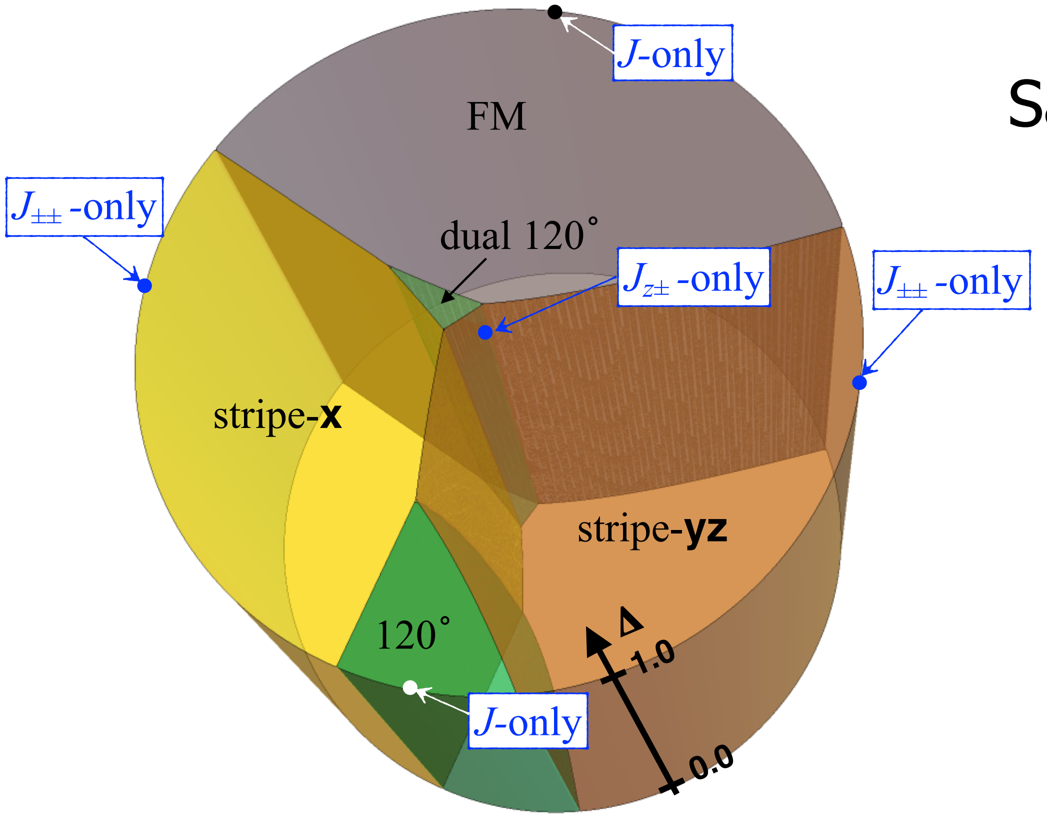
anisotropic-exchange magnets on a triangular lattice



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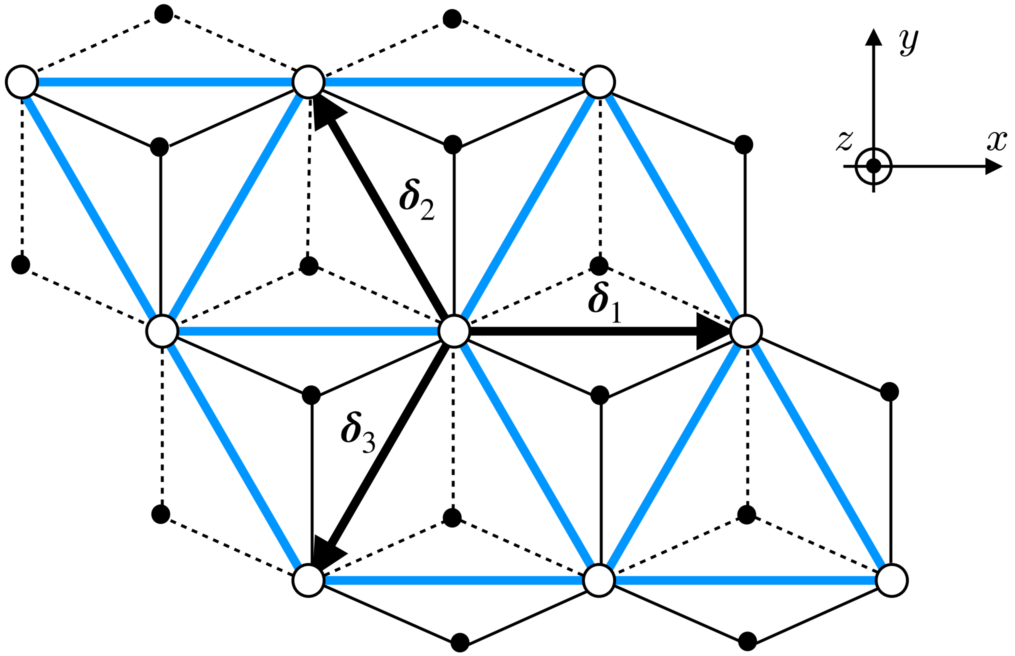

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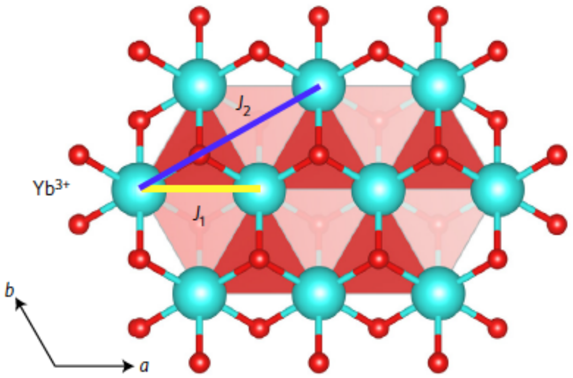

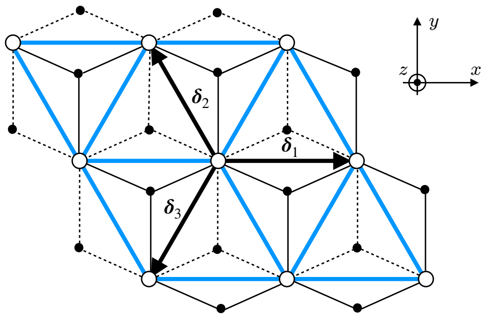

most general* n-n anisotropic model on the triangular lattice

-- no exact (Kitaev-like) solution
-- combination of
(a) bond-dependent (anisotropic) interactions, and
(b) frustrating [triangular-lattice] geometry
⇛ **something interesting**

(*) later

the team

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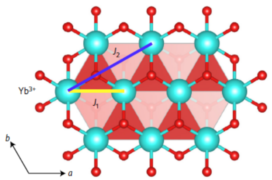
Zhenyue Zhu	Steven White	Pavel Maksimov
		
DMRG guru (AI now)		*(quasi-)classic guy [now Dubna]*

Z. Zhu *etal.*, PRL **119**, 157201 (2017); **120**, 207203 (2018); P. A. Maksimov *etal.*, PRX **9**, 021017 (2019).

papers



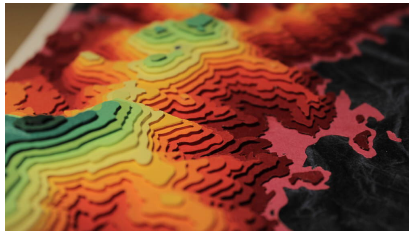
YbMgGaO$_4$

I. disorder-induced **mimicry** in YMGO

PRL **119**, 157201 (2017) PHYSICAL REVIEW LETTERS week ending 13 OCTOBER 2017

Disorder-Induced Mimicry of a Spin Liquid in YbMgGaO$_4$

Zhenyue Zhu, P. A. Maksimov, Steven R. White, and A. L. Chernyshev



II. phase diagram and **topography** of spin-liquid phase

PHYSICAL REVIEW LETTERS **120**, 207203 (2018)

Topography of Spin Liquids on a Triangular Lattice

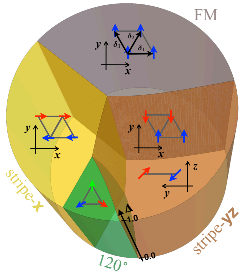
Zhenyue Zhu, P. A. Maksimov, Steven R. White, and A. L. Chernyshev

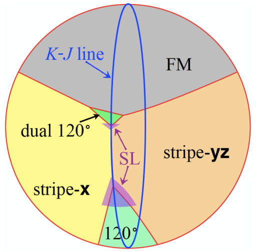
III. accidental degeneracies, **duality** of spin liquids

PHYSICAL REVIEW X **9**, 021017 (2019)

Anisotropic-Exchange Magnets on a Triangular Lattice: Spin Waves, Accidental Degeneracies, and Dual Spin Liquids

P. A. Maksimov, Zhenyue Zhu, Steven R. White, and A. L. Chernyshev

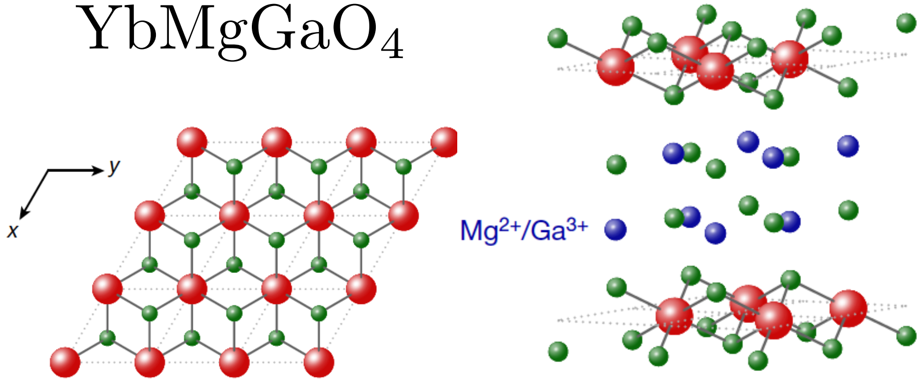




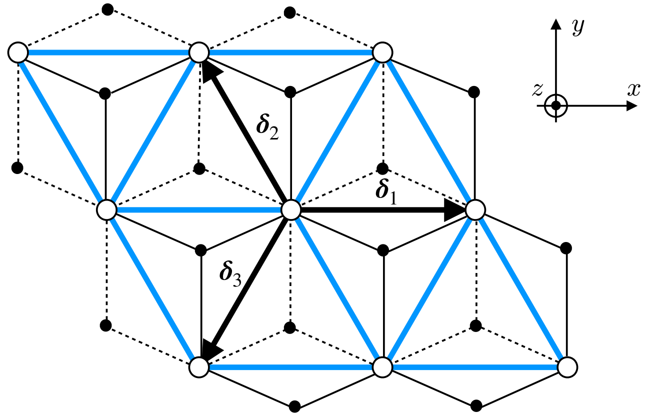
Z. Zhu *etal.*, PRL **119**, 157201 (2017); **120**, 207203 (2018); P. A. Maksimov *etal.*, PRX **9**, 021017 (2019).

materials, rare-earth: common model

$YbMgGaO_4$



$NaYbCh_2$ (Ch = O, S, Se)
$CeCd_3As_3$



lattice symmetry:

- in-plane **$2\pi/3$** rotation
- ***π*** rotation around the bond
- site inversion

○ rare-earth, Yb^{3+}, ***J****=7/2*, + crystal-field splitting
○ lowest doublet, effective *S=1/2*
○ anisotropic exchanges … (compare to Heisenberg)

○ octahedral environment …
○ lattice symmetries ⇛ **four** terms in the exchange matrix
○ mostly nearest-neighbor exchanges (*f*-electrons)

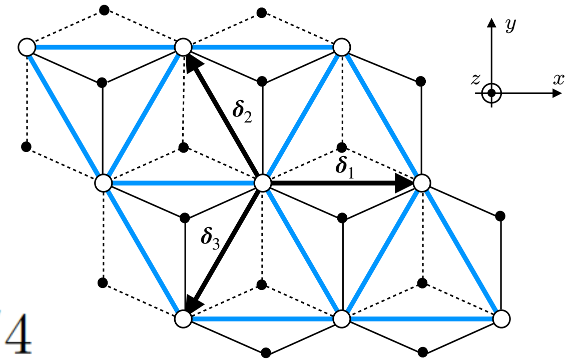
$$\hat{\mathcal{H}}_{12} = \mathbf{S}_1^0 \begin{pmatrix} J_{xx} & \cancel{J_{xy}} & \cancel{J_{xz}} \\ \cancel{J_{yx}} & J_{yy} & J_{yz} \\ \cancel{J_{zx}} & J_{zy} & J_{zz} \end{pmatrix} \mathbf{S}_2^0$$

Y. Li *etal*, Sci. Rep. **5**, 16419 (2015); M. Baenitz *etal*, PRB **98**, 220409(R) (2018); Ding *etal.*, arXiv:1901.07810; Ranjith *etal.*, PRB **99**, 180401 (2019).

XXZ-J_{++}-J_{z+} model

$$\hat{\mathcal{H}} = \sum_{\langle ij \rangle} \mathbf{S}_i^{\mathrm{T}} \hat{\boldsymbol{J}}_{ij} \mathbf{S}_j$$

bond-$\boldsymbol{\delta}_1$: $\hat{\mathcal{H}}_{ij} = \mathbf{S}_i^{\mathrm{T}} \hat{\boldsymbol{J}}_1 \mathbf{S}_j = \mathbf{S}_i^{\mathrm{T}} \begin{pmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & J_{yz} \\ 0 & J_{zy} & J_{zz} \end{pmatrix} \mathbf{S}_j$



- parametrization: $J_{zz} = \Delta \cdot J, \quad J = (J_{xx} + J_{yy})/2, \quad J_{\pm\pm} = (J_{xx} - J_{yy})/4$
  $J_{zy} = J_{zy} = J_{z\pm}$
- other bonds: $\hat{\boldsymbol{J}}_\alpha = \hat{\mathbf{R}}_\alpha^{-1} \hat{\boldsymbol{J}}_1 \hat{\mathbf{R}}_\alpha \qquad \hat{\mathbf{R}}_\alpha = \begin{pmatrix} \cos\tilde{\varphi}_\alpha & \sin\tilde{\varphi}_\alpha & 0 \\ -\sin\tilde{\varphi}_\alpha & \cos\tilde{\varphi}_\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$\mathcal{H} = \sum_{\langle ij \rangle} \boxed{J\left[S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z\right]} \text{ XXZ} \qquad \boxed{\tilde{\varphi}_\alpha = \{0, -2\pi/3, 2\pi/3\}}$$

bond-dependent

$$\boxed{\begin{aligned} &+ 2J_{\pm\pm}\left[\cos\tilde{\varphi}_\alpha \left(S_i^x S_j^x - S_i^y S_j^y\right) - \sin\tilde{\varphi}_\alpha \left(S_i^x S_j^y + S_i^y S_j^x\right)\right] \\ &+ J_{z\pm}\left[\cos\tilde{\varphi}_\alpha \left(S_i^y S_j^z + S_i^z S_j^y\right) - \sin\tilde{\varphi}_\alpha \left(S_i^x S_j^z + S_i^z S_j^x\right)\right] \end{aligned}}$$

- XXZ part ⇛ **(*): 0< Δ < 1**
- **Δ > 1,** "Ising"-like
- **Δ < 0,** less physically motivated?

Zhu *etal.*, PRL **119**, 157201 (2017); **120**, 207203 (2018); P. A. Maksimov *etal.*, PRX **9**, 021017 (2019).

