UNIVERSITY OF ICELAND

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Venice, Italy

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A driven-dissipative spin chain model based on exciton-polariton condensates

arXiv:1704.04811 [cond-mat.quant-gas]

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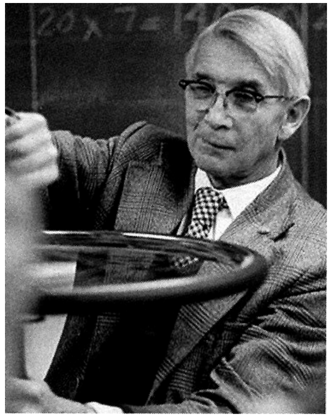
Outline

- A view on spin lattices:
  - **From** linear **to** non-linear
  - **From** conservative **to** dissipative
- Spin-bifurcating (SB) polariton condensates: Slight experimental look
- Theory of chain (1D) coupled SB polariton condensates: Exact solutions
- Agreement between experiment and numerical results
- Beyond 1D lattices: Potential combinatorial supersolvers

Spin lattices: Interest and potential


Ernst Ising

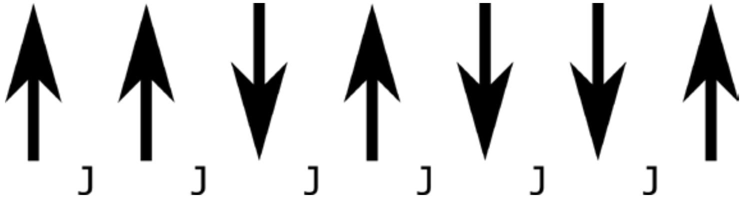
"Ising. Please solve this." – Wilhelm Lenz 1925
(Not Emil Lenz known for **Lenz's Law**)


Lars Onsager

Ising model: $$H = J \sum_{\langle m \rangle} S_{nz} S_{mz}$$

Classical binary spin values (± 1). Nearest neighbor coupling (J)
Configuration probability given by Boltzmann distribution.

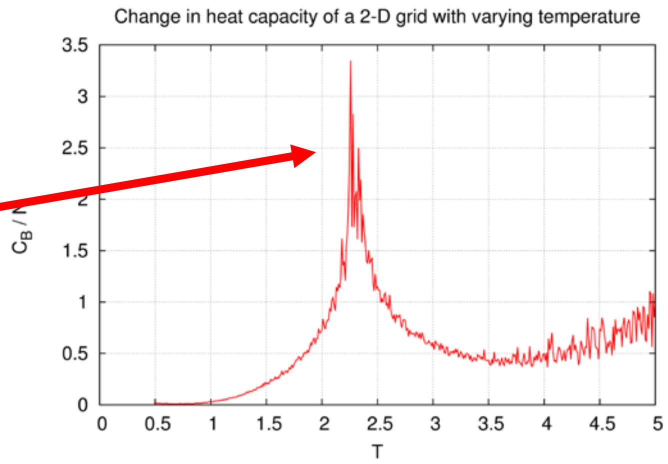
1D chain of spins: Not much going on. No phase transitions.



(>1)D lattices of spins: **Transitions** from disordered to ordered states.
Square arranged spins solved exactly by Lars Onsager in 1949

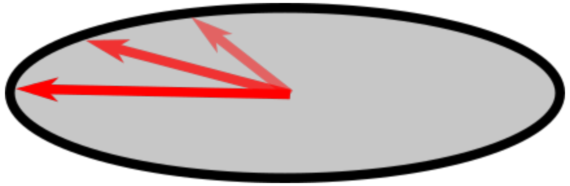
Ferromagnetic to paramagnetic at Curie temperature:

Onsager: $$T_c = \frac{2J}{k_B \ln(1+\sqrt{2})}$$

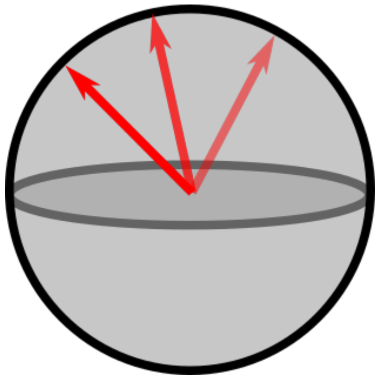


Continuous spins orientations instead of binary:

xy-spin (2D) → XY model

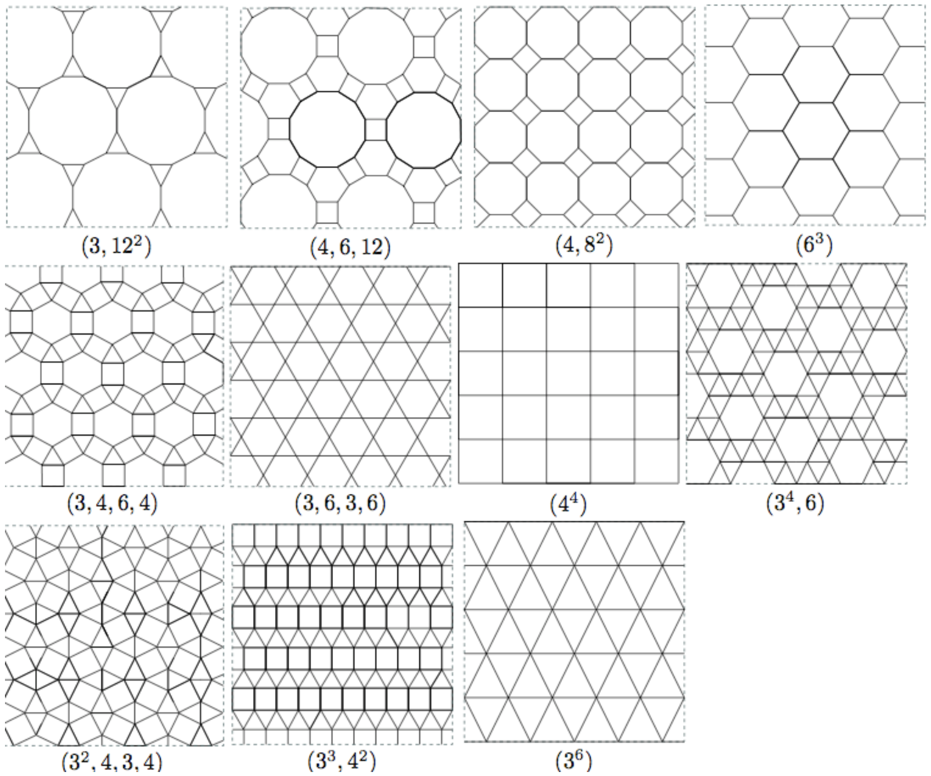


Full spin (3D) → Heisenberg model



More complex lattices:
Higher number of nearest neighbors

Archimedean Lattices:
Study of order and connectivity



Classical

$$H = J \sum_{\langle m \rangle} \mathbf{S}_n \cdot \mathbf{S}_m$$

Quantum picture

$$\hat{H} = J \sum_{\langle m \rangle} \hat{\sigma}_n \cdot \hat{\sigma}_m$$

Antiferromagnetic ground state energy different! Quantum order

$$E_0 = -NJ/4$$

$$E_0 = NJ(\frac{1}{4} - \ln 2)$$ **(Spin ½ models)**

Bethe ansatz: Bethe, H.A.: Z. Phys. 71, 205–226 (1931)

Jordan-Wigner transformation: Jordan, P., Wigner, E.: Z. Phys. 47, 631 (1928) 61

Lieb, E., Schultz, T., Mattis, D.: Ann. Phys. 16, 407 (1961) 61, 67

Motivation for studying spin lattices:

- Spin ice
- Quantum computation
- Models for complex problems
- Spin glasses
- Spin liquids

