon and Electronic Chern Insulators

Generally no quantized response

Interactions generically present and may be crucial

Magnon-Magnon Interactions

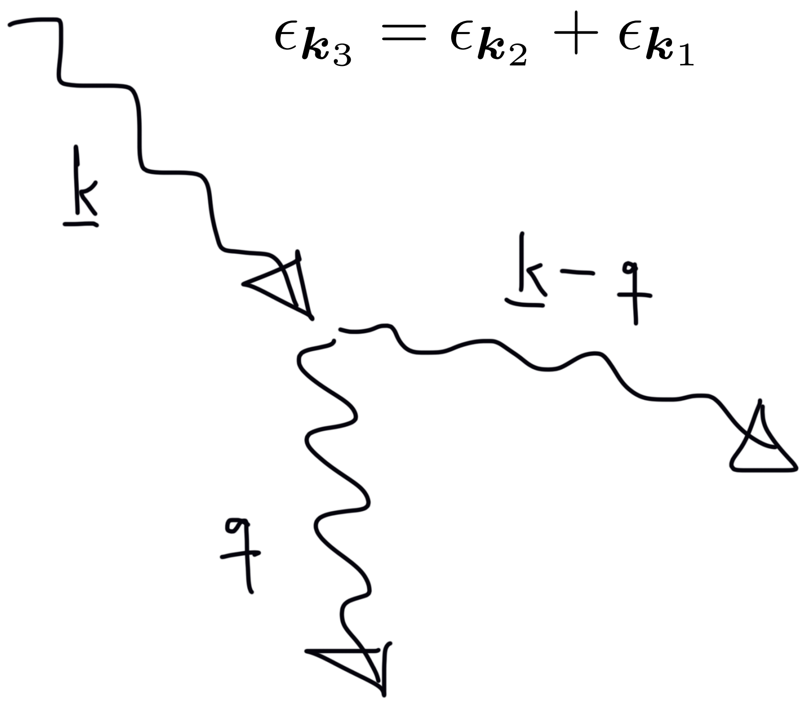
Magnon-magnon interactions from Holstein-Primakoff beyond 1/S

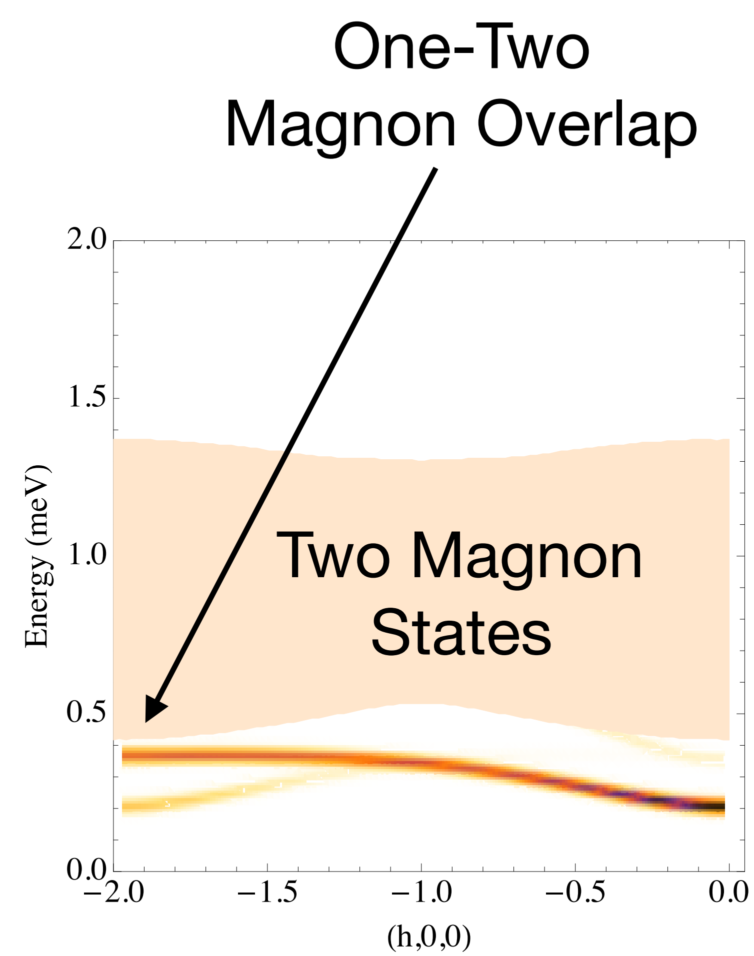
$$\mathcal{H}_3 = \frac{1}{2}\sum_{\boldsymbol{k}_\mu} V_3(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)(a^\dagger_{\boldsymbol{k}_1} a^\dagger_{\boldsymbol{k}_2} a_{\boldsymbol{k}_3} + \text{h.c.}) + \dots$$

Generally number non-conserving terms

Single particle picture may not survive in any detail

$$\epsilon_{\boldsymbol{k}_3} = \epsilon_{\boldsymbol{k}_2} + \epsilon_{\boldsymbol{k}_1}$$





