Exciton-Polariton Josephson Junctions at Finite Temperatures

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Daejeon, South Korea, 2017

Problems Under Discussion

1. Non-equilibrium effects with exciton-polariton BEC

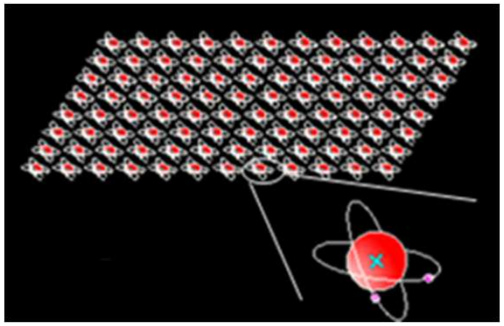
- Exiton-polaritons in 1D potentials ;
- Permanent Rabi oscillations with exciton-polaritons and polariton laser with dynamical parity-time (PT) symmetry.

2. Quantum effects with equilibrium exciton-polariton BEC

- Quantum tunneling problem with exciton-polariton BEC,
- Quantum simulators problem with exciton polaritons

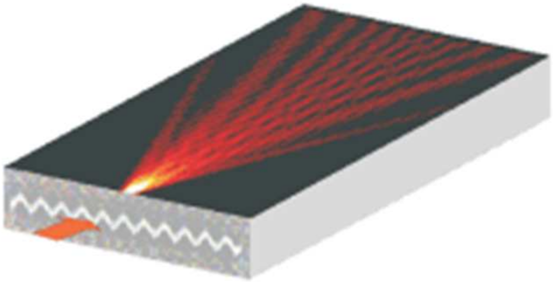
Quantum Simulators

Lattices of atoms



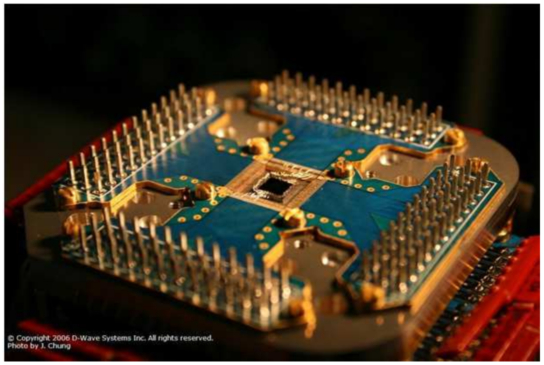
Atomic circuits

Waveguides and circuits



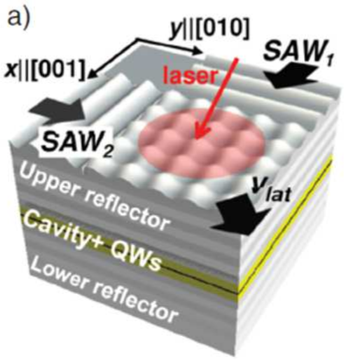
Photonic circuits

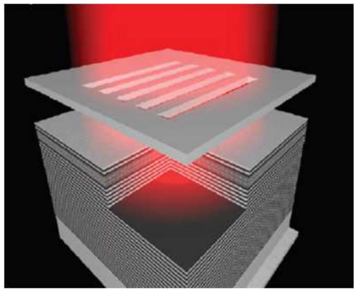
D-Wave machine



Superconductor circuits

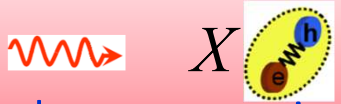
What is about polaritons?



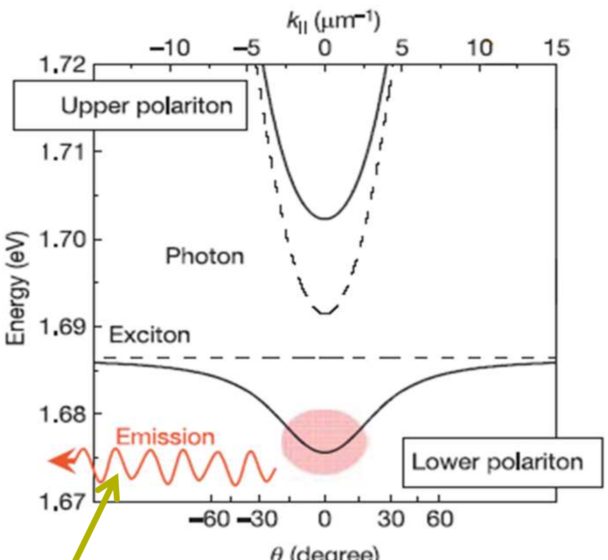


Non-Equilibrium Exciton-Polariton BEC

J. J. Hopfield, *Phys. Rev.* **112** (5), 1555–1567 (1958),
V.M. Agranovich, JETP **37**, 430 (1959),
L.V. Keldysh, et al., JETP, **36**, 1193 (1968).

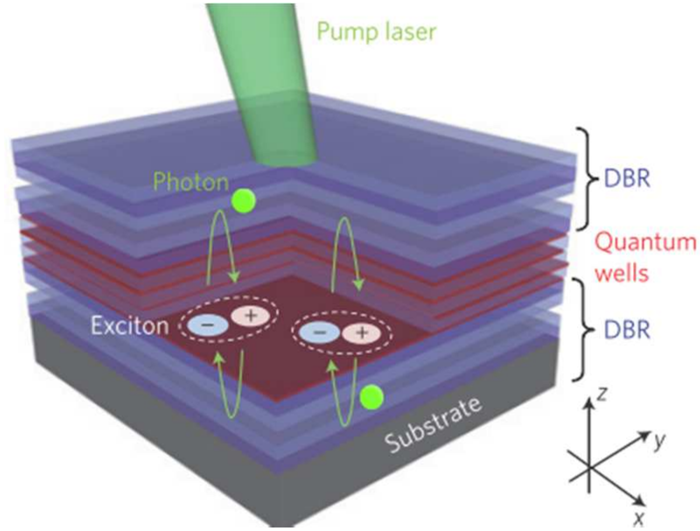
Polariton = C  X
photon exciton

C и X - Hopfield coefficients



LB polariton dispersion in CdTe microcavity

Coherent irradiation



Semiconductor Microcavity

J. Kasprzak et al., *Nature* **443**, 409 (2006) - **CdTe**

Small effective photon mass m_{ph}

$$\mathrm{m}_{\mathrm{pol}} \approx 2\mathrm{m}_{\mathrm{ph}} \sim 10^{-36}\ \mathbf{kg}$$