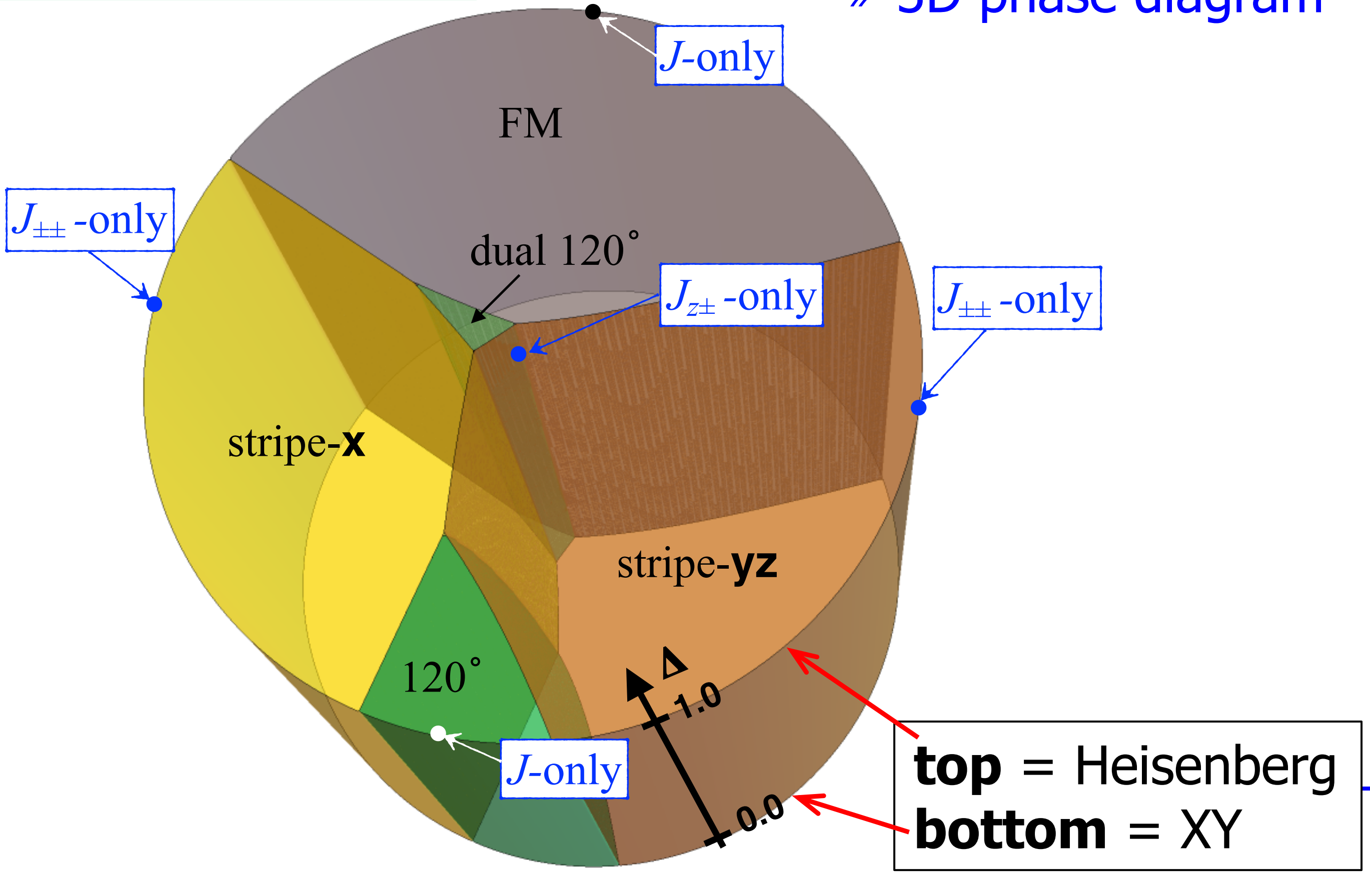
XXZ-J_{++}-J_{z+} model

$$\mathcal{H} = \sum_{\langle ij\rangle} \underbrace{J\left[S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z\right]}_{\text{XXZ}} \qquad \tilde{\varphi}_\alpha = \{0, -2\pi/3, 2\pi/3\}$$

$$\underbrace{+\, 2J_{\pm\pm}\left[\cos\tilde{\varphi}_\alpha \left(S_i^x S_j^x - S_i^y S_j^y\right) - \sin\tilde{\varphi}_\alpha \left(S_i^x S_j^y + S_i^y S_j^x\right)\right] + J_{z\pm}\left[\cos\tilde{\varphi}_\alpha \left(S_i^y S_j^z + S_i^z S_j^y\right) - \sin\tilde{\varphi}_\alpha \left(S_i^x S_j^z + S_i^z S_j^x\right)\right]}_{\text{bond-dependent}}$$

- XXZ part ⇛ **(*): Δ < 1**

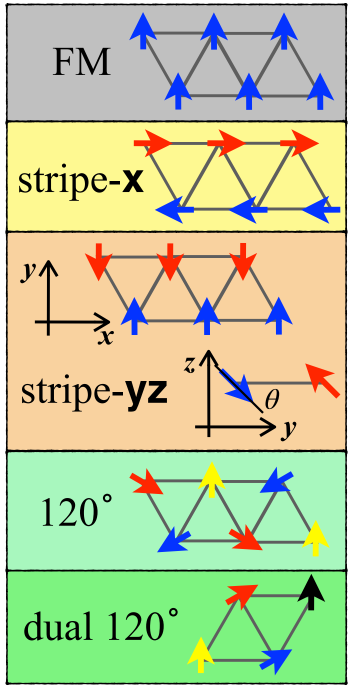
- **four** parameters ⇛ 3D phase diagram



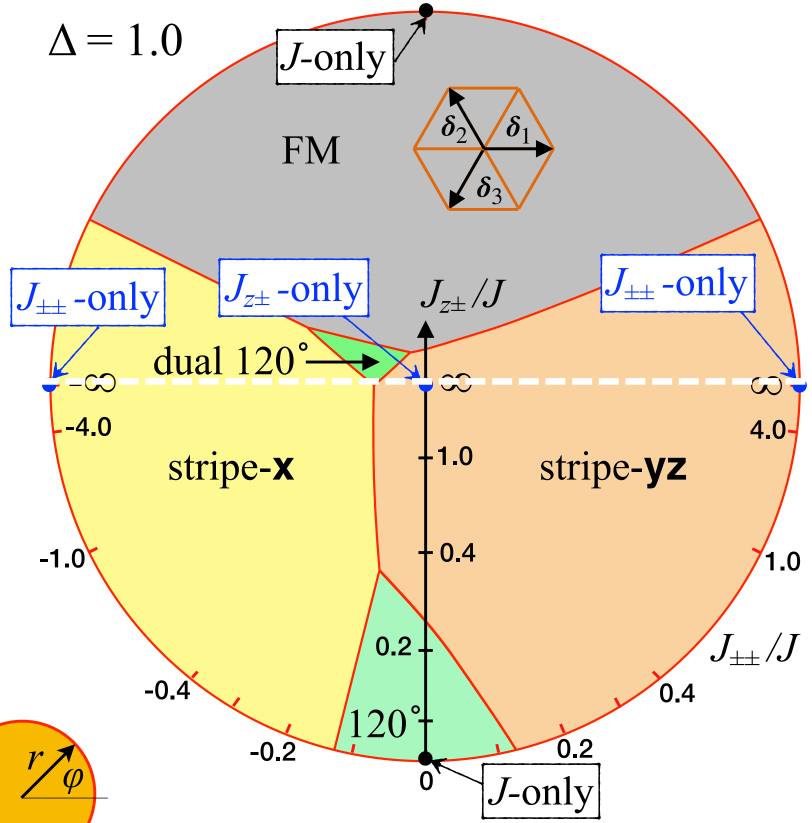
 Zhu *et al.*, PRL **119**, 157201 (2017); **120**, 207

0^{th}-order phase diagram: classical, single-Q

- parametrization of Δ = const "slice": $J_{z\pm}$ -radial, $J_{\pm\pm}$ -polar

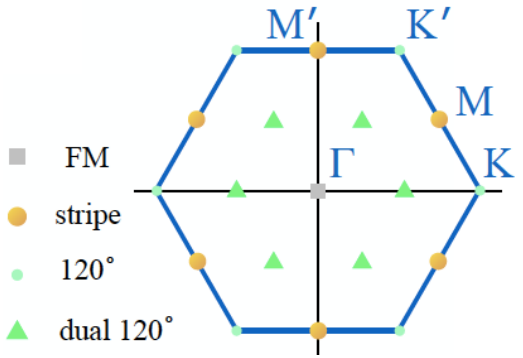


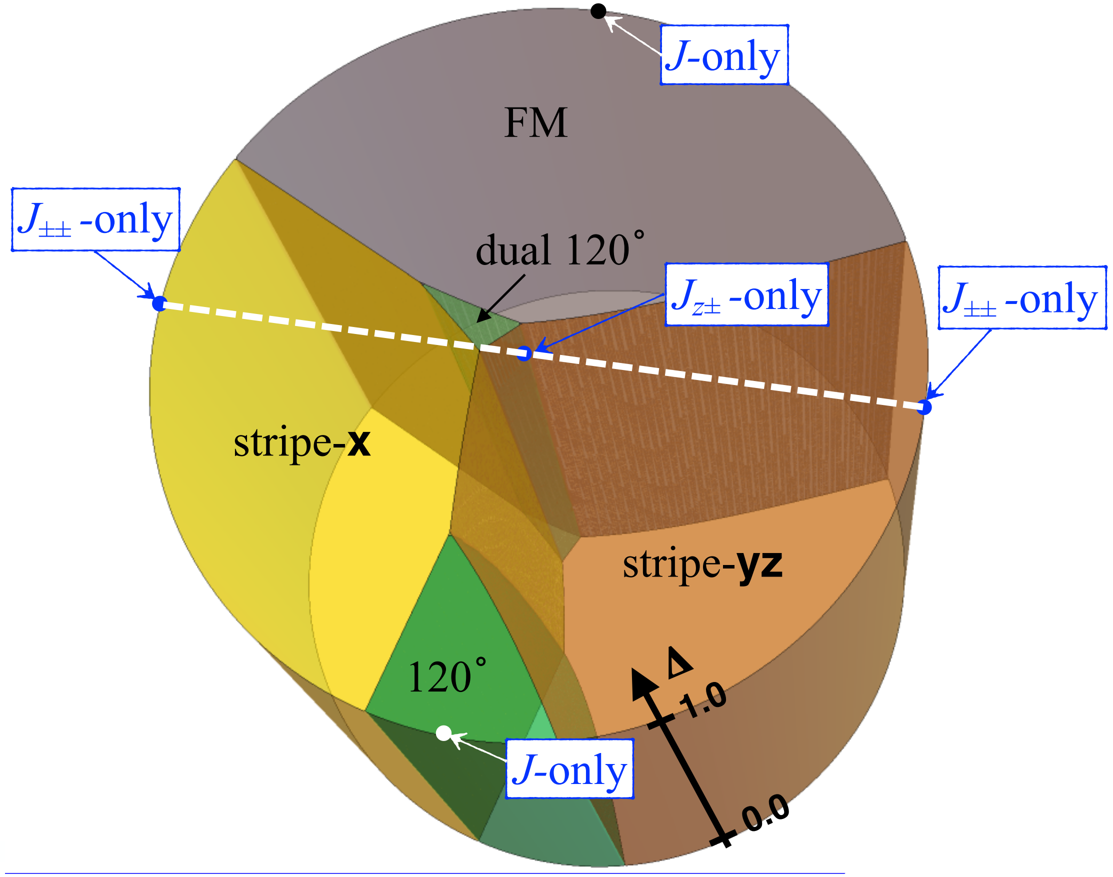
$$5J_{z\pm} = \sqrt{1-r^2}$$
$$2J_{\pm\pm} = r\cos\varphi$$
$$J = -r\sin\varphi$$

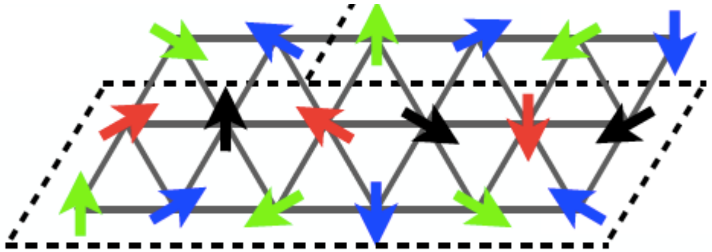


five single-Q states

some are obvious, some are not ...







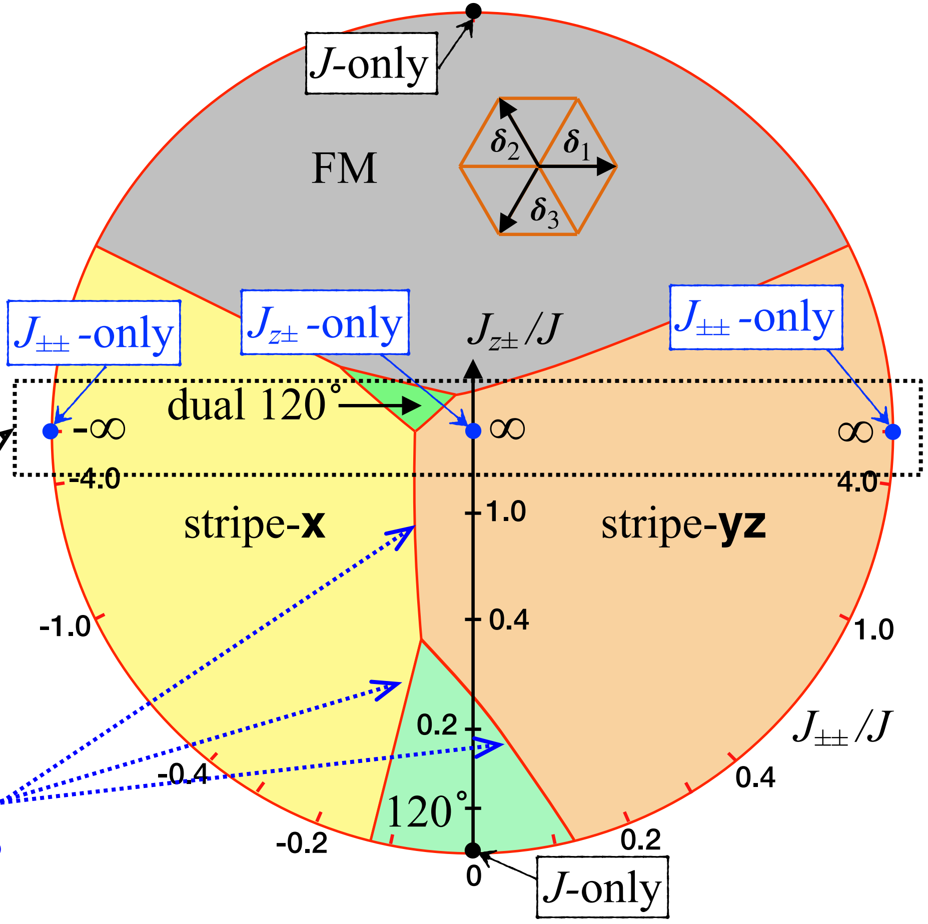
 Zhu *et al.*, PRL **119**,

where is YMGO or $NaYbO_2$?
stay tuned …

look for a spin liquid?