Decay Kinematically Allowed

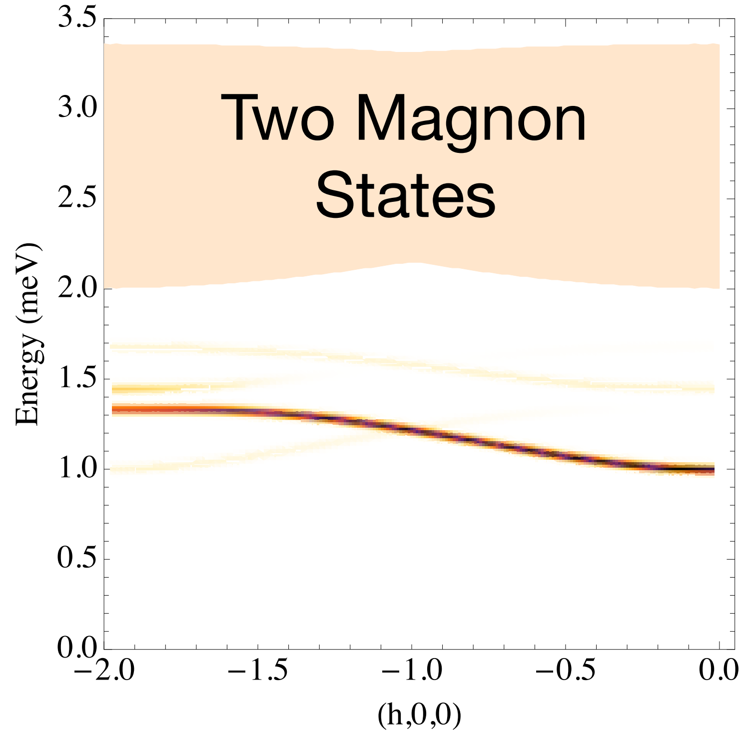
Magnon edge states are nonuniversal

But can they be experimentally robust quasiparticles?

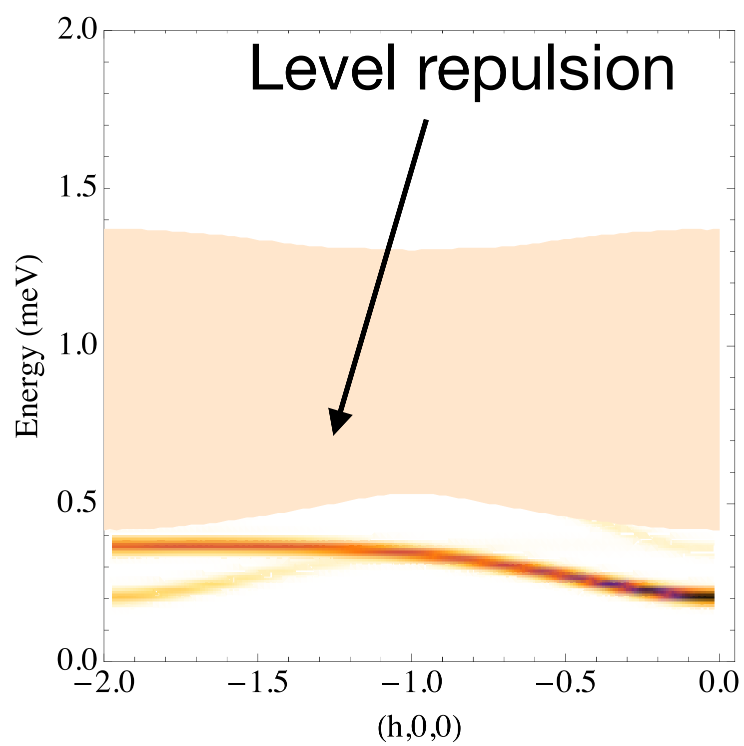
Chernyshev, Zhitomirsky (2013)

