Interactive optomechanical coupling with nonlinear polaritonic systems

Nataliya Bobrovska

Michał Matuszewski

Timothy Liew

NANYANG TECHNOLOGICAL UNIVERSITY

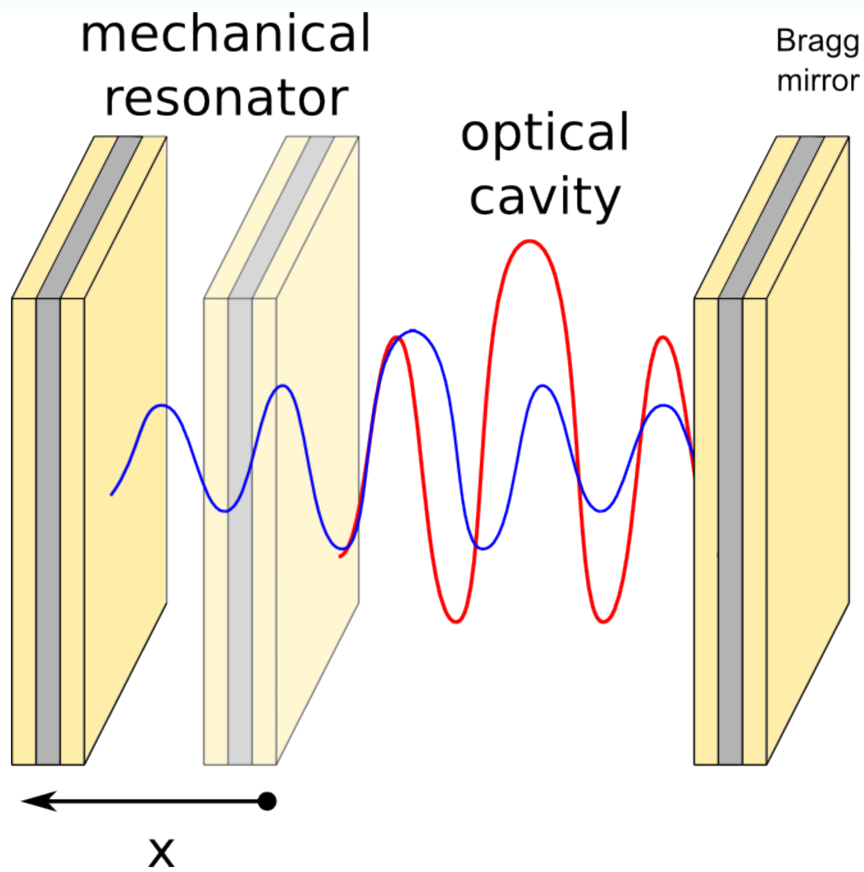
Oleksander Kyriienko

Niels Bohr Institutet

*NB, M. Matuszewski, T. C. H. Liew, O. Kyriienko, Phys. Rev. B **95**, 085309 (2017)*

Cavity optomechanics

Optical cavity parameter depends
from the position of mechanical resonator

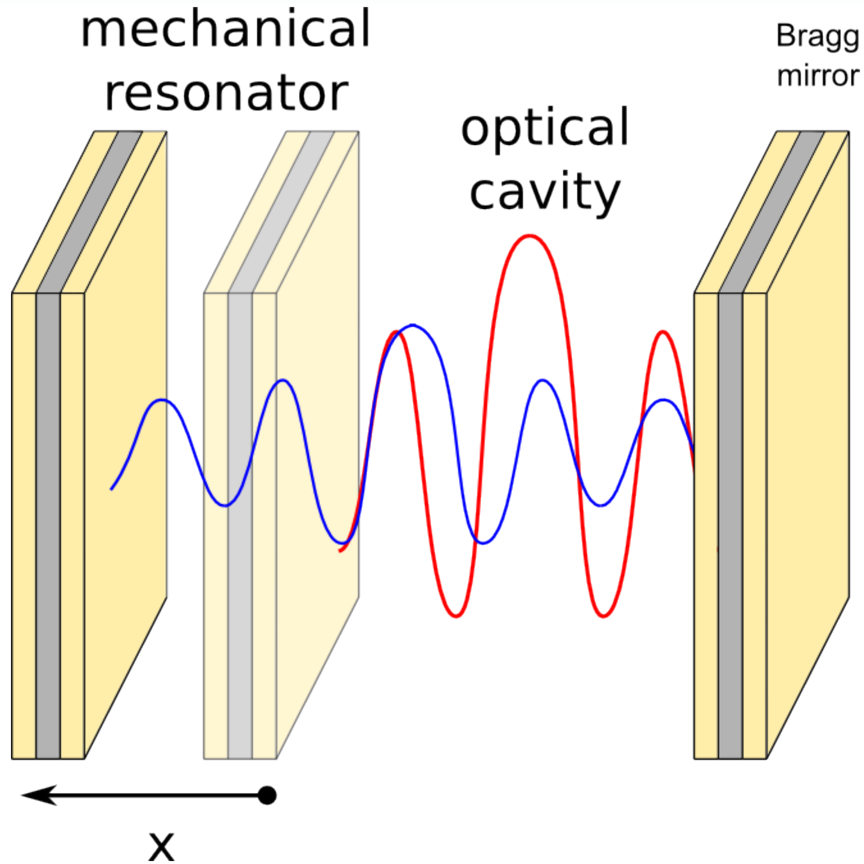


Types of optomechanical coupling:

- Dispersive coupling (mechanical modulation of the cavity photon frequency)
- Dissipative coupling (mechanical modulation of the cavity damping rate)

Cavity optomechanics

Optical cavity parameter depends
from the position of mechanical resonator

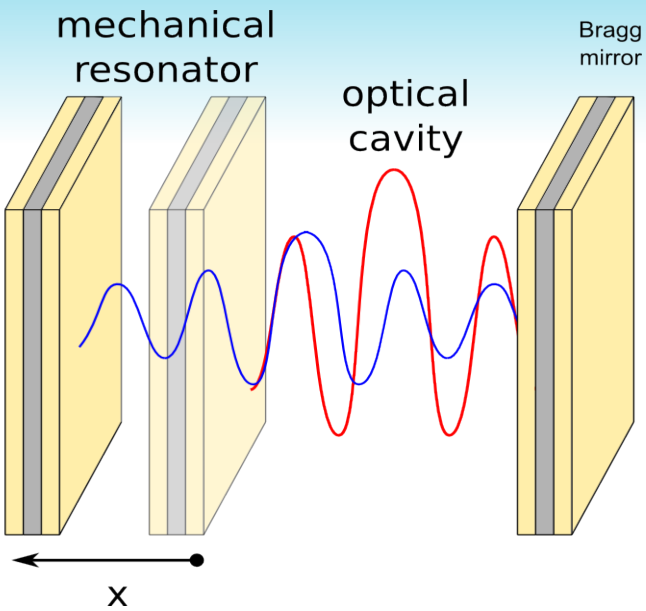


Types of optomechanical coupling:

- Dispersive coupling (mechanical modulation of the cavity photon frequency)
- Dissipative coupling (mechanical modulation of the cavity damping rate)
- **Interactive** coupling (dependence of nonlinearity on the mechanical oscillator position)

Model - general idea

$$\hat{\mathcal{H}} = E_{\mathrm{C}}\hat{a}_{\mathrm{C}}^{\dagger}\hat{a}_{\mathrm{C}} + p^2/2m + m\omega_m^2 x^2/2 + U(x)\hat{a}_{\mathrm{C}}^{\dagger}\hat{a}_{\mathrm{C}}^{\dagger}\hat{a}_{\mathrm{C}}\hat{a}_{\mathrm{C}}$$

