Topological and geometrical effects in electronic and photonic honeycomb lattices

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Strong contribution from A. Nalitov, now in Southampton

- **Introduction.**
- **Valley Hall effect.**
- **Quantum Valley Hall effect.**
- **Quantum fluids in topological systems.**
- **Quantum Anomalous Hall effect: the role of the Spin Orbit Coupling winding number.**
- **All Optical control of topological phase transitions in the Photonic Quantum Anomalous Hall effect.**
- **Perfect Valley filter.**

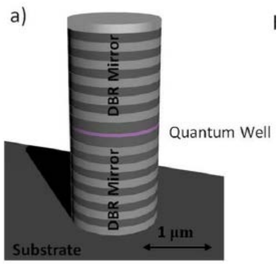
2D Photonic lattices

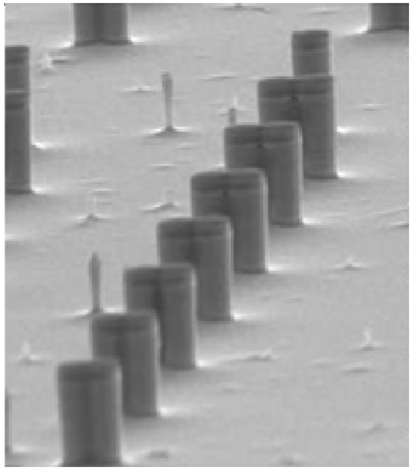
Photonic crystal slabs.
Guided modes in a layer with total internal reflection+periodic modulation of the optical index.

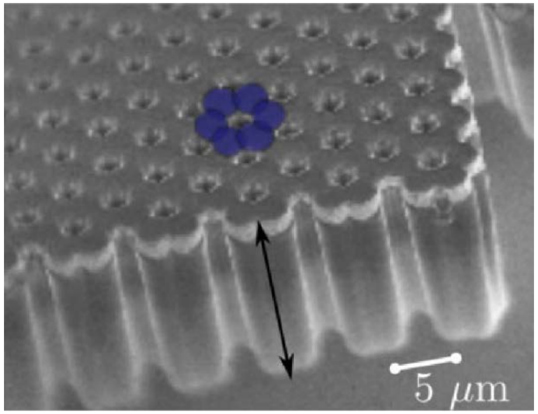
Possibility to write down effective Hamiltonians near specific points of the dispersion (such as crossing points).

Often, but not always, TE and TM modes are far away the one from the other.

Lattice based on the coupling between 0 D photonic modes (Photonic atoms).







Exciton+photon→ Polariton

Interacting photons.

Zeeman splitting under magnetic field.

Good description with tight binding approach,
but radiative modes, TE and TM modes are close.

Chern number and classification of Insulators

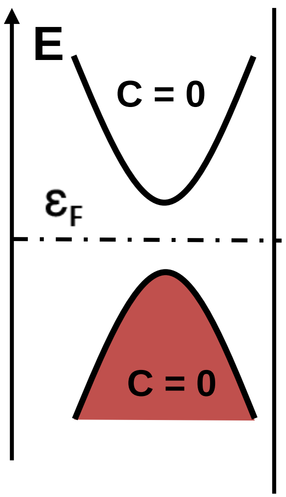
Berry phase accumulated by a Bloch wave function over a whole band in the complete Brillouin zone is quantized:

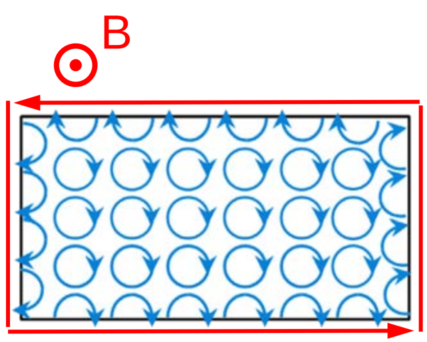
Chern number

which characterizes the chirality of the band.

e- band structures in SC ⟷ photon dispersion in a periodic media

e- insulator

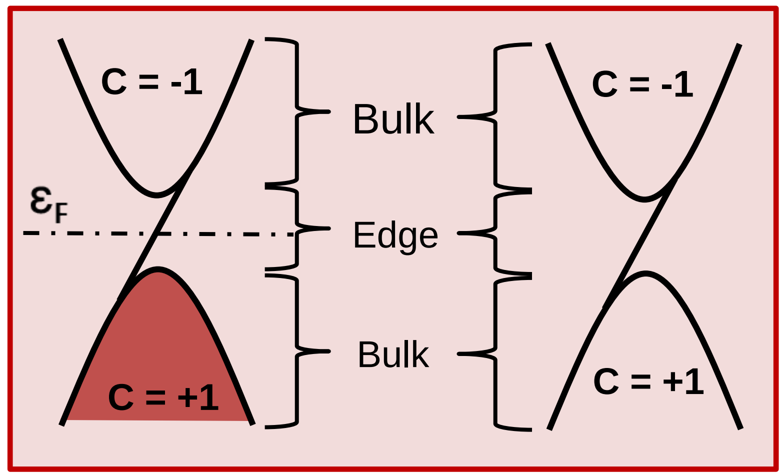




ex: Quantum Hall Effect K.v.Klitzing (1980)

e- $\mathbb{Z}$ topological *insulator*

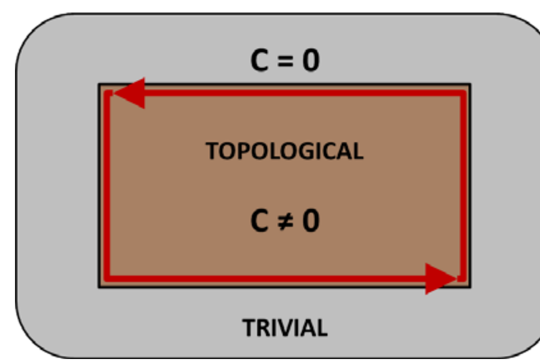
Photonic C-I



$$C_n = \frac{1}{2\pi i}\int_{BZ} \mathbf{F}_n(\mathbf{k})\, d\mathbf{k}$$

A gap should close to change topology.
The vacuum is trivial. Gap Closure on the interface.

One way edge modes, which cannot be elastically scattered.



Berry curvature and Chern number

- Spinor Wave function in a lattice $\psi_{\vec{k}} = u_{\vec{k}} e^{i\vec{k}\vec{r}} = \begin{pmatrix} u_{\vec{k}}^1 \\ u_{\vec{k}}^2 \end{pmatrix} e^{i\vec{k}\vec{r}}$

Table B1 | Comparison of the Berry phase for Bloch wavefunctions and the Aharonov–Bohm phase.

Vector potential	$A(r)$	$\mathcal{A}(k) = \langle u(k) \vert i\nabla_k \vert u(k) \rangle$	Berry connection
Aharonov–Bohm phase	$\oint A(r) \cdot dl$	$\oint \mathcal{A}(k) \cdot dl$	Berry phase
Magnetic field	$B(r) = \nabla_r \times A(r)$	$\mathcal{F}(k) = \nabla_k \times \mathcal{A}(k)$	Berry curvature
Magnetic flux	$\iint B(r) \cdot ds$	$\iint \mathcal{F}(k) \cdot ds$	Berry flux
Magnetic monopoles	$\# = \frac{e}{h} \oiint B(r) \cdot ds$	$C = \frac{1}{2\pi} \oiint \mathcal{F}(k) \cdot ds$	Chern number

The Berry connection measures the local change in phase of wavefunctions in momentum space, where $i\nabla_k$ is a Hermitian operator. Similar to the vector potential and Aharonov–Bohm phase, Berry connection and Berry phase are gauge dependent (that is, $u(k) \rightarrow e^{i\phi(k)} u(k)$). The rest of the quantities are gauge-invariant. The Berry phase is defined only up to multiples of 2π. The phase and flux can be connected through Stokes' theorem. Here, $u(k)$ is the spatially periodic part of the Bloch function; the inner product of $\langle\rangle$ is done in real space. The one-dimensional Berry phase is also known as the Zak phase.

Table from L. Lu, J.D. Joannopoulos, M. Soljacic, Nat. Phot. 8, 821 (2014).

$$u_{\vec{k}-\vec{k}_c} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{1+\rho_c} \\ \sqrt{1-\rho_c}\, e^{im\varphi} \end{pmatrix}$$

Winding of the wave function around a singularity in reciprocal space :
Berry curvature and non-zero Chern Number.

$$\rho_c \xrightarrow[|k| \to k_c]{} 1$$

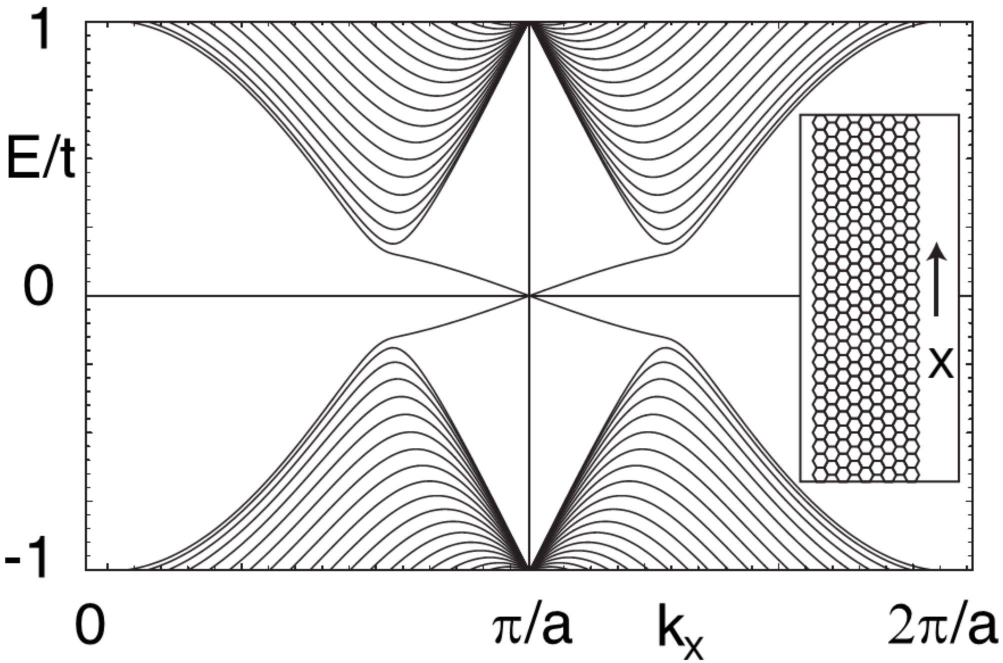
Half Vortex in momentum space.

Quantum **Spin** Hall Effect

Spin Chern number. $1/2(C_+ - C_-)$ is quantized, $\mathbb{Z}_2$ topological invariant.
No magnetic field.

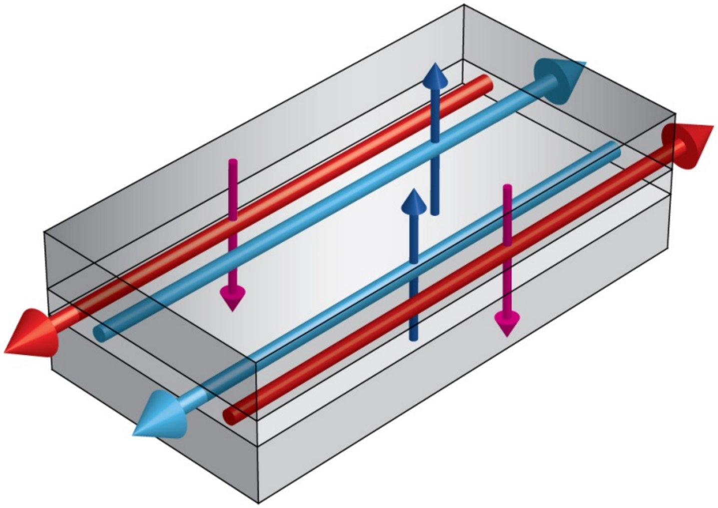
$$\mathcal{H}_0 = -i\hbar v_F \psi^\dagger(\sigma_x \tau_z \partial_x + \sigma_y \partial_y)\psi.$$

$$\boxed{\mathcal{H}_{\rm SO} = \Delta_{so}\psi^\dagger \sigma_z \tau_z s_z \psi.}$$



C.L.Kane, E.J.Mele,
Phys. Rev. Lett. **95**, 226801 (2005)

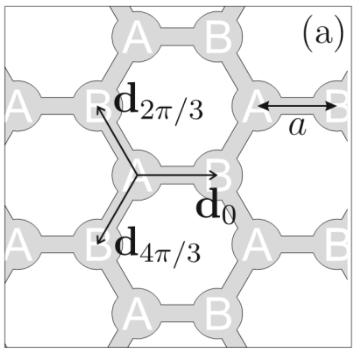
$\mathbb{Z}_2$ topological insulator



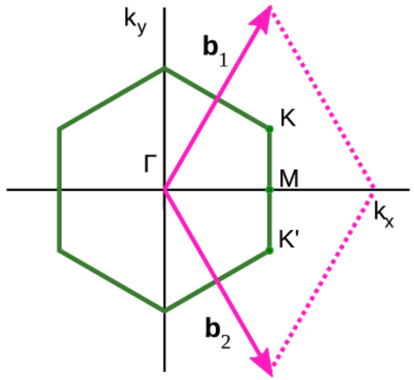
Graphene Hamiltonian (no spin)

Honeycomb lattice:

real space



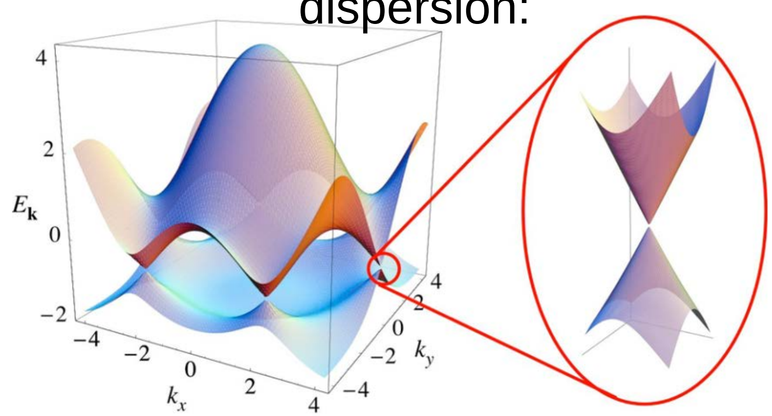
reciprocal space



Tight-binding Hamiltonian *(Wallace 1946):*

$$H_{graphene} = -\begin{pmatrix} 0 & Jf_k \\ Jf_k^+ & 0 \end{pmatrix} \begin{matrix} A \\ B \end{matrix} \qquad f_{\mathbf{k}} = \sum_{j=1}^{3} \exp(-i\mathbf{k}\mathbf{d}_{\varphi_j})$$

dispersion:



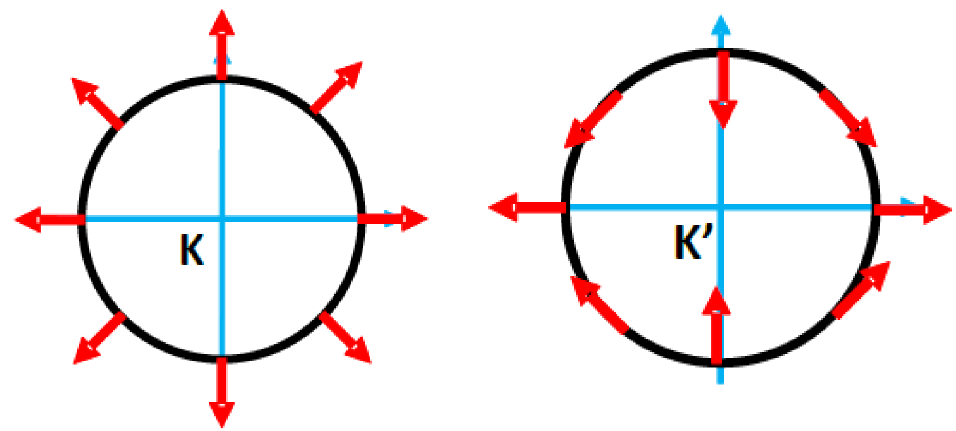
$$H_{graphene} = -J\left(\mathrm{Re}(f_k)\sigma_x - \mathrm{Im}(f_k)\sigma_y\right) = -\vec{\Omega}_{graphene}.\vec{\sigma}$$

$$\vec{\Omega}_{graphene} = J\left(\mathrm{Re}(f_k), -\mathrm{Im}(f_k), 0\right)^T$$

$\vec{\sigma}$ Sublattice pseudo-spin

Close to K or K'=-K $\quad f_{K,K'} = 0 \quad \tau_z = \pm 1$

$$\vec{\Omega}_{graphene} \equiv \left(\tau_z k_x, k_y, 0\right)^T = k\left(\cos(\tau_z\varphi), \sin(\tau_z\varphi), 0\right)^T$$



Monopolar

Dresselhauss

Opposite winding at K and K'

Staggered honeycomb lattice

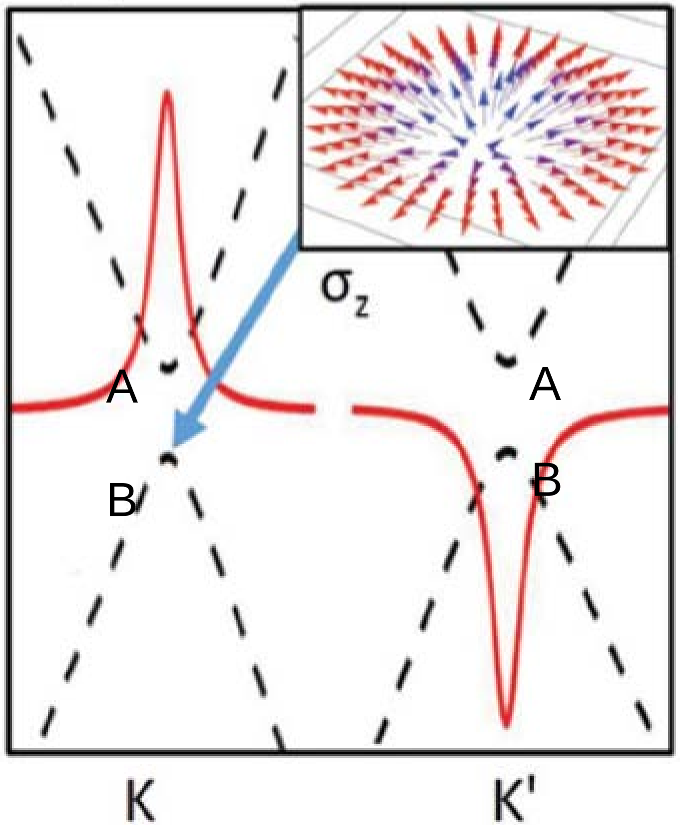
Let us make A and B different.

$$H_{staggered} = -\begin{pmatrix} -\Delta & Jf_k \\ Jf_k^+ & \Delta \end{pmatrix} \approx -\left(J\left(\tau_z k_x \sigma_x + k_y \sigma_y\right) + \Delta \sigma_z\right)$$

Di Xiao, Wang Yao, and Qian Niu, Phys. Rev. Lett. 99, 236809, (2007).

- Massive Dirac Hamiltonian.

- Gap opening.
- Berry curvature of opposite sign at K and K'.
- Valley dependent magnetic moment and selection rules for optical absorption.



Haldane model 1988

$$H_{Haldane} = -\left(J\left(\tau_z k_x \sigma_x + k_y \sigma_y\right) + \Delta \tau_z \sigma_z\right)$$

- Berry curvature of same sign at K and K'→ Non Zero Chern number.
- Quantum Anomalous Hall effect.

Staggered honeycomb lattice: Valley Hall Effect

Physical implementations :

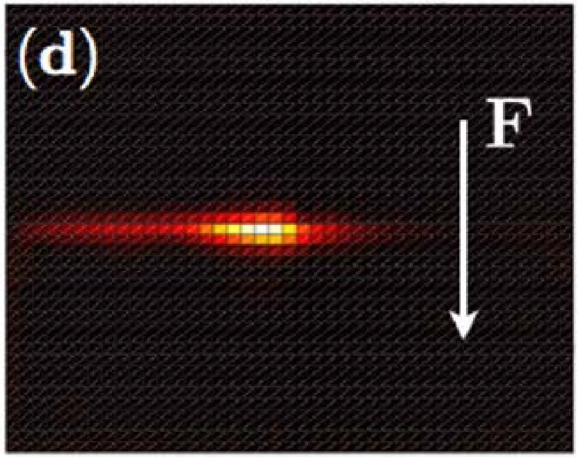
- Transitional Metal Dicalchogenide (TMD) monolayers (MoS2, MoSe2, WS2 ect…) .
- Bi-Layer graphene with applied electric field ….
- Silicene …
- Photonic lattices ?

Berry curvature=drift
Anomalous Hall effect,
Intrinsic Spin Hall effect for holes.

Valley dependent electron drift: Valley current, « Valleytronics ».

Observed in MoS2 transistors, *Science, 344, 1486, (2014).*

Photonics *Ozawa and Carusotto, PRL, 112 133902 (2014).*



Valley dependent drift of a pump spot in presence of a potential.

Quantum Valley Hall Effect

Di Xiao, Wang Yao, and Qian Niu, Phys. Rev. Lett. 99, 236809, (2007).
A. Rycerz, J. Tworzydlo, CWJ. Beenakker, Nat. Phys 3, 172, (2007).

Valley = pseudo-spin

Definition of a Valley Chern Number $C_K = -C_{K'} = 1$

$C_{KK'} = \frac{C_K - C_{K'}}{2} = 1$ Z_2 topological invariant, like in the Quantum Spin Hall effect.

But Valley states are not stationnary in vacuum. No « chiral » edge states at the edge of a TMD monolayer for instance.

However, interface states exist at the Zigzag interface between two materials of opposite $C_{KK'}$

Topological valley transport at bilayer graphene domain walls, Nature 520, 650–655 (30 April 2015).

Remark:

Topological states, but unprotected from inter-valley scattering.
Same as QSHE, un-protected from inter-spin scattering.