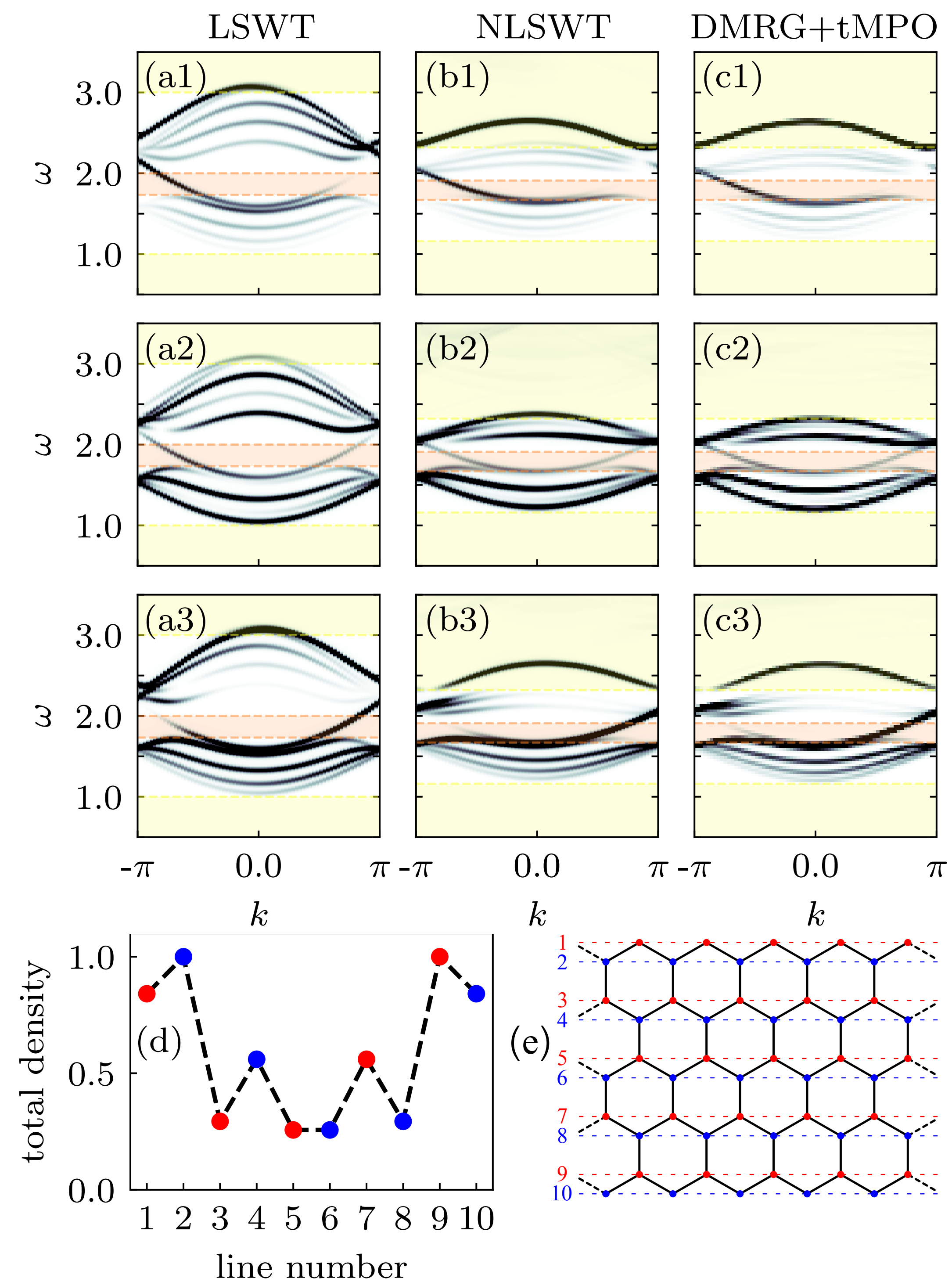
Magnon-Magnon Interactions

High Field



Low Field





DMRG calculation of magnon edge states

Surface states originating from magnon Chern bands can be experimentally robust states

PAM, X-Y Dong, M Gohlke, F Pollmann, R Moessner, K Penc PRB(R) 98, 060404 (2018)

Magnon Interactions in Topological Magnon Systems

The case of linear touching points

Work in collaboration with Jeff Rau
MPI PKS → University of Windsor, ON

PAM and J G Rau, PRB(R) 100, 100405 (2019)

Non-Hermitian Hamiltonian from Magnon Interactions

Strategy: focus on single magnon states and effectively trace over multi-magnon states

Expect effective non-Hermitian Hamiltonian for single magnons

$$H_{\text{eff}}(\boldsymbol{k},\omega) = H(\boldsymbol{k}) + \Sigma(\boldsymbol{k},\omega)$$

Effect of interactions encoded in self-energy defined from retarded Green's function

$$\boldsymbol{G}(\boldsymbol{k},\omega) \equiv \left[(\omega + i0^+)\boldsymbol{\sigma}_3 - (\boldsymbol{M}_{\boldsymbol{k}} + \boldsymbol{\Sigma}(\boldsymbol{k},\omega))\right]^{-1}.$$

Suppose self-energy is small so that single magnons have *quasi-normal modes*

$$\det\left((E_{\boldsymbol{k}} - i\Gamma_{\boldsymbol{k}} + i0^+)\boldsymbol{\sigma}_3 - [\boldsymbol{M}_{\boldsymbol{k}} + \boldsymbol{\Sigma}(\boldsymbol{k}, E_{\boldsymbol{k}} + i\Gamma_{\boldsymbol{k}})]\right) = 0$$

defined through poles of retarded Green's function.

$$E_{\boldsymbol{k}\alpha} - i\Gamma_{\boldsymbol{k}\alpha} = \epsilon_{\boldsymbol{k}\alpha} + \boldsymbol{V}^\dagger_{\boldsymbol{k}\alpha}\boldsymbol{\sigma}_3\boldsymbol{\Sigma}(\boldsymbol{k},\epsilon_{\boldsymbol{k}\alpha})\boldsymbol{V}_{\boldsymbol{k}\alpha}$$

Now consider touching point - local in energy and momentum and expand self-energy around this point

$$\boldsymbol{\Sigma}(\boldsymbol{k},\omega) \approx \boldsymbol{\Sigma}(\boldsymbol{k},\omega_0) + (\omega - \omega_0)\partial_\omega\boldsymbol{\Sigma}(\boldsymbol{k},\omega_0) + \cdots$$

Effective non-Hermitian Hamiltonian

Neglecting energy dependence of the self-energy arrive at non-Hermitian Hamiltonian

$$\boxed{\boldsymbol{M}_{\boldsymbol{k}}^{\mathrm{eff}} \equiv \left[\boldsymbol{M}_{\boldsymbol{k}} + \boldsymbol{\Sigma}'(\boldsymbol{k}, \omega_0)\right] + \boldsymbol{\Sigma}''(\boldsymbol{k}, \omega_0)}$$

Constraint from causality (no driving): imaginary parts of quasi-normal modes negative

Single magnons treated within non-Hermitian effective theory have renormalized band structure and finite lifetime

Picture can break down for strong coupling and around van Hove singularities in multi-magnon density of states

Non-Hermitian Topology

Focus on universal physics in vicinity of two-band touching

$$H_{\mathrm{NH}} = v\left(k_x \sigma_x + k_y \sigma_y\right) - i\left(a_0 + \boldsymbol{a} \cdot \boldsymbol{\sigma}\right)$$