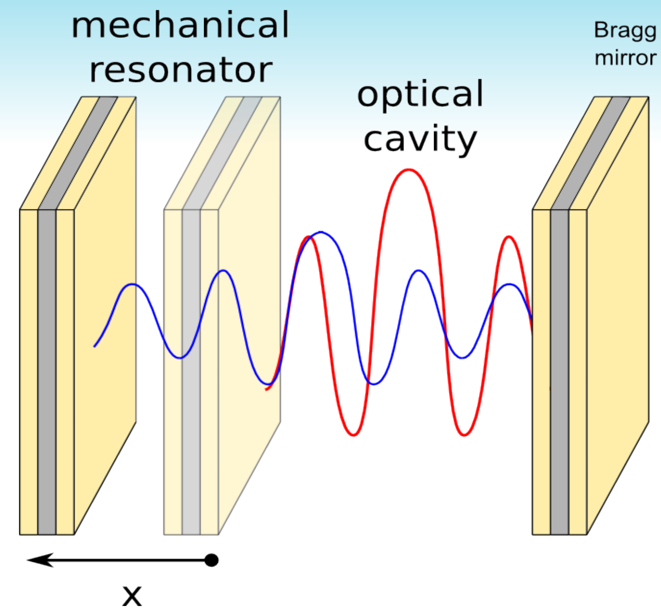


Model – general idea

Mechanical oscillator

$$\hat{\mathcal{H}} = E_{\mathrm{C}}\hat{a}_{\mathrm{C}}^{\dagger}\hat{a}_{\mathrm{C}} + \boxed{p^2/2m + m\omega_m^2 x^2/2} + U(x)\hat{a}_{\mathrm{C}}^{\dagger}\hat{a}_{\mathrm{C}}^{\dagger}\hat{a}_{\mathrm{C}}\hat{a}_{\mathrm{C}}$$

position depended Kerr nonlinearity

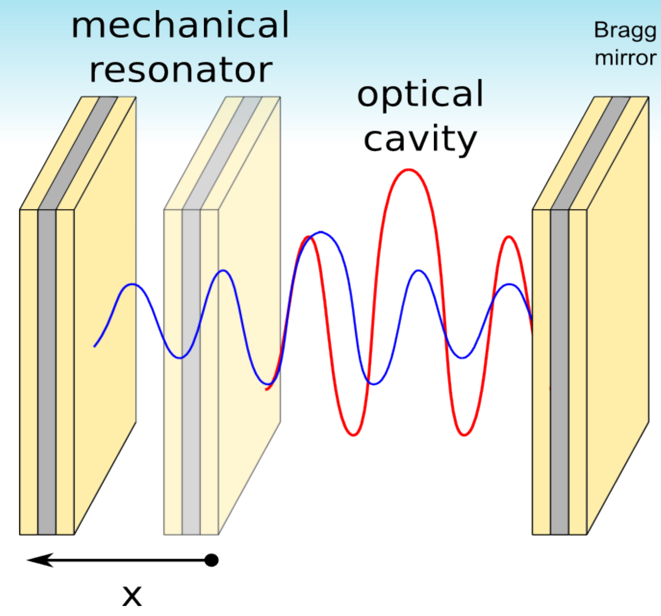


Model – general idea

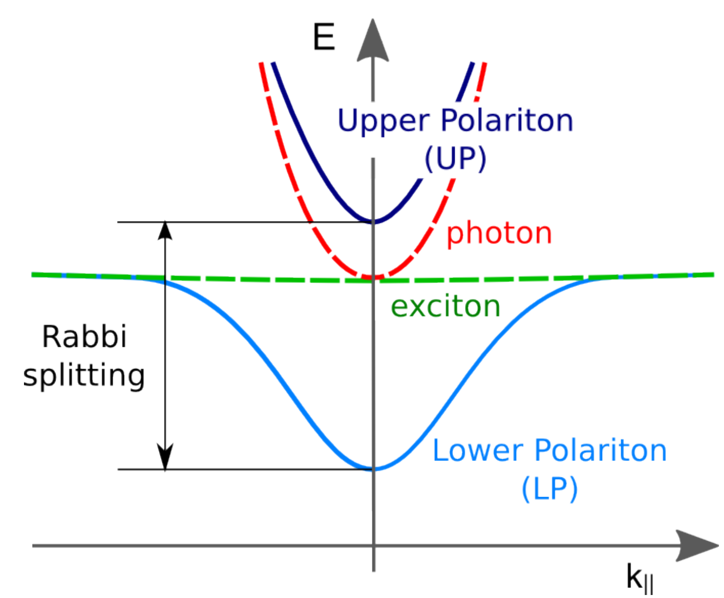
Mechanical oscillator

$$\hat{\mathcal{H}} = E_{\rm C}\hat{a}_{\rm C}^{\dagger}\hat{a}_{\rm C} + \boxed{p^2/2m + m\omega_m^2 x^2/2} + U(x)\hat{a}_{\rm C}^{\dagger}\hat{a}_{\rm C}^{\dagger}\hat{a}_{\rm C}\hat{a}_{\rm C}$$

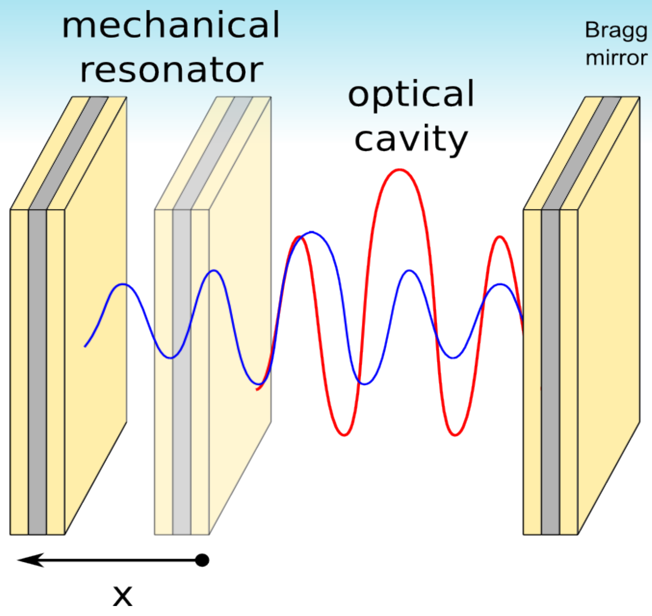
position depended
Kerr nonlinearity



Physical example – exciton-polariton system



Model – general idea



Mechanical oscillator

$$\hat{\mathcal{H}} = E_{\mathrm{C}}\hat{a}_{\mathrm{C}}^{\dagger}\hat{a}_{\mathrm{C}} + \boxed{p^2/2m + m\omega_m^2 x^2/2} + U(x)\hat{a}_{\mathrm{C}}^{\dagger}\hat{a}_{\mathrm{C}}^{\dagger}\hat{a}_{\mathrm{C}}\hat{a}_{\mathrm{C}}$$

position depended Kerr nonlinearity

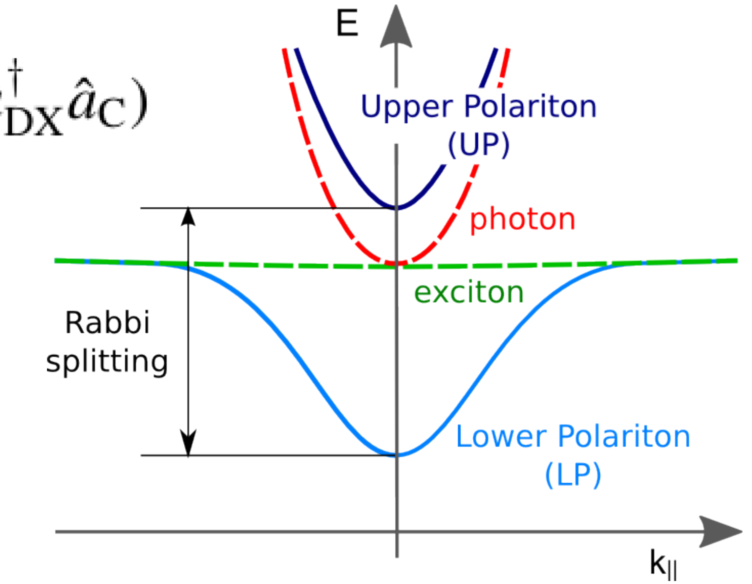
Physical example – exciton-polariton system