PRL **105**, 110502 (2010) PHYSICAL REVIEW LETTERS week ending 10 SEPTEMBER 2010

Quantum Computational Renormalization in the Haldane Phase

Stephen D. Bartlett,[1] Gavin K. Brennen,[2] Akimasa Miyake,[3] and Joseph M. Renes[4]

nature Vol 463|14 January 2010|**doi:10.1038/nature08680**

LETTERS

Time-reversal symr
Hall effect without

Yo Machida[1]†, Satoru Nakatsuji[1], Shigeki

Spin glasses: Experimental facts, theoretical concepts, and open questions

Non-equilibrium non-linear spin lattices offer new physics

ARTICLE doi:10.1038/nature09994

Quantum simulation of antiferromagnetic spin chains in an optical lattice

Jonathan Simon[1], Waseem S. Bakr[1], Ruichao Ma[1], M. Eric Tai[1], Philipp M. Preiss[1] & Markus Greiner[1]

nature physics LETTERS
PUBLISHED ONLINE: 11 APRIL 2010 | DOI: 10.1038/NPHYS1628

Direct observation of magnetic monopole defects in an artificial spin-ice system

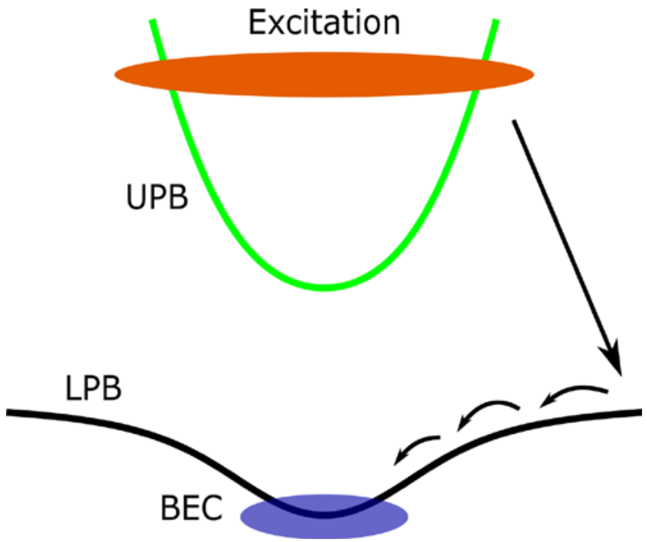
S. Ladak, D. E. Read, G. K. Perkins, L. F. Cohen and W. R. Branford*

Driven-dissipative polariton condensates

❖ Generated via external optical (or electrical) driving field.

❖ Stimulated scattering of the polaritons forms a macroscopic coherent state (condensate).

❖ Mean-field approach possible (Gross-Pitaevskii, Ginzburg-Landau equation).

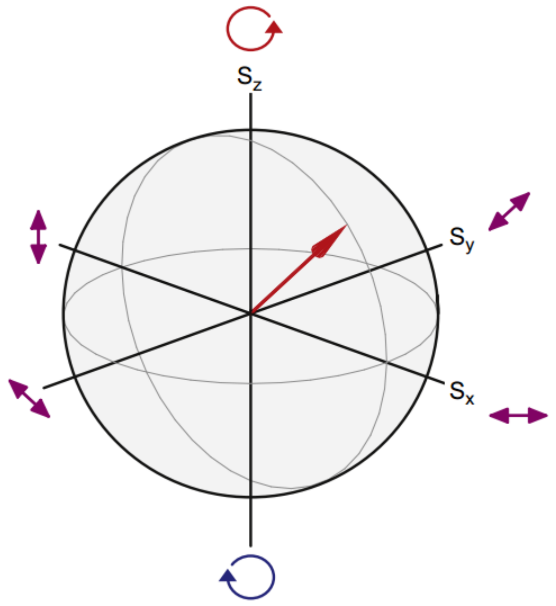


Spin structure

❖ Spin-1 quasiparticles (doublets)

❖ Explicitly related to the polarization of the emitted light ($\sigma_{\pm} = \pm S_z$).

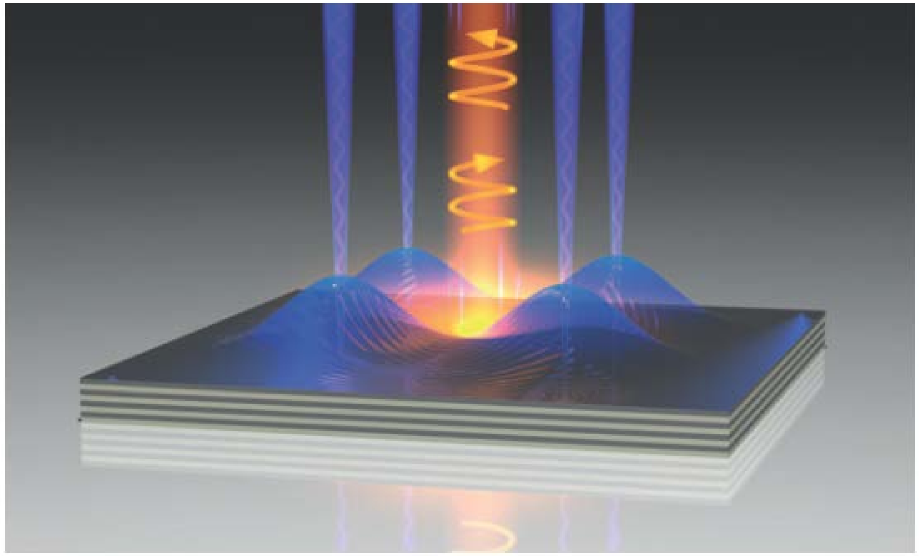
❖ Mean field spin of condensate → Pseudospin (Poincaré sphere, Bloch sphere).



PHYSICAL REVIEW X 5, 031002 (2015)

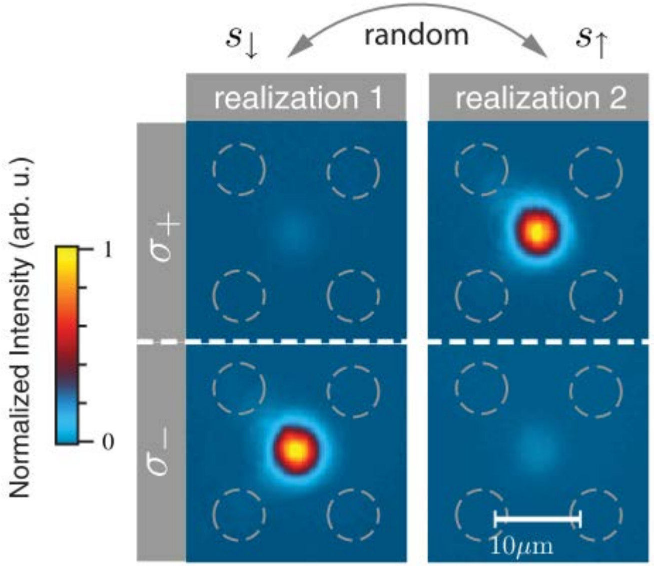
Spontaneous Spin Bifurcations and Ferromagnetic Phase Transitions in a Spinor Exciton-Polariton Condensate