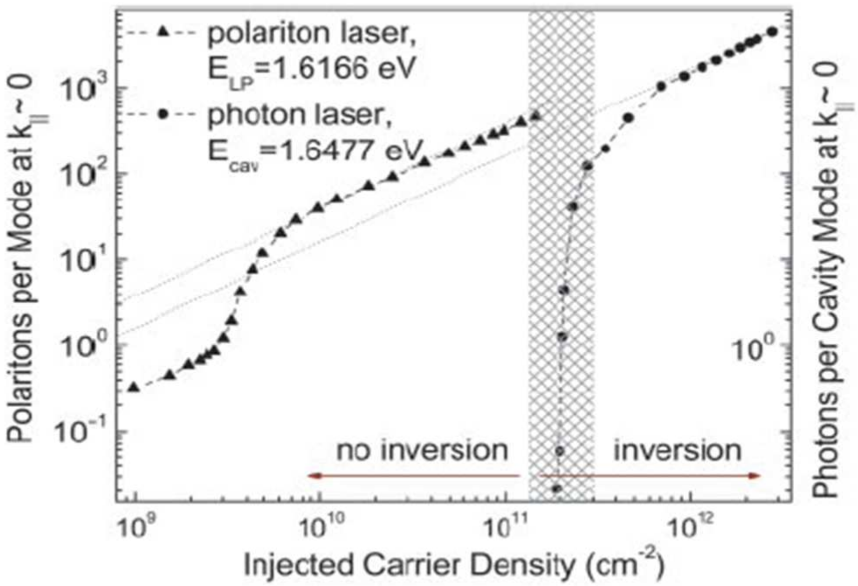
Temperature is

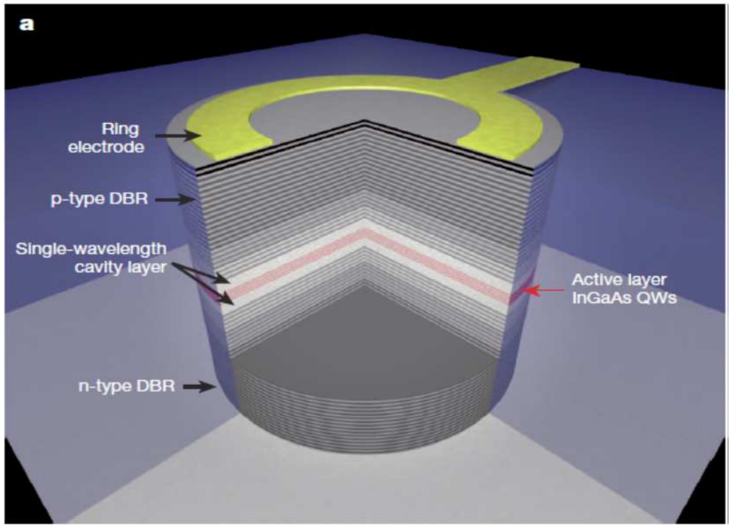
***T*= 5 — 20 K**

1. It is necessary to achieve strong coupling;

2. It is not necessary population inversion

Polariton Laser – New Quantum Source of Light





Advantages

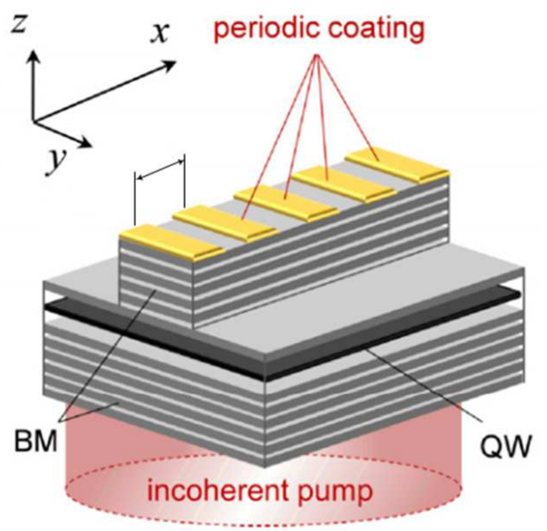
- High flexibility to external optical and/or electrical pump
- Low threshold, few mW,
- High nonlinearity: It is 3 orders bigger than in VCSEL,
- Fast switching (picoseconds) between the states.

Disadvantages

For typically used microcavities Short lifetime – up to tens of ps

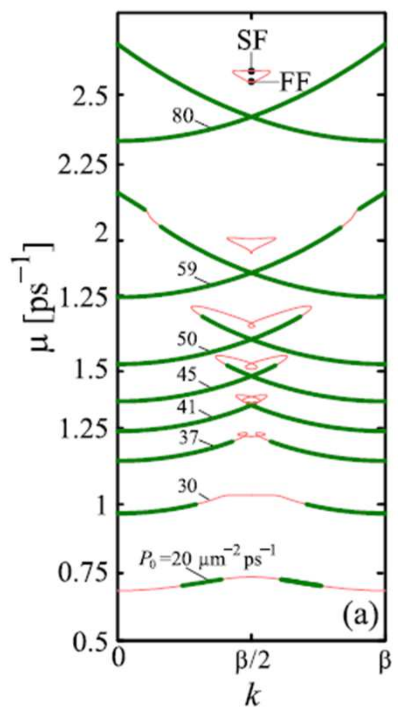
Lattices with Non-equilibrium Condensates

1D periodic condensate lattice

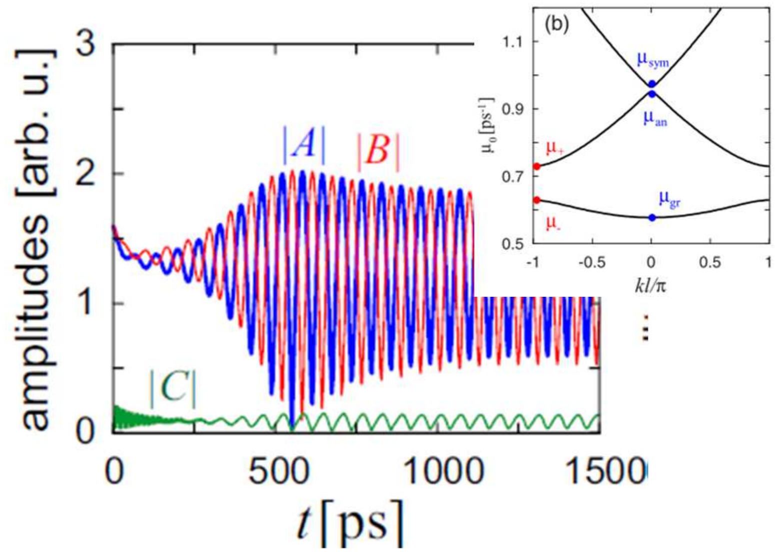


X. Ma, I. Yu. Chestnov, M. V. Charukhchyan, A Alodjants, O. A. Egorov, PRB, **91**, 214301 (2015)

I. Yu. Chestnov, A. V. Yulin, A. P. Alodjants, and O. A. Egorov, PRB, **94**, 094306 (2016)



Dynamically stable persistent oscillations

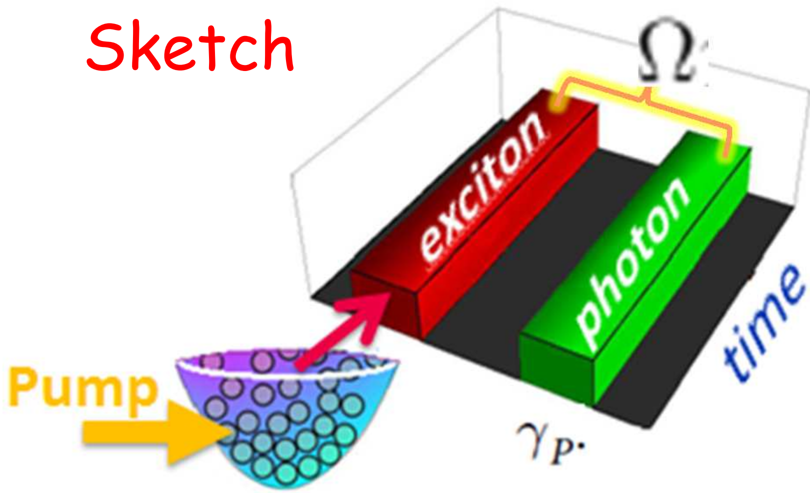


Bifurcation diagram showing the formation of the loops at BZ edge for fixed value of potential depth

✓ We use Wouters model of coupled GPE + reservor

Permanent Rabi Oscillations with Exciton Polaritons Possessing Dynamical PT-symmetry