In strong exciton-photon coupling regime, system can be rewritten in the polariton basis formed by superposition of cavity photon and exciton.

$$\hat{\mathcal{H}}_{\mathrm{pol}} = E_{\mathrm{C}}(x)\hat{a}_{\mathrm{C}}^{\dagger}\hat{a}_{\mathrm{C}} + E_{\mathrm{DX}}\hat{a}_{\mathrm{DX}}^{\dagger}\hat{a}_{\mathrm{DX}} + \frac{\Omega}{2}(\hat{a}_{\mathrm{C}}^{\dagger}\hat{a}_{\mathrm{DX}} + \hat{a}_{\mathrm{DX}}^{\dagger}\hat{a}_{\mathrm{C}})$$
$$+ U_{\mathrm{DX}}\hat{a}_{\mathrm{DX}}^{\dagger}\hat{a}_{\mathrm{DX}}^{\dagger}\hat{a}_{\mathrm{DX}}\hat{a}_{\mathrm{DX}}$$

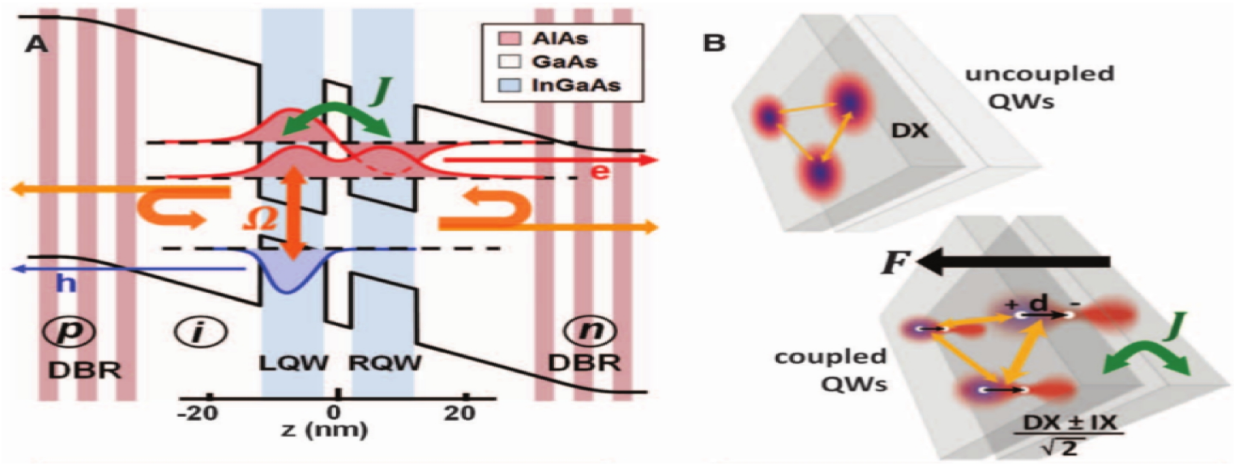
$$\Downarrow$$

$$\hat{\mathcal{H}}_{\mathrm{L}} = E_{\mathrm{L}}(x)\hat{a}_{\mathrm{L}}^{\dagger}\hat{a}_{\mathrm{L}} + U_{\mathrm{L}}(x)\hat{a}_{\mathrm{L}}^{\dagger}\hat{a}_{\mathrm{L}}^{\dagger}\hat{a}_{\mathrm{L}}\hat{a}_{\mathrm{L}}$$



Dipolariton system

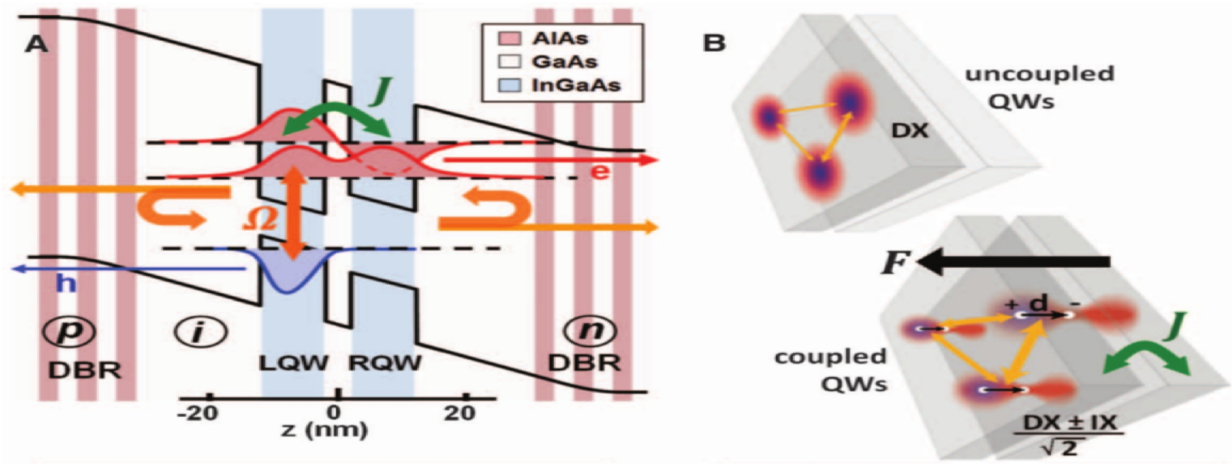
Nonlinearity can be enhanced by using coupled QW, where indirect excitons can be formed.



Cristofolini et al, Science (2012)

Dipolariton system

Nonlinearity can be enhanced by using coupled QW, where indirect excitons can be formed.