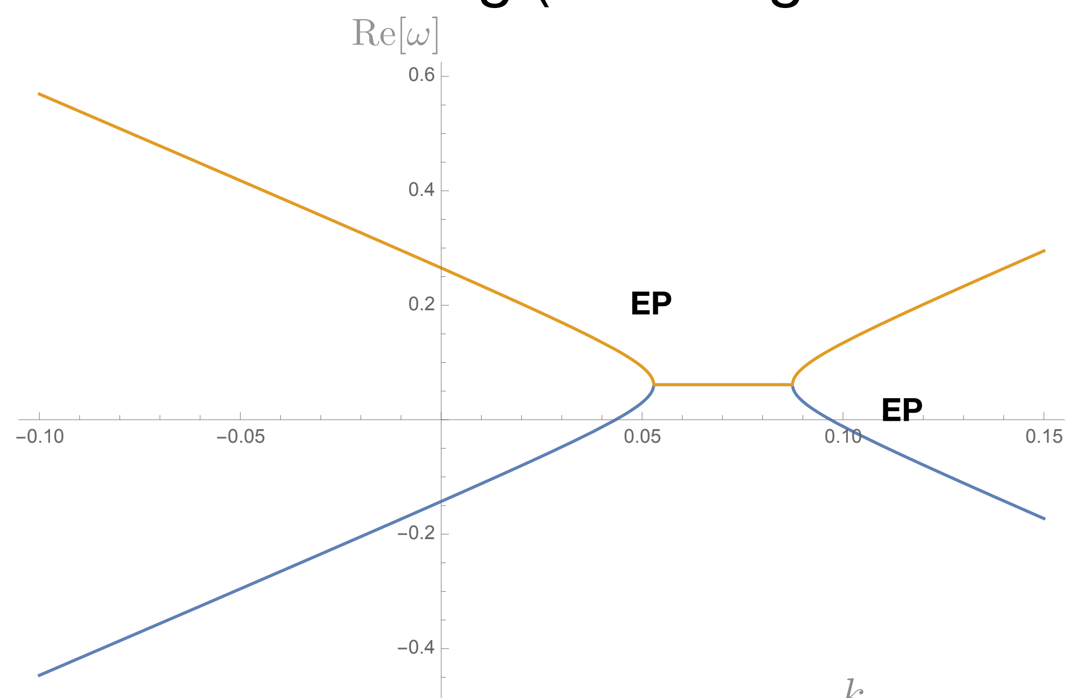
Four parameter non-hermitian part: a_0 constant damping

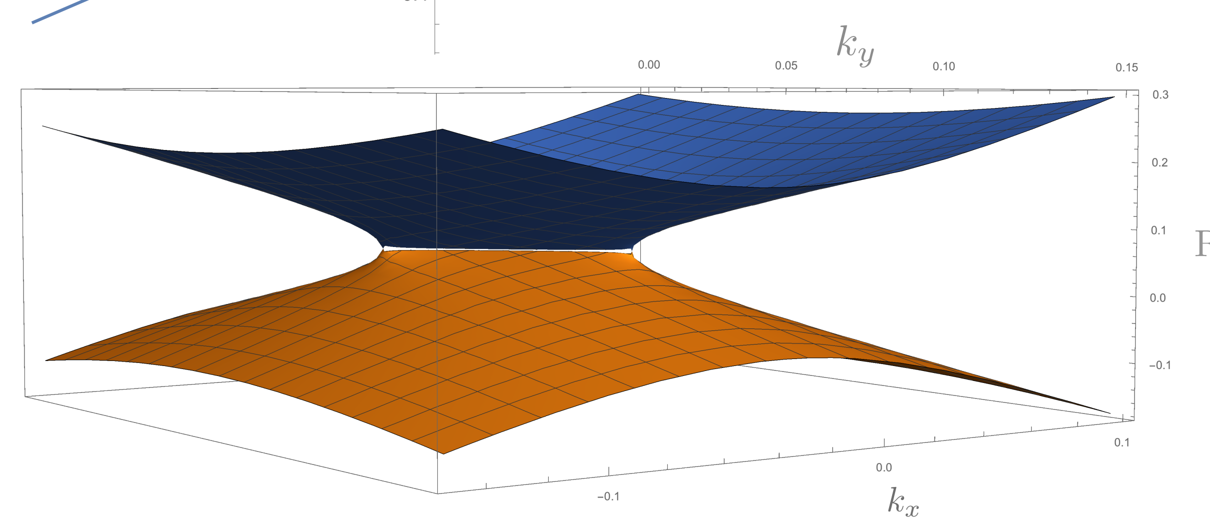
Vector part leads to splitting of touching point into line connecting (non-diagonalizable) exceptional points.