Sketch



Exciton Reservour

I. Yu. Chestnov, S. S. Demirchyan, A. P. Alodjants, Yuri G. Rubo, A. V. Kavokin, Scientific Reports Vol.6, p.9551 (2016)

Basic Equations

$$\dot{\phi} = -\frac{1}{2}\gamma_P \phi - i\Omega\chi$$ → Photonic Field

$$\dot{\chi} = \frac{1}{2}(p_X - \gamma_X)\chi + i\delta\chi - i\Omega\phi - ig_c|\chi|^2 - ig_R N$$ → Excitonic Field

$$\dot{N} = P - \gamma_R N - p_X|\chi|^2$$ → Excitonic Reservoir Particle Number

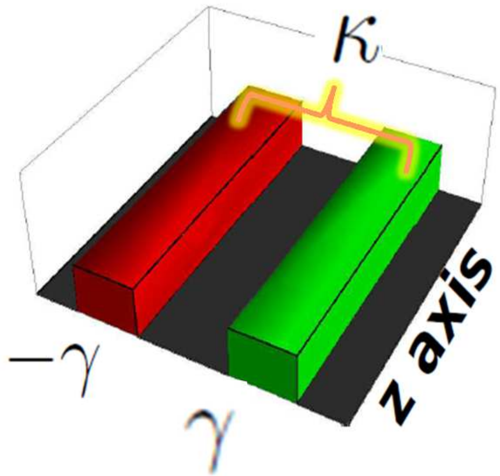
↑ Pump

Stimulated scattering from reservour

$$p_X \equiv p_1 = R_1 N \qquad p_X \equiv p_2 = R_2 N^2 |\chi|^2$$

PT-symmetry in Optics

Tunnely coupled waveguides



An illustration of PT -symmetric dimer waveguide. The green waveguide indicates associated **optical loss** γ while the red waveguide involves an equivalent amount of optical **gain** $-\gamma$
Light is transferred from one waveguide to the other via evanescent coupling . κ

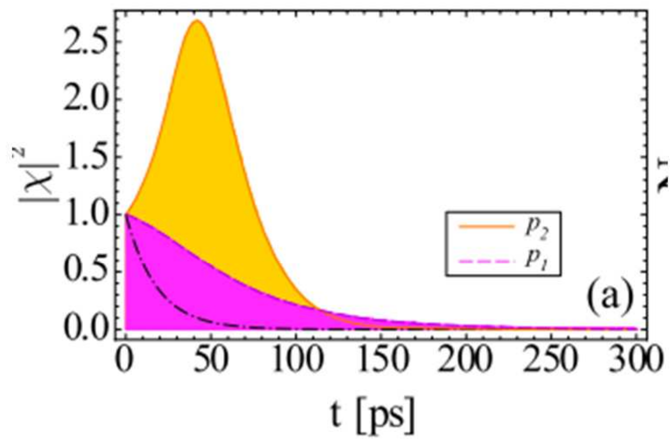
Hamiltonian is $\mathcal{H} = \begin{pmatrix} \epsilon_0 + i\gamma & \kappa \\ \kappa & \epsilon_0 - i\gamma \end{pmatrix}$

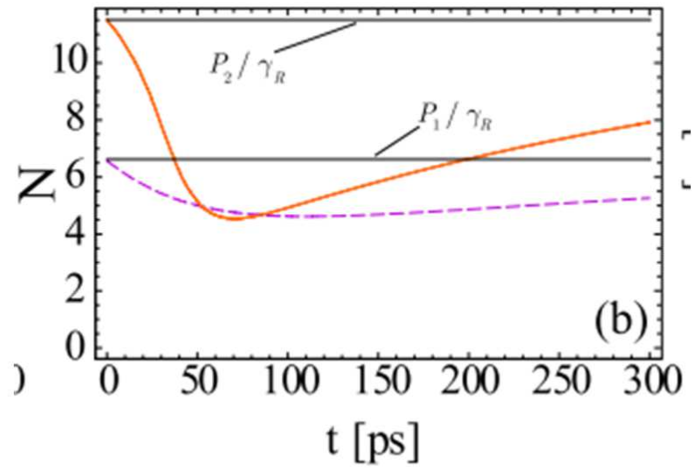
It is easy to see that the dimer is $\mathcal{PT}$-invariant: parity switches between the two waveguides (the diagonal terms in $\mathcal{H}$) while time reversal transforms them back by complex conjugation so that the combined action of $\mathcal{PT}$ leaves Hamiltonian unchanged

Eigenvalues $\mathcal{E}_\pm = \pm\sqrt{\kappa^2 - \gamma^2}$

The sharp transition from a real to a complex spectrum that takes place at $\gamma_{\mathcal{PT}} = \kappa$ is coined spontaneous $\mathcal{PT}$ -symmetry breaking.

Rabi Oscillations Below Threshold





Parameters are:

$\gamma_X = 0.01\,\mathrm{ps}^{-1}, \gamma_P = 0.1\,\mathrm{ps}^{-1}, \hbar\Omega = 2.5\,\mathrm{meV}, \gamma_R = 0.003\,\mathrm{ps}^{-1}, \hbar R_1 = 0.01\,\mu\mathrm{m}^2\mathrm{meV}$
$\hbar R_2 = 0.001\,\mu\mathrm{m}^6\mathrm{meV}$. $P_1 = 0.02\,\mu\mathrm{m}^{-2}\mathrm{ps}^{-1}$ (magenta curve) $P_2 = 0.035\,\mu\mathrm{m}^{-2}\mathrm{ps}^{-1}$ (yellow curve).
Initial conditions are: $\chi(0) = 0,\quad \phi(0) = 1, N(0) = P/\gamma_R.$

Eigen-frequencies are
$$\omega_{1,2} = \frac{i}{4}(\tilde{\gamma} - p_X) + \frac{\delta}{2} \pm \frac{1}{2}\sqrt{4\Omega^2 + \left(\delta - \frac{i}{2}(\gamma_P - \gamma_X + p_X)\right)^2},$$

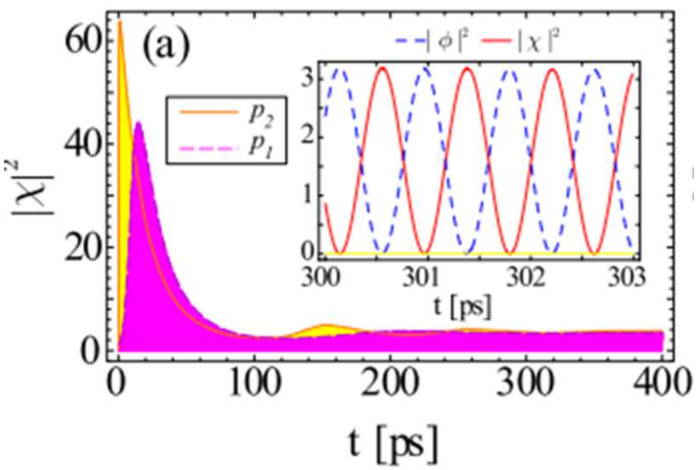
characterizing steady-state solutions $\chi = \chi_0 e^{i\omega t}$ and $\phi = \phi_0 e^{i\omega t}$. Physically, Eq. (3) determines frequencies of upper (ω_2) and lower (ω_1) polariton branches[34], measured with respect to the bare photonic frequency ω_P. In Eq. (3) we denoted $\tilde{\gamma} = \gamma_P + \gamma_X$. In this case, from Eq. (1c) one obtains $P_1^{th} = \tilde{\gamma}\gamma_R/R_1$.