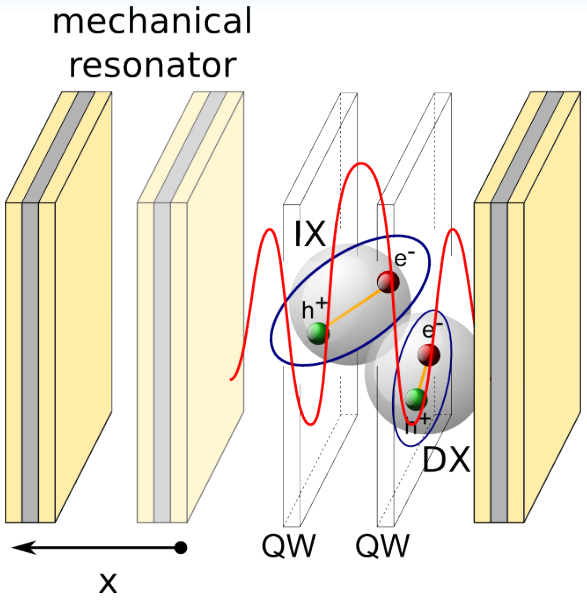


Hamiltonian for optomechanical coupling in dipolariton microcavity

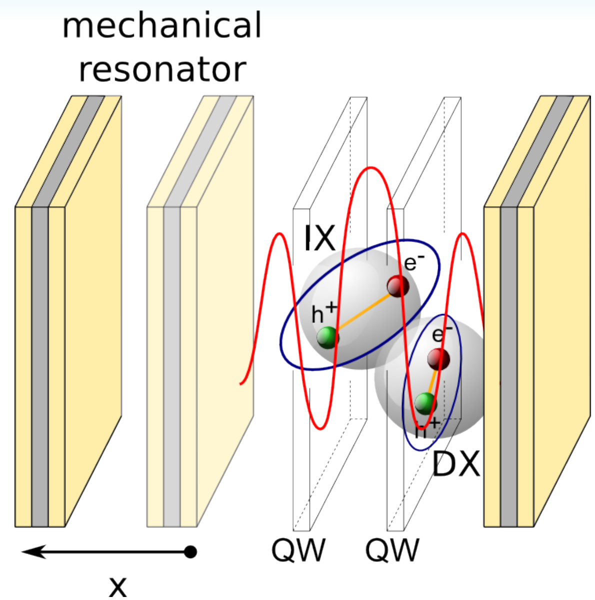
$$\hat{\mathcal{H}}_{\mathrm{dpl}} = E_{\mathrm{C}}(x)\hat{a}_{\mathrm{C}}^{\dagger}\hat{a}_{\mathrm{C}} + E_{\mathrm{DX}}\hat{a}_{\mathrm{DX}}^{\dagger}\hat{a}_{\mathrm{DX}} + E_{\mathrm{IX}}\hat{a}_{\mathrm{IX}}^{\dagger}\hat{a}_{\mathrm{IX}} + \hbar\omega_m\hat{b}^{\dagger}\hat{b}$$
$$+ \left(\frac{\Omega}{2}\hat{a}_{\mathrm{DX}}^{\dagger}\hat{a}_{\mathrm{C}} + \frac{J}{2}\hat{a}_{\mathrm{IX}}^{\dagger}\hat{a}_{\mathrm{DX}} + \mathrm{H.c.}\right)$$
$$+ U_{\mathrm{DX}}\hat{a}_{\mathrm{DX}}^{\dagger}\hat{a}_{\mathrm{DX}}^{\dagger}\hat{a}_{\mathrm{DX}}\hat{a}_{\mathrm{DX}} + U_{\mathrm{IX}}\hat{a}_{\mathrm{IX}}^{\dagger}\hat{a}_{\mathrm{IX}}^{\dagger}\hat{a}_{\mathrm{IX}}\hat{a}_{\mathrm{IX}}$$
$$+ U_{\mathrm{DI}}\hat{a}_{\mathrm{DX}}^{\dagger}\hat{a}_{\mathrm{IX}}^{\dagger}\hat{a}_{\mathrm{DX}}\hat{a}_{\mathrm{IX}}$$

Cristofolini et al, Science (2012)

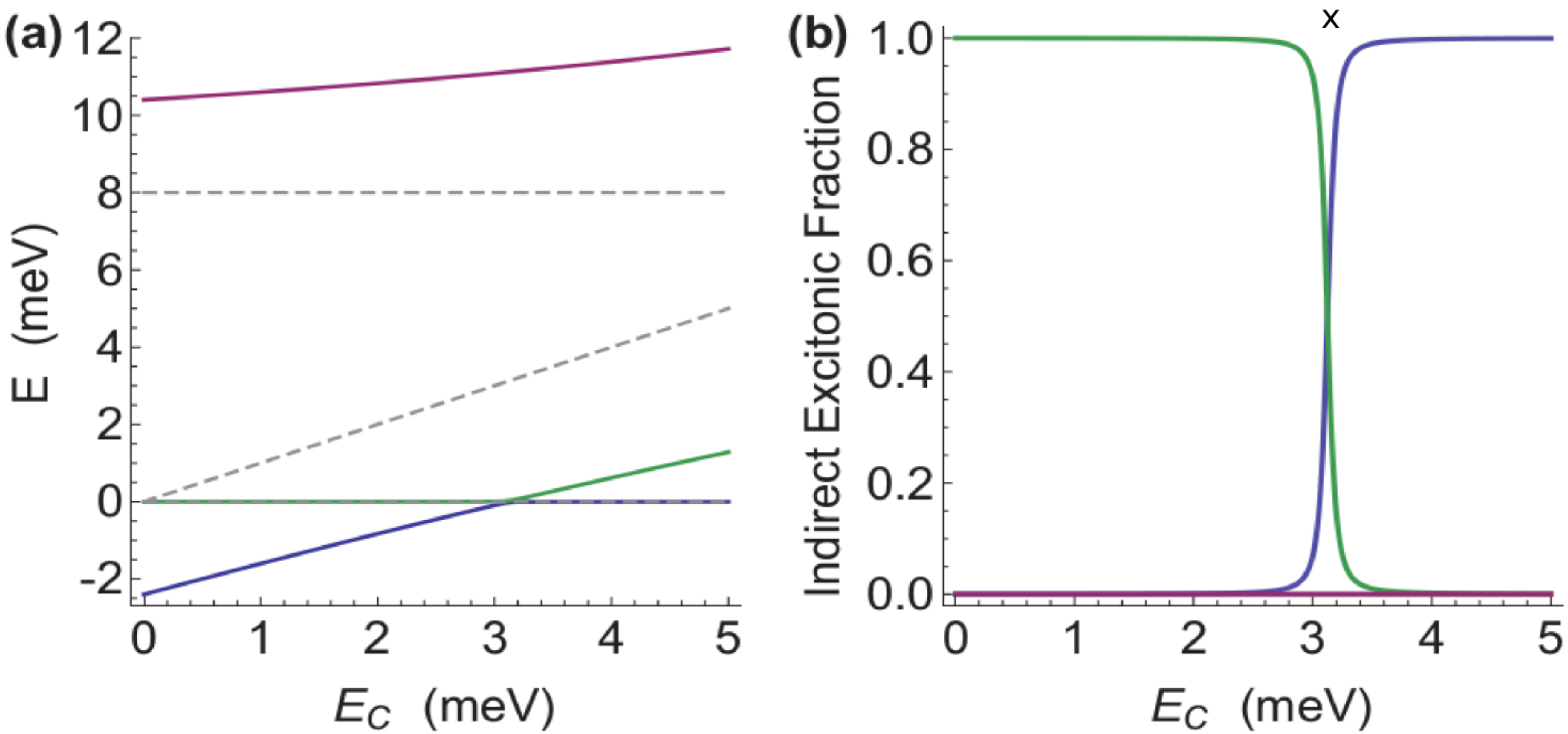
Direct and indirect exciton-polaritons in optomechanical microcavity



Direct and indirect exciton-polaritons in optomechanical microcavity

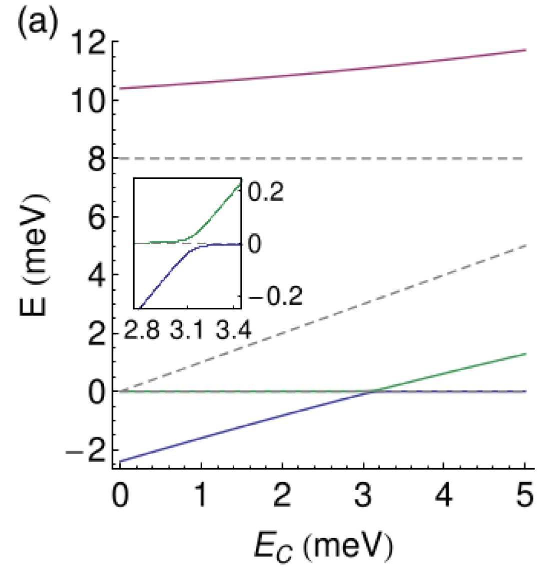


Dipolarion modes in optomechanical microcavity



The Hoplfield coefficients are very sensitive to the position of the mechanical resonator.