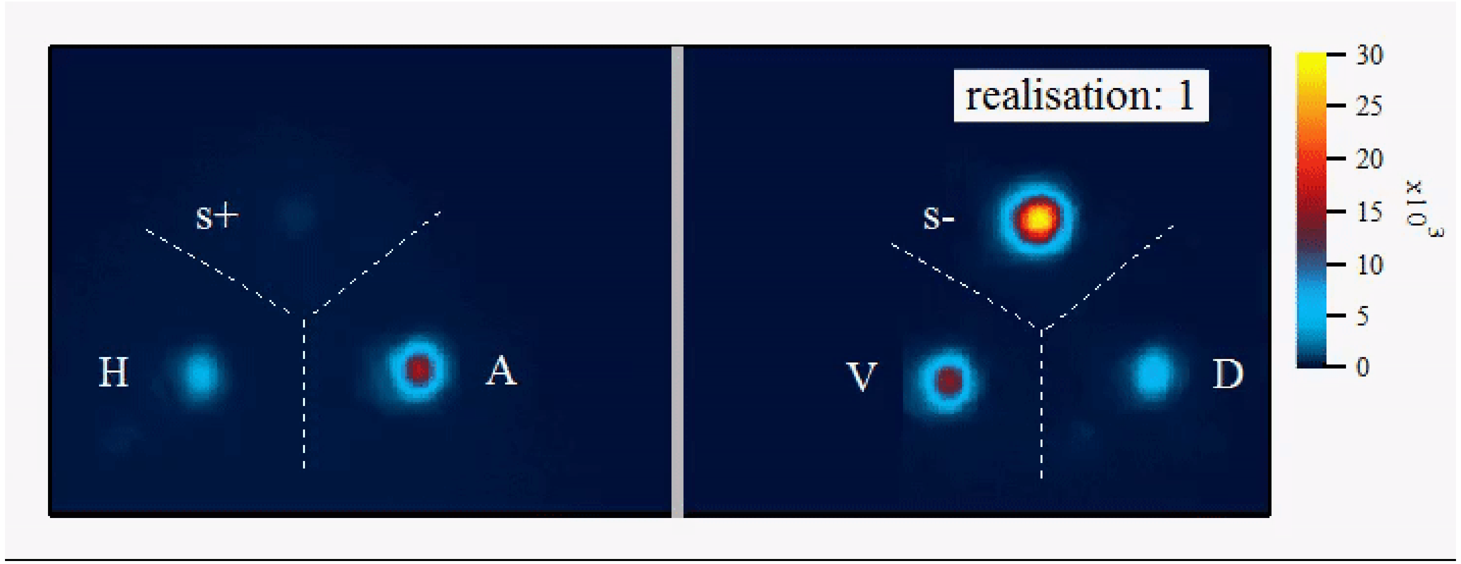
H. Ohadi,[1,*] A. Dreismann,[1] Y. G. Rubo,[2] F. Pinsker,[3,4] Y. del Valle-Inclan Redondo,[1] S. I. Tsintzos,[5] Z. Hatzopoulos,[5,6] P. G. Savvidis,[1,5,7] and J. J. Baumberg[1,†]



At a critical pump intensity the condensate collapses to either of the circularly polarized states

(Blue) Nonresonant **linearly polarized** excitation
(Orange) Exciton-polariton condensate



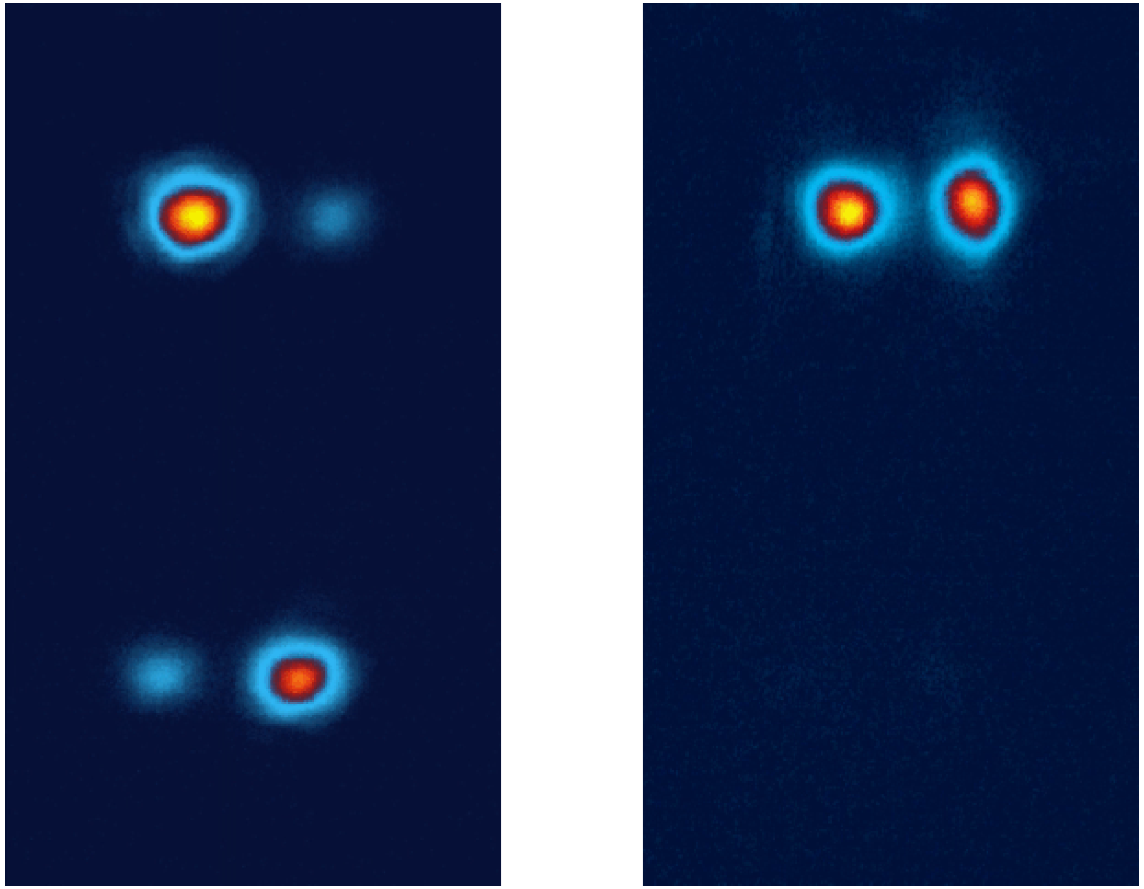


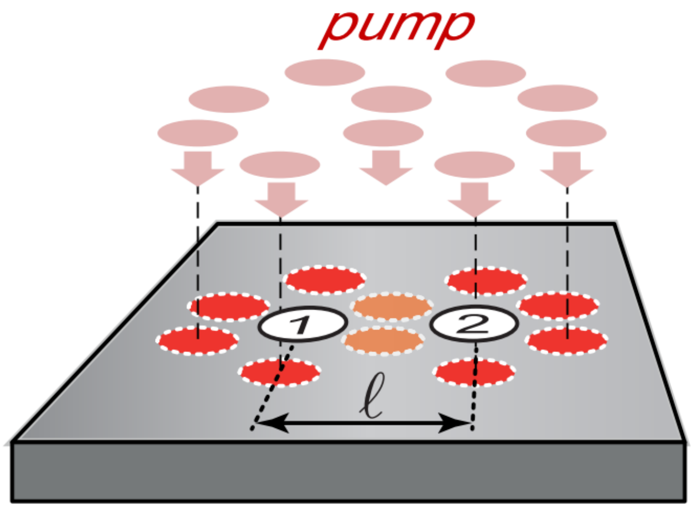
PRL 116, 106403 (2016) PHYSICAL REVIEW LETTERS week ending 11 MARCH 2016

Tunable Magnetic Alignment between Trapped Exciton-Polariton Condensates

H. Ohadi,[1*] Y. del Valle-Inclan Redondo,[1] A. Dreismann,[1] Y. G. Rubo,[2] F. Pinsker,[3] S. I. Tsintzos,[4] Z. Hatzopoulos,[4,5] P. G. Savvidis,[4,6] and J. J. Baumberg[1†]

AFM FM



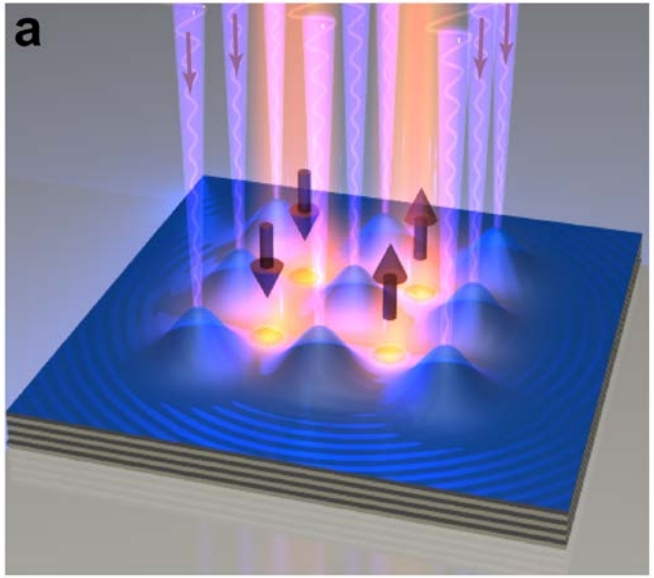


Pump spots between the condensates act as a tunable barrier, controlling the tunneling rate between the condensates.