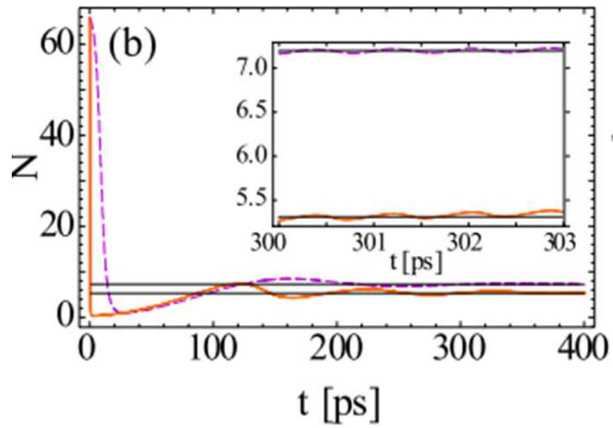
PT-symmetry condition

$$p_X - \gamma_X = \gamma_P.$$
$$\delta = 0,$$

Rabi Oscillations above threshold

$P = 0.2\,\mu\text{m}^{-2}\text{ps}^{-1}$





PT-symmetry condition

$$p_X - \gamma_X = \gamma_P.$$

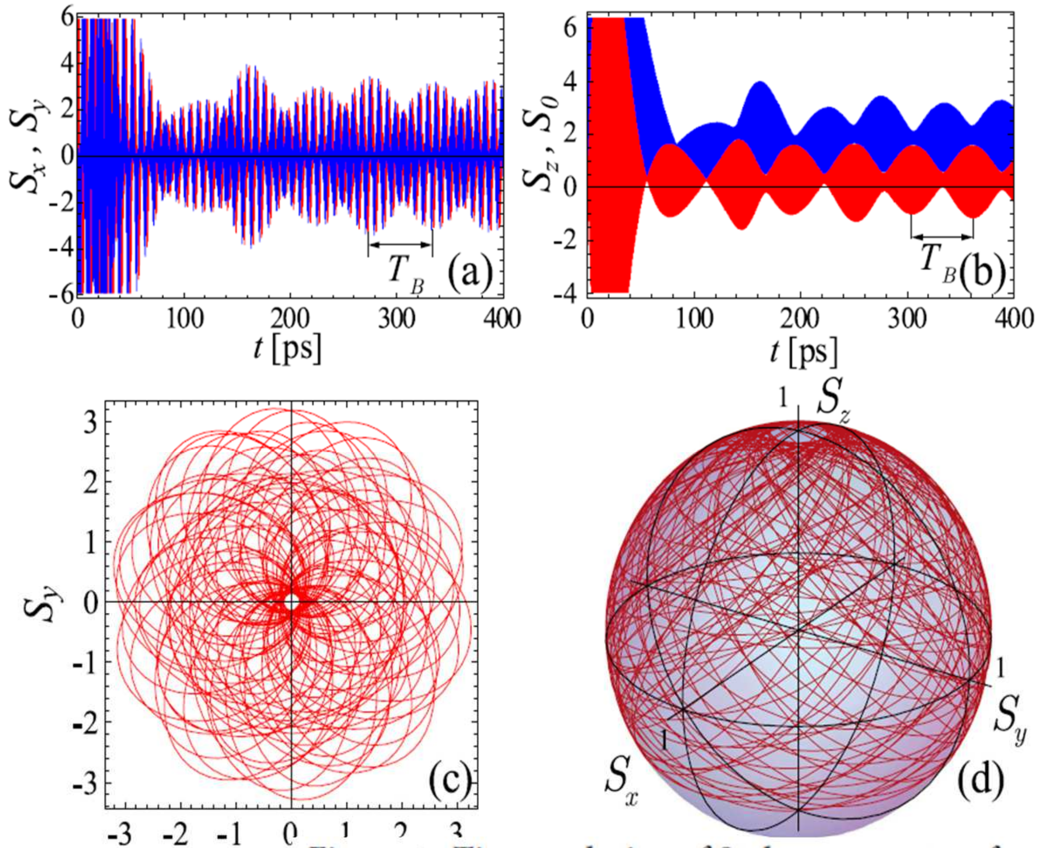
Eigen frequencies

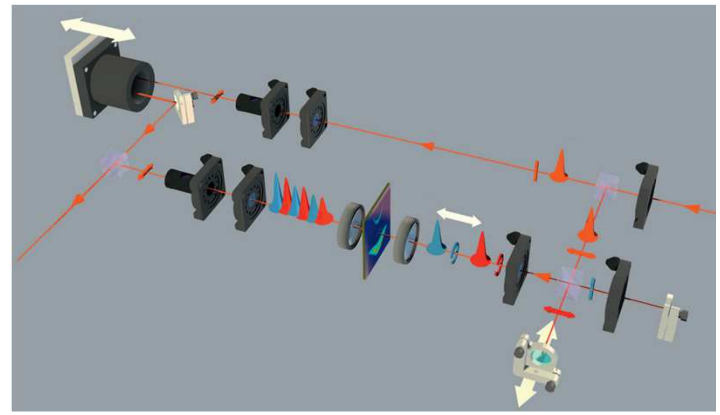
$$\omega_{1,2} = \pm\omega \equiv \pm\sqrt{\Omega^2 - \gamma_P^2/4}.$$

$R_1\overline{N} = \tilde{\gamma},$ that simply implies the balance between pump and loss rates

Polarization properties in the presence of magnetic field

Time evolution of the Stokes parameters





David Colas,et al,
Light: Science & Applications (2015)

$$T_B = 2\pi/\omega_B = 2\pi/(\omega_{R+} - \omega_{R-}) \approx \frac{2\pi\Omega}{\Delta\Delta_Z}$$

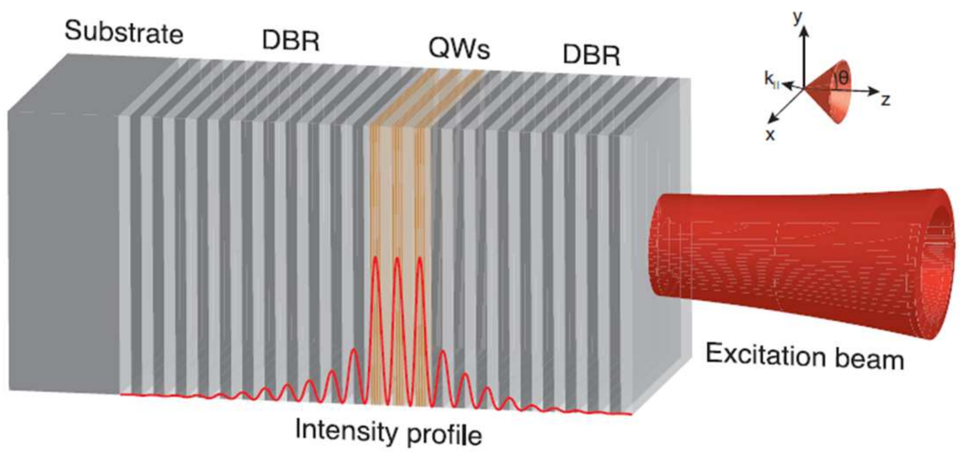
Figure 4. Time evolution of Stokes parameters for $\Delta_Z = 0.2\Omega$, $\Delta = 0.15\Omega$ for the case of the exciton state pumping by exciton-exciton scattering ($p_X[N] = p_2$). The panel (a) shows the behavior of S_x (red) and S_y (blue) components, while S_z (red) and S_0 (blue) component is shown in the panel (b). The evolution of polarization on (S_x, S_y)-plane and on Poincare sphere on the time scale of $T_B = 2\pi/\omega_B$ characterizing permanent Rabi oscillations beating period is shown in the panels (c,d), respectively. The parameters are the same as in Fig. 1, excepting $P = 0.2\,\mu\text{m}^{-2}\,\text{ps}^{-1}$.