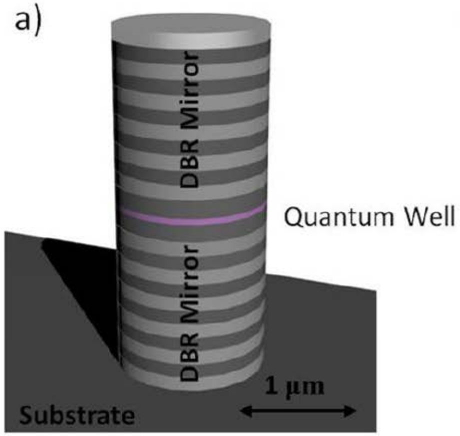
Photonic Quantum Valley Hall Effect

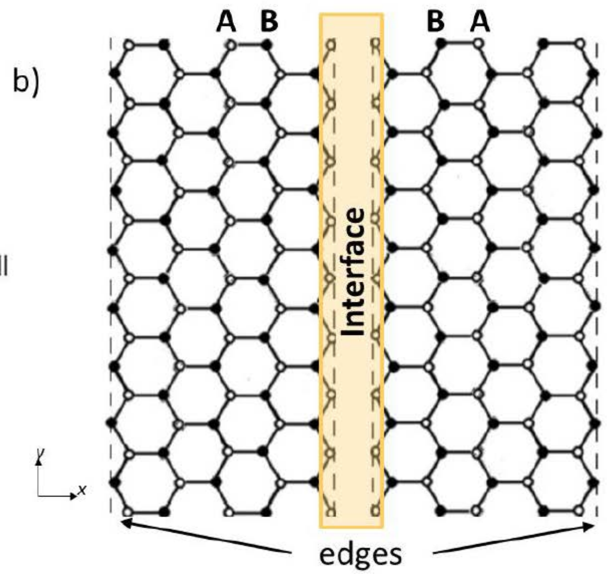
In photonic crystal slabs:

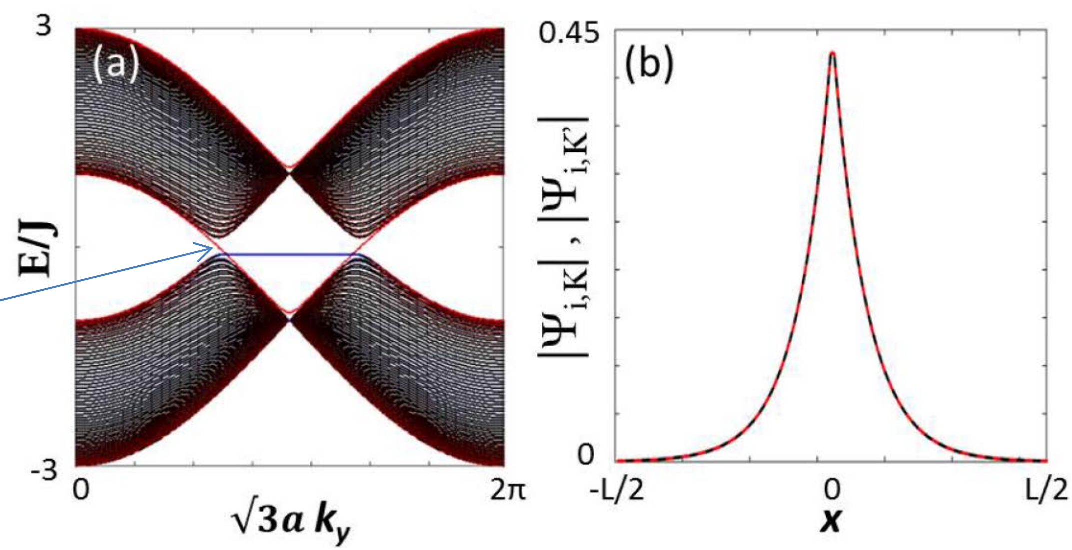
- L.-H. Wu and X. Hu, Phys. Rev. Lett. 114 , 223901 (2015).
- T. Ma, A. B. Khanikaev, S. H. Mousavi, and G. Shvets, Phys. Rev. Lett. 114 , 127401 (2015).
- T. Ma and G. Shvets, New Journal of Physics, 18, 025012 (2016).
- L. Xu, H. Wang, Y. D. Xu, H. Y. Chen, and J.-H. Jiang, arXiv:1601.03168.
- X.-D. Chen and J.-W. Dong, arXiv:1602.03352.
- … ,Hafezi, arXiv:1605.08822

Remark, even if it done in these works, the link with an effective Hamiltonian is not that direct.

Quantum Valley Hall effect based on photonic analogs of Transitional Metal Dichalcogenides



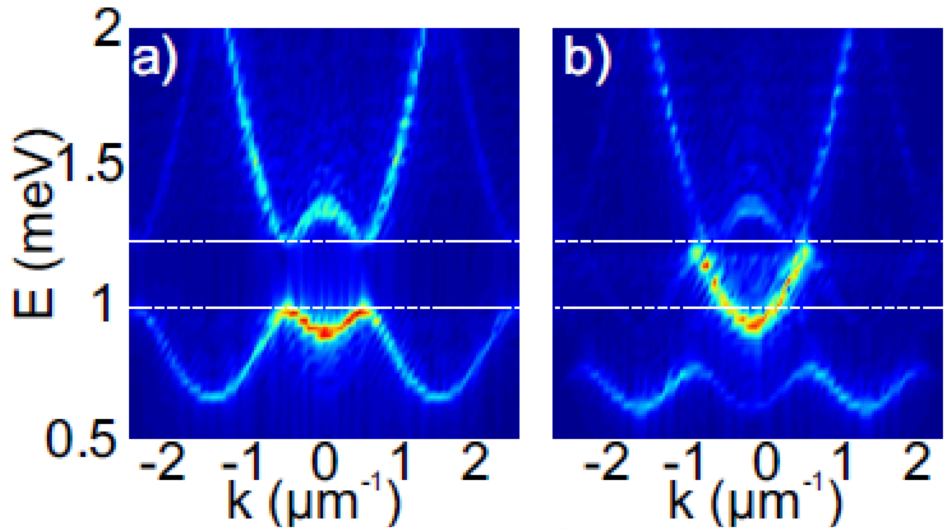


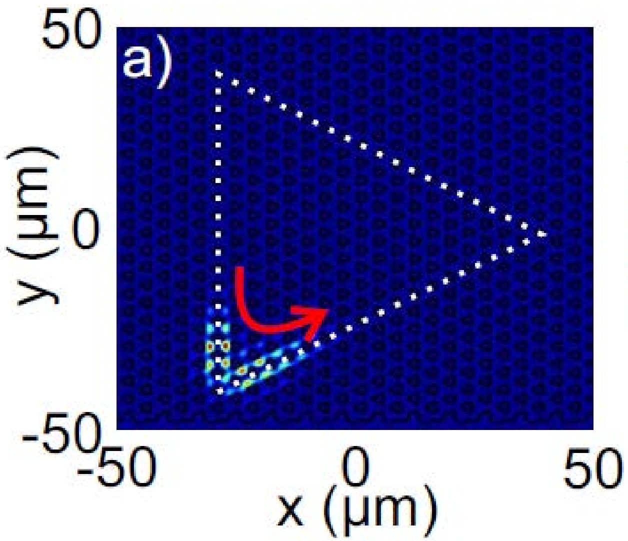


One valley, one group velocity.

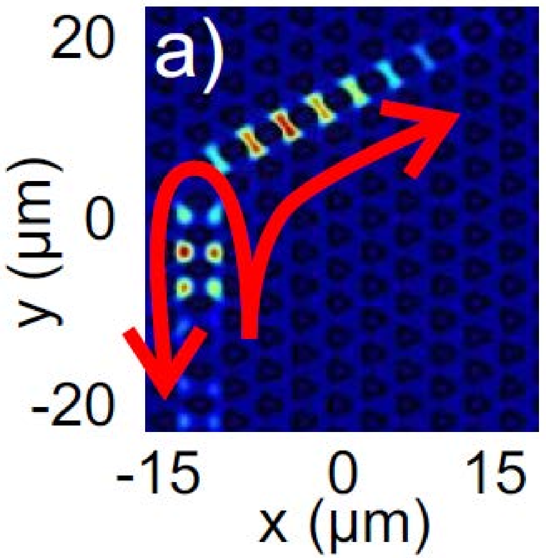
Absence of real topological protection

Numerical solution of Schrödinger equation

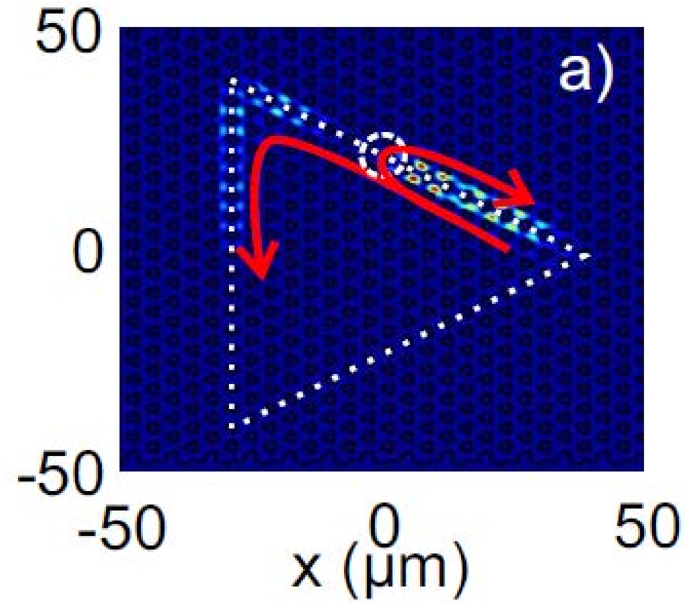




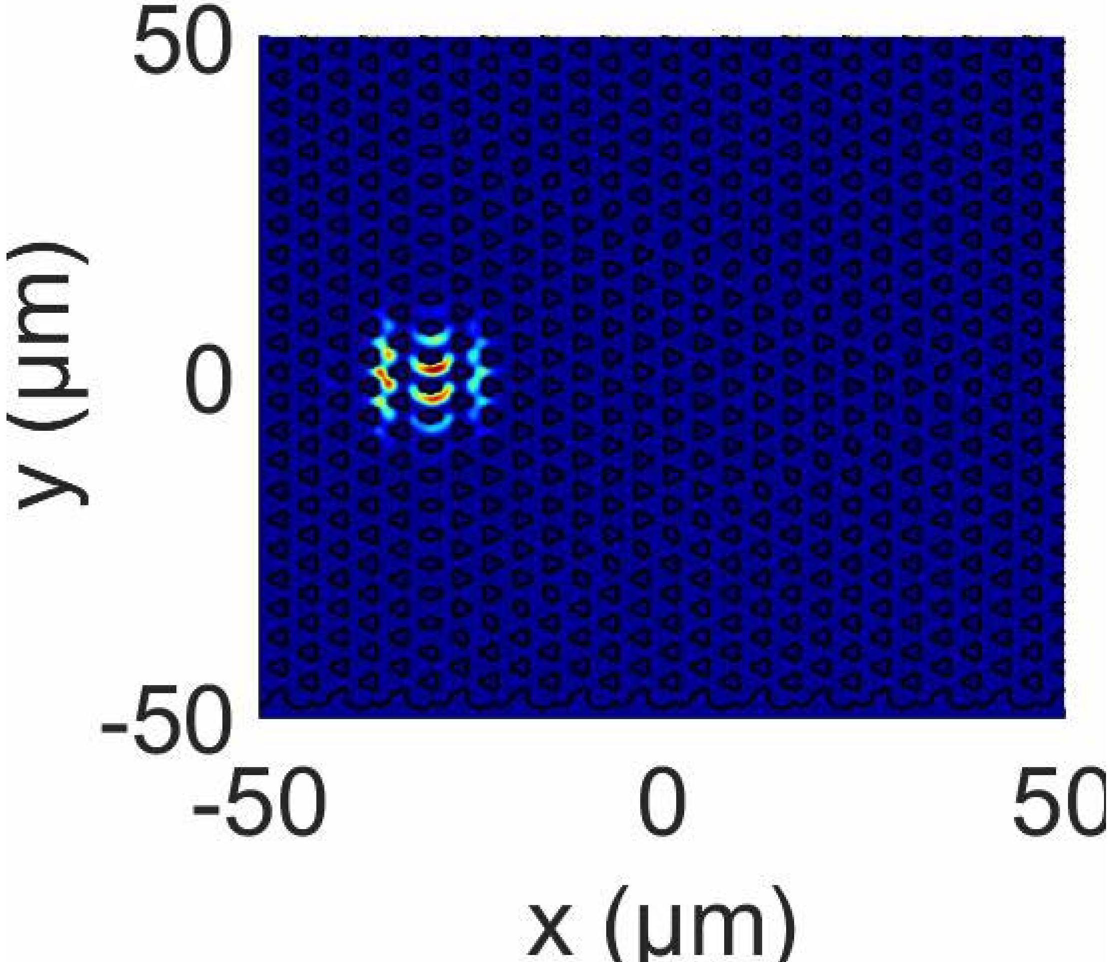
Perfect transmission trough a 120° corner.



Strong reflection trough a 60° corner.



Strong reflection at a defect


50
0
-50
y (μm)
-50
0
50
x (μm)

Quantum fluids in topological lattices

Condition: Presence of a Bose Einstein Condensate at the Gamma point described by a non-linear Schrödinger equation.

Two types of excitations:

Density waves (Bogolons): O. Bleu et al. PRB 2016.

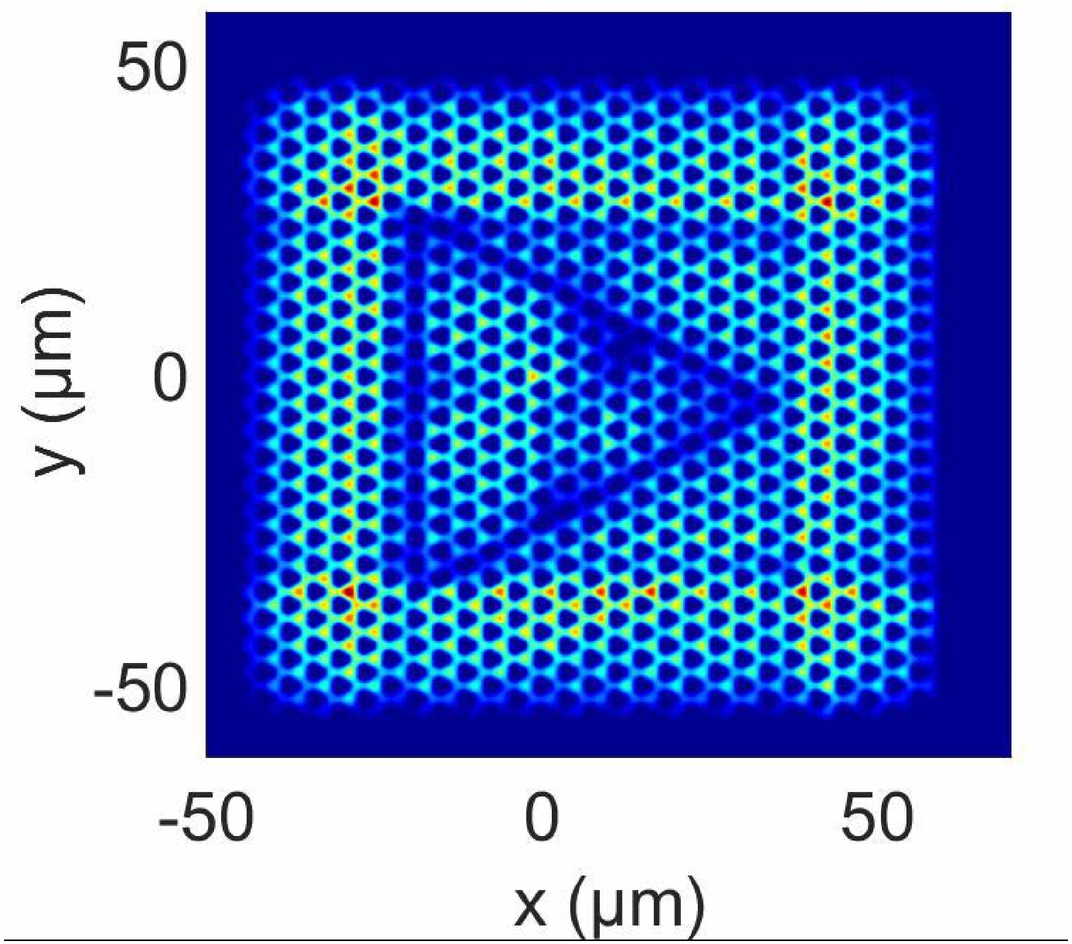
Topological defects.
Solitons in 1D: D. Solnyshkov et al, PRL 2016, PRL 2017.
Quantized vortices in 2D.

Staggered honeycomb lattice.

- The core of the vortex is composed by states near K and K' which posses an intrinisic angular momentum.
- The sense of rotation of the quantum vortex is linked with the Valley.
- The Valley imposes a well defined propagation direction on a TMD-TMD interface.

Winding - Valley coupling

Valley – Propagation direction coupling.



InterValley Scattering is suppressed, because the vortex angular momentum is a topologically protected quantity.

Combination of two topological protections of different origins.

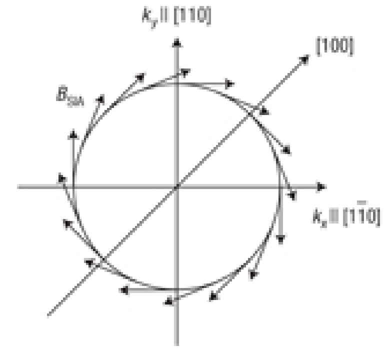
Honeycomb lattice for spinor particles

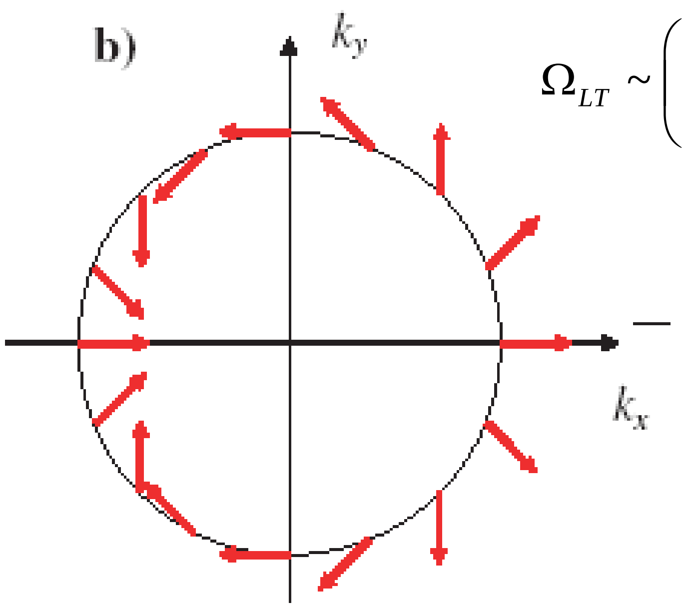
Photons and Polaritons
2 spin projections coupled by TE-TM Splitting

$$H = \begin{pmatrix} H_0(k) & \Omega_{LT}(k)e^{-2i\varphi} \\ \Omega_{LT}(k)e^{2i\varphi} & H_0(k) \end{pmatrix} = H_0\mathbf{I} + \vec{\Omega}_{LT} \cdot \vec{\boldsymbol{\sigma}}$$

The direction of the field depends on the wave vector

Spin-orbit interaction for electrons (Rashba)





$$\Omega_{LT} \sim \begin{pmatrix} \cos(2\varphi) \\ \sin(2\varphi) \end{pmatrix}$$

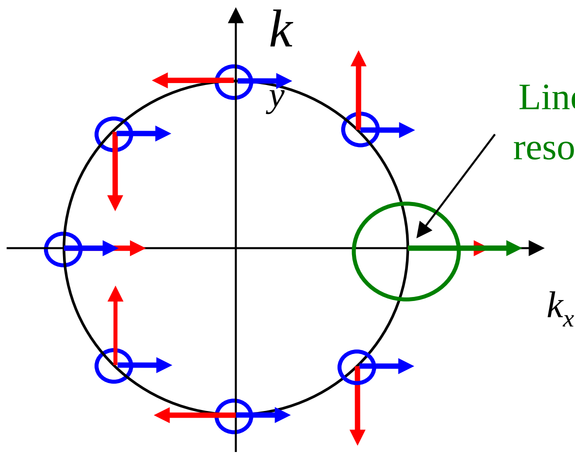
Effective in-plane magnetic field

Spin-Orbit coupling for photons

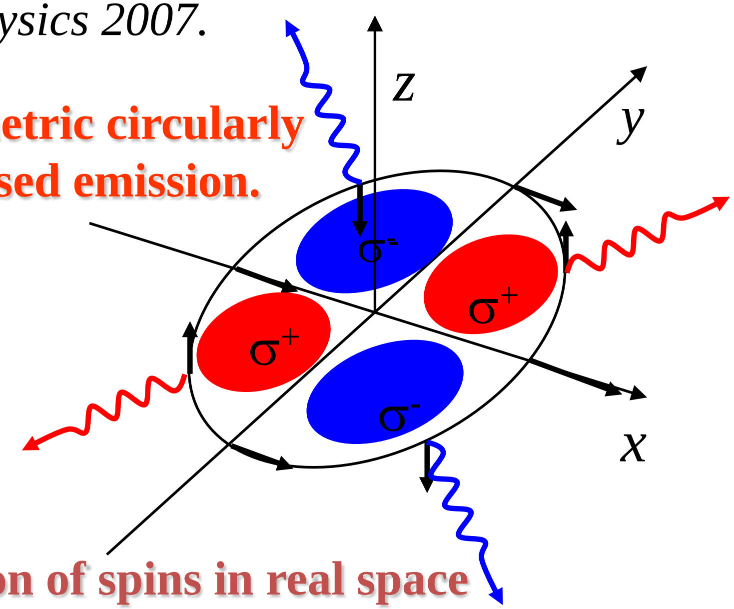
Planar cavity, effective field ~ k^2

TE-TM splitting Optical Spin Hall Effect

Linear Optical Spin Hall Effect, *Kavokin, Malpuech, Glazov PRL 2005.*
C. Leyder et al., Nature Physics 2007.

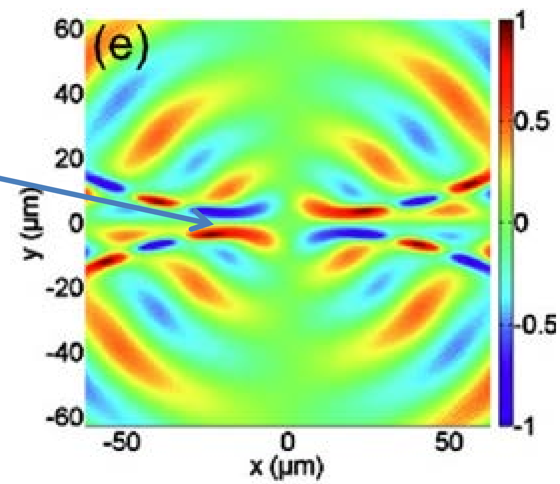


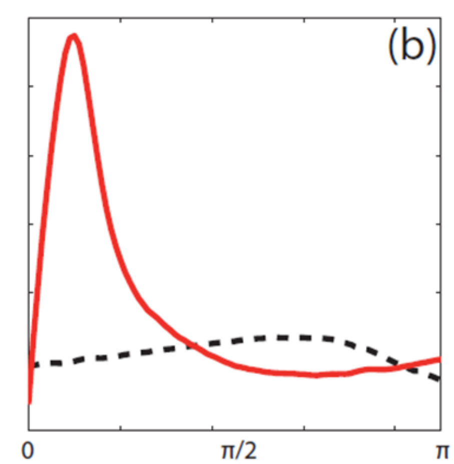
Effective field distribution
Spin distribution after Rayleigh scattering events



Non-linear regime:
Focusing of spin currents
PRL 110, 016404, (2013).