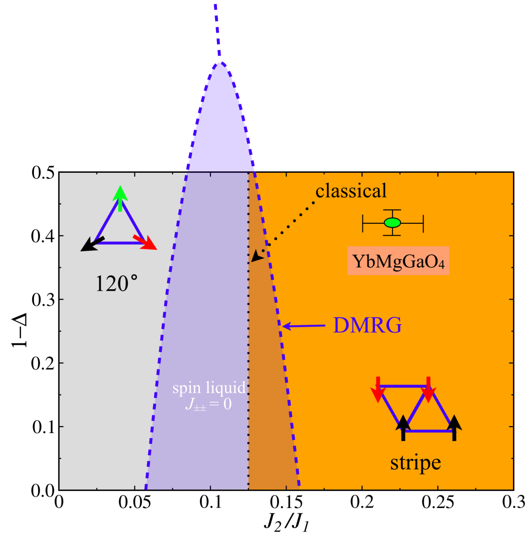
by analogy …

- **Kitaev**-like logic: bond-dependent terms dominate
- **conventional** logic:
  intermediate regions between ordered phases

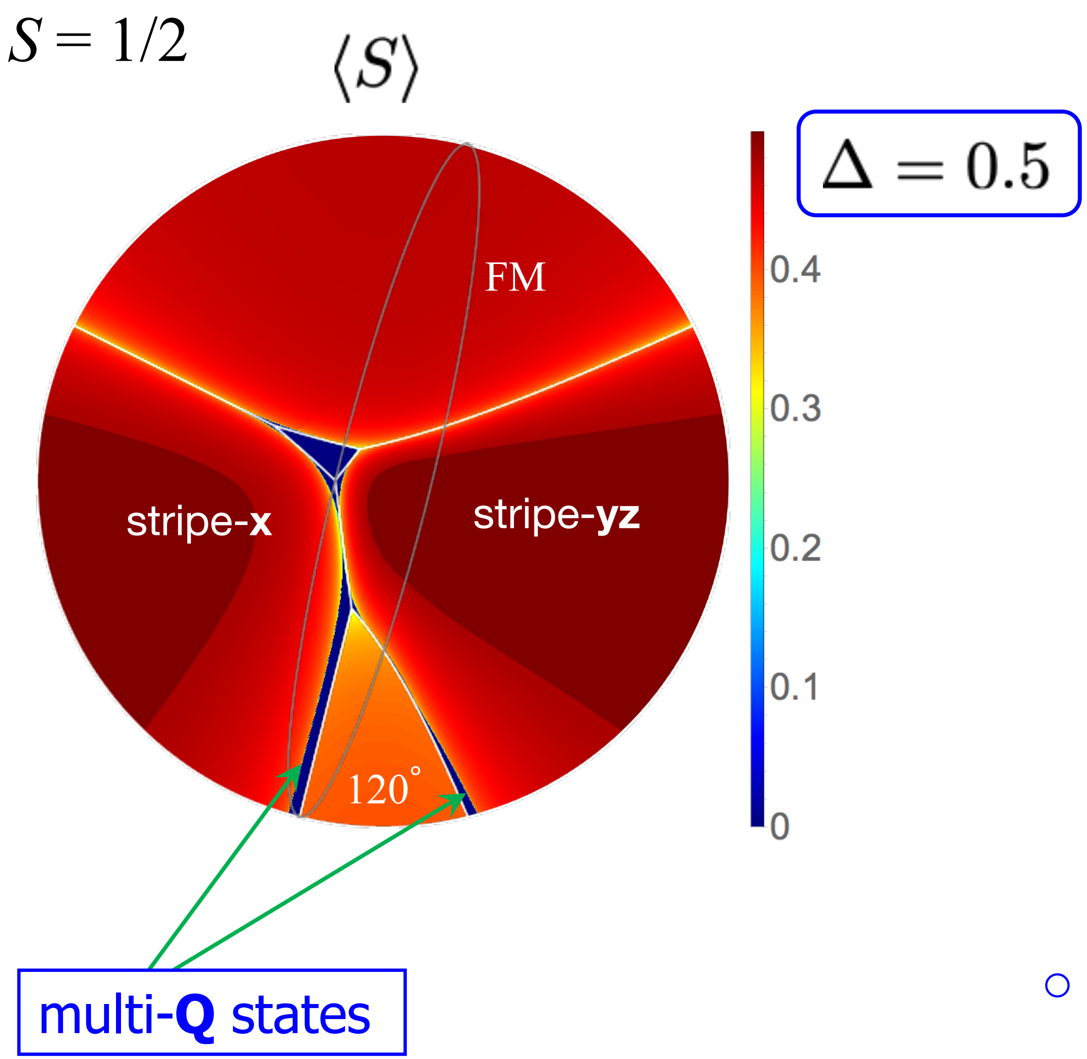


quasiclassics

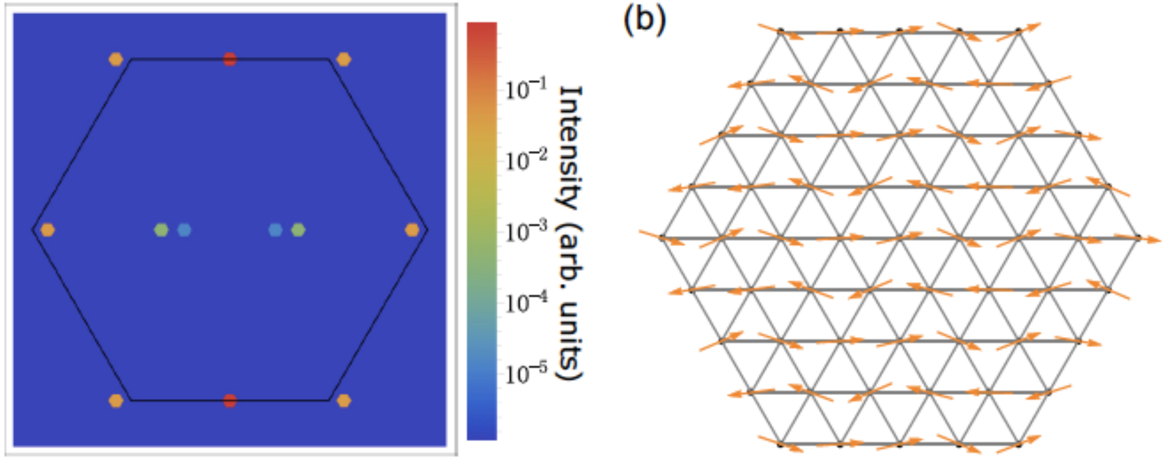
hints of an intermediate (non-magnetic?) states



quasiclassics: strongly-ordered phases …



- cut at *XXZ*-anisotropy Δ=0.5 (middle of the cylinder)
- **large ordered moment**:
  leaves little (almost no) hope for a spin liquid …
- some intermediate multi-**Q** states (modulated stripes)
- FM and 120° phases: preserve U(1), order-by-disorder

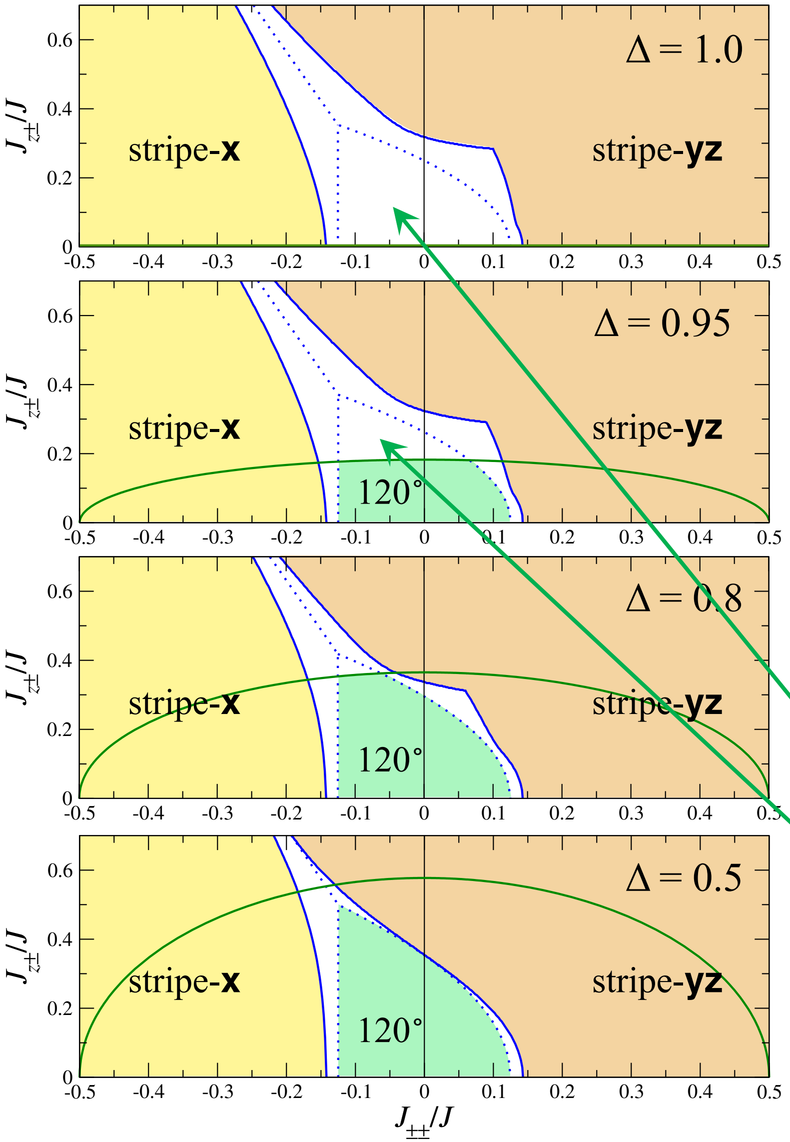


C. Liu, R. Yu, and X. Wang, Phys. Rev. B **95**, 174424 (2016).

- spectrum instability boundaries match numerical (MC), and indicate the same extra **Q** vectors

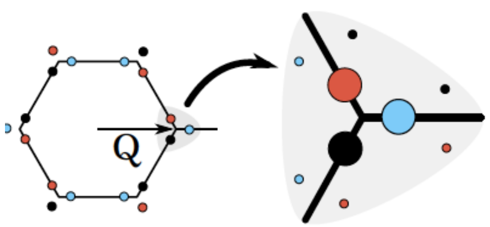
P. A. Maksimov *et al.*, PRX **9**, 021017 (2019).

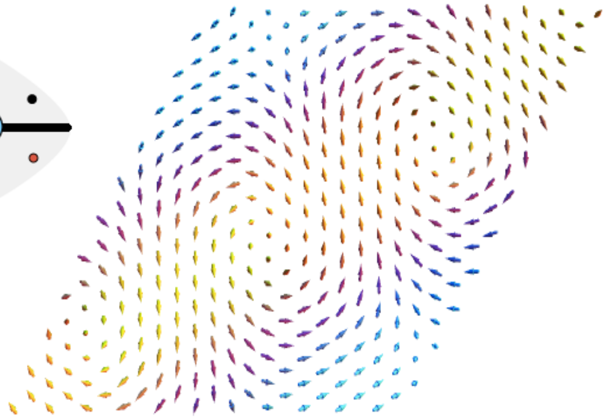
quasiclassics: spectrum instabilities …



- same cut at *XXZ*-anisotropy Δ=0.5, Cartesian coordinates
- spectrum instability: $\varepsilon_{\mathbf{k}}^2<0$ at some $\mathbf{k}$ point(s)
- stripe spectra: unstable within their nominal boundaries (toward multi-**Q**, modulated stripes)
- 120° spectrum: stable well **beyond** its boundaries
- such overlap of stable spectra of the competing phases: recipe for a **direct transition**
- larger Δ, stability to $J_{z\pm}$ shrinks: another multi-**Q**, modulated 120° state [Z_2 vortex]
- **may be an opening an SL in *S=1/2* case?**

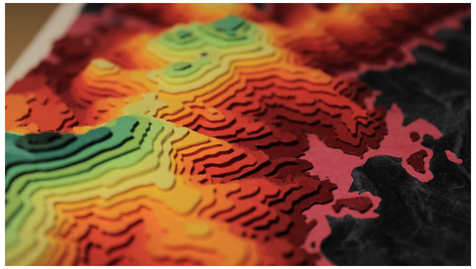
another multi-**Q** state



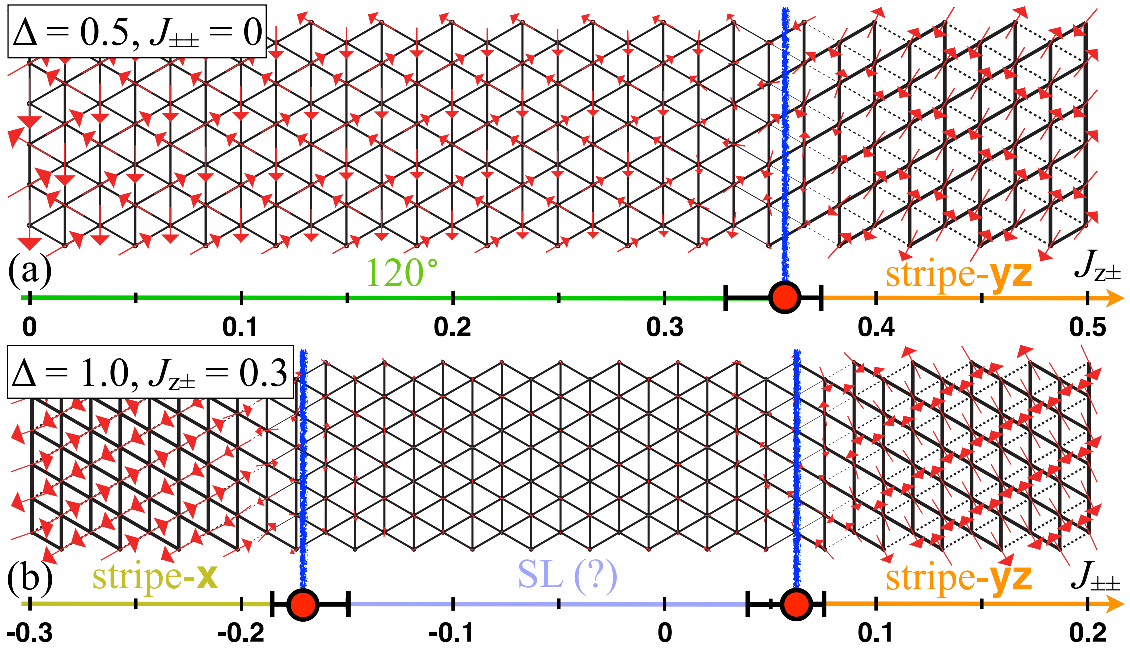


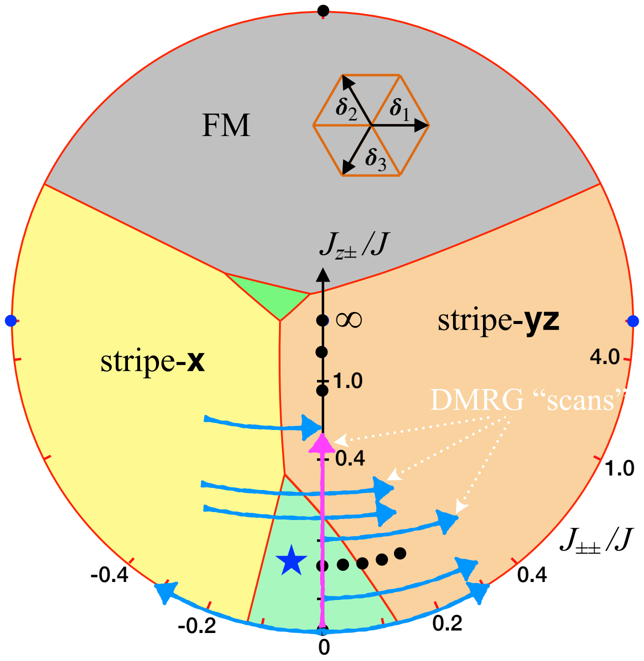
I. Rousochatzakis *etal.*, PRB **93**, 104417 (2016); M. Becker *etal.*, PRB **91**, 155135 (2015); M. Li *etal.*, PRR **1**, 013002 (2019).