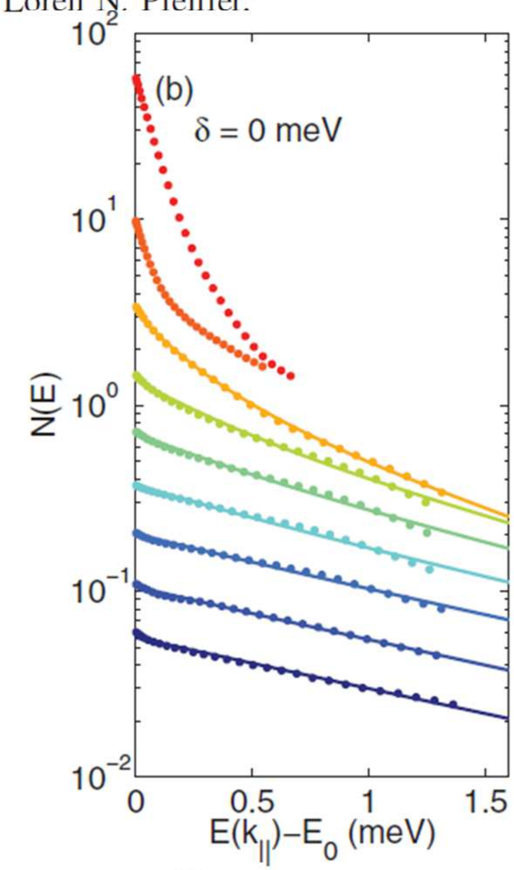
Selected for a Viewpoint in *Physics*
PRL **118,** 016602 (2017) PHYSICAL REVIEW LETTERS week ending 6 JANUARY 2017

Bose-Einstein Condensation of Long-Lifetime Polaritons in Thermal Equilibrium

Yongbao Sun,[1,*] Patrick Wen,[1] Yoseob Yoon,[1] Gangqiang Liu,[2] Mark Steger,[2] Loren N. Pfeiffer,[3] Ken West,[3] David W. Snoke,[2,†] and Keith A. Nelson[1]

Ga As microstructure





P_{BE} 443 mW

$T_{bath} = 12.5$ K

Polariton lifetime is about of **270 ps** at resonance

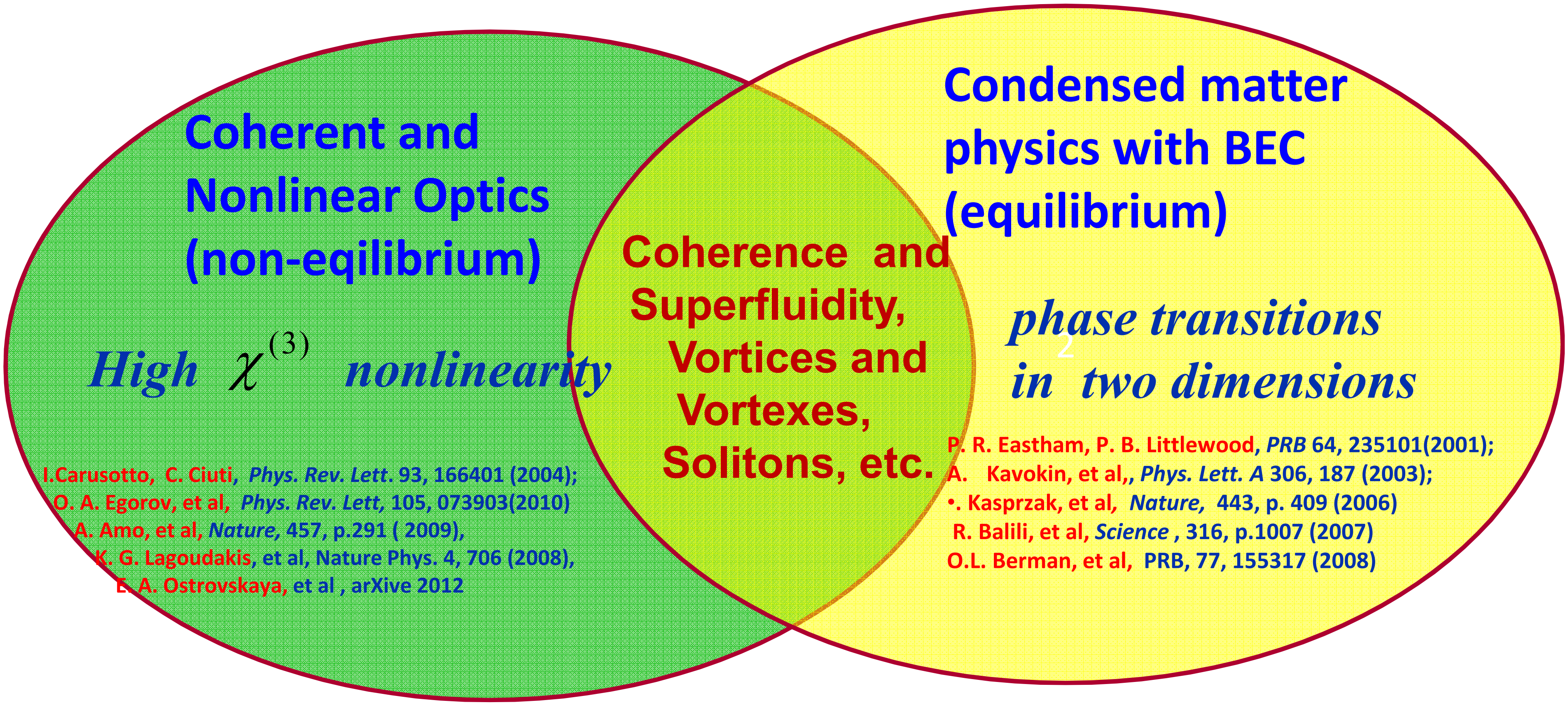
Current activity with polaritons in semiconductor structures

Polariton lifetime is **less than 10 ps,**

Polariton lifetime is **270 ps** at resonance,
PRL 118, 016602 (2017)

$\tau_{lifetime} \leq T_{therm}$

$\tau_{lifetime} >> T_{therm}$



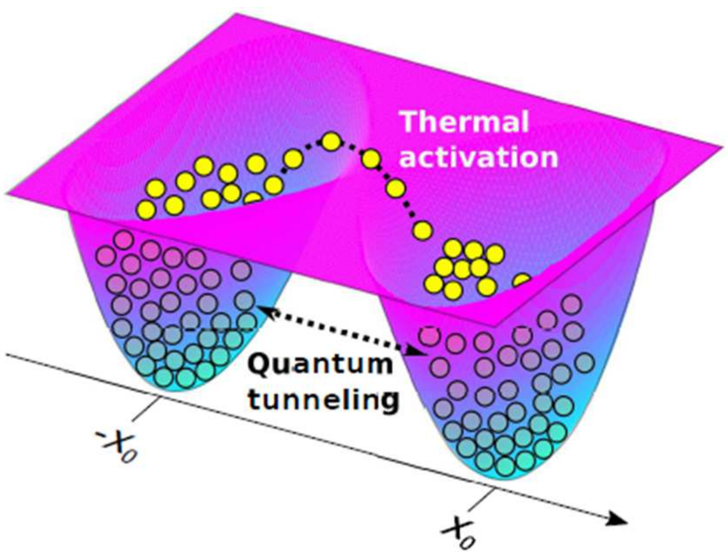
Josephson junction Problem at Finite Temperature

Josephson junctions in atomic BEC at (T=0)

Anthony J. Leggett , Rev. Mod. Phys. 73, 307 (2001)
Oliver Morsch, Markus Oberthaler, Rev. Mod. Phys., Vol. 78 (2006)

Josephson junctions in non-equilibrium Exiton-polariton BECs

I. Aleiner, B. Altshuler, Y. Rubo, Phys. Rev. B 85 (2012).
M. Abbarchi, et al. Nat. Phys. 9, 275 (2013).
K. Lagoudakis, et al, Phys. Rev. Lett. 105, 120403 (2010).
L. Dominici, et al, Phys. Rev. B 78 (2008).
M. Borgh, J. Keeling, J. & N. Berloff, Phys. Rev. B 81 (2010).



What is happen at equilibrium and at relatively high temperatures ?