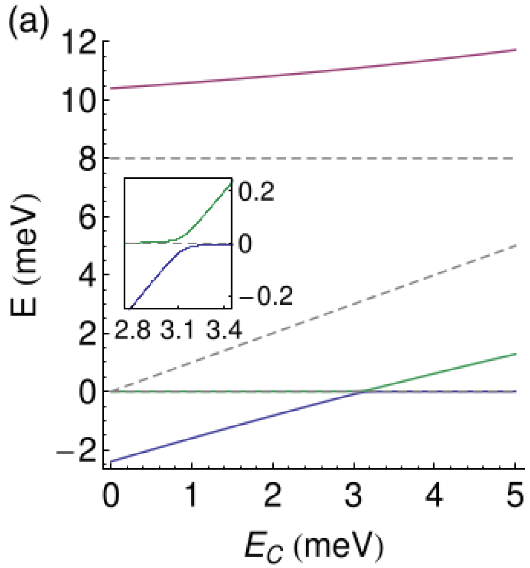
Dipolariton system



Dipolariton system

Lower polariton mode Hamiltonian:

$$\hat{\mathcal{H}}_{\mathrm{L}} = E_{\mathrm{L}}(x)\hat{\psi}^{\dagger}\hat{\psi} + \hbar\omega_m \hat{b}^{\dagger}\hat{b} + [U_{\mathrm{DX}}\beta_{\mathrm{D}}(x)^4 + U_{\mathrm{IX}}\beta_{\mathrm{I}}(x)^4 + U_{\mathrm{DI}}\beta_{\mathrm{D}}(x)^2\beta_{\mathrm{I}}(x)^2]\hat{\psi}^{\dagger}\hat{\psi}^{\dagger}\hat{\psi}\hat{\psi}$$

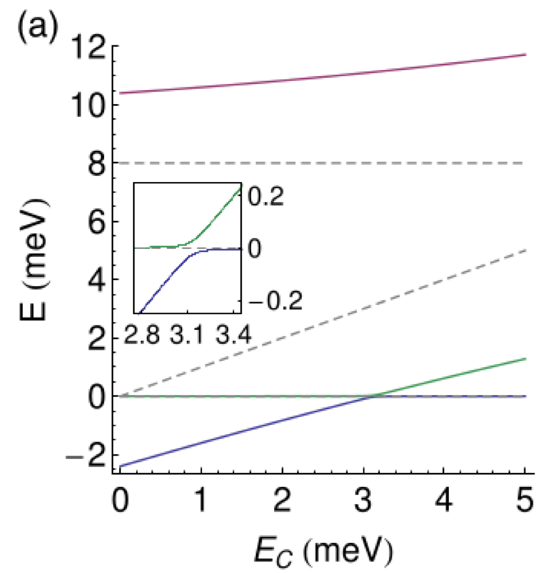


Dipolariton system

Lower polariton mode Hamiltonian:

$$\hat{\mathcal{H}}_{\mathrm{L}} = E_{\mathrm{L}}(x)\hat{\psi}^{\dagger}\hat{\psi} + \hbar\omega_m \hat{b}^{\dagger}\hat{b} + [U_{\mathrm{DX}}\beta_{\mathrm{D}}(x)^4 + U_{\mathrm{IX}}\beta_{\mathrm{I}}(x)^4 + U_{\mathrm{DI}}\beta_{\mathrm{D}}(x)^2\beta_{\mathrm{I}}(x)^2]\hat{\psi}^{\dagger}\hat{\psi}^{\dagger}\hat{\psi}\hat{\psi}$$

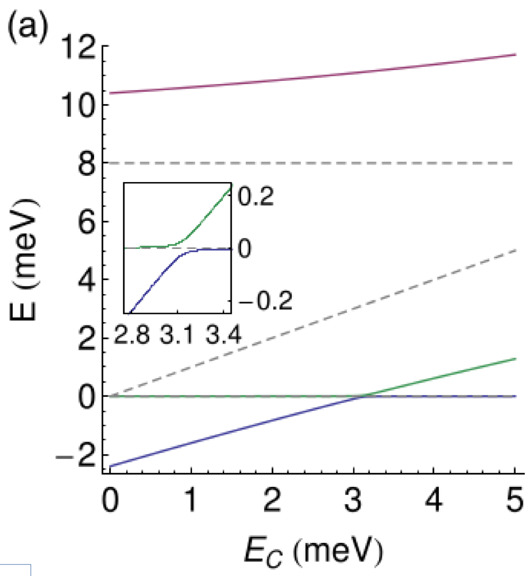
X-depended term can be expanded at x =0 (anticrossing point)



Dipolariton system

Lower polariton mode Hamiltonian:

$$\hat{\mathcal{H}}_\mathrm{L} = E_\mathrm{L}(x)\hat{\psi}^\dagger\hat{\psi} + \hbar\omega_m\hat{b}^\dagger\hat{b} + [U_\mathrm{DX}\beta_\mathrm{D}(x)^4 + U_\mathrm{IX}\beta_\mathrm{I}(x)^4 + U_\mathrm{DI}\beta_\mathrm{D}(x)^2\beta_\mathrm{I}(x)^2]\hat{\psi}^\dagger\hat{\psi}^\dagger\hat{\psi}\hat{\psi}$$



X-depended term can be expanded at x =0 (anticrossing point)

$$E_\mathrm{L}(x) \approx E_\mathrm{L}(0) + x\frac{\partial E_\mathrm{L}}{\partial x} = E_\mathrm{L}(0) - g_0(\hat{b}+\hat{b}^\dagger)\frac{\partial E_\mathrm{L}}{\partial E_\mathrm{C}},$$

Where x is replased by

$$\hat{x} = x_\mathrm{ZPF}(\hat{b}+\hat{b}^\dagger)$$

$$g_0 = -x_\mathrm{ZPF}\partial E_\mathrm{C}/\partial x.$$

Hopfield coefficients are

$$\beta_\mathrm{D,I}(x)^4 \approx \beta_\mathrm{D,I}(0)^4 - g_0(\hat{b}+\hat{b}^\dagger)\frac{\partial \beta^4_\mathrm{D,I}}{\partial E_\mathrm{C}}$$

$$\beta_\mathrm{D}(x)^2\beta_\mathrm{I}(x)^2 \approx \beta_\mathrm{D}(0)^2\beta_\mathrm{I}(0)^2 - g_0(\hat{b}+\hat{b}^\dagger)\frac{\partial\left(\beta^2_\mathrm{D}\beta^2_\mathrm{I}\right)}{\partial E_\mathrm{C}}$$

Dipolariton system

Finally the lower polariton mode Hamiltonian:

$$\hat{\mathcal{H}}_{\mathrm{L}} = E_{\mathrm{L}}(x)\hat{\psi}^{\dagger}\hat{\psi} + \hbar\omega_m \hat{b}^{\dagger}\hat{b} + [U_{\mathrm{DX}}\beta_{\mathrm{D}}(x)^4 + U_{\mathrm{IX}}\beta_{\mathrm{I}}(x)^4 + U_{\mathrm{DI}}\beta_{\mathrm{D}}(x)^2\beta_{\mathrm{I}}(x)^2]\hat{\psi}^{\dagger}\hat{\psi}^{\dagger}\hat{\psi}\hat{\psi}$$

$$\hat{\mathcal{H}}_{\mathrm{L}} = E_{\mathrm{L}}(0)\hat{\psi}^{\dagger}\hat{\psi} + \hbar\omega_m \hat{b}^{\dagger}\hat{b} + [U_{\mathrm{DX}}\beta_{\mathrm{D}}(0)^4 + U_{\mathrm{IX}}\beta_{\mathrm{I}}(0)^4 + U_{\mathrm{DI}}\beta_{\mathrm{D}}(0)^2\beta_{\mathrm{I}}(0)^2]\hat{\psi}^{\dagger}\hat{\psi}^{\dagger}\hat{\psi}\hat{\psi} + \hat{\xi}(\hat{b} + \hat{b}^{\dagger})$$

Dipolariton system

Finally the lower polariton mode Hamiltonian:

$$\hat{\mathcal{H}}_{\mathrm{L}} = E_{\mathrm{L}}(x)\hat{\psi}^{\dagger}\hat{\psi} + \hbar\omega_m \hat{b}^{\dagger}\hat{b} + [U_{\mathrm{DX}}\beta_{\mathrm{D}}(x)^4 + U_{\mathrm{IX}}\beta_{\mathrm{I}}(x)^4 + U_{\mathrm{DI}}\beta_{\mathrm{D}}(x)^2\beta_{\mathrm{I}}(x)^2]\hat{\psi}^{\dagger}\hat{\psi}^{\dagger}\hat{\psi}\hat{\psi}$$

$$\Downarrow$$

$$\hat{\mathcal{H}}_{\mathrm{L}} = E_{\mathrm{L}}(0)\hat{\psi}^{\dagger}\hat{\psi} + \hbar\omega_m \hat{b}^{\dagger}\hat{b} + [U_{\mathrm{DX}}\beta_{\mathrm{D}}(0)^4 + U_{\mathrm{IX}}\beta_{\mathrm{I}}(0)^4 + U_{\mathrm{DI}}\beta_{\mathrm{D}}(0)^2\beta_{\mathrm{I}}(0)^2]\hat{\psi}^{\dagger}\hat{\psi}^{\dagger}\hat{\psi}\hat{\psi} + \boxed{\hat{\xi}(\hat{b} + \hat{b}^{\dagger})}$$