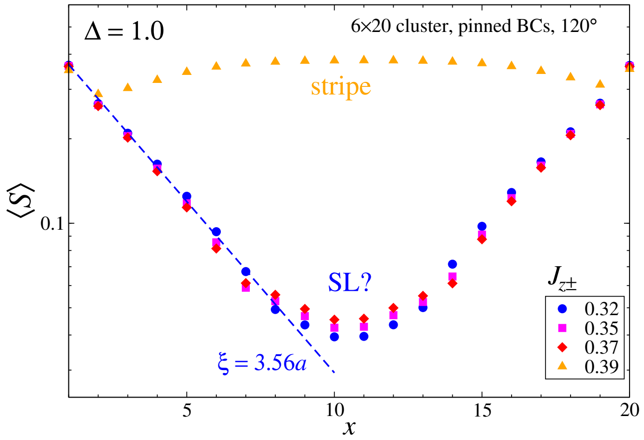
DMRG



DMRG: *S=1/2*, 3D phase space





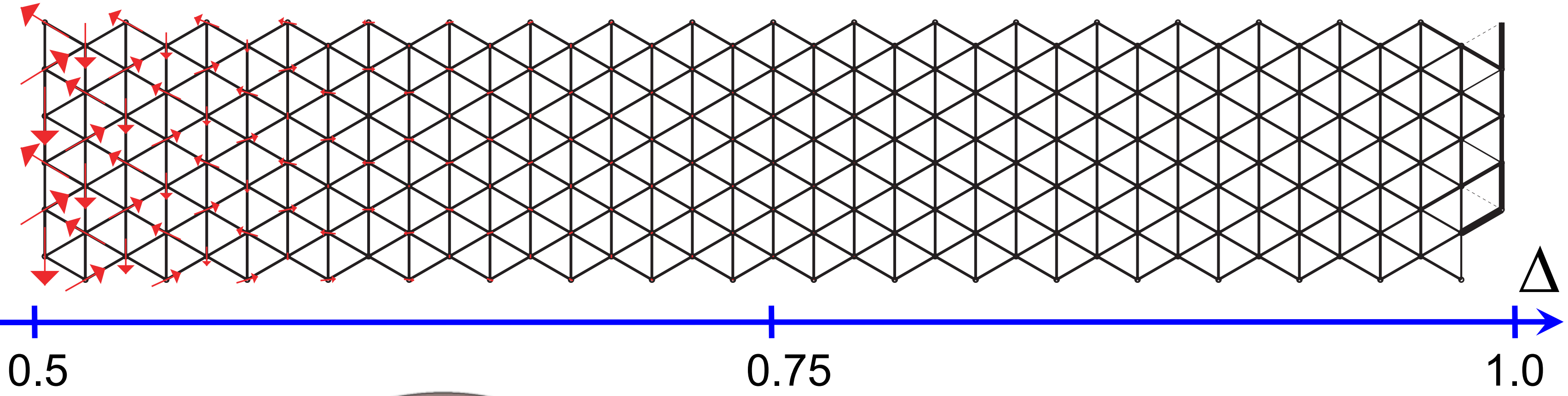


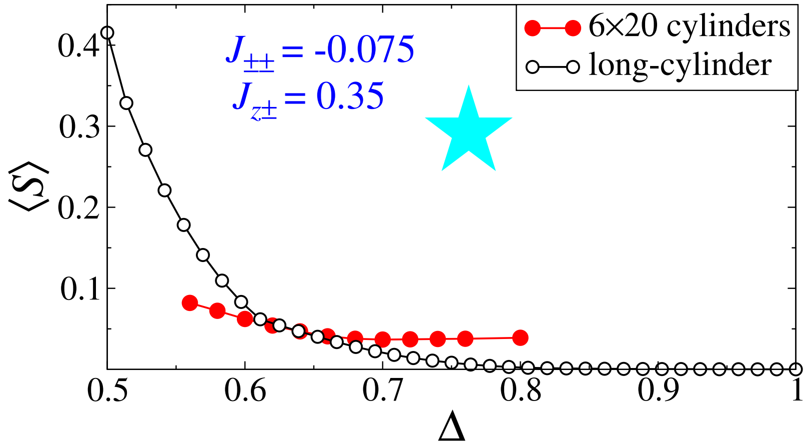
- **1D long-cylinder scans ["scans"]:** vary one parameter, play with BCs/range, read orders

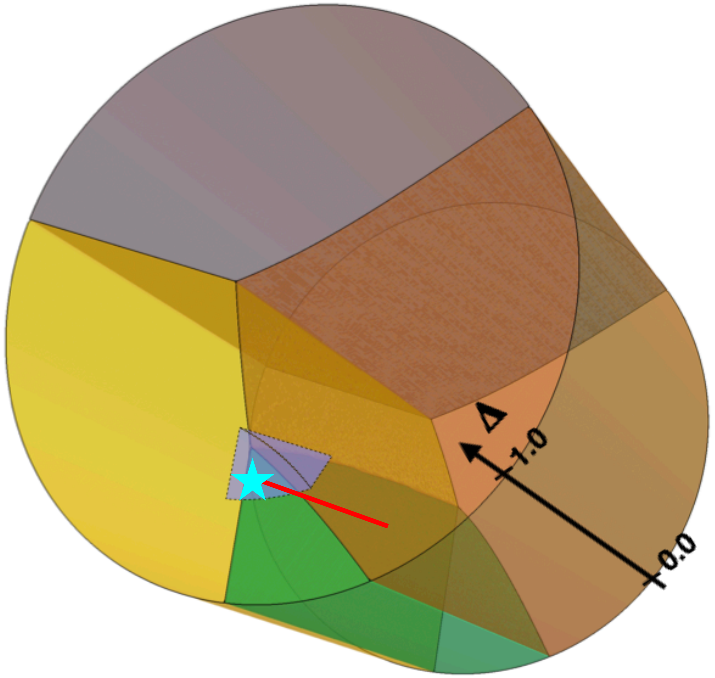
- confirmed that most transitions are direct [also in agreement with prior ED/DMRG]
- **unexpected indication of a SL**
- *1/L_y*-scaling in "non-scan" clusters, decay of correlations off boundary, exp vs power-law to confirm

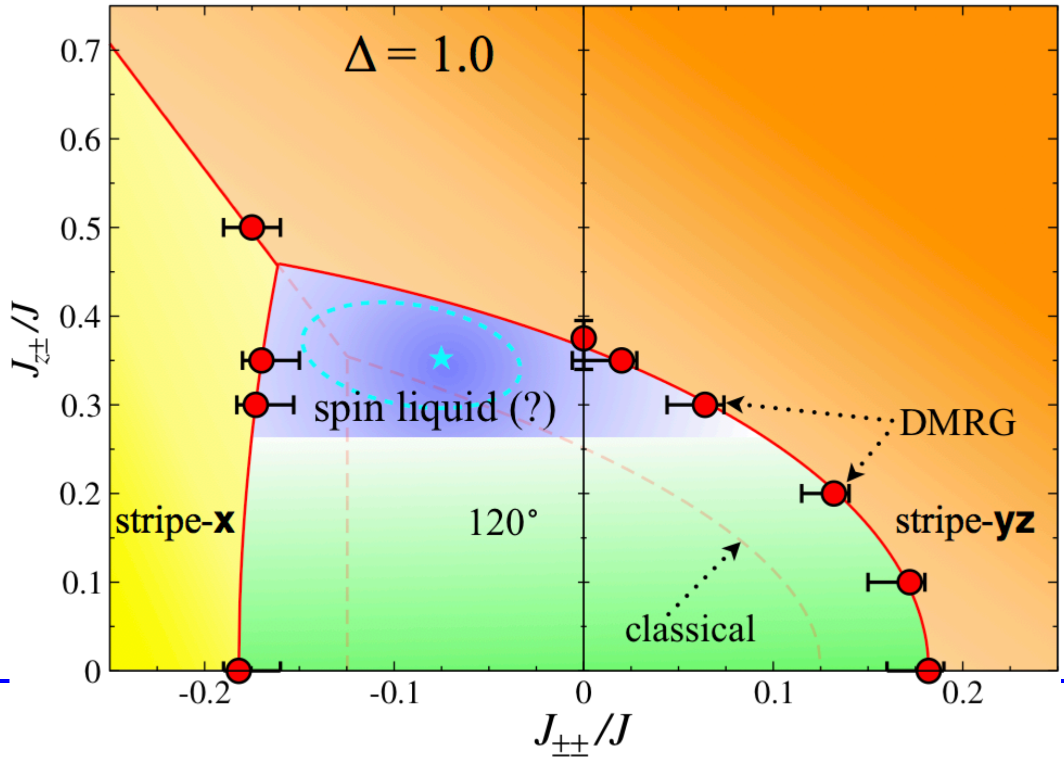
 Q. Luo *etal*, PRB **95**, 165110 (2017); Zhu *etal.*, PRL **120**, 207203 (2018); Maksimov *etal.*, PRX **9**, 021017 (2019).

spin liquid **#1**



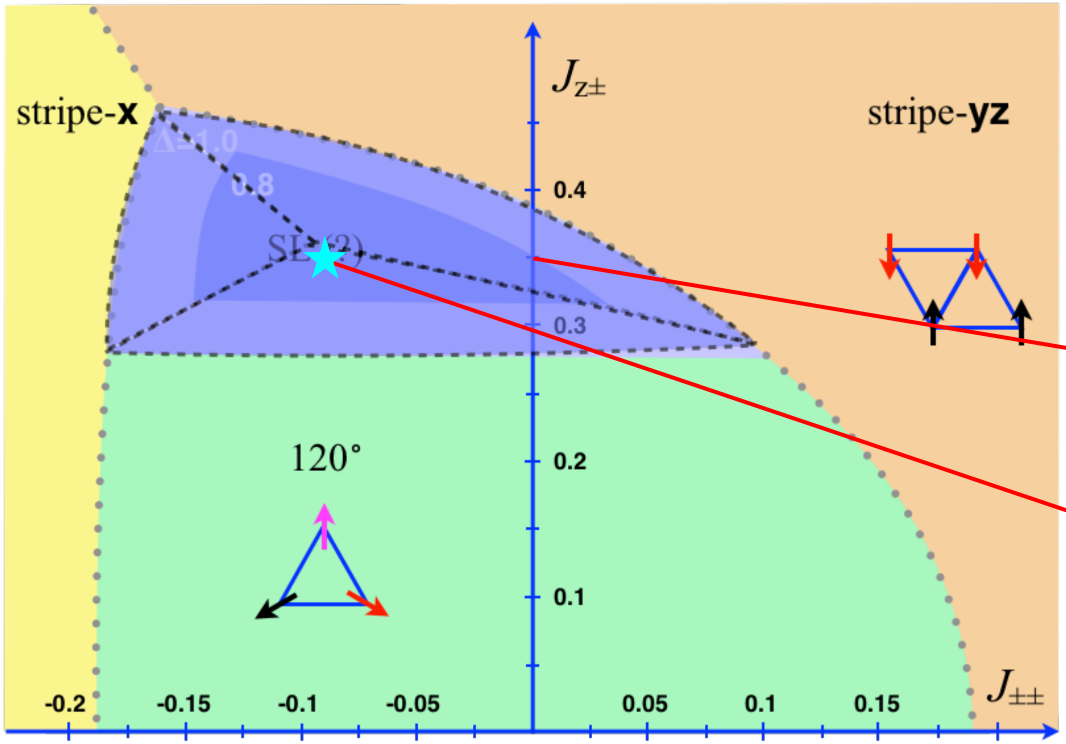


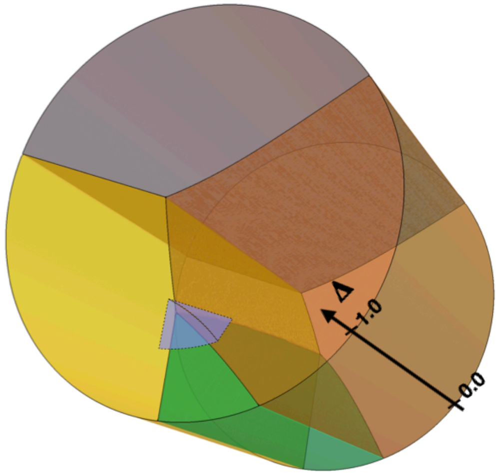




Zhu *etal.*, PRL **120**, 207203 (2018)