Length $2|\boldsymbol{a}|/v$

Arc perpendicular to $\boldsymbol{a}_\perp \equiv (a_x, a_y)$

Dispersion about EP $\mathrm{Re}[\omega] \sim |\boldsymbol{k}|^{1/2}$

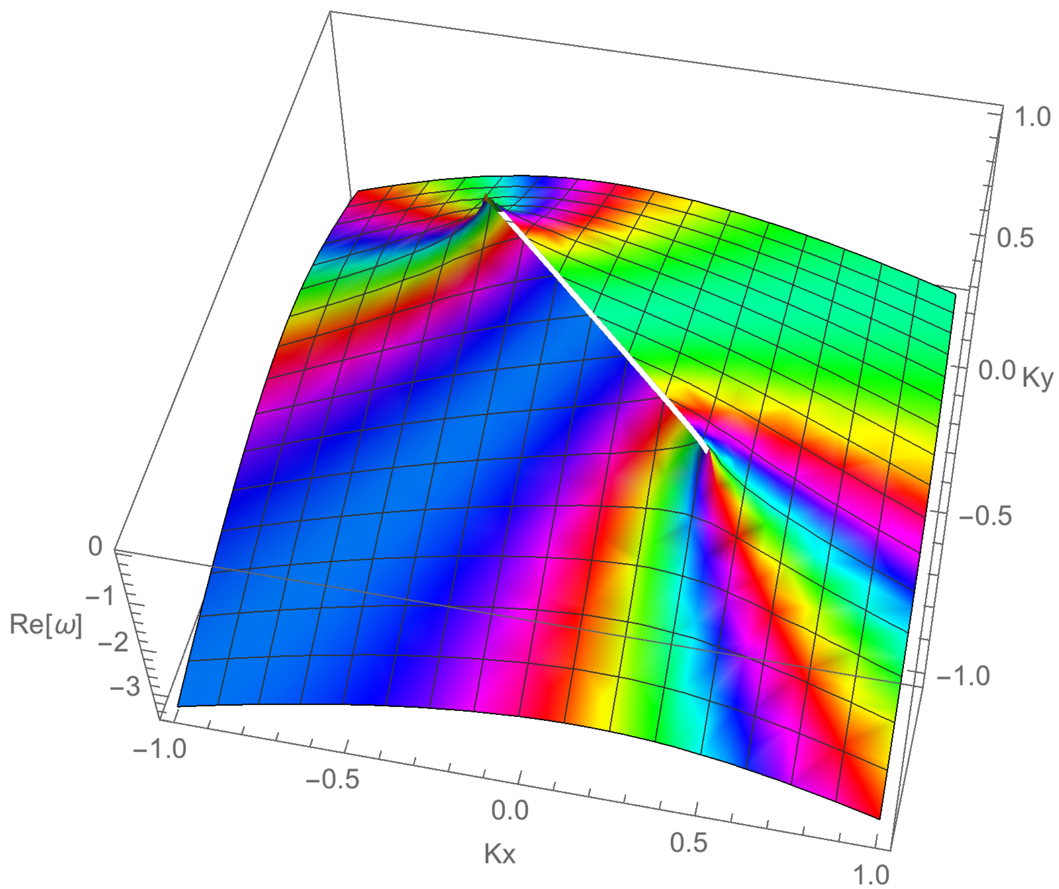




Exceptional points topologically protected - can only be annihilated in pairs

Non-Hermitian Topology II

Eigenvalues swap discontinuously passing through arc



Topological invariant

$$\nu_{nm} = -\frac{1}{2\pi} \oint_{\Gamma} \boldsymbol{\nabla}_{\boldsymbol{k}} \arg\left(E_n(\boldsymbol{k}) - E_m(\boldsymbol{k})\right) \cdot d\boldsymbol{k}$$

where loop is taken around exceptional point

$$\nu = \pm 1/2$$

Exceptional points topologically protected
- can only be annihilated in pairs

Appearance of exceptional points generic for Dirac touching points

Lifetime of magnon branch discontinuous passing through arc

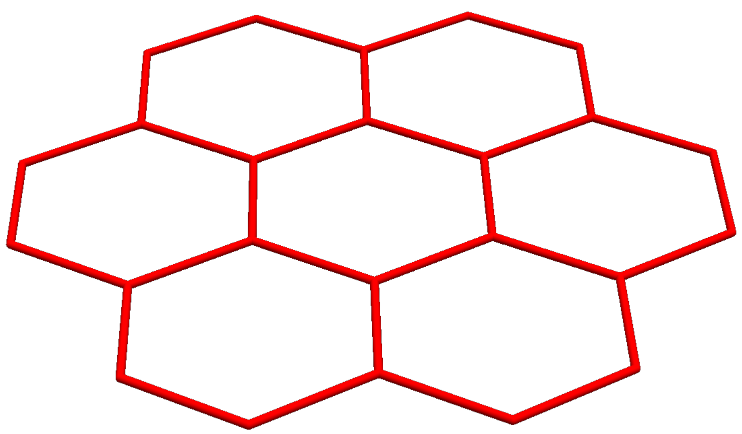
Shen, Zhen, Fu (2017)

Example: Honeycomb FM with in-plane DM

Illustration of non-Hermitian magnon topology:

$$H = -\sum_{n=1}^{3}\sum_{\langle ij\rangle_n} J_n \boldsymbol{S}_i \cdot \boldsymbol{S}_j + D\sum_{\langle ij\rangle_2}(-1)^i \hat{\boldsymbol{r}}_{ij}\cdot(\boldsymbol{S}_i \times \boldsymbol{S}_j)$$

$$J_2/J_1 = 0.2 \qquad J_3/J_1 = 0.1$$



Collinear ground state with full rotational symmetry

Put moment along x direction to preserve two-fold rotational symmetry and compute spin waves

Spectrum has symmetry protected Dirac points with energy independent of coupling D.

Dzyaloshinskii-Moriya little affects quadratic spin waves but leads to spontaneous magnon decay when interactions are considered.

Example continued…

Leading contribution to self-energy - bubble diagram

$$\Sigma^{-+}_{\alpha\beta}(\boldsymbol{k},\omega) = \frac{1}{N}\sum_{\boldsymbol{q}}\sum_{\rho\rho'} \frac{[W^{\alpha,\rho\rho'}_{\boldsymbol{k},\boldsymbol{q}}]^* W^{\beta,\rho\rho'}_{\boldsymbol{k},\boldsymbol{q}}}{\omega - \epsilon_{\boldsymbol{q}\alpha} - \epsilon_{\boldsymbol{k}-\boldsymbol{q},\beta} + i0^+},$$

For $D/J_1 = 0.125$ in vicinity of touching point

$$\Sigma^{-+}_{\alpha\beta}(K,\omega_0) \approx \frac{D^2}{J_1}\begin{pmatrix} 0.08 - 0.95i & -0.41 + 0.28i \\ -0.41 + 0.28i & 0.08 - 0.95i \end{pmatrix}_{\alpha\beta}$$