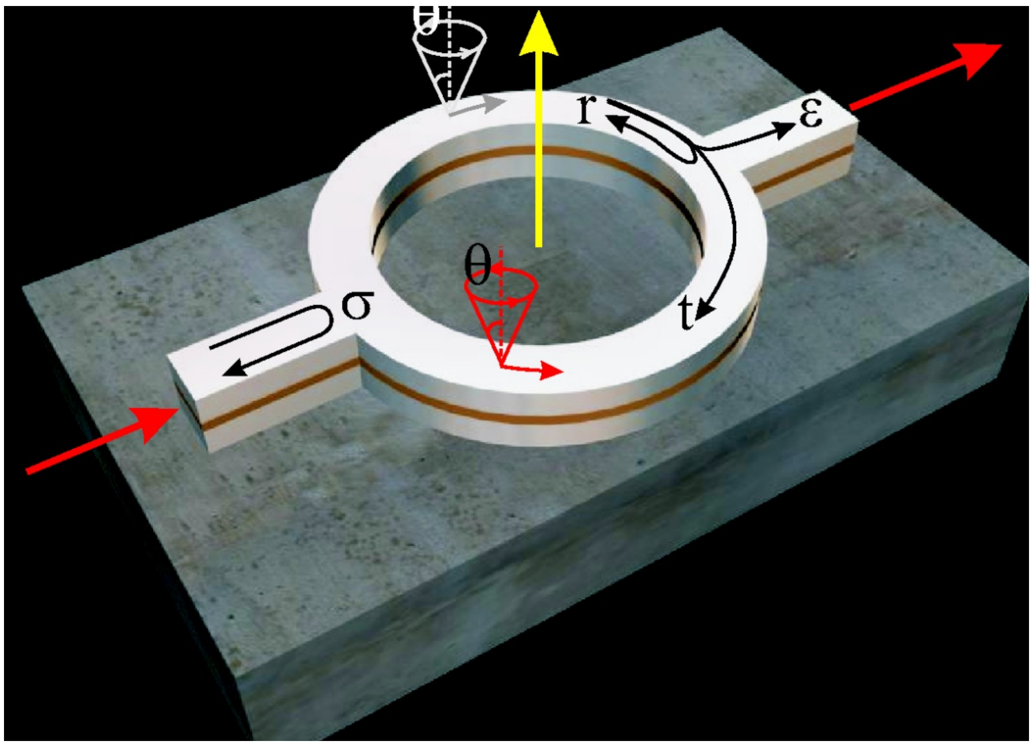
Emergence of magnetic monopole analogs (half solitons).
Nat. Phys. 8, 724, (2012).





TE-TM + Zeeman field in a ring gives Berry Phase

$$\vec{\Omega} = \Omega_z u_z + \Omega_{TETM}\left(\cos(2\varphi)u_x + \sin(2\varphi)u_y\right)$$



$$\psi = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{1+\rho_c} \\ \sqrt{1-\rho_c}\, e^{-2i\varphi} \end{pmatrix}$$

Berry phase accumulated on a round trip:

$$\varphi_{Berry} = 2\pi\rho_c$$

I.A. Shelykh, G. Pavlovic, D. Solnyshkov, G. Malpuech, PRL 102, 046407, (2009).

Photonic Topological Insulator Analog

Z-Type

Quantum anomalous Hall effect

PRL **100,** 013904 (2008) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2008

Possible Realization of Directional Optical Waveguides in Photonic Crystals with Broken Time-Reversal Symmetry

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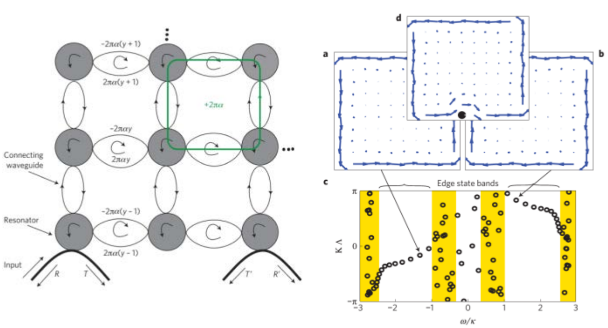
Department of Physics, Princeton University, Princeton, New Jersey 08544-0708, USA

(Received 23 March 2005; revised manuscript received 30 May 2007; published 10 January 2008)

We show how, in principle, to construct analogs of quantum Hall edge states in “photonic crystals” made with nonreciprocal (Faraday-effect) media. These form “one-way waveguides” that allow electromagnetic energy to flow in one direction only.

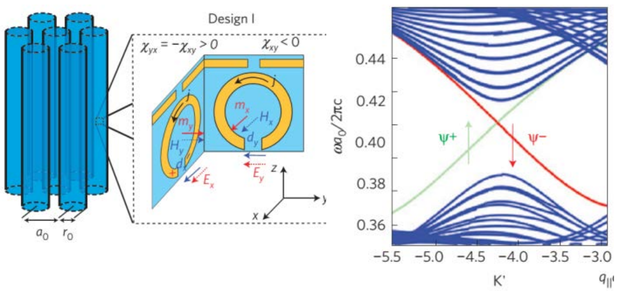
DOI: 10.1103/PhysRevLett.100.013904 PACS numbers: 42.70.Qs, 03.65.Vf

Si-based PTIs



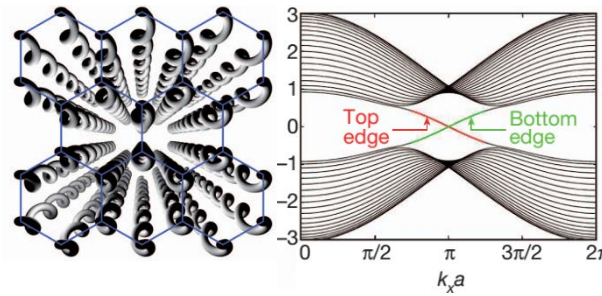
M. Hafezi et al,
Nature Physics **7**, 907-912 (2011)

PTIs with metamaterials



A.B. Khanikaev et al,
Nat Mater **12**, 233–239 (2012)

Floquet PTIs



M.C. Rechtsman et al,
Nature **496**, 196–200 (2013)

Observation, generalisation of the Haldane-Raghu proposal

nature Vol 461|8 October 2009|doi:10.1038/nature08293

LETTERS

Observation of unidirectional backscattering-immune topological electromagnetic states

Zheng Wang[1]*, Yidong Chong[1]†*, J. D. Joannopoulos[1] & Marin Soljačić[1]

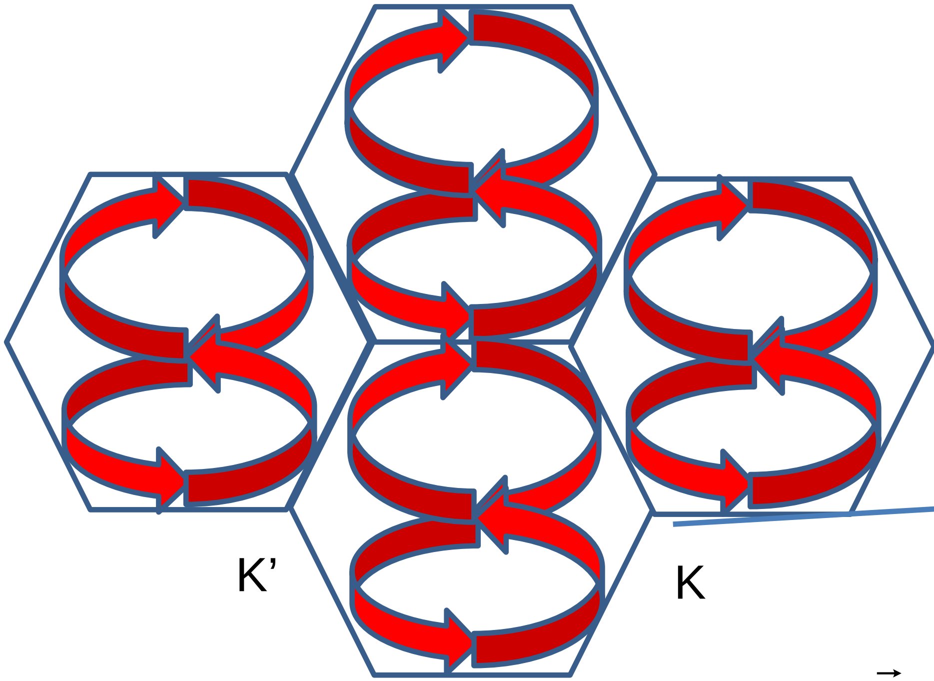
General receipe

- Pure TE or TM states in a photonic crystal waveguide.
- Degeneracy (band crossing) at the corner of the Brillouin Zone.
- Effective field along Z (Zeeman splitting, gyromagnetic …).

Gyromagnetic effects, limited to microwaves.

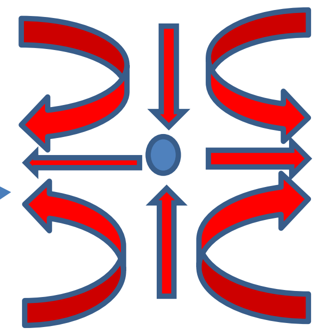
Haldane - Soljacic effect: how does it work ?

TE or TM states in reciprocal space:



At K and K', effective in plane fields cancel

Around K and K', Emergence of **Dresselhaus** gauge fields of opposite signs but with the same winding number.



$$\vec{\Omega}_K = \Omega_z u_z + \left(k_x - K_x\right) u_x - \left(k_y - K_y\right) u_y$$

Vortical Wave function

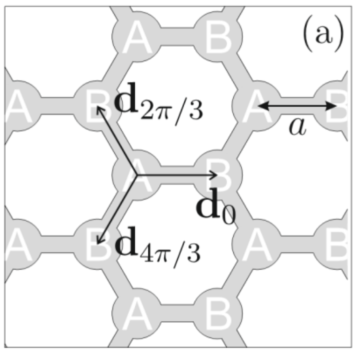
$$u_K(\vec{k}) = \begin{pmatrix} \sqrt{1 - k + K} \\ \sqrt{\left(k - K\right)} e^{i\varphi} \end{pmatrix}$$

Same winding, same circularity at K and K', same contribution to the Chern number: **Topological insulator.**

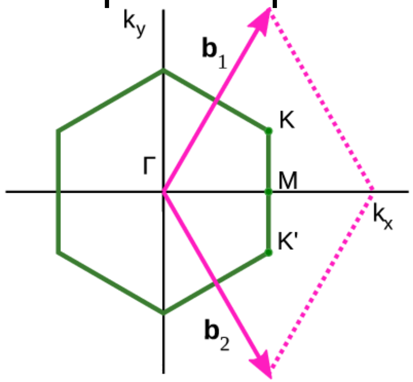
Polaritonic graphene

honeycomb lattice:

real space



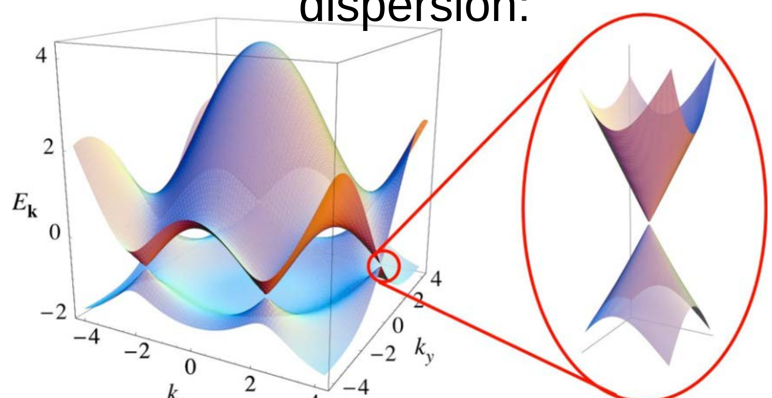
reciprocal space

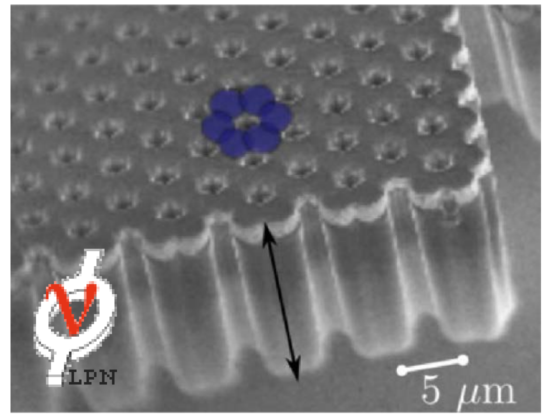


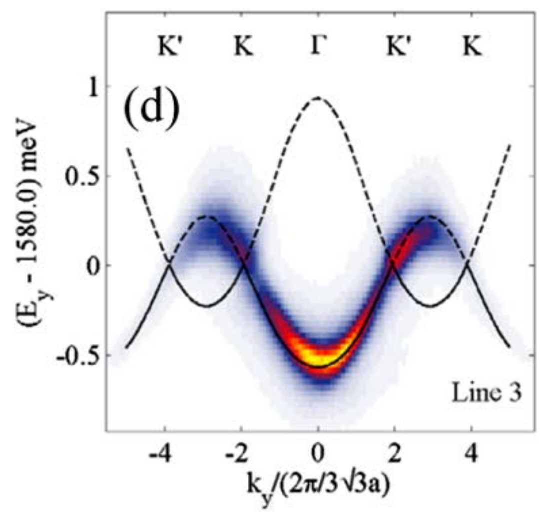
tight-binding Hamiltonian:

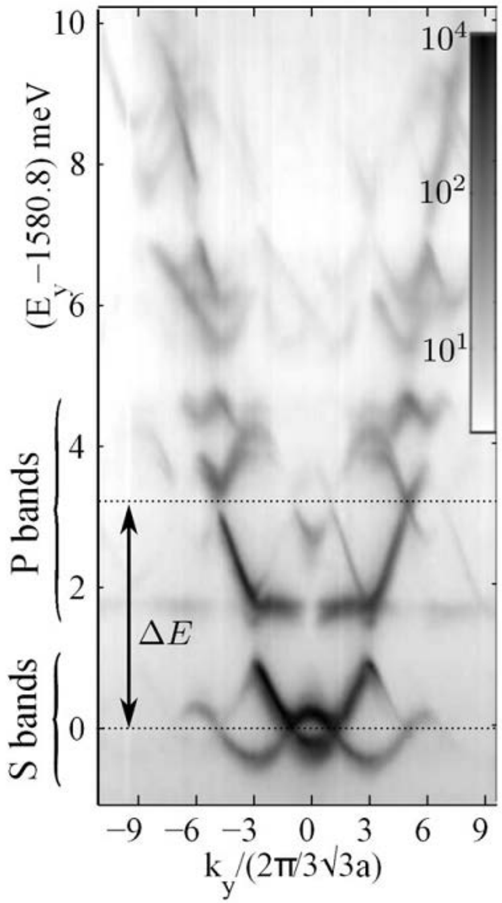
$$\mathrm{H}_{\mathbf{k}} = \begin{pmatrix} 0 & -Jf_{\mathbf{k}} \\ -Jf^{*}_{\mathbf{k}} & 0 \end{pmatrix} \begin{matrix} \mathrm{A} \\ \mathrm{B} \end{matrix} \qquad f_{\mathbf{k}} = \sum_{j=1}^{3} \exp(-i\mathbf{k}\mathbf{d}_{\varphi_j})$$

dispersion:









T.Jacqmin et al.,
Phys. Rev. Lett. **112** 116402 (2014)

Spin-orbit coupling

$\delta J = J_T - J_L$ Tunneling coeff depend on polarisation

V.G. Sala et al, PRX 2015

TE-TM splitting: tight-binding Hamiltonian:

$$\mathrm{H}_{\mathbf{k}} = \begin{pmatrix} E_0 & 0 & -Jf_{\mathbf{k}} & -\delta J f_{\mathbf{k}}^{+} \\ 0 & E_0 & -\delta J f_{\mathbf{k}}^{-} & -Jf_{\mathbf{k}} \\ -Jf_{\mathbf{k}}^{*} & -\delta J (f_{\mathbf{k}}^{-})^{*} & E_0 & 0 \\ -\delta J (f_{\mathbf{k}}^{+})^{*} & -Jf_{\mathbf{k}}^{*} & 0 & E_0 \end{pmatrix} \begin{matrix} |A,+\rangle \\ |A,-\rangle \\ |B,+\rangle \\ |B,-\rangle \end{matrix}$$

$$f_{\mathbf{k}} = \sum_{j=1}^{3} \exp(-\mathrm{i}\mathbf{k}\mathbf{d}_{\varphi_j}), \quad f_{\mathbf{k}}^{\pm} = \sum_{j=1}^{3} \exp(-\mathrm{i}\left[\mathbf{k}\mathbf{d}_{\varphi_j} \mp 2\varphi_j\right])$$

development near Dirac points:

$$H_{\mathbf{q}}^{(0)} = \hbar v_F \left(\tau_z q_x \sigma_x + q_y \sigma_y\right)$$

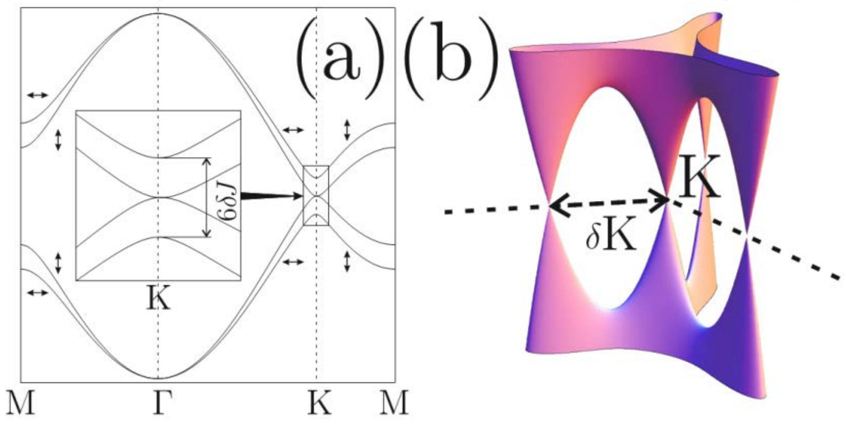
$$H_c^{SO} = -\Delta c \tau_z (q_x s_x - q_y s_y)/q.$$

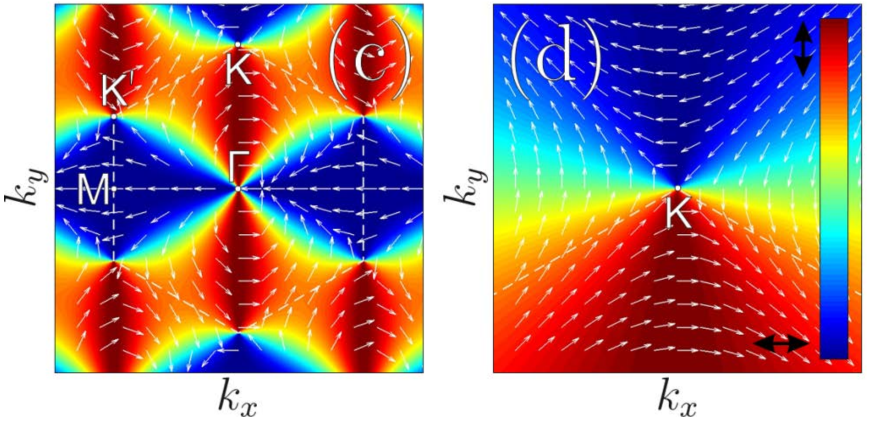
Dresselhaus-like field

$$H_{\mathbf{q}}^{\mathrm{SO}} = \frac{\Delta a}{2}\left[s_x\left(\tau_z q_y \sigma_y - q_x \sigma_x\right) - s_y\left(\tau_z q_x \sigma_y + q_y \sigma_x\right)\right]$$

trigonal warping

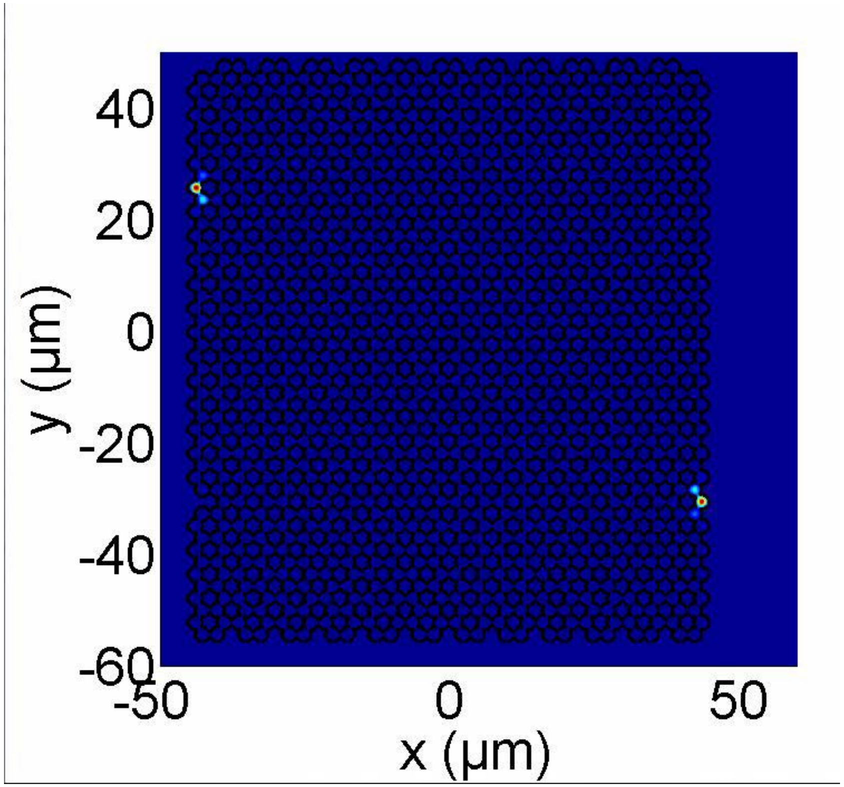
dispersion | trigonal warping | effective magnetic field acting on pseudospin

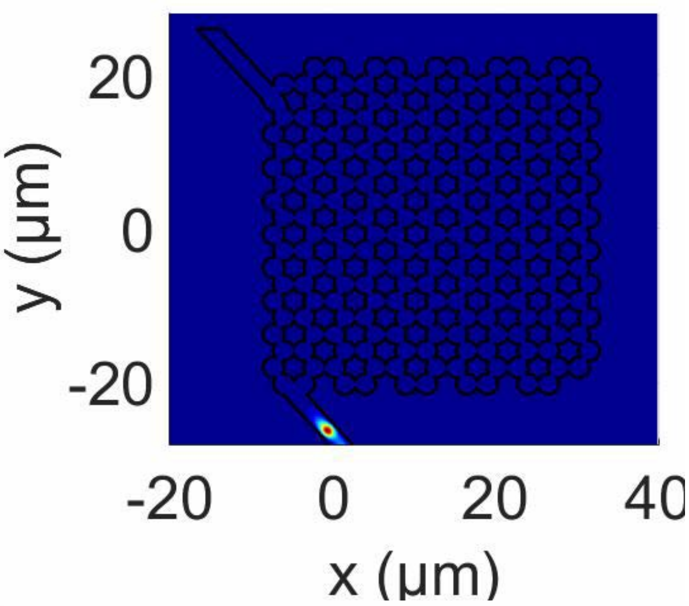


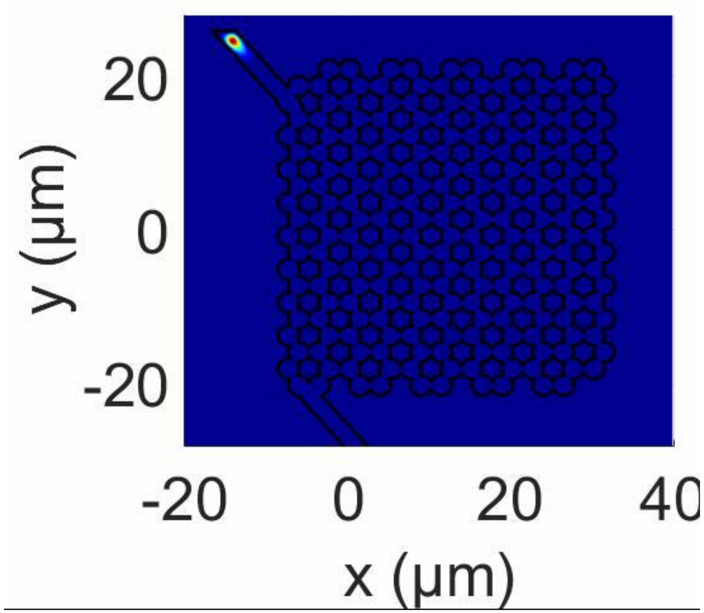


A. Nalitov et al. PRL, 114, 026803 (2015).

One way transport, Chiral Valve







A.Nalitov, D. Solnyshkov, and G. Malpuech, PRL 114, 116401, (2015).