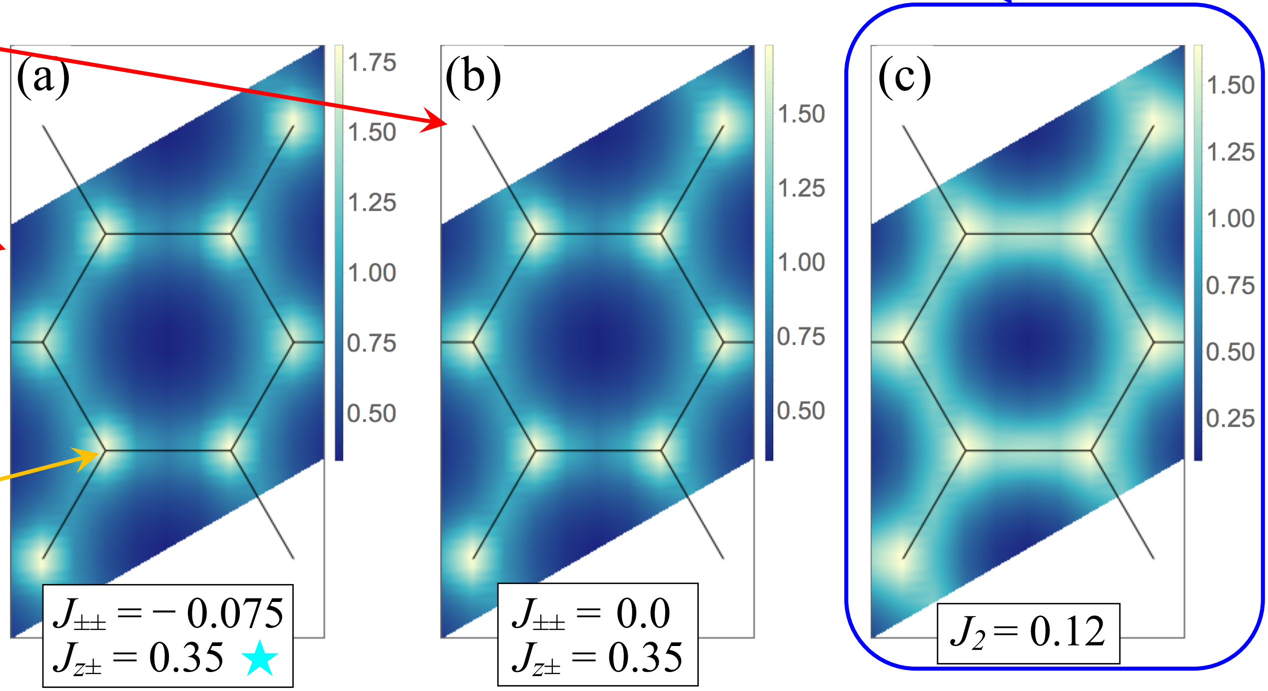
spin liquid **#1**: topographic map





- *S(**q**):* "molten 120", connection to J_1-J_2

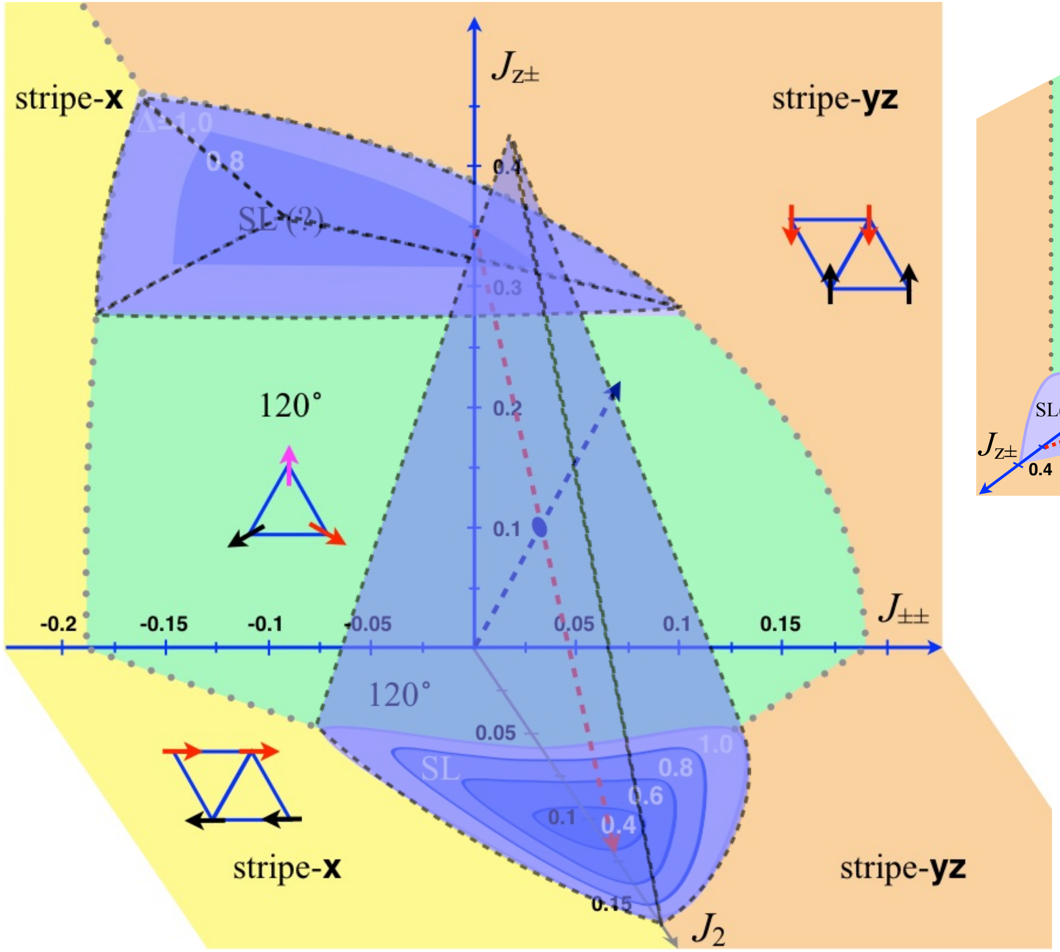
K-points

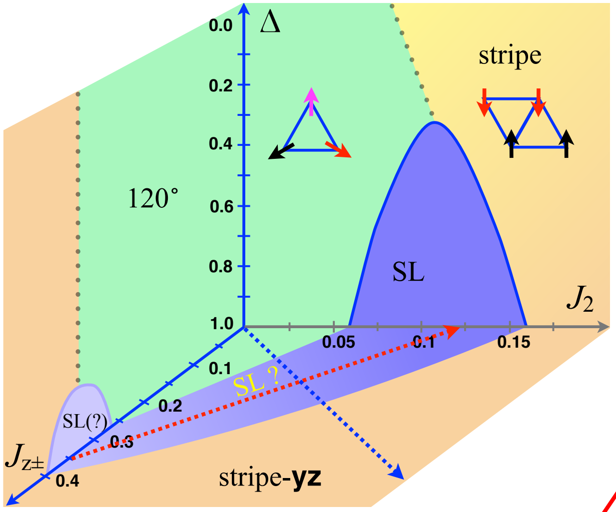


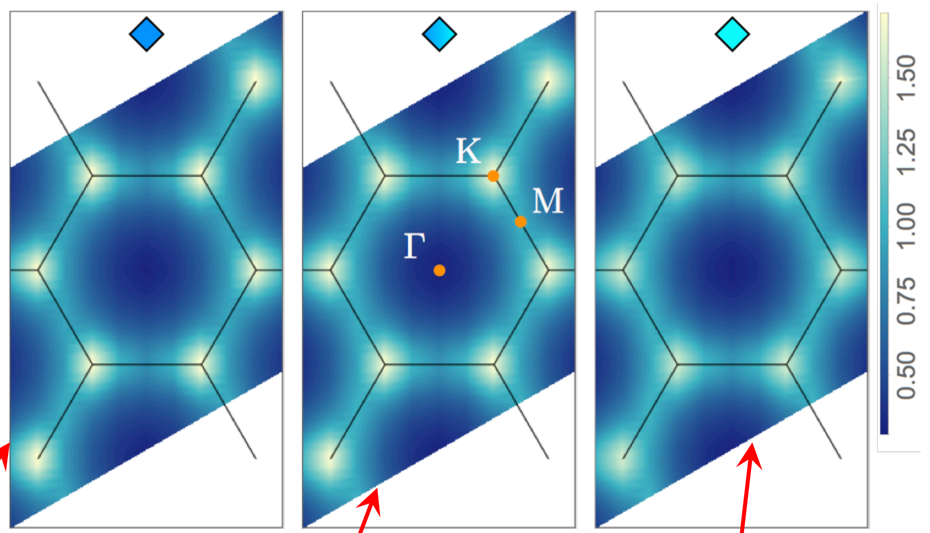
Zhu *etal.*, PRL **120**, 207203 (2018)

Z. Zhu and S. R. White, PRB **92**, 041105 (2015).

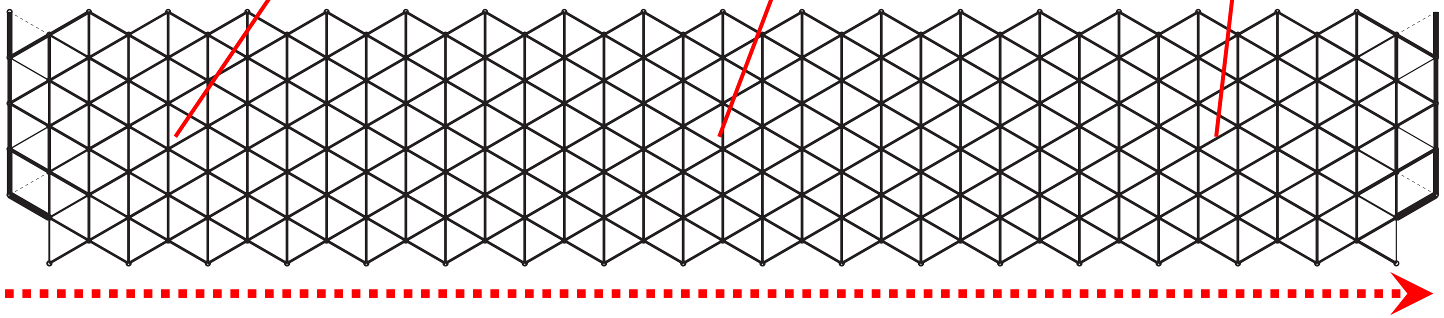
4D connection with the SL in J_1-J_2-XXZ





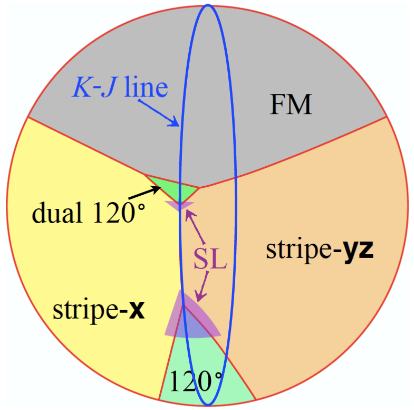


○ ⇛ **anisotropic and J_1-J_2 spin liquids are isomorphic** (= same phase)

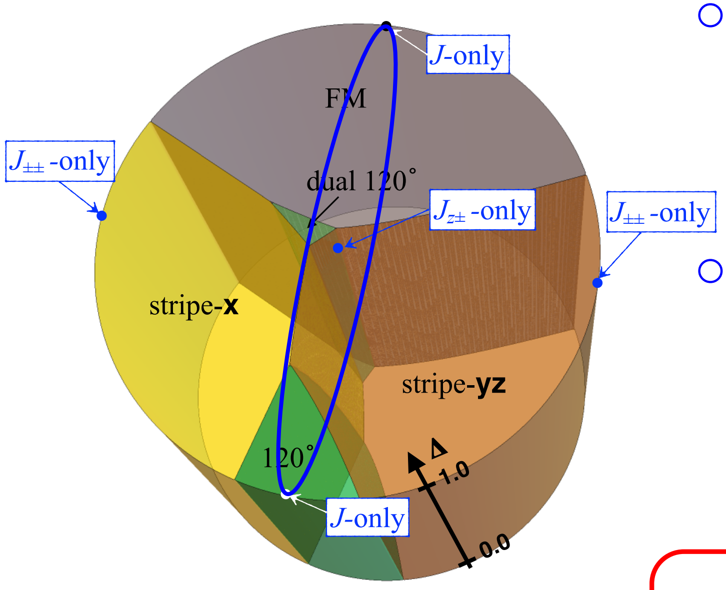


Z. Zhu *et al.*, PRL **120**, 207203 (2018).

duality



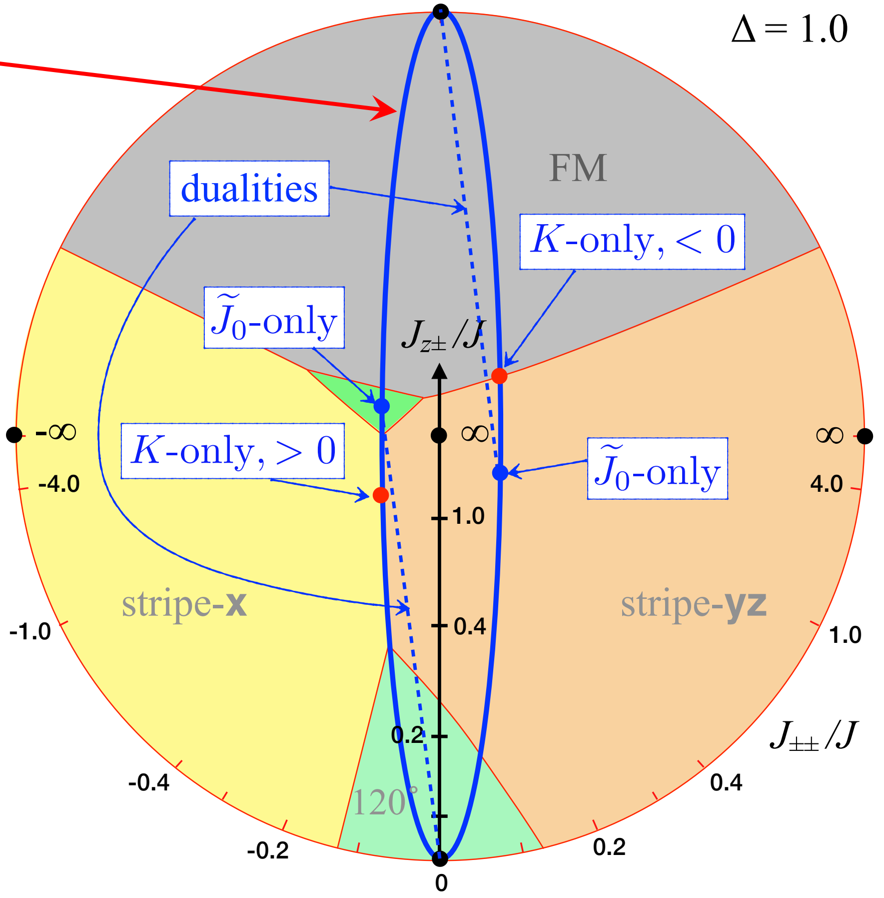
J-K-Γ-Γ′-model correspondence



- for Δ=**1.0** and ***$J_{z\pm}=2\sqrt{2}J_{\pm\pm}$* line**
  the model becomes ***J-K*** model
  (*Γ=Γ′=0*)

- ***elsewhere (not on that line):**
  ⇛ equivalent to ***J-K-Γ-Γ′*** model

$$\mathcal{H} = -J_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_{\gamma \| \langle ij \rangle} S_i^\gamma S_j^\gamma$$



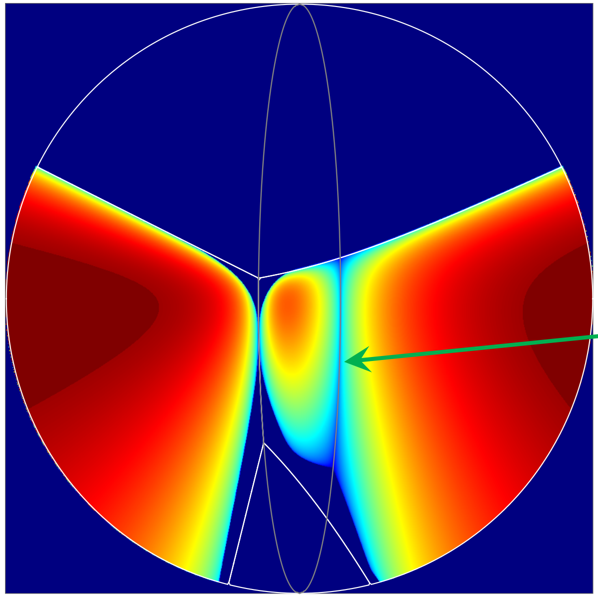
$$\mathcal{H} = \sum_{\langle ij \rangle} J\left[S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z\right] + 2J_{\pm\pm}\left[\cos\tilde{\varphi}_\alpha \left(S_i^x S_j^x - S_i^y S_j^y\right) - \sin\tilde{\varphi}_\alpha \left(S_i^x S_j^y + S_i^y S_j^x\right)\right] + J_{z\pm}\left[\cos\tilde{\varphi}_\alpha \left(S_i^y S_j^z + S_i^z S_j^y\right) - \sin\tilde{\varphi}_\alpha \left(S_i^x S_j^z + S_i^z S_j^x\right)\right]$$

⇛

$$\mathcal{H} = \sum_{\langle ij \rangle_\gamma} J_0 \mathbf{S}_i \cdot \mathbf{S}_j + K S_i^\gamma S_j^\gamma + \Gamma\left(S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha\right) + \Gamma'\left(S_i^\gamma S_j^\alpha + S_i^\gamma S_j^\beta + S_i^\alpha S_j^\gamma + S_i^\beta S_j^\gamma\right)$$

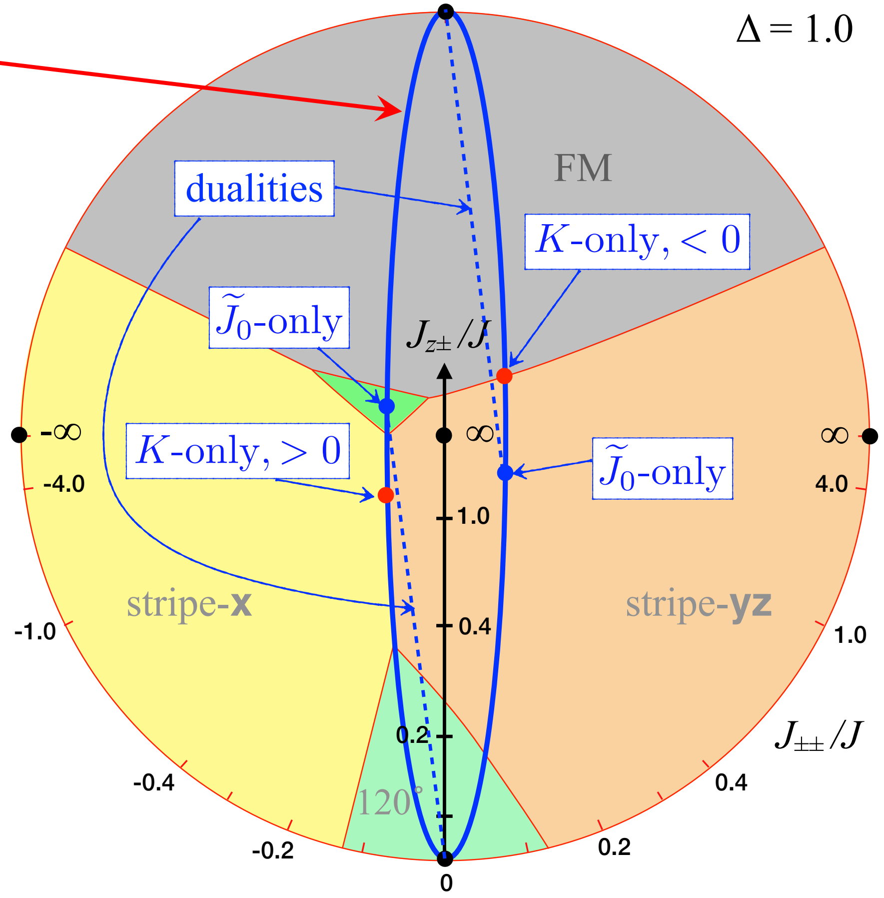
Chaloupka and Khaliullin, PRB **92**, 24413 ('15); J. G. Rau and M. J. P. Gingras, PRB **98**, 54408 ('18).