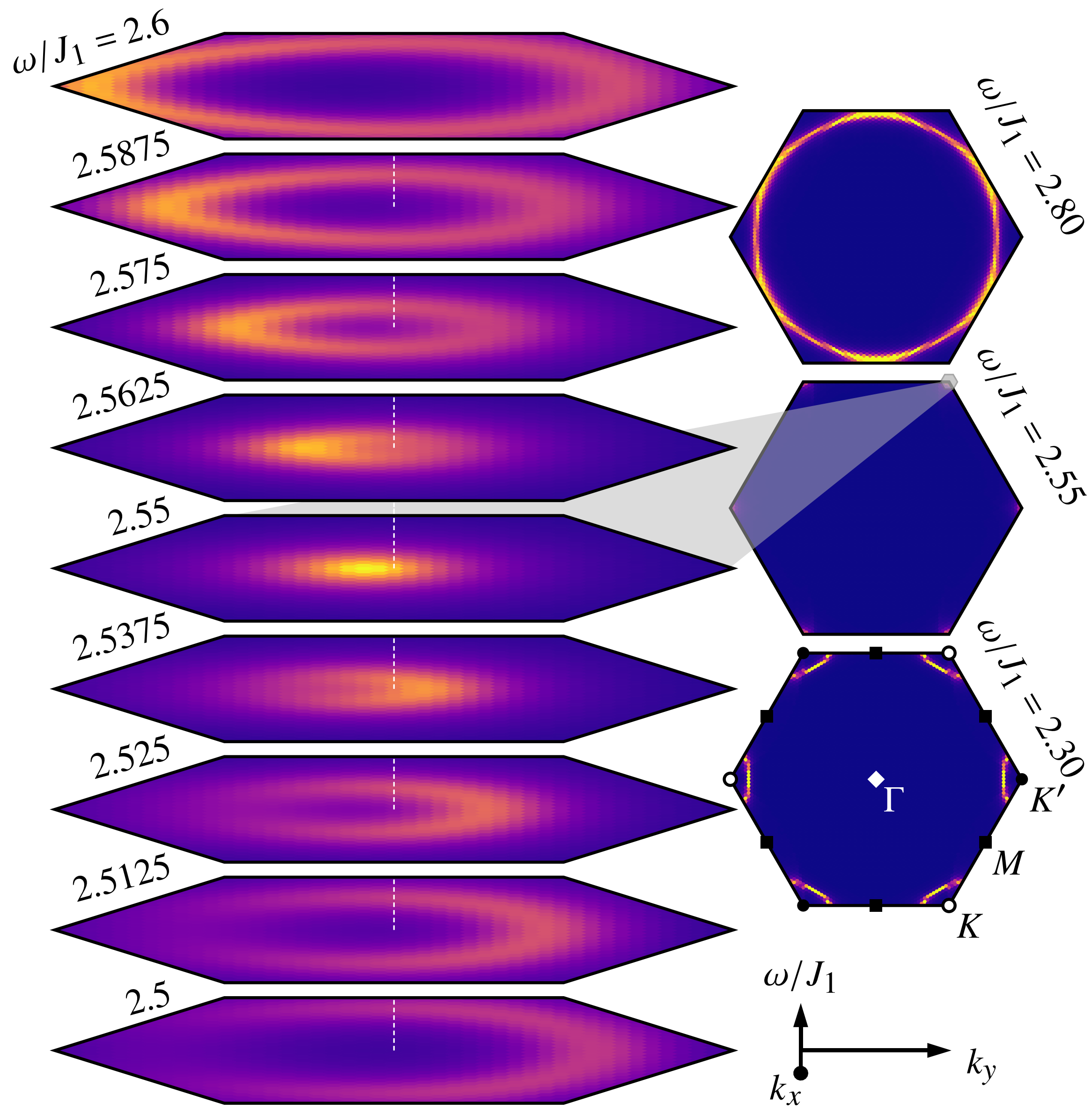
Significant off-diagonal elements relative to diagonal elements

This gives non-Hermiticity of effective single-magnon Hamiltonian

Spectral Function

Spectral function of honeycomb ferromagnet with DM on constant energy slices

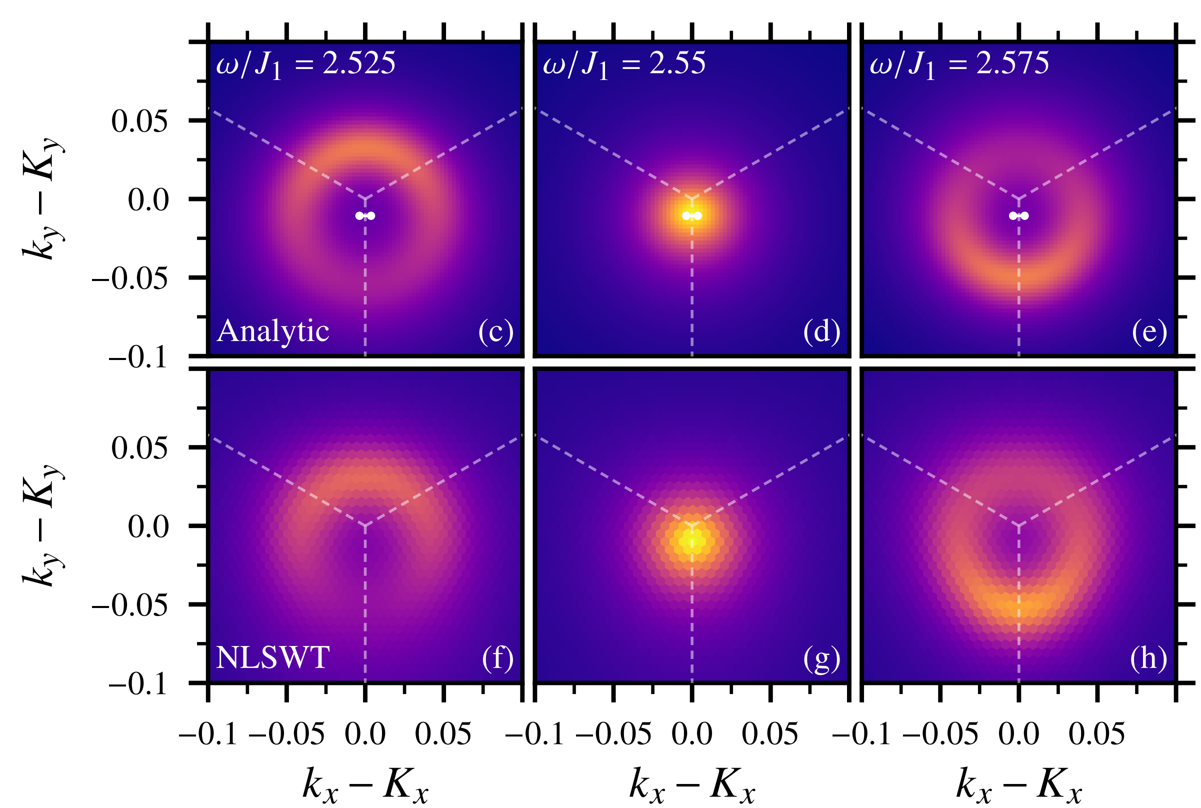
$$A(\boldsymbol{k},\omega) \equiv -\frac{1}{\pi}\mathrm{ImTr}[G(\boldsymbol{k},\omega)]$$