Interactive coupling

Dipolariton system

Finally the lower polariton mode Hamiltonian:

$$\hat{\mathcal{H}}_{\mathrm{L}} = E_{\mathrm{L}}(x)\hat{\psi}^{\dagger}\hat{\psi} + \hbar\omega_m \hat{b}^{\dagger}\hat{b} + [U_{\mathrm{DX}}\beta_{\mathrm{D}}(x)^4 + U_{\mathrm{IX}}\beta_{\mathrm{I}}(x)^4 + U_{\mathrm{DI}}\beta_{\mathrm{D}}(x)^2\beta_{\mathrm{I}}(x)^2]\hat{\psi}^{\dagger}\hat{\psi}^{\dagger}\hat{\psi}\hat{\psi}$$

$$\hat{\mathcal{H}}_{\mathrm{L}} = E_{\mathrm{L}}(0)\hat{\psi}^{\dagger}\hat{\psi} + \hbar\omega_m \hat{b}^{\dagger}\hat{b} + [U_{\mathrm{DX}}\beta_{\mathrm{D}}(0)^4 + U_{\mathrm{IX}}\beta_{\mathrm{I}}(0)^4 + U_{\mathrm{DI}}\beta_{\mathrm{D}}(0)^2\beta_{\mathrm{I}}(0)^2]\hat{\psi}^{\dagger}\hat{\psi}^{\dagger}\hat{\psi}\hat{\psi} + \hat{\xi}(\hat{b} + \hat{b}^{\dagger})$$

Interactive coupling

$$\hat{\xi} = -g_0\left(U_{\mathrm{DX}}\frac{\partial\beta_{\mathrm{D}}^4}{\partial E_{\mathrm{C}}} + U_{\mathrm{DI}}\frac{\partial\left(\beta_{\mathrm{D}}^2\beta_{\mathrm{I}}^2\right)}{\partial E_{\mathrm{C}}} + U_{\mathrm{IX}}\frac{\partial\beta_{\mathrm{I}}^4}{\partial E_{\mathrm{C}}}\right)\hat{\psi}^{\dagger}\hat{\psi}^{\dagger}\hat{\psi}\hat{\psi} - g_0\frac{\partial E_{\mathrm{L}}}{\partial E_{\mathrm{C}}}\hat{\psi}^{\dagger}\hat{\psi}.$$

Dipolariton system

Elimination phonon modes from

$$\hat{\mathcal{H}}_{\mathrm{L}} = E_{\mathrm{L}}(0)\hat{\psi}^{\dagger}\hat{\psi} + \hbar\omega_m \hat{b}^{\dagger}\hat{b} + [U_{\mathrm{DX}}\beta_{\mathrm{D}}(0)^4 + U_{\mathrm{IX}}\beta_{\mathrm{I}}(0)^4 + U_{\mathrm{DI}}\beta_{\mathrm{D}}(0)^2\beta_{\mathrm{I}}(0)^2]\hat{\psi}^{\dagger}\hat{\psi}^{\dagger}\hat{\psi}\hat{\psi} + \hat{\xi}(\hat{b} + \hat{b}^{\dagger})$$

By using the polaron (Schrieffer-Wolff) transformation

$$\hat{\mathcal{H}}'_{\mathrm{L}} = E_{\mathrm{L}}(0)\hat{\psi}^{\dagger}\hat{\psi} + \hbar\omega_m \hat{b}^{\dagger}\hat{b} + [U_{\mathrm{DX}}\beta_{\mathrm{D}}(0)^4 + U_{\mathrm{IX}}\beta_{\mathrm{I}}(0)^4 + U_{\mathrm{DI}}\beta_{\mathrm{D}}(0)^2\beta_{\mathrm{I}}(0)^2]\hat{\psi}^{\dagger}\hat{\psi}^{\dagger}\hat{\psi}\hat{\psi} - \frac{\hat{\xi}^2}{\hbar\omega_m}$$

The behavior of optomechanical dipolaritons in a quasi-1D system

Gross-Pitaevskii type equation

$$i\hbar\frac{\partial\psi(z,t)}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m_{\mathrm{LP}}} + \left(\alpha_{1\mathrm{D}} - \frac{\gamma_{1\mathrm{D}}^2}{\hbar\omega_m}\right) n - \frac{2\beta_{1\mathrm{D}}\gamma_{1\mathrm{D}} n^2 + \beta_{1\mathrm{D}}^2 n^3}{\hbar\omega_m}\right]\psi(z,t)$$

$$n = |\psi(z,t)|^2$$

$$\alpha_{1\mathrm{D}} \approx U_{\mathrm{IX}} A \beta_{\mathrm{I}}(0)^4/d$$

$$\beta_{1\mathrm{D}} \approx g_0 U_{\mathrm{IX}}/J(A/d)^{3/2}$$

$$\gamma_{1\mathrm{D}} = g_0(A/d)^{1/2}\partial E_{\mathrm{L}}/\partial E_{\mathrm{C}} \approx g_0(A/d)^{1/2}/2$$

The behavior of optomechanical dipolaritons in a quasi-1D system

Gross-Pitaevskii type equation

$$i\hbar\frac{\partial\psi(z,t)}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m_{\mathrm{LP}}} + \left(\alpha_{\mathrm{1D}} - \frac{\gamma_{\mathrm{1D}}^2}{\hbar\omega_m}\right) n - \frac{2\beta_{\mathrm{1D}}\gamma_{\mathrm{1D}} n^2 + \beta_{\mathrm{1D}}^2 n^3}{\hbar\omega_m}\right]\psi(z,t)$$