With Zeeman field topological gap opens: one way transport.

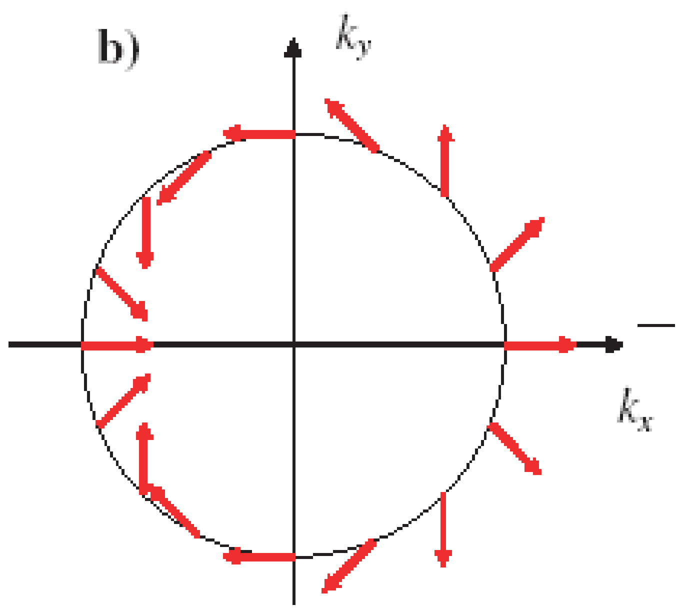
Comparison of electronic and photonic systems

O. Bleu et al, PRB 95, 115415 (2017).

Photons and electrons differ by the winding number of their Spin-Orbit coupling.

Photons: TE-TM induced SOC

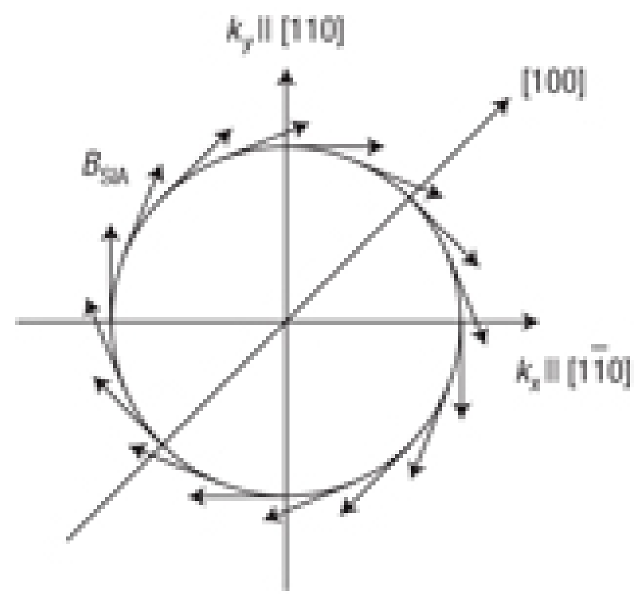
Winding number 2



$$\Omega_{LT} \sim \begin{pmatrix} \cos(2\varphi) \\ \sin(2\varphi) \end{pmatrix}$$

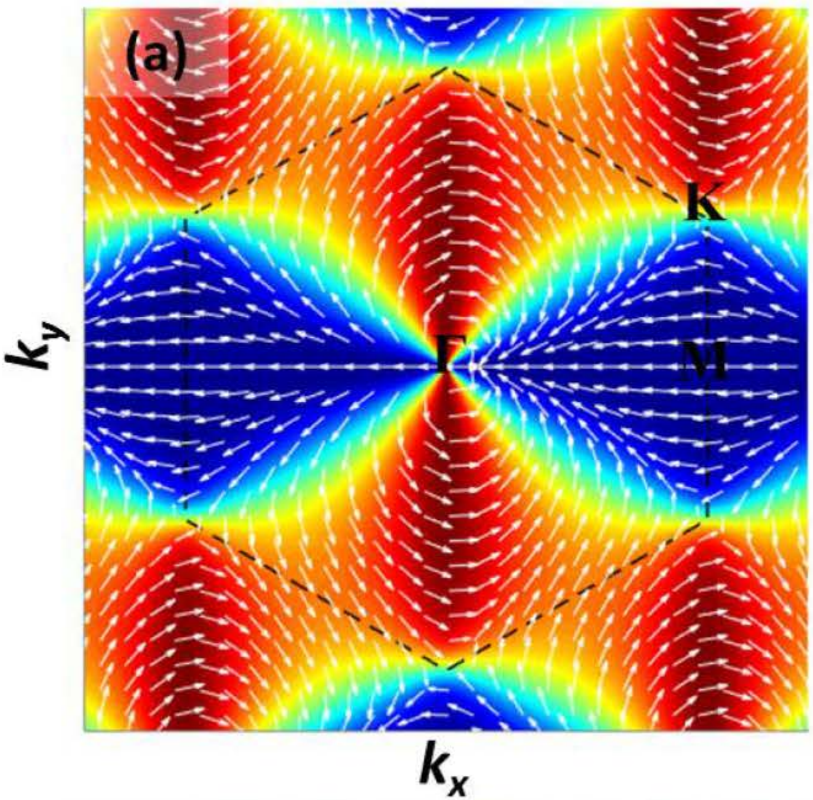
Electrons: Rashba SOC

Winding number 1



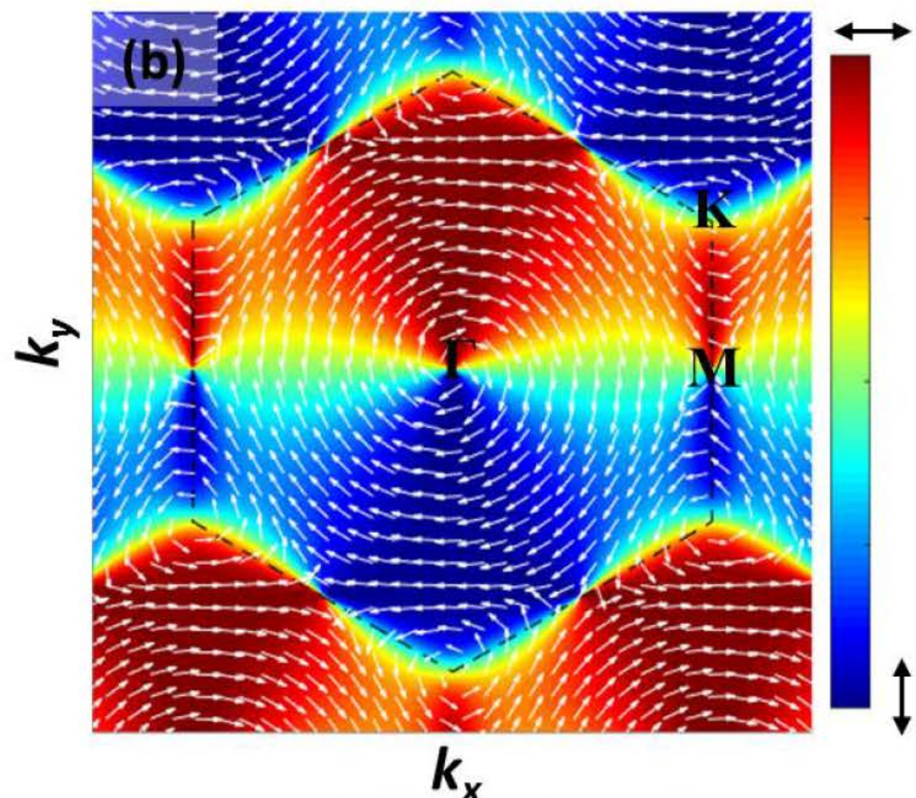
Honeycomb lattice
Effective field texture acting on the real electron and photon pseudo-spin

Photons: TE-TM induced SOC



Dresselhaus of same winding at K and K'

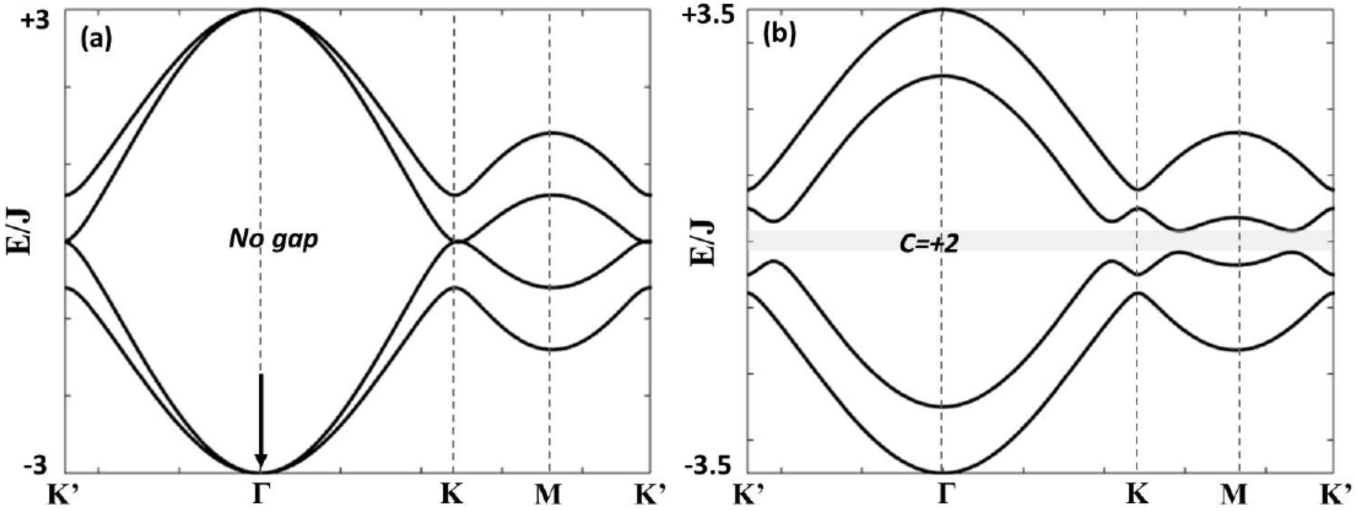
Electrons: Rashba SOC



Rashba of same winding at K and K'

Quantum Anomalous Hall effect (1988 Haldane)
Based on a combination of a Zeeman field and Rashba / TE-TM SOC

Zeeman field acting on the spin opens a gap
Berry curvature of same sign at K and K' → Non Zero Chern number.



For electrons in graphene … *Niu Q, Quantum anomalous Hall effect in graphene from Rashba and exchange effects, Phys. Rev. B, 82, 161414 (2010).*

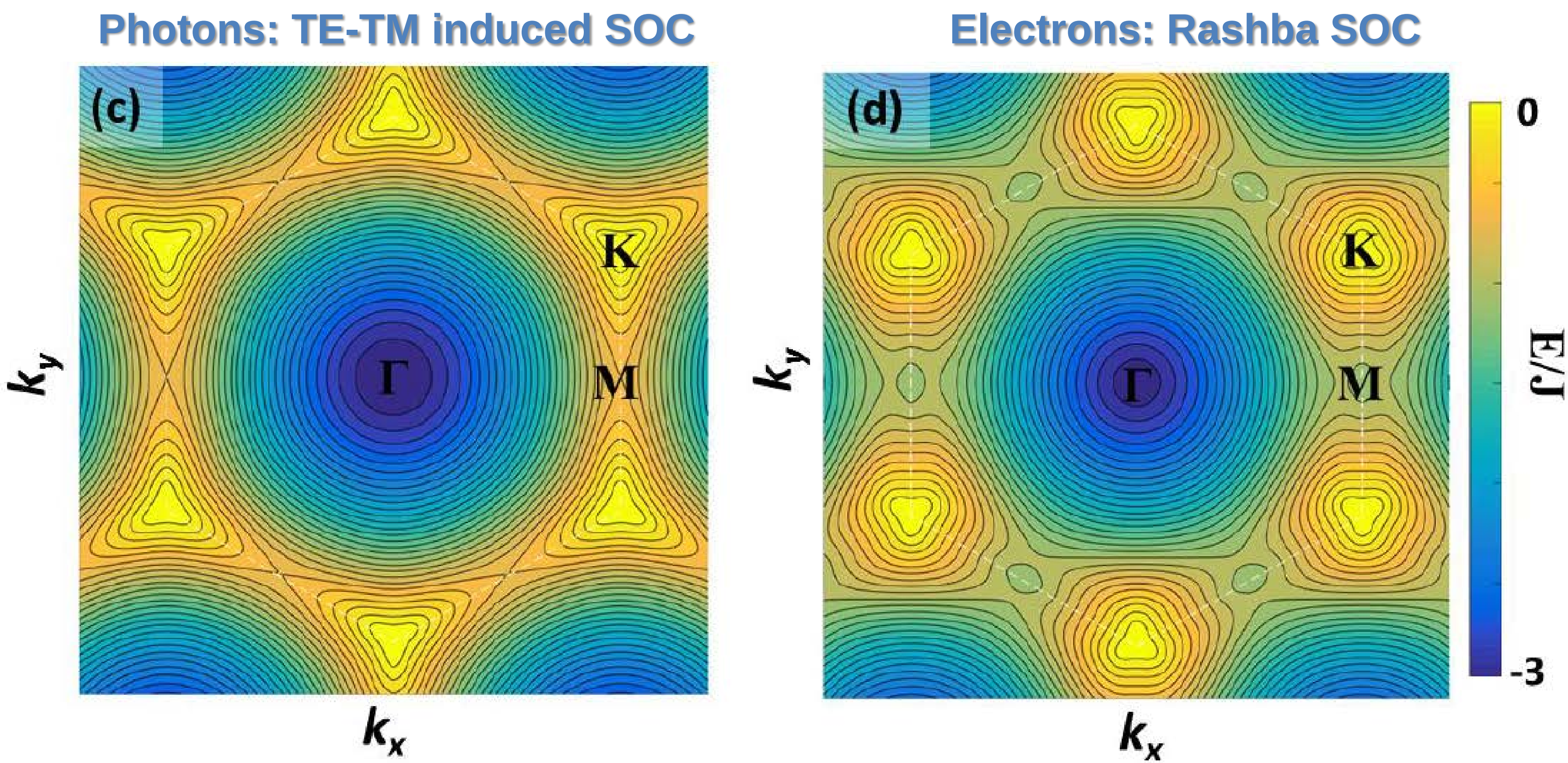
$$H_{Niu} \approx \lambda_R \left(k_y s_x + k_x s_y \right) + \Delta s_z$$

$$H_{Haldane} = J\left(\tau_z k_x \sigma_x + k_y \sigma_y \right) + \Delta \tau_z \sigma_z$$

For photons in graphene analogs. A. *Nalitov et al. PRL 2015.*

$$H_{Nalitov} \approx \lambda_{TETM} \tau_z \left(k_y s_x - k_x s_y \right) + \Delta s_z$$

Trigonal warping

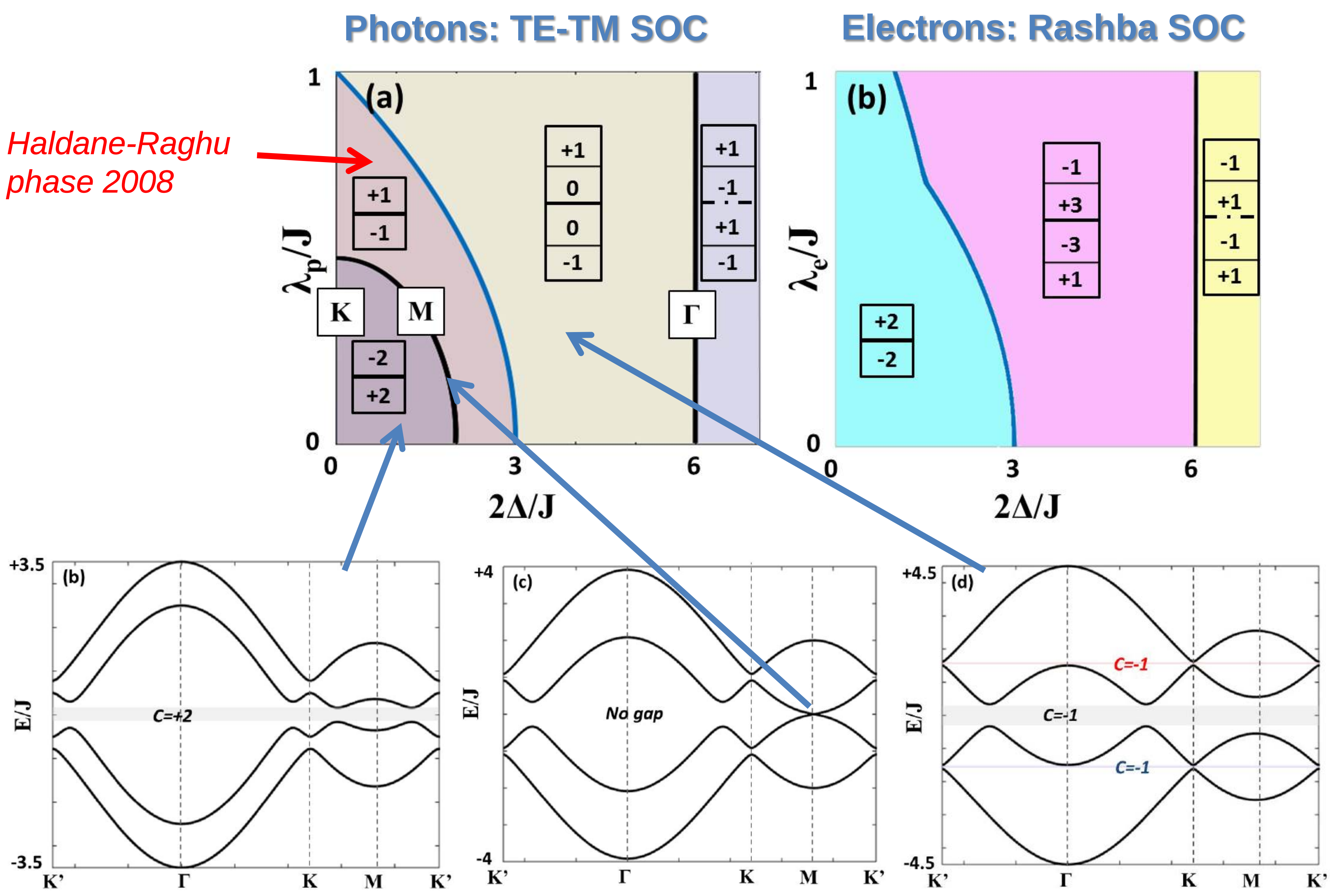


Mixing of lattice and polarisation spin near K: Dirac point splits in 4.

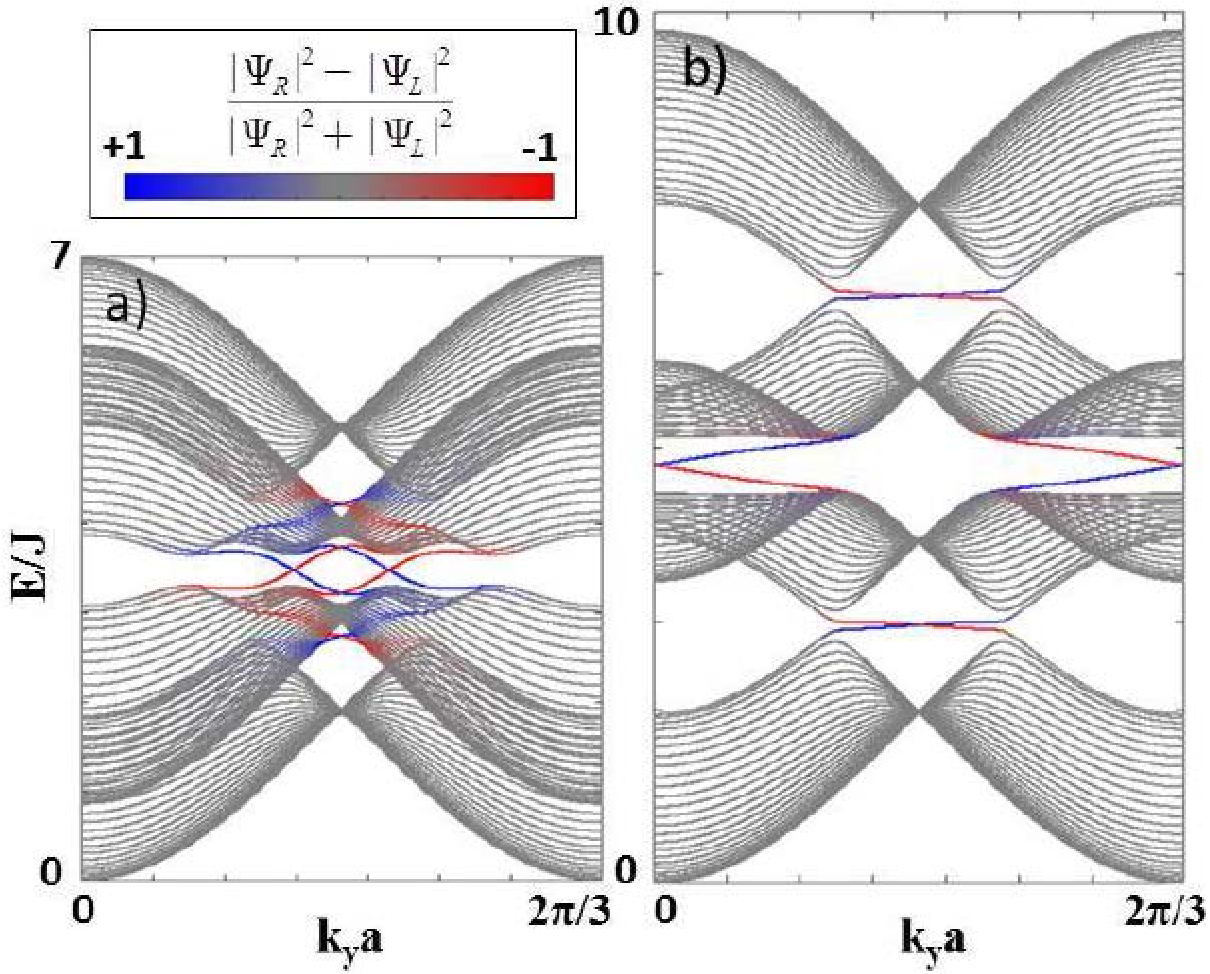
Total Chern number equal 2 in both cases.