For weak tunneling/coupling between the condensates an **antiferromagnetic** arrangement becomes dominant.

For large tunneling/coupling a **ferromagnetic** arrangement becomes dominant.

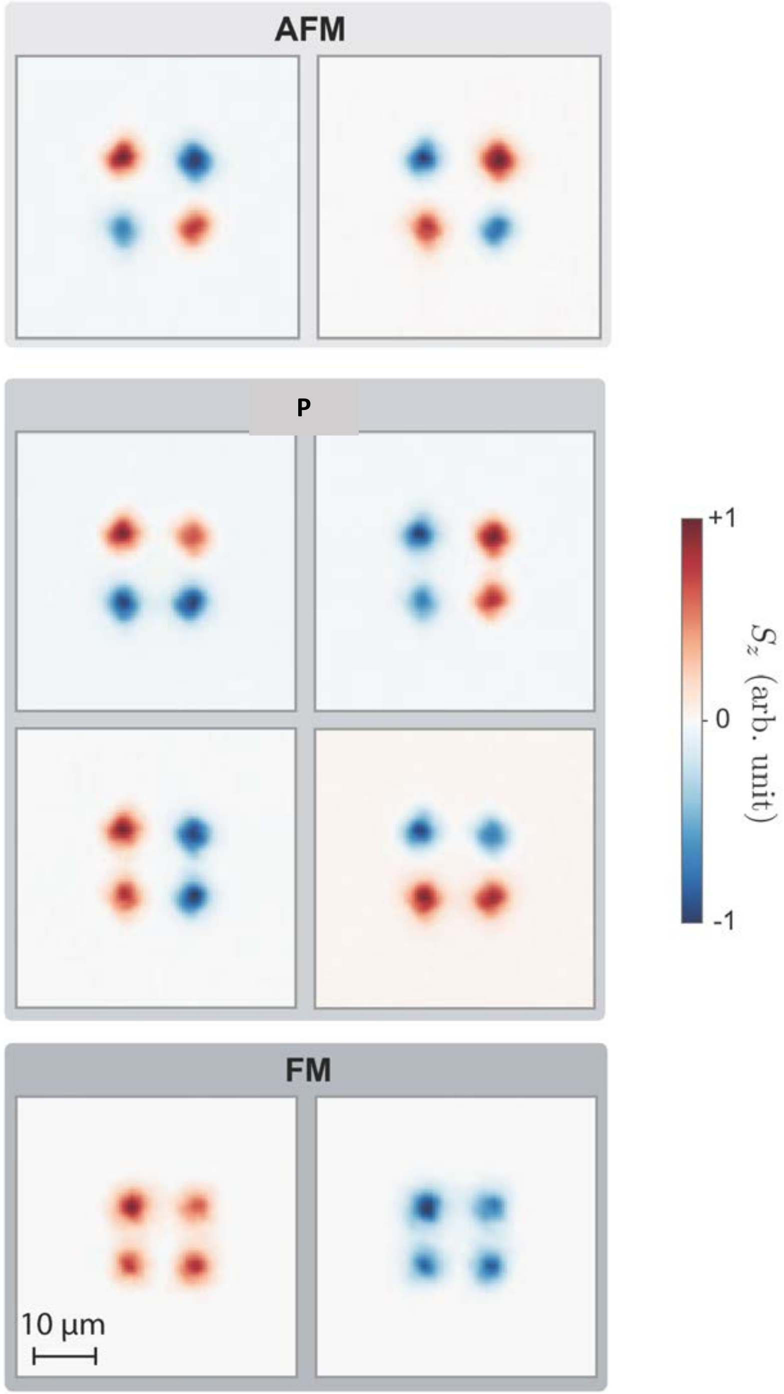
We consider next an experiment of a closed chain of 4 condensates with nearest neighbor couplings



Weak coupling

Intermediate coupling

Strong coupling



Can our theory predict the hierarchy of these spin patterns?

Circular polarization basis

$$\Psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix}$$

Total spin population

$$S = \frac{|\Psi_+|^2 + |\Psi_-|^2}{2}$$

z-spin population

$$S_z = \frac{|\Psi_+|^2 - |\Psi_-|^2}{2}$$

Gain-dissipation mechanism

$$g(S) = \Gamma - P(\mathbf{r}) + \eta S$$

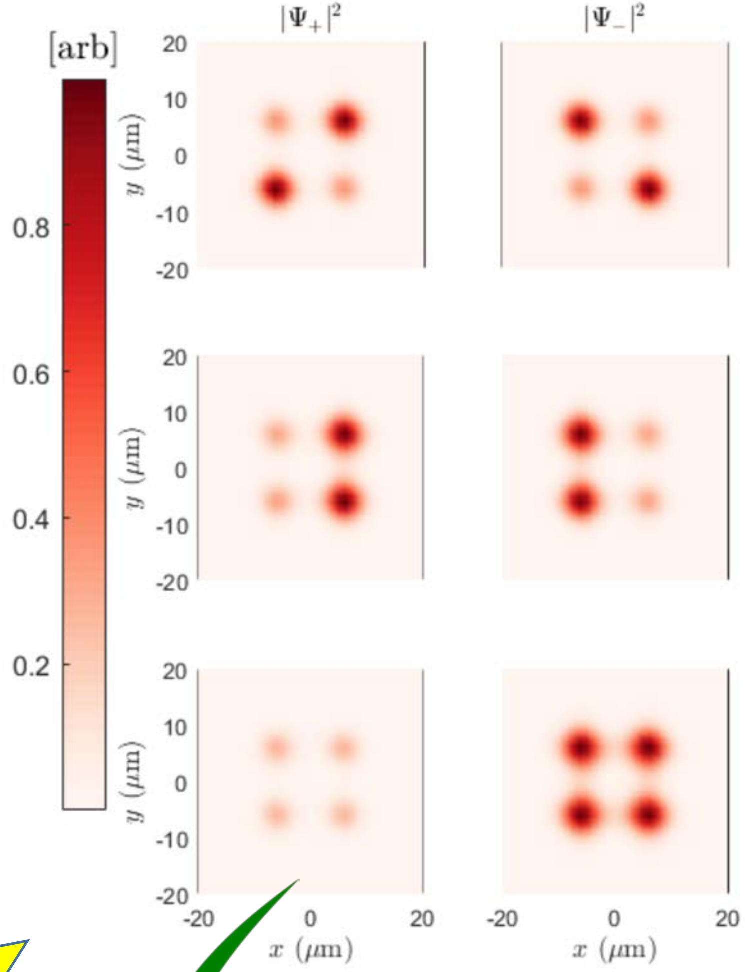
Reservoir blueshift

$$i\dot{\Psi} = \frac{1}{2}\left[-ig(S) + \bar{\alpha}S + \alpha S_z \ldots - g_P P(\mathbf{r}) - (\epsilon + i\gamma)\,\sigma_x\right]\Psi.$$