mechanical coupling

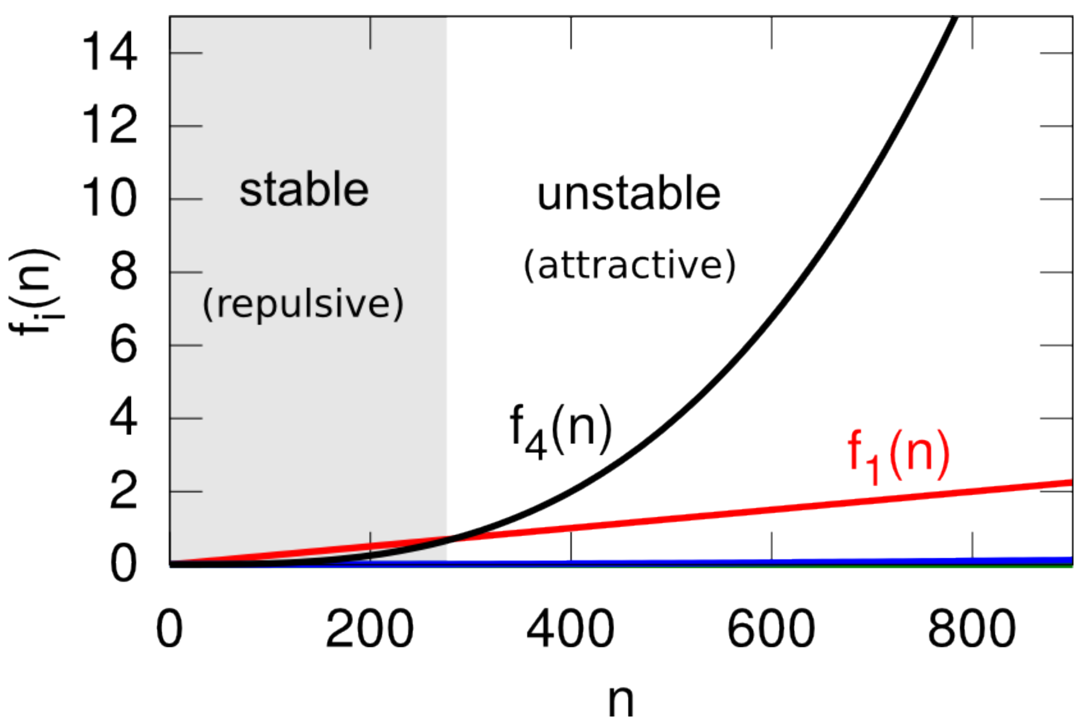
$$n = |\psi(z,t)|^2$$

The behavior of optomechanical dipolaritons in a quasi-1D system

Gross-Pitaevskii type equation

$$i\hbar\frac{\partial\psi(z,t)}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m_{\mathrm{LP}}} + \left(\alpha_{\mathrm{1D}} - \frac{\gamma_{\mathrm{1D}}^2}{\hbar\omega_m}\right)n - \frac{2\beta_{\mathrm{1D}}\gamma_{\mathrm{1D}}n^2 + \beta_{\mathrm{1D}}^2 n^3}{\hbar\omega_m}\right]\psi(z,t)$$

f1(n) f2(n) f3(n) f4(n)

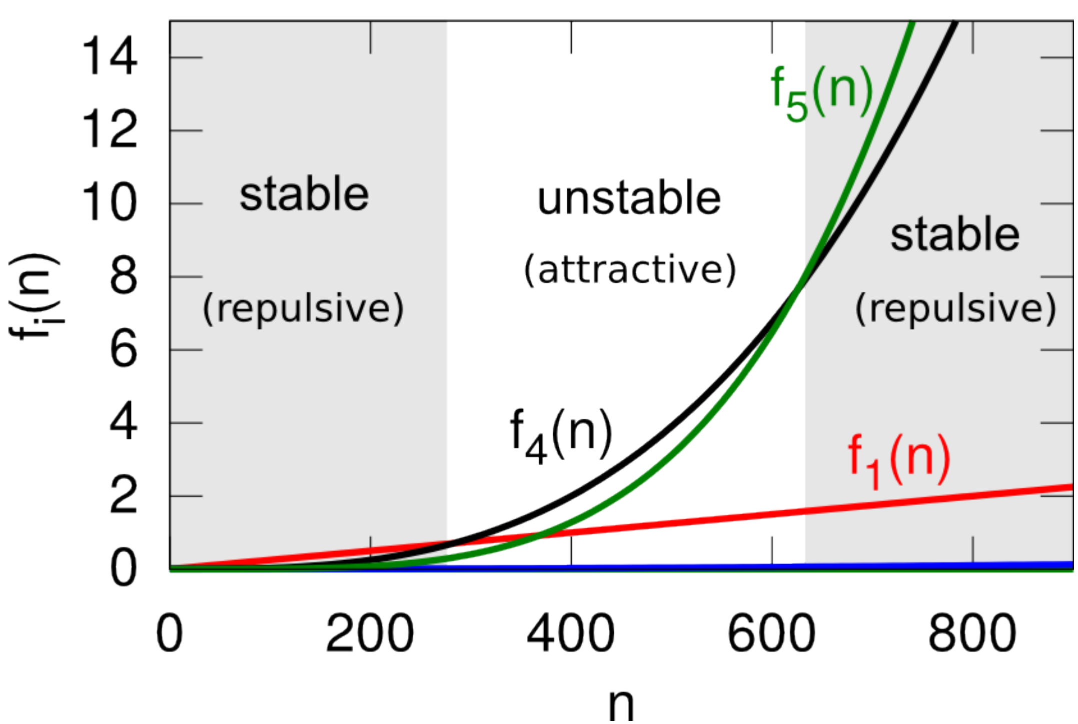


The behavior of optomechanical dipolaritons in a quasi-1D system

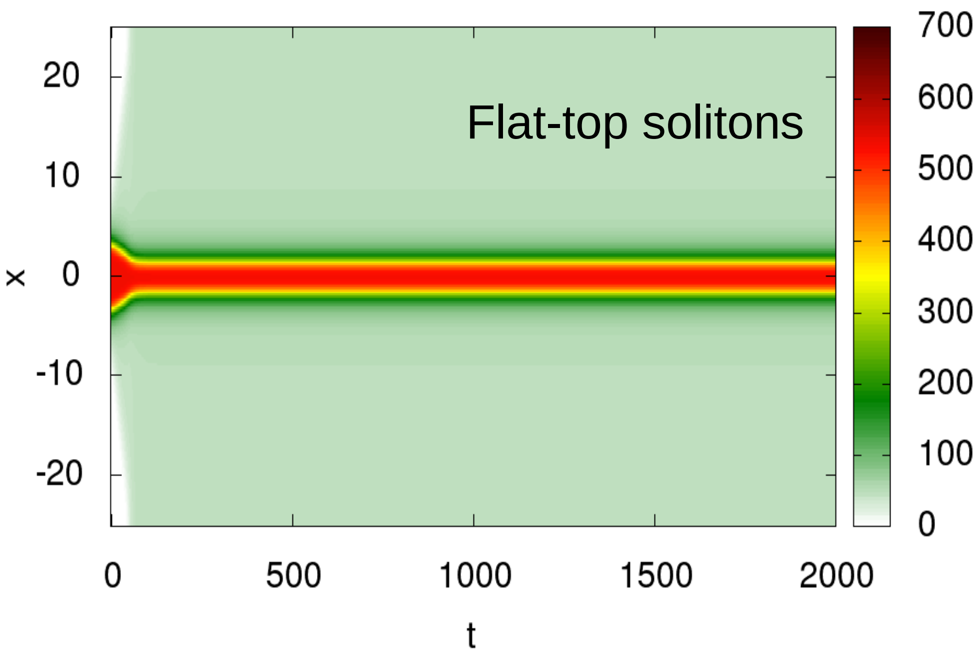
Gross-Pitaevskii type equation

$$i\hbar\frac{\partial\psi(z,t)}{\partial t}=\left[-\frac{\hbar^2\nabla^2}{2m_{\mathrm{LP}}}+\left(\alpha_{1\mathrm{D}}-\frac{\gamma_{1\mathrm{D}}^2}{\hbar\omega_m}\right)n-\frac{2\beta_{1\mathrm{D}}\gamma_{1\mathrm{D}}n^2+\beta_{1\mathrm{D}}^2n^3}{\hbar\omega_m}+\delta n^4\right]\psi(z,t)$$