But triangles are differently oriented.

Phase diagrams

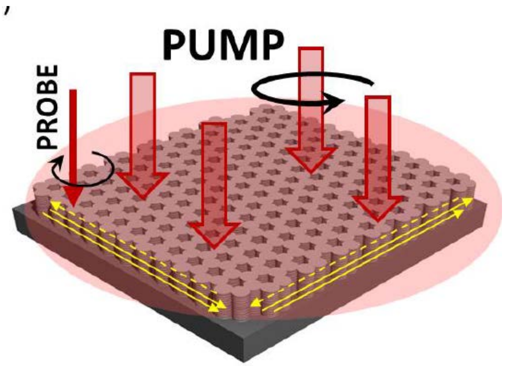


Edge states

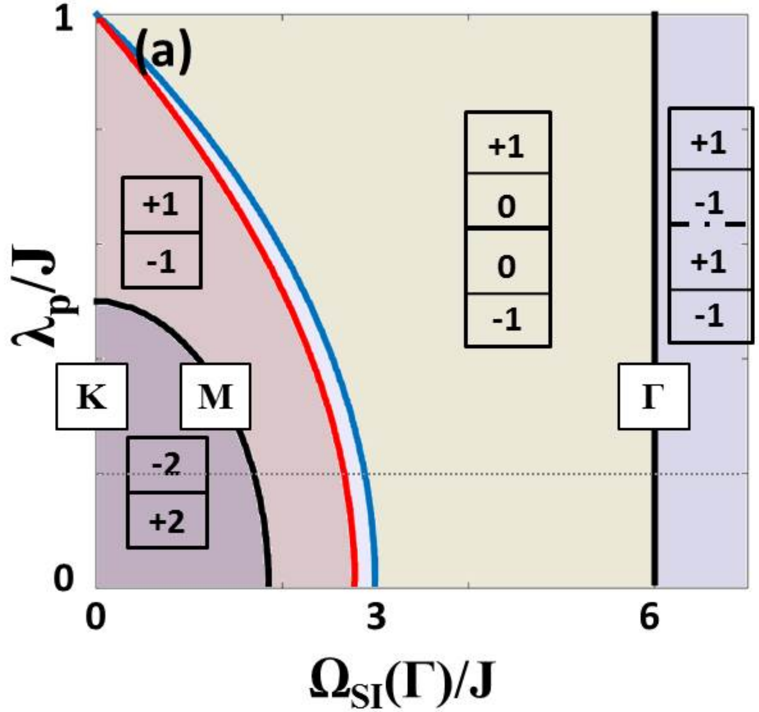


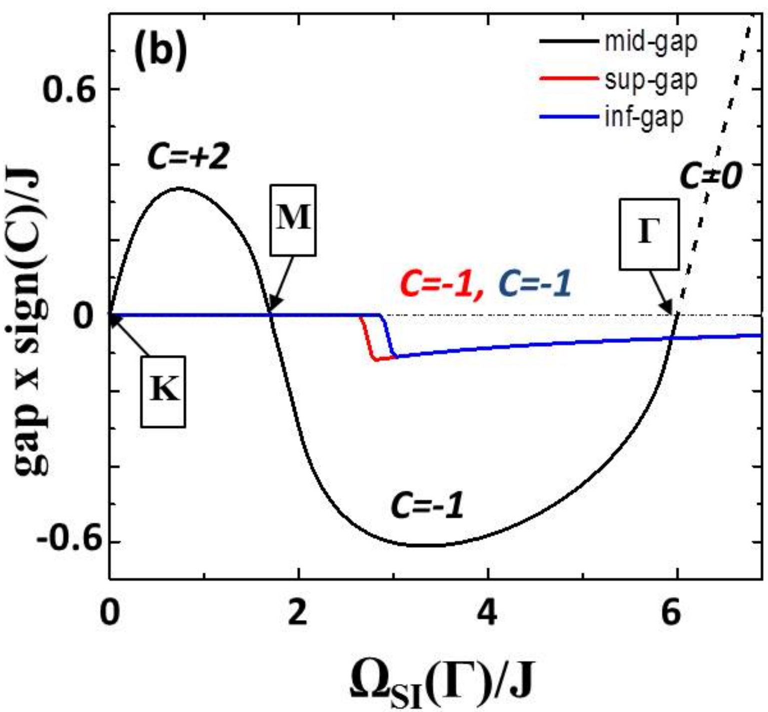
$\frac{|\Psi_R|^2 - |\Psi_L|^2}{|\Psi_R|^2 + |\Psi_L|^2}$
+1
-1
7
a)
E/J
0
0
$2\pi/3$
$k_y a$
10
b)
0
0
$2\pi/3$
$k_y a$

All optical control of the band topology

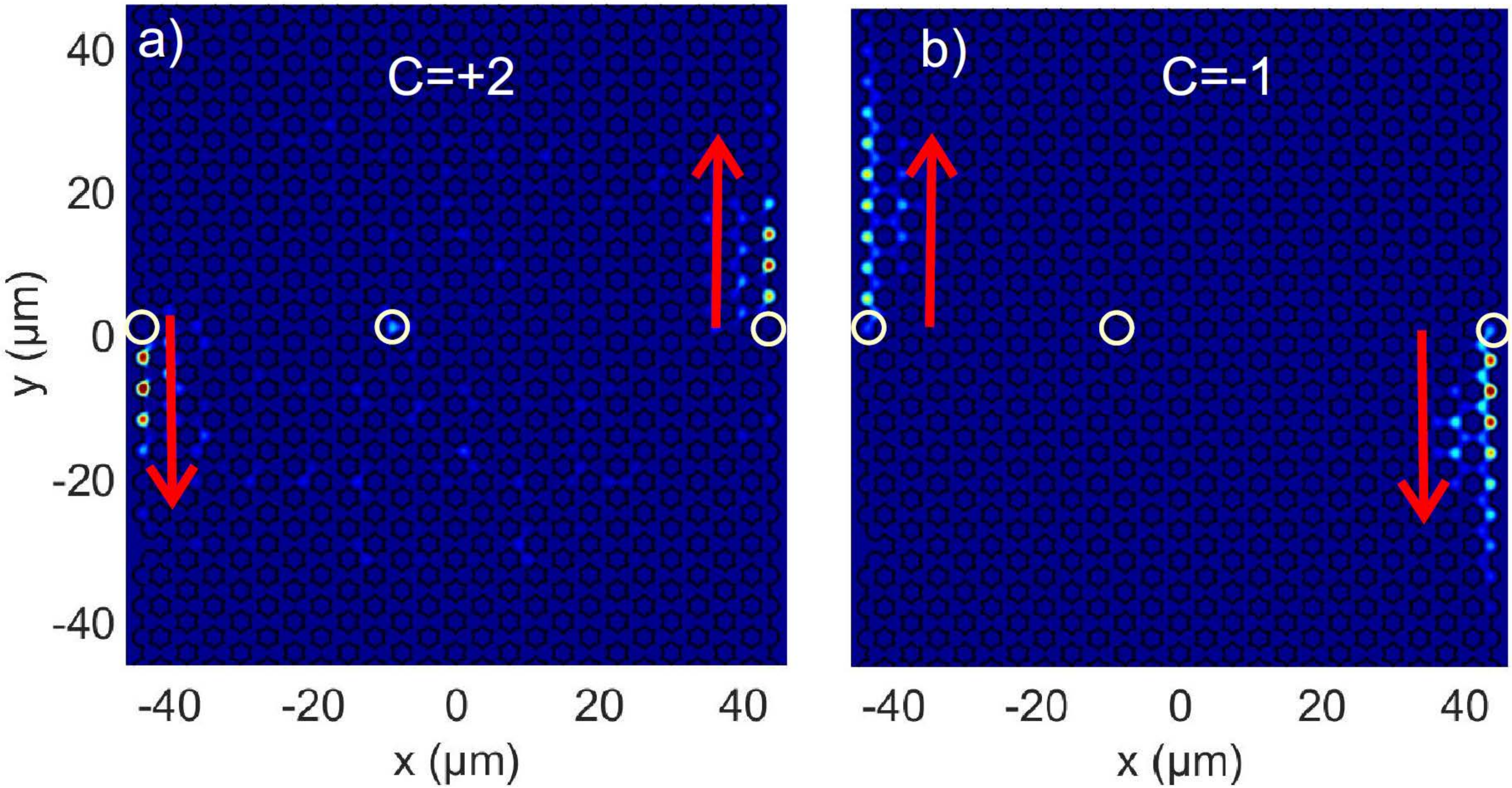


Resonant optical pumping at k=0.
Self induced Zeeman field.

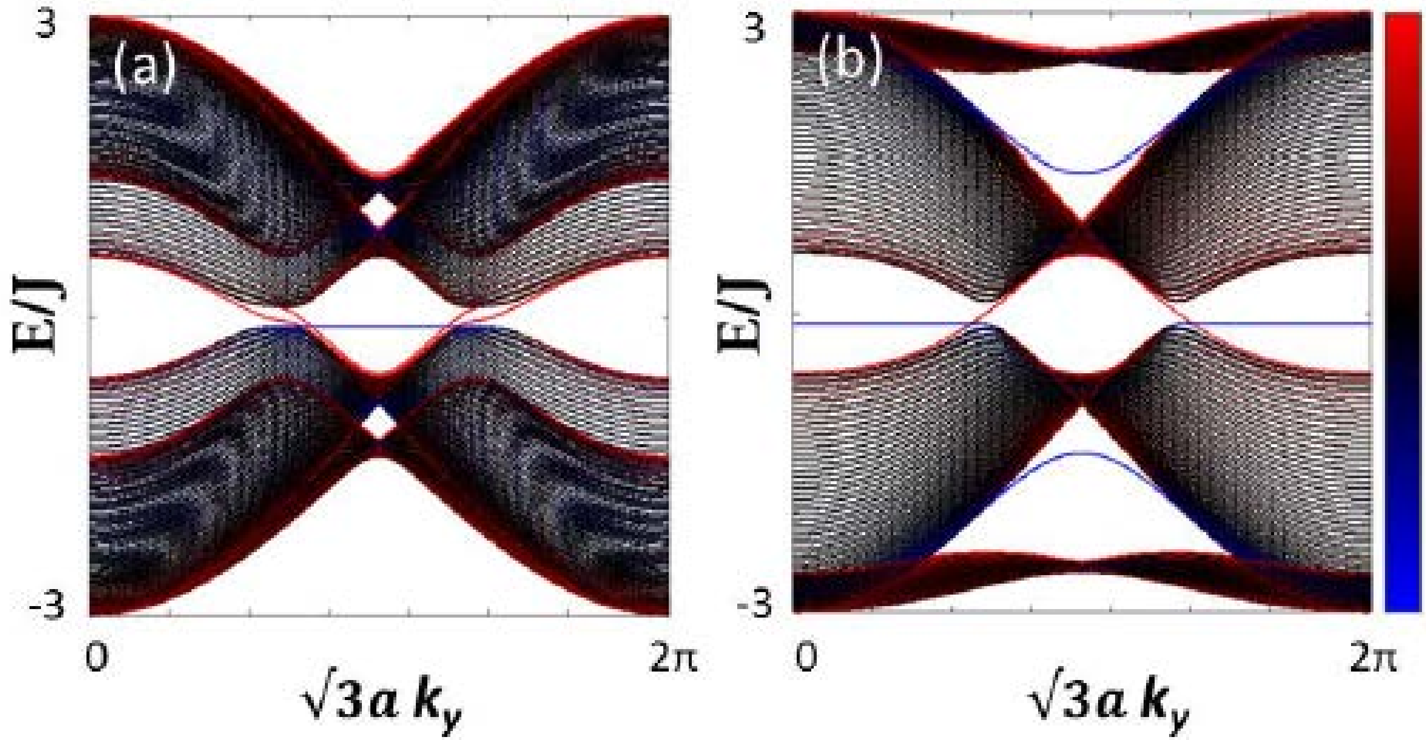




Chirality inversion by increasing the pump density

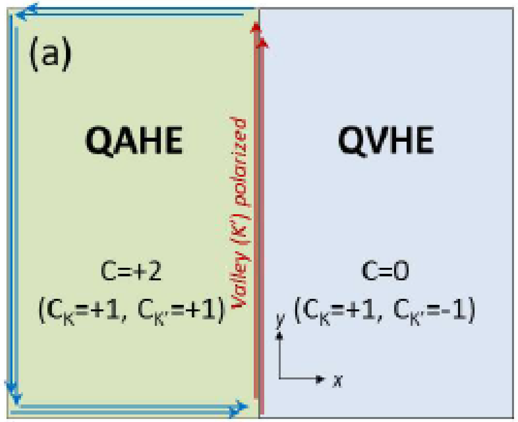


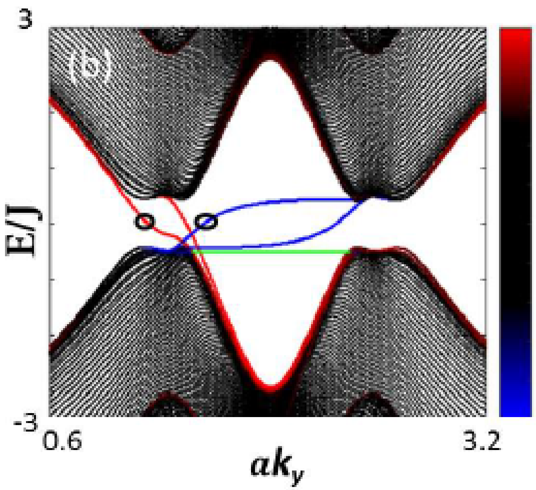
Let us put all together (staggering and TE-TM)

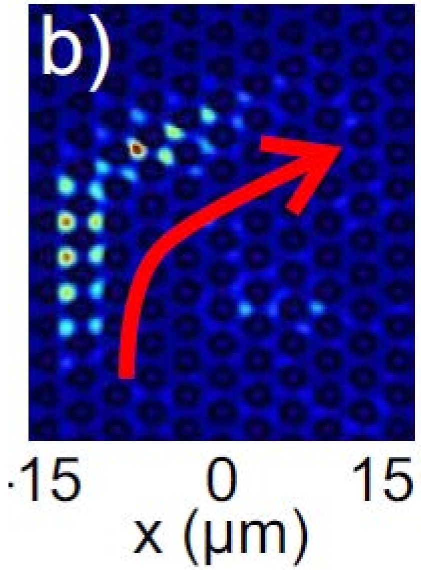


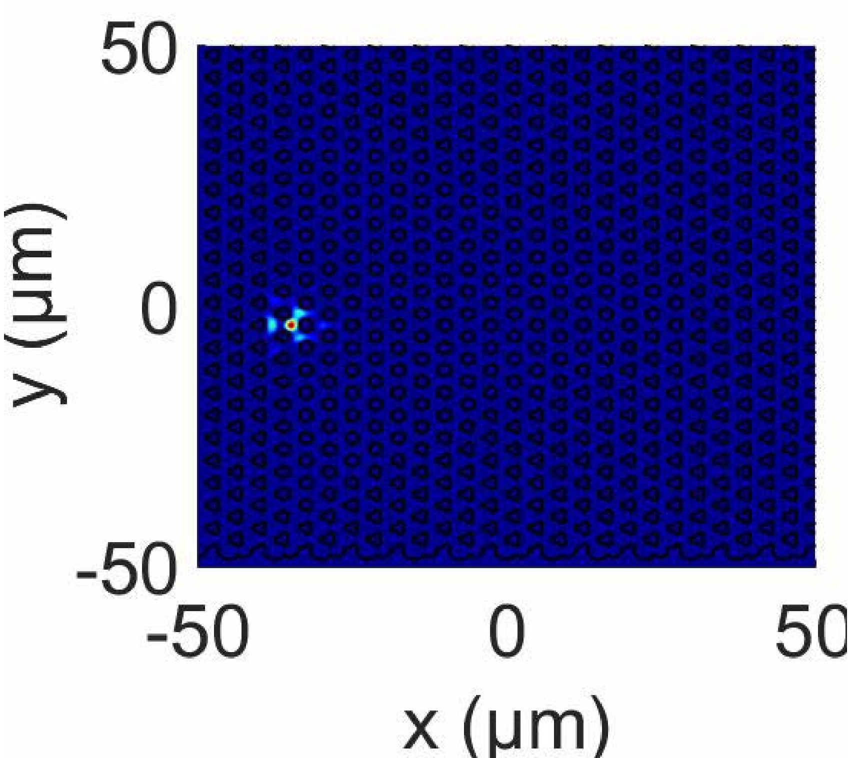
Changing the strength of the TE-TM SOC allows to control the number and direction of Valley polarized interface states.

Valley filter









O. Bleu et al., arXiv:1703.05104

Conclusions

- Quantum Valley Hall effect can be « easily » implemented using polariton graphene.
- Quantum Anomalous Hall Effect
  - Generic Hamitonian of photons in a lattice in presence of SOC.
  - Crucial difference with the electronic case: The SOC winding.
  - Richer phase diagram for the Photonic case.
  - All optical control because of Self Induced Zeeman splitting.
- One can combine QVHE and QAHE to make perfect valley filter.
- Quantum fluids in topological lattices.

Solitons in bosonic dimer chains

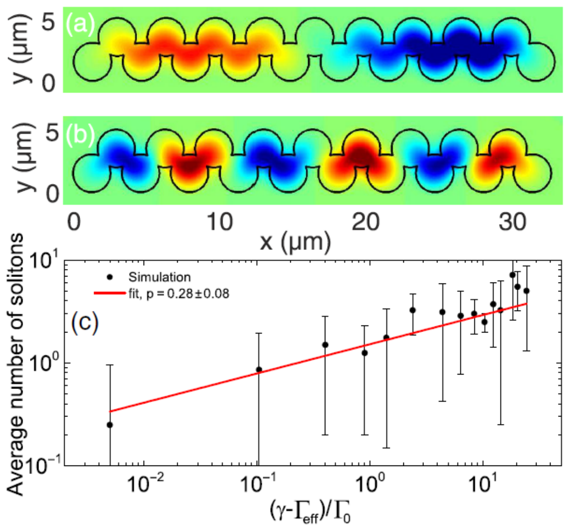
PRL 116, 046402 (2016) PHYSICAL REVIEW LETTERS week ending 29 JANUARY 2016

Kibble-Zurek Mechanism in Topologically Nontrivial Zigzag Chains of Polariton Micropillars

D. D. Solnyshkov,[1] A. V. Nalitov,[1,2] and G. Malpuech[1]
[1]*Institut Pascal, PHOTON-N2, Université Clermont Auvergne, CNRS, 4 Avenue Blaise Pascal, 63178 Aubière Cedex, France*
[2]*School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, United Kingdom*
(Received 15 June 2015; revised manuscript received 7 October 2015; published 29 January 2016)

Zig-zag chain + TE-TM

Existence of topologically protected phase singularities.



Spontaneous symmetry breaking in a topologically non trivial system.

Condensation in the topological edge states observed in arXiv:1704.07310.

PRL 118, 023901 (2017) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2017

Chirality of Topological Gap Solitons in Bosonic Dimer Chains

D. D. Solnyshkov, O. Bleu, B. Teklu, and G. Malpuech
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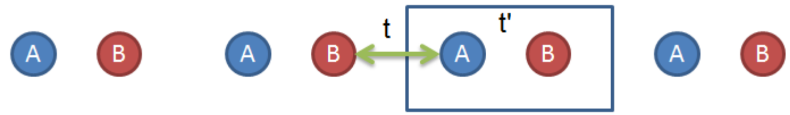
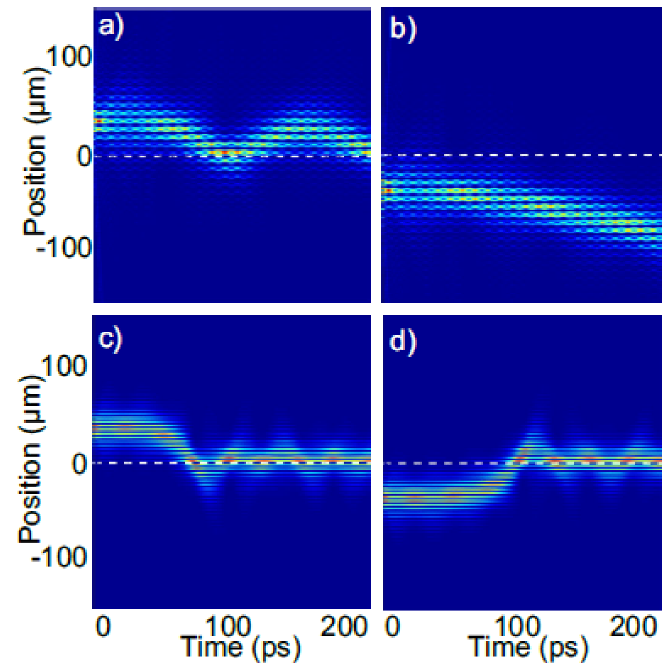
(Received 6 July 2016; published 12 January 2017)

We study gap solitons which appear in the topological gap of 1D bosonic dimer chains within the mean-field approximation. We find that such solitons have a nontrivial texture of the sublattice pseudospin. We reveal their chiral nature by demonstrating the anisotropy of their behavior in the presence of a localized energy potential.



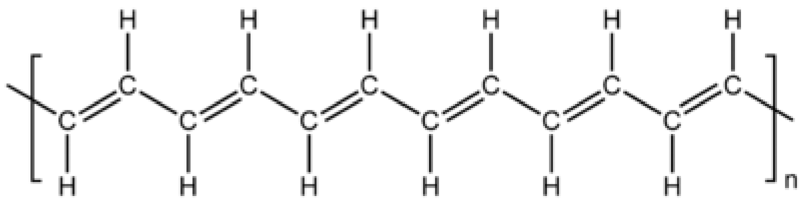
Chiral 1 D band

Chiral non-linear gap states

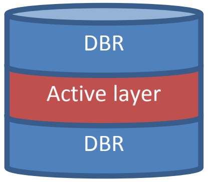


Observed in Science, 350, 182, 2015.