Polariton-polariton interactions

Birefringence

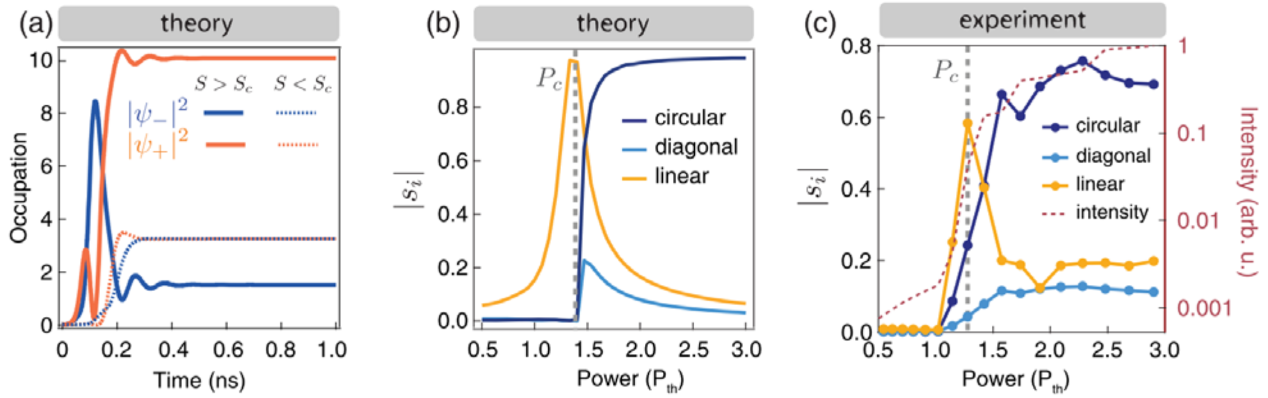
Numerical modeling



Too complicated to solve analytically. Tight binding model better!

$$i\dot{\Psi}_n = \frac{1}{2}\left[-ig(S_n) + \bar{\alpha}S_n + \alpha S_{zn}\hat{\sigma}_z - (\epsilon + i\gamma)\hat{\sigma}_x\right]\Psi_n - \frac{J}{2}\sum_{\langle m \rangle}\Psi_m$$

Single condensate: Spin bifurcation happens at a critical density given by $S_{\text{crit}} = \dfrac{\epsilon^2 + \gamma^2}{\alpha \epsilon}$



H. Ohadi, et.al. PRX, **5**, 031002 (2015)

What about a chain of condensates?
Let's work with the following ansatz.

$$\Psi_{n+1} = e^{i\varphi_{n+1}} \Psi_n \qquad \text{(FM bonds)}$$

$$\Psi_{n+1} = e^{i\varphi_{n+1}} \hat{\sigma}_x \Psi_n \qquad \text{(AFM bonds)}$$

Phase slip from site n to its neighbor

The n-coupled GP equations are now recast into n-**uncoupled** GP equations

$$i\dot{\Psi}_n = -\frac{i}{2}(g + i\omega_J)\Psi_n - \frac{i}{2}(\gamma - i\epsilon_J)\hat{\sigma}_x \Psi_n + \frac{1}{2}(\bar{\alpha} S_n + \alpha S_{zn} \hat{\sigma}_z)\Psi_n$$

New parameters: ϵ_J and ω_J

$$\epsilon_J = \epsilon + J(\delta_i e^{i\varphi_i} + \delta_j e^{i\varphi_j}),$$
$$\omega_J = -J((1-\delta_i)e^{i\varphi_i} + (1-\delta_j)e^{i\varphi_j})$$

δ_i is 0 for FM bonds and 1 for AFM bonds

Exploit the 1D nature of the chain and its periodicity

$$\epsilon_J^{\mathrm{FM}} = \epsilon, \quad \omega_J^{\mathrm{FM}} = -2J\cos\left(2\pi m/N\right)$$

Ferromagnetic

N: number of sites
$m \in Z$

$$\epsilon_J^{\mathrm{AFM}} = \epsilon + 2J\cos\left(2\pi m/N\right), \quad \omega_J^{\mathrm{AFM}} = 0$$

Antiferromagnetic

$$\epsilon_J^{\mathrm{P}} = \epsilon \pm J, \quad \omega_J^{\mathrm{P}} = \pm J$$

Pair-ferromagnetic

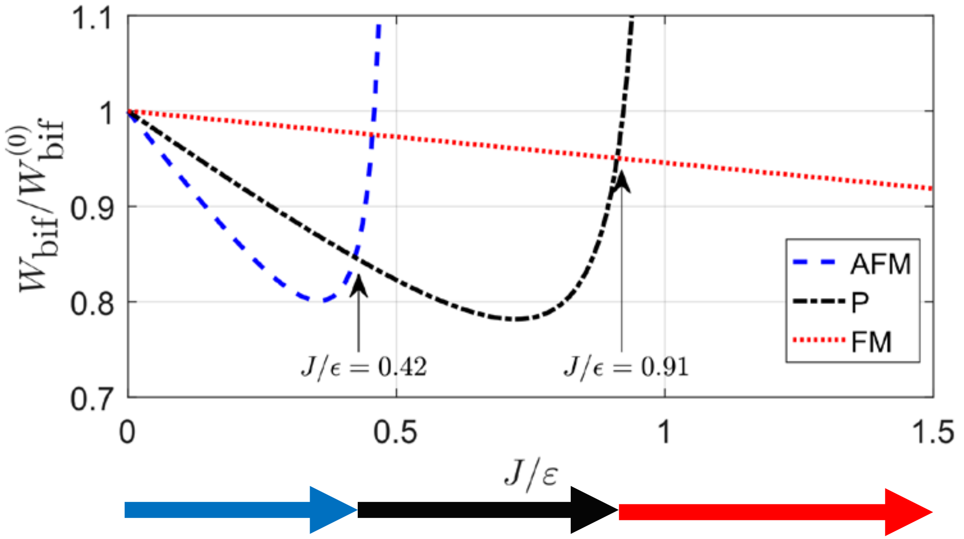
This is the complete set of stationary uniform solutions of our 1D spin model

arXiv:1704.04811 [cond-mat.quant-gas]

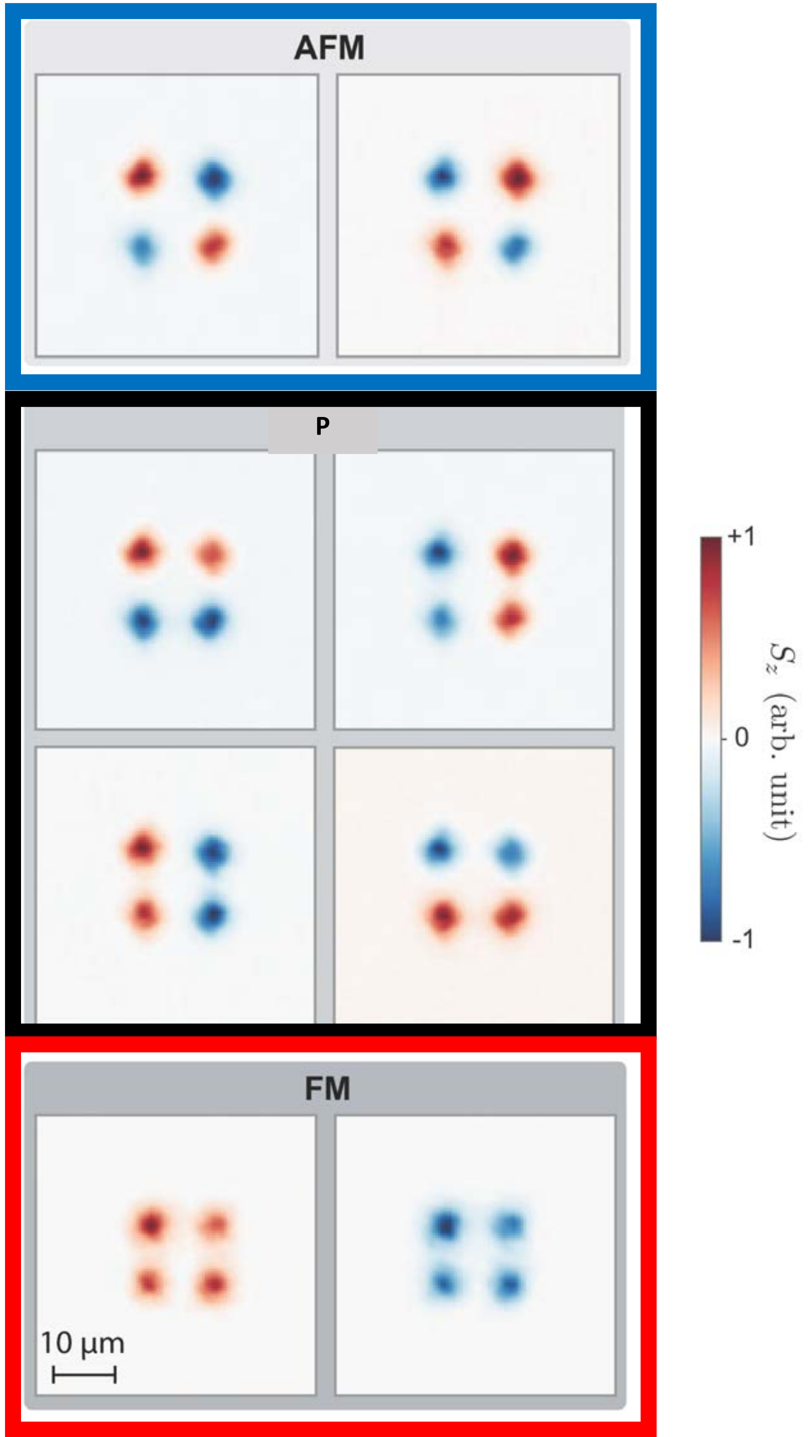
What about the hierarchy?