M. E. Lebedev, A Kavokin, A. Alodjants, et al, Scientific Reports, 2017

Dissipative tunneling problem

A. Larkin, & Y. Ovchinnikov.
Sov. JETP Letts., issue 37/7, p. 322 (1983).

Non-Standard Bose-Hubbard model

Hamiltonian

$$\hat{H} = \frac{A}{2}(\hat{\psi}_1^{\dagger 2}\hat{\psi}_1^2 + \hat{\psi}_2^{\dagger 2}\hat{\psi}_2^2) - \frac{G}{2}(\hat{\psi}_1^{\dagger}\hat{\psi}_2 + \hat{\psi}_1\hat{\psi}_2^{\dagger})$$

$$-\frac{\Gamma}{2}(\hat{\psi}_1^{\dagger 2}\hat{\psi}_1\hat{\psi}_2 + \hat{\psi}_1^{\dagger}\hat{\psi}_1^2\hat{\psi}_2^{\dagger} + \hat{\psi}_1^{\dagger}\hat{\psi}_2^{\dagger}\hat{\psi}_2^2 + \hat{\psi}_1\hat{\psi}_2^{\dagger 2}\hat{\psi}_2 + \frac{C}{2}(\hat{\psi}_1^{\dagger 2}\hat{\psi}_2^2 + 4\hat{\psi}_1^{\dagger}\hat{\psi}_1\hat{\psi}_2^{\dagger}\hat{\psi}_2 + \hat{\psi}_1^2\hat{\psi}_2^{\dagger 2}),$$

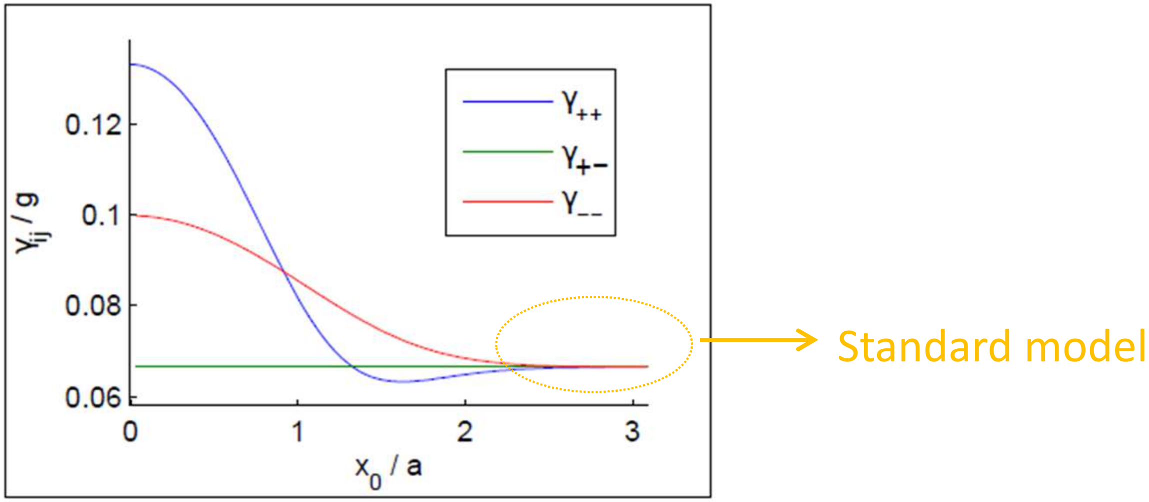
Definitions $\Gamma = \frac{1}{2}(\gamma_{++} - \gamma_{--})$; $C = \frac{1}{4}(\gamma_{++} + \gamma_{--} - 2\gamma_{+-})$; $G = \mu_- - \mu_+ - 2\Gamma$

$$A = \frac{1}{4}(\gamma_{++} + \gamma_{--} + 6\gamma_{+-});$$

$\Phi_\pm(x) = \pm\Phi_\pm(-x)$ are symmetrical and anti-symmetrical wave functions respectively

Coupling coefficients are

$$\gamma_{ij} = g\int \Phi_i^2\Phi_j^2 dx, i,j \in \{+,-\}$$



Mapping to Spin Model

Transformations for Pseudo-spin S_j and ϕ Phase

$$\hat{S}_x = \frac{1}{2}(\hat{\psi}_1^\dagger \hat{\psi}_2 + \hat{\psi}_1 \hat{\psi}_2^\dagger)$$

$$\hat{S}_y = \frac{i}{2}(\hat{\psi}_1^\dagger \hat{\psi}_2 - \hat{\psi}_1 \hat{\psi}_2^\dagger)$$

$$\hat{S}_z = \frac{1}{2}(\hat{\psi}_2^\dagger \hat{\psi}_2 - \hat{\psi}_1^\dagger \hat{\psi}_1)$$

Spin Hamiltonian

$$\hat{H} = \alpha \hat{S}_z^2 + \beta \hat{S}_x^2 - B\hat{S}_x$$

Cost function Hamiltonian ($\alpha \hat{S}_z^2 + \beta \hat{S}_x^2$)

"Kinetic energy" Hamiltonian ($B\hat{S}_x$)

with $\alpha = A - C = 2\gamma_\pm > 0$, $B = \Gamma N + G > 0$, $\beta = 2C > 0$

Mapping to Quantum Phase

Transformations for pseudo-spin S_j and ϕ Phase

$$\hat{S}_x = s\cos\phi - \sin\phi\frac{d}{d\phi}, \quad \hat{S}_y = s\sin\phi + \cos\phi\frac{d}{d\phi}, \quad \hat{S}_z = -i\frac{d}{d\phi},$$

where $s = N/2$

Quantum phase Hamiltonian

$$H = -(\alpha - \beta\sin^2\phi)\frac{d^2}{d\phi^2} - (\beta\ s\sin 2\phi - B\sin\phi)\frac{d}{d\phi} - Bs\cos\phi - \beta s^2\sin^2\phi - \beta s\cos^2\phi$$

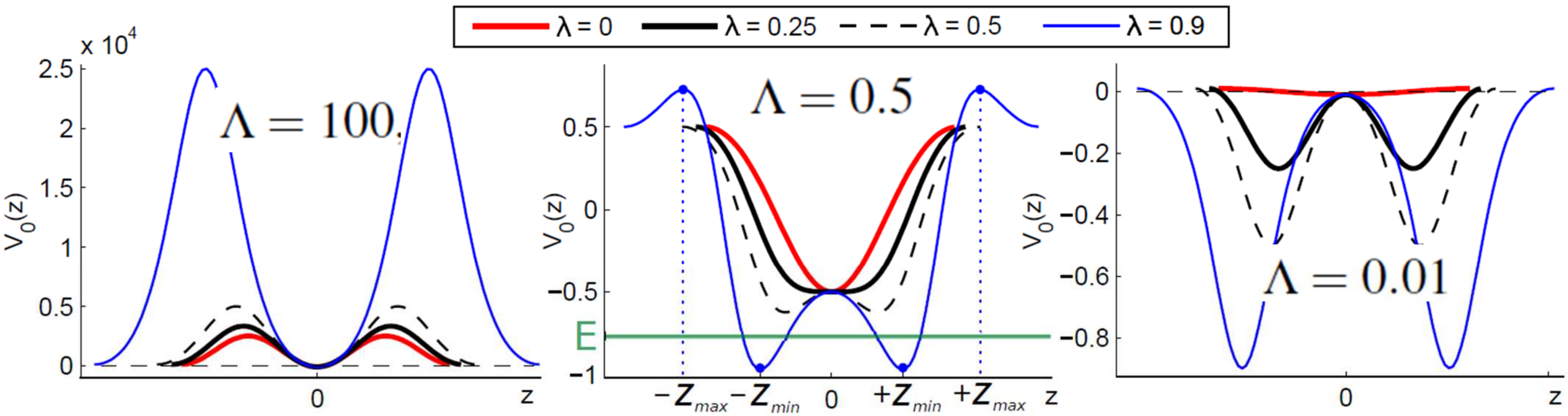
Schrodinger equation $$\alpha\frac{d^2\Psi}{dz^2} + (E - V(z))\Psi = 0.$$

with "mass" $m = \frac{\hbar^2}{2\alpha}$

and moving in potential $$V_0(z) = \frac{(\frac{1}{4}\Lambda^2 - \lambda(1-\lambda))\mathrm{sn}^2 z - \Lambda\mathrm{cn} z}{\mathrm{dn}^2 z}$$

where $\Lambda = \frac{B}{\alpha s}, \lambda = \frac{\beta}{\alpha}$ and $z = \int_0^\phi \frac{d\xi}{\sqrt{1-\lambda\sin^2\xi}}$ Is phase variable