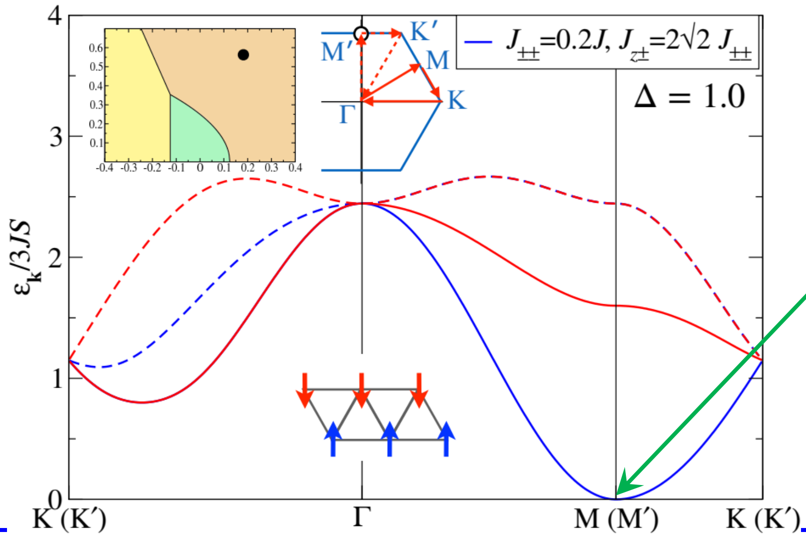
J-K-Γ-Γ′-model correspondence



- ***J-K*** model: emergent continuous symmetries (in the classical limit)

- **⇛ pseudo-Goldstone modes**
- **⇛ Mermin-Wagner, “scars” in T_N**



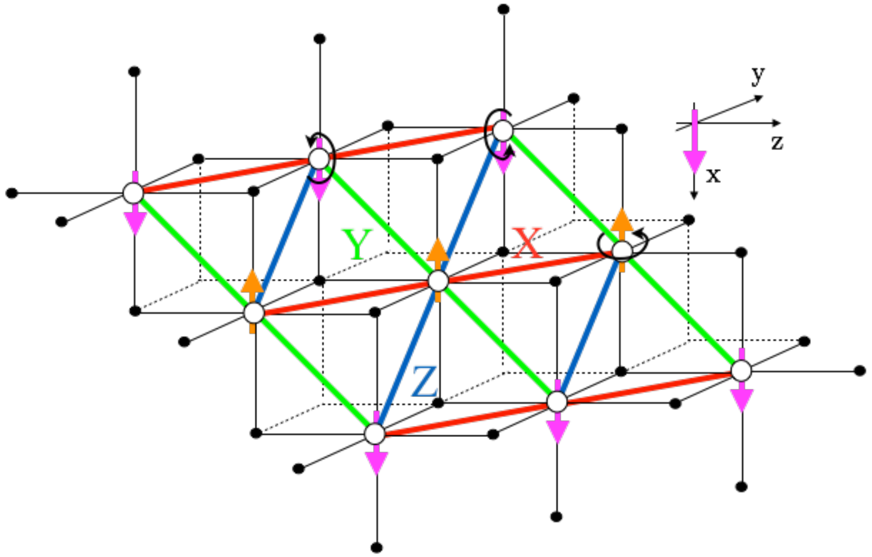


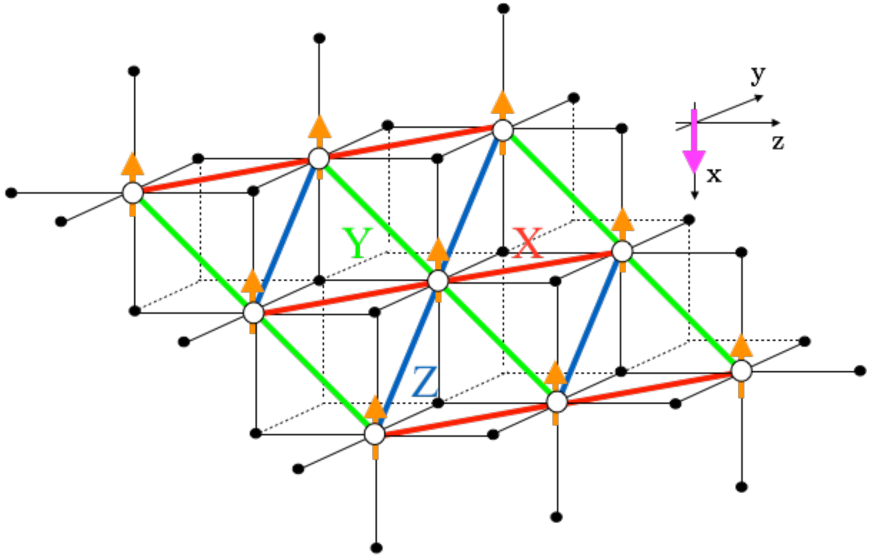
$$\mathcal{H} = -\widetilde{J}_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_{\gamma \| \langle ij \rangle} S_i^\gamma S_j^\gamma$$

Chaloupka and Khaliullin, PRB **92**, 24413 (’15); J. G. Rau and M. J. P. Gingras, PRB **98**, 54408 (’18).

Klein dualities

- four-sublattice transformation: $\mathbf{S}(\mathbf{r})$, $\mathbf{R}_X(\pi)\mathbf{S}(\mathbf{r}+\boldsymbol{\delta}_1)$, $\mathbf{R}_Y(\pi)\mathbf{S}(\mathbf{r}+\boldsymbol{\delta}_2)$, $\mathbf{R}_Z(\pi)\mathbf{S}(\mathbf{r}+\boldsymbol{\delta}_3)$
- that leaves the model invariant (e.g., stripe-yz ⇛ FM)





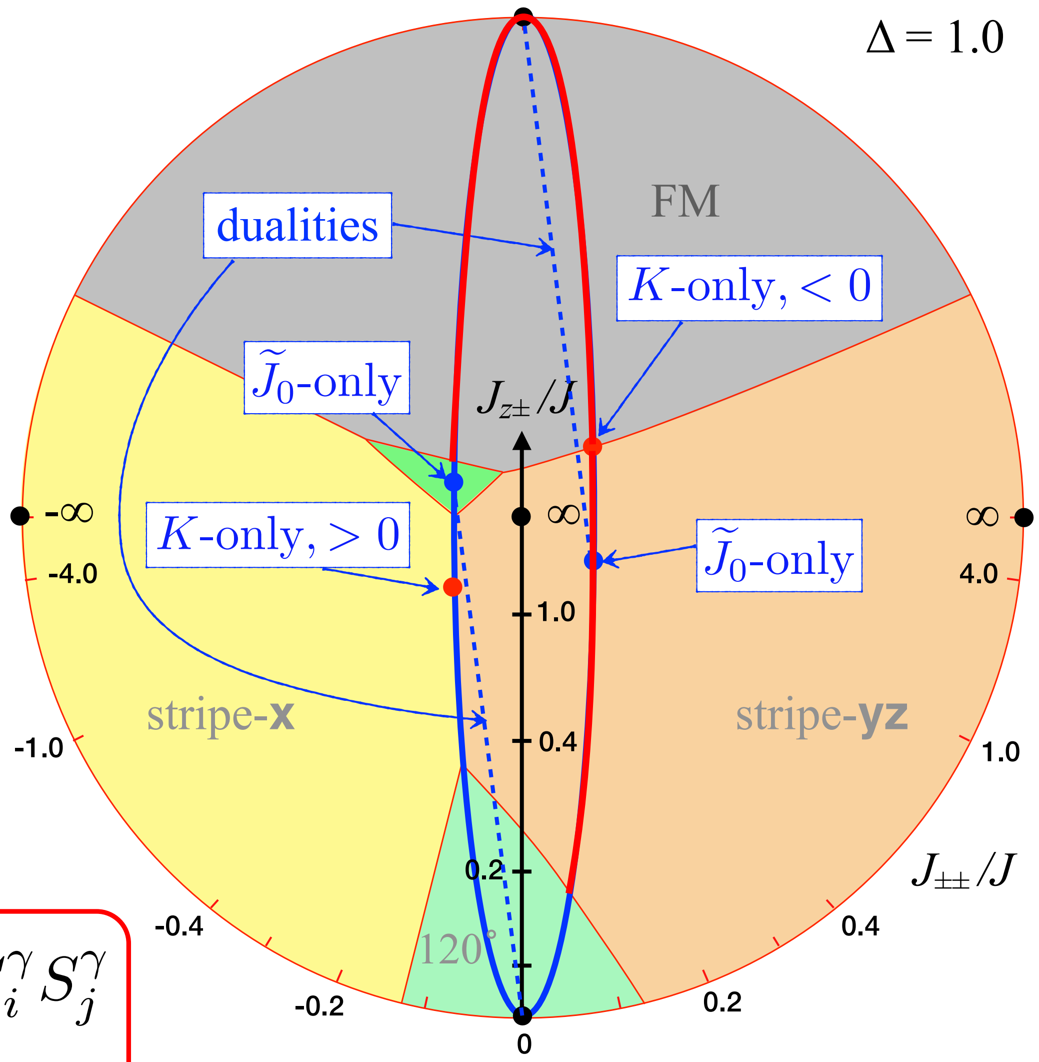
$$\widetilde{J}_0 = -J_0, \quad \widetilde{K} = 2J_0 + K,$$

$$\boxed{\mathcal{H} = -J_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_{\gamma \| \langle ij \rangle} S_i^\gamma S_j^\gamma} \rightarrow \boxed{\mathcal{H} = -\widetilde{J}_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - \widetilde{K} \sum_{\gamma \| \langle ij \rangle} S_i^\gamma S_j^\gamma}$$

Khaliullin, Prog. Th. Phys. **160**, 155 ('05); Chaloupka *etal*, PRL **105**, 27204 ('10); Kimchi and Vishwanath, PRB **89**, 014414 ('14).

Klein dualities: stripe-**yz** to FM

- dualities of different sectors
- **yz**-stripe sector ⇛ **duality to FM**

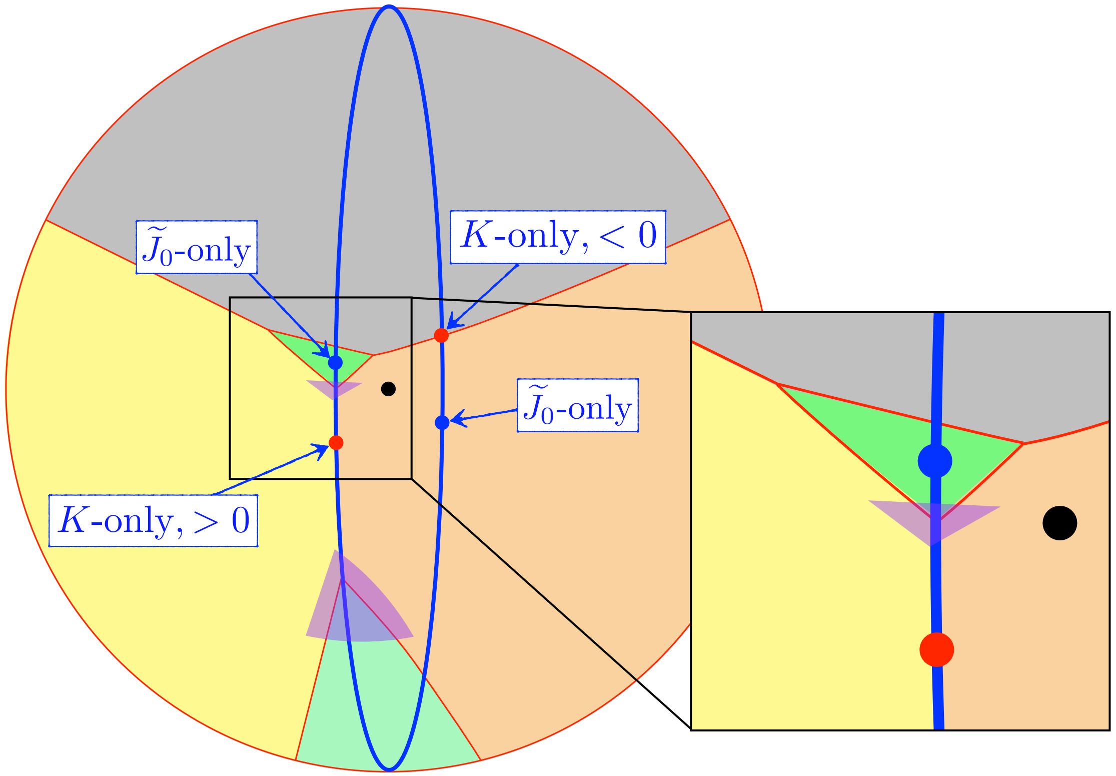


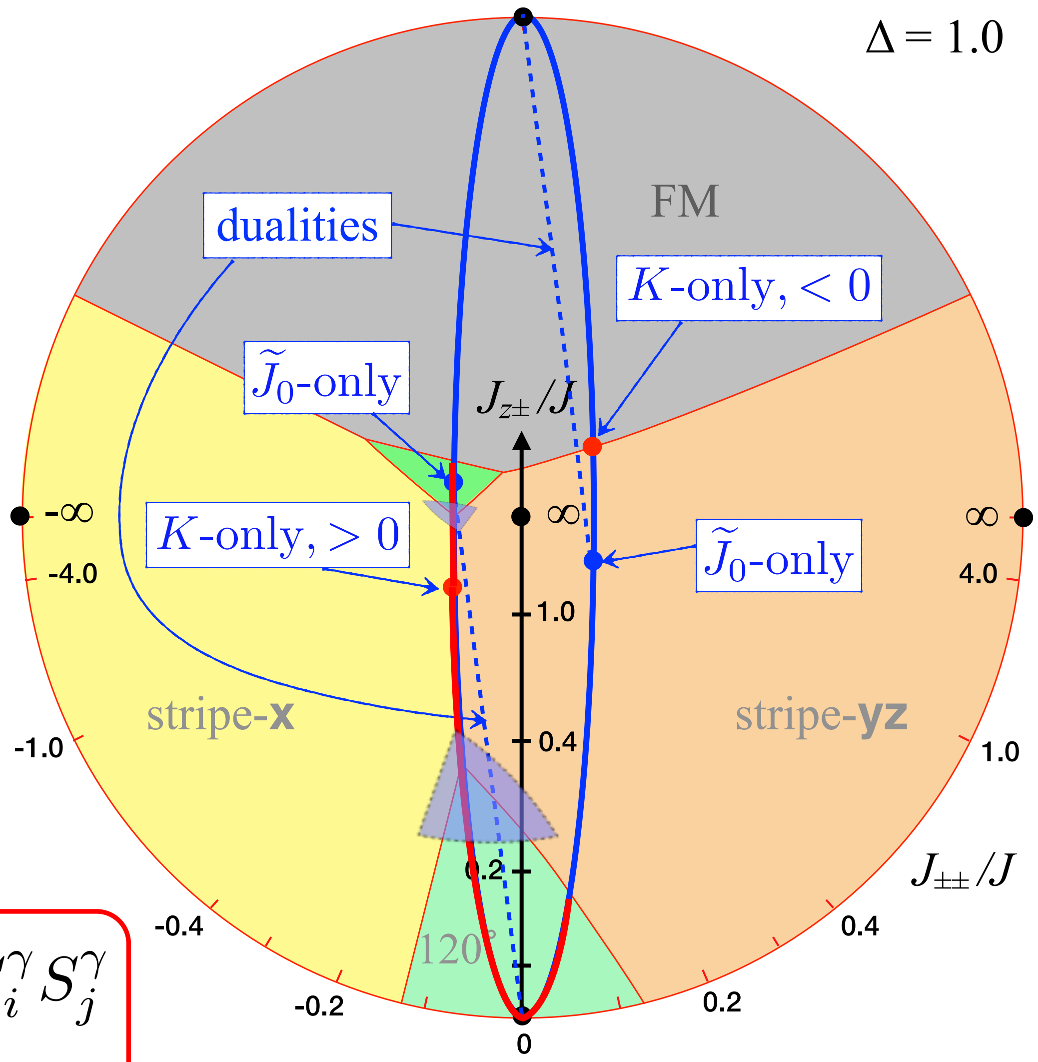
$$\mathcal{H} = -J_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_{\gamma \| \langle ij \rangle} S_i^{\gamma} S_j^{\gamma}$$

I. Rousochatzakis *etal*, PRB **93**, 104417 (2016); M. Becker *etal*, PRB **91**, 155135 (2015)

Klein dualities: spin liquid #2

- dualities of different sectors
- there should exist a **dual SL phase**

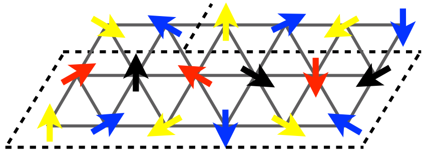


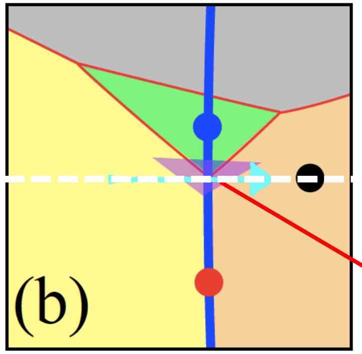


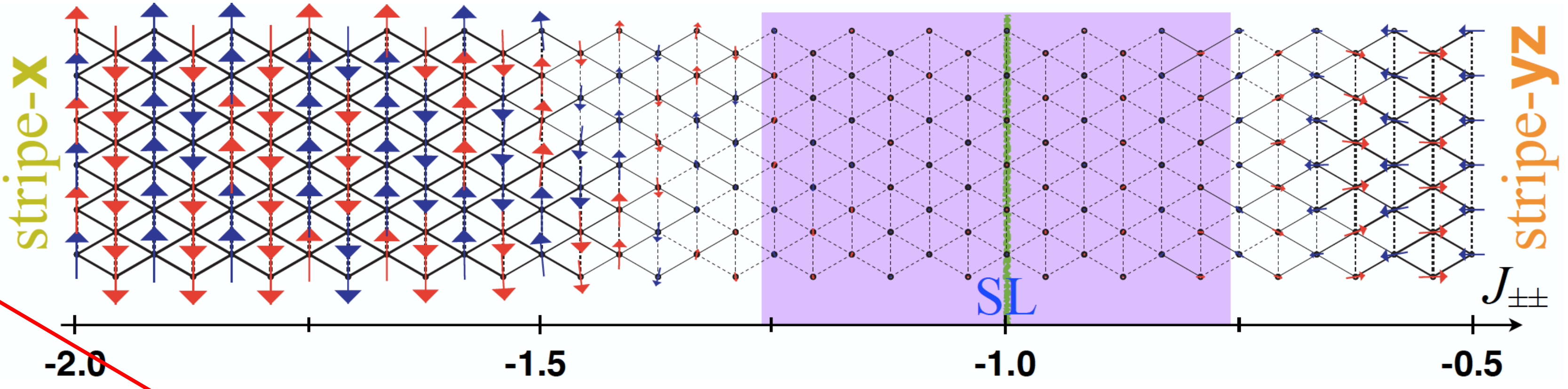
$$\mathcal{H} = -J_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_{\gamma \| \langle ij \rangle} S_i^\gamma S_j^\gamma$$