Solutions with modified ϵ_J can have lower bifurcation threshold as a function of coupling strength J.

$$W_{\text{bif}} = \Gamma - \gamma + \eta \frac{\gamma^2 + \epsilon_J^2}{\alpha \epsilon_J}$$

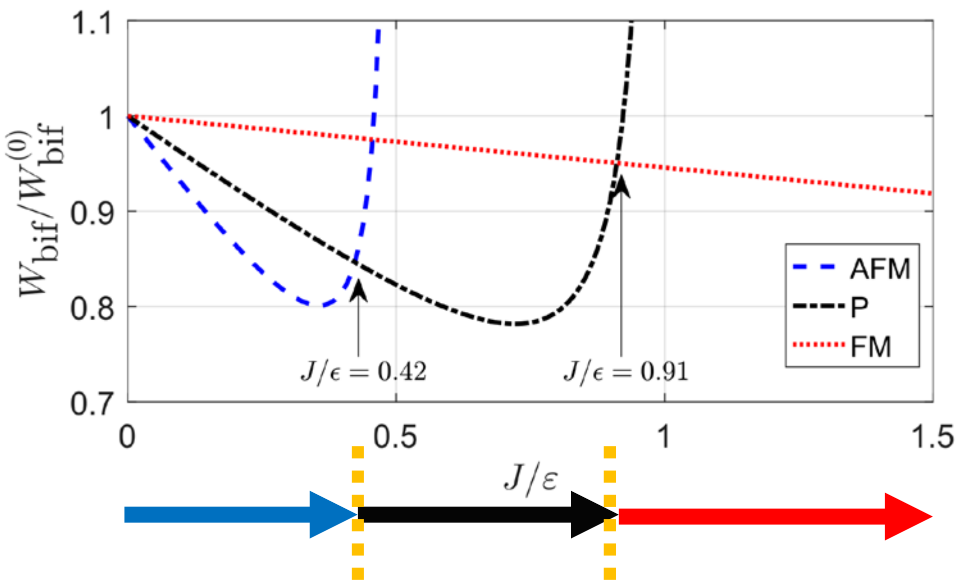


Equations predict the same hierarchy of spin pattern formation as seen in experiment

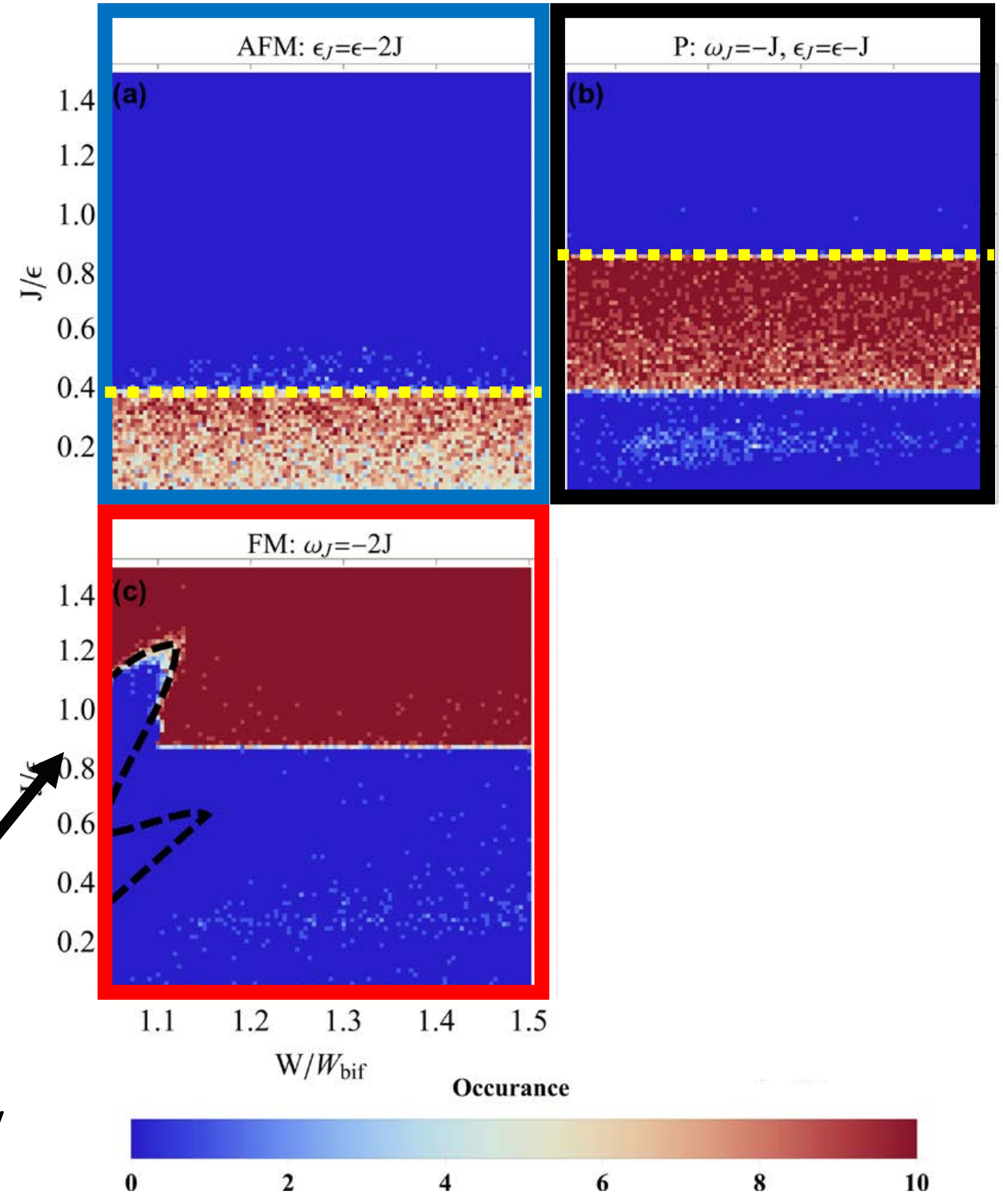


Monte-Carlo iterations of the GP-equation

Adiabatically increasing the pump intensity (W) for a given coupling value (J) results in a orderly phase map