Dimer chains

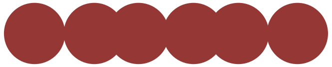


- Original study: polymers
  - Well-known SSH model
- Sites = pillar cavities
  - S-type band (the lowest one)



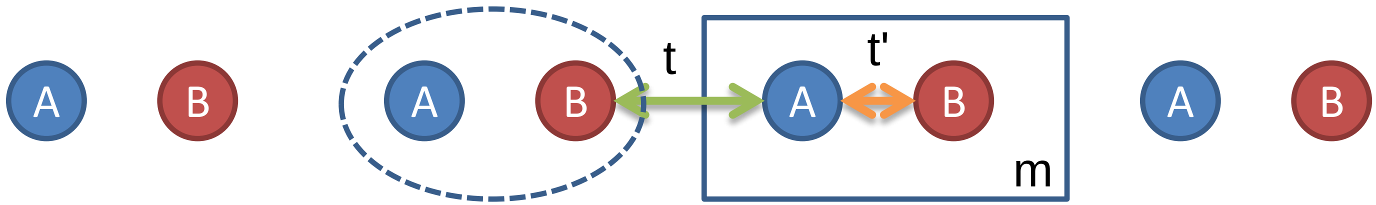
- Zigzag chain
  - Dimerization by polarisation effects.
  - Dimerization by existence of 2 orbitals.

- Straight chain
  - Dimerization introduced by fabrication

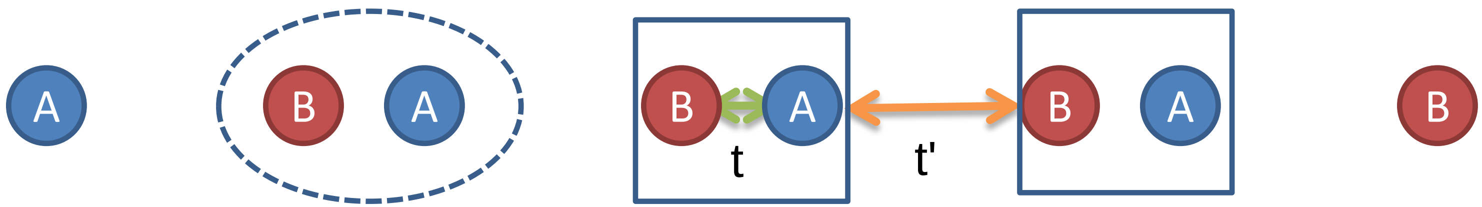


- Dimer chains in other systems
  - Acoustic waves, Atomic lattices, Waveguide arrays…

1D: Dimer chain and edge states



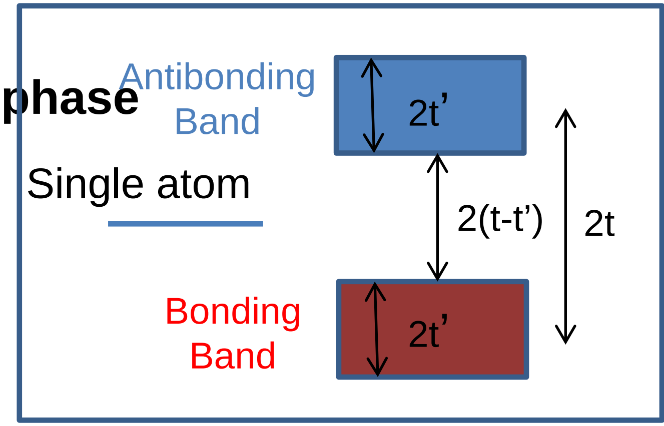
t'>t: tightly bound pairs = "molecules" **AB**, no «extra» atoms
Two bands: AB in phase/out of phase (like s and p states of a single site)



t'<t: tightly bound "molecules" **BA**; two «extra» atoms on the edges

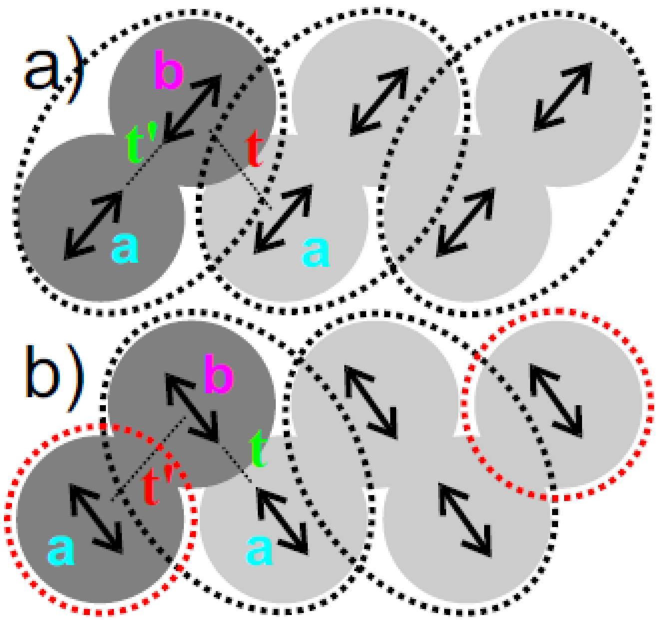
Topological quantity characterizing edge states – the **Zak phase**

$$\gamma_n = \int_{-\pi/a}^{\pi/a} \frac{2\pi}{a} \int_0^a u_{nk}^*(x)\, i \frac{\partial u_{nk}(x)}{\partial k}\, dx dk$$

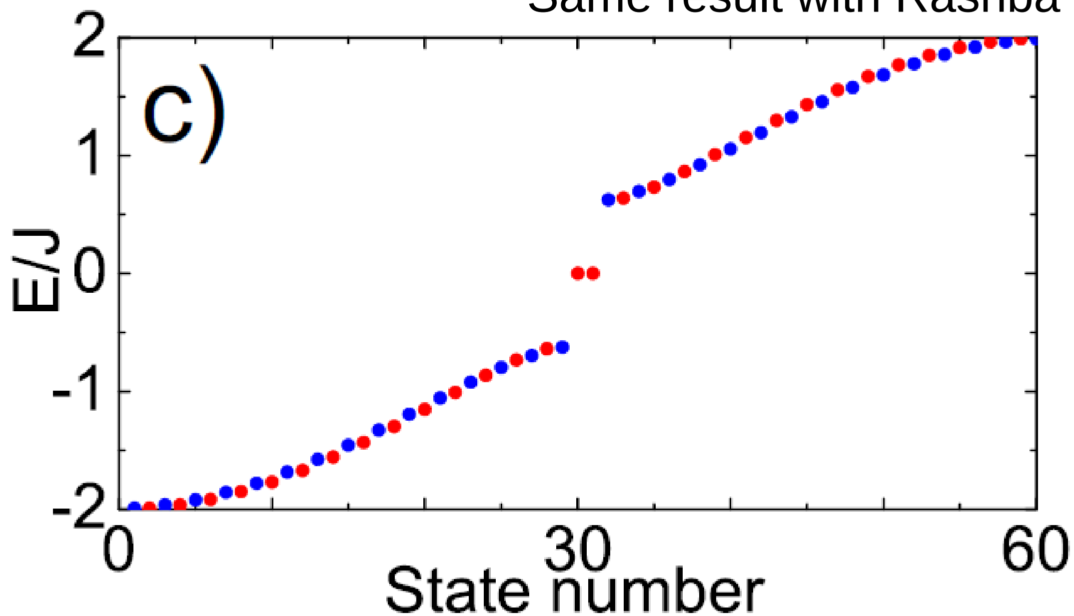


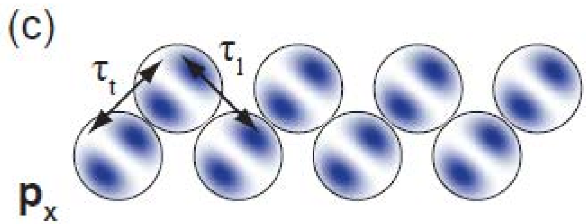
Dimerization of a zigzag chain for polaritons

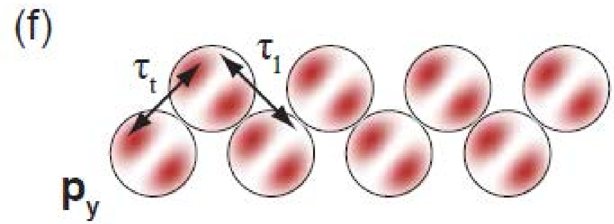
- Polarization-dependent coefficients t and t’
- Tight-binding calculation of the eigenstates
- 0 edge states in D-polar, 2 edge states in A-polar



Same result with Rashba field







Same can be done with p-orbitals
arXiv:1704.07310

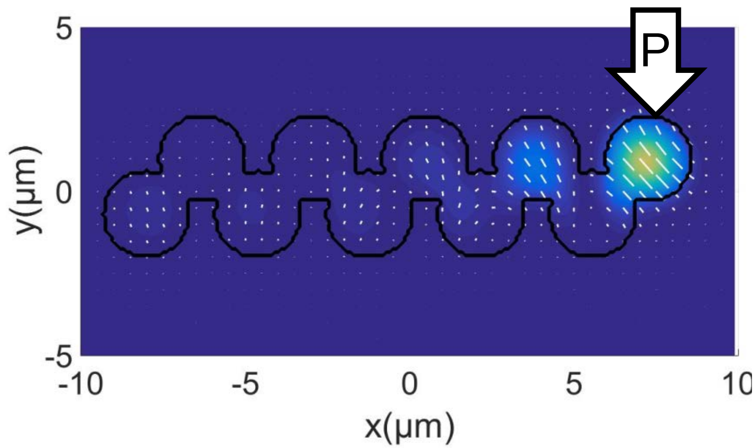
Edge states in the condensation

- Edge states favored by higher overlap
- Localized pumping

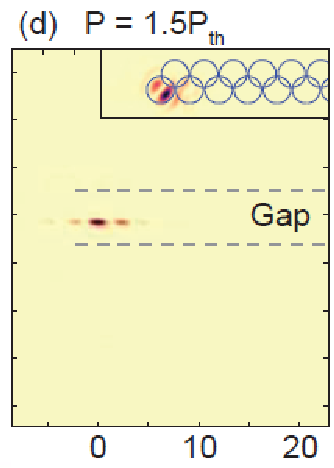
$$i\hbar\frac{\partial\psi_\pm}{\partial t} = -\left(1-i\Lambda\right)\frac{\hbar^2}{2m}\Delta\psi_\pm + \beta\left(\frac{\partial}{\partial x}\mp i\frac{\partial}{\partial y}\right)^2\psi_\mp \quad (4)$$

$$+U\psi_\pm - \frac{i\hbar}{2\tau}\psi_\pm + \left(\left(U_R + i\gamma(n)\right)\psi_\pm + \xi\right)\exp\left(-\frac{(\mathbf{r}-\mathbf{r}_0)^2}{\sigma^2}\right)$$

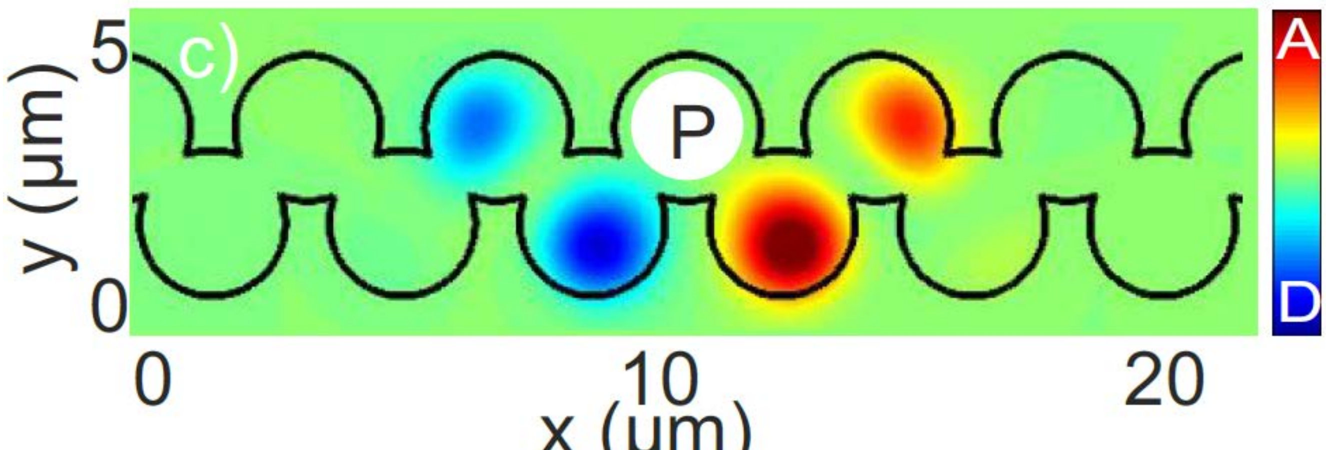
D. Solnyshkov et al., PRA 89, 033626 (2014).



Pump on the edge.
Condensation in the edge state.



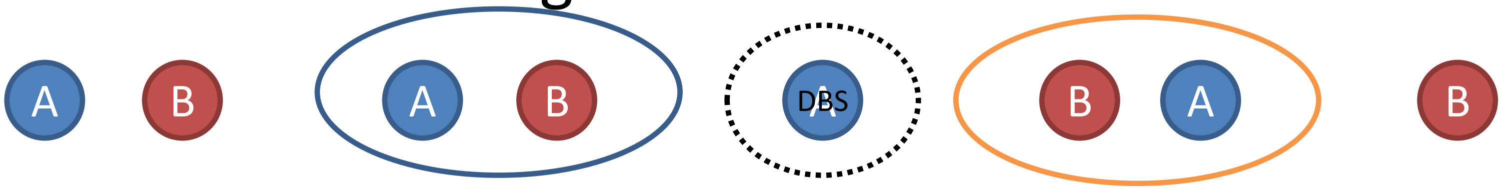
Observed in arXiv:1704.07310



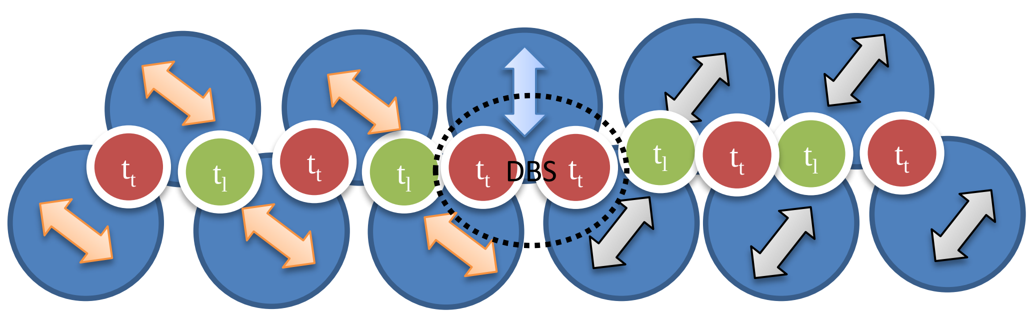
High reservoir cuts the chain into 2, and condensation occurs in the two self induced edges.

Solitonic solution

- Su-Schrieffer-Heeger soliton in CHx



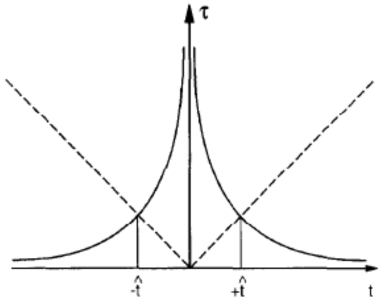
- Choice of polarization (A/D) = dimerization
- Dark-bright soliton in polariton system
- Kibble-Zurek mechanism or controlled creation



SSH, 1980

Dark-bright soliton on H/V basis

Kibble-Zurek Mechanis



Adiabaticity breakdown

- Symmetry-breaking phase transitions
  - Order parameter
  - Thermodynamic potential
- Critical point: potential almost flat
- Large fluctuations of the order parameter
  - « Fluctuation » here means « order »!
  - Low energy of the fluctuations, slow relaxation
- Adiabatic phase transition impossible
- Order parameter domains from fluctuations

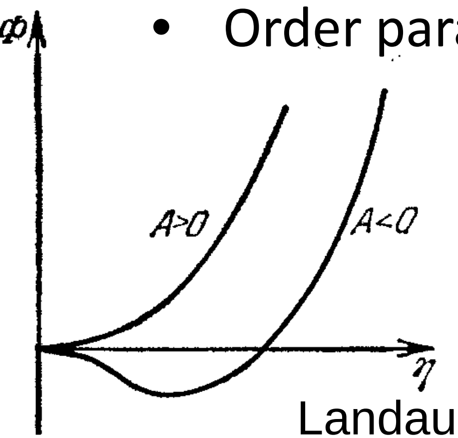


Рис. 62.

$$\Phi(\eta) = \Phi_0 + \alpha\eta + A\eta^2 + B\eta^3 + C\eta^4 + \dots$$

$$A(T) = a\frac{T - T_c}{T_c} = a\varepsilon \qquad \varepsilon = \frac{T - T_c}{T_c}$$

Landau & Lifshits, Statistical Physics (1939)

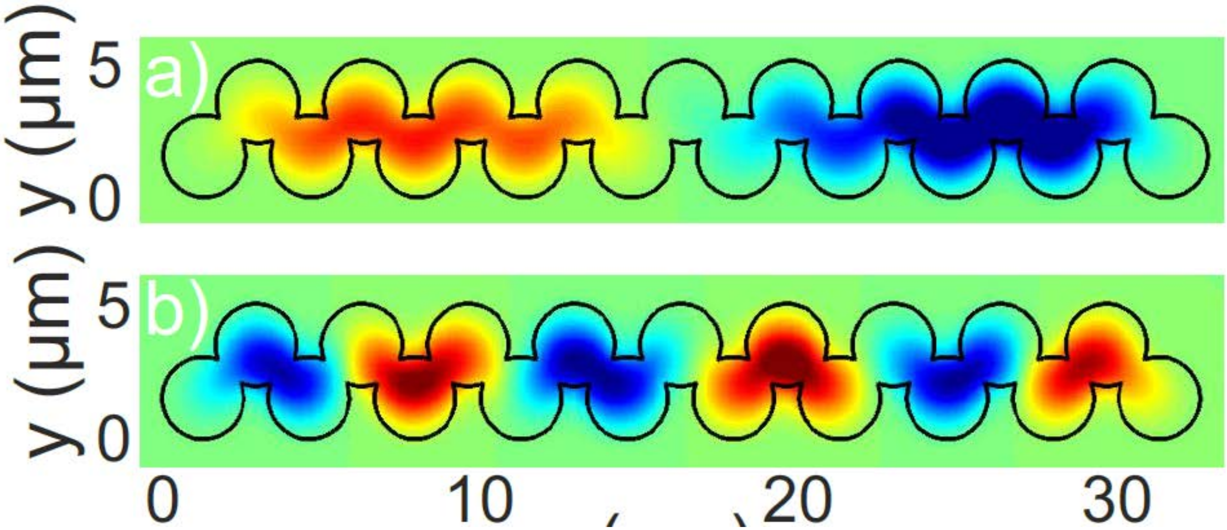
$$C > 0$$

Higgs, 1964

Topology of cosmic domains and strings, Kibble 1976

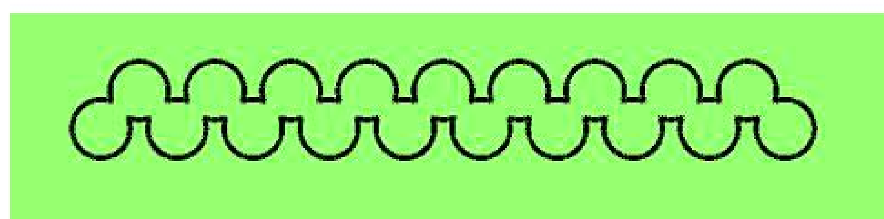
Solitons in polariton condensation (homogeneous pumping)

Plotting difference between diagonal polarization intensities I_A-I_D



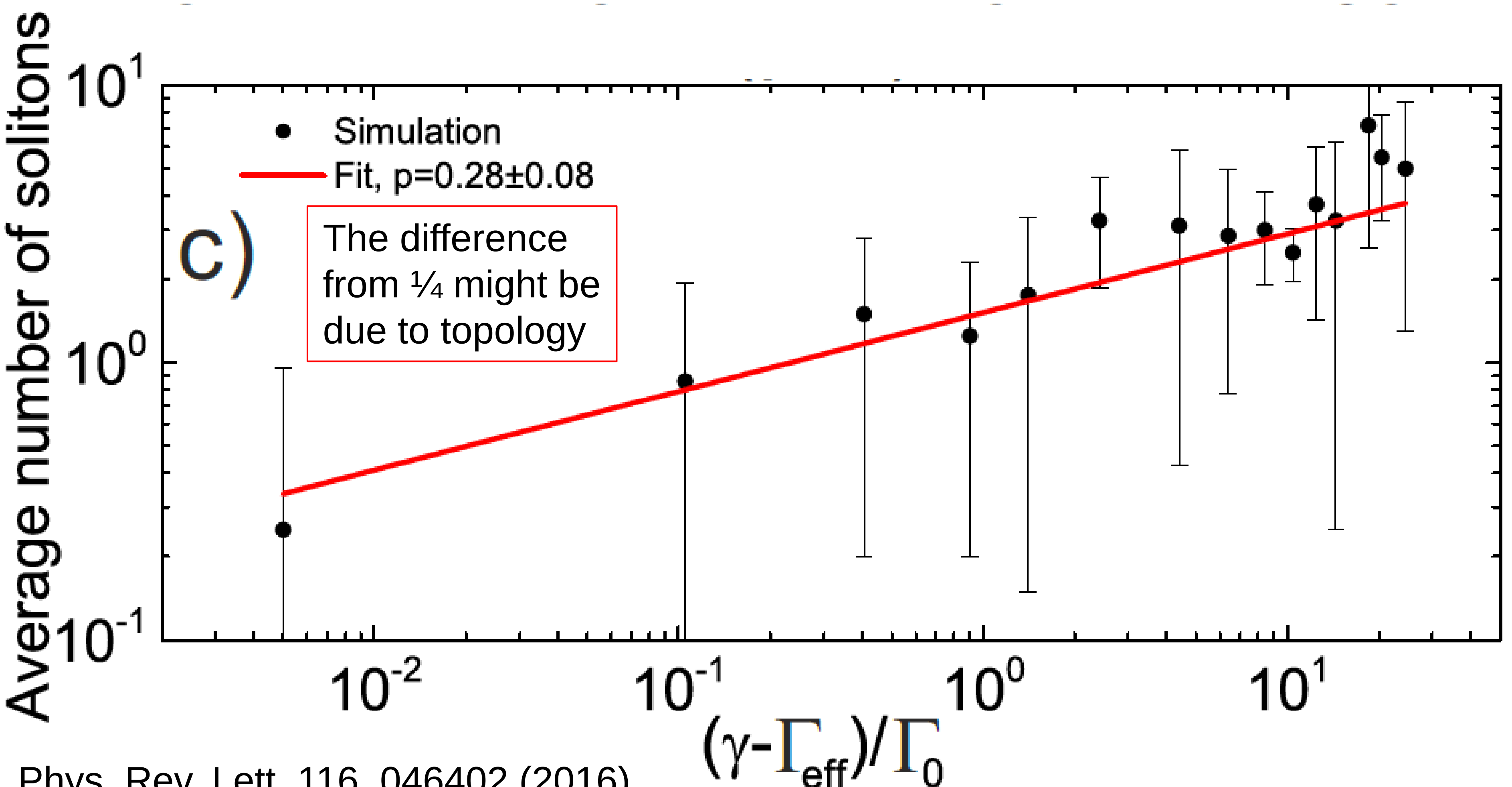
More solitons are observed on average for higher pumping…

Video of the condensation dynamics



KZM power law

The MF scaling exponent of ¼ is verified!