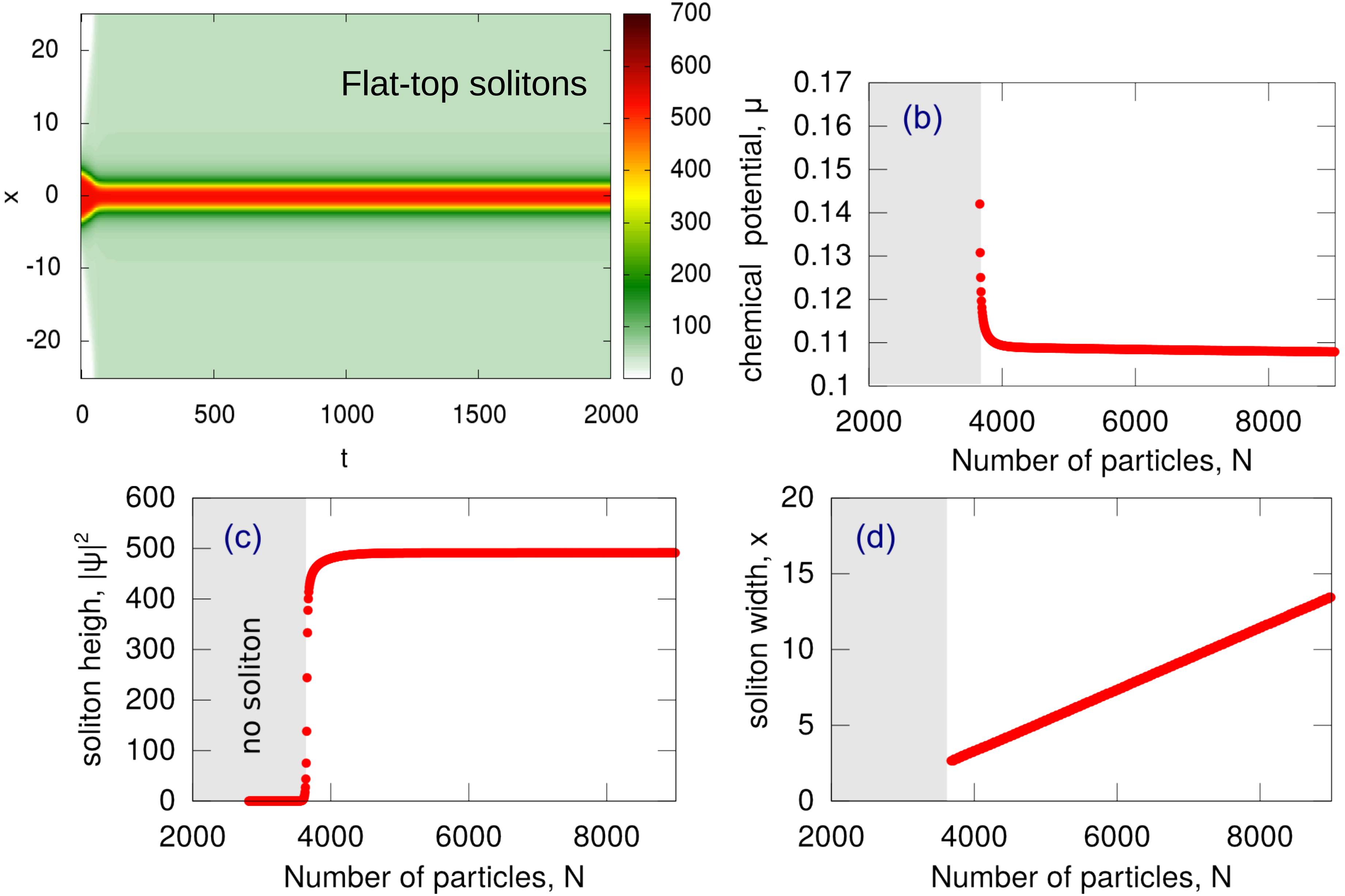
f1(n) f2(n) f3(n) f4(n) f5(n)



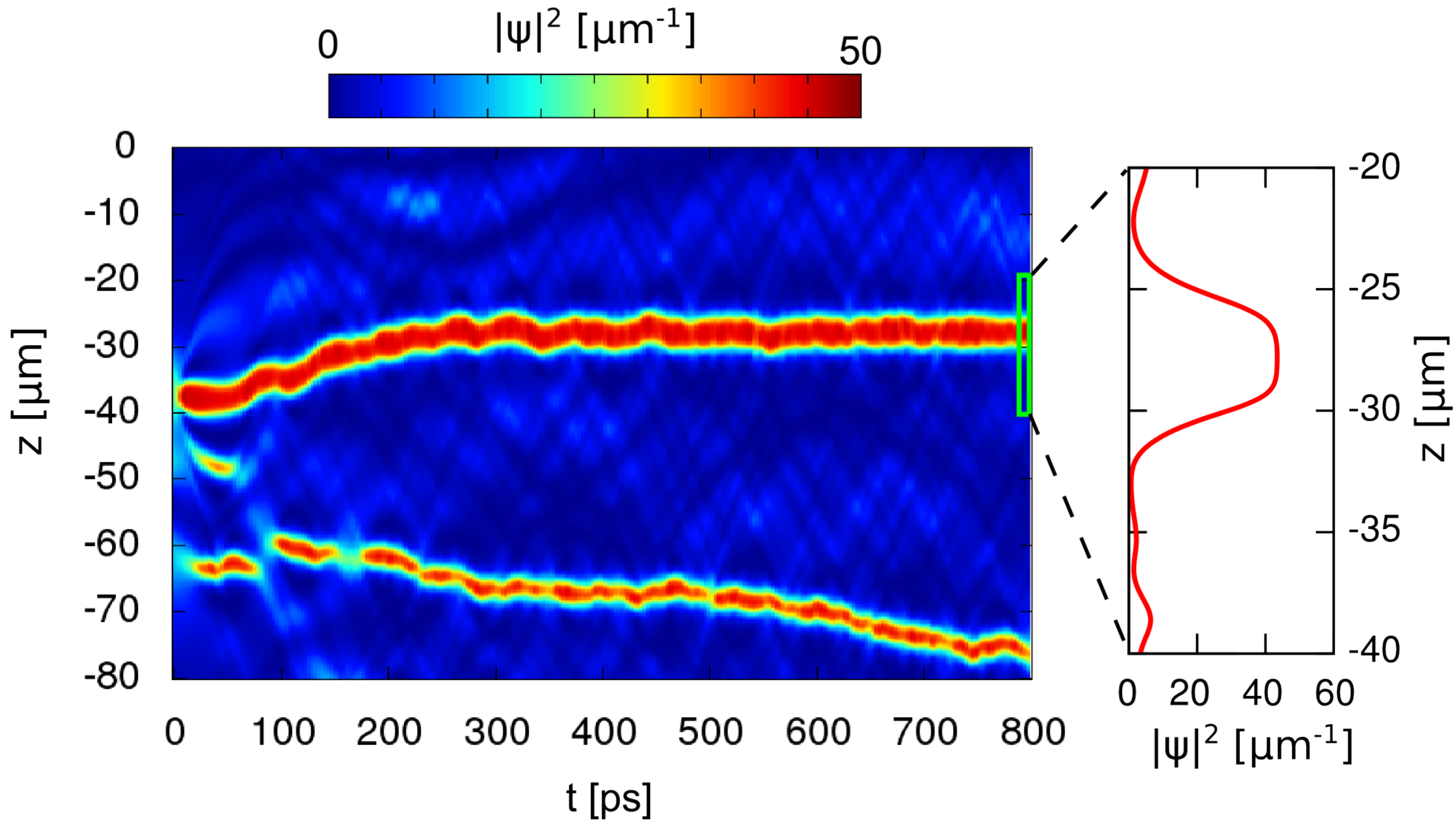
The formation of polariton condensate in the ground state (The imaginary time method)



The formation of polariton condensate in the ground state (The imaginary time method)



The formation of polariton condensate in the ground state (Real time)



Summary

We introduce new type of optomechanical coupling - the interactive optomechanical coupling, which appears in generic systems where the strength of nonlinearity is influenced by a mechanical motion.

As particular examples of systems where interactive coupling can be realized we describe exciton-polaritons and dipolaritons. In the latter case the steep dependence of Hopfield coefficients allows to attain large interactive coupling constant.

The attractive nonlinearityin the systems with interactive coupling may induce the spontaneous formation of flat-top solitons. .

Summary

We introduce new type of optomechanical coupling - the interactive optomechanical coupling, which appears in generic systems where the strength of nonlinearity is influenced by a mechanical motion.

As particular examples of systems where interactive coupling can be realized we describe exciton-polaritons and dipolaritons. In the latter case the steep dependence of Hopfield coefficients allows to attain large interactive coupling constant.

The attractive nonlinearityin the systems with interactive coupling may induce the spontaneous formation of flat-top solitons. .

Thank you for your attention!

NATIONAL SCIENCE CENTRE