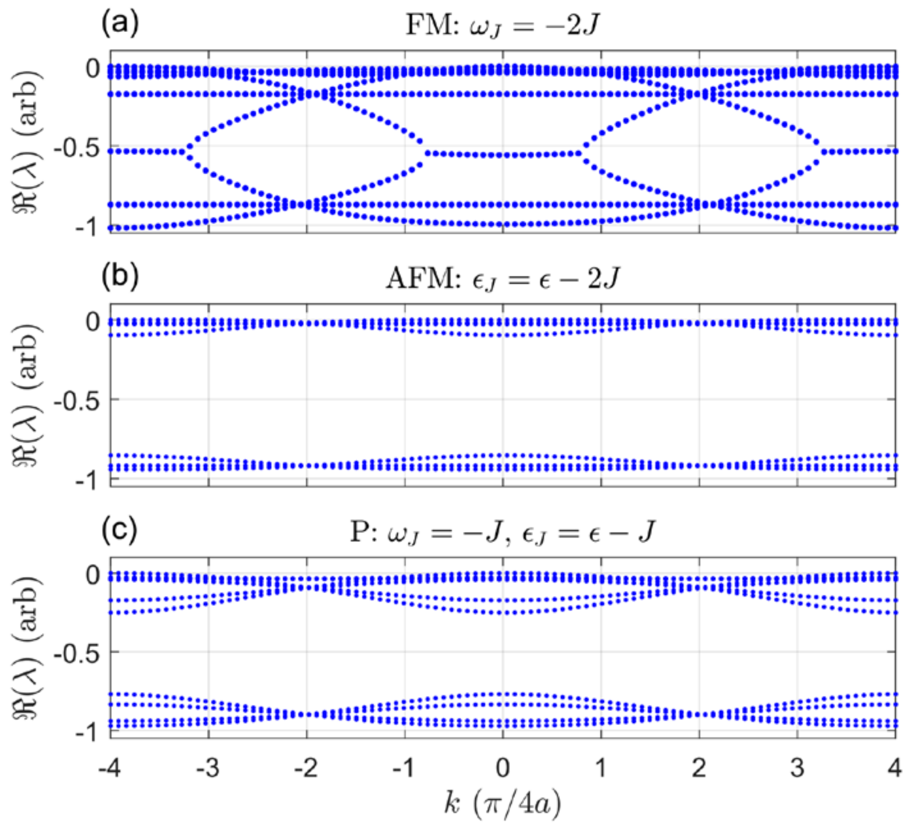


Small area of instability

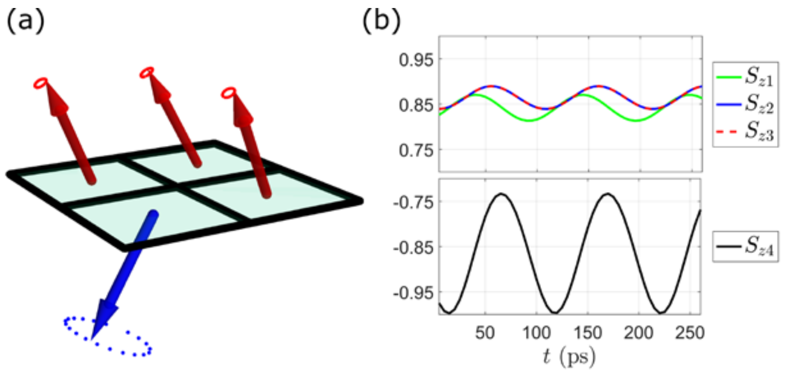


Complete stability against long wavelength fluctuations along the chain

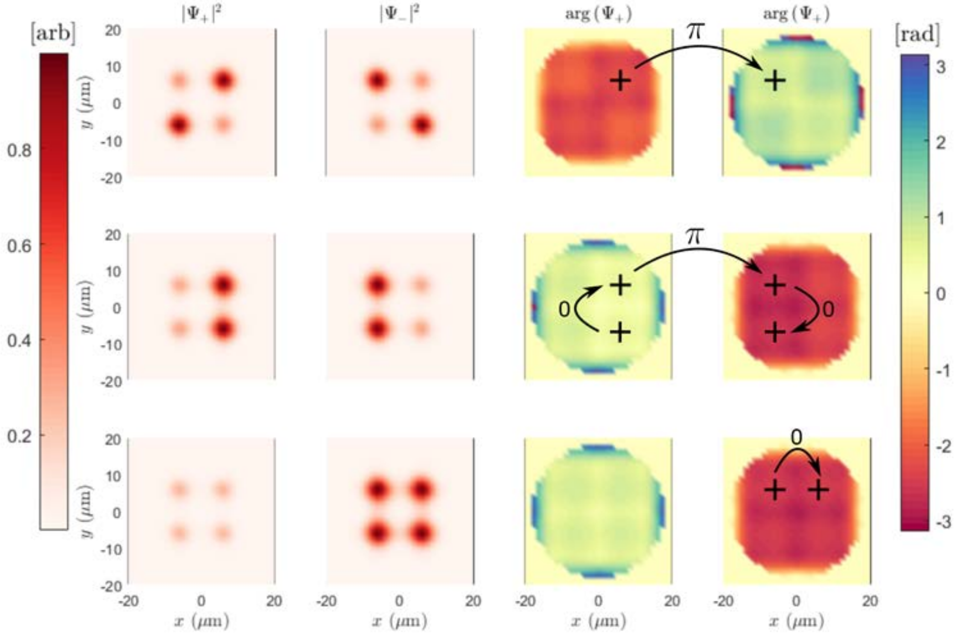
Lyapunov components:



Nonstationary behavior:
Limit cycle solutions exist

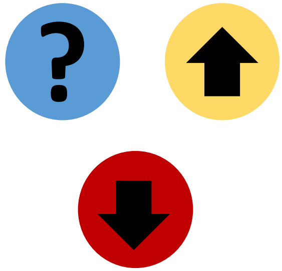


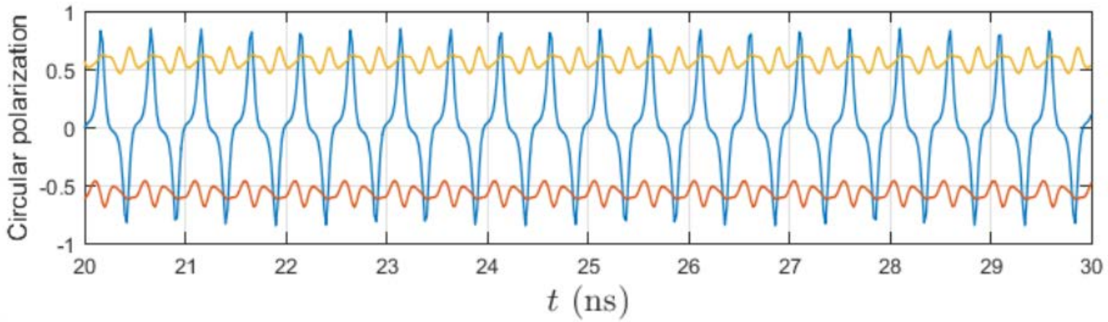
2D simulations (no tight binding) reveal same behavior



Higher number of neighbors?

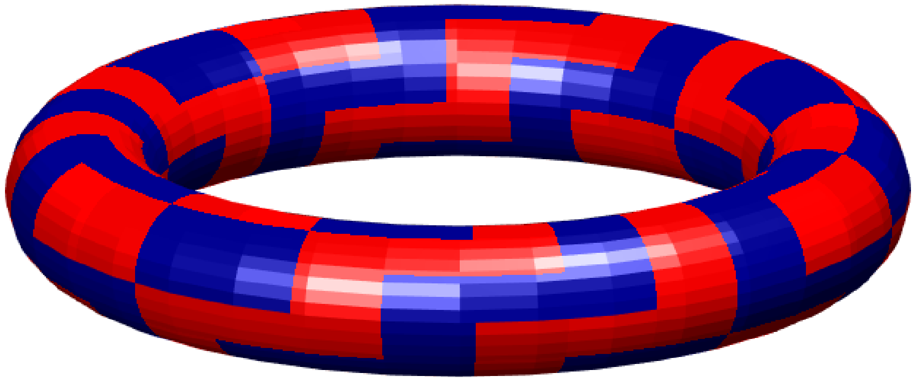
Frustration in AFM triangular lattices
Kagome lattices?