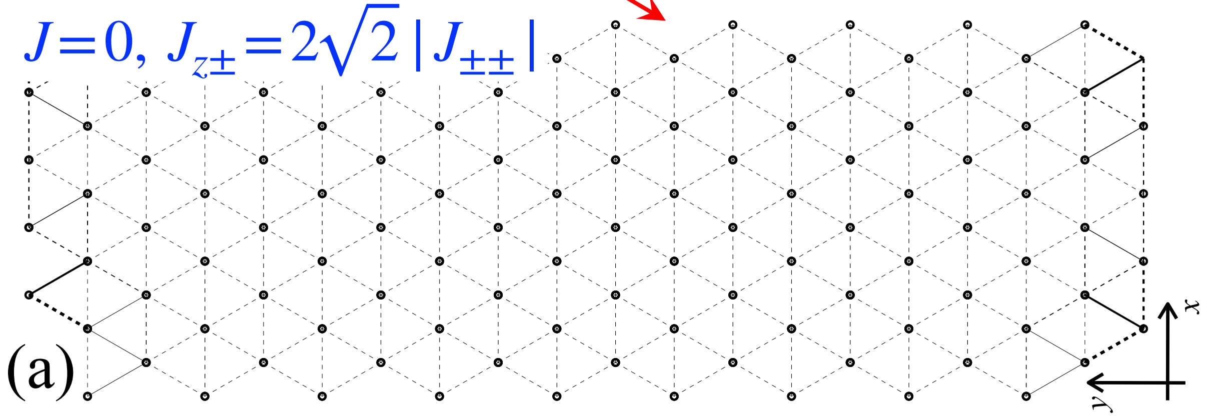
spin liquid **#2**

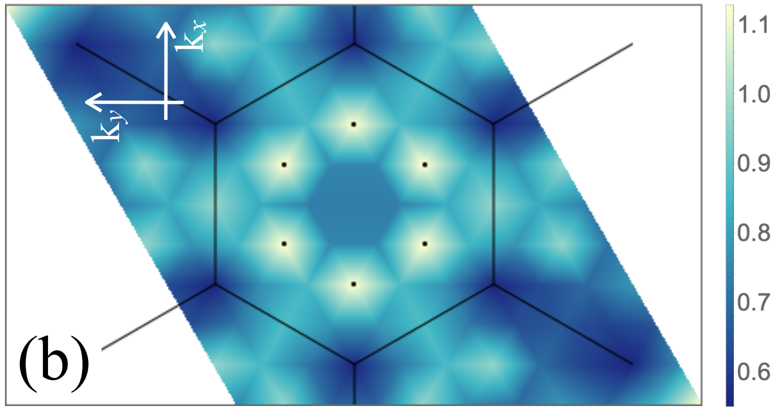
- DMRG "scans", "non-scans", $S(\boldsymbol{q})$
- SL = molten dual 120-phase
- mesmerizing, **all-anisotropic** model (only $J_{\pm\pm}$-$J_{z\pm}$) has an SL, exact solution?









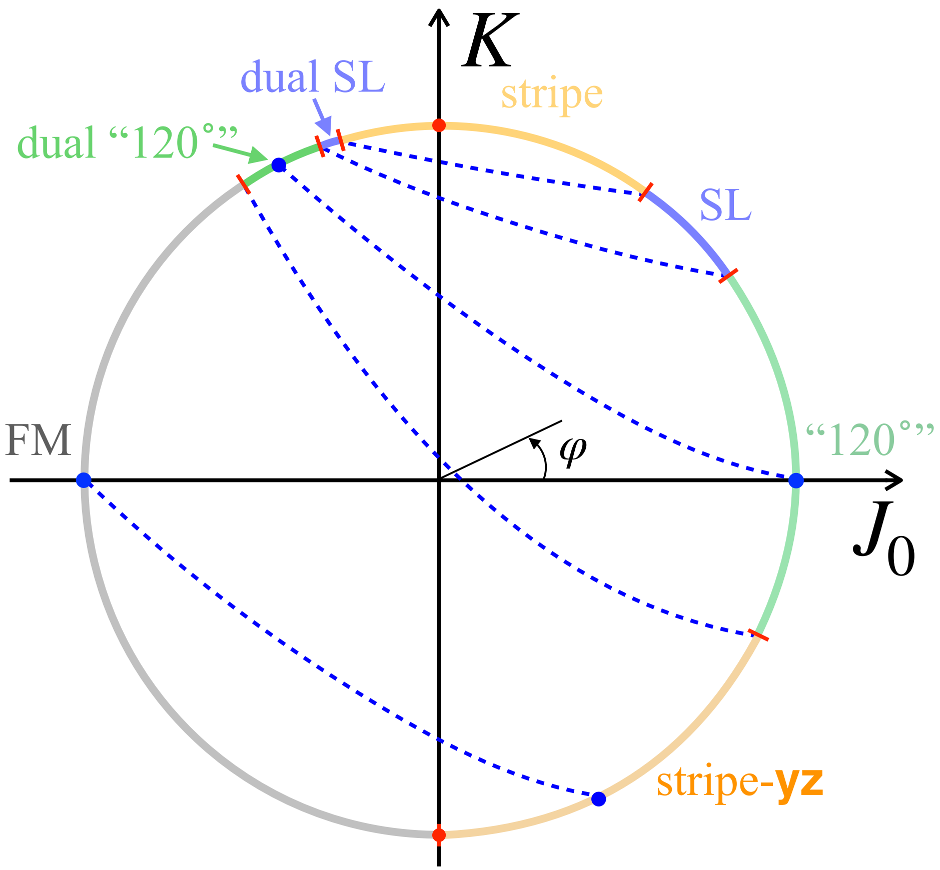


P. A. Maksimov *etal.*, PRX **9**, 021017 (2019).

by-product

- phase diagram of the triangular-lattice ***J-K*** model:
  no Kitaev-like solution, two (dual) SL phases (!)

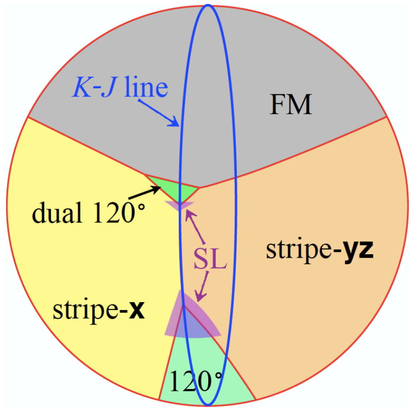
$$\boxed{\mathcal{H} = -J_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_{\gamma \| \langle ij \rangle} S_i^\gamma S_j^\gamma}$$



- "mesmerizing point", **not integrable (ED)**

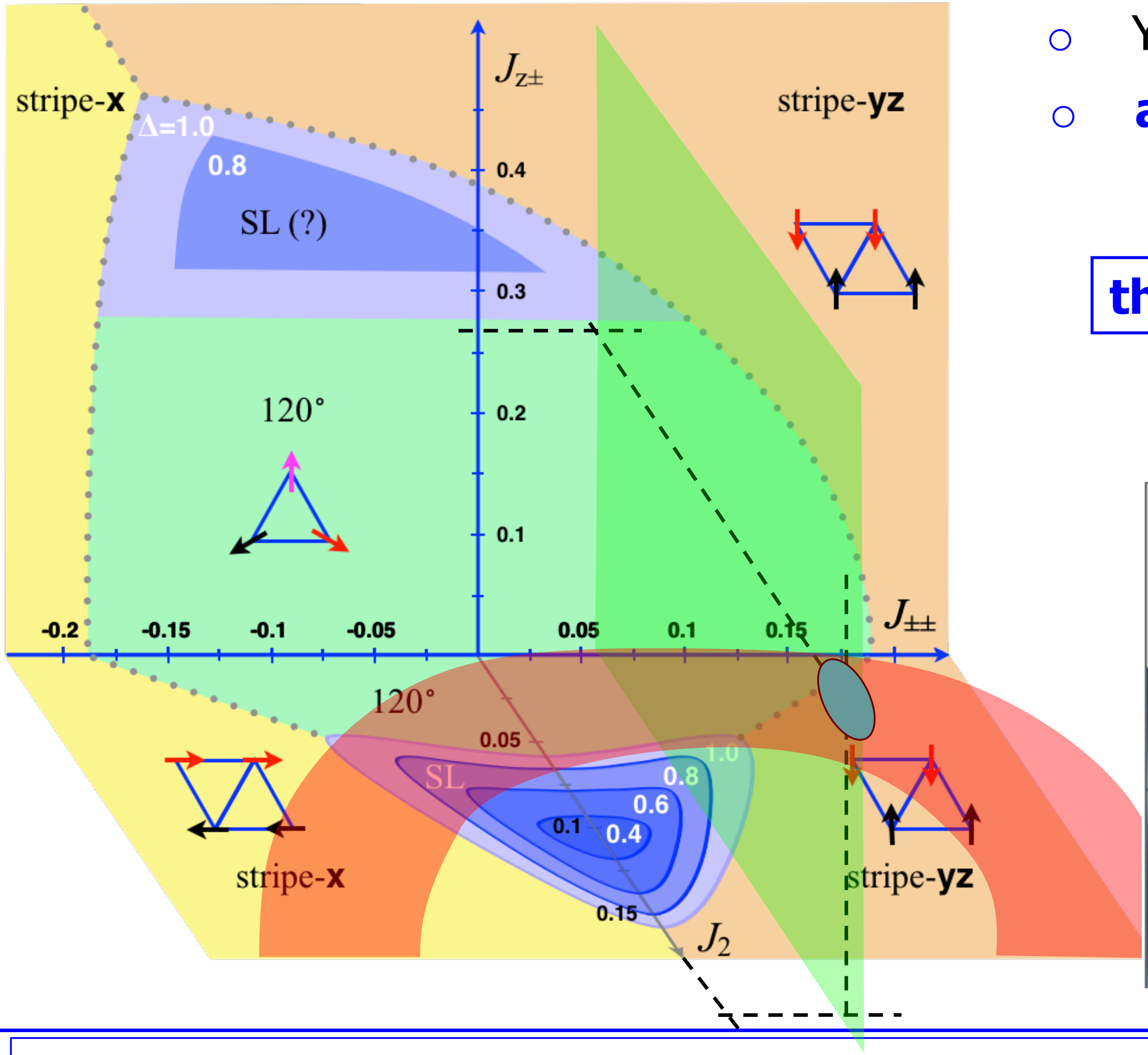
P. A. Maksimov *et al.*, PRX **9**, 021017 (2019).

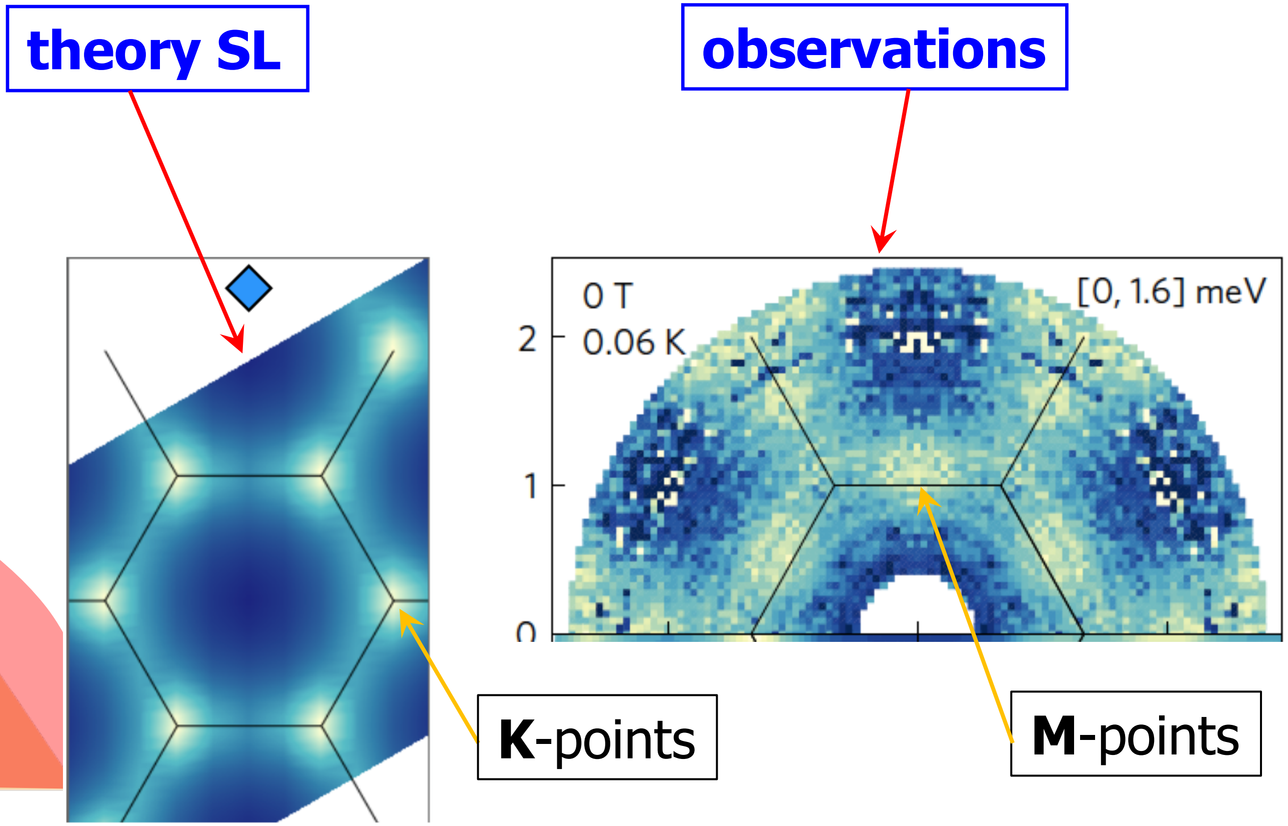
where are we?



“where is disorder-free YMGO?”