Effective Quantum Phase Potential



Rabi regime $\Lambda \gg 1.$

Josephson regime $1/N^2 \ll \Lambda < 1.$

Fock regime $\Lambda \ll 1/N^2$

$$\Lambda = \frac{2B}{\alpha N}$$

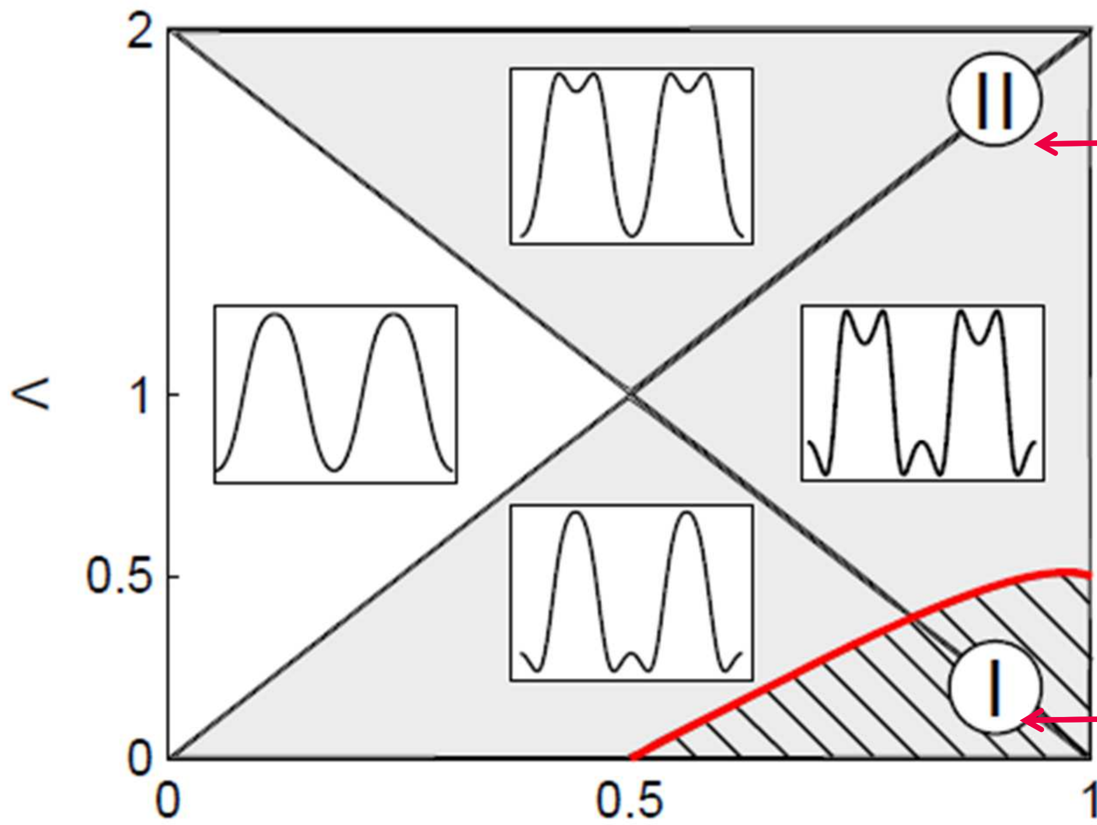
$$\lambda = \frac{\beta}{\alpha}$$

Phase Diagram

$$\hat{H} = \alpha \hat{S}_z^2 + \beta \hat{S}_x^2 - B\hat{S}_x$$

Cost function Hamiltonian

"Kinetic energy" Hamiltonian



Second order phase transition

First order phase transition

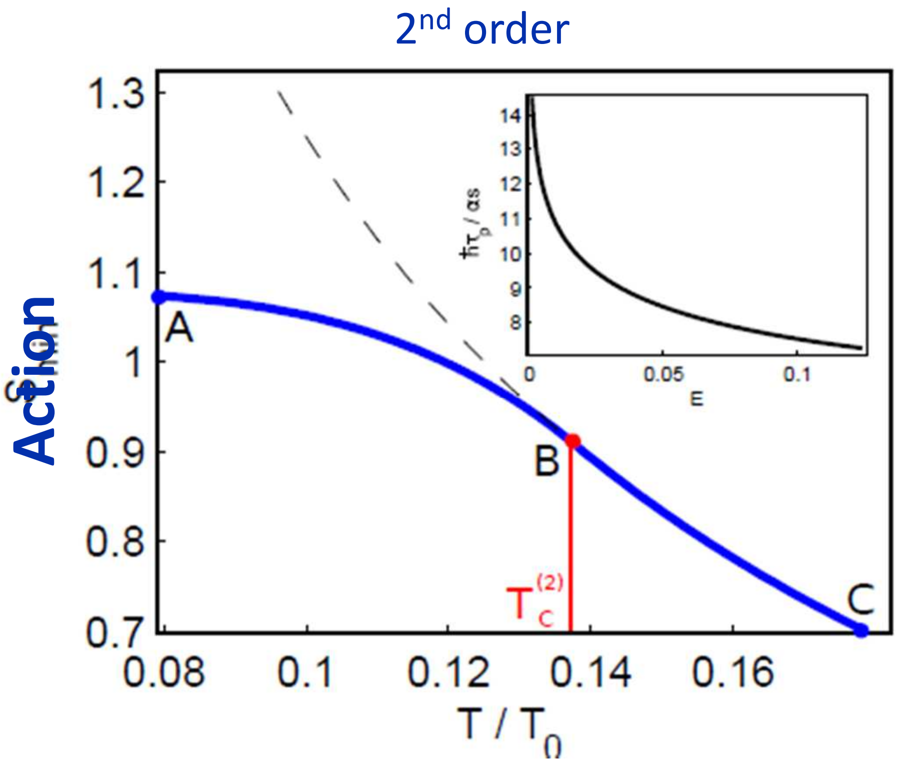
$$\lambda = \frac{\beta}{\alpha}$$

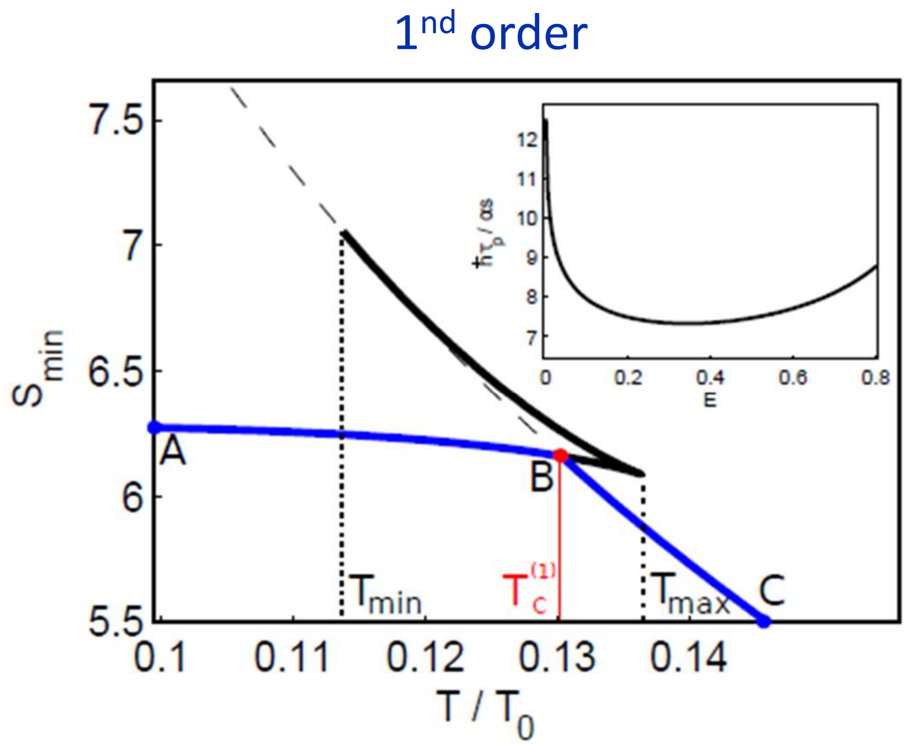
$$\Lambda = \frac{2B}{\alpha N}$$

Boundary condition

$$\Lambda = \frac{1 - 16\lambda + 16\lambda^2 + \sqrt{1 + 32\lambda - 32\lambda^2}}{4(2\lambda - 1)}$$