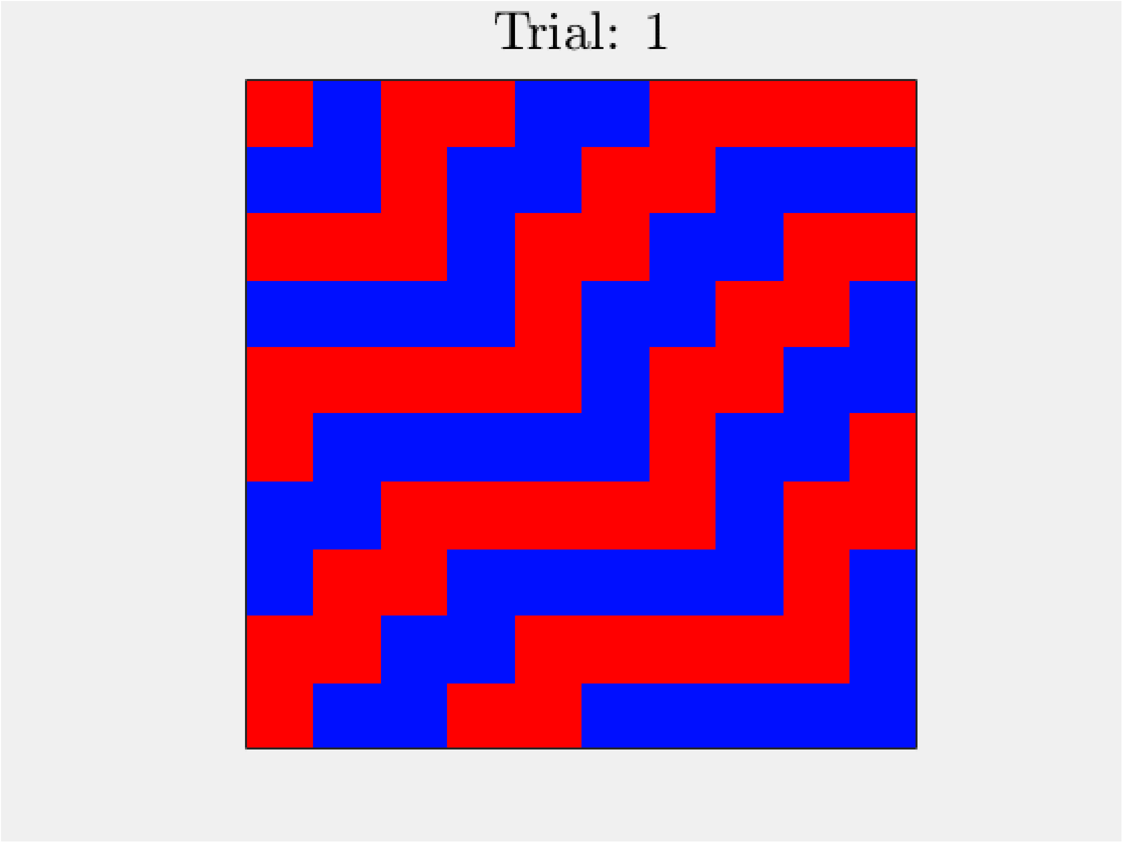




Square lattices: Super-degenerate P-states

Many different patterns for the same solution.
A fast way to solve a mathematical tiling problem on a torus. **#P-Complete problem**

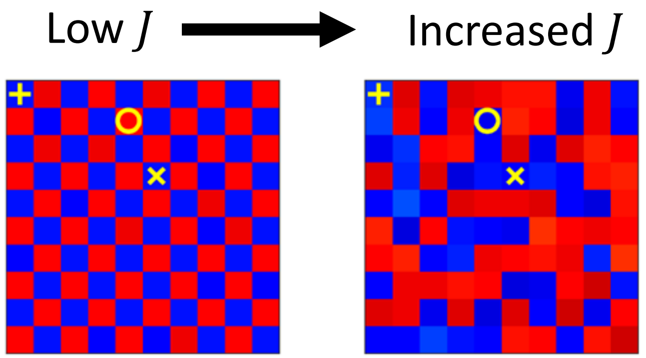


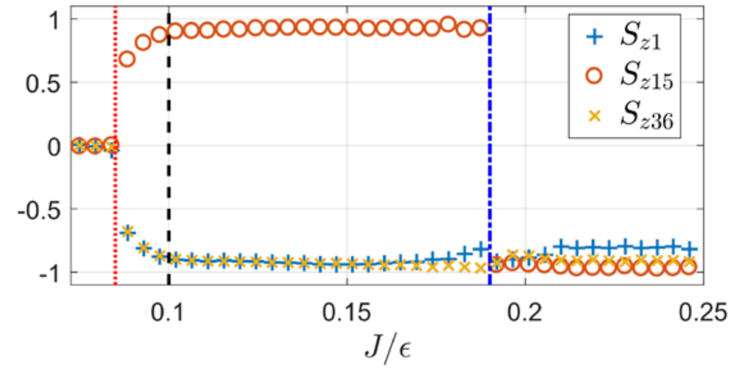


Glass formation

Ordered states break down into disordered ones.

Uniform couplings J. Random patterns appear and freeze for a critical coupling value.

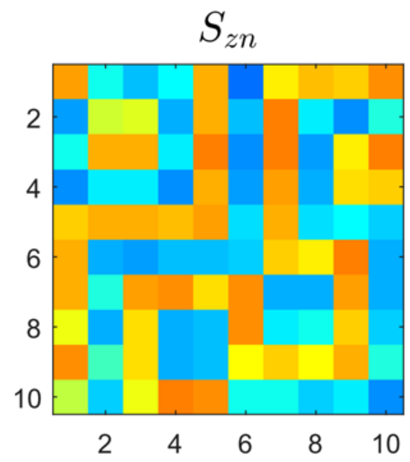


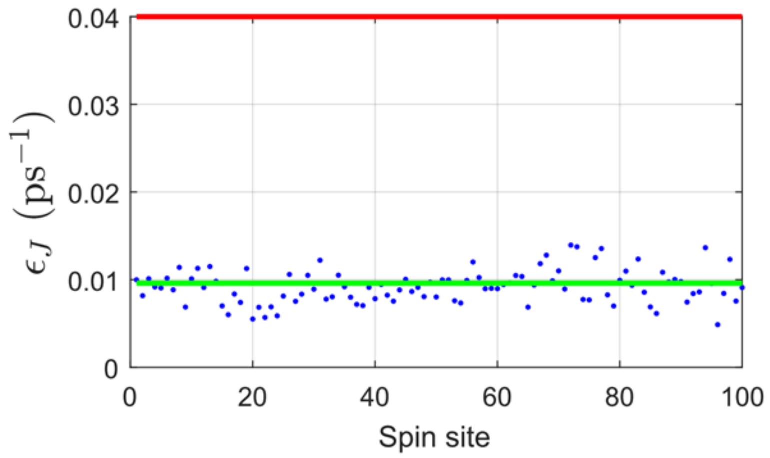


Powerful self-averaging

Random coupling values J_{ij} (Gaussian distributed)

System still seeks ordered patterns. Condensate intensities modified to have same energy and dissipation in the system (ϵ_J)





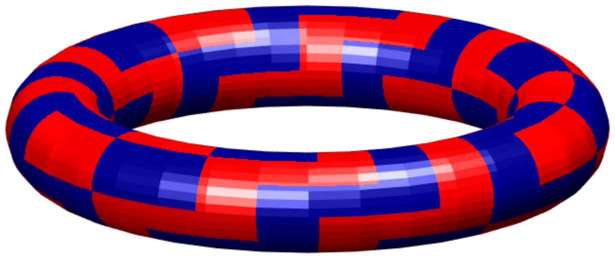
Conclusions:

We have developed, solved, and investigated a spin model for a driven-dissipative chain of polariton condensates

The condensates possess not only a spin degree of freedom but also a phase. This, along with interactions, allows the system to quickly find a favorable spin solution (a solution which bifurcates first).

The clear hierarchy of spin pattern formation is an important feature in spin-lattice simulators.

Continuation - questions:



More complicated patterns available for more nearest neighbors (square lattice)?

Triangular pattern of condensates, a way to study nonlinearities and frustration?

Inhomogeneous coupling distribution, an efficient method of finding solutions to graph theory problems?