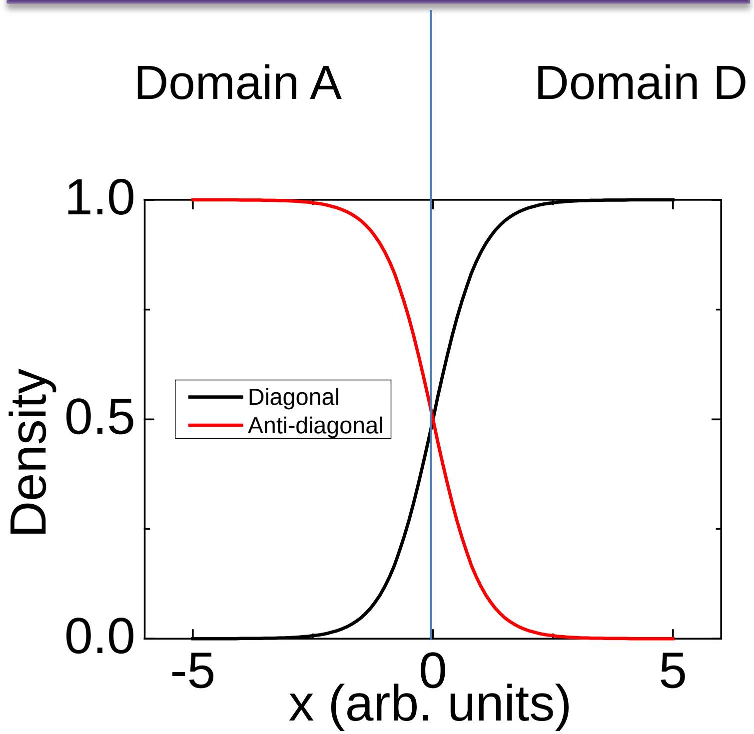


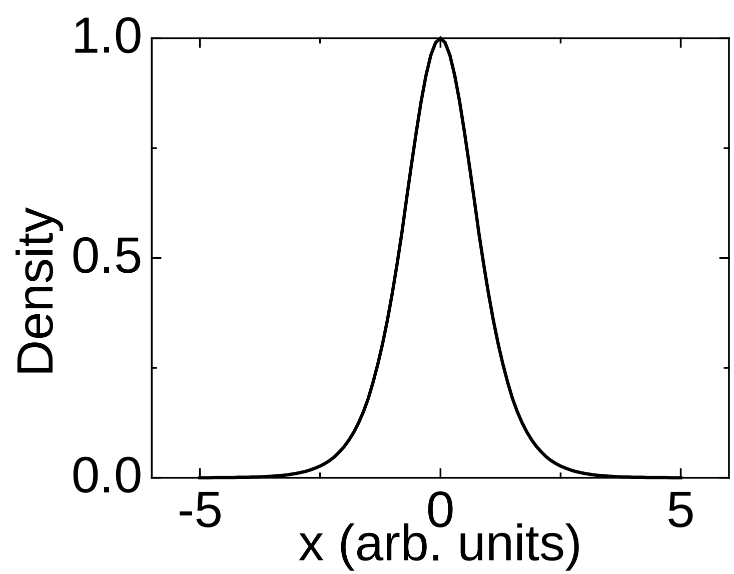
Phys. Rev. Lett. 116, 046402 (2016)

SSH (dark-bright) soliton vs Gap soliton

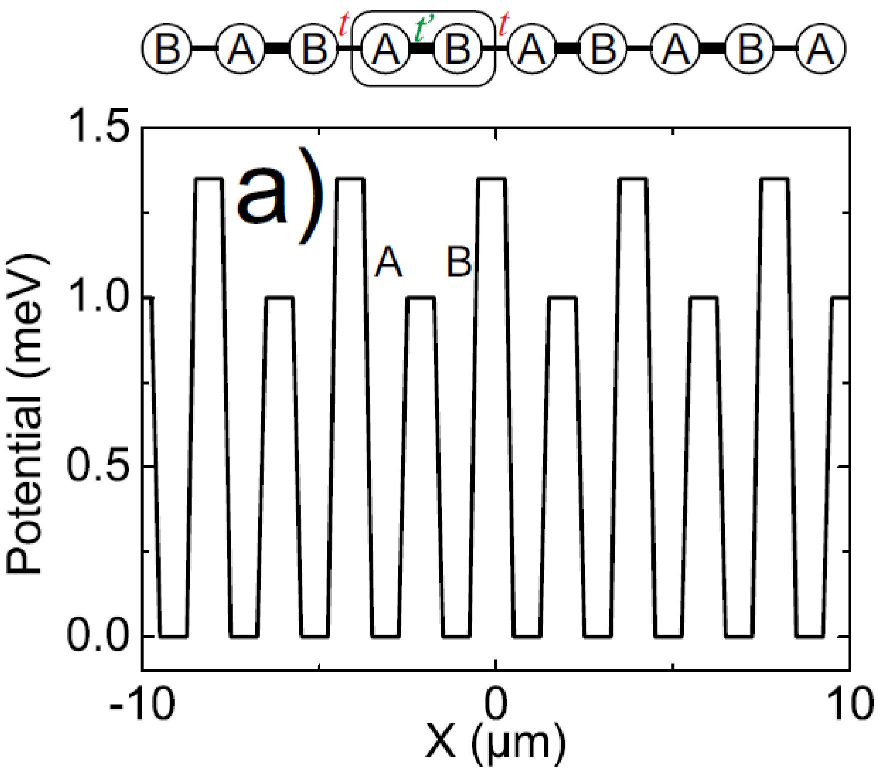
SSH soliton = domain wall in an extended condensate
Two different dimerization domains
Total density is **constant**
Dynamic dimerization required

Gap soliton = localized bright soliton
Repulsive interactions + negative mass
Negative mass = maximization of energy

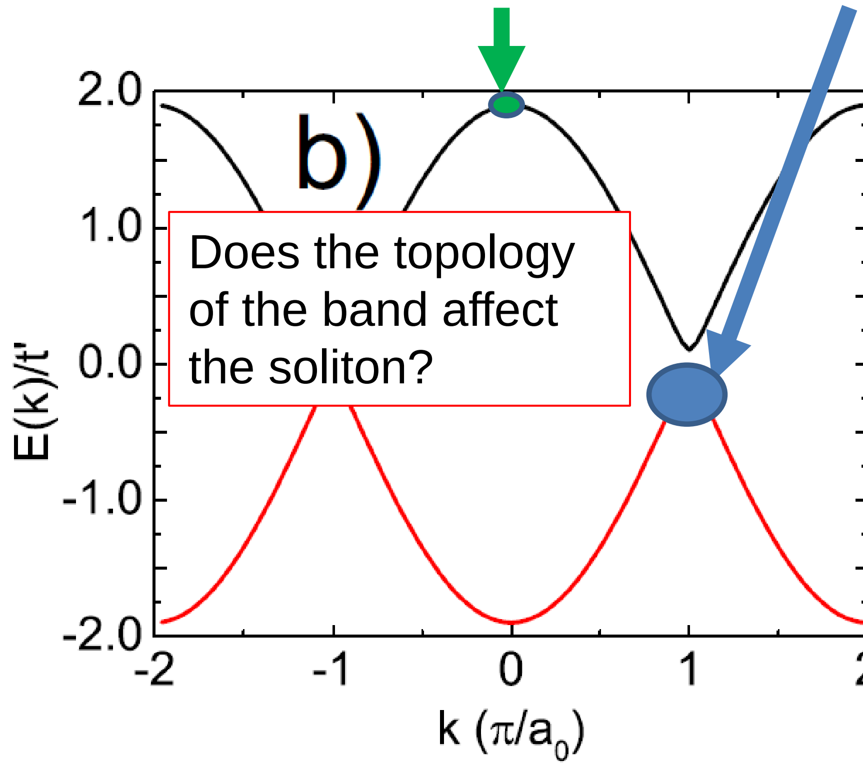




Straight dimer chain: gap soliton



Gap solitons possible for negative mass sta



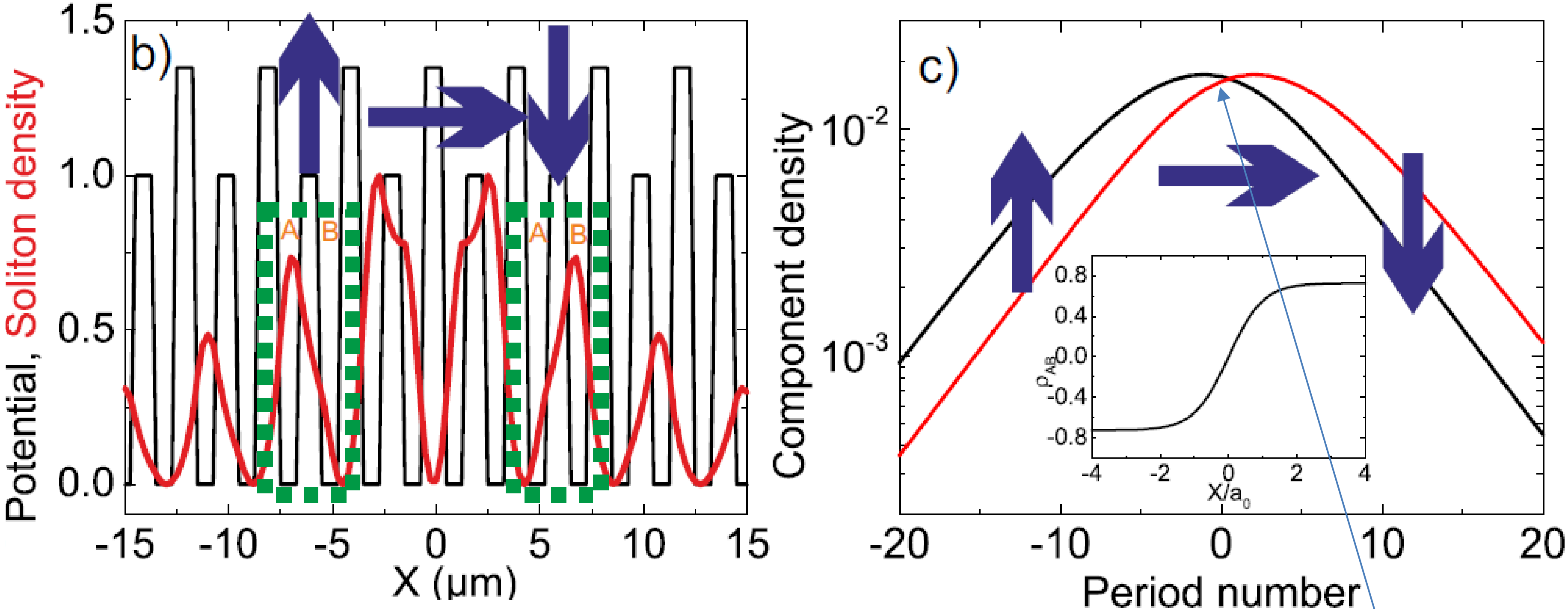
$$\hat{H}(k) = -\begin{pmatrix} 0 & t' + te^{-ika_0} \\ t' + te^{ika_0} & 0 \end{pmatrix}$$

Pseudospin: A and B atoms of a dimer

2x2 Hamiltonian favors pseudospin interpretation

$$E_{\pm}(k) = \pm\sqrt{t^2 + t'^2 + 2tt'\cos(ka_0)}$$

Gap soliton in the topological gap: variational approach



$$\psi(x,a,b) = 2\sqrt{n/a}\begin{pmatrix} 1/\cosh\left((x-b)/a\right) \\ 1/\cosh\left((x+b)/a\right) \end{pmatrix}$$

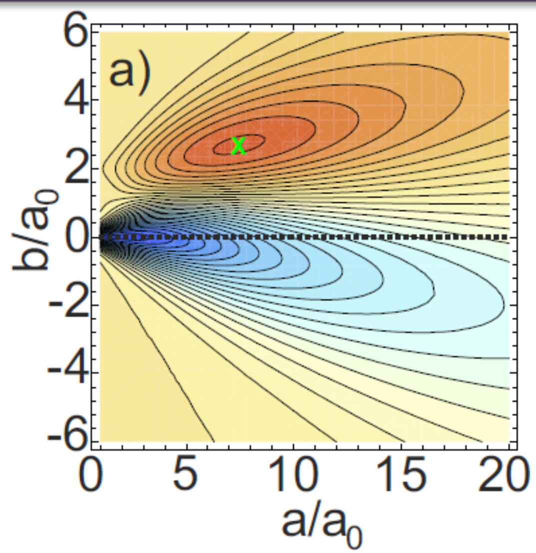
Displaced maxima for A and B components

Spin-anisotropic interactions

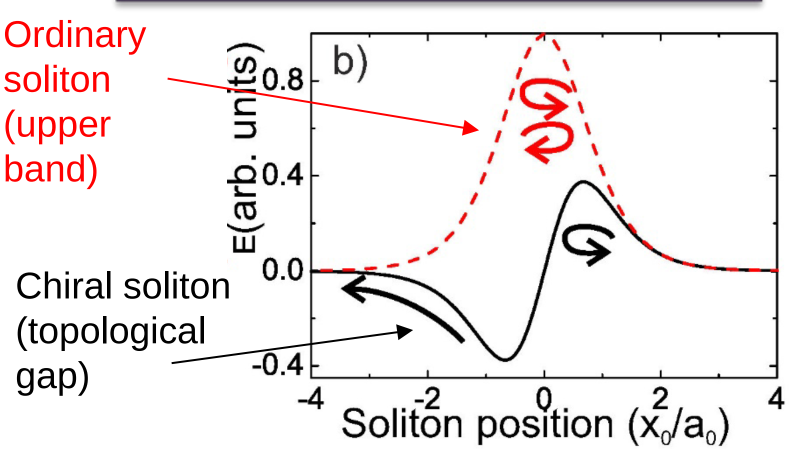
$$E_{int}(a) = \frac{1}{2}\alpha\int_{-\infty}^{+\infty}\left(|\psi_A|^4 + |\psi_B|^4\right)dx$$

Chiral behavior of the gap soliton

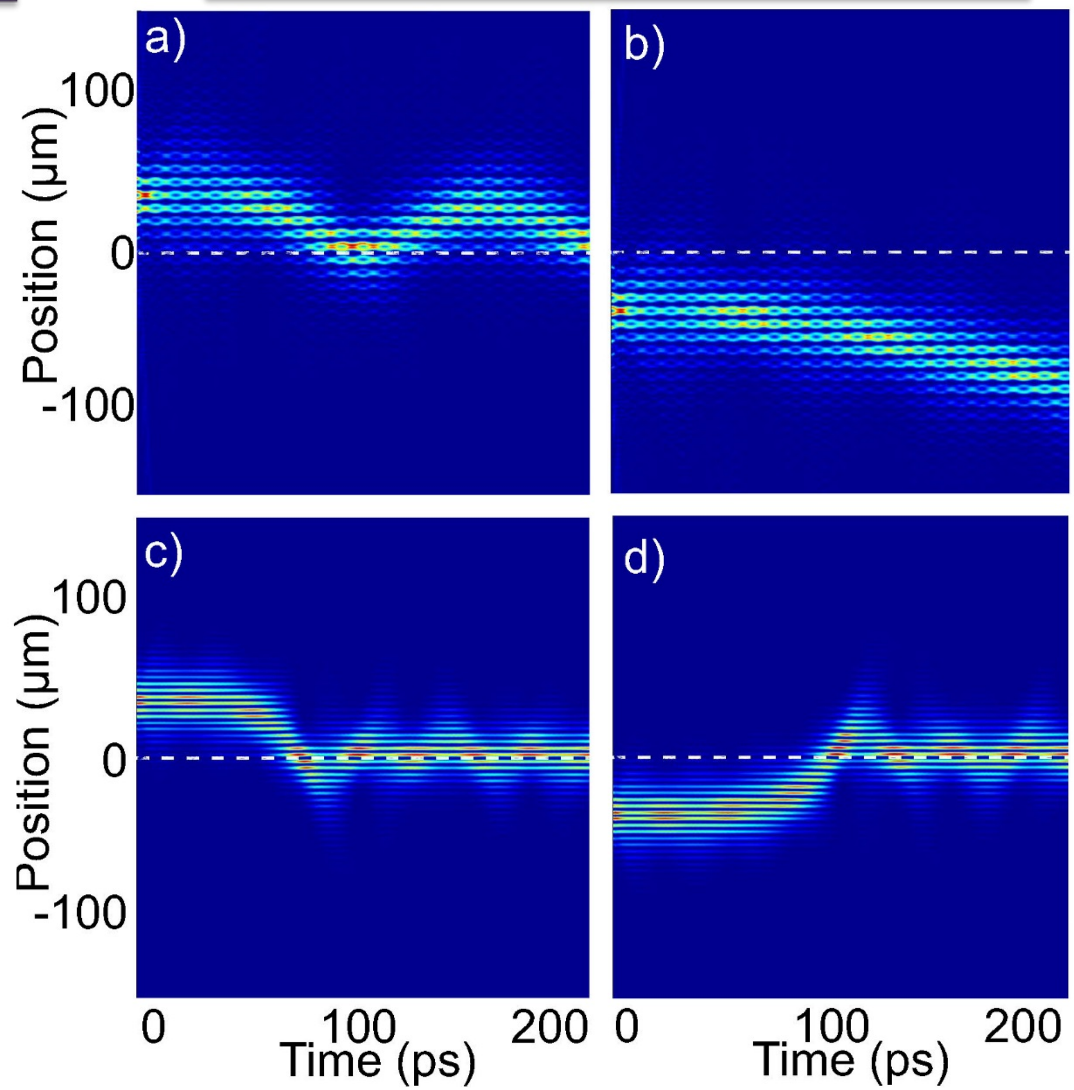
Only one type of soliton possible:



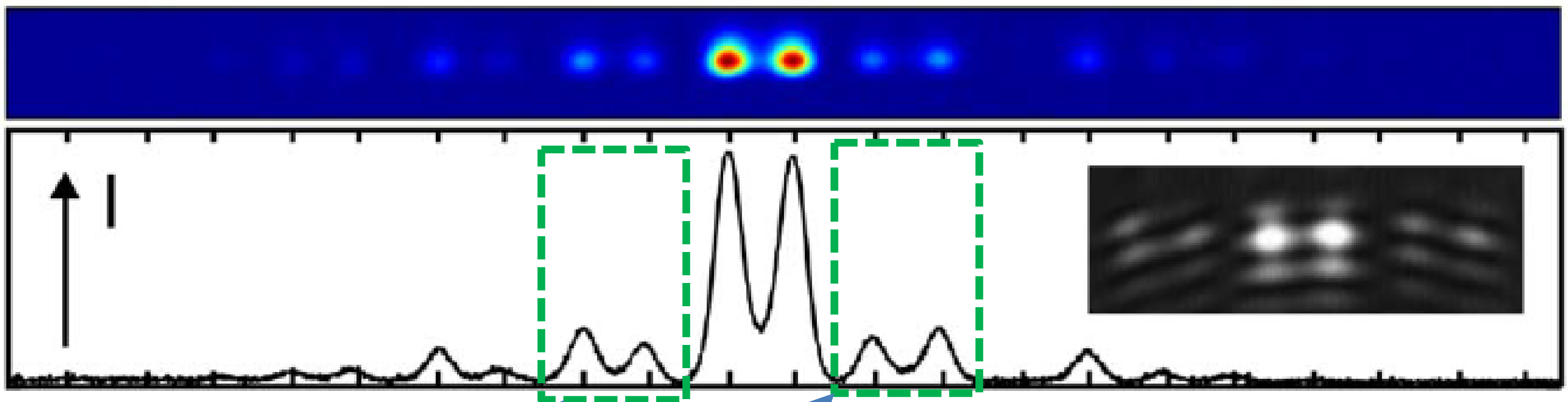
Energy in presence of a defect:



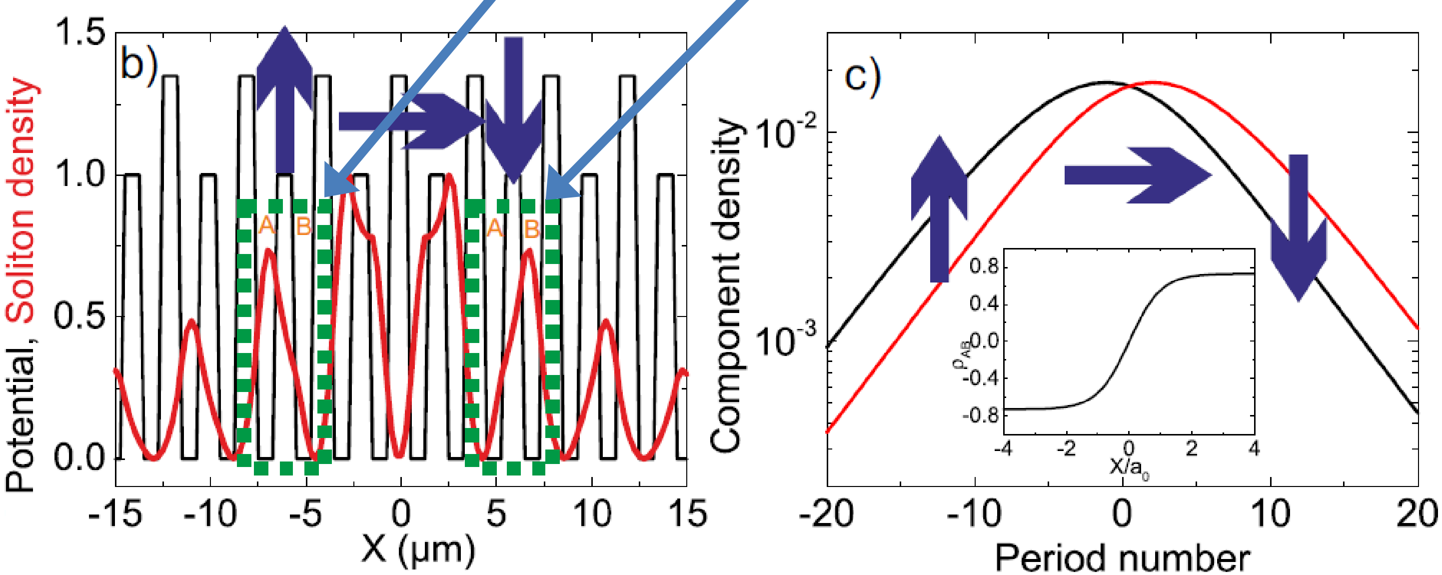
Dynamics of chiral and ordinary solitons