Quantum – Classical Phase Transitions



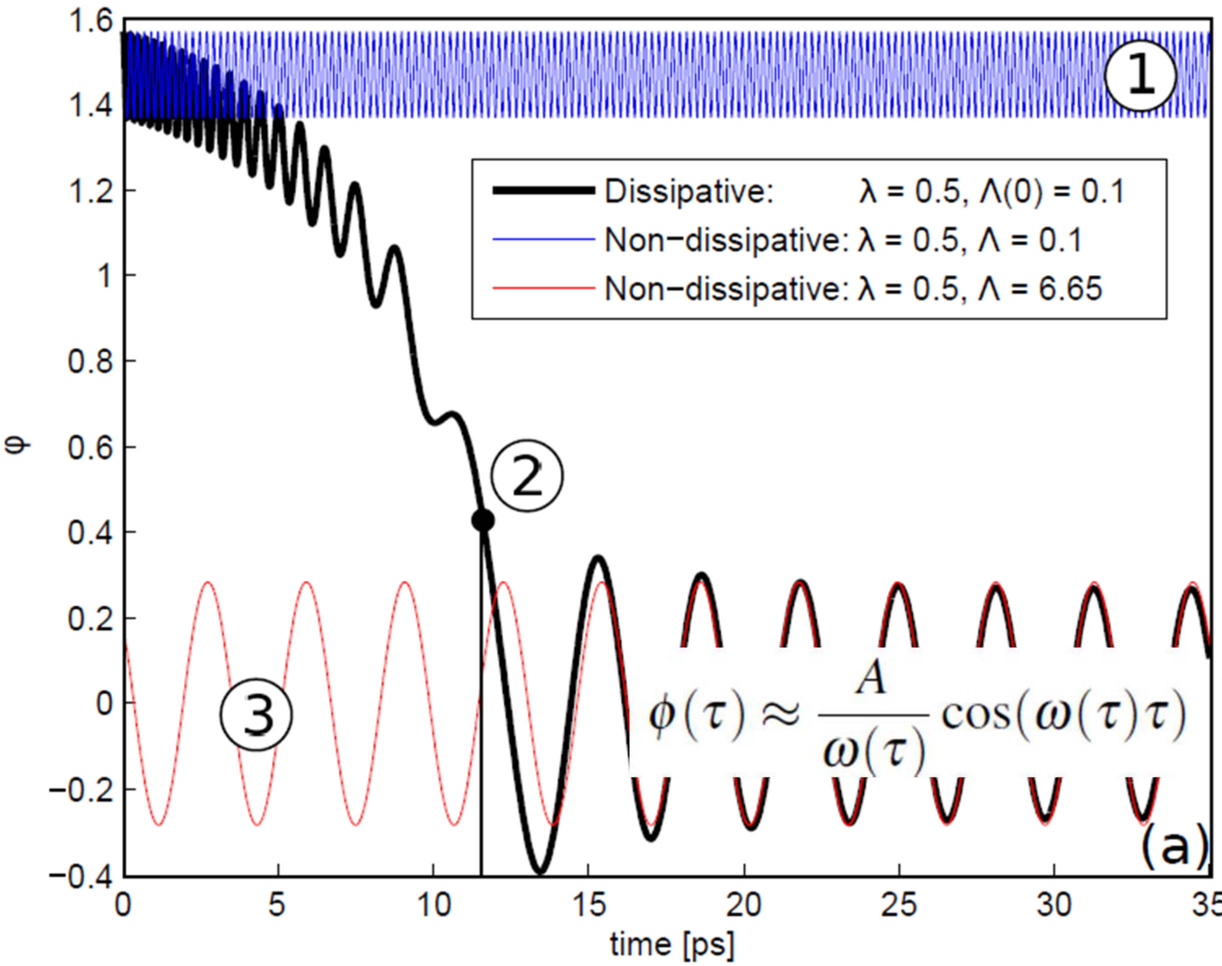


$T_0 = \alpha N / 2\pi k_B$ Is temperature of blue shift

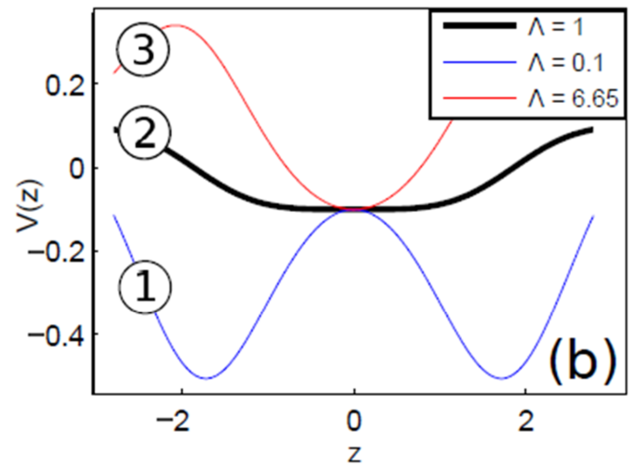
For narrow band semiconductors phase transition temperature is 0,5 – 2 Kelvins !
It is less that temperature of condensation , and $\tau_0 \approx 7ps.$

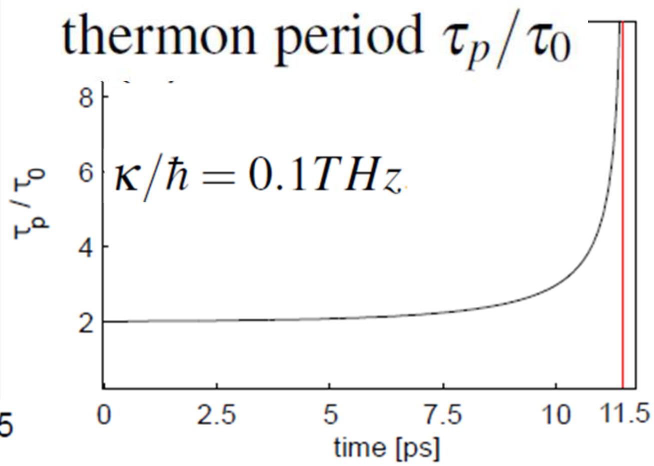
PT's in the Presence of Dissipation

Normalized phase difference as a function of time



Effective potential vs phase parameter z





Conclusions

We propose a physical mechanism which enables permanent Rabi oscillations in driven-dissipative condensates of exciton-polaritons in semiconductor microcavities subjected to external magnetic fields.

For polariton condensate in the lattice we demonstrate stable macroscopic oscillations, akin to nonlinear Josephson oscillations, between different spectral components of the polariton condensate in the momenta space

We consider finite temperature effects in a non-standard Bose-Hubbard model for an exciton polariton Josephson junction that is characterised by complicated potential energy landscapes (PEL) consisting of sets of barriers and wells.

We have shown that the transition between thermal activation (classical) and tunneling (quantum) regimes for quantum simulators exhibits universal features of the 1st and 2nd order phase transition depending on the PEL for two polariton condensates.

In the presence of dissipation the relative phase of two condensates exhibits non-equilibrium PT from the quantum regime characterized by efficient tunneling of polaritons to the regime of permanent Josephson or Rabi oscillations, where the tunneling is suppressed, respectively.

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Thank You!

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