- YMGO?: **well outside SL#1, in a stripe phase**
- **also: “wrong SL”!** [$S(\boldsymbol{q})$ different]

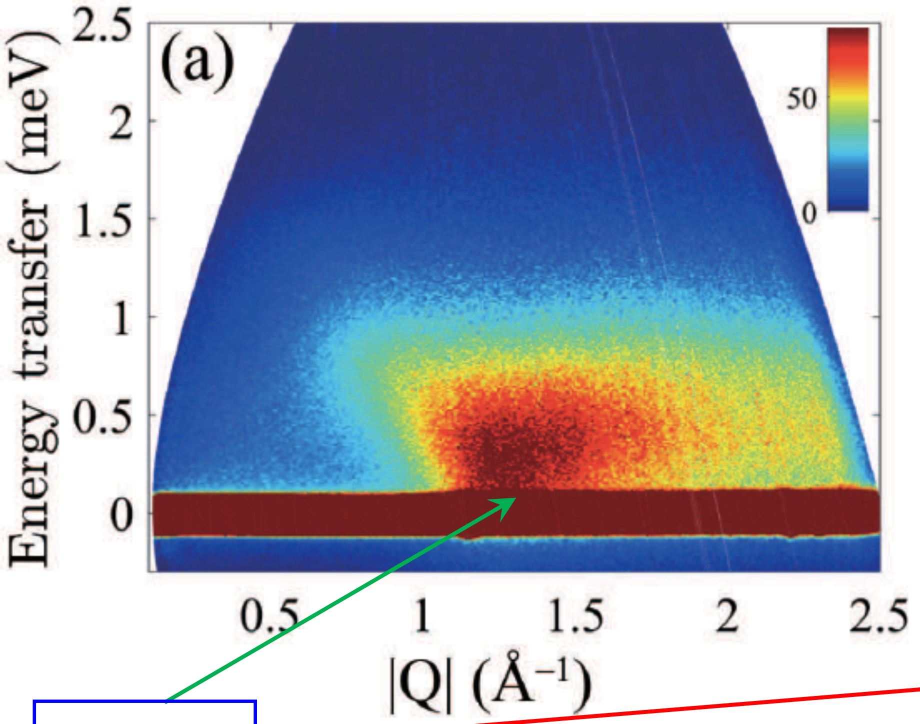


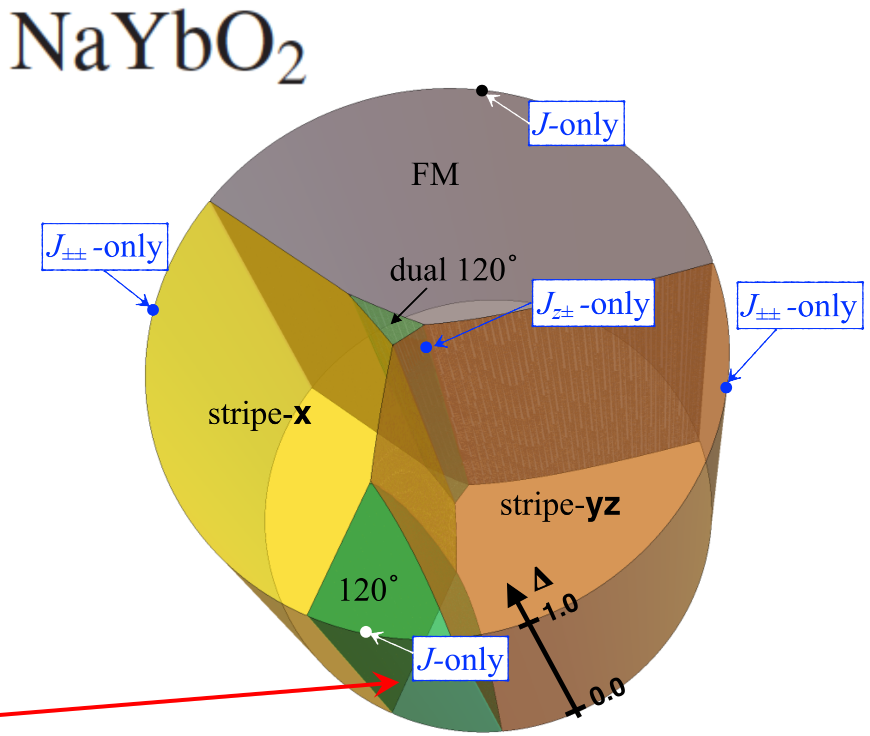


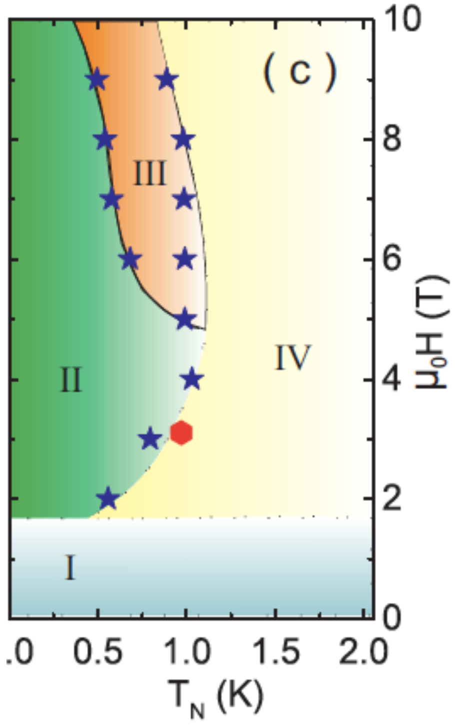
Z. Zhu, *etal*, PRL **119**, 157201 (2017); J. Iaconis *etal*, SciPost Phys. **4**, 3 (2018)

X. Zhang *etal*, Phys. Rev. X **8**, 031001 (2018).

new materials?







- possibly smaller *XXZ* anisotropy (Δ≈1), small other terms
- no ordering (**disordered 120° ? or SL#1**?); max intensity at the **K**-point
- consistent with being in 120° phase (and disordered)
- orders in small fields
- $NaYbS_2$: Δ≈0.3, also not ordered

Ding *etal.*, arXiv:1901.07810; Ranjith *etal.*, PRB **99**, 180401 (2019); Bordelon *etal.*, arXiv:1901.??; Baenitz *etal.*, PRB **98**, 220409(R) (2018).

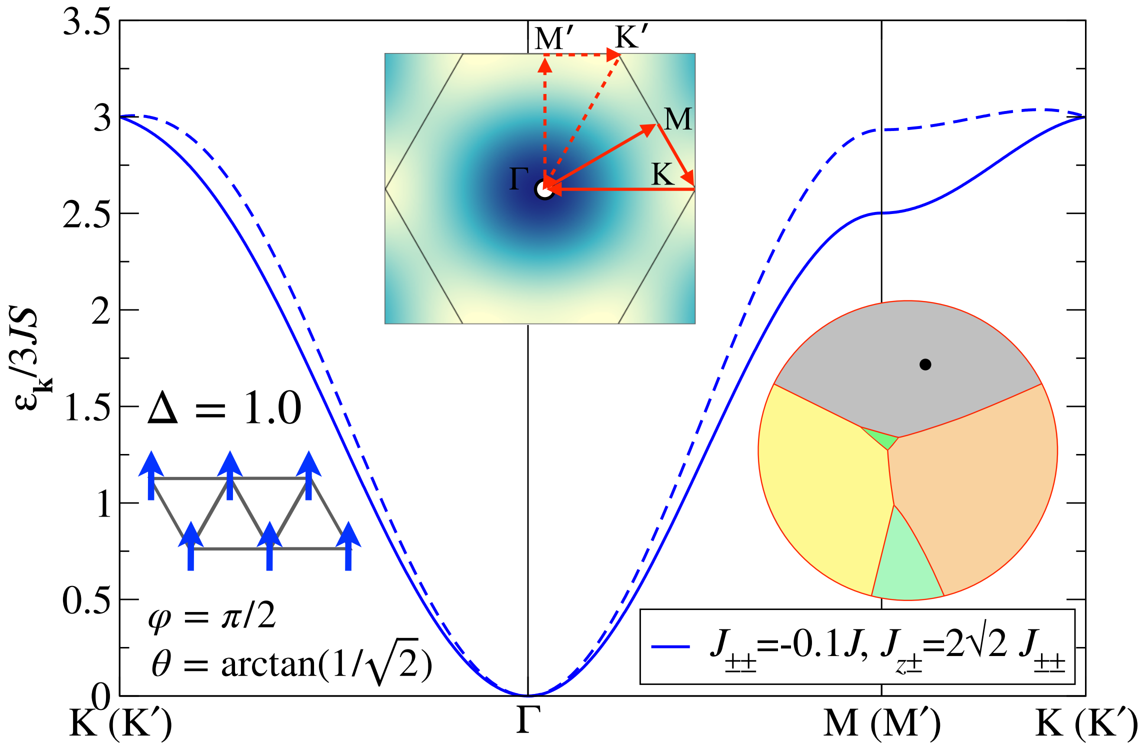
other issues

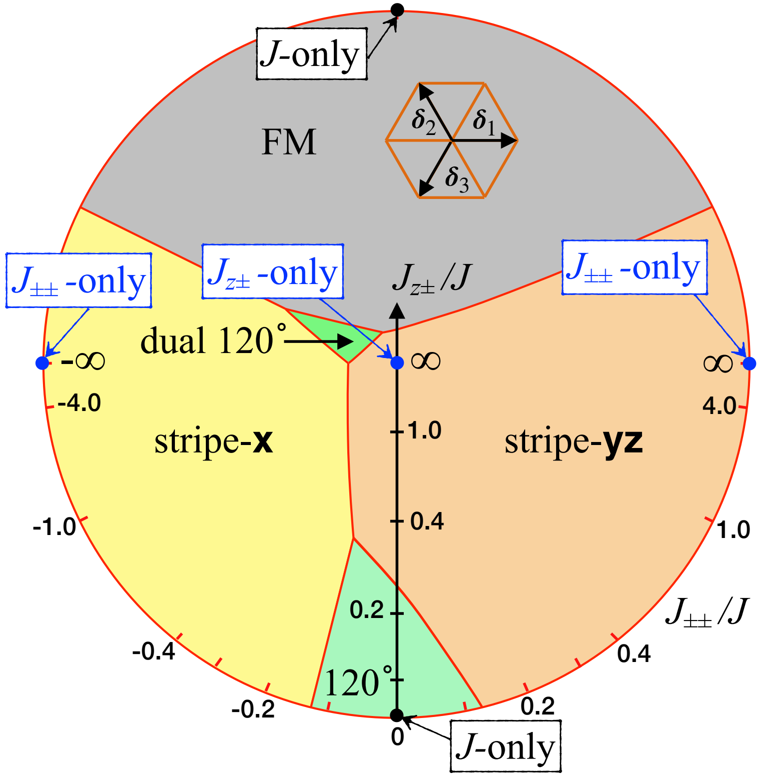
questions, future directions

- is agnosticism justified? can one explore all the phases? Yb^{3+} tend to be *XXZ* or isotropic.
- field-induced phases. YMGO may have one.

FF-word

- **Fluctuating Ferromagnet**: $Yb_2Ti_2O_7$ physics(*) of interacting magnons on a Bravais lattice

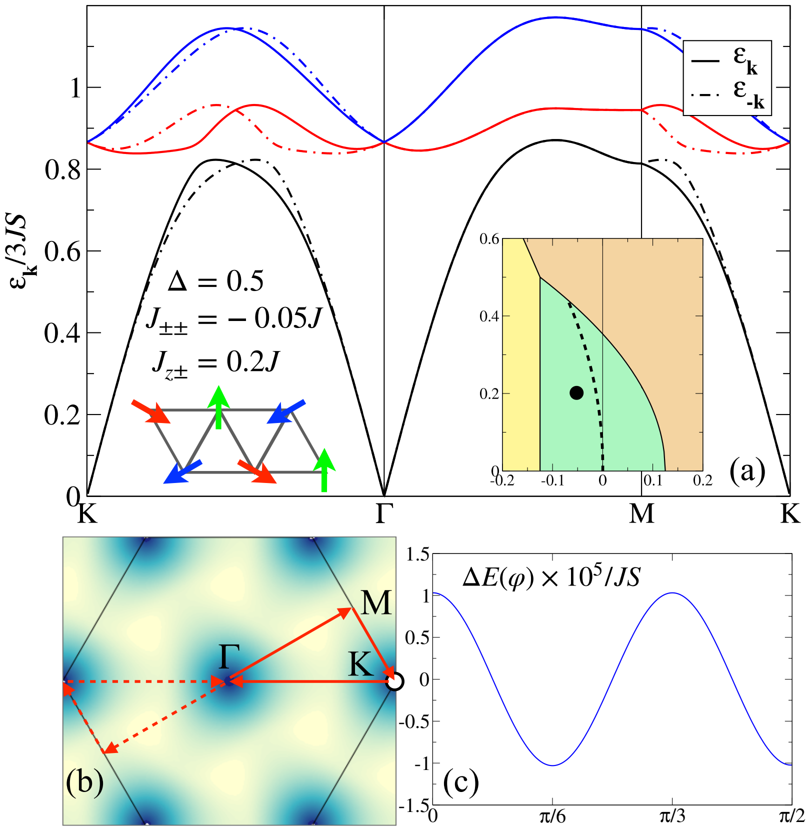


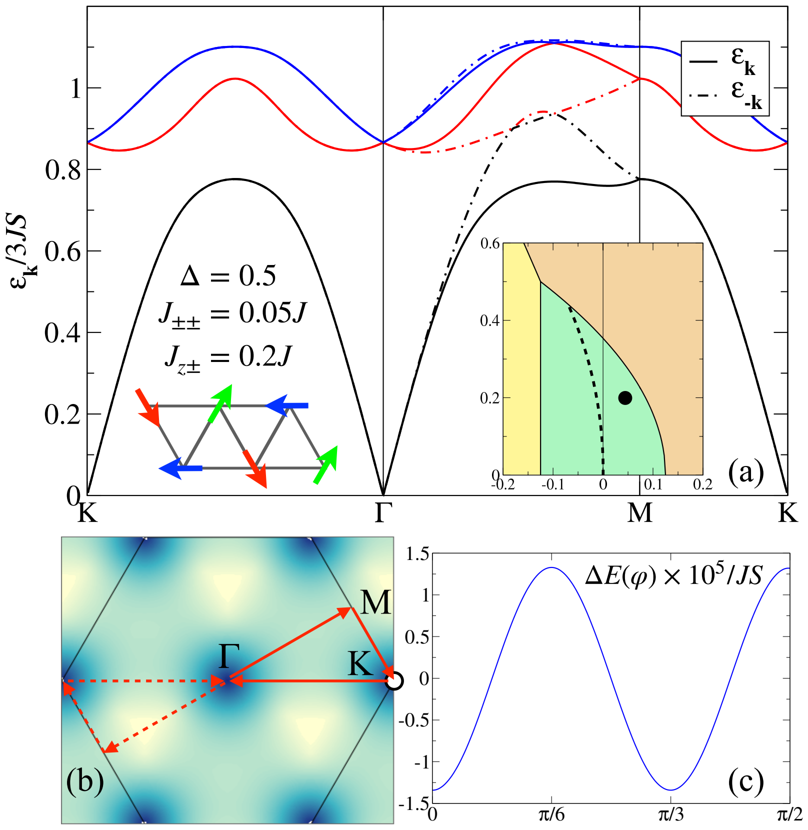


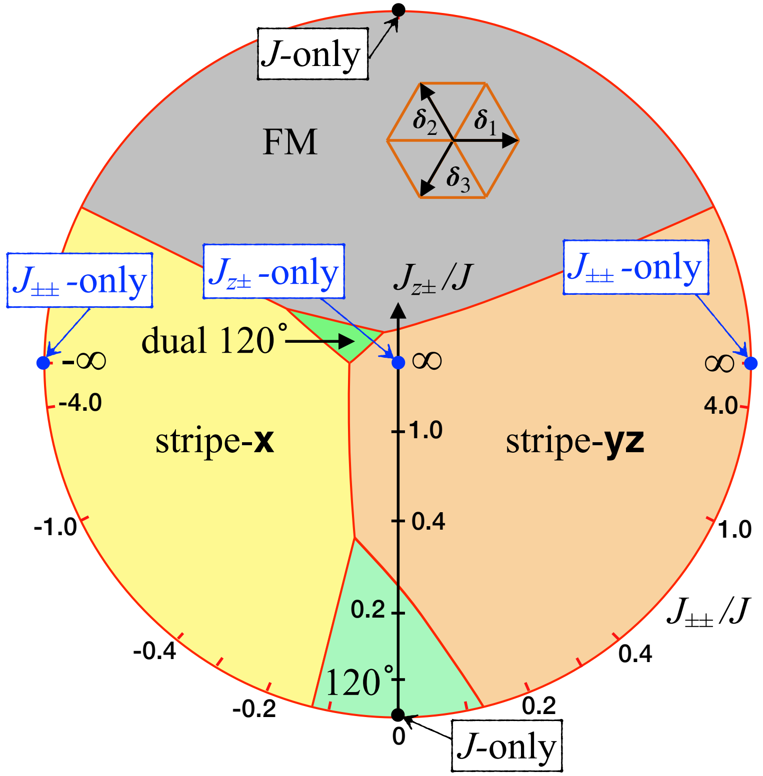
J. Rau *etal*, arXiv:1903.05098.

curiosities: order-by-disorder, non-reciprocity

- **ObD in 120° and FM phases**: classical 120° and FM phases are gapless despite no continuous symmetry in the Hamiltonian. GS "ignores" the bond-dependent terms, spectrum does not ⇛ ObD
- non-reciprocal spectrum in the 120° phase







conclusions

☑ phase diagram of the nearest-neighbor anisotropic-exchange triangular lattice model: **two spin liquids [non-Kitaev]**

☑ framework for many triangular-lattice based systems