Characteristic pattern of linewidth:

In lower band largest on one side of K point

In upper band largest on opposite side of K point

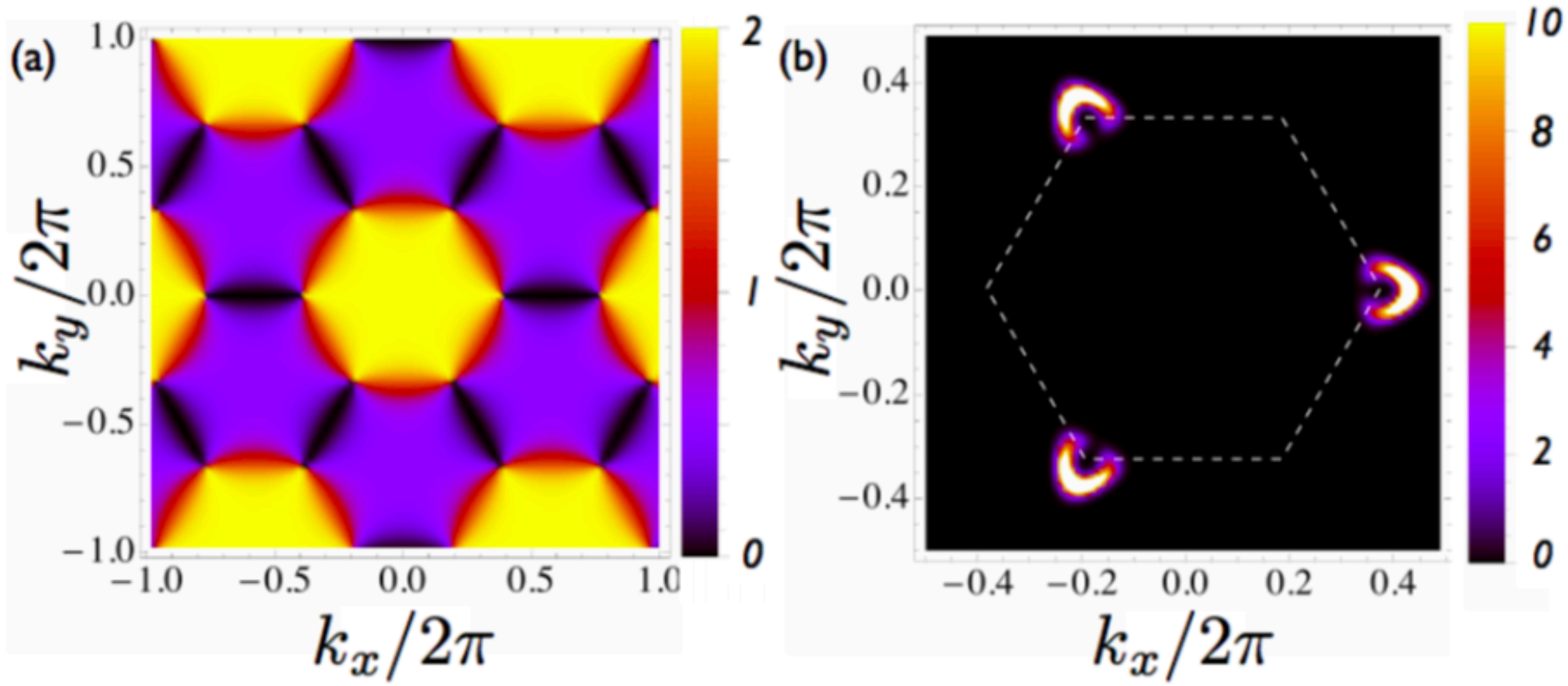


Agreement between effective non-Hermitian problem and full momentum-energy dependent spectral function

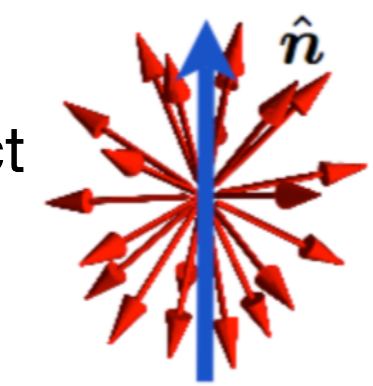
Neutron Scattering Cross Section: No Interactions

Characteristic anisotropy of neutron intensity around linear touching points

$$S^{\alpha\beta}(\boldsymbol{k},\omega) = \int d\omega e^{i\omega t} \langle S^{\alpha}_{-\boldsymbol{q}}(t) S^{\beta}_{\boldsymbol{q}} \rangle$$