Experimental observation in a waveguide array



A. Kanshu et al, Optics Letters 37, 1253 (2012)



The pseudospin-polarized nature and the associated chirality have not been understood

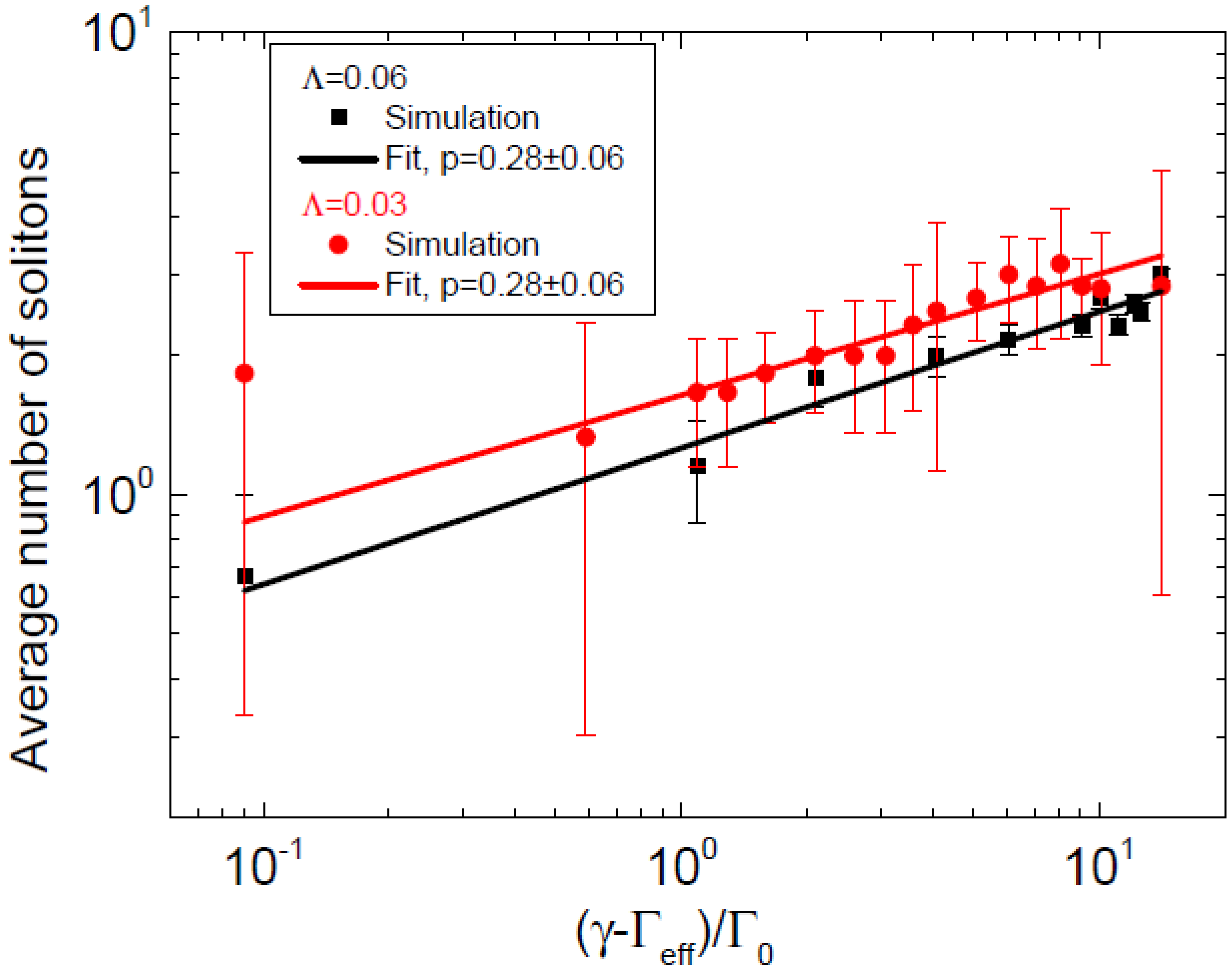
Conclusions

- TE-TM + Zigzag = dimerization
  - Stable dark-bright solitons
  - KZM formation
- Straight dimer chains = chiral solitons
  - Anisotropic behavior

Phys. Rev. Lett. 114, 116401 (2015);
Phys. Rev. B 93, 085438 (2016)
arXiv:1606.07410 (2016)

Phys. Rev. Lett. 116, 046402 (2016
arXiv:1607.01805 (2016)

Different relaxation constants



Higher relaxation means less solitons, but the same scaling exponent

One- word about 1D geometries

PRL 116, 046402 (2016) PHYSICAL REVIEW LETTERS week ending 29 JANUARY 2016

Kibble-Zurek Mechanism in Topologically Nontrivial Zigzag Chains of Polariton Micropillars

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[1]*Institut Pascal, PHOTON-N2, Université Clermont Auvergne, CNRS, 4 Avenue Blaise Pascal, 63178 Aubière Cedex, France*
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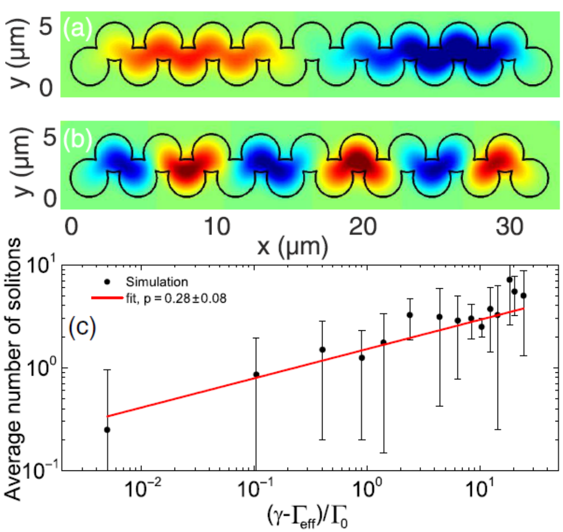
(Received 15 June 2015; revised manuscript received 7 October 2015; published 29 January 2016)

Zig-zag chain + TE-TM

Existence of topologically protected phase singularities.

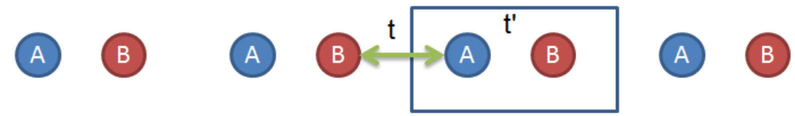


Spontaneous symmetry breaking in a topologically non trivial system.

Topological gap solitons in dimer chains

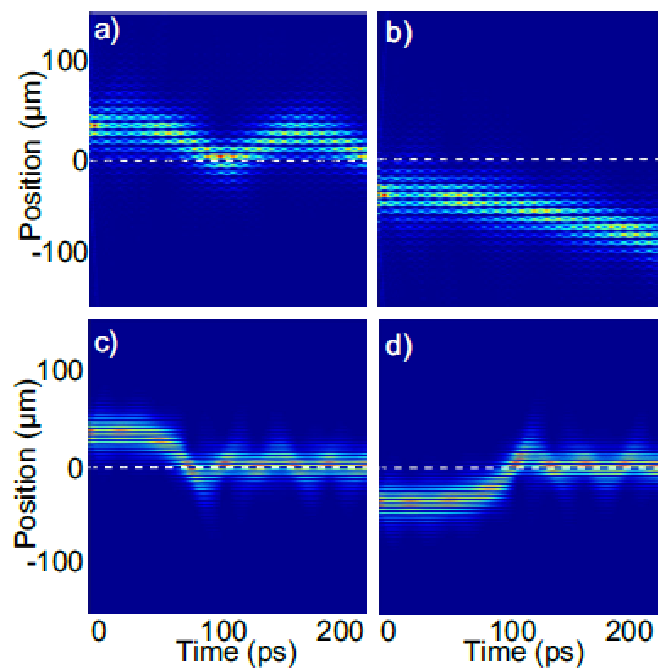
D. D. Solnyshkov, O. Bleu, B. Teklu, G. Malpuech
Institut Pascal, PHOTON-N2, Clermont Université, Blaise Pascal University, CNRS, 24 avenue des Landais, 63177 Aubière Cedex, France.

We study gap solitons which appear in the topological gap of 1D dimer chains. We find that such solitons have a non-trivial texture of the sublattice pseudospin. We reveal their chiral nature by demonstrating the anisotropy of their behavior in presence of effective magnetic fields.



Chiral 1 D band

Chiral non-linear gap states

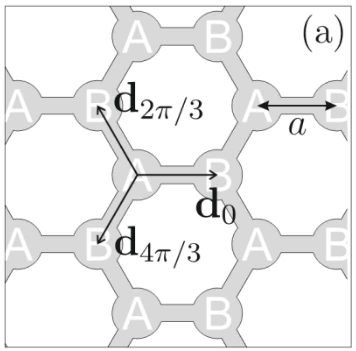


Observed in Science, 350, 182, 2015.

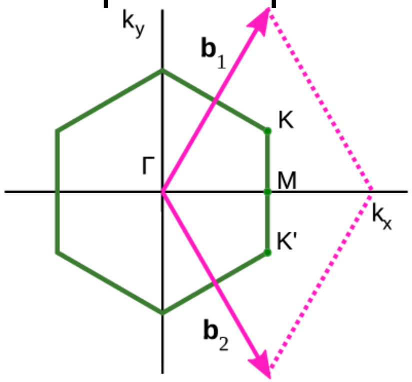
Polaritonic graphene

honeycomb lattice:

real space



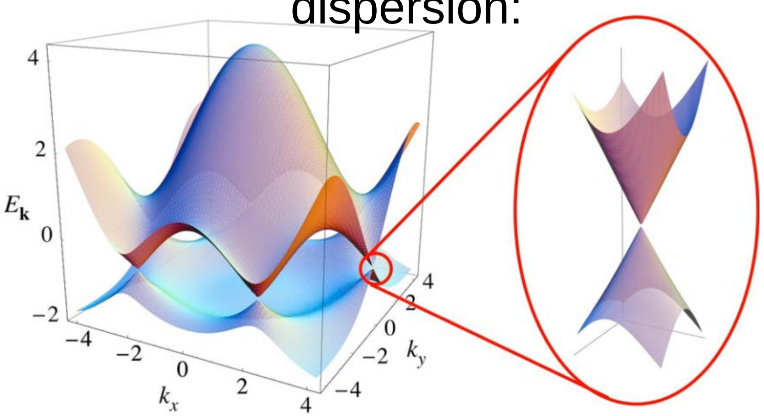
reciprocal space

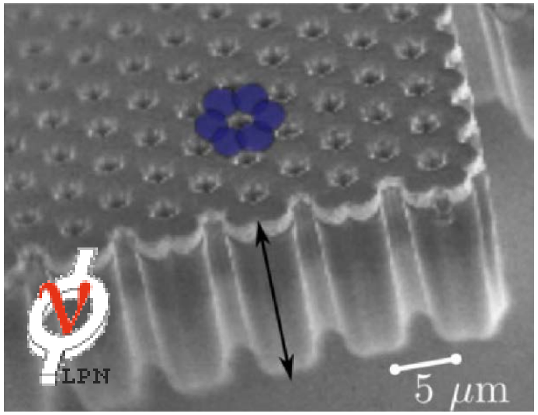


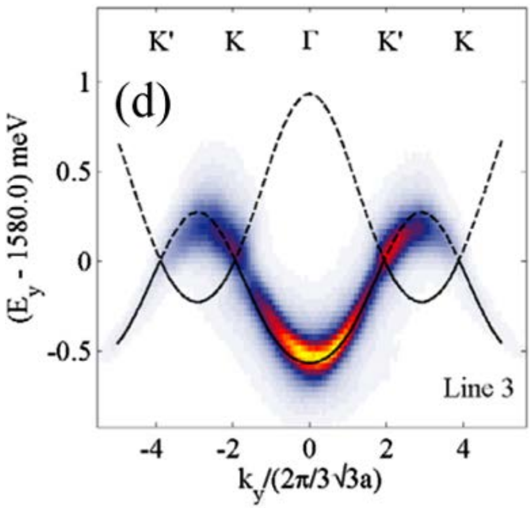
tight-binding Hamiltonian:

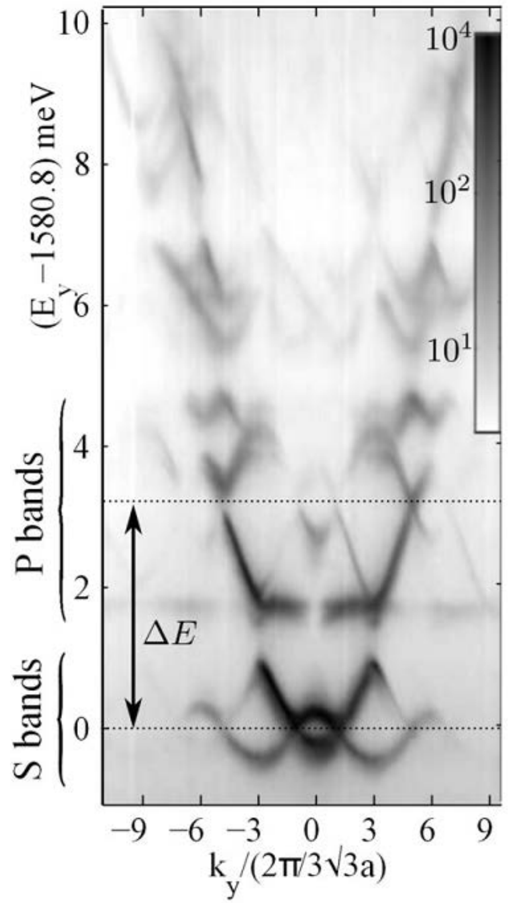
$$\mathrm{H}_{\mathbf{k}} = \begin{pmatrix} 0 & -J f_{\mathbf{k}} \\ -J f_{\mathbf{k}}^* & 0 \end{pmatrix} \begin{matrix} \text{A} \\ \text{B} \end{matrix} \qquad f_{\mathbf{k}} = \sum_{j=1}^{3} \exp(-i\mathbf{k}\mathbf{d}_{\varphi_j})$$

dispersion:



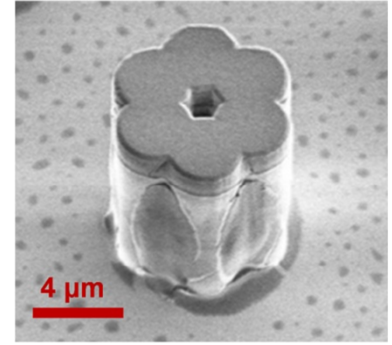




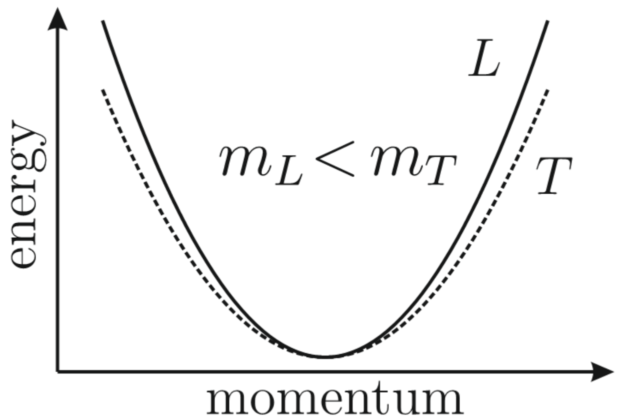


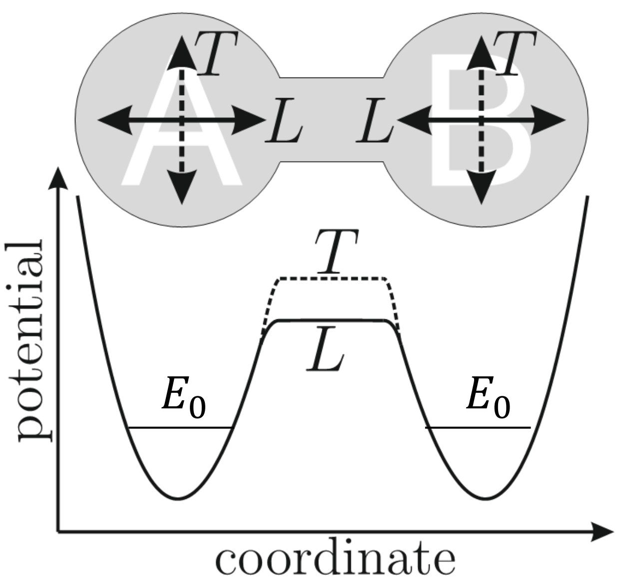
T. Jacqmin et al.,
Phys. Rev. Lett. **112** 116402 (2014)

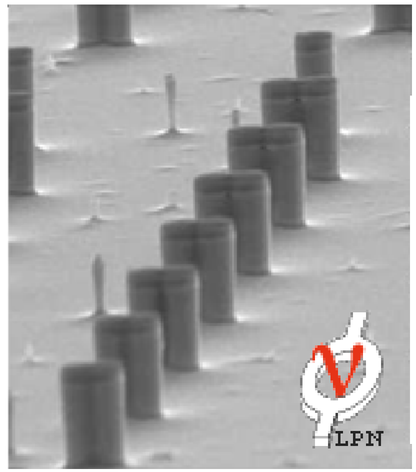
Polariton molecule: Tight binding approach



TE-TM splitting:

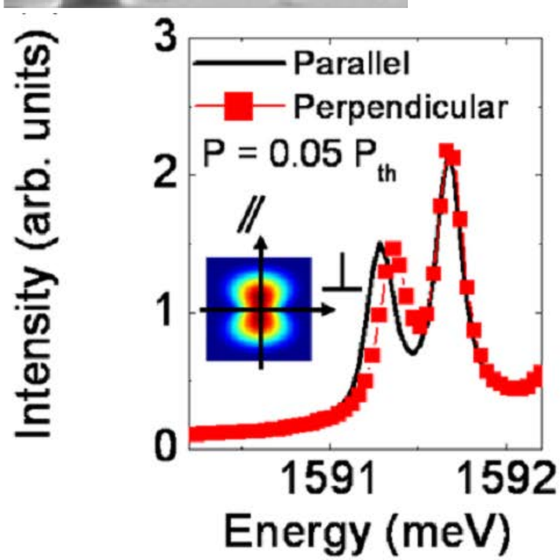


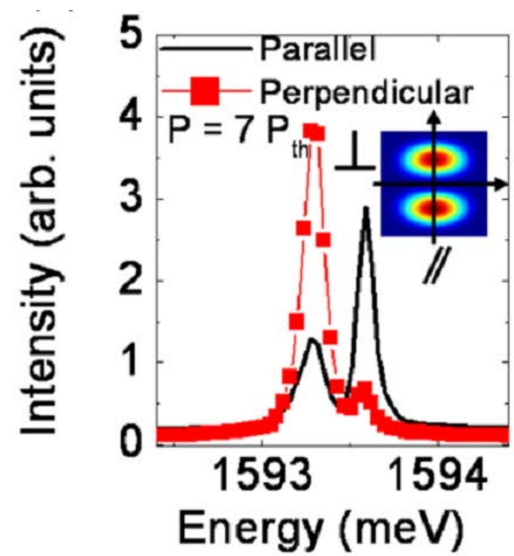




tight-binding Hamiltonian:

$$\mathrm{H} = \begin{pmatrix} E_0 & 0 & -J_L & 0 \\ 0 & E_0 & 0 & -J_T \\ -J_L & 0 & E_0 & 0 \\ 0 & -J_T & 0 & E_0 \end{pmatrix} \begin{matrix} |A, L\rangle \\ |A, T\rangle \\ |B, L\rangle \\ |B, T\rangle \end{matrix}$$





M. Galbiati et al., Phys. Rev. Lett. **108**, 126403 (2012)

Electrons versus Polaritons in 2D

- Electrons are charged which provides sensitivity to electric and magnetic field (Quantum Hall Effect). No difference in the absence of external field.

- Electrons are fermions, polariton bosons. No difference in the single particle limit.

- Electrons and polaritons demonstrate Zeeman splitting.

- Breaking of the potential symmetry leads to different types of effective spin-orbit coupling.

Scalar approximation, no external field, single particle limit, both cases are equivalent.

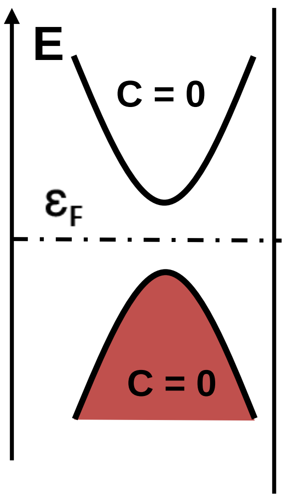
Berry phase accumulated by a Bloch wave function over a whole band in the complete Brillouin zone is quantized:

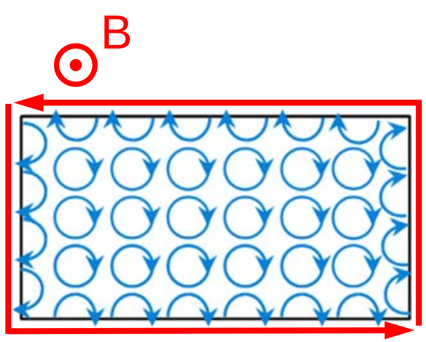
Chern number

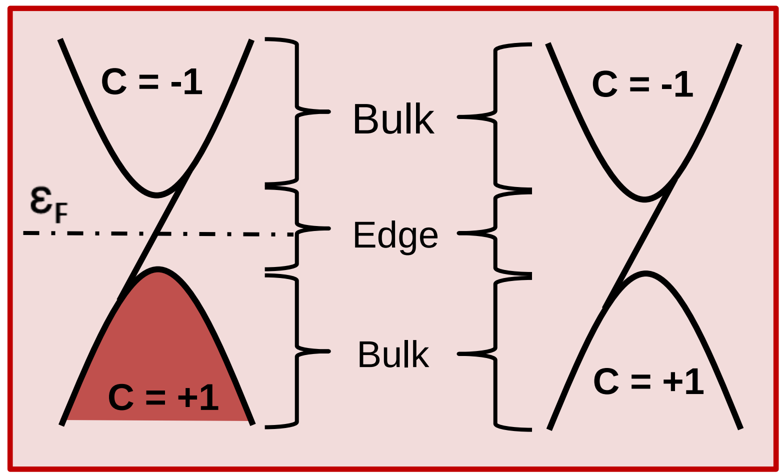
which characterizes the chirality of the band.

e- band structures in SC ⟷ photon dispersion in a periodic media

e- insulator *e- Chern insulator* *Photonic C-I*







$$C_n = \frac{1}{2\pi i}\int_{BZ} \mathbf{F}_n(\mathbf{k})\,d\mathbf{k}$$

ex: Quantum Hall Effect
K.v.Klitzing (1980)

A gap should close to change topology.
The vacuum is trivial. Gap Closure on the interface.

One way edge modes, which cannot be elastically scattered.