Comes from projection of momentum space pseudo-spin onto some direct determined by location of touching point in momentum space



$$H_{\text{eff}} = v\boldsymbol{k} \cdot \boldsymbol{\sigma}$$

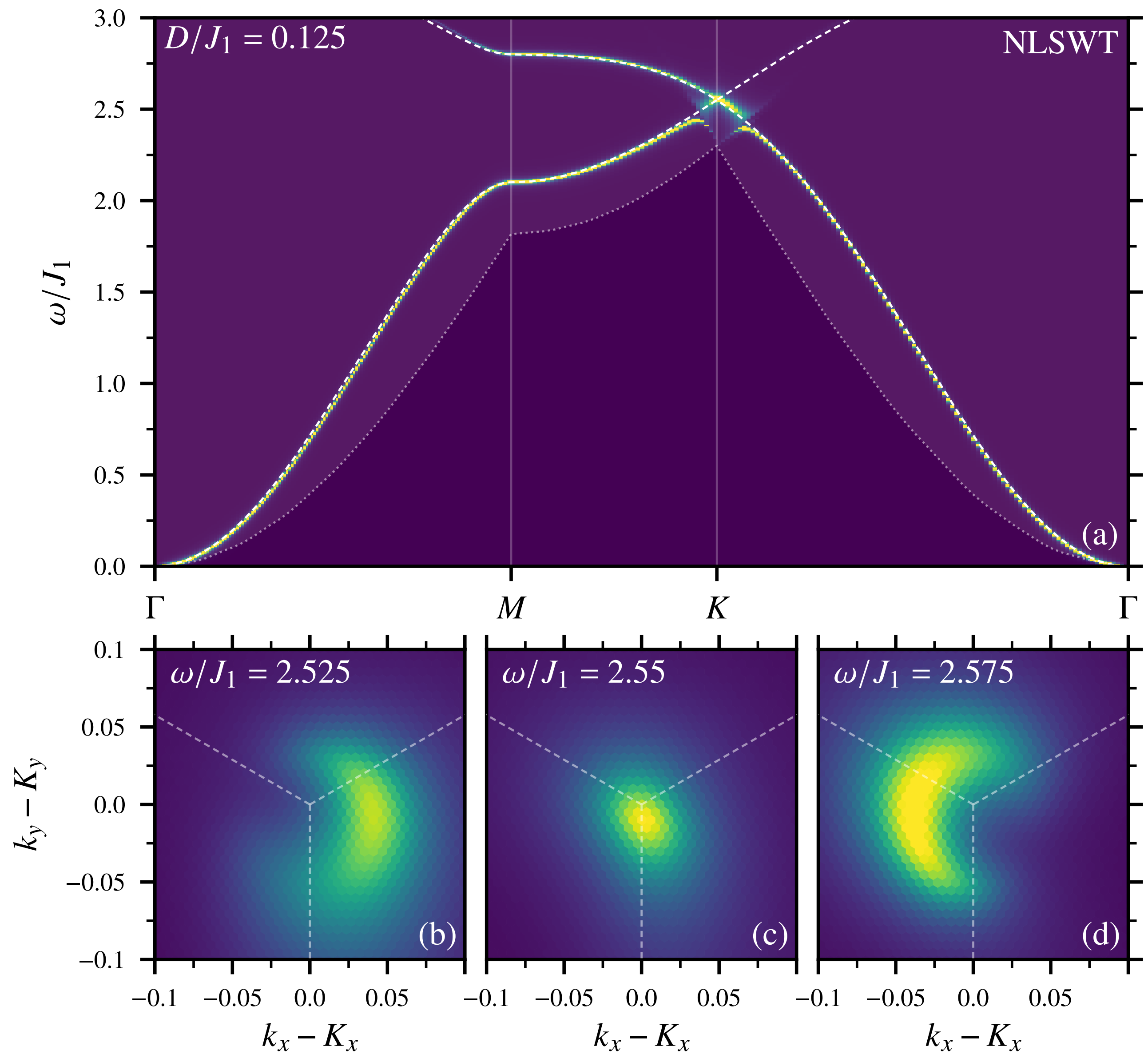
Shivam, Coldea, Moessner, PAM (2017)

Neutron Scattering Cross Section

Neutron measures dynamical structure factor not spectral function per se.

Around linear touching points, neutron intensity has characteristic anisotropy

High intensity on one side of K point in upper band, opposite side in lower band



But…

Anisotropy directions of intensity and line-shape are generally different - the latter following the direction of the exceptional line

Result is distortion of crescent pattern as shown

Outlook

Mechanism leading to exceptional points (2D) and lines (3D) around touching points is generic in the presence of magnon interactions

Magnitude of effect relative to diagonal broadening is model dependent

Discussion focused on spontaneous decay but temperature induced decay leads to similar physics

Winding signature of non-Hermitian topology in principle visible using inelastic neutron scattering, NRSE, perhaps THz spectroscopy

Magnetic field lifts two-magnon states relative to single magnon state

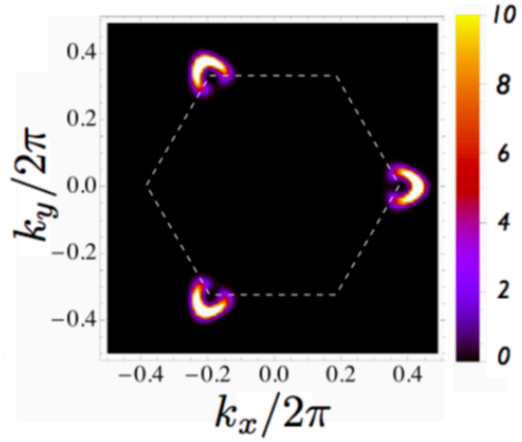
- In 2D potentially tunable van Hove singularities enhancing magnon decay
- For large enough field, can switch off magnon decay altogether

Summary

Chern Bands: Chiral edge states can be robust quasi-particles despite, or even because of magnon interactions

Magnon touching points - Dirac and Weyl:

Neutron intensities have characteristic features coming from pseudo-spin in momentum space



Neutron line shapes have characteristic pattern arising from presence of exceptional points